Enhancing the Magic Formula With Hierarchical Risk Parity (HRP) Approach: A Study on S&P 500 Stocks

Patapongrat Lertbuaban

WorldQuant University
patapongrat.lertbuaban@gmail.com

Abstract

Although Magic Formula is a popular investment strategy, there has been a notable lack of studies on the performance of Magic Formula on S&P 500 stocks during recent periods. This paper revisits the performance of Magic Formula from January 2011 to April 2024 by using survivorship bias-free financial data to simulate an investment portfolio, with a focus on risk evaluations. According to the analysis, the original Magic Formula achieved a 14.69% Compound Annual Growth Rate (CAGR), which was higher than the S&P 500 Index's CAGR of 13.16%. However, it showed higher volatility, larger maximum drawdowns, and a lower Sharpe ratio compared to the index. By incorporating the Hierarchical Risk Parity (HRP) approach, the revised Magic Formula produced better results, with a 15.86% CAGR, decreased volatility, and a higher Sharpe ratio, making it the top-performing strategy among the tested strategies.

Keywords: *Magic Formula, Hierarchical Risk Parity, Portfolio Optimization, Systematic Investing, Quantitative Investing, Backtesting*

1. Introduction

Systematic and quantitative investment strategies have been pivotal in the field of investment management. Many individuals and institutions are searching for strategies that can outperform the market. Among these, the Magic Formula is one of the strategies that offers a simple method for selecting undervalued stocks with the potential for high returns, considering the company's Return of Capital (ROC) and Earning Yield (EY). The formula was designed by Joel Greenblatt in order to simplify the process of finding investments without requiring in-depth financial analysis. However, investing in the stock market can be challenging in reality. For instance, when the Magic Formula strategy was applied in Thailand's stock market, it resulted in a larger drawdown compared to investing in the market index. This highlights the limitations of the Magic Formula strategy during market downturns. [11]. Furthermore, there has been a notable lack of studies on the performance of the Magic Formula on S&P 500 stocks during recent periods. These issues raise questions about the Magic Formula's ability to enhance portfolio performance and highlight the need for an evaluation and modification of the Magic Formula.

Therefore, this study aims to fill this gap by conducting a comprehensive backtesting analysis from 2011 to the present in the S&P 500 stock market. This will involve collecting survivorship bias-free financial and fundamental data to simulate the investment portfolio. Additionally, the portfolio's performance will be evaluated with a focus on risk measures, and the Magic Formula will be modified by using Hierarchical Risk Parity (HRP) approach. Then, the modified strategy will be compared against the market and conventional investment strategies, i.e., the simple price momentum strategies.

This study is organized into several main sections. The study starts with the theoretical framework. This section will explain the basic assumptions and mechanisms of the Magic Formula as well as review the recent literature. Next, the methodology section will describe how the data is collected, how the backtests are conducted, how the performance is evaluated, and how the modifications are done. Then, the results section will present the

performance of the Magic Formula in the S&P 500 stock market from 2011 to the present, as well as the performance of the modified strategy and the price momentum strategy. The discussion will then discuss the findings and any further potential modifications. Finally, the conclusion will summarize the study's findings and outcomes.

2. Theoretical Framework

The Magic Formula strategy is a value investing approach that uses a company's ROC and EY to rank stocks. The goal is to identify undervalued companies that have high potential for return. However, recent literature has produced mixed outcomes. This encourages further exploration into the strategy's performance and potential modifications. In this section, we will provide information on the history, mechanisms, and recent literature of the Magic Formula.

2.1. The History and Mechanisms of Magic Formula

Joel Greenbalt, a professional asset manager, wrote about the success of the Magic Formula in his book, which claims that it can provide an annual return of 30.8% throughout the 17-year period [5]. He published the criteria for the Magic Formula in his book, in which his criteria are listed below.

2.1.1. The Magic Formula Criteria: The formula from Joel Greenbalt [5] can be summarized as shown below.

1. Set a minimum market capitalization threshold, which is typically higher than $50 million.
2. Leave out financial and utility stocks.
3. Leave out foreign businesses (i.e., American Depositary Receipts).
4. Find the earnings yield of the company by dividing EBIT by enterprise value.
5. Calculate EBIT / (net fixed assets + working capital), which yields the company's return on capital.
6. Sort all the companies with higher market capitalization by their highest return on capital and highest earnings yield, expressed as percentages.
7. During a 12-month period, invest in 20–30 of the highest ranked companies, gaining 2-3 positions each month.
8. Once a year, rebalance the portfolio by selling the losers one week prior to the end of the year and the winners one week following.
9. Go on for a considerable amount of time (more than 5 years).

2.2. Global Market Performance

The Magic Formula has been evaluated in numerous foreign markets by many studies. For instance, research on Indonesian Islamic stocks by Setiawan, Alfianto Hendry, Resfa Fitri, Marhamah Muthohharoh, and Mohammad Iqbal Irfany (2023) [13] showed that the Magic Formula portfolio outperformed the benchmark index between June 2018 and May 2021, demonstrating its efficacy in the Indonesian market. Preet, Simmar, Ankita Gulati, Arnav Gupta, and Aadit Aggarwal (2021) [12] also conducted a study on the Indian stock market and discovered that the Magic Formula outperformed the market indices, particularly when utilizing a portfolio consisting of thirty stocks. This also emphasizes how important it is to choose stocks carefully. Furthermore, Blackburn, Douglas, and Nusret Cakici (2017) [1] use data between 1991 and 2016 to validate Greenblatt's Magic Formula. Their analysis includes double sorts and Fama-MacBeth regressions, which prove the formula's efficacy over traditional metrics, e.g., size and book-to-market ratios.

On the other hand, some studies have found that the Magic Formula may not work. For example, a study conducted by Vestre, Tobias, and Viktor M. Wikheim (2022) [15] on the Oslo Stock Exchange demonstrated that, even though the strategy has the potential to generate significant excess returns, transaction costs may limit its efficacy. Additionally, another critical perspective was provided by Alexander Gunnar Juliao de Paula (2016) [6] from the study on the Brazilian stock market, which showed that the Magic Formula outperformed benchmarks but did not guarantee that it would consistently generate excess returns (i.e., alpha). Hence, this raises questions about the Magic Formula's ability to enhance portfolio performance.

2.3. S&P 500 Market Performance

The actual performance of the Magic Formula depends on how it compares to the indices, e.g., the S&P 500. According to Geyfman, Victoria, Hayden Wimmer, and Roy Rada (2016) [4], it demonstrated a strong value premium among large-value stocks even during market downturns, with exceptional performance from 2007 to 2014. Unfortunately, in the last few decades, there has been a noticeable absence of research on the effectiveness of the Magic Formula with S&P 500 stocks. This highlights the need to conduct an analysis of the Magic Formula's performance with S&P 500 stocks over the past ten years.

2.4. Modifications and Enhancements

To address the limitations of the Magic Formula, researchers have suggested some modifications. According to Luo Min’s (2018) [10] research, Chinese A-shares returns could be increased by including factors such as market size and weight. This highlights the necessity of weight allocation. Moreover, Indrapratama, Arif, and Erman Arif Sumirat (2022) [7] demonstrated the advantages of utilizing extra financial metrics by demonstrating that combining the Magic Formula with the Acquirer's Multiple could produce returns that are higher than those of the Indonesian market and conventional equity funds. Also, Føleide Lars (2016) [3] modified the formula by using a 110/10 Market-Neutral Long/Short portfolio strategy with the Magic Formula, which can increase returns to 24.7% while lowering the standard deviation and increasing the Sharpe ratio, indicating a more effective risk-adjusted performance.

Furthermore, Ljungberg, Axel, and Anton Högstedt (2021) [9] investigate how modern portfolio theory (MPT) can be used to improve the Magic Formula investment strategy. They apply efficient frontiers and create two alternative portfolios every year, one aiming for a maximum Sharpe ratio and the other for a minimum variance, by changing the asset weights in portfolios chosen by the Magic Formula according to MPT principles. According to their research, these MPT-weighted portfolios from 2011 to 2020 consistently produced higher risk-adjusted returns, suggesting that modern portfolio theory can effectively improve the Magic Formula. Also, Sjöbeck, Erik, and Joel Verngren (2019) [14] analyze the performance of Joel Greenblatt's Magic Formula on the Swedish stock market, particularly its efficacy when combined with a momentum strategy. Their aim is to assess whether the Magic Formula alone or in combination with momentum can outperform the benchmark OMX30 index in terms of risk-adjusted returns. The findings indicate that both strategies outperform the benchmark, with the combination of Magic Formula and momentum achieving even higher risk-adjusted returns. This result implies that these investment strategies could be useful instruments for individual investors looking to increase their savings and have a more comfortable retirement.

2.5. Risk Analysis

Some literature has studied the relationship between risk and return when utilizing the Magic Formula. According to Burhanuddina, Muhammad Ikhsan, and Rofikoh Rokhima's [2] 12-year study on the Indonesian Stock Exchange, the Magic Formula strategy can yield higher average returns. However, there is a greater risk associated with it when compared to the market.

2.6. Addressing the Gap

These studies reach different conclusions about the Magic Formula's efficacy. Even though it can yield substantial returns, there are still a lot of unanswered questions, especially about how it performs in bear markets and in real-world trading situations. Furthermore, studies on the Magic Formula's performance with S&P 500 stocks have been noticeably lacking recently. This emphasizes the need for a thorough analysis of the Magic Formula's performance with the S&P 500, particularly over the previous few decades, as well as for determining how well it performs under various scenarios.

The objective of this study is to conduct a thorough backtest of the S&P 500 market from 2011 to the present in order to bridge the gaps between the theory of the Magic Formula and the reality of its application in a variety of market conditions. This involves collecting survivorship bias-free financial and fundamental data to simulate investment strategies under practical trading conditions.

Furthermore, the study will evaluate the portfolio's performance throughout the study period, with a significant focus on risk measures such as drawdowns and other downside statistics, to measure the strategy's resilience against market volatility. Potential enhancements to the Magic Formula will also be explored by applying the Hierarchical Risk Parity (HRP) technique and eventually comparing these modifications against conventional investment approaches, i.e., the simple price momentum strategy. These approaches ensure clear results from the Magic Formula's performance and offer insights that could bridge the gap between theoretical expectations and real-world investment outcomes.

3. Methodology

3.1. Data Collection

The financial and fundamental data will be gathered from EODHistoricalData in order to perform a comprehensive backtest in the S&P 500 stock market from January 2011 to April 2024. The components of the S&P 500 for the given time frame will be obtained. This will include delisted stocks, which are resilient to survivorship bias. Our choice of this period is due to the fact that it was more difficult to obtain reliable and accessible fundamental data before this period, which might increase the bias. Additionally, it is more effective to assess the investment strategy using the most recent period, as the market microstructure may have changed.

To demonstrate the procedure, we take any day in 2011 and analyze all stocks that were included in the S&P 500 up to that point, i.e., we do not include stocks that were added in 2020 or that were not yet listed. By ensuring that they are excluded from our backtesting universe for 2011, this method helps to mitigate the survivorship bias. Moreover, we keep delisted stocks in our dataset until they are removed from the market [8].

The OHLC data (open, high, low, and close) as well as the adjusted close and volume are included in the financial data that was retrieved. In order to determine the company's ROC and EY, we will obtain the fundamental data, which consists of total assets, total current liabilities, total liabilities, cash and cash equivalents, outstanding common stock shares, and EBIT (earnings before interest and taxes). After that, the Pandas library will be mostly used to clean and organize the data. Then, it is time to backtest and assess the investment performance. The procedure will be covered in the following section.

3.2. Backtesting Condition

To backtest the performance of the magic formula strategy, we will simulate the portfolio that will use the magic formula to buy stocks in the S&P 500 market. This simulation will include some practical conditions for backtesting, as recommended by the study "The Performance of Momentum Investing Strategies on S&P 500 Stocks: A 2001–2024 Review," published by Investhematic in 2024 [8]. We will also apply the Magic Formula standards described in Section 2.1.1. However, instead of adding two or three positions every month, we will invest in 30 of the formula's top-ranked companies to simplify the process. We will then rebalance the portfolio every six months, instead of selling the losers one week before the year ends and the winners the week after. Furthermore, each stock will be bought with equal weight. We will also consider the trading commission in order to mimic the real trading scenario. However, in this backtest, we will not take taxes or slippage into account. The backtesting conditions are summarized in Table 3.2.1 below.

Table 3.2.1. Magic Formula Backtesting Conditions

Condition	Specifics
Universe	● Survivorship bias-free S&P 500 stocks
Period	● 2011/01/03 - 2024/04/26
Criteria	● According to the Magic Formula criteria listed in Section 2.1.1
Allocation	● Buy the top 30 stocks ranked by the Magic Formula score ● Allocate equally among top stocks
Rebalancing Period	● Six months
Entry and Exit	● Enter or exit the position at the opening price on the rebalancing day
Assumptions	● Long only ● Trading commission of 0.2% in both direction ● No slippage or taxes

3.3. Modification to the Magic Formula Strategy: Incorporating the Hierarchical Risk Parity (HRP) Algorithm

In order to potentially enhance the performance of Magic Formula, we will use portfolio optimization techniques to allocate the funds more effectively. In this study, we will select the stocks using the same criteria as before. In addition, we will apply the Hierarchical Risk Parity (HRP) approach invented by Marcos Lopez de Prado [17]. The algorithm will be used from the PyPortfolioOpt library [16] to calculate the optimal weight for each stock. Note that the past six-month returns are used to calculate the weight used for the next six-month period. This approach ensures that the performance due to the diversification still outperforms out-of-sample.

The Hudson & Thames publication by Aditya Vyas has summarized the HRP algorithm [18]. In summary, the Hierarchical Risk Parity (HRP) algorithm is one of the methods for

portfolio optimization that improves the traditional risk parity approach by modifying the structure of the data. The HRP algorithm begins by calculating the correlation matrix of the underlying asset returns, where all assets are grouped based on similarities. Then, we will have a dendrogram that represents the hierarchical relationship between the assets. After that, the correlation matrix is reordered to ensure that related assets are closer together in the correlation matrix. This step is known as quasi-diagonalization, in which the goal is to have larger covariances along the diagonal of the matrix and smaller covariances on the off-diagonal of the matrix.

Subsequently, the algorithm recursively bisects the clusters from the top of the hierarchical tree in order to assign the weights to each asset within each sub-cluster. This process ensures that each asset is considered based on the relationships within its cluster. Then, the weights are normalized and become the weights that we will use for our portfolio allocation.

The backtesting conditions of the modified Magic Formula strategy are summarized in Table 3.3.1 below.

Table 3.3.1. Modified Magic Formula Backtesting Conditions

Condition	Specifics
Universe	● Survivorship bias-free S&P 500 stocks
Period	● 2011/01/03 - 2024/04/26
Criteria	● According to the Magic Formula criteria listed in Section 2.1.1
Allocation	● Buy the top 30 stocks ranked by the Magic Formula score ● Allocate the funds according to the weight obtained by the Hierarchical Risk Parity (HRP) approach
Rebalancing Period	● Six months
Entry and Exit	● Enter or exit the position at the opening price on the rebalancing day
Assumptions	● Long only ● Trading commission of 0.2% in both direction ● No slippage or taxes

3.4. Comparing the Modified Strategy with the Price Momentum Strategy

To ensure that the increased complexity is worth it, the modified strategy will be compared to the simple price momentum strategy. The backtest conditions are the same as in the Magic Formula strategy, except for the fact that the price momentum strategies will use the previous six-month cumulative returns to rank the stocks. Note that both equal-weight allocation and the HRP approach will be used. The backtesting conditions of the price momentum strategy are summarized in Table 3.4.1 below.

Table 3.4.1. Price Momentum Backtesting Conditions

Condition	Specifics
Universe	• Survivorship bias-free S&P 500 stocks
Period	• 2011/01/03 - 2024/04/26
Criteria	• Filter the top 30 stocks that have had the highest returns over the past six months
Allocation	• Buy the top 30 stocks • Simple strategy: Allocate equally among top stocks • Modified strategy: Use the HRP approach
Rebalancing Period	• Six months
Entry and Exit	• Enter or exit the position at the opening price on the rebalancing day
Assumptions	• Long only • Trading commission of 0.2% in both direction • No slippage or taxes

4. Results

The performance of price momentum, modified price momentum (i.e., using the HRP approach), magic formula, and modified magic formula (i.e., using HRP) on S&P 500 stocks between January 2011 and April 2024 is summarized in Table 4.1 below.

Table 4.1. Backtesting Results

Metric	Modified Magic Formula Strategy	Modified Price Momentum Strategy	Magic Formula Strategy	Price Momentum Strategy	S&P 500 Index (Total Return)
CAGR	15.86%	**16.49%**	14.69%	16.42%	13.16%
Cumulative Return	611.01%	**663.69%**	521.02%	657.66%	419.24%
Annualized Volatility	**17.01%**	19.78%	20.25%	23.00%	17.36%
Sharpe Ratio	**0.95**	0.87	0.78	0.78	0.8
Max. Drawdown	-35.64%	-34.53%	-35.64%	-36.76%	**-33.79%**
Longest DD (Days)	545	**477**	831	808	708

It's worth noting that the maximum drawdown of both the modified and original Magic Formula strategies is the same. This is because equal weight allocation is used when the HRP algorithm cannot find the optimal weight during some periods, due to the clustering problem. As the period with the largest drawdown is the same for both strategies and equal weight is used during that period, the maximum drawdown of both strategies is the same.

The backtesting results, using Quanstats [19], are shown in Figures 4.1 to 4.4 below. The completed results can be found in the appendix.

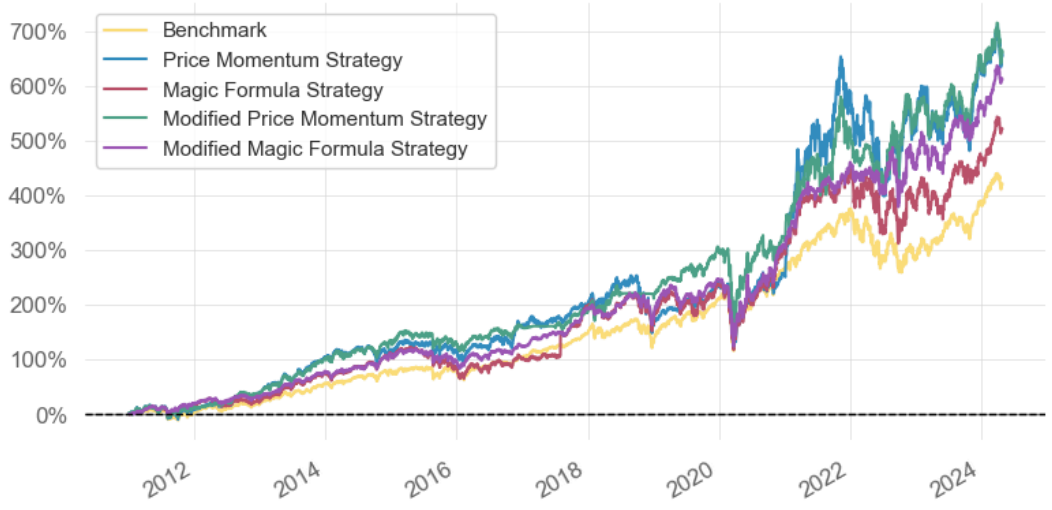

Figure 4.1. Cumulative Returns of All Strategies

Figure 4.2. Cumulative Returns of All Strategies (Volatility Matched)

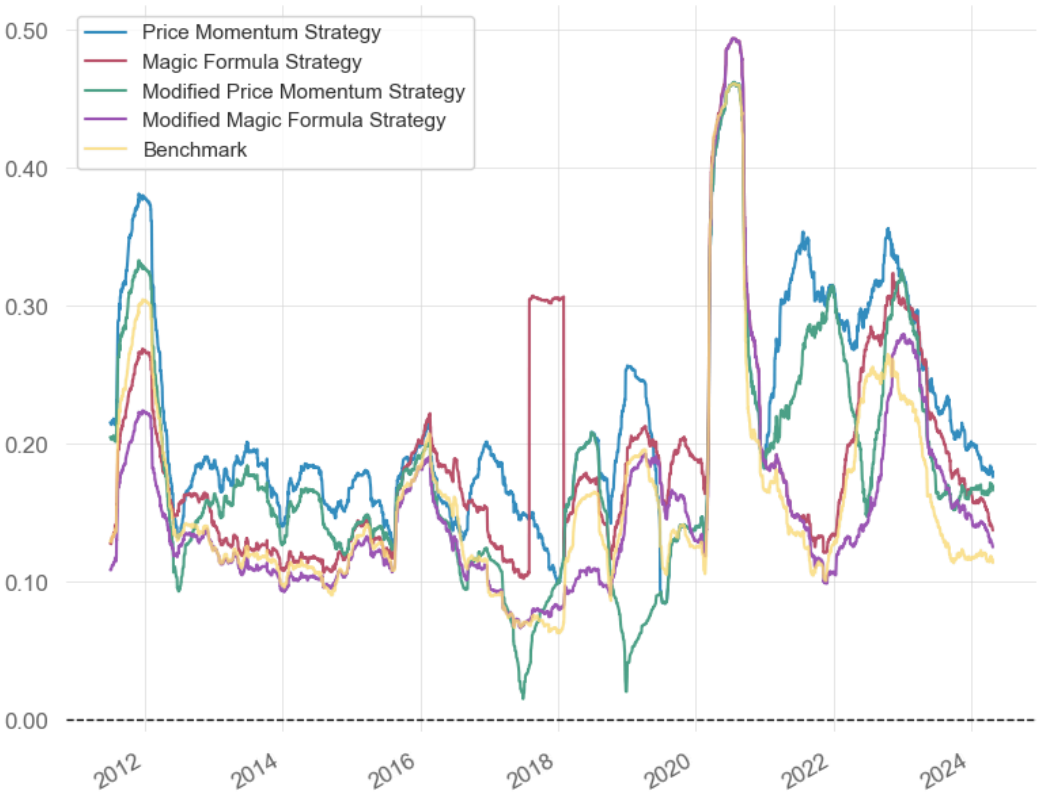

Figure 4.3. Rolling Volatility (6-Months)

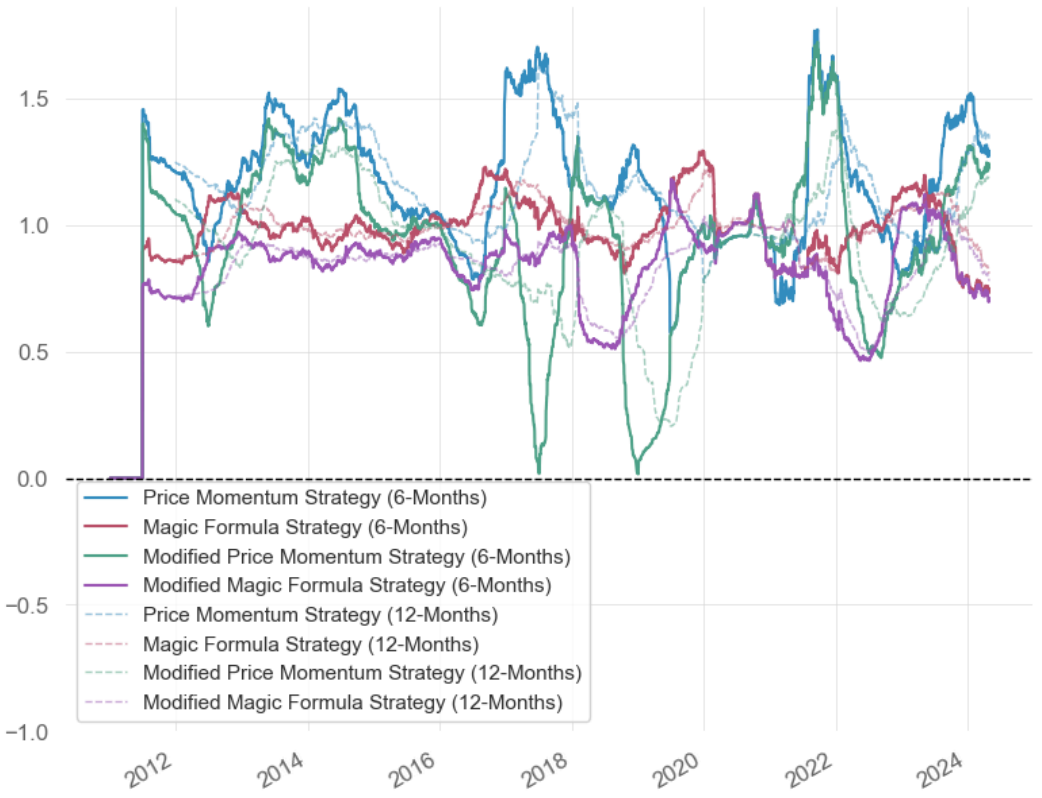

Figure 4.4. Rolling Beta to Benchmark

5. Discussion

Based on the results, the original Magic Formula achieved a Compound Annual Growth Rate (CAGR) of 14.69%, which is higher than the S&P 500 Index's 13.16% CAGR over the past 12 years. However, after taking the risk measure into account, we observe that the annualized volatility and the maximum drawdown of the Magic Formula are greater than those of the S&P 500 index. Additionally, when comparing the Sharpe ratio, the Magic Formula has a lower Sharpe ratio than simply buying and holding the S&P 500 Index. These findings are consistent with the study in Thailand's stock market by Peeratiyuth Koedkao (2020) [11].

By incorporating the HRP algorithm into the Magic Formula, we have observed that the strategy is capable of achieving a 15.86% CAGR, which outperforms both the index and the original Magic Formula. Additionally, the Magic Formula with the HRP algorithm has resulted in lower annualized volatility when compared to the original Magic Formula and the index. It is remarkable to note that the Sharpe ratio has increased to a value of 0.95, which makes this strategy the best-performing one among all the strategies that have been studied so far, including the price momentum strategy with the HRP algorithm.

Based on the data presented in Figures 4.1 and 4.2, it is apparent that the modified price momentum strategy has shown the highest cumulative returns. However, if we consider the risk associated with the strategy, i.e., we match the volatility with the returns, the modified Mafic Formula becomes the best-performing strategy.

It has also been observed that investing in the S&P 500 index and holding it for a long term still has the lowest maximum drawdown compared to other investment strategies. It also has the least number of days in drawdown when compared to the original magic formula and price momentum strategies. However, when the HRP algorithm is applied, the maximum drawdown and the longest drawdown are reduced, indicating that the portfolio allocation is also significant in terms of risk management. This can also be seen in Figures 4.3 and 4.4, where the modified magic formula has lower rolling volatility and beta than other strategies on average. This is due to the use of the HRP algorithm, which enables the strategy to manage risk more effectively.

Although the backtesting results imply that the Magic Formula combined with the HRP algorithm has outperformed the benchmark over the past 12 years, it is worth noting that backtesting is not a research tool, and past success is not a guarantee of future performance. This strategy can serve as a starting point for using the Magic Formula and HRP algorithm, but it cannot guarantee its success in the future. In fact, backtesting can lead to overfitting problems. To enhance the effectiveness of the backtest further, Marcos Lopez De Prado's study [20] suggests using techniques like combinatorial purged cross-validation to address the overfitting issue. Alternatively, we can determine the optimal trading strategy without using historical data by deriving it from underlying stochastic processes.

6. Conclusion

Different studies on the Magic Formula have arrived at varying conclusions about its effectiveness. While it can generate significant returns, there are several unanswered questions regarding its performance in real-world trading situations. Moreover, recent studies on the Magic Formula's performance with S&P 500 stocks have been insufficient, which highlights the necessity of conducting a thorough analysis of the Magic Formula's performance with the S&P 500, particularly over the past few decades. This study aims to fill this gap by conducting a comprehensive backtesting analysis of the S&P 500 stock market from January 2011 to April 2024. The study involves gathering survivorship bias-free financial and fundamental data to simulate the investment portfolio. Additionally, the portfolio's performance is evaluated with a focus on risk measures. The

Magic Formula is then modified using the Hierarchical Risk Parity (HRP) approach, and compared with the market and conventional investment strategies, i.e., the price momentum strategies.

According to the analysis, the study shows that the original Magic Formula achieved a Compound Annual Growth Rate (CAGR) of 14.69%, which is better than the S&P 500 Index's CAGR of 13.16% over the past 12 years. However, when looking at risk factors, the Magic Formula had higher annualized volatility and greater maximum drawdown compared to the S&P 500 index. Additionally, when examining the Sharpe ratio, which measures risk-adjusted returns, the Magic Formula performed worse than simply buying and holding the S&P 500 Index. By incorporating the Hierarchical Risk Parity (HRP) algorithm into the Magic Formula, the modified strategy achieved a 15.86% CAGR, outperforming both the index and the original Magic Formula. This modified version also showed lower annualized volatility and a higher Sharpe ratio of 0.95, making it the most effective among all tested strategies, including the price momentum strategy with the HRP algorithm.

It should be noted that while this strategy can serve as a starting point for using the Magic Formula and HRP algorithm, it does not ensure future success. To further strengthen the reliability of the backtest, techniques like combinatorial purged cross-validation can be applied to reduce overfitting problems. Alternatively, the optimal trading strategy could be identified without relying on historical simulation by deriving it from the defined underlying stochastic processes.

Appendix

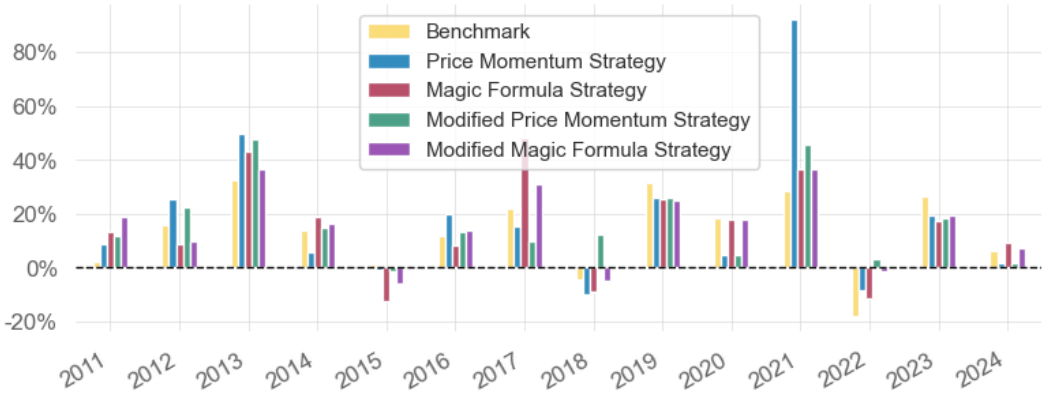

Figure A1. EOY Returns vs Benchmark

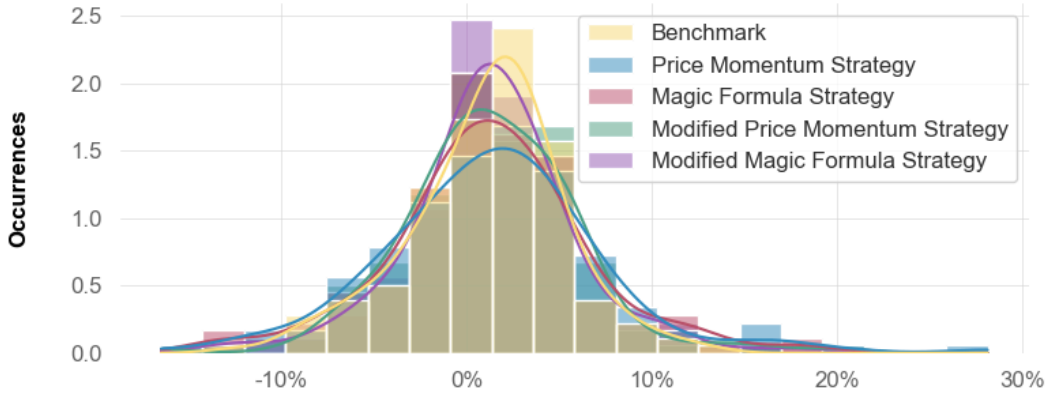

Figure A2. Distribution of Monthly Returns

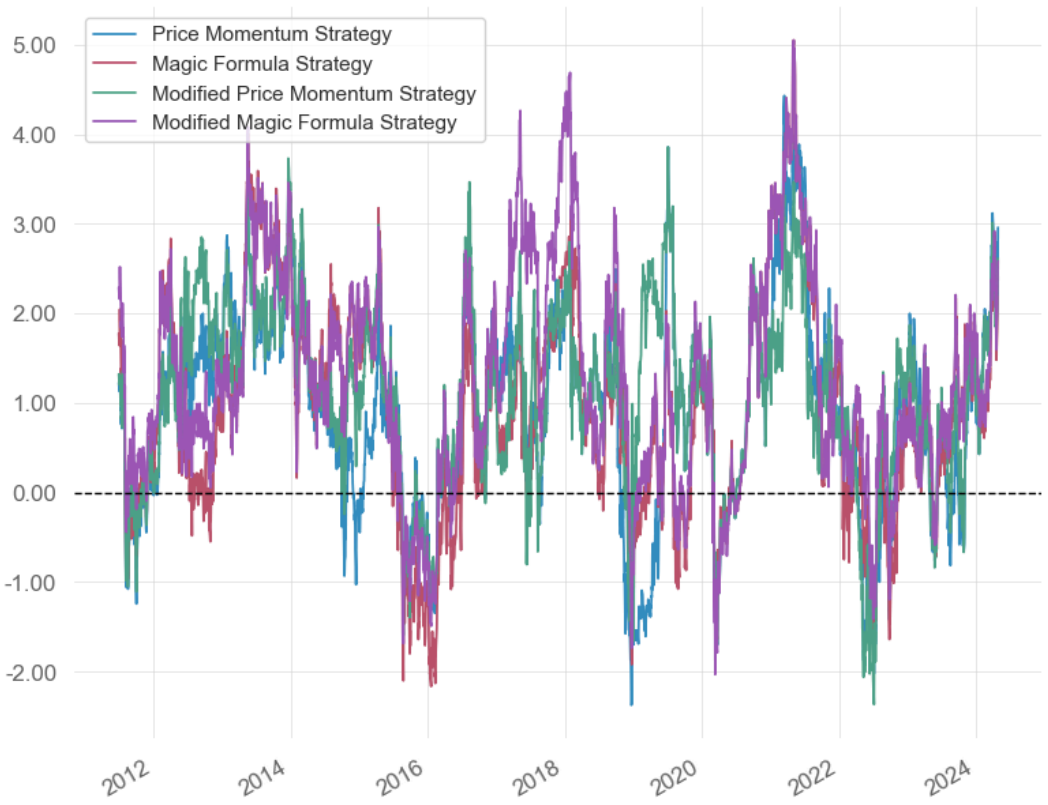

Figure A3. Rolling Sharpe (6-Months)

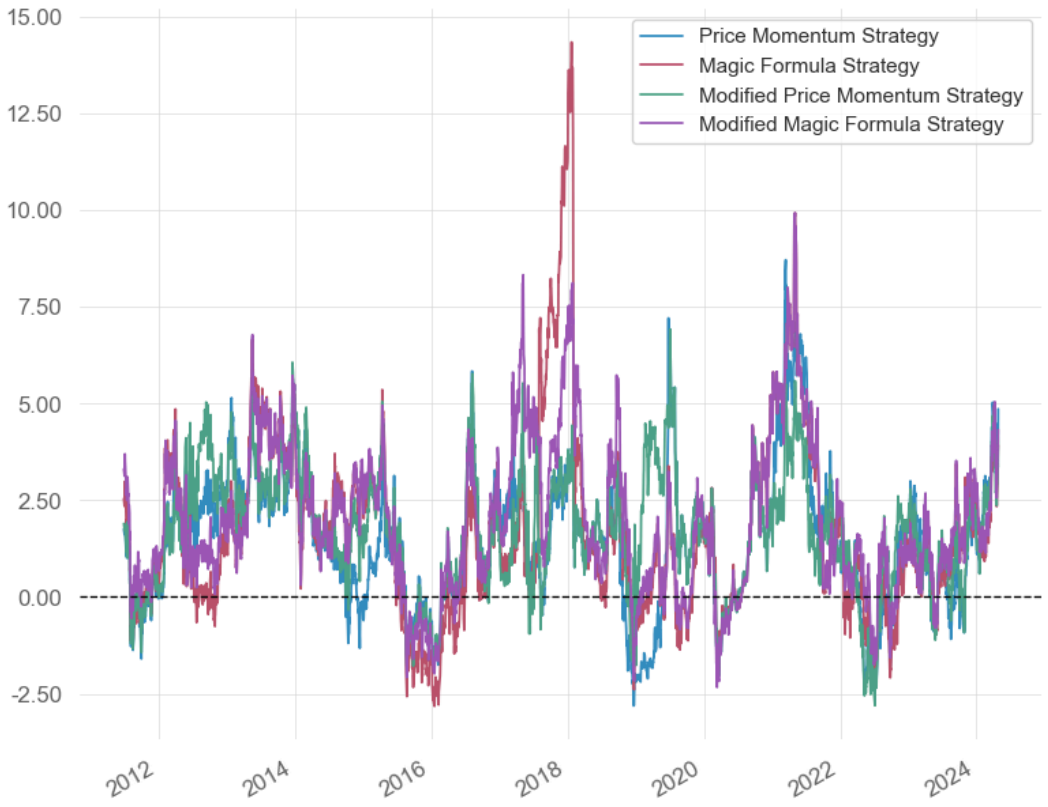

Figure A4. Rolling Sortino (6-Months)

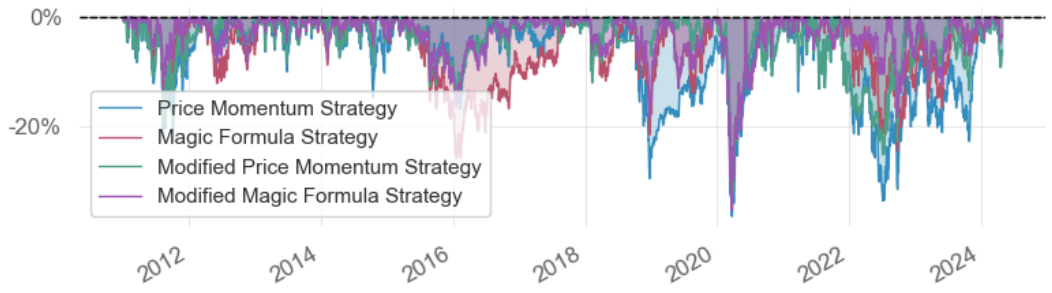

Figure A5. Underwater Plot

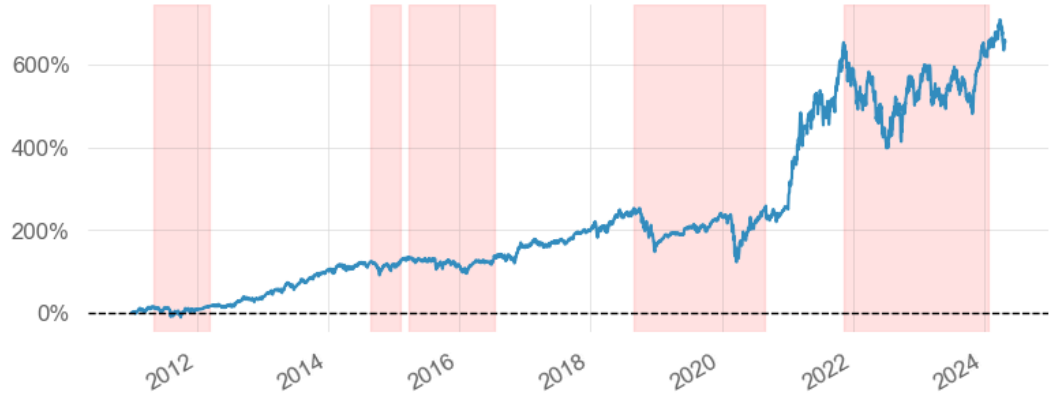

Figure A6. Price Momentum Strategy - Worst 5 Drawdown Periods

Figure A7. Magic Formula Strategy - Worst 5 Drawdown Periods

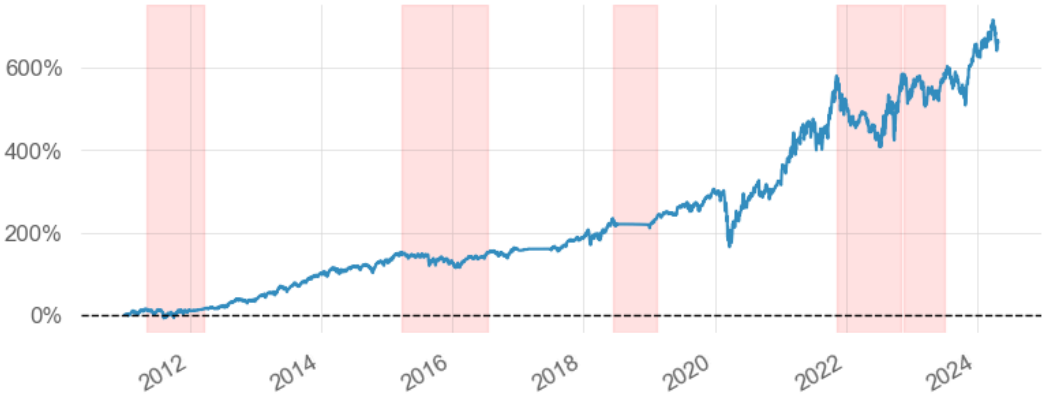

Figure A8. Modified Price Momentum Strategy - Worst 5 Drawdown Periods

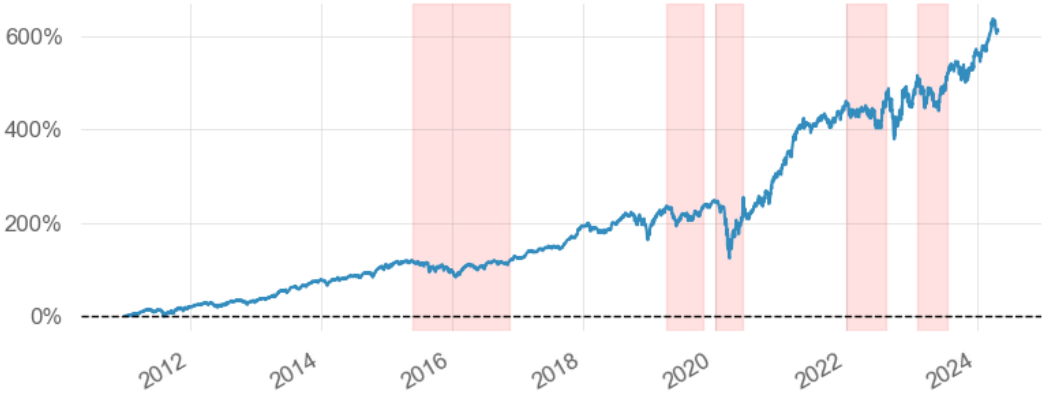

Figure A9. Modified Magic Formula Strategy - Worst 5 Drawdown Periods

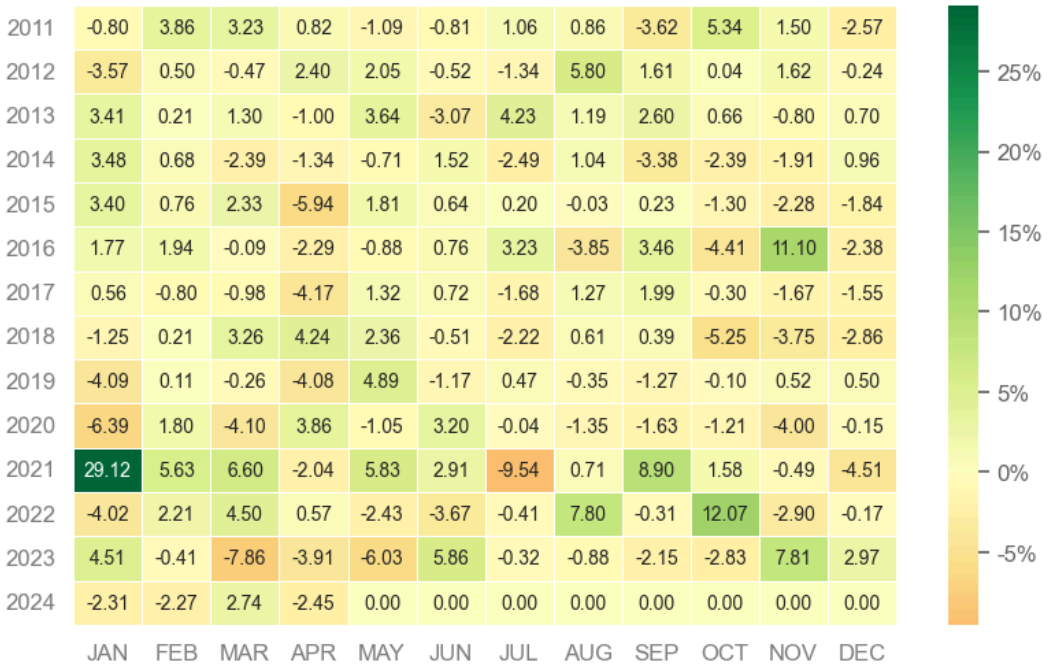

Figure A10. Price Momentum Strategy - Monthly Active Returns (%)

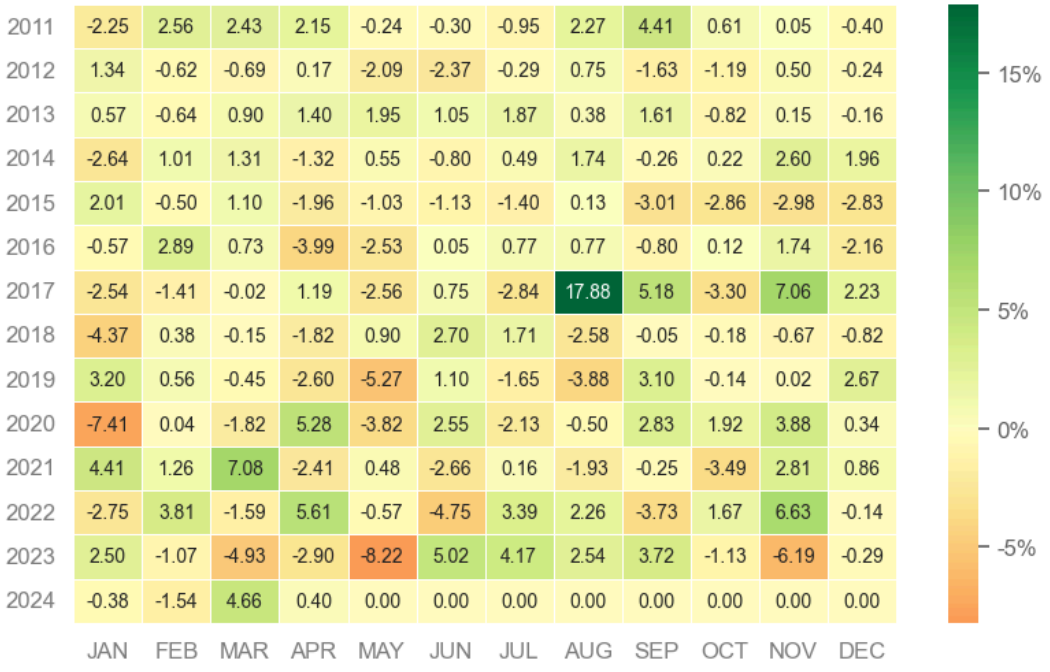

Figure A11. Magic Formula Strategy - Monthly Active Returns (%)

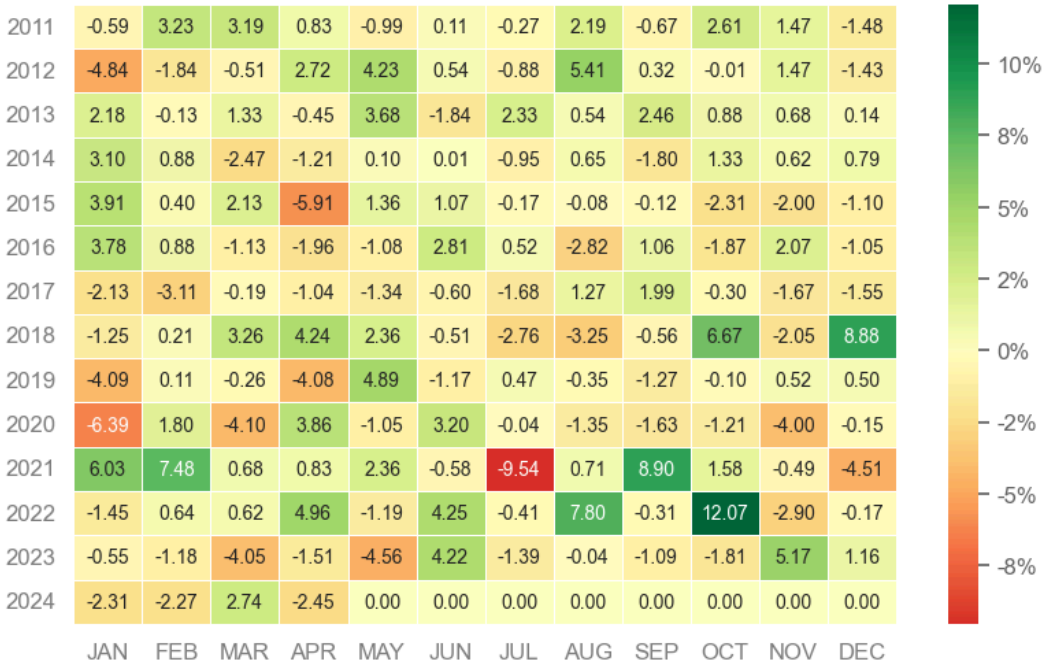

Figure A12. Modified Price Momentum Strategy - Monthly Active Returns (%)

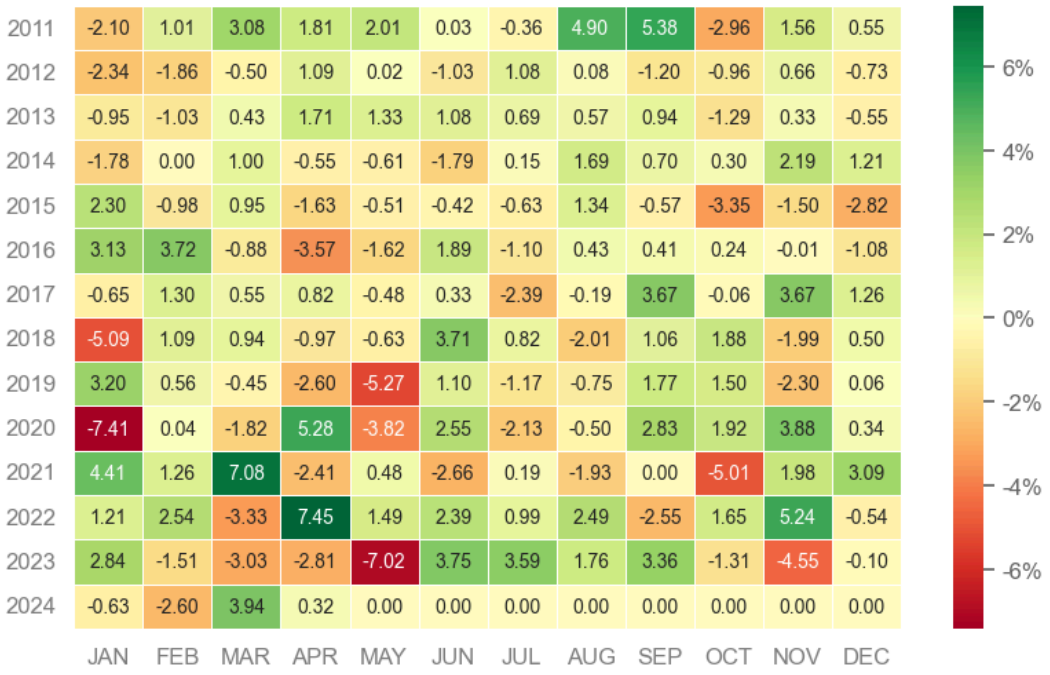

Figure A13. Modified Magic Formula Strategy - Monthly Active Returns (%)

References

[1] Blackburn, Douglas W., and Nusret Cakici. “The Magic Formula: Value, Profitability, and the Cross Section of Global Stock Returns.” *SSRN Electronic Journal*, Elsevier BV, 2017. https://doi.org/10.2139/ssrn.2956448.

[2] Burhanuddina, Muhammad Ikhsan, and Rofikoh Rokhima. "Analysis of Magic Formula Investment Strategy Applied on Indonesia Stock Exchange in 2007-2019." *Predictors of Managers' Career Success: The case of the Egyptian Service Sector*: 87.

[3] Føleide, Lars. “Magic formula combined with long/short portfolio optimization.” MS thesis, 2016.

[4] Geyfman, Victoria, Hayden Wimmer, and Roy Rada. “The Use of Accounting Screens for Separating Winners from Losers among the S&P 500 Stocks.” *Journal of Accounting and Finance*, vol. 16, no. 1, Jan. 2016, pp. 45–60.

[5] Greenblatt, Joel. *The Little Book That Still Beats the Market*. John Wiley and Sons, 2010.

[6] Gunnar, Alexander. “Backtesting the Magic Formula in the Brazilian Stock Market.” 2016.

[7] Indrapratama, Arif, and Erman Arif Sumirat. “Implementation of Magic Formula and Acquirer’s Multiple Stock Investment Strategy in the Indonesia Stock Exchange.” *International Journal of Current Science Research and Review*, vol. 05, no. 12, Everant Journals, Dec. 2022. https://doi.org/10.47191/ijcsrr/v5-i12-10.

[8] Investhematic. “The Performance of Momentum Investing Strategies on S&P 500 Stocks: A 2001–2024 Review.” *Medium*, 25 Mar. 2024, medium.com/@investhematic/the-performance-of-momentum-investing-strategies-on-s-p-500-stocks-a-2001-2024-review-c1c3cff06486.

[9] Ljungberg, Axel, and Anton Högstedt. *Modern Portfolio Theory Combined With Magic Formula: A study on how Modern Portfolio Theory can improve an established investment strategy*, 2021.

[10] Luo, Min. "Case Study of Magic Formula Based on Value Investment in Chinese A-shares Market." *Advances in Computational Science and Computing*. Springer International Publishing, 2019.

[11] Peeratiyuth, Koedkao. “Update : Magic Formula สูตรการลงทุนอันแสนมหัศจรรย์ของ Joel Greenblatt.” *SiamQuant*, 14 May 2020, www.siamquant.com/magic-formula-update-2020.

[12] Preet, Simmar, Ankita Gulati, Arnav Gupta, and Aadit Aggarwal "Back Testing Magic Formula on Indian Stock Markets: An Analysis of Magic Formula Strategy." *Paideuma Journal of Research* 14.10, 2021.

[13] Setiawan, Alfianto Hendry, Resfa Fitri, Marhamah Muthohharoh, and Mohammad Iqbal Irfany. “Investment Strategy on Indonesia Islamic Stocks Using Greenblatt Magic Formula.” *International Journal of Financial, Accounting, and Management*, vol. 5, no. 3, Goodwood Publishing, Dec. 2023, pp. 281–96. https://doi.org/10.35912/ijfam.v5i3.1322.

[14] Sjöbeck, Erik, and Joel Verngren. "Magic Formula has its magic and Momentum has its moments: A study on magic formula and momentum on the Swedish stock market." 2019.

[15] Vestre, Tobias, and Viktor M. Wikheim. *The Magic Formula: An empirical study of Joel Greenblatt's magic formula, backtested on the Oslo Stock Exchange*. MS thesis. 2022.

[16] Robert Andrew Martin. PyPortfolioOpt: Portfolio optimization in Python. Journal of Open Source Software, 6(61), 3066, https://doi.org/10.21105/joss.03066.

[17] López de Prado, Marcos. "Building Diversified Portfolios That Outperform Out of Sample." *The Journal of Portfolio Management*, vol. 42, no. 4, With Intelligence LLC, May 2016, pp. 59–69. https://doi.org/10.3905/jpm.2016.42.4.059.

[18] Vyas, Aditya. "The Hierarchical Risk Parity Algorithm: An Introduction." *Hudson & Thames*, 5 Feb. 2023, hudsonthames.org/an-introduction-to-the-hierarchical-risk-parity-algorithm.

[19] *QuantStats*. github.com/ranaroussi/quantstats.

[20] De Prado, Marcos Lopez. *Advances in Financial Machine Learning*. John Wiley and Sons, 2018.