



## AN ABSTRACT OF THE DISSERTATION OF

Parisa Ataei for the degree of Doctor of Philosophy in Computer Science  
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Eric Walkingshaw

In this thesis I present the variational database management system, a formal framework and its implementation for representing variation in relational databases and managing variational information needs. A variational database is intended to support any kind of variation in a database. Specific kinds of variation in databases have already been studied and are well-supported, for example, schema evolution systems address the variation of a database's schema over time and data integration systems address variation caused by accessing data from multiple data sources simultaneously. However, many other kinds of variation in databases arise in practice, and different kinds of variation often interact, but these scenarios are not well-supported by the existing work. For example, neither the schema evolution systems nor the database integration systems can address variation that arises when data sources combined in one database evolve over time.

This thesis collects a large amount of work: It defines the variational database framework and the syntax and [specific kind of] semantics of the variational relational algebra, a query language for variational databases. It also defines the requirements of a generic variational database framework that makes the framework expressive enough to encode any kind of variation in databases. Additionally, it [shows/proves] that the introduced framework satisfies all these needs. It presents two use cases of the variational database framework that are based on existing data sets and scenarios that are partially supported by existing techniques. It presents the variational database management system which is the implementation of variational databases and variational relational algebra as an abstract layer written in Haskell on top of a traditional RDBMS. It also presents several theoretical results related to the framework and query language, such as syntax-based equivalence rules that preserve the semantics of a query, a type system for ensuring that a variational query is well formed with respect to the underlying variational schema, and a confluence property of the variational relational algebra type system and semantics with respect to the relational algebra type system and semantics.

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# Theory and Implementation of a Variational Database Management System

by

Parisa Ataei

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degree of

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APPROVED:

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Major Professor, representing Computer Science

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Director of the School of Electrical Engineering and Computer Science

---

Dean of the Graduate School

I understand that my dissertation will become part of the permanent collection of Oregon State University libraries. My signature below authorizes release of my dissertation to any reader upon request.

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Parisa Ataei, Author

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[Eric. Committee. jeff. abu. parents. friends. ]

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## Chapter 1: Introduction

Managing variation in databases is a perennial problem in database literature and appears in different forms and contexts [50, 9, 14, 24, 11] and it is unavoidable [48]. Variation in databases mainly arises when multiple database instances conceptually represent the same database but differ in their schema, content, or constraints. Specific kinds of variation in databases have been addressed by context-specific solutions, such as schema evolution [39, 13, 6, 47, 41], data integration [20], and database versioning [12, 31]. However, there are no generic solution that addresses all kind of variation in databases. We motivate the need for a generic solution to variation in databases in Section 1.1.

The major contribution of this thesis is the *variational database* framework, a generic relational database framework that explicitly accounts for database variation, and *variational relational algebra*, a query language for our framework that allows for information extraction from a variational database. The framework is generic because it can encode any kind of variation in databases. Additionally and more importantly, it is designed such that it can satisfy any information need that a user may have in a variational database scenario. Based on information needs in a variational database scenario, we define requirements of a variational database framework in Section 1.1 and throughout the thesis, we show that our framework satisfies these requirements.

In addition to a formal description of the variational database framework and variational relational algebra and some theoretical results, this thesis distributes and presents two variational data sets (including both a variational database and a set of queries) as well as a *variational database management system* that implements the variational database framework as an abstraction layer on top of a traditional relational database management system in Haskell. Section 1.2 enumerates the specific contributions in the context of an outline of the structure of the rest of the thesis.

Just as variation is ubiquitous in software, it is also ubiquitous in the relational databases that software systems rely on. Database researchers have long studied different kinds of variation in databases, such as through work on database evolution [13, 39, 41], database versioning [12, 31], and data integration [20]. However, work in the databases community does not identify *variation* as a general, orthogonal concern that arises in many different contexts. This is a problem since it means that tools and techniques developed for one kind of database variation cannot be easily reused for another. More concretely for software developers, and especially software product line (SPL) practitioners, the lack of a general representation of database variation means that many kinds of variation that arise in software do not map cleanly onto corresponding variation encodings in the databases they use.

In contrast, SPL researchers have invested significant effort in studying *variation* (or *variability*) as a general phenomenon and concern in software. Although many kinds of software variation are possible, most can be roughly organized into variation in *time* or *space* [53]. Variation in *time* refers to the evolution of a system

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and is addressed by revision control systems and configuration management [? ], while variation in *space* refers to the simultaneous development and maintenance of related systems with different feature sets and is the traditional focus of research on SPLs [? ]. Multiple lines of work in the SPL community have sought to develop general purpose representations of variation in software, such as delta-oriented programming [? ] and the choice calculus [21], among others. These can be used to unify both kinds of variation in software, enabling reuse of analyses across dimensions and enabling new kinds of analyses that consider variation in both time and space simultaneously [53].

Because the existing data variation models developed in the databases community do not align with all of the variation scenarios that arise in software, SPL researchers have identified the need for more general encodings of variation in data models and database schema. To this end, they have developed encodings of data models that allow for arbitrary variation by annotating different elements of the model with features from the SPL [46, 44, 2]. However, these solutions address only variation in the data model but do not extend to the level of the data or queries. The lack of variation support in queries leads to unsafe techniques such as encoding different variants of query through string munging, while the lack of variation support in data precludes testing with multiple variants of a database at once.

In previous work, we have developed *variational databases (VDBs)* and *variational queries* [8, 7] to encode general variation in the representation and use of relational databases. Our work extends ideas developed in the SPL community

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to relational databases. Conceptually, a VDB represents potentially many different plain relational databases at the same time. Similarly, a variational query represents potentially many different queries, each one corresponding to a variant of the VDB. Together, VDBs and variational queries enable safely and efficiently working with many variants of a relational database at once, and reliably integrating the variants of a database with the corresponding variants of an SPL. We are currently implementing these ideas in *VDBMS*, a practical implementation of variational databases as a lightweight wrapper on top of a traditional relational database management system.

However, the generic and expressive approach of VDB in dealing with database variation creates new complexity and costs which raises the question: Is explicitly encoding variation in databases actually a good idea? With this question in mind, in this paper, we:

- Show the feasibility of VDB by systematically generating two VDBs from realistic scenarios of database variation in time and space (Section ??).
- Illustrate the applicability of variational queries by encoding information needs for the developed VDBs using scenarios described in the literature (Section ??).
- Discuss the tradeoffs of explicitly encoding variation in databases (Section ??).

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## 1.1 Motivation and Impact

Managing variation in databases is a perennial problem in database literature and appears in different forms and contexts [50, 9, 14, 24, 11]. Variation in databases mainly arises when multiple database instances conceptually represent the same database, but, differ in their schema or content. The variation in schema and/or content occurs in two *dimensions*: time and space. Variation in space refers to different variants of database that coexist in parallel while variation in time refers to the evolution of database, similar to variation observed in software [53]. Note that variation in a database can occur due to both dimensions at the same time.

Existing work on variation in databases addresses specific kinds of variation in a context and proposes solutions specific to the context of the variation, such as schema evolution [39, 13, 6, 47, 41], data integration [20], and database versioning [12, 31]. Unfortunately, some of these tools do not address all their user’s needs. Furthermore, they are all *variation-unaware*, i.e., they dismiss that they are addressing a specific kind of a bigger problem. Thus, not only they cannot address a new kind of database variation but they also cannot address the interaction of different kinds of database variation since they assume that each kind of variation is *isolated* from another kinds. This costs database researchers to develop a new system for every individual kind of data variation and forces developers and DBAs to use manually unsafe workaround.

For example, schema evolution is a kind of schematic variation in databases over time that is well-supported [39, 13, 6, 47, 41]. Changes applied to the schema

over time are *variation* in the database and every time the database evolves, a new *variant* is generated. Current solutions addressing schema evolution dismiss that it is a kind of variation, thus, they *simulate* the effect of variation by relying on temporal nature of schema evolution and using timestamps [39, 13, 6, 47] or keeping an external file of time-line history of changes applied to the database [41].

Unlike schema evolution, some kinds of database variation are partially supported. For example, while developing software for multiple clients simultaneously, an approach called software product line (SPL) [18], a different database schema is required for each client due to client’s different business requirements and environments [46]. SPL researchers have developed encodings of data models that allow for arbitrary variation by annotating different elements of the model with features from the SPL [46, 44, 2]. Thus, they can generate a database schema *variant* for each software *variant* requested by a client and generated by the SPL. However, these solutions address *only* variation in the data model but do not extend it to the level of the data or queries. The lack of variation support in queries leads to unsafe techniques such as encoding different variants of query through string munging, while the lack of variation support in data precludes testing with multiple variants of a database at once.

The situation exacerbates even more when two kinds of variation interact and create a new kind of database variation: the evolution of the database used in development of software by an SPL. This is due to the evolution of software and its artifacts; an inevitable phenomena [29]. This is where even previous context-specific solutions like schema evolution tools fail because the context-specific ap-

proaches assume that the kind of variation that they address is completely isolated from other kinds of variation. We motivate this case through an example in Section 1.1. Note that new kinds of variation may arise that are not necessarily the interaction of already-addressed variation in database. We discuss such a scenario in Section 1.1.2.

As we have shown, variation in databases is abundant and inexorable [48]; impacts DBAs, data analysts, and developers significantly [29]; and appears in different contexts. Current methods are all extremely tailored to a specific context. Consequently, they fail to address the interaction of their specific kind of database variation with other kinds. Hence, the challenge becomes defining a variation-aware database that can model different kinds of variation in different contexts such that it satisfies different specialists’ needs at different stages, e.g., development, information extraction, deployment, and testing.

### 1.1.1 Motivating Example

In this section, we motivate the interaction of two kinds of variation in databases resulting in a new kind: database-backed software produced by an SPL and schema evolution. An SPL uses a set of boolean variables called *features* to indicate the functionalities that each software variant requires. Consider an SPL that generates management software for companies. It has a feature *edu* that indicates whether a company provides educational means such as courses for its employees. Software variants in which *edu* is disabled (i.e., *edu* = **false**) only provide basic function-

Table 1.1: Schema variants of the employee database developed for multiple software variants by an SPL. Note that an **educational** database variant must contain a **basic** database variant too.

(a) Database schema variants for basic software variants.

Temporal Features	basic Database Schema Variants
$V_1$	<i>engineerpersonnel</i> ( <i>empno</i> , <i>name</i> , <i>hiredate</i> , <i>title</i> , <i>deptname</i> ) <i>otherpersonnel</i> ( <i>empno</i> , <i>name</i> , <i>hiredate</i> , <i>title</i> , <i>deptname</i> ) <i>job</i> ( <i>title</i> , <i>salary</i> )
$V_2$	<i>empacct</i> ( <i>empno</i> , <i>name</i> , <i>hiredate</i> , <i>title</i> , <i>deptname</i> ) <i>job</i> ( <i>title</i> , <i>salary</i> )
$V_3$	<i>empacct</i> ( <i>empno</i> , <i>name</i> , <i>hiredate</i> , <i>title</i> , <i>deptno</i> ) <i>job</i> ( <i>title</i> , <i>salary</i> ) <i>dept</i> ( <i>deptname</i> , <i>deptno</i> , <i>managerno</i> ) <i>empbio</i> ( <i>empno</i> , <i>sex</i> , <i>birthdate</i> )
$V_4$	<i>empacct</i> ( <i>empno</i> , <i>hiredate</i> , <i>title</i> , <i>deptno</i> , <i>std</i> , <i>instr</i> ) <i>job</i> ( <i>title</i> , <i>salary</i> ) <i>dept</i> ( <i>deptname</i> , <i>deptno</i> , <i>managerno</i> ) <i>empbio</i> ( <i>empno</i> , <i>sex</i> , <i>birthdate</i> , <i>name</i> )
$V_5$	<i>empacct</i> ( <i>empno</i> , <i>hiredate</i> , <i>title</i> , <i>deptno</i> , <i>std</i> , <i>instr</i> , <i>salary</i> ) <i>dept</i> ( <i>deptname</i> , <i>deptno</i> , <i>managerno</i> , <i>stdnum</i> , <i>instrnum</i> ) <i>empbio</i> ( <i>empno</i> , <i>sex</i> , <i>birthdate</i> , <i>firstname</i> , <i>lastname</i> )

(b) Database schema variants for educational software variants.

Temporal Feature	educational Database Schema Variants
$T_1$	<i>course</i> ( <i>coursename</i> , <i>teacherno</i> ) <i>student</i> ( <i>studentno</i> , <i>coursename</i> )
$T_2$	<i>course</i> ( <i>courseno</i> , <i>coursename</i> , <i>teacherno</i> ) <i>student</i> ( <i>studentno</i> , <i>courseno</i> )
$T_3$	<i>course</i> ( <i>courseno</i> , <i>coursename</i> ) <i>teach</i> ( <i>teacherno</i> , <i>courseno</i> ) <i>student</i> ( <i>studentno</i> , <i>courseno</i> , <i>grade</i> )
$T_4$	<i>ecourse</i> ( <i>courseno</i> , <i>coursename</i> ) <i>course</i> ( <i>courseno</i> , <i>coursename</i> , <i>time</i> , <i>class</i> ) <i>teach</i> ( <i>teacherno</i> , <i>courseno</i> ) <i>student</i> ( <i>studentno</i> , <i>courseno</i> , <i>grade</i> )
$T_5$	<i>ecourse</i> ( <i>courseno</i> , <i>coursename</i> , <i>deptno</i> ) <i>course</i> ( <i>courseno</i> , <i>coursename</i> , <i>time</i> , <i>class</i> , <i>deptno</i> ) <i>teach</i> ( <i>teacherno</i> , <i>courseno</i> ) <i>take</i> ( <i>studentno</i> , <i>courseno</i> , <i>grade</i> )

alities while ones in which *edu* is enabled provide educational functionalities in addition to the basic ones. Thus, this SPL yields two types of variants: **basic** and **educational**.

Each variant of this SPL needs a database to store information about employees, but SPL features impact the database: While **basic** variants do not need to store any education-related records **educational** variants do. refs here have changed.

We visualize the impact of features on the schema variants in Table 1.1: It has two schema types: **basic**, shown in Table 1.1a, and **educational**, shown in Table 1.1b. A **basic** schema variant contains only the schema in one of the cells in Table 1.1a while an **educational** schema variant consists of two sub-schemas: one from the **basic** and another from Table 1.1b. For example, the yellow highlighted cells include relation schemas for an **educational** schema variant.

Rows of Table 1.1 indicate the evolution of schema variants in time. To capture the evolution of the software and its database we add two disjoint sets of features which again are boolean variables. The temporal feature sets are disjoint to allow individual paces for the evolution of each type of schema. For example, yellow cells of Table 1.1 show a valid schema variant even though the **basic** and **educational** sub-schemas are not in the same row.

refs here have changed. mot to mot-basic

Now, consider the following scenario: In the initial design of the **basic** database, SPL DBAs settle on three tables *engineerpersonnel*, *otherpersonnel*, and *job*; shown in Table 1.1a and associated with feature  $V_1$ . After some time, they decide

to refactor the schema to remove redundant tables, thus, they combine the two relations *engineerpersonnel* and *otherpersonnel* into one, *empacct*; associated with feature  $V_2$ . Since some clients' software relies on a previous design the two schemas have to coexist in parallel. Therefore, the existence (presence) of *engineerpersonnel* and *otherpersonnel* relations is *variational*, i.e., they only exist in the **basic** schema when  $V_1 = \text{true}$ . This scenario describes *component evolution*: developers update, refactor, and improve components including the database [29].

Now, consider the case where a client that previously requested a **basic** variant of the management software has recently added courses to educate its employees in specific subjects. Hence, an SPL developer needs to enable the *edu* feature for this client, forcing the adjustment of the schema variant to **educational**. This case describes *product evolution*: database evolution in SPL resulted from clients adding/removing features/components [29].

The situation is further complicated since the **basic** and **educational** schemas are interdependent: Consider the **basic** schema variant for feature  $V_4$ . Attributes *std* and *instr* only exists in the *empacct* relation when *edu* = **true**, represented by a dash-underline, otherwise the *empacct* relation has only four attributes: *empno*, *hiredate*, *title*, and *deptno*. Hence, the presence of attributes *std* and *instr* in *empacct* relation is *variational*, i.e., they only exist in *empacct* relation when *edu* = **true**.

Our example demonstrates how different kinds of variation interact with each other, an indispensable consequence of modern software development. The described interaction is similar to a recent scenario we discussed with an industry

contact in Section 1.1.2.

### 1.1.2 New Instance of Data Variation in Industry

New variational scenarios could appear, either from combination of other scenarios or even a new scenario could reveal itself. For example, the following is a scenario we recently discussed with an industry contact: A software company develops software for different networking companies and analyzes data from its clients to advise them accordingly. The company records information from each of its clients' networks in databases customized to the particular hardware, operating systems, etc. that each client uses. The company analysts need to query information from all clients who agreed to share their information, but the same information need will be represented differently for each client. This problem is essentially a combination of the SPL variation problem (the company develops and maintains many databases that vary in structure and content) and the data integration problem (querying over many databases that vary in structure and content). However, neither the existing solutions from the SPL community nor database integration address both sides of the problem. Currently the company manually maintains variant schemas and queries, but this does not take advantage of sharing and is a major maintenance challenge. With a database encoding that supports explicit variation in schemas, content, and queries, the company could maintain a single variational database that can be configured for each client, import shared data into a variational database, and write variational queries over the variational database to analyze the data,

significantly reducing redundancy across clients.

### 1.1.3 Requirements of a Variation-Aware Database Framework

For a variation-aware framework to be expressive enough to encode any kind of variation in databases, it must satisfy some requirements. Thus, we define the requirements that make a database framework variational through studying different kinds of variation in databases. A variational artifact, including databases, encompasses multiple *variants* of the artifact and provides a way to distinguish between different variants that are all encoded in one place, the variational artifact. It also provides a way to get the variants from the variational artifact, we call this *configuration function*. These requirements that help distinguish a database framework that simulates the effect of variation compared to one that is variational are listed below:

- (R0) *All database variants must be accessible at a given time.* For example, in our motivating example, a company that started with  $V_1$  of the **basic** schema evolves over time but its different branches adopt the new schema at different paces, thus, it requires access to all variants of the **basic** schema.
- (R1) *The query language must allow for querying multiple database variants simultaneously. Additionally, it must allow for filtering tuples to specific variants.* That is, the framework must provide a query language that allows users to query multiple database variants at the same time in addition to giving them the freedom to choose the variants that they want to query. For example,



an SPL tester that is testing a piece of code for the not highlighted variants of the software in Table 1.1 needs to write queries that exclude the variants associated with yellow cells of Table 1.1.

- (R2)** *Every piece of data must clearly state the variant it belongs to and this information must be kept throughout the entire framework.* Continuing the example of the SPL tester, they need to know the variant that some results belong to in order to be able to debug the software correctly and accordingly.
- (R3)** *The variational database must provide a way for generating database and query variants.* For example, the SPL developers need to deploy the management software for each client, thus, they need to configure the database schema and its queries in the code for each software variant.

Throughout the thesis, we show how the proposed variational database framework satisfies these requirements via examples, proofs, and tests.

## 1.2 Contributions and Outline of this Thesis

[to be filled out when I have the chapters.]

The high level goal of this thesis is to emphasize the need for a variation-aware database framework and to present one such framework. Therefore, in addition to the formal definition of the framework and query language, it also provides variational data sets (including both the variational database and a set of queries) to illustrate the feasibility of the proposed framework. Furthermore, it illustrates var-

ious approaches to implement such a framework and compares their performance.

The rest of this section describes the structure of this thesis, enumerating the specific contribution that each chapter makes.

Chapter 2 (*Background*) introduces several concepts and terms that are the basis of this thesis. It describes types and how to interpret them. It explains relational databases with assumptions that are held throughout the thesis and relational algebra. It also describes various ways of incorporating variation into elements of a database.

[Chapter 3]

[Chapter 4]

Chapter 5 aims at guiding an expert through generating a VDB and writing variational queries for a variation scenario where unfortunately the database variants do not exist and thus, generating the VDB requires the expert knowledge and cannot be automated. It details two such variation scenario and introduces two use cases of VDB, one over space (adopts the email SPL described by Hall [28] and explained in Section 5.1) and another over time (adopts the evolution of an employee schema described by Moon et al. [41] and explained in Section 5.2), in addition to how a VDB can store all database variants in a single database and how variational queries can capture various information needs over different database variants in a single query. It also describes how the VDBs were systematically generated and how the variational queries were adapted and adjusted from papers describing the variation scenario. The last section of this chapter, Section 5.3, discusses the question “should variation be encoded explicitly in databases?” through

the lessons learned while generating these use cases.

[Chapter 6]

Chapter 7 (*Related Work*) collects research related to different kinds of variation in databases and other related variational research.

Finally, Chapter 8 (*Conclusion*) briefly presents ...

## Chapter 2: Background

The core of this thesis is injecting a new aspect to relational databases: *variation*. Thus, the goals of this chapter are twofold: first, to introduce how variation is encoded and represented in our variational database framework; second, to provide the reader with the concepts and notations used to build up the main contributions of this thesis, mainly relational databases and relational algebra. And additionally, developed approaches of converting non-variational components into variational counterparts by using the introduced encoding of variation. Throughout the thesis, we discuss relational concepts and their variational counterparts. For clarity or when it is unclear from context, we use an underline to distinguish a non-variational entity from its variational counterpart, e.g.,  $\underline{x}$  is a non-variational entity while  $x$  is its variational counterpart, if it exists. Section 2.1 describes the database model of relational databases and the specification of the structured used to store the data [1]. Section 2.2 describes the relational algebra to query relational databases [1]. Then, Section 2.3 defines our choice of encoding for the variation space used in the variational database as propositional formulas of boolean variables [8, 7]. Finally, we introduce main techniques used to incorporate variation into our variational database framework. Section 2.4 introduces variation into sets which forms the basis of the variational database framework [23, 56, 7] and Section 2.5 describes the formula choice calculus used to incorporate variation into

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and nothing  
added  
to a type.

types!!

relational algebra [32].

## 2.1 Relational Databases

We introduce relational databases concepts that make up the bones of this thesis. To ease in the reader into relational databases we first provide an example in Table 2.1 which represents an database instance corresponding to  $V_2$  introduced in our motivating example in Table 1.1a. Intuitively, the data is represented in tables where each row contains data about a specific object or sets of object, then, rows with uniform structure and meaning are grouped into a table [1]. Each table is called a relation and has a name, for example, the relation *job* contains rows specifying the salary of a position in a company. The columns form the structure of the table and are called attributes, for example, the relation *job* has two attributes: *title* and *salary*. The values of attributes are taken from a set of constants called the domain. For simplicity, we do not distinguish between different types of values such as strings, numbers, dates.<sup>1</sup> Finally, we differentiate between the database schema, which is the structure data is stored, and the database instance, which is the actual content of the database. For example, Table 2.1b and Table 2.1c illustrate the database instance while Table 2.1a shows the database schema. This differentiation can be viewed as the differentiation of types and values in programming languages. For example,  $x$  is variable of type *Int* and may

read up to  
the formal  
definitions.

<sup>1</sup>This design decision was made to make proofs easier, however, our database system implementation distinguishes between different kinds of values and considers a unique domain for each.

Table 2.1: An example of a relational database corresponding to  $V_2$  of our motivating example given in Table 1.1a.

(a) The schema of a relational database.

*empacct* (*empno*, *name*, *hiredate*, *title*, *deptname*)  
*job* (*title*, *salary*)

(b) The *empacct* table.

<i>empacct</i>	<i>empno</i>	<i>name</i>	<i>hiredate</i>	<i>title</i>	<i>deptname</i>
	10001	Georgi Facello	1986-06-26	Senior Engineer	Development
	10002	Bezalel Simmel	1985-11-21	Staff	Sales
	...	...	...	...	...
	499998	Patricia Breugel	1993-10-13	Senior Staff	Finance
	499999	Sachin Tsukuda	1997-11-30	Engineer	Production

(c) The *job* table.

<i>job</i>	<i>title</i>	<i>salary</i>
	Assistant Engineer	61594
	Senior Engineer	96646
	...	...
	Staff	77935
	Technique Leader	58345

have value 15.

We now shift our focus to the formal definitions of a relational database. A relational database  $\underline{D}$  stores information in a structured manner by forcing data to conform to a *schema*  $\underline{S}$  that is a finite set  $\{\underline{s}_1, \dots, \underline{s}_n\}$  of *relation schemas*. A relation schema is defined as  $\underline{s} = r(\underline{a}_1, \dots, \underline{a}_k)$  where each  $\underline{a}_i \in \underline{\mathbf{A}}$  is an *attribute* contained in the relation named  $r$  and  $\underline{\mathbf{A}}$  is a fixed countably infinite set of attributes. We assume a total order  $\leq_{\underline{\mathbf{A}}}$  on  $\underline{\mathbf{A}}$ , and assume for simplicity that sets of attributes are sorted according to  $\leq_{\underline{\mathbf{A}}}$ .

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The content of database  $\underline{D}$  is stored in the form of *tuples*. A tuple  $\underline{u}$  is a sorted set of *values*. We do not distinguish between different types of values within a relational database. The order of values within a tuple correspond to the order of attributes in its corresponding relation schema, that is, given tuple  $\underline{u} = (\underline{v}_1, \dots, \underline{v}_k)$  in the relation with relation schema  $r(\underline{a}_1, \dots, \underline{a}_k)$ ,  $\underline{v}_i$  corresponds to attribute  $\underline{a}_i$ . A *relation content*,  $\underline{U}$ , is the set of all tuples  $\{\underline{u}_1, \dots, \underline{u}_m\}$  corresponding to a particular relation. The operation  $att(i)$  returns the attribute corresponding to index  $i$  in a tuple, implicitly looking up the attribute in the corresponding relation schema. A *table*  $\underline{t} = (\underline{s}, \underline{U})$  is a pair of a relation schema and relation content. A *database instance*,  $\underline{\mathcal{I}}$ , of the database  $\underline{D}$  with schema  $\underline{S}$ , is a set of tables  $\{\underline{t}_1, \dots, \underline{t}_n\}$  for each relation in  $\underline{S}$ . For brevity, when it is clear from context, we refer to a database instance as simply a *database*.

## 2.2 The SPC Relational Algebra

Having introduced relational databases we now shift gears into querying these databases, that is, extracting information from tables. Importantly and almost across all relational query languages, the result of a query is also a table [1]. We base our variational query language on the SPC relational algebra. Three primitive operators form the SPC algebra: *selection*, *projection*, and *cross-product* (or Cartesian product) [1]. We introduce these operators through Example 2.2.1 by stating an intent and then building up a query to extract the information required by the intent.

add type system

maybe add semantics later on

please read this section completely.

**Example 2.2.1** Consider the database instance given in Table 2.1. We want to get a list of employees (by their names) whose salary is more than 65000 dollars. As the first step we use the selection operator to get the salaries that are more than 65000 dollars.

$$\underline{Q_1} = \sigma_{salary \geq 65000}(job)$$

A sample of the results returned by the query  $\underline{Q_1}$  is given in Table 2.2a. Next a set of wide tuples is created by taking the cross-product of  $\underline{Q_1}$  and *empacct*.

$$\underline{Q_2} = \underline{Q_1} \times empacct$$

A sample of the results returned by the query  $\underline{Q_2}$  is given in Table 2.2b. However, looking closely at these results there is no connection between an employee in the *empacct* relation and their salary in the *job* relation. Thus, we have to perform another selection to connect each employee with their title.

$$\underline{Q_3} = \sigma_{empacct.title=job.title}(\underline{Q_2})$$

A sample of the results returned by the query  $\underline{Q_3}$  is given in Table 2.2c. At this point, we are only interested in two attributes, that is, name and salary. Thus, we use projection to discard the unneeded columns.

$$\underline{Q_4} = \pi_{name,salary}(\underline{Q_3})$$

A sample of the results returned by the query  $\underline{Q_4}$  is given in Table 2.2d.

We now embark on the the syntax of SPC relational algebra extended with join, set operations, and empty relation. This syntax is defined in Figure 2.1. The set operations, union and intersection, require two subqueries to have the same relation schema and simply apply the operation, either union or intersection, to the tuples returned by the subqueries. The *join* operation is equivalent to selec-



Table 2.2: Results of each step of building the final query in Example 2.2.1.

(a) Result of the query  $\sigma_{salary \geq 65000}(job)$ .

<i>title</i>	<i>salary</i>
Senior Staff	77935
Senior Engineer	96646
...	...
Staff	77935
Engineer	96646

(b) Result of the query  $(\sigma_{salary \geq 65000}(job)) \times empacct$ .

<i>title</i>	<i>salary</i>	<i>empno</i>	<i>name</i>	<i>hiredate</i>	<i>title</i>	<i>deptname</i>
Senior Engineer	96646	13094	Sanjay Servieres	1986-01-01	Engineer	Research
Staff	77935	16099	Mohan Ferretti	1987-09-20	Senior Staff	Human Resources
...	...	...	...	...	...	...
Engineer	80324	19162	Chinho Fadgyas	1986-05-19	Technique Leader	Production
Senior Staff	88070	22255	Kristian Merel	1986-09-12	Senior Engineer	Development

(c) Result of the query  $\sigma_{empacct.title=job.title}((\sigma_{salary \geq 65000}(job)) \times empacct)$ .

<i>title</i>	<i>salary</i>	<i>empno</i>	<i>name</i>	<i>hiredate</i>	<i>title</i>	<i>deptname</i>
Engineer	96646	13094	Sanjay Servieres	1986-01-01	Engineer	Research
Senior Staff	77935	16099	Mohan Ferretti	1987-09-20	Senior Staff	Human Resources
...	...	...	...	...	...	...
Senior Engineer	96646	22255	Kristian Merel	1986-09-12	Senior Engineer	Development
Staff	77935	43670	JoAnna Randi	1987-10-18	Staff	Marketing

(d) Result of the query  $\pi_{name,salary}(\sigma_{empacct.title=job.title}((\sigma_{salary \geq 65000}(job)) \times empacct))$ .

<i>name</i>	<i>salary</i>
Sanjay Servieres	96646
Mohan Ferretti	77935
...	...
Kristian Mere	96646
JoAnna Randi	77935

**Operators:**

$$\begin{aligned} \bullet &:= < \mid \leq \mid = \mid \neq \mid > \mid \geq \\ \circ &:= \cup \mid \cap \end{aligned}$$

**Variational conditions:**

$$\underline{\theta} \in \underline{\Theta} := \text{true} \mid \text{false} \mid a \bullet k \mid a \bullet a \mid \neg \underline{\theta} \mid \underline{\theta} \vee \underline{\theta}$$

**Relational queries:**

$$\begin{aligned} \underline{q} \in \underline{\mathbf{Q}} &:= \underline{r} && \text{Relation} \\ &| \pi_{\underline{A}} \underline{q} && \text{Projection} \\ &| \sigma_{\underline{\theta}} \underline{q} && \text{Selection} \\ &| \underline{q} \times \underline{q} && \text{Cross-Product} \\ &| \underline{q} \circ \underline{q} && \text{Set Operation} \\ &| \underline{q} \bowtie_{\underline{\theta}} \underline{q} && \text{Join} \\ &| \varepsilon && \text{Empty Relation} \end{aligned}$$

Figure 2.1: Syntax of relational algebra.

tion applied to cross-product, that is,  $\underline{q}_1 \bowtie_{\underline{\theta}} \underline{q}_2 = \sigma_{\underline{\theta}}(\underline{q}_1 \times \underline{q}_2)$ . For example,  $\underline{Q}_3$  in Example 2.2.1 can be rewritten as

$$\underline{Q}_3' = (\sigma_{\text{salary} \geq 65000}(\text{job})) \bowtie_{\text{empacct.title}=\text{job.title}} \text{empacct} \quad .$$

Throughout our examples, omitting the condition of join implies it is a natural join, that is, join on the shared attribute of the two subqueries. For example,  $\underline{Q}_3'$  can be rewritten using the natural join

$$\underline{Q}_3'' = (\sigma_{\text{salary} \geq 65000}(\text{job})) \bowtie \text{empacct} \quad .$$

We also extend relational algebra such that projection of an empty set of attributes is a valid query that returns an empty set of tuples. We define the *empty* query  $\varepsilon$  as shorthand for projecting an empty set of attributes, that is,  $\varepsilon = \pi_{\{\}} \underline{q}$ . Note that we do not extend the notation of using underline for relational algebra

operators. Instead, relational algebra operators are overloaded and are used as both plain relational and variational operators. It should be clear from context when an operation is variational or not.

Although we do not consider renaming of queries in the formal definition of relational algebra we consider for our implementation. Furthermore, we use it to rename subqueries of our examples to make them easier to understand. For example, query  $\underline{Q_3''}$  can be written as:

$$\begin{aligned}\underline{Q_3''} &= \underline{temp} \bowtie empacct \\ \underline{temp} &\leftarrow \sigma_{salary \geq 65000}(job)\end{aligned}$$

Making this renaming explicit is necessary to avoid names conflicting in some cases.

### 2.3 Variation Space and Encoding

In this section, we describe how we represent variation throughout this thesis. We encode variation in terms of *features*. The *feature space*,  $\mathbf{F}$ , of a variational database is a closed set of boolean variables called features. A feature  $f \in \mathbf{F}$  can be enabled (i.e.,  $f = \mathbf{true}$ ) or disabled ( $f = \mathbf{false}$ ). Features describe the variation in a given variational scenario. For example, in the context of schema evolution, features can be generated from version numbers (e.g., features  $V_1$  to  $V_5$  and  $T_1$  to  $T_5$  in the motivating example, Table 1.1); for SPLs, the features can be adopted from the SPL feature set (e.g., the *edu* feature in our motivating example, Table 1.1);

**Feature expression syntax:**

$f \in \mathbf{F}$		<i>Feature Name</i>
$b \in \mathbf{B} := \text{true} \mid \text{false}$		<i>Boolean Value</i>
$e \in \mathbf{E} := b \mid f \mid \neg e \mid e \wedge e \mid e \vee e$		<i>Feature Expression</i>
$c \in \mathbf{C} : \mathbf{F} \rightarrow \mathbf{B}$		<i>Configuration</i>

**Evaluation of feature expressions:**

$$\begin{aligned}
\mathbb{E}[\cdot] &: \mathbf{E} \rightarrow \mathbf{C} \rightarrow \mathbf{B} \\
\mathbb{E}[b]_c &= b \\
\mathbb{E}[f]_c &= c \ f \\
\mathbb{E}[\neg f]_c &= \neg \mathbb{E}[f]_c \\
\mathbb{E}[e_1 \wedge e_2]_c &= \mathbb{E}[e_1]_c \wedge \mathbb{E}[e_2]_c \\
\mathbb{E}[e_1 \vee e_2]_c &= \mathbb{E}[e_1]_c \vee \mathbb{E}[e_2]_c
\end{aligned}$$

**Relations over feature expressions:**

$$\begin{aligned}
e_1 \equiv e_2 &\text{ iff } \forall c \in \mathbf{C}. \mathbb{E}[e_1]_c = \mathbb{E}[e_2]_c \\
\text{sat}(e) &\text{ iff } \exists c \in \mathbf{C}. \mathbb{E}[e]_c = \text{true} \\
\text{unsat}(e) &\text{ iff } \forall c \in \mathbf{C}. \mathbb{E}[e]_c = \text{false}
\end{aligned}$$

**Syntactic sugar for mutually exclusive features:**

$$\begin{aligned}
f_1 \oplus f_2 \oplus \dots \oplus f_n &= (f_1 \wedge \neg f_2 \wedge \dots \wedge \neg f_n) \\
&\quad \vee (\neg f_1 \wedge f_2 \wedge \dots \wedge \neg f_n) \\
&\quad \vee (\neg f_1 \wedge \neg f_2 \wedge \dots \wedge f_n)
\end{aligned}$$

Figure 2.2: Feature expression syntax, relations, syntactic sugar, and evaluation.

and for data integration, the features may represent different data sources. Note that it is easy to extend features to multi-valued variables that have a finite set of countable values. Consider our motivating example, Table 1.1. We could choose to have a feature  $V$  as a multi-valued variable which takes one of the values in set  $\{1, 2, 3, 4, 5\}$ . This new multi-valued feature  $V$  is equivalent to having five boolean variables  $V_1, V_2, V_3, V_4$ , and  $V_5$ .

features as  
multivalued  
variables  
added.

Features are used at variation points to indicate which variants a particular element belongs to. Thus, enabling or disabling each of the features in the feature set produces a particular database *variant* where all variation has been removed. A *configuration* is a *total* function that maps every feature in the feature set to a boolean value and is denoted by  $c \in \mathbf{C} : \mathbf{F} \rightarrow \mathbf{B}$ . We represent a configuration by the set of enabled features. For example, in our motivating scenario, the configuration  $\{V_2, T_3, edu\}$  represents a database variant where only features  $V_2, T_3$ , and  $edu$  are enabled (and the rest are disabled). This database variant contains relation schemas in the yellow cells of Table 1.1. We refer to a variant by the configuration that produces it. For example, variant  $\{V_2, T_3, edu\}$  refers to the variant produced by applying that configuration.

When describing variation points in the database, we need to refer to subsets of the configuration space. We do this with propositional formulas of features. Thus, such a propositional formula defines a condition that holds for a subset of configurations and their corresponding variants. For example, the propositional formula  $\neg edu$  represents all variants of our motivating example where the  $edu$  feature is disabled, that is, variant schemas of Table 1.1b. We call a

propositional formula of features a *feature expression* and define it formally in Figure 2.2. Additionally, in Figure 2.2, we formally define the evaluation function of feature expressions  $\mathbb{E}[\![e]\!]_c : \mathbf{E} \rightarrow \mathbf{C} \rightarrow \mathbf{B}$ . This function simply substitutes each feature  $f$  in the expression  $e$  with the boolean value given by configuration  $c$  and then simplifies the propositional formula to a boolean value. For example,  $\mathbb{E}[\![f_1 \vee f_2]\!]_{\{f_1\}} = \mathbf{true} \vee \mathbf{false} = \mathbf{true}$ , while  $\mathbb{E}[\![f_1 \vee f_2]\!]_{\{\}} = \mathbf{false} \vee \mathbf{false} = \mathbf{false}$ . Furthermore, in Figure 2.2, we define a binary *equivalence relation* ( $\equiv$ ) relation on feature expressions corresponding to logical equivalence, unary *sat* and *unsat* predicates that determine whether a feature expression is satisfiable or unsatisfiable, and syntactic sugar  $\oplus$  for mutually exclusive features, respectively.

To incorporate feature expressions into the database, we *annotate* database elements (including attributes, relations, and tuples) with feature expressions. An *annotated element*  $x$  with feature expression  $e$  is denoted by  $x^e$ . The feature expression attached to an element is called its *presence condition* since it determines the condition (set of configurations) under which the element is present in the database. For example, assuming  $\mathbf{F} = \{f_1, f_2\}$ , the annotated number  $2^{f_1 \vee f_2}$  is present in variants  $\{f_1\}$ ,  $\{f_2\}$ , and  $\{f_1, f_2\}$  but not in variant  $\{\}$ . The operation  $pc(x^e) = e$  returns the presence condition of an annotated element.

No matter the context, features often have a relationship with each other that constrains the set of possible configurations. For example, only one of the temporal features of  $V_1$ – $V_5$  can be **true** for a given variant. This relationship is captured by a feature expression, called a *feature model* and denoted by  $m$ , which restricts the set of *valid configurations*. That is, a configuration  $c$  is only valid if  $\mathbb{E}[\![m]\!]_c = \mathbf{true}$ .

added  $\oplus$  as syntactic sugar

you had **E** instead of **F**. was that a typo or am I making a mistake?

parisa, fix vldb if it's a typo.

For example, the restriction that at a given time only one of temporal features  $V_1$ – $V_5$  can be enabled is represented by the feature model  $V_1 \oplus V_2 \oplus V_3 \oplus V_4 \oplus V_5$ .

## 2.4 Variational Set

A *variational set* (*v-set*)  $X = \{x_1^{e_1}, \dots, x_n^{e_n}\}$  is a set of annotated elements [23, 56, 7]. We typically omit the presence condition **true** in a variational set, e.g.,  $4^{\mathbf{true}} = 4$ . Conceptually, a variational set represents many different plain sets simultaneously. These plain sets can be generated by *configuring* a variational set with a configuration. This is done by the *variational set configuration* function  $\mathbb{X}[[X]]_c : \mathcal{X} \rightarrow \mathbf{C} \rightarrow \underline{\mathcal{X}}$ , defined in Figure 2.3. The configuration function evaluates the presence condition  $e_i$  of each element  $x_i$  of the variational set with the configuration  $c$ . If the evaluation results in **true** it includes  $x_i$  in the plain set and otherwise it does not. Example 2.4.1 illustrates the configuration of a variational set for all possible configurations.

revised this  
to refer to  
the figure.

**Example 2.4.1** Assume we have the feature space  $\mathbf{F} = \{f_1, f_2\}$  and the variational set  $X_1 = \{2^{f_1}, 3^{f_2}, 4\}$ .  $X_1$  represents four plain sets:

it'd be nice  
to have the  
entire ex in  
the same  
page.

$$\mathbb{X}[[X_1]]_c = \begin{cases} \{2, 3, 4\}, & c = \{f_1, f_2\} \\ \{2, 4\}, & c = \{f_1\} \\ \{3, 4\}, & c = \{f_2\} \\ \{4\}, & c = \{\} \end{cases}$$

**Configuration of an annotated element:**

$$\begin{aligned} \mathbf{x}[\![\cdot]\!] &: \mathbf{X} \rightarrow \mathbf{C} \rightarrow \underline{\mathbf{X}}_{\perp} \\ \mathbf{x}[x^e]_c &= \begin{cases} x, & \text{if } \mathbb{E}[e]_c = \mathbf{true} \\ \perp, & \text{otherwise} \end{cases} \end{aligned}$$

**Configuration of (annotated) variational sets:**

$$\begin{aligned} \mathbb{X}[\![\cdot]\!] &: \mathcal{X} \rightarrow \mathbf{C} \rightarrow \underline{\mathcal{X}} \\ \mathbb{X}[\![x_1^{e_1}, x_2^{e_2}, \dots, x_n^{e_n}]\!]_c &= \otimes \{ \mathbf{x}[x_1^{e_1}]_c, \mathbf{x}[x_2^{e_2}]_c, \dots, \mathbf{x}[x_n^{e_n}]_c \} \\ \mathbb{X}[\![\{ \}]\!]_c &= \{ \} \\ \mathbb{X}[X^e]_c &= \mathbb{X}[\![\downarrow(X^e)]\!]_c \\ \mathbb{X}[\![\{ \}^{\mathbf{true}}]\!]_c &= \{ \} \\ \mathbb{X}[\![\{ \}^{\mathbf{false}}]\!]_c &= \{ \} \end{aligned}$$

**Dropping bots from a plain set:**

$$\otimes \{x_1, x_2, \dots, x_n\} = \{x_i \mid x_i \neq \perp\}$$

**Normalization of (annotated) variational sets:**

$$\begin{aligned} \downarrow(\{x_1^{e_1}, x_2^{e_2}, \dots, x_n^{e_n}\}) &= \{x_i^{e_i} \mid 1 \leq i \leq n, \text{sat}(e_i)\} \\ \downarrow(X^e) &= \{x_i^{e_i \wedge e} \mid x_i^{e_i} \in X^e, \text{sat}(e_i \wedge e)\} \end{aligned}$$

**Operations over (annotated) variational sets:**

$$\begin{aligned} X_1 \cup X_2 &= \{x^{e_1} \mid x^{e_1} \in \downarrow(X_1), \exists e_2. x^{e_2} \notin \downarrow(X_2)\} \\ &\quad \cup \{x^{e_2} \mid x^{e_2} \in \downarrow(X_2), \exists e_1. x^{e_1} \notin \downarrow(X_1)\} \\ &\quad \cup \{x^{e_1 \vee e_2} \mid x^{e_1} \in \downarrow(X_1), x^{e_2} \in \downarrow(X_2)\} \\ X_1 \cap X_2 &= \{x^{e_1 \wedge e_2} \mid x^{e_1} \in X_1, x^{e_2} \in X_2, \text{sat}(e_1 \wedge e_2)\} \\ X_1 \times X_2 &= \{(x_1, x_2)^{e_1 \wedge e_2} \mid x_1^{e_1} \in X_1, x_2^{e_2} \in X_2\} \\ X_1 \equiv X_2 &\text{ iff } \forall x^e \in (\downarrow(X_1) \cup \downarrow(X_2)), x^{e_1} \in \downarrow(X_1), x^{e_2} \in \downarrow(X_2). e_1 \equiv e_2, e \equiv e_1 \end{aligned}$$

Figure 2.3: Configuration of variational set and annotated variational set, normalization of variational sets and annotated variational sets, and operations over variational sets. The operations on variational sets are overloaded, that is,  $\cup$  is used to both denote the union of variational sets and plain sets.



This states that, for example, configuring  $X_1$  for the variant that enables both  $f_1$  and  $f_2$  (that is,  $f_1 = \text{true}, f_2 = \text{true}$ ) results in the plain set  $\mathbb{X}[[X_1]]_{\{f_1, f_2\}} = \{2, 3, 4\}$ .

Following database notational conventions we drop the brackets of a variational set when used in database schema definitions and queries.

A variational set itself can also be annotated with a feature expression.  $X^e = \{x_1^{e_1}, \dots, x_n^{e_n}\}^e$  is an *annotated variational set*. Annotating a variational set with the feature expression  $e$  means that all elements in the variational set are only present when  $e$  evaluates to **true**. The *normalization* operation  $\downarrow(X^e)$  applies this restriction by pushing it into the presence conditions of the individual elements:  $\downarrow(X^e) = \{x_i^{e_i \wedge e} \mid x_i^{e_i} \in X^e, \text{sat}(e_i \wedge e)\}$ . Thus, we consider both variational sets and annotated variational sets to belong to the set of variational set  $\mathcal{X}$ , that is, we consider them to have the same type. Note that the *normalization* operation also removes elements with unsatisfiable presence conditions and may also be applied to an unannotated variational set  $X$  since  $X^{\text{true}} = X$ . For example, the annotated variational set  $X_1 = \{2^{f_1}, 3^{\neg f_2}, 4, 5^{f_3}\}^{f_1 \wedge f_2}$  indicates that all the elements of the set can only exist when both  $f_1$  and  $f_2$  are enabled. Thus, normalizing the variational set  $X_1$  results in  $\{2^{f_1 \wedge f_2}, 4^{f_1 \wedge f_2}, 5^{f_1 \wedge f_2 \wedge f_3}\}$ . The element 3 is dropped since  $\neg \text{sat}(pc(3, X_1))$ , where  $pc(3, X_1) = \neg f_2 \wedge (f_1 \wedge f_2)$ . Note that we use the function  $pc(x, X^e)$  to return the presence condition of a unique variational element within a bigger variational structure.

added that both v-set and annot v-set are of the same type.

In Figure 2.3, we also define several operations, such as union and intersection, over variational sets; these operations are used in Section 4.1.3. The semantics of a variational set operation is equivalent to applying the corresponding plain

set operation to every corresponding variant of the argument variational sets. For example, the union of two variational sets  $X_1 \cup X_2$  should produce a new variational set  $X_3$  such that  $\forall c \in \mathbf{C}. \mathbb{X}[[X_3]]_c = \mathbb{X}[[X_1]]_c \cup \mathbb{X}[[X_2]]_c$ , where  $\cup$  is the plain set union operation. This property must hold for all operations over variational sets, that is, for all possible operations,  $\odot$ , defined on variational sets the property  $(P_1) : \forall c \in \mathbf{C}. \mathbb{X}[\downarrow(X_1) \odot \downarrow(X_2)]_c = \mathbb{X}[[X_1]]_c \odot \mathbb{X}[[X_2]]_c$  must hold, where  $\odot$  is the counterpart operation on plain sets.<sup>2</sup>

added the  
general  
property.

## 2.5 The Formula Choice Calculus

[formula choice calculus]

The choice calculus [54, 21] is a metalanguage for describing variation in programs and its elements such as data structures [56, 23]. In the choice calculus, variation is represented in-place as choices between alternative subexpressions. For example, the variational expression  $e = A\langle 1, 2 \rangle + B\langle 3, 4 \rangle + A\langle 5, 6 \rangle$  contains three choices. Each choice has an associated *dimension*, which is used to synchronize the choice with other choices in different parts of the expression. For example, expression  $e$  contains two dimensions,  $A$  and  $B$ , and the two choices in dimension  $A$  are synchronized. Therefore, the variational expression  $e$  represents four different plain expressions, depending on whether the left or right alternatives are selected from each dimension. Assuming that dimensions may be set to boolean values where **true** indicates the left alternative and **false** indicates the right alternative,

---

<sup>2</sup>This property is proved for the operations we define over variational sets in Coq proof assistant [? ].

we have: (1)  $1 + 3 + 5$ , when  $A$  and  $B$  are **true**, (2)  $1 + 4 + 5$ , when  $A$  is **true** and  $B$  is **false**, (3)  $2 + 3 + 6$ , when  $A$  is **false** and  $B$  is **true**, and (4)  $2 + 4 + 6$ , when  $A$  and  $B$  are **false**.

The formula choice calculus [32] extends the choice calculus by allowing dimensions to be propositional formulas. For example, the variational expression  $e' = A \vee B \langle 1, 2 \rangle$  represents two plain expressions: (1) 1, when  $A \vee B$  evaluates to **true** and (2) 1, when  $A \vee B$  evaluates to **false**.

A formula  $e$  can also be used to *annotate/tag* an element  $x$  which is denoted by  $x^e$ . For example, the variational expression  $1^{A \vee B}$  states that the expression 1 is present when  $A \vee B$  evaluates to **true** and otherwise it is absent.

## Chapter 3: The Variational Database Framework

[needs. must have configuration. ]

### 3.1 Variational Schema

[vsch]

#### 3.1.1 Variational Schema Configuration

[vsch configuration.]

### 3.2 Variational Table

[vtab]

#### 3.2.1 Variational Table Configuration

[vtab configuration]

### 3.3 Variational Database

[vdb]

### 3.3.1 Variational Database Configuration

[vdb configuration]

## 3.4 Properties of a Variational Database Framework

[well-formed vdb properties.context-specific properties.]

[show that they hold for vdb.]

the following is taken from VaMoS

In this section, we describe a set of basic properties that a well-formed VDB should satisfy. These checks ensure that presence conditions are consistent and satisfiable, which ensures that each element is present in at least one variant. In the following,  $sat(e)$  denotes a satisfiability check that returns **true** if the feature expression  $e$  is satisfiable and **false** otherwise.

A well-formed v-schema should have the following properties:

1. There is at least one valid configuration of the feature model  $m$ :  $sat(m)$
2. Every relation  $r$  is present in at least one configuration of the variational schema:  $\forall r \in S, sat(m \wedge pc(r))$
3. Every attribute  $a$  in every relation  $r$  is present in at least one configuration of the variational schema:  $\forall a \in r, \forall r \in S, sat(m \wedge pc(r) \wedge pc(a))$

4. If  $S_c$  denotes the expected plain relational schema for configuration  $c$  of the variational schema  $S$ , then configuring the variational schema with that configuration, written  $\llbracket S \rrbracket_c$ , actually yields that variant:  $\forall c \in \mathbf{C}, \llbracket S \rrbracket_c = S_c$

At the data level, a well-formed VDB should have these properties:

1. Every tuple  $u$  in relation  $r$  is present in at least one variant:  $\forall u \in r, \forall r \in S, \text{sat}(m \wedge pc(r) \wedge pc(u))$
2. For every tuple  $u$  in relation  $r$ , if an attribute  $a$  in  $r$  is not present in any variants of the tuple, then the value of that attribute in the tuple, written  $value_u(a)$ , should be NULL:  $\forall u \in r, \forall a \in r, \forall r \in S, \neg \text{sat}(m \wedge pc(r) \wedge pc(a) \wedge pc(u)) \Rightarrow value_u(a) = \text{NULL}$

We implemented these checks in our VDBMS tool and verified that both use cases described in this paper satisfy all of them. Depending on the context of the VDB, more specialized properties can be checked too. For example, if temporal variability in a database is accumulated over variants (i.e. old data is included in more recent variants in addition to newly added data), it is desirable to ensure that older variants are subsets of newer variants. This property should hold for our employee data set. To check this, assume that configurations  $c_1, c_2, \dots$  represent time-ordered configurations, then check  $\forall c_i, c_j \in \mathbf{C}, i \leq j, \llbracket D \rrbracket_{c_i} \subseteq \llbracket D \rrbracket_{c_j}$ , where  $\llbracket D \rrbracket_c$  denotes configuring the VDB instance  $D$  for configuration  $c$ .

## Chapter 4: The Variational Query Language

[vql]

### 4.1 Variational Relational Algebra

[vra]

#### 4.1.1 VRA Configuration

[vra configuration]

#### 4.1.2 VRA Semantics

[vra semantics]

#### 4.1.3 VRA Type System

[type sys]

#### 4.1.4 VRA Variation-Minimization Rules

[rules]

## 4.2 Variational Query Language Properties

[prop. show for vra.]



## Chapter 5: Variational Database Use Cases

Thus far we introduced the variational database framework and variational queries. However, the question becomes: “How useable is the variational database framework?”, “How hard is it to generate a VDB and write variational queries?”, “Can a VDB be generated automatically? And if so, what is required to make this process automated?”. In this chapter, we aim at answering these questions. Thus, the goal of this chapter is twofold: first, how generating a VDB can be made automatic; second, to guid an expert through both generating a VDB from a variation scenario when it cannot be done automatically and writing variational queries that express expert’s information need over multiple database variants in the variation scenario.

A VDB can be generated automatically when the main variational information and database variants are available, that is, when the closed set of features, the feature model, and closed set of database variants are provided. Unfortunately, in practice, these information and encodings are not available since existing work only focuses on studying a specific kind of variation in databases and does not encode variation inside the database, instead, it addresses the problem with tools that simulate the effect of the specific kind of variation. Consequently, we describe how we systematically generated two variational databases from real world scenarios where variation appears in databases. We take a scenario where variation over

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to here.

either time or space exists in the database, use the schema variants to generate the variational schema, and attach feature expressions to tables and tuples to populate the VDB with data for each use case.

Additionally, variation in software affects not only databases but also how developers and database administrators interact with databases. Since different software variants have different information needs, developers must often write and maintain different queries for different software variants. Moreover, even if a particular information need is similar across variants, different variants of a query may need to be created and maintained to account for structural differences in the schema for each variant. Creating and maintaining different queries for each variant is tedious and error-prone, and potentially even intractable for large and open-ended configuration spaces, such as most open-source projects [48].

Thus, for each use case we present a set of variational queries and we illustrate how VRA realizes the information needs of the different variants of the database and potentially the corresponding software systems. It achieves this level of expressiveness by accounting for variation explicitly and linking variation in software and databases to queries by using the same feature names and configuration space. We present only a sample of the queries, yet we provide the full query sets online.<sup>1</sup> The full query sets capture all of the information needs described in the papers that we base our variation scenarios on. It is important to note that this makes our query sets potentially biased toward queries containing more variation points since

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<sup>1</sup>Complete sets of queries in both formats are available at: <https://zenodo.org/record/4321921>.

the focus of the papers is on variational parts of the system. A complete query set, capturing *all* information needs for each scenario might contain more plain queries, that is, queries that perform the same way over all variants. However, we do not believe this bias is harmful for the role the case studies are intended to serve, namely, motivating and evaluating variational database systems. For this role, queries that contain variation are more useful than plain queries, and additional plain queries can likely be more easily generated if needed.

We distribute the v-queries in two formats: (1) VRA, encoded in the format used by our VDBMS tool, and (2) plain SQL queries with embedded `#ifdef` annotations to capture variation points. The SQL format provides queries for studying variational data independently of VDBMS tool and will be more immediately useful for other researchers studying variational data independently of our VDBMS tool, but we use VRA in this thesis for its brevity because it is much more concise.

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here

We first focus on variation in databases over space, Section 5.1. Section 5.1.1 describes the variation scenario from Hall [28] that is the basis of this use case including the feature set and feature model. Then, Section 5.1.2 and Section 5.1.3 describe generating the variational schema for the described variation scenario and populating the email SPL VDB with Enron email data,<sup>2</sup> respectively. Finally, Section 5.1.4 describes how variational queries capture the information need adapted from feature interactions described by Hall [28]. We then switch focus to variation in databases over time, Section 5.2. Again, Section 5.2.1 describes the evolution

<sup>2</sup><http://www.ahschulz.de/enron-email-data/>

of an employee database as the variation scenario from Moon et al. [41] that is the basis of this use case. Then, Section 5.2.2 and Section 5.2.3 describe generating the variational schema for the described variation scenario and populating the employee VDB with a well-known employee data set,<sup>3</sup> respectively. Finally, Section 5.2.4 describes the adapted and adjusted queries from Moon et al. [41]. At the end of this chapter, Section 5.3, we discuss the trade offs of using variational databases and we answer the question: “Should variation be encoded explicitly in databases?”.

to here.

We distribute the VDBs, SQL scripts for generating them, and queries of our use cases.<sup>4</sup> We distribute the VDBs in both MySQL and Postgres in two forms, one intended for use with our VDBMS tool, and one intended for more general-purpose research on variation in databases. We distribute the variational queries as simple `#ifdef`-annotated SQL files to promote their broad reuse in the design and evaluation of other systems for managing variational relational data.

## 5.1 Variation in Space: Email SPL Use Case

In our first case study, we focus on variation that occurs in “space”, that is, where multiple software variants are developed and maintained in parallel. In software, variation in space corresponds to a SPL, where many distinct variants (products) can be produced from a single shared code base by enabling or disabling features. A variety of representations and tools have been developed for indicating which

<sup>3</sup>[https://github.com/datacharmer/test\\_db](https://github.com/datacharmer/test_db)

<sup>4</sup>Available at: <https://zenodo.org/record/4321921>

code belongs to which feature(s) and supporting the process of configuring a SPL to obtain a particular variant.

Naturally, different variants of a SPL have different information needs. For example, an optional feature in the SPL may require a corresponding attribute or relation in the database that is not needed by the other features in the SPL. Currently, there is no good solution to managing the varying information needs of different variants at the level of the database. One possible solution is to manually maintain a separate database schema for each variant of the SPL. This works for some SPLs where the number of products is relatively small and the developer has control over the configuration process. However, it does not scale to open-source SPLs or other scenarios where the number of products is large and/or configuration is out of the developer's hands. Another possible solution is to use and maintain a single universal schema that includes all of the relations and attributes used by any feature in the SPL. In this solution, every product will use the same database schema regardless of the features that are enabled. This solves the problem of scaling to large numbers of products but is dangerous because it means that potentially several attributes and relations will be unused in any given product. Unused attributes will typically be populated by `NULL` values, which are a well-known source of errors in relational databases [1].

VDBs solve the problem by allowing the structure of a relational database to vary in a synchronous way with the SPL. Attributes and relations may be annotated by presence conditions to indicate in which feature(s) those attributes and relations are needed. An implementation of the VDB model might use a

universal schema under-the-hood to realize VDBs on top of a standard relational database management system (indeed, this is exactly how our prototype VDBMS implementation works), but by capturing the variation in the schema explicitly, we can validate (potentially variational) queries against the relevant variants of the variational schema to statically ensure that no `NULL` values will be referenced.

The email SPL use case shows the use of VDB to encode the variational information needs of a database-backed SPL. We consider an email SPL that has been used in several previous SPL research projects (e.g. [5, 3]). It develops a variational schema that captures the information needs of a SPL based on Hall’s decomposition of an email system into its component features [28]. The email SPL has been used in several previous SPL research projects (e.g. [4, 3]). The variational email database is populated using the Enron email dataset, adapted to fit our variational schema [45]. Our use case is formed by systematically combining two pre-existing works:

1. We use Hall’s decomposition of an email system into its component features [28] as high-level specification of a SPL.
2. We use the Enron email dataset<sup>5</sup> as a source of a realistic email database.

In combining these works, we show how variation in space in an email SPL requires corresponding variation in a supporting database, how we can link the variation in the software to variation in the database, and how all of these variants can be encoded in a single VDB.

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<sup>5</sup><http://www.ahschulz.de/enron-email-data/>

### 5.1.1 Variation Scenario: An Email SPL

The email SPL consists of the following features from Hall [28]:

- *addressbook*, users can maintain lists of known email addresses with corresponding aliases, which may be used in place of recipient addresses;
- *signature*, messages may be digitally signed and verified using cryptographic keys;
- *encryption*, messages may be encrypted before sending and decrypted upon receipt using cryptographic keys;
- *autoresponder*, users can enable automatically generated email responses to incoming messages;
- *forwardmessages*, users can forward all incoming messages automatically to another address;
- *remailmessage*, users may send messages anonymously;
- *filtermessages*, incoming messages can be filtered according to a provided white list of known sender address suffixes; and
- *mailhost*, a list of known users is maintained and known users may retrieve messages on demand while messages sent to unknown users are rejected.

Note that Hall's decomposition separates *signature* and *encryption* into two features each (corresponding to signing and verifying, encrypting and decrypting).

Table 5.1: Original Enron email dataset schema.

*employeelist*(*eid*, *firstname*, *lastname*, *email\_id*, *email2*, *email3*, *email4*, *folder*, *status*)  
*messages*(*mid*, *sender*, *date*, *message\_id*, *subject*, *body*, *folder*)  
*recipientinfo*(*rid*, *mid*, *rtype*, *rvalue*)  
*referenceinfo*(*rid*, *mid*, *reference*)

Since these pairs of features must always be enabled together and they are so closely conceptually related, we reduce them to one feature each for simplicity.

The listed features are used in presence conditions within the variational schema for the email VDB, linking the software variation to variation in the database. In the email SPL, each feature is optional and independent, resulting in the simple feature model  $m_{en} = \mathbf{true}$ , given as a feature expression. The feature model  $m_{en}$  is used as the root presence condition of the variational schema for the email VDB, implicitly applying it to all relations, attributes, and tuples in the database.

### 5.1.2 Generating Variational Schema of the Email SPL VDB

To produce a variational schema for the email VDB, we start from plain schema of the Enron email dataset shown in Table 5.1, then systematically adjust its schema to align with the information needs of the email SPL described by Hall [28]. The *employeelist* table contains information about the employees of the company including the employee identification number (*eid*), their first name and last name (*firstname* and *lastname*), their primary email address (*email\_id*), alternative email addresses (e.g. *email2*), a path to the folder that contains their data (*folder*), and their last status in the company (*status*). The *messages* table contains



Table 5.2: Variational schema of the email VDB with feature model  $m_{en}$ . Presence conditions are colored blue for clarity.

```

employeeelist(eid, firstname, lastname, email_id, folder, status, verification_keysignature,
              public_keyencryption)
messages(mid, sender, date, message_id, subject, body, folder, is_system_notification,
         is_encryptedencryption, is_autoresponseautoresponder, is_signedsignature,
         is_forward_msgforwardmessages)
recipientinfo(rid, mid, rtype, rvalue)
forward_msg(eid, forwardaddr)forwardmessages
mailhost(eid, username, mailhost)mailhost
filter_msg(eid, suffix)filtermessages
reemail_msg(eid, pseudonym)reemailmessage
auto_msg(eid, subject, body)autoresponder
alias(eid, email, nickname)addressbook

```

information about the email messages including the message ID ( $mid$ ), the sender of the message ( $sender$ ), the date ( $date$ ), the internal message ID ( $message\_id$ ), the subject and body of the message ( $subject$  and  $body$ ), and the exact folder of the email ( $folder$ ). The *recipientinfo* table contains information about the recipient of a message including the recipient ID ( $rid$ ), the message ID ( $mid$ ), the type of the message ( $rtype$ ), and the email address of the recipient ( $rvalue$ ). The *referenceinfo* table contains messages that have been referenced in other email messages, for example, in a forwarded message; it contains a reference-info ID ( $rid$ ), the message ID ( $mid$ ), and the entire message (*reference*). This table simply backs up the emails.

From this starting point, we introduce new attributes and relations that are needed to implement the features in the email SPL. We attach presence conditions to new attributes and relations corresponding to the features they are needed to

support, which ensure they will *not* be present in configurations that do not include the relevant features. The resulting variational schema is given in Table 5.2.

For example, consider the *signature* feature. In the software, implementing this feature requires new operations for signing an email before sending it out and for verifying the signature of a received email. These new operations suggest new information needs: we need a way to indicate that a message has been signed, and we need access to each user’s public key to verify those signatures (private keys used to sign a message would not be stored in the database). These needs are reflected in the variational schema by the new attributes *verification\_key* and *is\_signed*, added to the relations *employeeelist* and *messages*, respectively. The new attributes are annotated by the *signature* presence condition, indicating that they correspond to the *signature* feature and are unused in configurations that exclude this feature. Additionally, several features require adding entirely new relations. For example, when the *forward\_msg* feature is enabled, the system must keep track of which users have forwarding enabled and the address to forward the messages to. This need is reflected by the new *forward\_msg* relation, which is correspondingly annotated by the *forward\_msg* presence condition.

A main focus of Hall’s decomposition [28] is on the many feature interactions. Several of the features may interact in undesirable ways if special precautions are not taken. For example, any combination of the *forward\_msg*, *reply\_msg*, and *autoresponder* features can trigger an infinite messaging loop if users configure the features in the wrong way; preventing this creates an information need to identify auto-generated emails, which is realized in the variational schema by attributes like

*is\_forward\_msg* and *is\_autoresponse*. As another example consider the interaction that occurs between the *signature* and *remail\_msg* features: the *remail\_msg* feature enables anonymously sending messages by replacing the sender with a pseudonym, but this prevents the recipient from being able to verify a signed email. Furthermore, consider the interaction that occurs between the *signature* and *forward\_msg* features: if Sarah signs a message and sends it to Ina, and Ina forwards the message to Philippe, then the signature verification operation may incorrectly interpret Ina as the sender rather than Sarah and fail to verify the message.

For each feature, we (1) enumerated the operations that must be supported both to implement the feature itself and to resolve undesirable feature interactions, (2) identified the information needs to implement these operations, and (3) extended the variational schema to satisfy these information needs. We make similar changes made to accommodate all features and their interactions.

For brevity, we omit some attributes and relations from the original schema that are irrelevant to the email SPL described by Hall [28], such as the *referenceinfo* relation and alternative email addresses.

We distribute the variational schema for the email VDB in two formats. First, we provide the schema in the encoding used by our prototype VDBMS tool. Second, we provide the variational schema in plain SQL. The SQL encoding is given by a “universal” schema containing the relations and attributes of all variants, plus a relation *vdb\_pcs* (*element\_id*, *pres\_cond*) that captures all of the relevant presence conditions: that of the variational schema itself (i.e. the feature model), and

those of each relation and attribute.<sup>6</sup> The *element\_id* of the feature model is *variational\_schema*; the *element\_id* of a relation *r* is its name *r*, and of attribute *a* in relation *r* is *r.a*. The plain SQL encoding of the variational schema supports the use of the use cases for research on the effective management of variation in databases independent of VDBMS.

### 5.1.3 Populating the Email SPL VDB

The final step to create the email VDB is to populate the database with data from the Enron email dataset, adapted to fit our variational schema [45]. For evaluation purposes, we want the data from the dataset to be distributed across multiple variants of the VDB. To simulate this, we identified five plausible configurations of the email SPL, which we divide the data among. The five configurations of the email SPL we considered are:

- *basic email*, which includes only basic email functionality and does not include any of the optional features from the SPL.
- *enhanced email*, which extends *basic email* by enabling two of the most commonly used email features, *forwardmessages* and *filtermessages*.
- *privacy-focused email*, which extends *basic email* with features that focus on privacy, specifically, the *signature*, *encryption*, and *remailmessage* features.
- *business email*, which extends *basic email* with features tailored to an en-

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<sup>6</sup>All encodings are available at: <https://zenodo.org/record/4321921>.

vironment where most emails are expected to be among users within the same business network, specifically, *addressbook*, *signature*, *encryption*, *autoresponder*, and *mailhost*.

- *premium email*, in which all of the optional features in the SPL are enabled.

For all variants, any features that are not enabled are disabled.

The original Enron dataset has 150 employees with 252,759 email messages. We load this data into the *employeelist* and *messages* tables defined in Section 5.1.2, initializing all attributes that are not present in the original dataset to `NULL`.

For the *employeelist* table, we construct five views corresponding to the five variants of the email system described above. We allocate 30 employees to each view based on their employee ID, that is, the first 30 employees sorted by employee ID are associated with the basic email variant, the next 30 with the enhanced email variant, and so on. The presence condition for each tuple is set to the conjunction of features enabled in that view. We then modify each of the views of the *employeelist* table by adding randomly generated values for attributes associated with the enabled features; e.g., in the view for the privacy-focused variant, we populate the *verification\_key* and *public\_key* attributes. Any attribute that is not present in the given tuple due to a conflicting presence condition will remain `NULL`. For example, both the *verification\_key* and *public\_key* attributes remain `NULL` for employees in the enhanced variant view since the presence condition does not include the corresponding features.

For the *messages* table, we again create five views corresponding to each of

the variants. Each tuple is added to the view of the variant that contains the message’s sender, which updates the tuple’s presence condition accordingly. The *messages* table also contains several additional attributes corresponding to optional features, which we populate in a systematic way. We set *is\_signed* to **true** if the message sender has the *signature* feature enabled, and we set *is\_encrypted* to **true** if *both* the message sender and recipient have *encryption* enabled. We populate the *is\_forward\_msg*, *is\_autoresponse*, and *is\_system\_notification* attributes by doing a lightweight analysis of message subjects to determine whether the email is any of these special kinds of messages; for example, if the subject begins with “FWD”, we set the *is\_forward\_msg* attribute to **true**. If a forward or auto-reply message was sent by a user that does not have the corresponding feature enabled, we filter it out of the dataset. After filtering, the *messages* relation contains 99,727 messages. For each forward or auto-reply message, we also add a tuple with the relevant information to the new *forward\_msg* and *auto\_msg* tables. For employees belonging to database variants that enable *remailmessage*, *autoresponder*, *addressbook*, or *mailhost* we randomly generate tuples in the tables that are specific to each of these features. Finally, the *recipientinfo* relation is imported directly from the dataset. We set each tuple’s presence condition to a conjunction of the presence conditions of the sender and recipient.

We provide SQL scripts to automate the creation of views for each variant and to automate the population of these views with tuples from the original dataset, which also sets each tuple’s presence condition. The resulting database is distributed in two forms, one with the embedded variational schema as described in Sec-

tion 5.1.2, and one without the embedded schema for use with our VDBMS tool in which the variational schema is provided separately.<sup>7</sup> We have tested the email SPL VDB for the properties described in Section 3.4 and all of them hold.

#### 5.1.4 Email Query Set

To produce a set of queries for the email SPL use case, we collected all of the information needs that we could identify in the description of the email SPL by Hall [28]. In order to make the information needs more concrete, we viewed the requirements of the email SPL mostly through the lens of constructing an email header. An email header includes all of the relevant information needed to send an email and is used by email systems and clients to ensure that an email is sent to the right place and interpreted correctly. More specifically, the email header includes the sender and receiver of the email, whether an email is signed and the location of a signature verification key, whether an email is encrypted and the location of the corresponding public key, the subject and body of the email, the mail host it belongs to, whether the email should be filtered, and so on. Although there is obviously other infrastructure involved, the fundamental information needs of an email system can be understood by considering how to construct email headers that ensures the email would get where it needs to go and be interpreted correctly on the other end.

Hall’s decomposition focuses on enumerating the features of the email SPL and

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<sup>7</sup>Both the scripts and different encodings of the email SPL VDB are available at: <https://zenodo.org/record/4321921>.

enumerating the potential interactions of those features. We deduce the information need for each feature by asking: “what information is needed to modify the email header in a way that incorporates the new functionality?”. We deduce the information need for each interaction by asking: “what information is needed to modify the email header in a way that avoids the undesirable feature interaction?”. We can then translate these information needs into queries on the underlying variational database.

In total, we provide 27 queries for the email SPL. This consists of 1 query for constructing the basic email header, 8 queries for realizing the information needs corresponding to each feature, and 18 queries for realizing the information needs to correctly handle the feature interactions described by Hall.

We start by presenting the query to assemble the basic email header,  $Q_{basic}$ . This corresponds to the information need of a system with no features enabled. We use  $X$  to stand for the specific message ID ( $mid$ ) of the email whose header we want to construct.

$$Q_{basic} = \pi_{sender, rvalue, subject, body} mes\_rec$$

$$mes\_rec \leftarrow (\sigma_{mid=X} messages) \bowtie recipientinfo$$

This query extracts the sender, recipient, subject, and body of the email to populate the header. The projection is applied to an intermediate result  $mes\_rec$  constructed by joining the  $messages$  table with the  $recipientinfo$  table on recipient IDs; we reuse this intermediate result also in subsequent queries.



Taking  $Q_{basic}$  as our starting point, we next construct our set of 8 *single-feature queries* that capture the information needs specific to each feature. When a feature is enabled in the SPL, more information is needed to construct the header of email  $X$ . For example, if the feature *filtermessages* is enabled, then the query  $Q_{filter}$  extends  $Q_{basic}$  with the *suffix* attribute used in filtering. This additional information allows the system to filter a message if its address contains any of the suffixes set by the receiver.

$$\begin{aligned}
 Q_{filter} &= \pi_{sender, rvalue, suffix, subject, body} temp \\
 temp &\leftarrow mes\_rec\_emp \bowtie filter\_msg \\
 mes\_rec\_emp &\leftarrow mes\_rec \bowtie_{rvalue=email\_id} employeelist
 \end{aligned}$$

We can construct a query that retrieves the required header information whether *filtermessages* is enabled or not by combining  $Q_{basic}$  and  $Q_{filter}$  in a choice, as  $Q_{bf} = filtermessages \langle Q_{filter}, Q_{basic} \rangle$ . Although we do not show the process in this thesis, we can use equivalence laws from the choice calculus [21, 32] to factor commonalities out of choices and reduce redundancy in queries like  $Q_{bf}$ . The other single-feature queries are written similarly.

As another example of a single-feature query,  $Q_{forward}$  captures the information needs for implementing the *forwardmessages* feature. It is similar to the previous queries except that it extracts the *forwardaddr* from the *auto\_msg* table, which is needed to construct the message header for the new email to be forwarded when

email  $X$  is received by a user with a *forwardaddr* set.

$$Q_{forward} = \pi_{rvalue, forwardaddr, subject, body} temp$$

$$temp \leftarrow mes\_rec\_emp \bowtie_{employeeid.eid=forward\_msg.eid} auto\_msg$$

The other single-feature queries are similar to those shown here.

Besides single-feature queries, we also provide queries that gather information needed to identify and address the undesirable feature interactions described by Hall [28]. Out of Hall’s 27 feature interactions, we determined 16 of them to have corresponding information needs related to the database; 2 of the interactions require 2 separate queries to resolve. Therefore, we define and provide 18 queries addressing all 16 of the relevant feature interactions. As before, we deduced the information needs through the lens of constructing an email header; in these cases, the header would correspond to an email produced after successfully resolving the interaction. However, some interactions can only be detected but not automatically resolved. In these cases, we constructed a query that would retrieve the relevant information to detect and report the issue.

One undesirable feature interaction occurs between the two features *signature* and *forwardmessages*: if Philippe signs a message and sends it to Sarah, and Sarah forwards the message to an alternate address Sarah-2, then signature verification may incorrectly interpret Sarah as the sender rather than Philippe and fail to verify the message (Hall’s interaction #4). A solution to this interaction is to embed the original sender’s verification information into the email header of the forwarded

message so that it can be used to verify the message, rather than relying solely on the message's "from" field.

Below, we show a variational query  $Q_{sf}$  that includes four variants corresponding to whether *signature* and *forwardmessages* are enabled or not independently. The information need for resolving the interaction is satisfied by the first alternative of the outermost choice with condition  $signature \wedge forwardmessages$ . The alternatives of the choices nested to the right satisfy the information needs for when only *signature* is enabled, only *forwardmessages* is enabled, or neither is enabled ( $Q_{basic}$ ). We don't show the single-feature  $Q_{sig}$  query, but it is similar to other single-feature queries shown above.

$$\begin{aligned}
Q_{sf} = & \text{signature} \wedge \text{forwardmessages} \\
& \langle \pi_{rvalue, forwardaddr, emp1.is\_signed, emp1.verification\_key} temp \\
& , \text{signature} \langle Q_{sig}, \text{forwardmessages} \langle Q_{forward}, Q_{basic} \rangle \rangle \rangle \\
temp \leftarrow & (((\sigma_{mid=X} \text{messages}) \bowtie \text{recipientinfo}) \\
& \bowtie_{sender=emp1.email\_id} (\rho_{emp1} \text{employeelist})) \\
& \bowtie_{rvalue=emp2.email\_id} (\rho_{emp2} \text{employeelist})) \bowtie \text{forward\_msg}
\end{aligned}$$

The query  $Q_{sf}$  also resolves another consequence of the interaction between these two features. This time Sam successfully verifies message  $X$  and forwards it to Sam2 which changes the header in the system such that it states message  $X$  has been successfully verified, thus, the message could be altered by hackers while it is being forwarded (Hall's interaction #27). The system can use  $Q_{sf}$  to generate

the correct header in this scenario again.

Some feature interactions require more than one query to satisfy their information need. For example, assume both *encryption* and *forwardmessages* are enabled. Philippe sends an encrypted email  $X$  to Sarah; upon receiving it the message is decrypted and forwarded it to Sarah-2 (Hall's interaction #9). This violates the intention of encrypting the message and the system should warn the user. Queries  $Q_{ef}$  and  $Q'_{ef}$  satisfy the information need for this interaction when a message is encrypted or unencrypted, respectively.

$$\begin{aligned}
Q_{ef} &= \text{encryption} \wedge \text{forwardmessages} \\
&\quad \langle \pi_{rvalue}(\sigma_{mid=X \wedge is\_encrypted} \text{messages}) \\
&\quad , \text{encryption} \langle Q_{encrypt}, \text{forwardmessages} \langle Q_{forward}, Q_{basic} \rangle \rangle \rangle \\
Q'_{ef} &= \text{encryption} \wedge \text{forwardmessages} \\
&\quad \langle \text{temp}, \text{encryption} \langle Q_{encrypt}, \text{forwardmessages} \langle Q_{forward}, Q_{basic} \rangle \rangle \rangle \\
\text{temp} &\leftarrow \pi_{rvalue, forwardaddr, subject, body}(\sigma_{mid=X \wedge \neg is\_encrypted} \\
&\quad (\text{mes\_rec\_emp} \bowtie_{employee\text{list}.eid=forward\_msg.eid} \text{forward\_msg}))
\end{aligned}$$

However, managing feature interactions is not necessarily complicated. Some interactions simply require projecting more attributes from the corresponding single-feature queries. For example, assume both *filtermessages* and *mailhost* features are enabled. Philippe sends a message to a non-existent user in a mailhost that he has filtered. The mailhost generates a non-delivery notification and sends it to

Philippe, but he never receives it since it is filtered out (Hall’s interaction #26). The system can check the *is\_system\_notification* attribute for the  $Q_{filter}$  query and decide whether to filter a message or not. Therefore, we can resolve this interaction by extending the single-feature query for *filtermessages* to  $Q'_{filter}$ .

$$Q'_{filter} = \pi_{sender, rvalue, suffix, is\_system\_notification, subject, body} temp$$

$$temp \leftarrow mes\_rec\_emp \bowtie_{employeeelist.eid=filter\_msg.eid} filter\_msg$$

Overall, for the 18 interaction queries we provide, 12 have 4 variants, 3 have 3 variants, 2 have 2 variants, and 1 has 1 variant.

## 5.2 Variation in Time: Employee Use Case

In our second case study, we focus on variation that occurs in “time”, that is, where the software variants are produced sequentially by incrementally extending and modifying the previous variant in order to accommodate new features or changing business requirements. Although new variants conceptually replace older variants, in practice, older variants must often be maintained in parallel; external dependencies, requirements, and other issues may prevent clients from updating to the latest version. Variation in software over time directly affects the databases such software depends on [48], and dealing with such changes is a well-studied problem in the database community known as *database evolution* [43].

Although research on database evolution has produced a variety of solutions for

managing database variation over time, these solutions do not treat variation as an orthogonal property and so cannot also accommodate variation in space. The goal of our work on variational databases is not to directly compete with database evolution solutions for time-only variation scenarios, but rather to present a more general model of database variation that can accommodate variation in both time and space, and that integrates with related software via feature annotations.

We demonstrate variation in time by using a VDB to encode an employee database evolution scenario systematically adapted from Moon et al. [41] and populated by a dataset that is widely used in databases research.<sup>8</sup>

### 5.2.1 Variation Scenario: An Evolving Employee Database

Moon et al. [41] describe an evolution scenario in which the schema of a company’s employee management system changes over time, yielding the five versions of the schema shown in Table 5.3. In  $V_1$ , employees are split into two separate relations for engineer and non-engineer personnel. In  $V_2$ , these two tables are merged into one relation, *empacct*. In  $V_3$ , departments are factored out of the *empacct* relation and into a new *dept* relation to reduce redundancy in the database. In  $V_4$ , the company decides to start collecting more personal information about their employees and stores all personal information in the new relation *empbio*. Finally, in  $V_5$ , the company decides to decouple salaries from job titles and instead base salaries on individual employee’s qualifications and performance; this leads to dropping the

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<sup>8</sup>[https://github.com/datacharmer/test\\_db](https://github.com/datacharmer/test_db)

Table 5.3: Evolution of an employee database schema from Moon et al. [41].

Version	Schema
$V_1$	<i>engineerpersonnel</i> ( <i>empno</i> , <i>name</i> , <i>hiredate</i> , <i>title</i> , <i>deptname</i> ) <i>otherpersonnel</i> ( <i>empno</i> , <i>name</i> , <i>hiredate</i> , <i>title</i> , <i>deptname</i> ) <i>job</i> ( <i>title</i> , <i>salary</i> )
$V_2$	<i>empacct</i> ( <i>empno</i> , <i>name</i> , <i>hiredate</i> , <i>title</i> , <i>deptname</i> ) <i>job</i> ( <i>title</i> , <i>salary</i> )
$V_3$	<i>empacct</i> ( <i>empno</i> , <i>name</i> , <i>hiredate</i> , <i>title</i> , <i>deptno</i> ) <i>job</i> ( <i>title</i> , <i>salary</i> ) <i>dept</i> ( <i>deptname</i> , <i>deptno</i> , <i>managerno</i> )
$V_4$	<i>empacct</i> ( <i>empno</i> , <i>hiredate</i> , <i>title</i> , <i>deptno</i> ) <i>job</i> ( <i>title</i> , <i>salary</i> ) <i>dept</i> ( <i>deptname</i> , <i>deptno</i> , <i>managerno</i> ) <i>empbio</i> ( <i>empno</i> , <i>sex</i> , <i>birthdate</i> , <i>name</i> )
$V_5$	<i>empacct</i> ( <i>empno</i> , <i>hiredate</i> , <i>title</i> , <i>deptno</i> , <i>salary</i> ) <i>dept</i> ( <i>deptname</i> , <i>deptno</i> , <i>managerno</i> ) <i>empbio</i> ( <i>empno</i> , <i>sex</i> , <i>birthdate</i> , <i>firstname</i> , <i>lastname</i> )

*job* relation and adding a new *salary* attribute to the *empacct* relation. This version also separates the *name* attribute in *empbio* into *firstname* and *lastname* attributes.

We associate a feature with each version of the schema, named  $V_1 \dots V_5$ . These features are mutually exclusive since only one version of the schema is valid at a time. This yields the feature model  $m_{emp}$ . Also, note that the feature model represent a restriction on the entire database.

$$\begin{aligned}
m_{emp} = & (V_1 \wedge \neg V_2 \wedge \neg V_3 \wedge \neg V_4 \wedge \neg V_5) \\
& \vee (\neg V_1 \wedge V_2 \wedge \neg V_3 \wedge \neg V_4 \wedge \neg V_5) \vee (\neg V_1 \wedge \neg V_2 \wedge V_3 \wedge \neg V_4 \wedge \neg V_5) \\
& \vee (\neg V_1 \wedge \neg V_2 \wedge \neg V_3 \wedge V_4 \wedge \neg V_5) \vee (\neg V_1 \wedge \neg V_2 \wedge \neg V_3 \wedge \neg V_4 \wedge V_5)
\end{aligned}$$

Table 5.4: Employee v-schema with feature model.  $m_{emp}$ .

$engineerpersonnel(empno, name, hiredate, title, deptname)^{V_1}$
$otherpersonnel(empno, name, hiredate, title, deptname)^{V_1}$
$empacct(empno, name^{V_2 \vee V_3}, hiredate, title,$ $deptname^{V_2}, deptno^{V_3 \vee V_4 \vee V_5}, salary^{V_5})^{V_2 \vee V_3 \vee V_4 \vee V_5}$
$job(title, salary)^{V_2 \vee V_3 \vee V_4}$
$dept(deptname, deptno, managerno)^{V_3 \vee V_4 \vee V_5}$
$empbio(empno, sex, birthdate, name^{V_4}, firstname^{V_5}, lastname^{V_5})^{V_4 \vee V_5}$

### 5.2.2 Generating Variational Schema of the Employee VDB

The variational schema for this scenario is given in Table 5.4. It encodes all five of the schema versions in Table 5.3 and was systematically generated by the following process. First, generate a universal schema from all of the plain schema versions; the universal schema contains every relation and attribute appearing in any of the five versions. Then, annotate the attributes and relations in the universal schema according to the versions they are present in. For example, the *empacct* relation is present in versions  $V_2$ – $V_5$ , so it will be annotated by the feature expression  $V_2 \vee V_3 \vee V_4 \vee V_5$ , while the *salary* attribute within the *empacct* relation is present only in version  $V_5$ , so it will be annotated by simply  $V_5$ . The overall variational schema will be annotated by the feature model  $m_{emp}$ , described in Section 5.2.1. Since the presence conditions of attributes are implicitly conjuncted with the presence condition of their relation that contains them, we can avoid redundant annotations when an attribute is present in all instances of its parent relation. For example, the *empbio* relation is present in  $V_4 \vee V_5$ , and the *birthdate*



attribute is present in the same versions, so we do not need to redundantly annotate *birthdate*.

Similar to the email SPL VDB, we distribute the variational schema for the employee VDB in two formats: First, we provide the schema in the encoding used by our prototype VDBMS tool. Second, we provide a direct encoding in SQL that generates the universal schema for the VDB in either MySQL or Postgres. The variability of the schema is embedded within the employee VDB using the same encoding as described at the end of Section 5.1.2.<sup>9</sup>

### 5.2.3 Populating the Employee VDB

Finally, we populate the employee VDB using data from the widely used employee database linked to in this subsection’s lede. This database contains information for 240,124 employees. To simulate the evolution of the database over time, we divide the employees into five roughly equal groups based on their hire date within the company. For example, the first group consists of employees hired before 1988-01-01, while the second group contains employees hired from 1988-01-01 to 1991-01-01. Each group is assumed to have been hired during the lifetime of a particular version of the database, and is therefore added to that version of the database and *also* to all subsequent versions of the database. This simulates the fact that as a database evolves, older records are typically forward propagated to the new schema [43]. Thus,  $V_5$  contains the records for all 240,124 employees,

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<sup>9</sup>All encodings of the employee variational schema are available at: <https://zenodo.org/record/4321921>.

while older versions will contain progressively fewer records. The final employee VDB has 954,762 employee due to this forward propagation, despite having the same number of employees as the original database.

The schema of the employee database used to populate the employee VDB is different from all versions of the variational schema, yet it includes all required information. Thus, we manually mapped data from the original schema onto each version of the variational schema.

We provide SQL scripts of required queries to automatically generate the employee VDB. We also provide SQL scripts to automate the separation of each group of employees into views according to their hire date and populating those views from data in the employee database.<sup>10</sup>

As for any VDB, if an attribute is not present in any of the variants covered by a tuple’s presence condition, that attribute will be set to NULL in the tuple. We do this even though the relevant information may be contained in the original employee database to ensure that we have a consistent VDB. For example, while inserting tuples into the  $V_4$  view of the *empbio* table, we always insert NULL values attributes *firstname* and *lastname*. We also provide the final employee VDB in four flavors: both with and without the embedded schema, and in both cases, encoded in MySQL and PostgreSQL format.<sup>11</sup> We have tested the employee VDB for the properties described in Section 3.4 and all of them hold.

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<sup>10</sup>All the scripts are available at: <https://zenodo.org/record/4321921>.

<sup>11</sup>Both formats are available at: <https://zenodo.org/record/4321921>.

### 5.2.4 Employee Query Set

For this use case, we have a set of existing plain queries to start from. Moon et al. [41] provides 12 queries to evaluate the Prima schema evolution system. We adapt these queries to fit our encoding of the employee VDB described in Section 5.2. We provide the queries in both the VRA format usable by VDBMS and as `#ifdef`-annotated SQL.<sup>12</sup> We give an example of query written as `#ifdef`-annotated at the end of this section. 9 of these queries have one variant, 2 have two variants, and 1 has three variants.

Moon’s queries are of two types: 6 retrieve data valid on a particular date (corresponding to  $V_3$  in our encoding), while 6 retrieve data valid on or after that date ( $V_3$ – $V_5$  in our encoding). For example, one query expresses the intent “return the salary of employee number 10004” at a time corresponding to  $V_3$ , which we encode:

$$empQ_1 = \pi_{salary^{V_3}} (\sigma_{empno=10004} empacct) \bowtie_{empacct.title=job.title} job.$$

Note that the presence condition of the only attribute *salary* determines the presence condition of the resulting table.

In general and for simplicity, the shared part of presence conditions of projected attributes is factored out and applied to the entire table. Assume the returned table as a result of query has the schema  $(a_1^{e \wedge e_1}, a_2^{e \wedge e_2})$ . The shared restriction can be factored out and applied to the entire table, i.e.,  $(a_1^{e_1}, a_2^{e_2})^e$ .

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<sup>12</sup>All queries are available at: <https://zenodo.org/record/4321921>.

We encode the same intent, but for all times at or after  $V_3$  as follows:

$$\begin{aligned} empQ_2 = V_3 \vee V_4 \vee V_5 \langle \\ \pi_{salary}(V_3 \vee V_4 \langle ((\sigma_{empno=10004} empacct)) \bowtie job, \sigma_{empno=10004} empacct \rangle) \\ , \varepsilon \rangle \end{aligned}$$

There are a variety of ways we could encode both  $empQ_1$  and  $empQ_2$ . For  $empQ_1$  we could equivalently have embedded the projection in a choice,  $V_3 \langle \pi_{salary}(\dots), \varepsilon \rangle$ , however attaching the presence condition to the only projected attribute determines the presence condition of the resulting table and so achieves the same effect. In  $empQ_2$  we use choices to structure the query since we have to project on a different intermediate result for  $V_5$  than for  $V_3$  and  $V_4$ .

The feature expression  $V_3 \vee V_4 \vee V_5$  determines the database variants to be inquired. Since the schema of *empacct* and *job* tables are the same in variants  $V_3$  and  $V_4$  they both have the same query. Note that one could move the condition  $V_3 \vee V_4 \vee V_5$  to the projected attribute which results in  $empQ'_2$ , however, this query is wrong because the last alternative of the choice projects attribute *salary* from an empty relation which is incorrect. It is important to understand that the behavior of an empty relation is exactly the same as its behavior in relational algebra and one should be careful of using it in operations such as projection, selection, and join.

$$\begin{aligned}
empQ'_2 &= \pi_{salary}^{V_3 \vee V_4 \vee V_5} \\
&\quad (V_3 \vee V_4 \langle (\sigma_{empno=10004} empacct) \bowtie_{empacct.title=job.title} job \\
&\quad \quad , V_5 \langle \sigma_{empno=10004} empacct, \varepsilon \rangle \rangle)
\end{aligned}$$

As another example, the following query realizes the intent to “return the name of the manager of department d001” during the time frame of  $V_3$ – $V_5$ :

$$\begin{aligned}
empQ_3 &= V_3 \vee V_4 \vee V_5 \langle \\
&\quad \pi_{name,firstname,lastname} (V_3 \langle empacct, empbio \rangle \bowtie_{empno=managerno} \\
&\quad \quad (\sigma_{deptno="d001"} dept)) \\
&\quad , \varepsilon \rangle
\end{aligned}$$

Note that even though the attributes *name*, *firstname*, and *lastname* are not present in all three of the variants corresponding to  $V_3$ – $V_5$ , the VRA encoding permits omitting presence conditions that can be completely determined by the presence conditions of the corresponding relations or attributes in the variational schema. So,  $empQ_3$  is equivalent to the following query in which the presence conditions of the attributes from the variational schema are listed explicitly in the

projection:

$$\begin{aligned} empQ'_3 = & V_3 \vee V_4 \vee V_5 \langle \pi_{name^{V_3 \vee V_4}, firstname^{V_5}, lastname^{V_5}} \\ & (V_3 \langle empacct, empbio \rangle \bowtie_{empno=managerno} (\sigma_{deptno="d001"} dept)), \varepsilon \rangle \end{aligned}$$

Allowing developers to encode variation in variational queries based on their preference makes VRA more flexible and easy to use. Also, variational queries are statically type-checked to ensure that the variation encoded in them does not conflict the variation encoded in the v-schema.

Finally, we want to briefly illustrate what queries look like in the `#ifdef`-annotated SQL format that we distribute as a potentially more portable and easy-to-use format for other researchers. Below is the query  $empQ_3$  in this format.

```
#ifdef V3 || V4 || v5
  #ifdef V3 || V4
SELECT name
  #else
SELECT firstname , lastname
  #endif
FROM
  #ifdef V3
    empacct
  #else
    empbio
```

```
#endif
JOIN (SELECT * FROM dept WHERE deptno='d001')
ON empno=manageno
```

### 5.3 Discussion: Should Variation Be Encoded Explicitly in Databases?

In this section we discuss the use cases and our encodings of VDB and variational queries in the context of the question posed in the title of this paper: *Should variation be encoded explicitly in databases?*

**Expressiveness of explicit variation.** The use cases in Chapter 5 show that by treating variation as an orthogonal concern and embedding it directly in databases and queries (via presence conditions and choices), one can encode data variation scenarios in both time and space. In fact, VDBs and variational queries are *maximally expressive* in the sense that any set of plain relational databases can be encoded as a single VDB and any set of plain queries over the variants of a VDB can be encoded as a variational query.<sup>13</sup>

The expressiveness of our approach is its main advantage over other ways to manage database variation. When working with a form of variation that already has its own specialized solution (e.g. schema evolution, data integration), the ex-

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<sup>13</sup>The expressiveness of VDBs and variational queries can be proved by construction. For VDBs, one can simply take the union of all relations, attributes, and tuples across all variants, then attach presence conditions corresponding to which variants each is present in. For variational queries, all variants can be organized under a tree of choices that similarly organizes the variants in the appropriate way.

pressiveness of explicit variation is probably not worth the additional complexity. The expressiveness of explicit variation is most useful when working with a form of variation that is not well supported (e.g. query-level variation in SPLs), or when combining multiple forms of variation in one database (e.g. during SPL evolution).

We expect that ill-supported forms of variation are common in industry and justify the expressiveness of explicit variation. For example, the following is a scenario we recently discussed with an industry contact: A software company develops software for different networking companies and analyzes data from its clients to advise them accordingly. The company records information from each of its clients' networks in databases customized to the particular hardware, operating systems, etc. that each client uses. The company analysts need to query information from all clients who agreed to share their information, but the same information need will be represented differently for each client. This problem is essentially a combination of the SPL variation problem (the company develops and maintains many databases that vary in structure and content) and the data integration problem (querying over many databases that vary in structure and content). However, neither the existing solutions from the SPL community nor database integration address both sides of the problem. Currently the company manually maintains variant schemas and queries, but this does not take advantage of sharing and is a major maintenance challenge. With a database encoding that supports explicit variation in schemas, content, and queries, the company could maintain a single variational database that can be configured for each client, import shared data into a VDB, and write variational queries over the VDB to analyze the data,



significantly reducing redundancy across clients.

**Complexity of explicit variation.** The generality of explicit variation comes at the cost of increased complexity. The complexity introduced by presence conditions and choices is similar to the complexity introduced by variation annotations in annotative approaches to SPL implementation [35]. There is widespread acknowledgment that unrestricted use of variation annotations, such as the C Pre-processor’s `#ifdef`-notation [27], makes software difficult to understand [37] and is error prone [26]. However, so-called *disciplined* use of variation annotations, where annotations are used in a way that is consistent with the object language syntax of variants, may suffer less from such issues [38]. In VDBs, and in the VRA notation for variational queries, annotations are disciplined since presence conditions and choices are integrated into the existing syntax of relational database schemas and relational algebra. Note that annotation discipline is not enforced in the `#ifdef`-annotated SQL notation that we use to distribute the variational queries associated with our use cases.

Subjectively, the development of our use cases suggests that the impact of variation annotations on understandability is moderate for variational schemas and VDBs, and significant for variational queries written in VRA, despite the fact that such annotations are disciplined. That is, we believe that presence conditions make clear the structural and content variation in our example VDBs without significantly impacting the understandability of the overall structure and content of the variant databases. However, the understandability of variational queries do

seem to be significantly impacted by the use of presence conditions and choices, despite the fact that their use is disciplined in the VRA notation.

It is possible that a more restrictive and/or coarse-grained form of variation in variational queries would make them easier to understand at the cost of increased redundancy and (potentially) reduced expressiveness. This tradeoff is one we already made when considering how to encode variation in the *content* of a VDB. Specifically, we do not support cell-level variation in a VDB (e.g. choices within individual cells). This does not reduce the expressiveness of content variation in VDBs since cell-level variation can be simulated by row variation, but it does increase redundancy since all non-varied cells in the row must be duplicated. Similarly, variation in queries could be restricted to expression-level choices, with no choices or annotations in conditions or attribute lists. This would likely make understanding individual query variants easier at the cost of increasing redundancy among the alternatives of each choice.

Alternatively, the understandability of variational queries could be improved through tooling, for example, using background colors [25], virtual separation of concerns [34], or view-based editing [55, 49]. Future work should validate our subjective assessment of the understandability VDBs and variational queries, and explore techniques for improving this concern.

**Analyzability of explicit variation.** The relationship of our work to alternative approaches can be viewed through the lens of annotative vs. compositional variation, familiar to the SPL community [35]. VDBs and variational queries rely

on generic annotations embedded directly in schemas and queries, respectively, while approaches from the databases community often express variation through separate artifacts, such as views [10]. Annotative vs. compositional representations often exhibit the same tradeoff between expressiveness and complexity described above: annotative variation tends to be general and expressive, while compositional variation tends to be more restrictive but support modular reasoning [35]. Traditionally, another advantage of compositional approaches is that they are more analyzable thanks to the ability to analyze components separately (i.e. *feature-based* analysis [52]), a benefit shared by database views. However, in the last decade there has been a significant amount of work in the SPL community to improve the analyzability of annotative variation by analyzing whole variational artifacts directly (i.e. *family-based* analysis [52]). Although not presented here, we build directly on this body of work, especially work on variational typing [15, 16], to enable efficiently checking variational queries against all variants of a VDB, among other properties. Thus, the increased complexity of explicit variation annotations does not prevent us from verifying its correctness.

## Chapter 6: Variational Database Management System (VDBMS)

[vdbms]

### 6.1 Implemented Approaches

[apps]

### 6.2 Experiments

[exp.]

## Chapter 7: Related Work

[related work! have to work on this!]

### 7.1 Instances of Variation in Databases

[schema evolution. database versioning. data integration. data provenance.]

### 7.2 Instances of Database Variation Resulted from Software Development

[SPL. data model. query.]

### 7.3 Variational Research

[blah]

from vamos

We proposed encoding variation explicitly in database schemas and queries in [7] and proposed applying this idea to database-backed SPLs in [8]. Our previous work uses slightly different encodings; the one presented here is the basis of our VDBMS implementation. This is the first work that provides use cases for VDBs.

The SPL community has a tradition of developing and distributing use cases to support research on software variation. For example, SPL2go [51] catalogs

the source code and variability models of a large number of SPLs. Additionally, specific projects, such as Apel et al.’s [5] work on SPL verification, often distribute use cases along with study results. However, there are no existing datasets or use cases that include corresponding relational databases and queries, despite their ubiquity in modern software.

Many researchers have recognized the need to manage structural variation in the databases that SPLs rely on. Abo Zaid and De Troyer [2] argue for modeling data variability as part of a model-oriented SPL process. Their *variable data models* link features to concepts in a data model so that specialized data models can be generated for different products. Khedri and Khosravi [36] address data model variability in the context of delta-oriented programming. They define delta modules that can incrementally generate a relational database schema, and so can be used to generate different schemas for each variant of a SPL. Humblet et al. [33] present a tool to manage variation in the schema of a relational database used by a SPL. Their tool enables linking features to elements of a schema, then generating different variants of the schema for different products. Schäler et al. [44] generate a variable database schema from a given global schema and software configurations by mapping schema element to features. Siegmund et al. [46] emphasize the need for variable database schema in SPLs and propose two decomposition approaches: (1) *physical* where database sub-schemas associated with a feature are stored in physical files and (2) *virtual* where a global entity-relation model of a schema is annotated with features. All of these approaches address the issue of *structural* database variation in SPLs and provide a way to derive a schema per variant,

which is also achievable by configuring a VDB. The work of Humblet et al. [33] is most similar to our notion of a variational schema since it is an annotative approach [35] that directly associates schema elements with features. Abo Zaid and De Troyer [2] is also annotative, but operates at the higher level of a data model that may only later be realized as a relational database. Khedri and Khosravi [36] is a compositional approach [35] to generating database schemas. None of these approaches consider *content-level* variation, which is captured by VDBs and observable in our use cases, nor do they consider how to express queries over databases with structural variation, which is addressed by our *v-queries*.

While the previous approaches all address data variation in space, Herrmann et al. [29] emphasize that as an SPL evolves over time, so does its database. Their approach adapts work on database evolution to SPLs, enabling the safe evolution of all deployed products.

Database researchers have studied several kinds of variation in both time and space. There is a substantial body of work on *schema evolution* and *database migration* [19, 41, 30, 42], which corresponds to variation in time. Typically the goal of such work is to safely migrate existing databases forward to new versions of the schema as it evolves. Work on *database versioning* [12, 31] extends this idea to a database’s content. In a versioned database, content changes can be sent between different instances of a database, similar to a distributed revision control system. All of this work is different from variational databases because it encodes a less general notion of variation and does not support querying multiple versions of the database at once. Work on *data integration* can be viewed as managing variation in

space [20]. In data integration, the goal is to combine data from disparate sources and provide a unified interface for querying. This is different from VDBs, which make differences between variants explicit.

The representation of v-schemas and variational tables is based on previous work on variational sets [23], which is part of a larger effort toward developing safe and efficient variational data structures [56, 40]. The central motivation of work on variational data structures is that many applications can benefit from maintaining and computing with variation at runtime [22, 17]. The ability to maintain and query several variants of a database at once extends the idea of computing with variation to relational databases.



## Chapter 8: Conclusion

[conclusion]

from vamos

We provide two use cases that illustrate how software variation leads to corresponding variation in relational databases. These use cases demonstrate the feasibility of VDBs and v-queries to capture the data needs of variational software systems. We argue that effectively managing such variation is an open problem, and we believe that these use cases will form a useful basis for evaluating research that addresses it, such as our own VDBMS framework.

VDBs encode variation explicitly in the structure and content of databases. This is a source of complexity that may impact understandability, as can be observed in our use cases. However, it also has several advantages: it is general in the sense that any set of variant databases and queries can be encoded as a VDB and v-queries, and it enables directly associating variation in databases to variation in software. By applying variational typing to variational queries, this generality does not come at the cost of safety. Future work can explore how tooling can mitigate the usability concerns using techniques that have been developed in the SPL community.

## Bibliography

- [1] Serge Abiteboul, Richard Hull, and Victor Vianu. *Foundations of Databases: The Logical Level*. Addison-Wesley, 1994.
- [2] Lamia Abo Zaid and Olga De Troyer. Towards modeling data variability in software product lines. In Terry Halpin, Selmin Nurcan, John Krogstie, Pnina Soffer, Erik Proper, Rainer Schmidt, and Ilia Bider, editors, *Enterprise, Business-Process and Information Systems Modeling*, pages 453–467, Berlin, Heidelberg, 2011. Springer. ISBN 978-3-642-21759-3.
- [3] Mustafa Al-Hajjaji, Thomas Thüm, Malte Lochau, Jens Meinicke, and Gunter Saake. Effective Product-Line Testing Using Similarity-Based Product Prioritization. *Software & Systems Modeling*, 18(1):499–521, 2019.
- [4] S. Apel, A. von Rhein, P. Wendler, A. Größlinger, and D. Beyer. Strategies for Product-Line Verification: Case Studies and Experiments. In *IEEE Int. Conf. on Software Engineering*, pages 482–491, 2013.
- [5] Sven Apel, Alexander von Rhein, Philipp Wendler, Armin Größlinger, and Dirk Beyer. Strategies for product-line verification: case studies and experiments. In *2013 35th International Conference on Software Engineering (ICSE)*, pages 482–491. IEEE, 2013.
- [6] Gad Ariav. Temporally oriented data definitions: Managing schema evolution in temporally oriented databases. *Data & Knowledge Engineering*, 6(6):451 – 467, 1991. ISSN 0169-023X. doi: [https://doi.org/10.1016/0169-023X\(91\)90023-Q](https://doi.org/10.1016/0169-023X(91)90023-Q). URL <http://www.sciencedirect.com/science/article/pii/0169023X9190023Q>.
- [7] Parisa Ataei, Arash Termehchy, and Eric Walkingshaw. Variational Databases. In *Int. Symp. on Database Programming Languages (DBPL)*, pages 11:1–11:4. ACM, 2017.
- [8] Parisa Ataei, Arash Termehchy, and Eric Walkingshaw. Managing Structurally Heterogeneous Databases in Software Product Lines. In *VLDB Workshop: Polystores and Other Systems for Heterogeneous Data (Poly)*, 2018.

- [9] Parisa Ataei, Qiaoran Li, and Eric Walkingshaw. Should variation be encoded explicitly in databases? In *15th International Working Conference on Variability Modelling of Software-Intensive Systems*, VaMoS'21, New York, NY, USA, 2021. Association for Computing Machinery. ISBN 9781450388245. doi: 10.1145/3442391.3442395. URL <https://doi.org/10.1145/3442391.3442395>.
- [10] François Bancilhon and Nicolas Spyratos. Update Semantics of Relational Views. *ACM Transactions on Database Systems (TODS)*, 6(4):557–575, 1981.
- [11] Anant P. Bhardwaj, Souvik Bhattacharjee, Amit Chavan, Amol Deshpande, Aaron J. Elmore, Samuel Madden, and Aditya G. Parameswaran. Datahub: Collaborative data science & dataset version management at scale. In *CIDR 2015, Seventh Biennial Conference on Innovative Data Systems Research, Asilomar, CA, USA, January 4-7, 2015, Online Proceedings*. [www.cidrdb.org](http://cidrdb.org), 2015. URL [http://cidrdb.org/cidr2015/Papers/CIDR15\\_Paper18.pdf](http://cidrdb.org/cidr2015/Papers/CIDR15_Paper18.pdf).
- [12] Souvik Bhattacharjee, Amit Chavan, Silu Huang, Amol Deshpande, and Aditya Parameswaran. Principles of dataset versioning: Exploring the recreation/storage tradeoff. *Proc. VLDB Endow.*, 8(12):1346–1357, August 2015. ISSN 2150-8097. doi: 10.14778/2824032.2824035. URL <http://dx.doi.org/10.14778/2824032.2824035>.
- [13] Cristina De Castro, Fabio Grandi, and Maria Rita Scalas. Schema versioning for multitemporal relational databases. *Information Systems*, 22(5):249 – 290, 1997. ISSN 0306-4379. doi: [https://doi.org/10.1016/S0306-4379\(97\)00017-3](https://doi.org/10.1016/S0306-4379(97)00017-3). URL <http://www.sciencedirect.com/science/article/pii/S0306437997000173>.
- [14] Badrish Chandramouli, Johannes Gehrke, Jonathan Goldstein, Donald Kossmann, Justin J. Levandoski, Renato Marroquin, and Wenlei Xie. READY: completeness is in the eye of the beholder. In *CIDR 2017, 8th Biennial Conference on Innovative Data Systems Research, Chaminade, CA, USA, January 8-11, 2017, Online Proceedings*. [www.cidrdb.org](http://cidrdb.org), 2017. URL <http://cidrdb.org/cidr2017/papers/p18-chandramouli-cidr17.pdf>.
- [15] Sheng Chen, Martin Erwig, and Eric Walkingshaw. An Error-Tolerant Type System for Variational Lambda Calculus. In *ACM SIGPLAN Int. Conf. on Functional Programming (ICFP)*, pages 29–40, 2012.

- [16] Sheng Chen, Martin Erwig, and Eric Walkingshaw. Extending Type Inference to Variational Programs. *ACM Trans. on Programming Languages and Systems (TOPLAS)*, 36(1):1:1–1:54, 2014.
- [17] Sheng Chen, Martin Erwig, and Eric Walkingshaw. A Calculus for Variational Programming. In *European Conf. on Object-Oriented Programming (ECOOP)*, volume 56 of *LIPICs*, pages 6:1–6:26, 2016.
- [18] Paul Clements and Linda Northrop. *Software Product Lines: Practices and Patterns*. Addison-Wesley, Boston, MA, 2001. ISBN 0-201-70332-7.
- [19] Carlo A. Curino, Hyun J. Moon, and Carlo Zaniolo. Graceful database schema evolution: The prism workbench. *Proc. VLDB Endow.*, 1(1):761–772, August 2008. ISSN 2150-8097. doi: 10.14778/1453856.1453939. URL <http://dx.doi.org/10.14778/1453856.1453939>.
- [20] AnHai Doan, Alon Halevy, and Zachary Ives. *Principles of Data Integration*. Morgan Kaufmann Publishers Inc., San Francisco, CA, USA, 1st edition, 2012. ISBN 0124160441, 9780124160446.
- [21] Martin Erwig and Eric Walkingshaw. The Choice Calculus: A Representation for Software Variation. *ACM Trans. on Software Engineering and Methodology (TOSEM)*, 21(1):6:1–6:27, 2011.
- [22] Martin Erwig and Eric Walkingshaw. Variation Programming with the Choice Calculus. In *Generative and Transformational Techniques in Software Engineering IV (GTTSE 2011), Revised and Extended Papers*, volume 7680 of *LNCS*, pages 55–99, 2013.
- [23] Martin Erwig, Eric Walkingshaw, and Sheng Chen. An Abstract Representation of Variational Graphs. In *Int. Work. on Feature-Oriented Software Development (FOSD)*, pages 25–32. ACM, 2013.
- [24] Mina Farid, Alexandra Roatis, Ihab F. Ilyas, Hella-Franziska Hoffmann, and Xu Chu. Clams: Bringing quality to data lakes. In *Proceedings of the 2016 International Conference on Management of Data, SIGMOD '16*, page 2089–2092, New York, NY, USA, 2016. Association for Computing Machinery. ISBN 9781450335317. doi: 10.1145/2882903.2899391. URL <https://doi.org/10.1145/2882903.2899391>.

- [25] Janet Feigenspan, Christian Kästner, Sven Apel, Jörg Liebig, Michael Schulze, Raimund Dachzelt, Maria Papendieck, Thomas Leich, and Gunter Saake. Do Background Colors Improve Program Comprehension in the `#ifdef` Hell? *Empirical Software Engineering*, 18(4):699–745, 2013.
- [26] Gabriel Ferreira, Momin Malik, Christian Kästner, Juergen Pfeffer, and Sven Apel. Do `#ifdefs` Influence the Occurrence of Vulnerabilities? An Empirical Study of the Linux Kernel. In *Int. Software Product Line Conf.*, 2016.
- [27] GNU Project. *The C Preprocessor*. Free Software Foundation, 2009. <http://gcc.gnu.org/onlinedocs/cpp/>.
- [28] Robert J. Hall. Fundamental Nonmodularity in Electronic Mail. *Automated Software Engineering*, 12(1):41–79, 2005.
- [29] Kai Herrmann, Jan Reimann, Hannes Voigt, Birgit Demuth, Stefan Fromm, Robert Stelzmann, and Wolfgang Lehner. Database evolution for software product lines. In *DATA*, 2015.
- [30] Jean-Marc Hick and Jean-Luc Hainaut. Database application evolution: A transformational approach. *Data & Knowledge Engineering*, 59(3):534 – 558, 2006. ISSN 0169-023X. doi: <https://doi.org/10.1016/j.datak.2005.10.003>. URL <http://www.sciencedirect.com/science/article/pii/S0169023X05001631>. Including: ER 2003.
- [31] Silu Huang, Liqi Xu, Jialin Liu, Aaron J. Elmore, and Aditya Parameswaran. Orpheusdb: Bolt-on versioning for relational databases. *Proc. VLDB Endow.*, 10(10):1130–1141, June 2017. ISSN 2150-8097. URL <http://dl.acm.org/citation.cfm?id=3115404.3115417>.
- [32] Spencer Hubbard and Eric Walkingshaw. Formula Choice Calculus. In *Int. Work. on Feature-Oriented Software Development (FOSD)*, pages 49–57. ACM, 2016.
- [33] Mathieu Humblet, Dang Vinh Tran, Jens H. Weber, and Anthony Cleve. Variability management in database applications. In *Proceedings of the 1st International Workshop on Variability and Complexity in Software Design, VACE '16*, pages 21–27, New York, NY, USA, 2016. ACM. ISBN 978-1-4503-4176-9. doi: 10.1145/2897045.2897050. URL <http://doi.acm.org/10.1145/2897045.2897050>.

- [34] Christian Kästner and Sven Apel. Virtual Separation of Concerns—A Second Chance for Preprocessors. *Journal of Object Technology*, 8(6):59–78, 2009.
- [35] Christian Kästner, Sven Apel, and Martin Kuhlemann. Granularity in Software Product Lines. In *IEEE Int. Conf. on Software Engineering*, pages 311–320, 2008.
- [36] Niloofar Khedri and Ramtin Khosravi. Handling database schema variability in software product lines. In *Asia-Pacific Software Engineering Conference (APSEC)*, pages 331–338, 2013. doi: 10.1109/APSEC.2013.52. URL <https://doi.org/10.1109/APSEC.2013.52>.
- [37] Duc Le, Eric Walkingshaw, and Martin Erwig. #ifdef Confirmed Harmful: Promoting Understandable Software Variation. In *IEEE Int. Symp. on Visual Languages and Human-Centric Computing (VL/HCC)*, pages 143–150, 2011.
- [38] Jörg Liebig, Christian Kästner, and Sven Apel. Analyzing the discipline of preprocessor annotations in 30 million lines of c code. In *Int. Conf. on Aspect-Oriented Software Development*, pages 191–202, 3 2011.
- [39] Edwin McKenzie and Richard Thomas Snodgrass. Schema evolution and the relational algebra. *Inf. Syst.*, 15(2):207–232, May 1990. ISSN 0306-4379. doi: 10.1016/0306-4379(90)90036-O. URL [http://dx.doi.org/10.1016/0306-4379\(90\)90036-O](http://dx.doi.org/10.1016/0306-4379(90)90036-O).
- [40] Meng Meng, Jens Meinicke, Chu-Pan Wong, Eric Walkingshaw, and Christian Kästner. A Choice of Variational Stacks: Exploring Variational Data Structures. In *Int. Work. on Variability Modelling of Software-Intensive Systems (VaMoS)*, pages 28–35. ACM, 2017.
- [41] Hyun J. Moon, Carlo A. Curino, Alin Deutsch, Chien-Yi Hou, and Carlo Zaniolo. Managing and querying transaction-time databases under schema evolution. *Proc. VLDB Endow.*, 1(1):882–895, August 2008. ISSN 2150-8097. doi: 10.14778/1453856.1453952. URL <http://dx.doi.org/10.14778/1453856.1453952>.
- [42] Sudha Ram and Ganesan Shankaranarayanan. Research issues in database schema evolution: the road not taken. 2003.

- [43] John F Roddick. A survey of schema versioning issues for database systems. *Information and Software Technology*, 37(7):383 – 393, 1995. ISSN 0950-5849. doi: [https://doi.org/10.1016/0950-5849\(95\)91494-K](https://doi.org/10.1016/0950-5849(95)91494-K). URL <http://www.sciencedirect.com/science/article/pii/095058499591494K>.
- [44] Martin Schäler, Thomas Leich, Marko Rosenmüller, and Gunter Saake. Building information system variants with tailored database schemas using features. In Jolita Ralyté, Xavier Franch, Sjaak Brinkkemper, and Stanislaw Wrycza, editors, *Advanced Information Systems Engineering*, pages 597–612, Berlin, Heidelberg, 2012. Springer. ISBN 978-3-642-31095-9.
- [45] Jitesh Shetty and Jafar Adibi. The Enron Email Dataset: Database Schema and Brief Statistical Report. Technical report, Information Sciences Institute, University of Southern California, 2004.
- [46] Norbert Siegmund, Christian Kästner, Marko Rosenmüller, Florian Heidenreich, Sven Apel, and Gunter Saake. Bridging the Gap Between Variability in Client Application and Database Schema. In *13. GI-Fachtagung Datenbanksysteme für Business, Technologie und Web (BTW)*, pages 297–306. Gesellschaft für Informatik (GI), 2009.
- [47] Richard Thomas Snodgrass. *The TSQL2 Temporal Query Language*. Kluwer Academic Publishers, USA, 1995. ISBN 0792396146.
- [48] Micheal Stonebraker, Dong Deng, and Micheal L. Brodie. Database decay and how to avoid it. In *Big Data (Big Data), 2016 IEEE International Conference*. IEEE, 2016. doi: 10.1109/BigData.2016.7840584.
- [49] Ștefan Stănciulescu, Thorsten Berger, Eric Walkingshaw, and Andrzej Wasowski. Concepts, Operations, and Feasibility of a Projection-Based Variation Control System. In *IEEE Int. Conf. on Software Maintenance and Evolution (ICSME)*, pages 323–333, 2016.
- [50] U. Störl, D. Müller, A. Tekleab, S. Tolale, J. Stenzel, M. Klettke, and S. Scherzinger. Curating variational data in application development. In *2018 IEEE 34th International Conference on Data Engineering (ICDE)*, pages 1605–1608, 2018. doi: 10.1109/ICDE.2018.00187.
- [51] Thomas Thüm and Fabian Benduhn. SPL2go: An Online Repository for Open-Source Software Product Lines, 2011. <http://spl2go.cs.ovgu.de>.

- [52] Thomas Thüm, Sven Apel, Christian Kästner, Ina Schaefer, and Gunter Saake. A Classification and Survey of Analysis Strategies for Software Product Lines. *ACM Computing Surveys (CSUR)*, 47(1):6, 2014.
- [53] Thomas Thüm, Leopoldo Teixeira, Klaus Schmid, Eric Walkingshaw, Mukelabai Mukelabai, Mahsa Varshosaz, Goetz Botterweck, Ina Schaefer, and Timo Kehrer. Toward Efficient Analysis of Variation in Time and Space. In *Int. Work. on Variability and Evolution of Software Intensive Systems (VariVolution)*, 2019.
- [54] Eric Walkingshaw. *The Choice Calculus: A Formal Language of Variation*. PhD thesis, Oregon State University, 2013. <http://hdl.handle.net/1957/40652>.
- [55] Eric Walkingshaw and Klaus Ostermann. Projectional Editing of Variational Software. In *ACM SIGPLAN Int. Conf. on Generative Programming: Concepts and Experiences (GPCE)*, pages 29–38, 2014.
- [56] Eric Walkingshaw, Christian Kästner, Martin Erwig, Sven Apel, and Eric Bodden. Variational Data Structures: Exploring Trade-Offs in Computing with Variability. In *ACM SIGPLAN Symp. on New Ideas in Programming and Reflections on Software (Onward!)*, pages 213–226, 2014.



## APPENDICES

