**CSCI 4525/5525: Written Assignment for Unit 2: Logic**

**Assignment**

Complete the following exercises from *Artificial Intelligence: A Modern Approach*. They are reprinted here for your convenience (and because the digital “global edition” has different problems than the physical copy, so this should clarify things—do the problems here!) The numbers I use here are from the physical edition of the book.

Also, because I can’t help but add my two-cents, I added some notes and changed the formatting a little bit to some of the questions. But the actual problems are the same!

**Question 1** *(8.9 from AI: A Modern Approach, page 316) –* ***19 points total, 1 point for each sub-part.***

This exercise uses the **function** *MapColor* [in the predicate logic notion of the term “function”], and **predicates** *In(x,y),* *Borders(x,y),* and *Country(x),* whose arguments are geographical regions, along with **constant symbols** for various regions. In each of the following we give an English sentence and a number of candidate logical expressions. For each of the logical expressions, choose between one of the following three options:

(1) it correctly expresses the English sentence;

(2) the logic is syntactically invalid and therefore meaningless; or

(3) the logic is syntactically valid but does not express the meaning of the English sentence.

You do not need to show your work for this problem, though if you wish to explain your reasoning (as well as specifying 1, 2, or 3) for each logical statement, you are welcome to do so.

1. **Paris and Marseilles are both in France.**
   1. *In(Paris ∧ Marseilles, France)*
      1. *(1)*
   2. *In(Paris, France) ∧ In(Marseilles, France).*
      1. *(1)*
   3. *In(Paris, France) ∨ In(Marseilles, France).*
      1. *(3)*
2. **There is a country that borders both Iraq and Pakistan**
   1. *∃c Country(c) ∧ Border(c, Iraq) ∧ Border(c, Pakistan).*
      1. *(1)*
      2. *There exist a c which is a country and c borders Iraq and c borders Pakistan.*
   2. *∃c Country(c) ⇒ [Border(c, Iraq) ∧ Border(c, Pakistan)].*
      1. *(2)*
      2. *If there exist a c where c is a country, then c borders Iraq and c borders Pakistan.*
   3. *[∃c Country(c)] ⇒ [Border(c, Iraq) ∧ Border(c, Pakistan)].*
      1. *(2)*
      2. *If there is a country C then c borders Iraq and Pakistan.*
   4. *∃c Border(Country(c), Iraq ∧ Pakistan).*
      1. *(1)*
3. **All countries that border Ecuador are in South America**
   1. *∀c Country(c) ∧ Border(c, Ecuador) ⇒ In(c, SouthAmerica).*
      1. *(1)*
      2. *Every c that is a country and country c border Ecuador, then c is in South America.*
   2. *∀c Country(c) ⇒ [Border(c, Ecuador) ⇒ In(c, SouthAmerica)].*
      1. *(1)*
      2. *If every country c, then c borders Ecuador, then c is in South America.*
   3. *∀c [Country(c) ⇒ Border(c, Ecuador)] ⇒ In(c, SouthAmerica).*
      1. *(1)*
      2. *All counties c border Ecuador, then c is in South America.*
   4. *∀c Country(c) ∧ Border(c, Ecuador) ∧ In(c, SouthAmerica).*
      1. *(2)*
      2. *Every country c and c borders Ecuador and c is in South America. All countries c border Ecuador and are in South America.*
4. **No region in South America borders any region in Europe**
   1. *¬[∃c,d In(c, SouthAmerica) ∧ In(d,Europe) ∧ Borders(c,d)].*
      1. *(2)*
      2. *There does not exist a c or d where c is not in South America or d is not in Europe or c does not border d.*
   2. *∀c,d [In(c, SouthAmerica) ∧ In(d,Europe)] ⇒ ¬ Borders(c,d).*
      1. *(1)*
      2. *For all c and d, c in South American and d in Europe implies c does not border d.*
   3. *¬∀c In(c, SouthAmerica) ⇒ ∃d In(d, Europe) ∧ ¬ Borders(c,d).*
      1. *(2)*
      2. *There exist a c, where c is in South America, which implies there exist a d in Europe and it does not border c.*
   4. *∀c In(c, SouthAmerica) ⇒ ∀d In(d, Europe) ⇒ ¬Borders(c,d).*
      1. *(1)*
      2. *Every c in South America implies every d in Europe implies c does not border d.*
5. **No two adjacent countries have the same map color:**
   1. *∀x,y ¬Country(x) ∨ ¬Country(y) ∨ ¬Borders(x,y) ∨*

*¬(MapColor(x) = MapColor(y)).*

* + 1. *(2)*
    2. *Every x and y, there is not a country x or not a country y or x does not boarder y or map color of x does not equal the map color of y.*
  1. *∀x,y (Country(x) ∧ Country(y) ∧ Borders(x,y) ∧ ¬(x = y)) ⇒*

*¬(MapColor(x) = MapColor(y)).*

* + 1. *(1)*
    2. *All x and y there is a country x and y that border each other and x and y are not equal to each other, then map color of x and y are not equal.*
  1. *∀x,y Country(x) ∧ Country(y) ∧ Borders(x,y) ∧*

*¬(MapColor(x) = MapColor(y)).*

* + 1. *(2)*
    2. *All x and y there are country x and y that border each other and the map color of x and y are not equal.*
  1. *∀x,y (Country(x) ∧ Country(y) ∧ Borders(x,y)) ⇒*

*MapColor(x* ***≠y)***

*(1)*

***All x and y, there is a country x and country y that border each other that implies the map colors are not equal.***

**Question 2** *(9.4 from AI: A Modern Approach, page 361) –* ***8 points total. (2 points each).***

For each pair of atomic sentences, give the most general unifier if it exists:

1. *P(A, B, B), P(x, y, z).*
   1. *{x/A, y/B, z/B}*
2. *Q(y, G(A, B)), Q(G(x, x), y).*
   1. *the unifier does not exist*
3. *Older(Father(y), y), Older(Father(x), John).*
   1. *{x/y}*
   2. *Older(Father(x), x), Older(Father(x), John)*
   3. *{y/x, x/John}*
   4. *{y/John, x/John}*
4. *Knows(Father(y),y), Knows(x, x)*
   1. *the unifier does not exist*

**Question 3** *(9.10 from AI: A Modern Approach, page 362) –* ***8 points total.***

A popular children’s riddle is “Brothers and sisters have I none, but that man’s father is my father’s son.” Use the rules of the family domain (Section 8.3.2 on page 301) to show who that man is. You may apply any of the inference methods described in this chapter. Why do you think that this riddle is difficult?

“Brothers and sisters have I none, but that man’s father is my father’s son.”

The man is me. The riddle makes you believe that “I” and “man” are two separate objects or predicates.

I was able to figure out the riddle, but couldn’t figure out how to write in proper predicates. Below is all my scratch paper to prove that I tried to write the proof. I’m terrible at proofs.

Text, letter

Description automatically generated

**Hint**: For this problem, it is recommended that you use resolution to get practice with resolution theorem proving. However, a typical proof by contradiction is not needed to solve this riddle. You already have enough information to find the answer in the knowledge base, so there is no need for a query.

In a typical proof by contradiction using resolution, you would negate the query and apply resolution until you derive the empty clause. But in this case no query is needed. This can be solved using a direct proof. Just apply resolution and stop when you have derived a new fact which represents the answer to the riddle.

**Students enrolled in CSCI 5525 must also complete:**

**Question 4** *(9.20 from AI: A Modern Approach, page 364-365) –* ***12 points total, four points for each part.***

Let ℒ be the first-order language with a single predicate *S(p,q)*, meaning “p shaves q.” Assume a domain of people.

1. Consider the sentence “There exists a person P who shaves every one who does not shave themselves, and only people that do not shave themselves.” Express this in ℒ.
2. Convert the sentence in (a) to clausal form.
3. Construct a resolution proof to show that the clauses in (b) are inherently inconsistent. \*Note: you do not need any additional axioms.)

**Submission**

Please observe these requirements in your submission:

* Submission must include your name and which section of the class you are enrolled in (i.e. 4525 or 5525).
* Submissions must be typed.
* Submissions must be submitted as PDF files.
* Submissions must be uploaded to Moodle on time

**Grading**

Problems will be graded using the following simple grading procedure, applied to each part (e.g. part a, part b, etc.) of the problem (except where otherwise noted above):

* Problem not attempted or does not demonstrate significant effort: no credit.
* Problem thoroughly attempted, but answer is incorrect or incomplete: half credit.
* Problem thoroughly attempted, and answer is correct and complete: full credit.

**You must show your work on all problems to receive credit**. Simply giving the answer (correct or incorrect) will earn no credit for that problem.