Estimation and Control with All Kinds of Data

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***Abstract*—In many estimation and control applications, several kinds of sensor data may be present (continuous-time, discrete-time with several sampling intervals, discrete-event data, and opportunistic data. A unified framework for dealing with all these kinds of data is presented by regarding all data as opportunistic. A general algorithm is presented and an example application is illustrated by simulation.**

INTRODUCTION

Many applications of estimation techniques may entail several types of data:

- Continuous-time data.
- Discrete-time data with several sampling intervals (which may or may not be commensurable).
- Discrete-event data, such as occurrence of a level crossing.
- Opportunistic data. Measurements taken at available opportunities, the times of which are not known until they occur.

The issue of with a variety of data sources has long been recognized and has been the subject of a number of reported investigations dating back to 1987 or earlier. [1-8].

Since it is all but certain that any estimation and control algorithm will be implemented in a digital computer, there is no practical need for dealing with continuous-time data. The concept of continuous-time data is nevertheless a convenient abstraction and is worth retaining for some purposes.

Assuming digital implementation of the estimation and control algorithm, it is appropriate to use a discrete-time model for the dynamic process under consideration. We assume that the algorithm is implemented in the computer at a fixed cycle period T. With state-of-the art hardware, T can be assumed to be much smaller than the intervals between data observations. Thus each occurrence of data to be processed can be recognized as a discrete event. This recognition forms the basis of the estimation technique presented in this paper.

GENERAL ESTIMATION ALGORITHM

For purposes of exposition, we assume that the dynamic process, the state x_n of which is to be estimated, is linear and time-invariant, and governed by the difference equation

$$x_{n+1} = \Phi x_n + \Gamma u_n + F v_n \tag{1}$$

where Φ and Γ are state transition and control distribution matrices, v_n is random noise and F is the noise distribution matrix.

For a linear,time-invariant system

$$\dot{x} = Ax + Bu \tag{2}$$

sampled at a fixed time interval T

$$\Phi = e^{AT}, \quad \Gamma = \int_0^T e^{At} dt B \tag{3}$$

The general estimation algorithm is based on several assumptions:

- The occurrence of an observation datum is a discrete “data event”.
- The estimation algorithm is implemented by the processing computer at in a fixed cycle of time duration T which is shorter than the fastest data sampling interval in the system.
- If data from sensor i is the result of periodic sampling at an interval T_i, we assume that $T_i = N_i T$, i.e., that the sampling interval for this source is an integer multiple of the computer cycle T.
  There may be several sources of periodically sampled data. The sampling intervals for these sources may be, but need not be the same length. The cycle length N_i is controlled by a counter in the central processor.
- If a sensor detects the attainment of a fixed level L_i

  $$|y - L_i| < \epsilon$$

  the corresponding flag is set and the reported data is L_i
- If a sensor “fires” as a result of an opportunistic event, the corresponding flag is set and the corresponding datum is recorded.

The processing of a “data event” (i.e., instant of periodic sampling, or other discrete event) entails two operations: (1) setting a “data flag” denoting occurrence of the event, and (2) recording the numerical value y_i of the data.

At the beginning of the nth short cycle the vector of data flags is scanned. If the data flag corresponding to the ith sensor is set, the data is processed, entailing updating the estimate of the state of the process and the corresponding covariance matrix, using the standard Kalman filtering algorithm:

$$K_n = P_n C_i' / (C_i' P_n C_i + W_i) \tag{4}$$
$$\hat{x}_n = \tilde{x}_n + K_n(y_n - C_i \tilde{x}_n \tag{5}$$
$$\hat{P}_n = \tilde{P}_n - K_n C_i \tilde{P}_n \tag{6}$$

where $\tilde{x}_n, \tilde{P}_n$, are the a priori state estimates and covariance matrix, $\hat{x}_n, \hat{P}_n$, are the a posteriori state estimates and covariance matrix, K_n is the Kalman gain matrix, and W_i is the variance of the ith sensor error. Note that each processed observation is a scalar, so no matrix inversion is entailed. In the (unlikely) case that two or more sensors "fire" concurrently, the observations are nevertheless processed sequentially. Finally, the data flag for the ith sensor is reset in preparation for a subsequent observation.

When the data for the nth short cycle is processed, the control signal u_n is computed and the state and covariance matrices are updated for the elapse of time in accordance with the standard algorithm:

$$\tilde{x}_{n+1} = \Phi \hat{x}_n + \Gamma u_n \tag{7}$$
$$\tilde{P}_{n+1} = \Phi \hat{P} \Phi' + F V F' \tag{8}$$

A computation flow chart for the algorithm is given in Figure **??**.

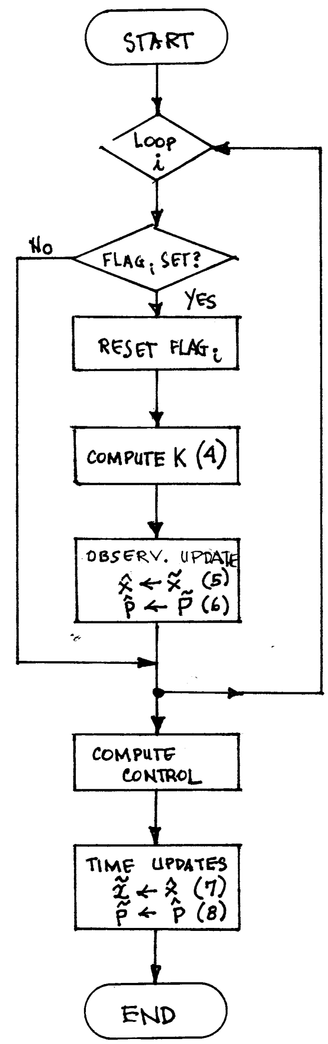


Fig. 1. Flow Chart for Estimation and Control Algorithm

Simplifications to the Algorithm

Short-time approximations

Under the reasonable assumption that the cycle time T is very short relative to the dynamics of the process, it is reasonable to approximate the state-transition and control distribution matrices as follows:

$$\Phi \approx I + AT, \quad \Gamma \approx BT \tag{9}$$

These approximations are equivalent to the numerical integration of the continuous-time state and covariance matrices by the first-order Euler algorithm.

Constant covariance matrix approximation

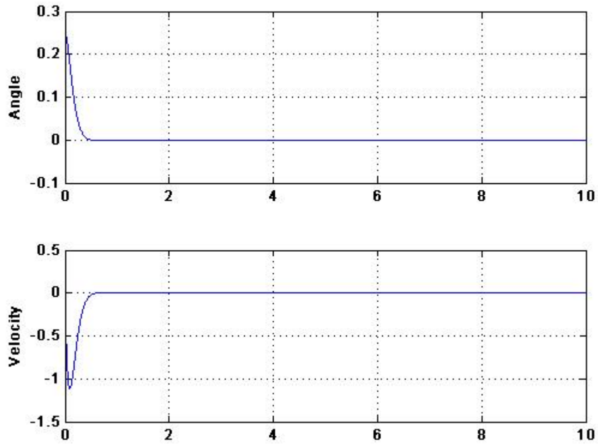
Computing the gain matrix K by (4) and updating the covariance matrix by (5) are the most computationally intensive of all the operations in the algorithm. In a a process entailing only one sampling interval, the covariance matrix converges to a constant matrix in the steady state. It is often acceptable to use the steady-state covariance matrix (which can be computed off-line) as an approximation as an alterative to performing the updating calculations. This greatly simplifies the algorithm, since the Kalman filter gain matrix is also constant. When the observations entail different sampling intervals, and/or include discrete-event data and opportunistic data, the covariance matrix does not converge to a constant. When there is no discrete-event or opportunistic data, the covariance matrix converges to a periodic matrix having a period which is the least common multiple of the individual sampling periods in the process. When discrete-event and opportunistic data is present, however, the covariance matrix is not periodic, but it may be become stationary. This can possibly be determined by simulation. And a possible approximation which might give acceptable results would be to use the stationary mean of the covariance matrix, which could greatly reduce the computational burden without a significant reduction in performance.

Example

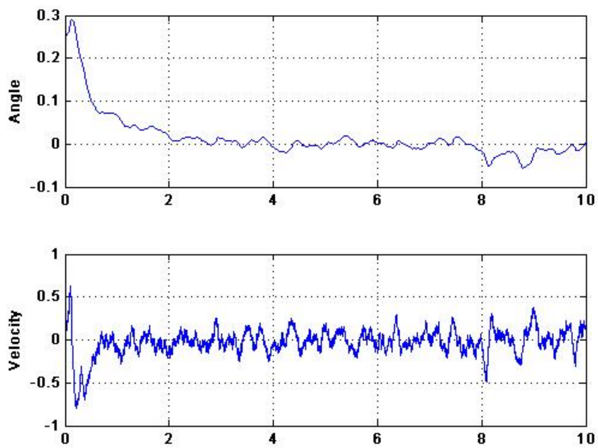
Inverted Pendulum

To illustrate the application of the above algorithm we consider an inverted pendulum driven by a d-c motor. In the absence of process noise, the dynamics of the process are given by

$$\dot{x}_1 = x_2 \tag{10}$$
$$\dot{x}_2 = \Omega^2 x_1 - \alpha x_2 + \beta u \tag{11}$$



(a)



(b)

Fig. 2. Performance with full-state feedback control. (a) No noise on input (b) Noisy input

with $\Omega^2 = g/L$ and α and β being related to the electrical characteristics of the motor. For our example we use:

$$\Omega^2 = 36, \alpha = 5, \beta = 1$$

For these numerical values the matrices representing the dynamics of the process are

$$A = \begin{bmatrix} 0 & 1 \\ 36 & -5 \end{bmatrix}, \quad B = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

The full-state feedback control law for the process is

$$u = -G_1 x_1 - G_2 x2$$

with $G_1 = 161, \quad G_2 = 15$. For purpose of comparison, the transient response for this control law with noise-free sensors of both angular displacement and velocity, and an initial angular error of 0.1 rad is shown in Fig.**??** in the absence of process noise, and in the presence of process noise.

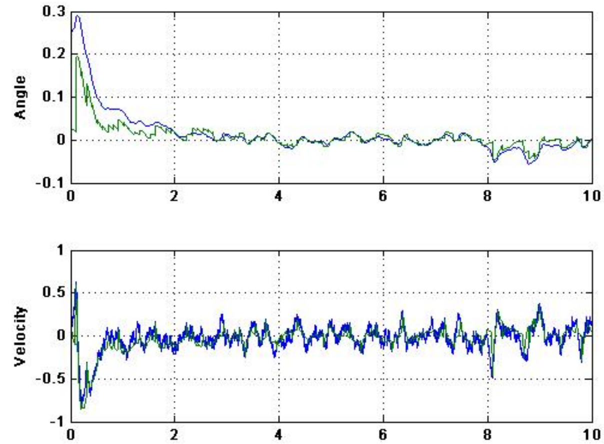


Fig. 3. Performance simulation with sampled velocity sensor and null position detector

To illustrate the application of the method of this paper we consider the following complement of sensors:

- A velocity sensor with a sampling interval $T = .001$ and a nominal noise variance of .01 rad/sec. (These parameters might be characteristic of an acoustical velocity sensor.)
- A null position detector, which is device (such a a Hall effect sensor, or an LED/Phototransistor) that can detect the proximity of the pendulum to the vertical. The sensor has detection range $\epsilon = .001$ rad and a nominal variance of .01.

The performance of the control system using this complement of sensors with the same process noise as was used for full-state feedback is shown in Fig.**??** The blue curve shows the simulated actual state; the green line shows the estimated state, as produced by the observer; the green curve shows the state estimates produced by the observer. The transient response with the observer present is scarcely different from that of full-state feedback, and the state estimate produced by the observer is a fairly faithful representation of the simulated actual state.

A segment of the steady-state transients is shown in Fig.**??**. The actual state (blue) is noisy the noisy trace; the observed state follows the model between observations, at which points it is corrected at the velocity sampling instants, and during the intervals when the null position sensor is active, i.e, when the angle is within ϵ.

To investigate the effect on performance of using fixed gains instead of updating the covariance matrix at each step, the covariance matrix was calculated in the steady state and approximately average, to give a covariance matrix of

$$P = \begin{bmatrix} .02 & .05 \\ .05 & .50 \end{bmatrix}$$

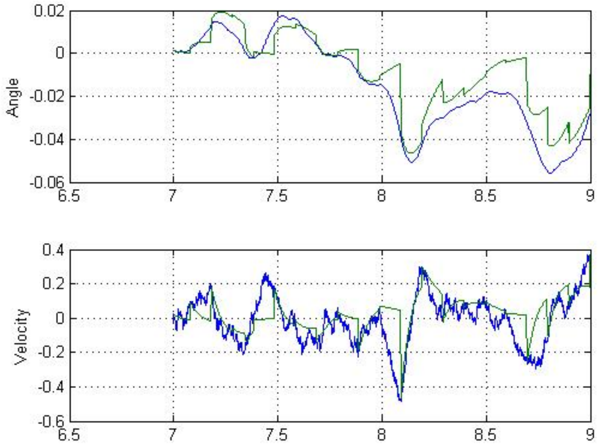


Fig. 4. Segment of performance simulation with sampled velocity sensor and null position detector

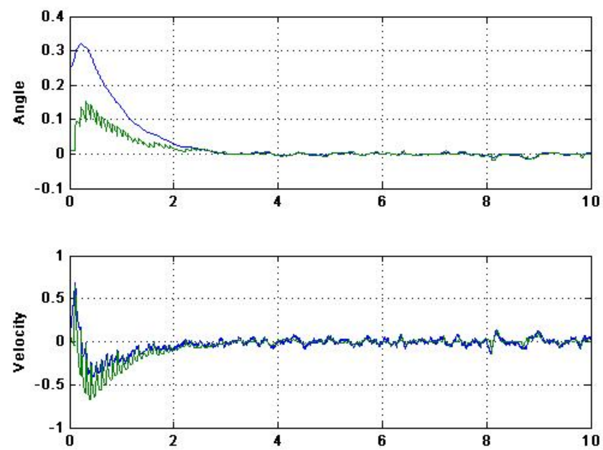


Fig. 5. Performance simulation with sampled velocity sensor and null position detector, using fixed observer gains.

which was used to calculate the fixed gains for each measurement type. The simulated results are shown in Fig. **??** with a detail in Fig. **??**. It is seen that the use of the fixed gains results in a larger error during the transient interval, as would be expected, but not appreciably different in the steady state. In fact, as the detail shows, the error appears to be smaller by a factor of about 2 when the fixed-gain observer is used.

Conclusions

The technique presented in this paper is fairly simple and readily applied in practical situations. The basic idea is based on the concept of a short clock cycle of duration T at which the state and covariance matrices are updated with whatever data (if any) is present at that time.

Although the technique is illustrated with a linear system with a linear control law, there is no apparent reason for it not to be valid for nonlinear systems using "standard" observer-based control law design technique.

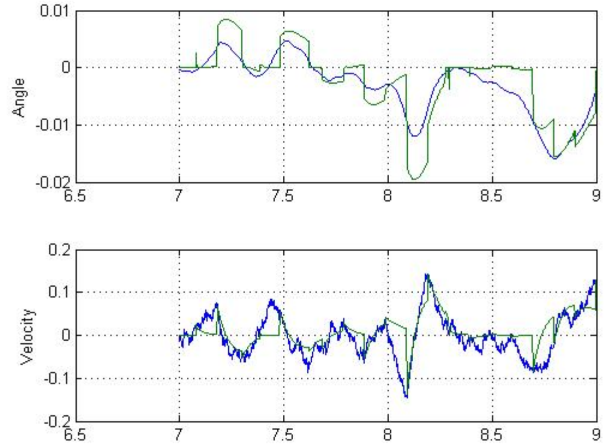


Fig. 6. Segment of performance simulation with sampled velocity sensor and null position detector, using fixed observer gains.

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