COEN 169

Vector Space Model

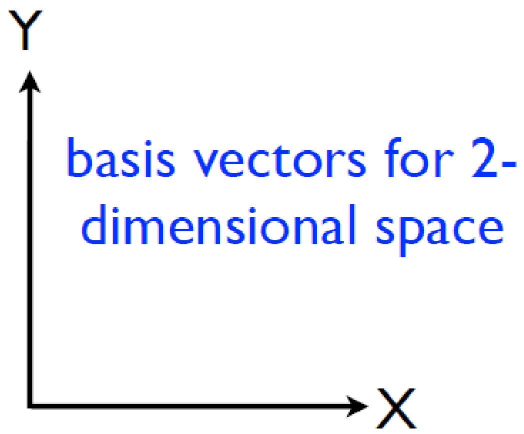
Yi Fang

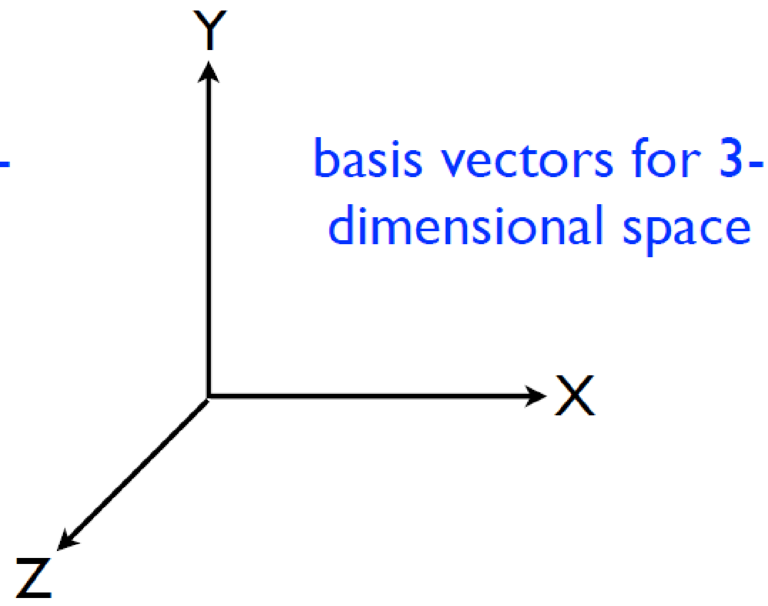
Department of Computer Engineering

Santa Clara University

What is a Vector Space?

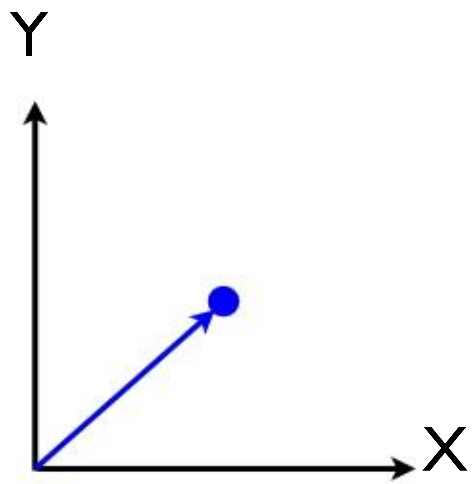
- Formally, a vector space is defined by a set of linearly independent basis vectors
- The basis vectors correspond to the dimensions or directions of the vector space

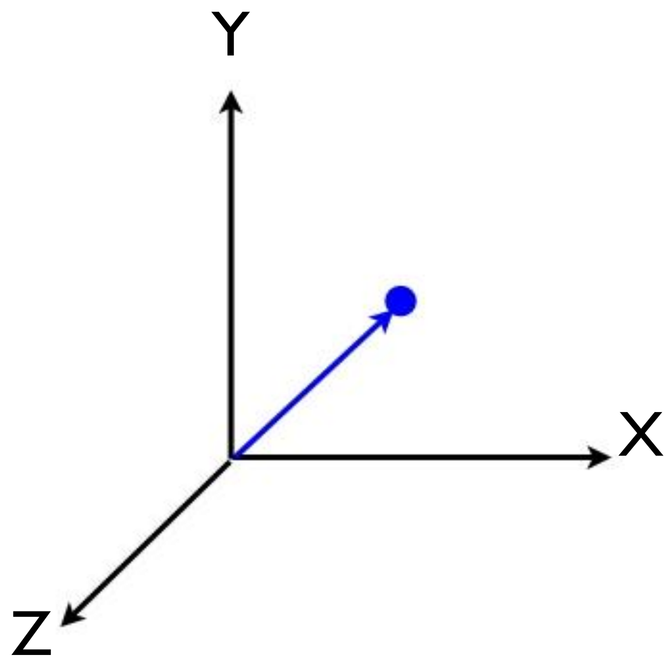




What is a Vector?

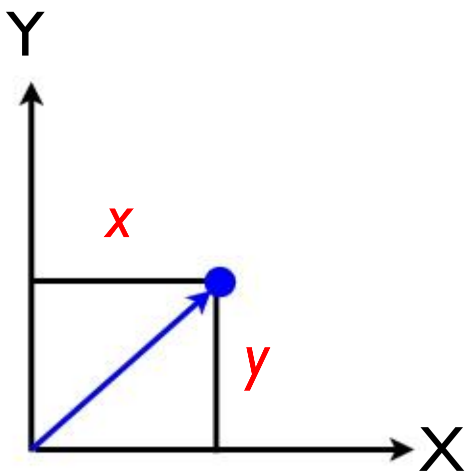
- A vector is a point in a vector space and has length (from the origin to the point) and direction

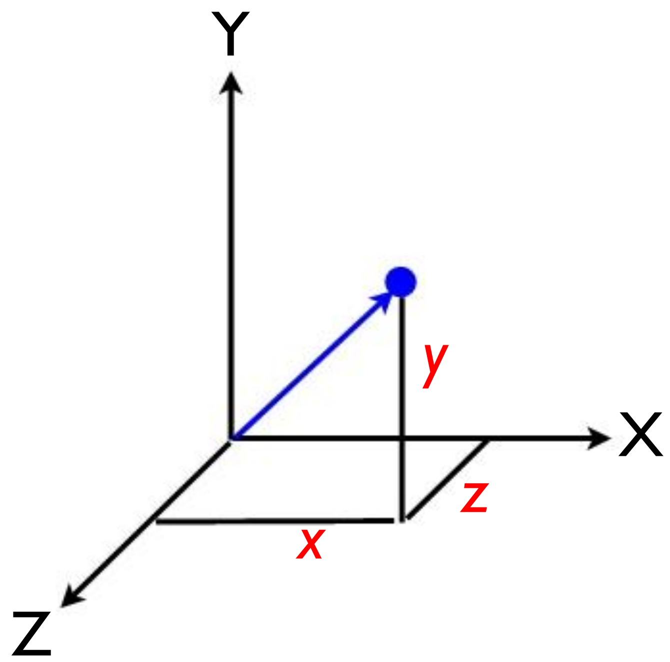




What is a Vector?

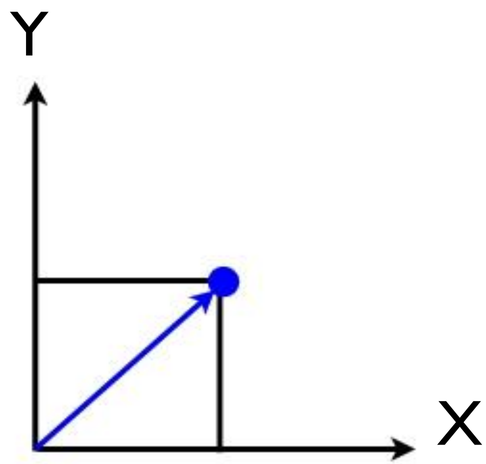
- A 2-dimensional vector can be written as [*x*,*y*]
- A 3-dimensional vector can be written as [*x*,*y*,*z*]



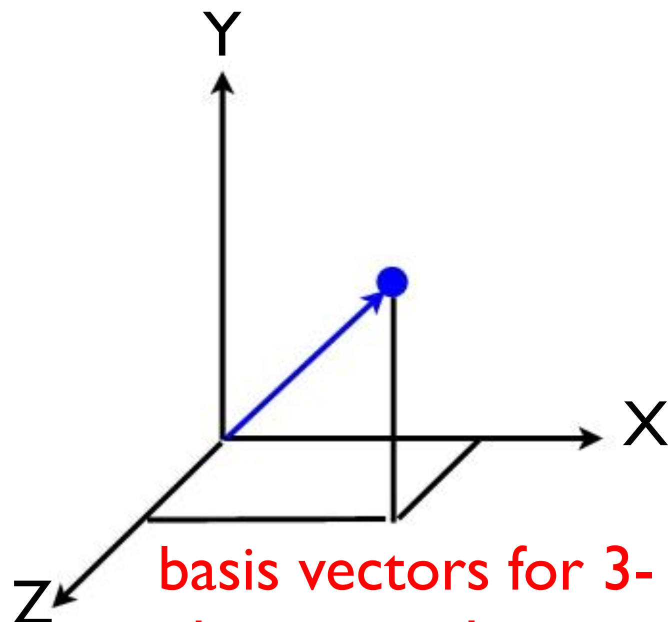


What is a Vector Space?

- The basis vectors are linearly independent because knowing a vector's value on one dimension doesn't say anything about its value along another dimension



basis vectors for 2-dimensional space



basis vectors for 3-dimensional space

Binary Text Representation

document-term matrix

	a	*aardvark*	*abacus*	*abba*	*able*	*zoom*
doc_1	1	0	0	0	0	1
doc_2	0	0	0	0	1	1
::	::	::	::	::	::	0
doc_m	0	0	1	1	0	0

- 1 = the word appears in the document
- 0 = the word does <u>not</u> appear in the document
- Does not represent word frequency, word location, or word order information

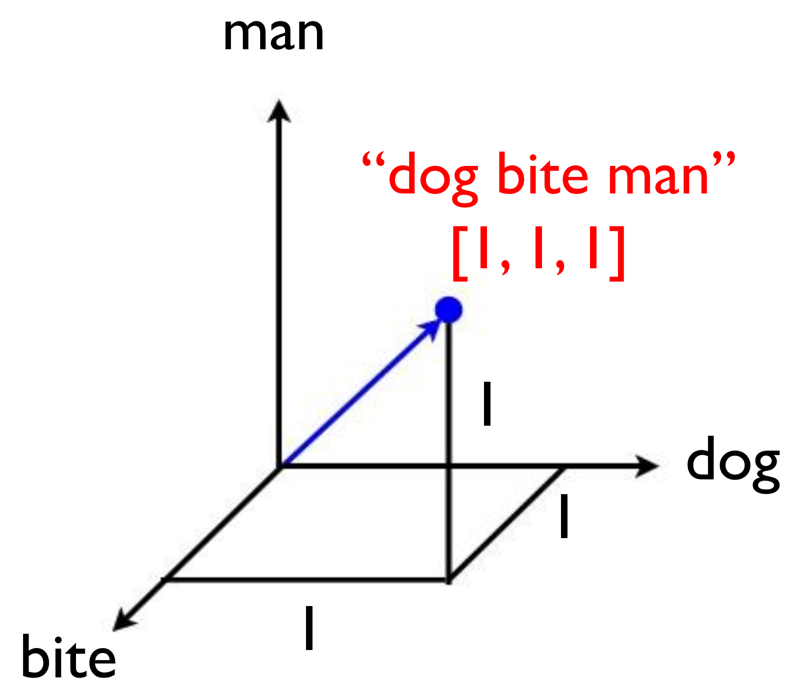
Vector Space Representation

- Let V denote the size of the indexed vocabulary
  - V = the number of unique terms,
  - V = the number of unique stems, etc...
  - V = the number of unique terms excluding stopwords,
- Any arbitrary span of text (i.e., a document, or a query) can be represented as a vector in V-dimensional space
- For simplicity, let's assume three index terms: dog, bite, man (i.e., V=3)
- Why? Because it's easy to visualize 3-D space

Vector Space Representation with binary weights

- 1 = the term appears at least once
- 0 = the term does <u>not</u> appear

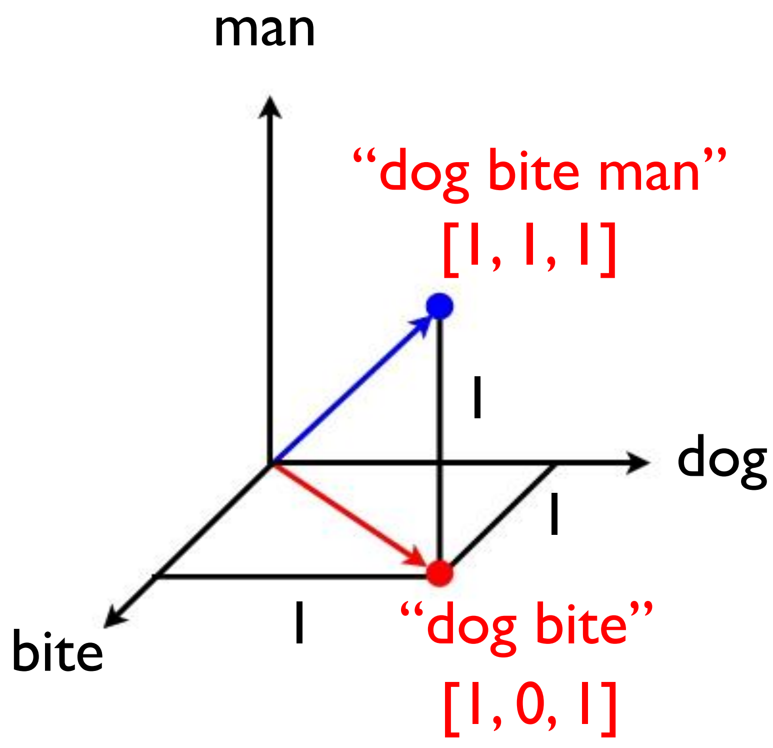
	dog	*man*	*bite*
doc_1	1	1	1



Vector Space Representation with binary weights

- 1 = the term appears at least once
- 0 = the term does not appear

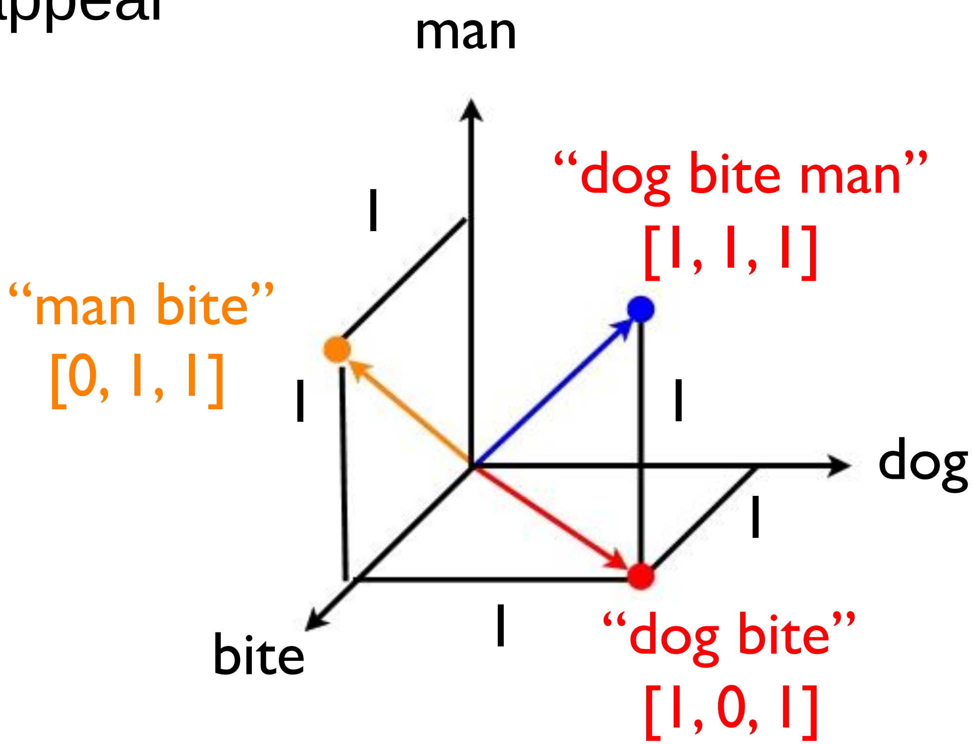
	dog	*man*	*bite*
doc_1	1	1	1
doc_2	1	0	1



Vector Space Representation with binary weights

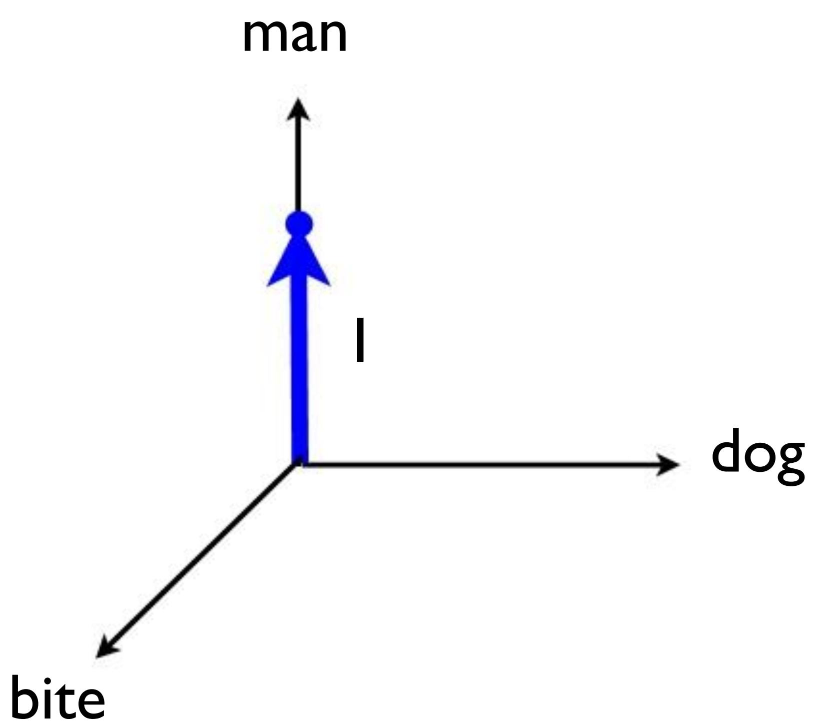
- 1 = the term appears at least once
- 0 = the term does <u>not</u> appear

	dog	*man*	*bite*
doc_1	1	1	1
doc_2	1	0	1
doc_3	0	1	1



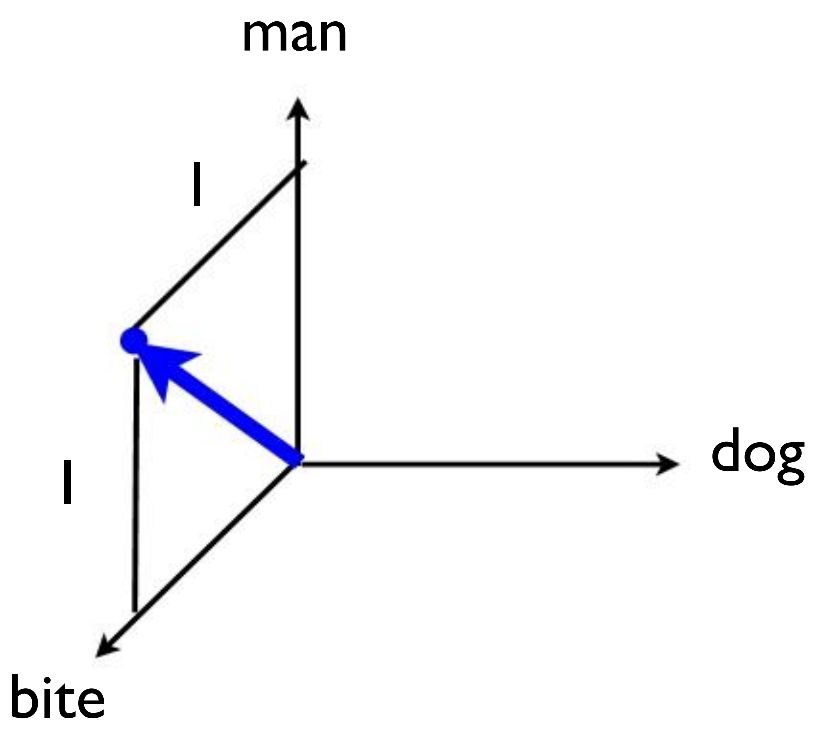
Vector Space Representation with binary weights

- What span(s) of text does this vector represent?



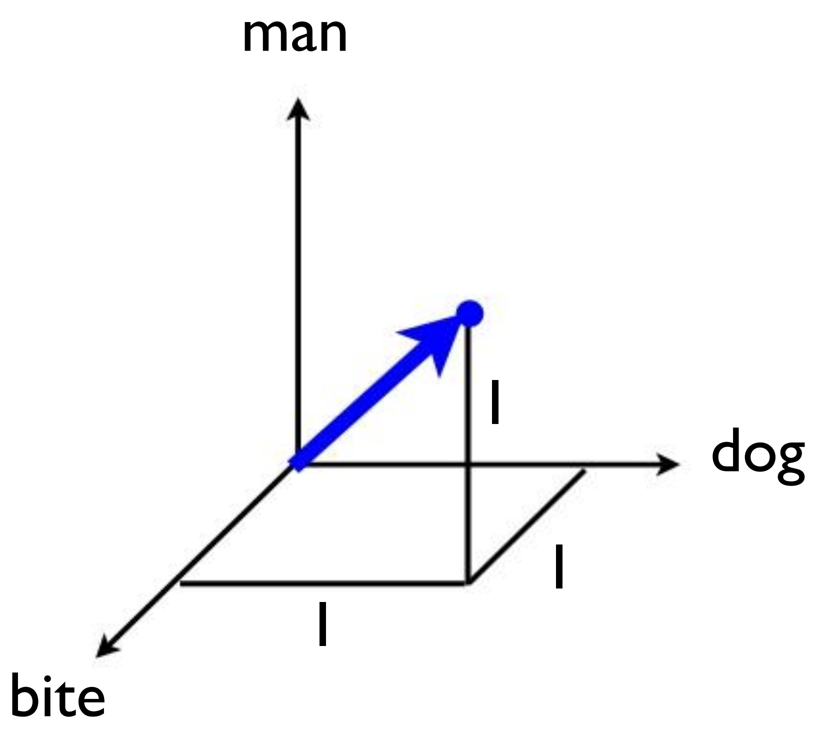
Vector Space Representation with binary weights

- What span(s) of text does this vector represent?



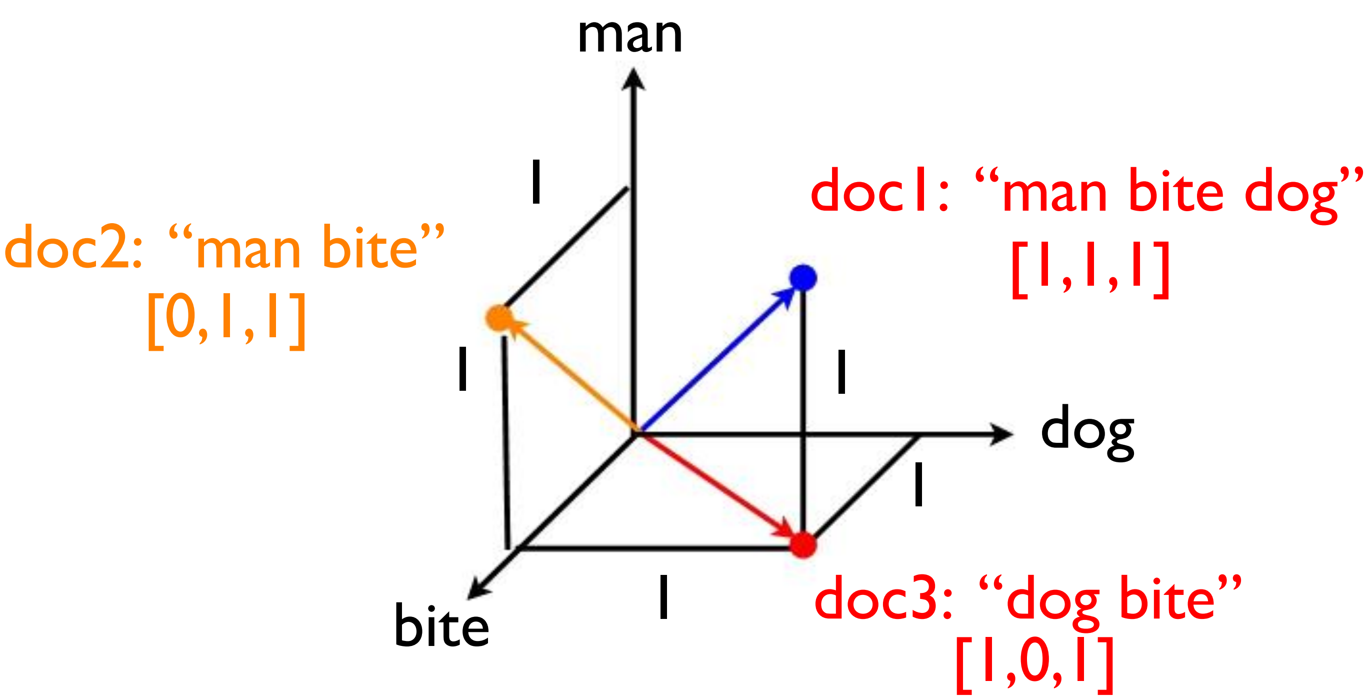
Vector Space Representation with binary weights

- What span(s) of text does this vector represent?



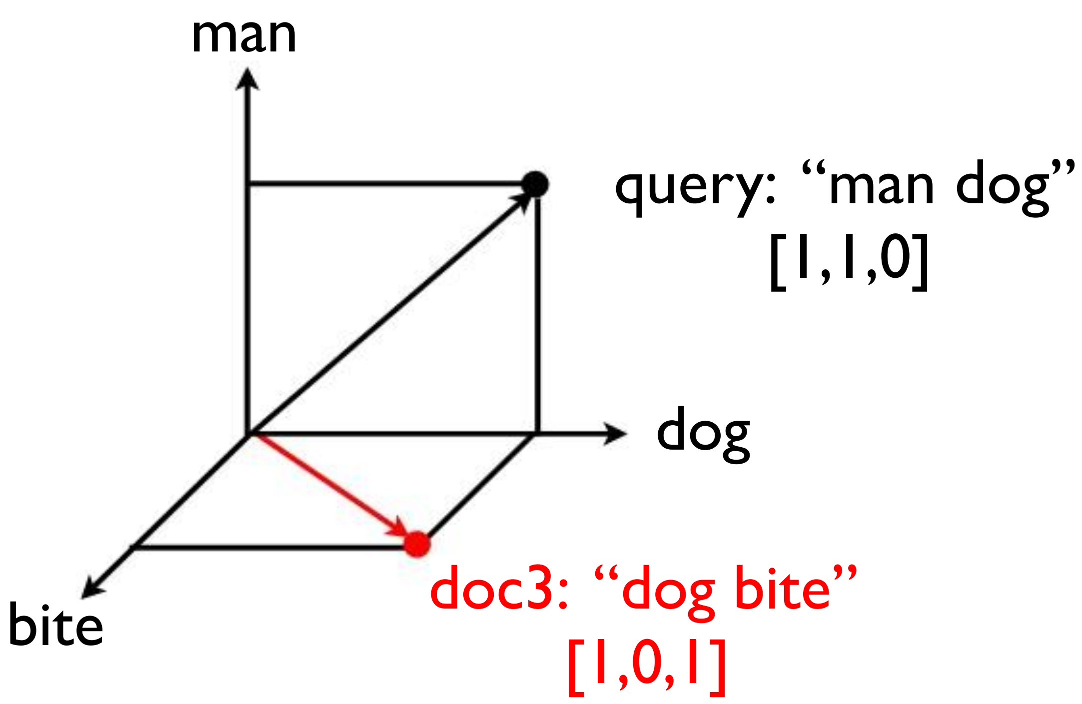
Vector Space Representation

- Any span of text is a vector in V-dimensional space, where V is the size of the vocabulary



Vector Space Representation

- A query is a vector in V-dimensional space, where V is the number of terms in the vocabulary



Vector Space Similarity

- The vector space model ranks documents based on the vector-space similarity between the query vector and the document vector
- There are many ways to compute the similarity between two vectors
- One way is to compute the vector product (inner product)

$$\sum_{i=1}^{V} a_i \times b_i$$

The Inner Product

- Multiply corresponding components and then sum of those products

$$\sum_{i=1}^{V} a_i \times b_i$$

	d_1	d_2	$d_1 \times d_2$
a	1	1	1
aardvark	0	1	0
abacus	1	1	1
abba	1	0	0
able	0	1	0
::	::	::	::
zoom	0	0	0
		inner product =>	2

The Inner Product

- When using 0's and 1's, this is just the number of terms in common between the query and the document

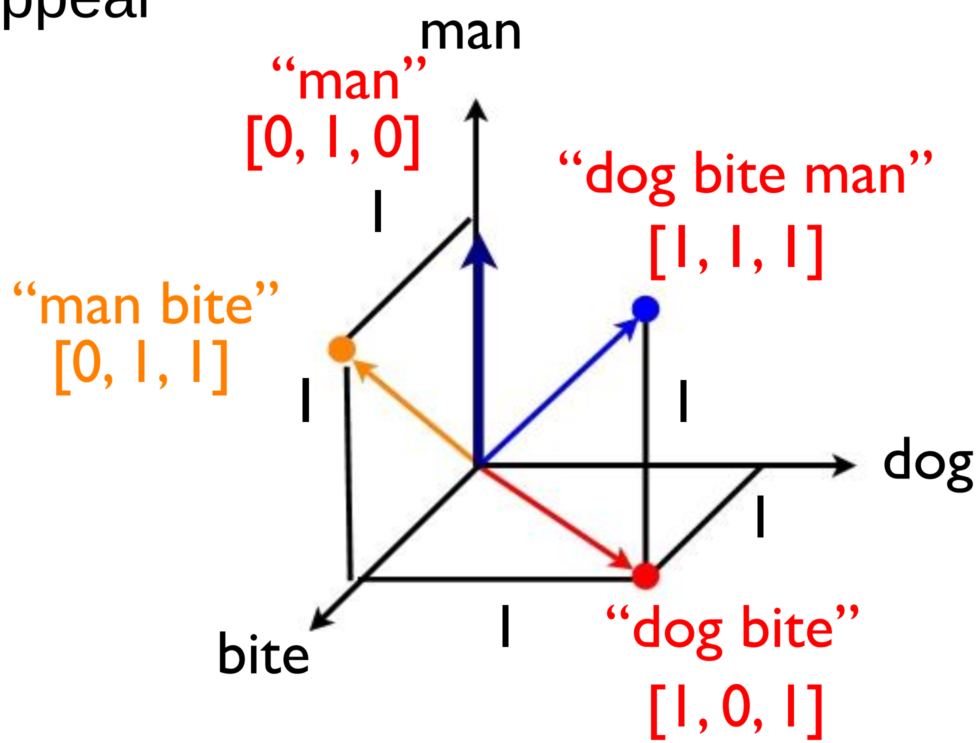
$$\sum_{i=1}^{V} a_i \times b_i$$

	d_1	d_2	$d_1 \times d_2$
a	1	1	1
aardvark	0	1	0
abacus	1	1	1
abba	1	0	0
able	0	1	0
::	::	::	::
zoom	0	0	0
		inner product =>	2

The Inner Product

- 1 = the term appears at least once
- 0 = the term does not appear

	dog	*man*	*bite*
doc_1	1	1	1
doc_2	1	0	1
doc_3	0	1	1
doc_4	0	1	0



The Inner Product

- Multiply corresponding components and then sum those products
- Using a binary representation, the inner product corresponds to the number of terms appearing (at least once) in both spans of text
- Scoring documents based on their inner-product with the query has one major issue. Any ideas?

The Inner Product

- What is more relevant to a query?
  - A 50-word document which contains 3 of the query-terms?
  - A 100-word document which contains 3 of the query-terms?
- The inner-product doesn’t account for the fact that documents have widely varying lengths
- All things being equal, longer documents are more likely to have the query-terms
- So, the inner-product favors long documents

The Cosine Similarity

$$\frac{\sum_{i=1}^{V} x_i \times y_i}{\sqrt{\sum_{i=1}^{V} x_i^2} \times \sqrt{\sum_{i=1}^{V} y_i^2}}$$

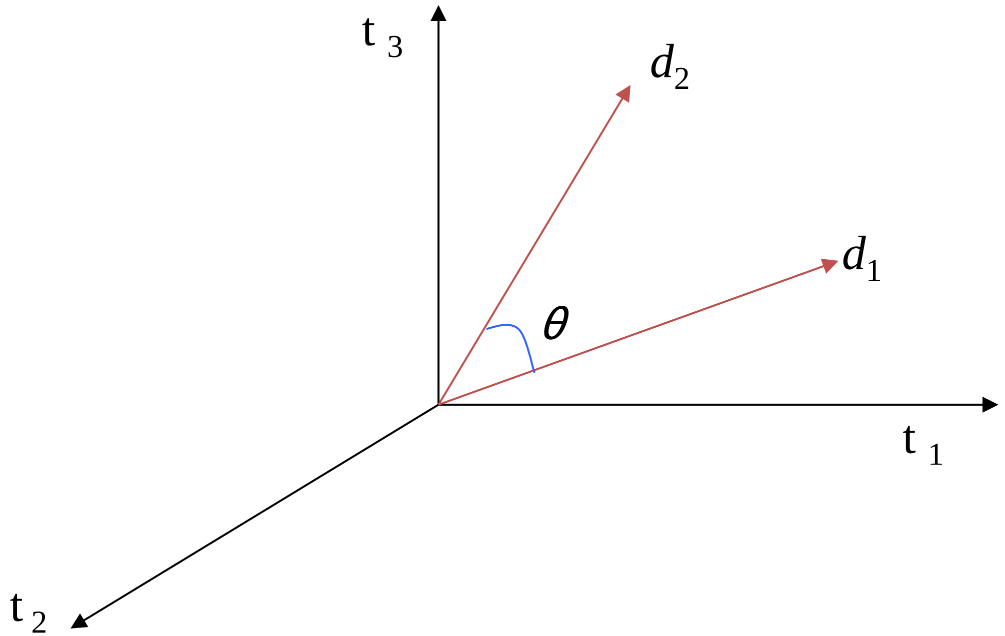
length of vector x

length of vector y

- Measures the cosine of the angle between the two vectors
- The numerator is the inner product
- The denominator “normalizes” for document length
- Ranges from 0 to 1 (equals 1 if the vectors are identical)
- Determines whether the two vectors are pointing in the same direction

The Cosine Similarity

- Distance between vectors d_1 and d_2 *captured* by the cosine of the angle x between them.
- Note – this is *similarity*, not distance



Exercise

cosine(“dog bite” , “man dog”) = ?

cosine([1,0,1] , [1,1,0]) =

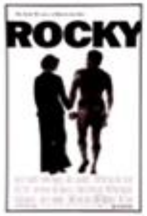
$$\frac{(1 \times 1) + (0 \times 1) + (1 \times 0)}{\sqrt{1^2 + 0^2 + 1^2} \times \sqrt{1^2 + 1^2 + 0^2}} = 0.5$$

Vector Space Representation

	a	*aardvark*	*abacus*	*abba*	*able*	*zoom*
doc_1	1	0	0	0	0	1
doc_2	0	0	0	0	1	1
::	::	::	::	::	::	0
doc_m	0	0	1	1	0	0

	a	*aardvark*	*abacus*	*abba*	*able*	*zoom*
query	0	1	0	0	1	1

- So far, we've assumed binary vectors
- 0's and 1's indicate whether the term occurs (at least once) in the document/query
- Let's explore a more sophisticated representation



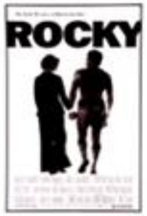
Term-Weighting

what are the most important terms?

- Movie: Rocky (1976)

- Plot:

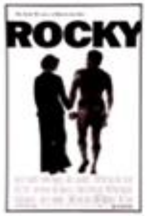
  Rocky Balboa is a struggling boxer trying to make the big time. Working in a meat factory in Philadelphia for a pittance, he also earns extra cash as a debt collector. When heavyweight champion Apollo Creed visits Philadelphia, his managers want to set up an exhibition match between Creed and a struggling boxer, touting the fight as a chance for a "nobody" to become a "somebody". The match is supposed to be easily won by Creed, but someone forgot to tell Rocky, who sees this as his only shot at the big time. Rocky Balboa is a small-time boxer who lives in an apartment in Philadelphia, Pennsylvania, and his career has so far not gotten off the canvas. Rocky earns a living by collecting debts for a loan shark named Gazzo, but Gazzo doesn't think Rocky has the viciousness it takes to beat up deadbeats. Rocky still boxes every once in a while to keep his boxing skills sharp, and his ex-trainer, Mickey, believes he could've made it to the top if he was willing to work for it. Rocky, goes to a pet store that sells pet supplies, and this is where he meets a young woman named Adrian, who is extremely shy, with no ability to talk to men. Rocky befriends her. Adrain later surprised Rocky with a dog from the pet shop that Rocky had befriended. Adrian's brother Paulie, who works for a meat packing company, is thrilled that someone has become interested in Adrian, and Adrian spends Thanksgiving with Rocky. Later, they go to Rocky's apartment, where Adrian explains that she has never been in a man's apartment before. Rocky sets her mind at ease, and they become lovers. Current world heavyweight boxing champion Apollo Creed comes up with the idea of giving an unknown a shot at the title. Apollo checks out the Philadelphia boxing scene, and chooses Rocky. Fight promoter Jergens gets things in gear, and Rocky starts training with Mickey. After a lot of training, Rocky is ready for the match, and he wants to prove that he can go the distance with Apollo. The 'Italian Stallion', Rocky Balboa, is an aspiring boxer in downtown Philadelphia. His one chance to make a better life for himself is through his boxing and Adrian, a girl who works in the local pet store. Through a publicity stunt, Rocky is set up to fight Apollo Creed, the current heavyweight champion who is already set to win. But Rocky really needs to triumph, against all the odds...



Term-Frequency

how important is a term?

rank	term	freq.	rank	term	freq.
1	a	22	16	creed	5
2	rocky	19	17	philadelphia	5
3	to	18	18	has	4
4	the	17	19	pet	4
5	is	11	20	boxing	4
6	and	10	21	up	4
7	in	10	22	an	4
8	for	7	23	boxer	4
9	his	7	24	s	3
10	he	6	25	balboa	3
11	adrian	6	26	it	3
12	with	6	27	heavyweigh	3
13	who	6	28	champion	3
14	that	5	29	fight	3
15	apollo	5	30	become	3



Term-Frequency

how important is a term?

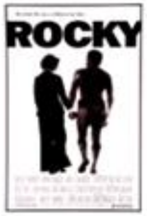
rank	term	freq.	rank	term	freq.
1	a	22	16	creed	5
2	rocky	19	17	philadelphia	5
3	to	18	18	has	4
4	the	17	19	pet	4
5	is	11	20	boxing	4
6	and	10	21	up	4
7	in	10	22	an	4
8	for	7	23	boxer	4
9	his	7	24	s	3
10	he	6	25	balboa	3
11	adrian	6	26	it	3
12	with	6	27	heavyweigh	3
13	who	6	28	champion	3
14	that	5	29	fight	3
15	apollo	5	30	become	3

Inverse Document Frequency (IDF)

how important is a term?

$$idf_t = \log(\frac{N}{df_t})$$

- N = number of documents in the collection
- df_t = number of documents in which term t appears



Inverse Document Frequency (IDF)

how important is a term?

rank	term	idf	rank	term	idf
1	doesn	11.66	16	creed	6.84
2	adrain	10.96	17	paulie	6.82
3	viciousness	9.95	18	packing	6.81
4	deadbeats	9.86	19	boxes	6.75
5	touting	9.64	20	forgot	6.72
6	jergens	9.35	21	ease	6.53
7	gazzo	9.21	22	thanksgivin	6.52
8	pittance	9.05	23	earns	6.51
9	balboa	8.61	24	pennsylvani	6.50
10	heavyweigh	7.18	25	promoter	6.43
11	stallion	7.17	26	befriended	6.38
12	canvas	7.10	27	exhibition	6.31
13	ve	6.96	28	collecting	6.23
14	managers	6.88	29	philadelphia	6.19
15	apollo	6.84	30	gear	6.18

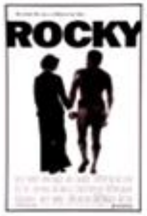
TF.IDF

how important is a term?

$$tf_t \times idf_t$$

greater when the term is frequent in in the document

greater when the term is rare in the collection (does not appear in many documents)



TF.IDF

how important is a term?

rank	term	tf.idf	rank	term	tf.idf
1	rocky	96.72	16	meat	11.76
2	apollo	34.20	17	doesn	11.66
3	creed	34.18	18	adrain	10.96
4	philadelphia	30.95	19	fight	10.02
5	adrian	26.44	20	viciousness	9.95
6	balboa	25.83	21	deadbeats	9.86
7	boxing	22.37	22	touting	9.64
8	boxer	22.19	23	current	9.57
9	heavyweigh	21.54	24	jergens	9.35
10	pet	21.17	25	s	9.29
11	gazzo	18.43	26	struggling	9.21
12	champion	15.08	27	training	9.17
13	match	13.96	28	pittance	9.05
14	earns	13.01	29	become	8.96
15	apartment	11.82	30	mickey	8.96

TF.IDF/Caricature Analogy

- TF.IDF: accentuates terms that are frequent in the document, but not frequent in general
- Caricature: exaggerates traits that are characteristic of the person (compared to the average)

Queries as TF.IDF Vectors

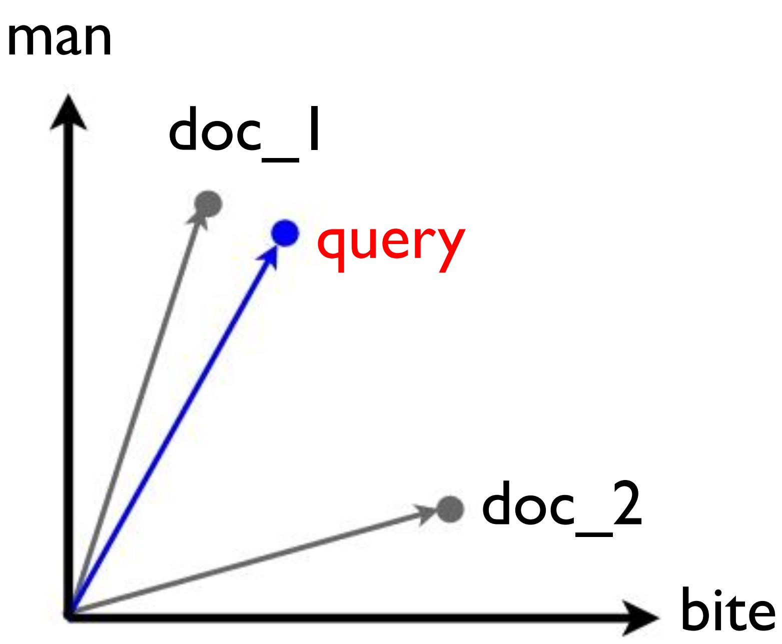
- Terms tend to appear only once in the query
- TF usually equals 1
- IDF is computed using the collection statistics

$$idf_t = \log\left(\frac{N}{df_t}\right)$$

- N is the total number of documents in the corpus
- Terms appearing in fewer documents get a higher weight

Putting Everything Together

- Rank documents based on cosine similarity to the query



TF.IDF

$$tf_t \times log\left(\frac{N}{df_t}\right)$$

term	tf	N	df	idf	tf.idf
rocky	19	230721	1420	5.09	96.72
philadelphia	5	230721	473	6.19	30.95
boxer	4	230721	900	5.55	22.19
fight	3	230721	8170	3.34	10.02
mickey	2	230721	2621	4.48	8.96
for	7	230721	117137	0.68	4.75

TF.IDF

- Many variants of this formula have been proposed
- However, they all have two components in common:
  - TF: favors terms that are frequent in the document
  - IDF: favors terms that do not occur in many documents

$$tf_t \times log\left(\frac{N}{df_t}\right)$$

Sub-linear TF Scaling

- Suppose ‘rocky’ occurs twice in document A and once in document B
- Is A twice as much about rocky than B?
- Suppose ‘rocky’ occurs 20 times in document A and 10 times in document B
- Is A twice as much about rocky than B?

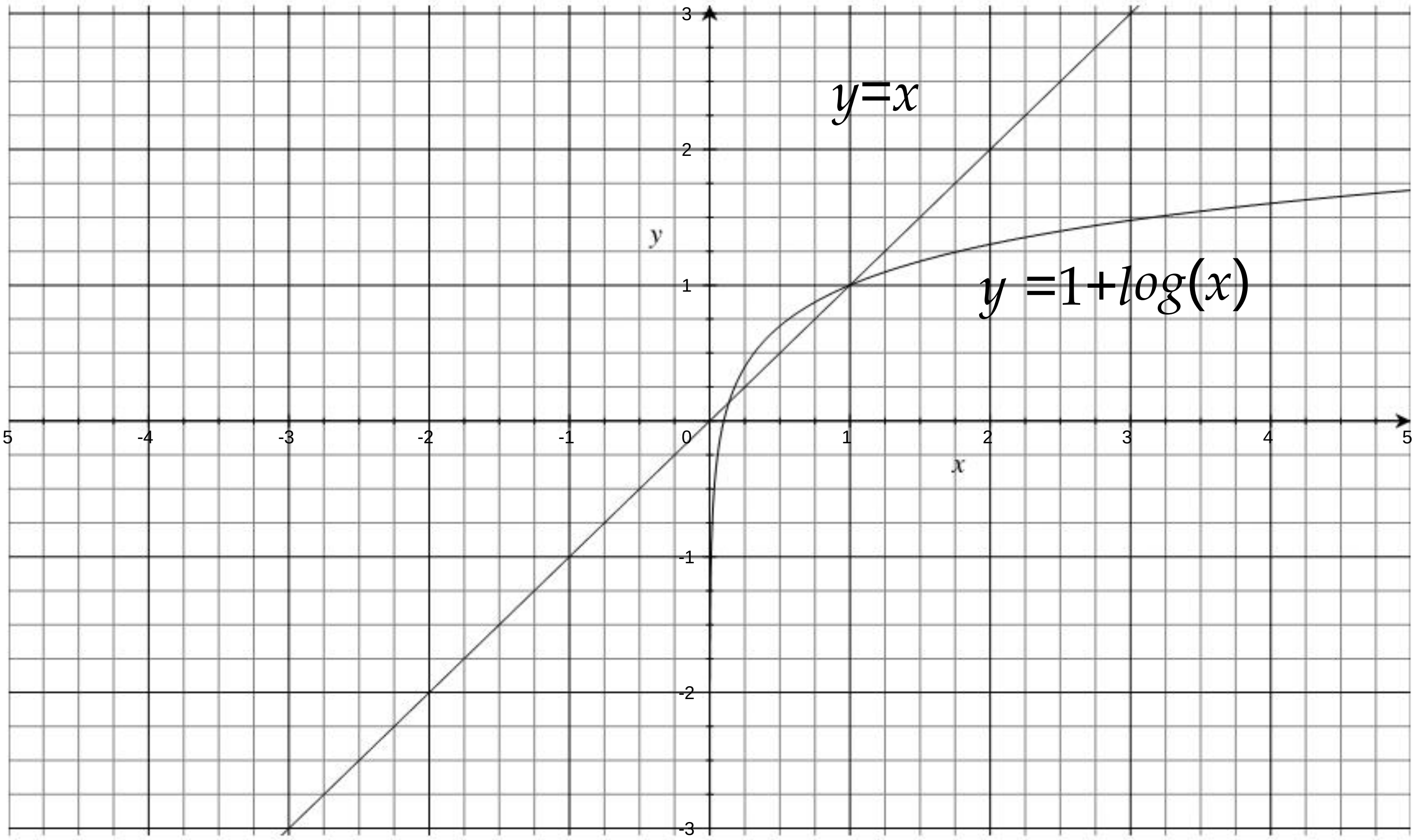
Sub-linear TF Scaling

- It turns out that IR systems are more effective when they assume this is not the case

Sub-linear TF Scaling

- Assumption:
  - A document that contains ‘rocky’ 5 times is more about rocky than one that contains ‘rocky’ 1 time
  - How much more?
  - Roughly, 5 times more
  - A document that contains ‘rocky’ 50 times is more about rocky than one that contains ‘rocky’ 10 times
  - How much more?
  - Not 5 times more. Less.

Sub-linear TF Scaling



TF.IDF

what are the most important terms?

$$(1 + log(tf_t)) \times log\left(\frac{N}{df_t}\right)$$

term	tf	fw	N	df	idf	tf.idf
rocky	19	3.94	230721	1420	5.09	20.08
philadelphia	5	2.61	230721	473	6.19	16.15
boxer	4	2.39	230721	900	5.55	13.24
fight	3	2.10	230721	8170	3.34	7.01
mickey	2	1.69	230721	2621	4.48	7.58
for	7	2.95	230721	117137	0.68	2.00

TF.IDF

what are the most important terms?

	$tf_t \times log\left(\frac{N}{df_t}\right)$	$(1 + log(tf_t)) \times log\left(\frac{N}{df_t}\right)$
term	**tf.idf (linear tf)**	**tf.idf (sub-linear tf)**
rocky	96.72	20.08
philadelphia	30.95	16.15
boxer	22.19	13.24
fight	10.02	7.01
mickey	8.96	7.58
for	4.75	2.00

Vector Space Model

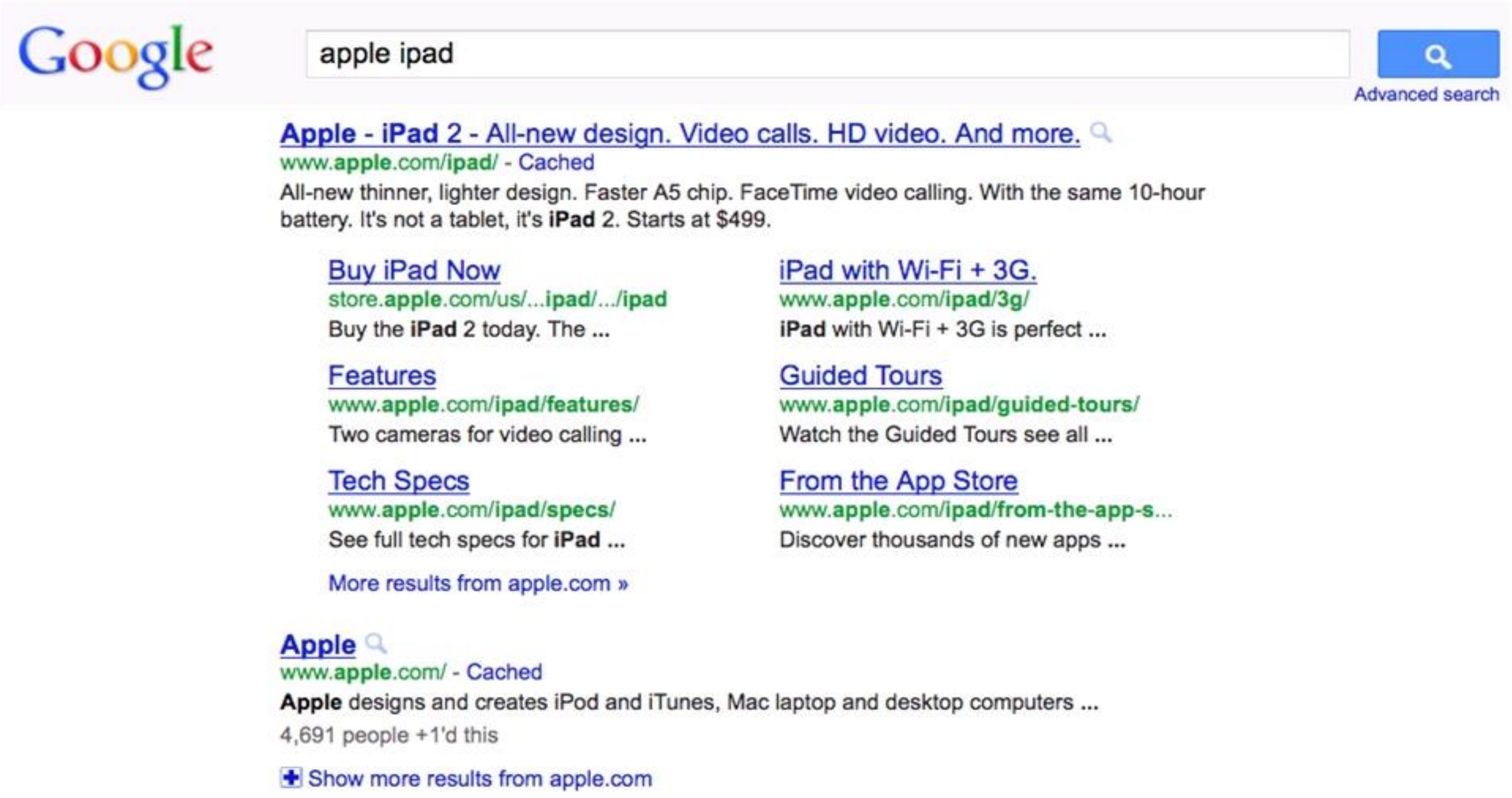
- Any text can be seen as a vector in V-dimensional space
  - a document
  - a query
  - a sentence
  - a word
  - an entire encyclopedia
- Rank documents based on their cosine similarity to query
- If a document is similar to the query, it is likely to be relevant (remember: topical relevance!)

Vector Space Representation

- A powerful tool!
- A lot of problems in IR can be cast as:
  - Find me ______ that is similar to ______ !
- As long as ______ and ______ are associated with text, one potential solution is:
  - represent these items as tf.idf term-weight vectors and compute their cosine similarity
  - return the items with the highest similarity

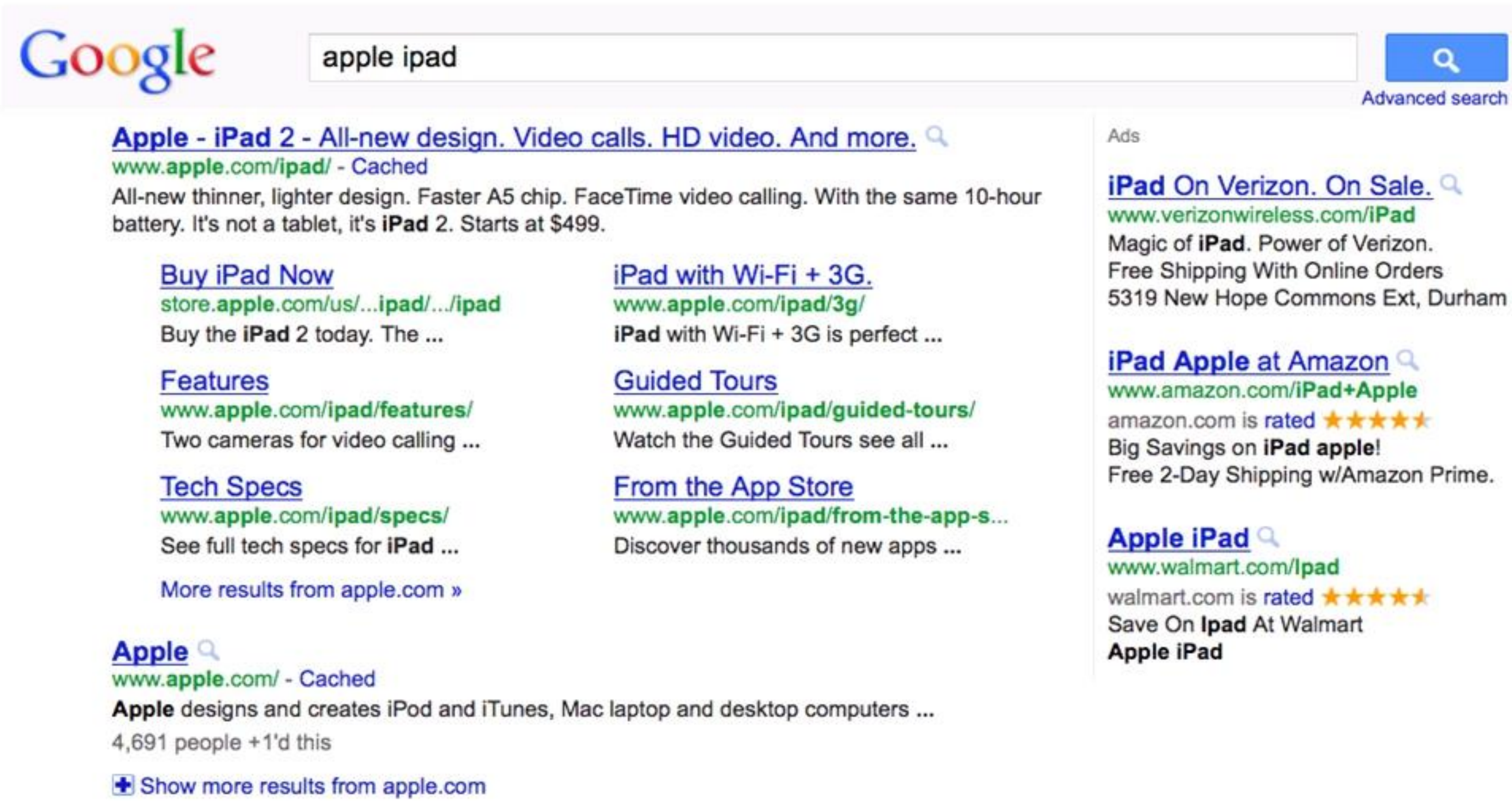
Vector Space Representation

- Find me documents that are similar to this query



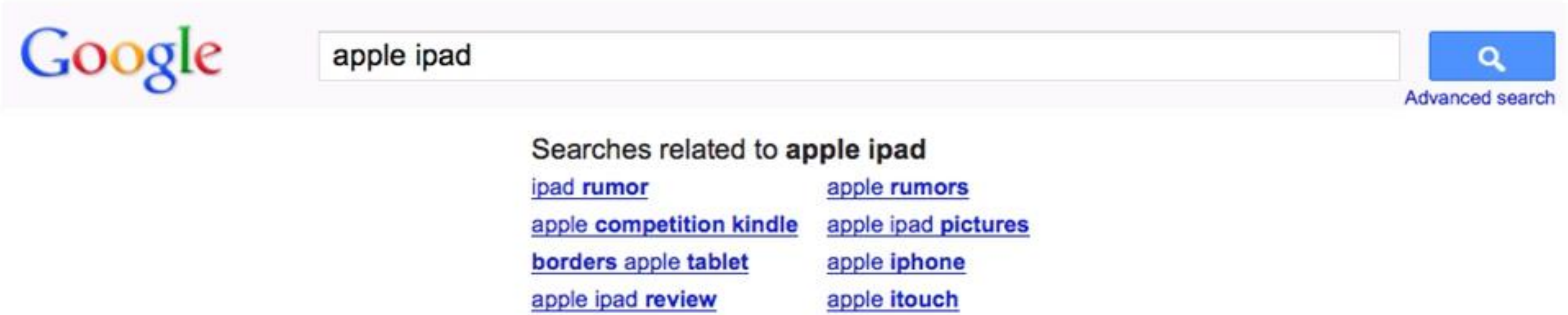
Vector Space Representation

- Find me ads that are similar to these results



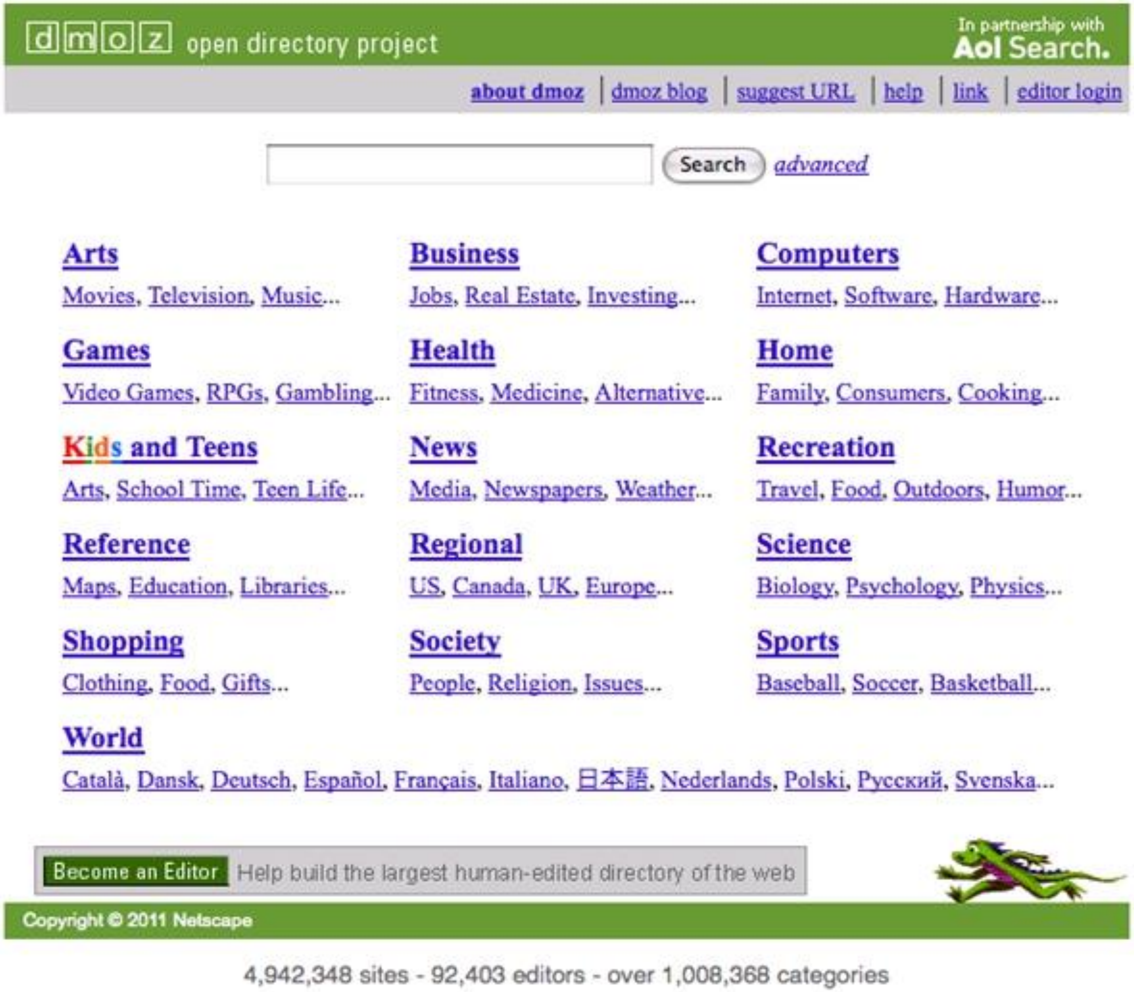
Vector Space Representation

- Find me queries that are similar to this query



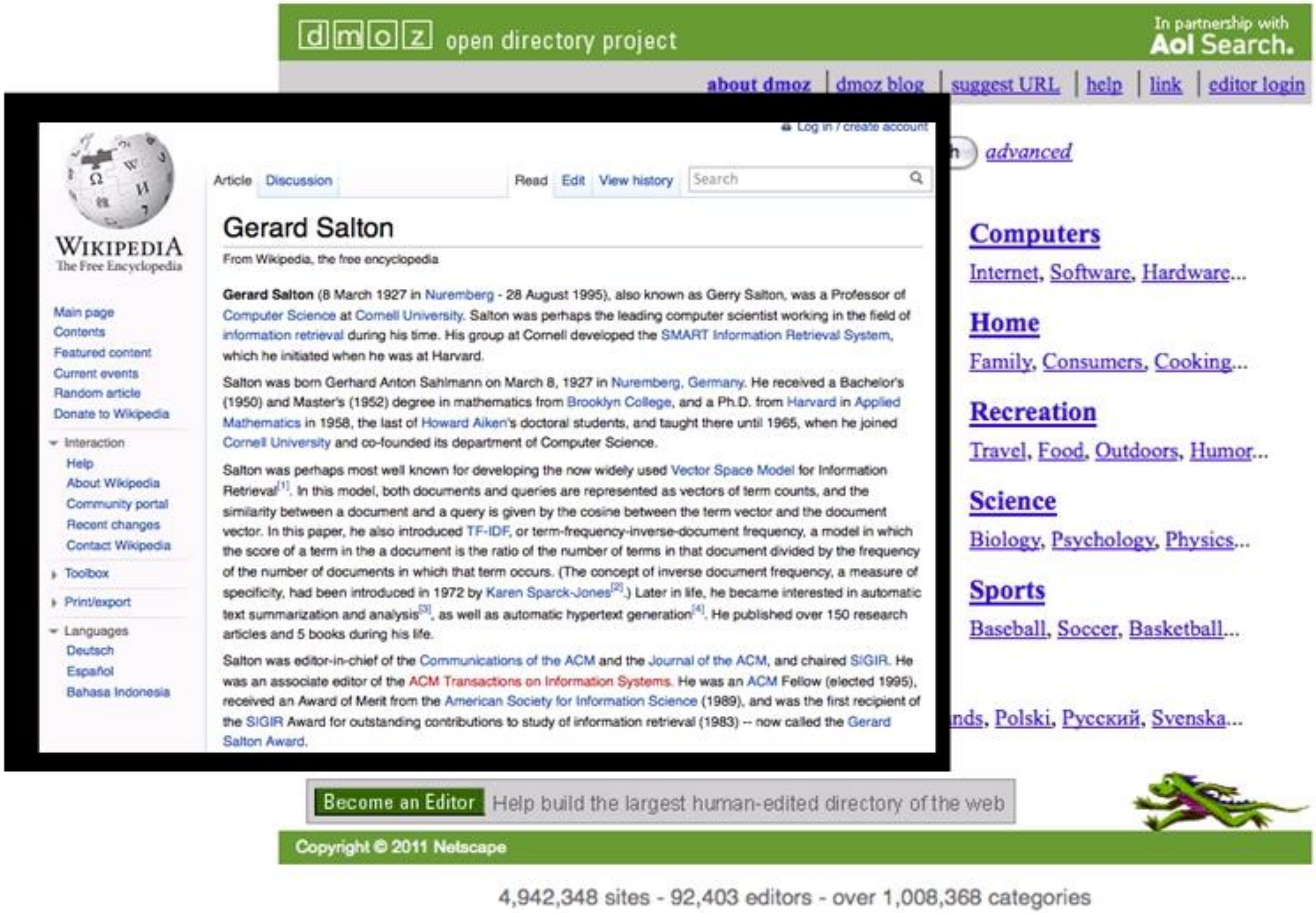
Vector Space Representation

- Categorization: automatically assigning a document to a category



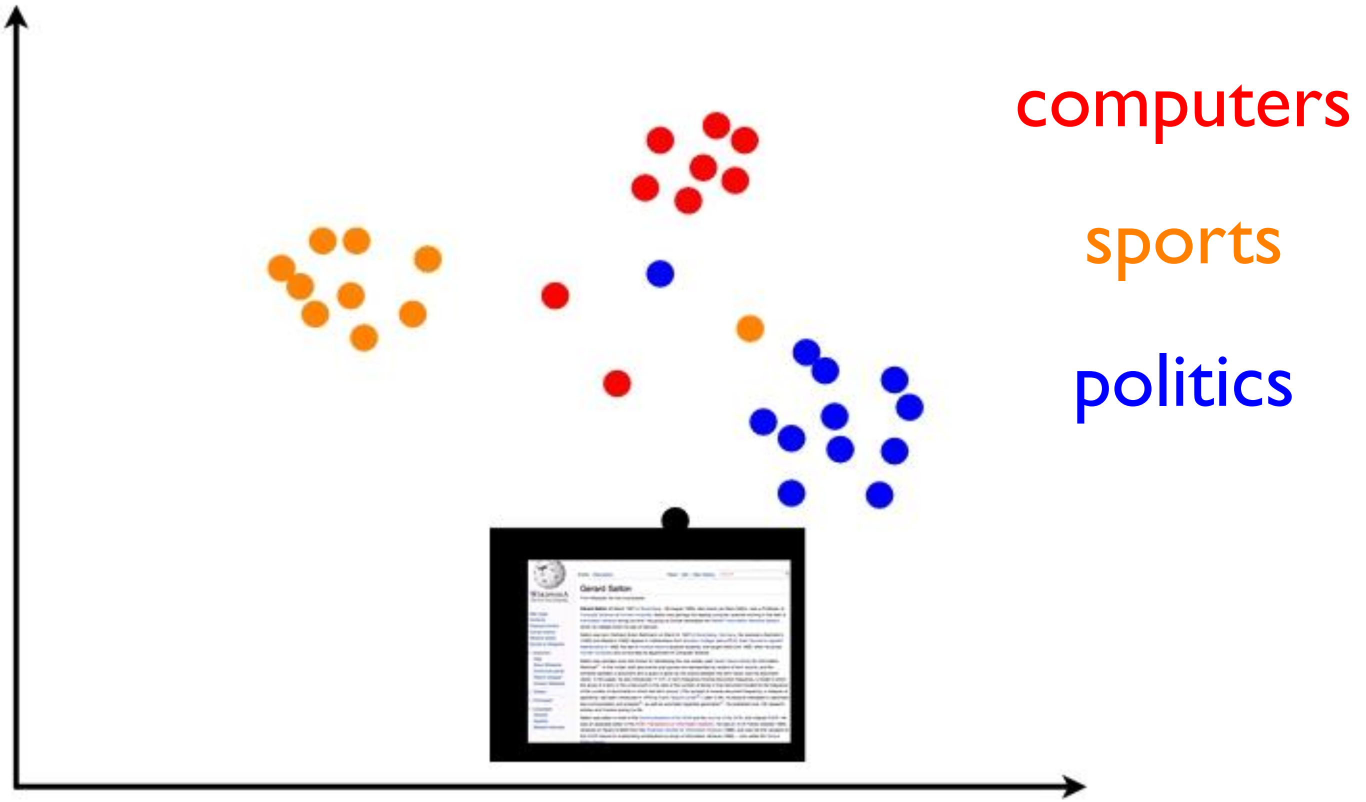
Vector Space Representation

- Find me documents (with a known category assignment) that are similar to this document



Vector Space Representation

- Find me documents (with a known category assignment) that are similar to this document



Advertisement Placement

- Find me ads similar to this this document

Anatidaephobia - The Fear That You are Being Watched by a Duck

December 08, 2008 by Tammy Duffey

Single page Font Size Read comments (44) Share

WAS THIS HELPFUL?
137
CLICK TO VOTE

Popular searches: YouTube | Rihanna | Tiger Woods | Search more

What Is Anatidaephobia?

Anatidaephobia is defined as a pervasive, irrational fear that one is being watched by a duck. The anatidaephobic individual fears that no matter where they are or what they are doing, a duck watches.

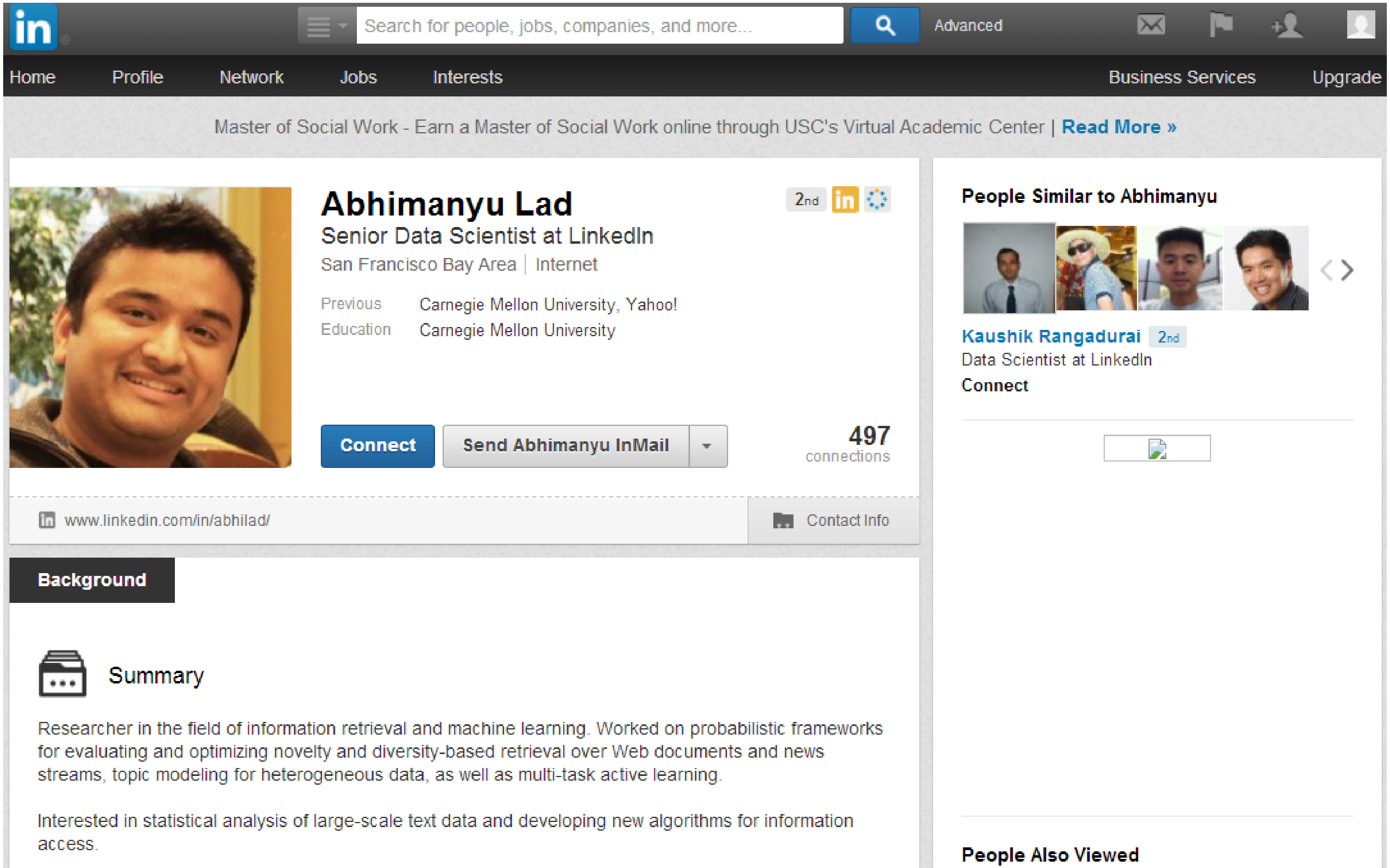
Anatidaephobia is derived from the Greek word "anatidae", meaning ducks, geese or swans and "phobos" meaning fear.

What Causes Anatidaephobia?

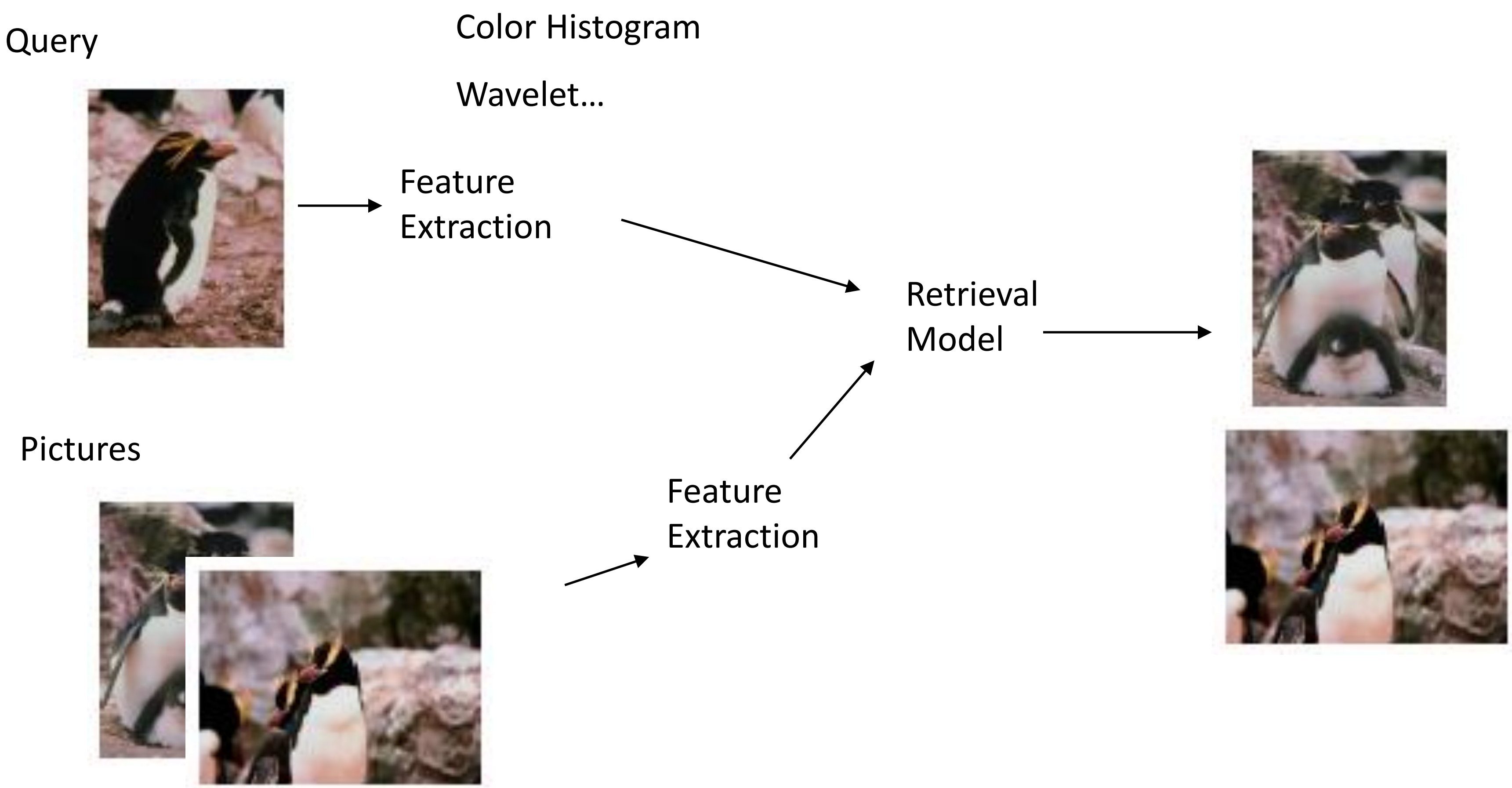
As with all phobias, the person coping with Anatidaephobia has experienced a real-life trauma. For the anatidaephobic individual, this trauma most likely occurred during childhood.

Perhaps the individual was intensely frightened by some species of water fowl. Geese and swans are relatively well known for their aggressive tendencies and perhaps the anatidaephobic person was actually bitten or flapped at. Of course, the Far Side comics did little to minimize the fear of being watched by a duck.

Similar People Recommendation



Multimedia Retrieval



SMART weightings

- Named after a widely used IR system
- Library of weightings schemes fitting the Vector Space Model (cosine similarity)
- Based on the following weighting:

$$w(t,d) = \frac{tf'_{t,d} \times idf'_t}{norm'_d}$$

SMART Weighting Scheme

Term frequency		Document frequency		Normalization	
n (natural)	$\mathrm{tf}_{t,d}$	n (no)	1	n (none)	1
l (logarithm)	$1 + \log(\mathrm{tf}_{t,d})$	t (idf)	$\log \frac{N}{\mathrm{df}_t}$	c (cosine)	$\frac{1}{\sqrt{w_1^2 + w_2^2 + \ldots + w_M^2}}$
a (augmented)	$0.5 + \frac{0.5 \times \mathrm{tf}_{t,d}}{\max_t(\mathrm{tf}_{t,d})}$	p (prob idf)	$\max\{0, \log \frac{N - \mathrm{df}_t}{\mathrm{df}_t}\}$	u (pivoted unique)	$1/u$ (Section 6.4.4)
b (boolean)	$\begin{cases} 1 & \text{if } \mathrm{tf}_{t,d} > 0 \\ 0 & \text{otherwise} \end{cases}$			b (byte size)	$1/\mathit{CharLength}^{\alpha}$, $\alpha < 1$
L (log ave)	$\frac{1+\log(\mathrm{tf}_{t,d})}{1+\log(\mathrm{ave}_{t \in d}(\mathrm{tf}_{t,d}))}$				

► **Figure 6.7** SMART notation for tf-idf variants. Here *CharLength* is the number of characters in the document.

Okapi BM25

Okapi BM25 is one of the most popular ranking functions in practice

$$\text{score}(D,Q) = \sum_{i=1}^{n} \text{IDF}(q_i) \cdot \frac{f(q_i, D) \cdot (k_1 + 1)}{f(q_i, D) + k_1 \cdot \left(1 - b + b \cdot \frac{|D|}{\text{avgdl}}\right)},$$

$f(q_i, D)$ Term frequency

$\text{IDF}(q_i)$ Inverse document frequency

$k_1 \in [1.2, 2.0]$ $b = 0.75$