Generative Adversarial Nets and Distances Between Distributions

(from thispersondoesnotexist.com)

CPSC 340, Fall 2021

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(we definitely won't get through all these slides, but let's see!)

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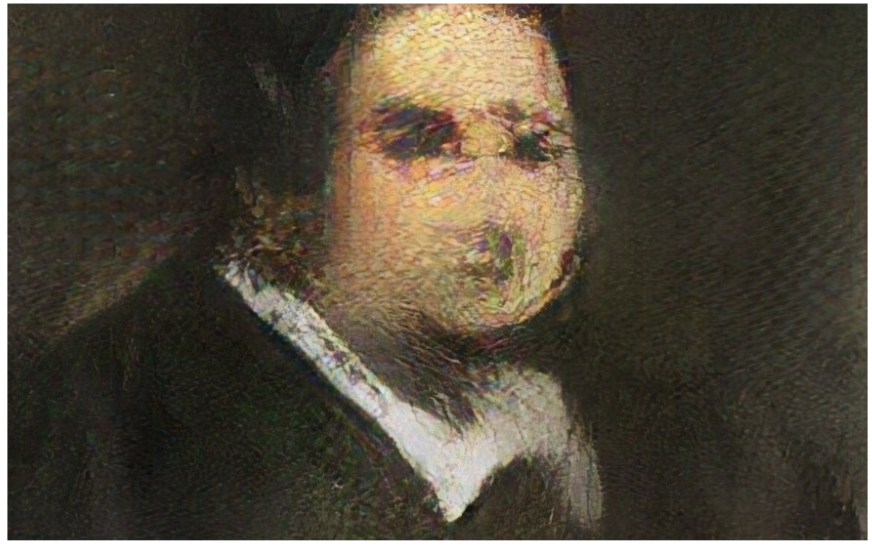
Why produce samples?



Is artificial intelligence set to become art's next medium?

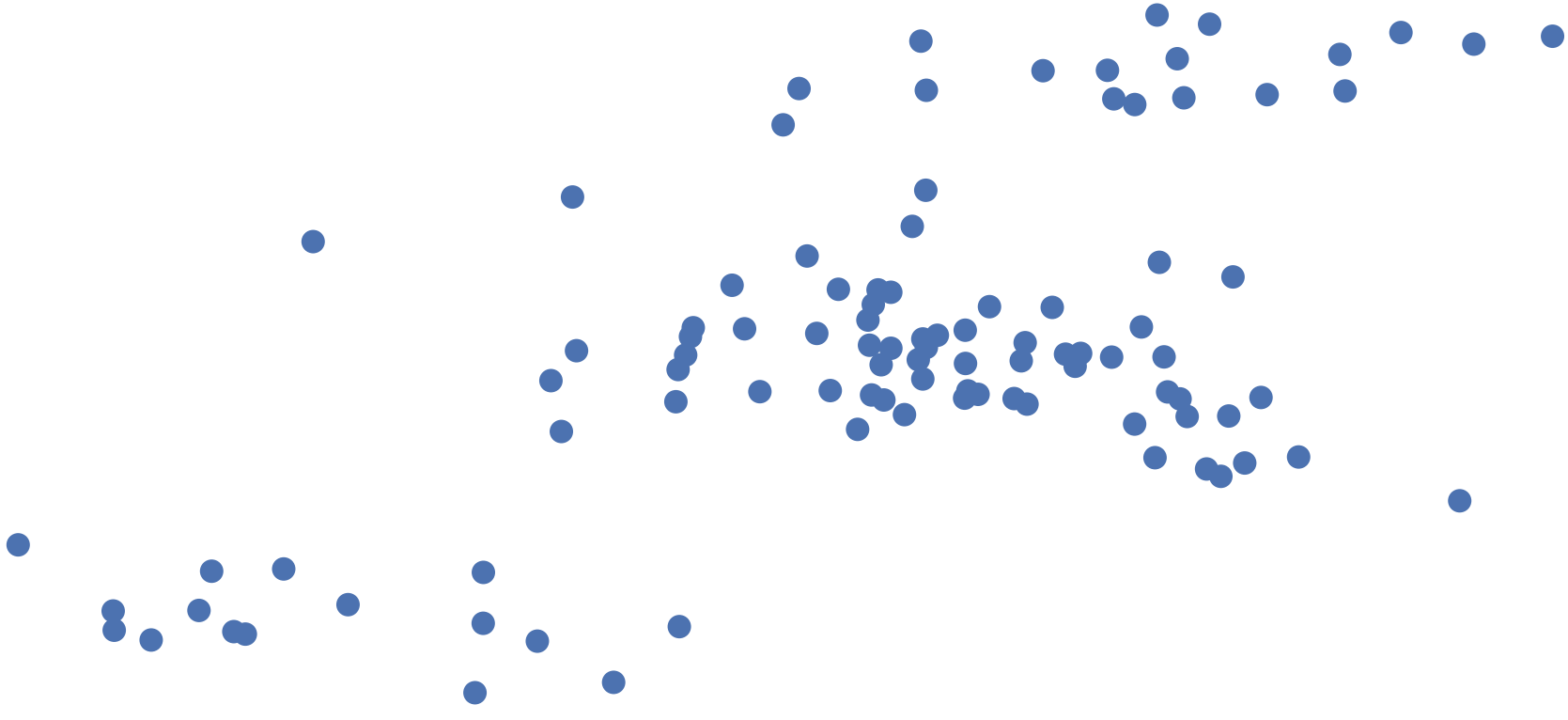
AI artwork sells for $432,500 — nearly 45 times its high estimate — as Christie's becomes the first auction house to offer a work of art created by an algorithm

The portrait in its gilt frame depicts a portly gentleman, possibly French and — to judge by his dark frockcoat and plain white collar — a man of the church. The work appears unfinished: the facial features are somewhat indistinct and there are blank areas of canvas. Oddly, the whole composition is displaced slightly to the north-west. A label on the wall states that the sitter is a man named Edmond Belamy, but the giveaway clue as to the origins of the work is the artist's signature at the bottom right. In cursive Gallic script it reads:

$$\min_G \max_D \mathbb{E}_x[\log(D(x))] + \mathbb{E}_z[\log(1 - D(G(z)))]$$

Image © Obvious

Generative models: a traditional way



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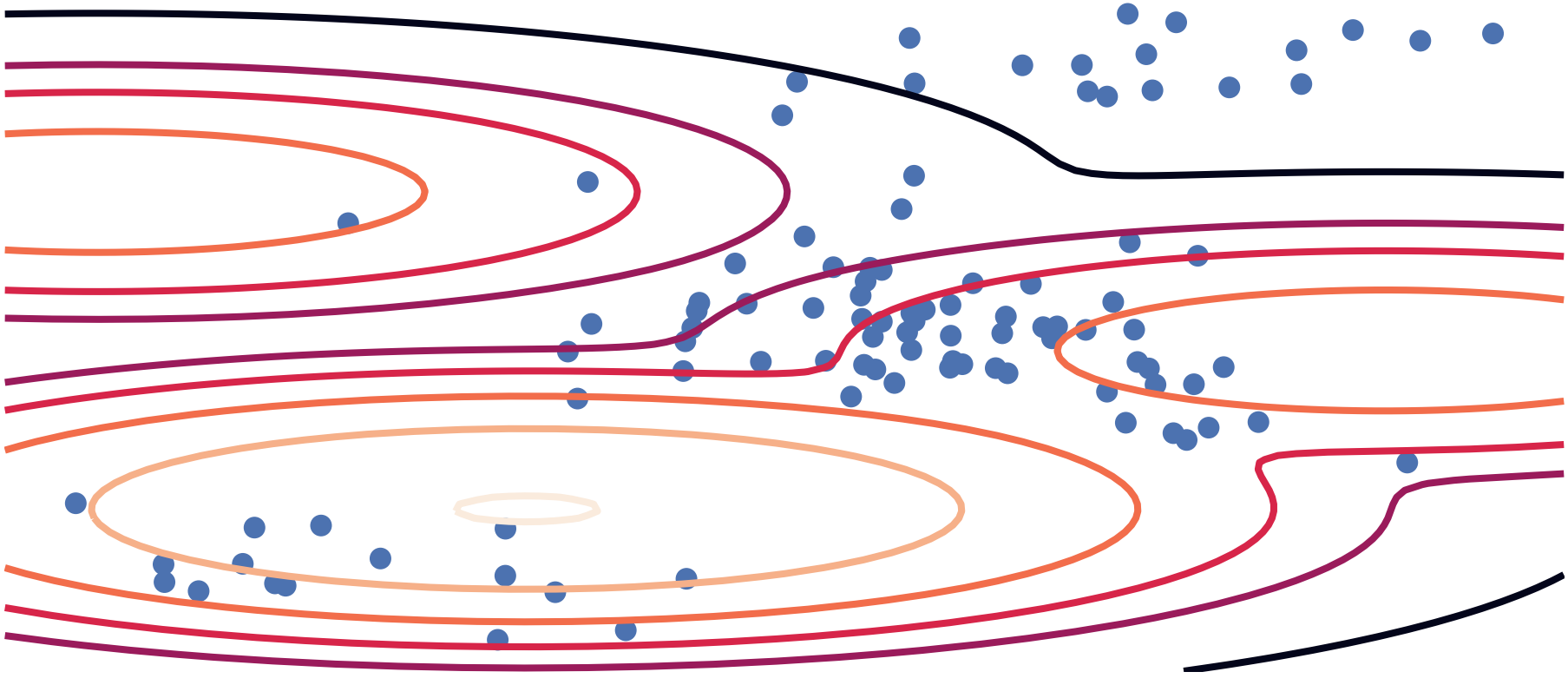

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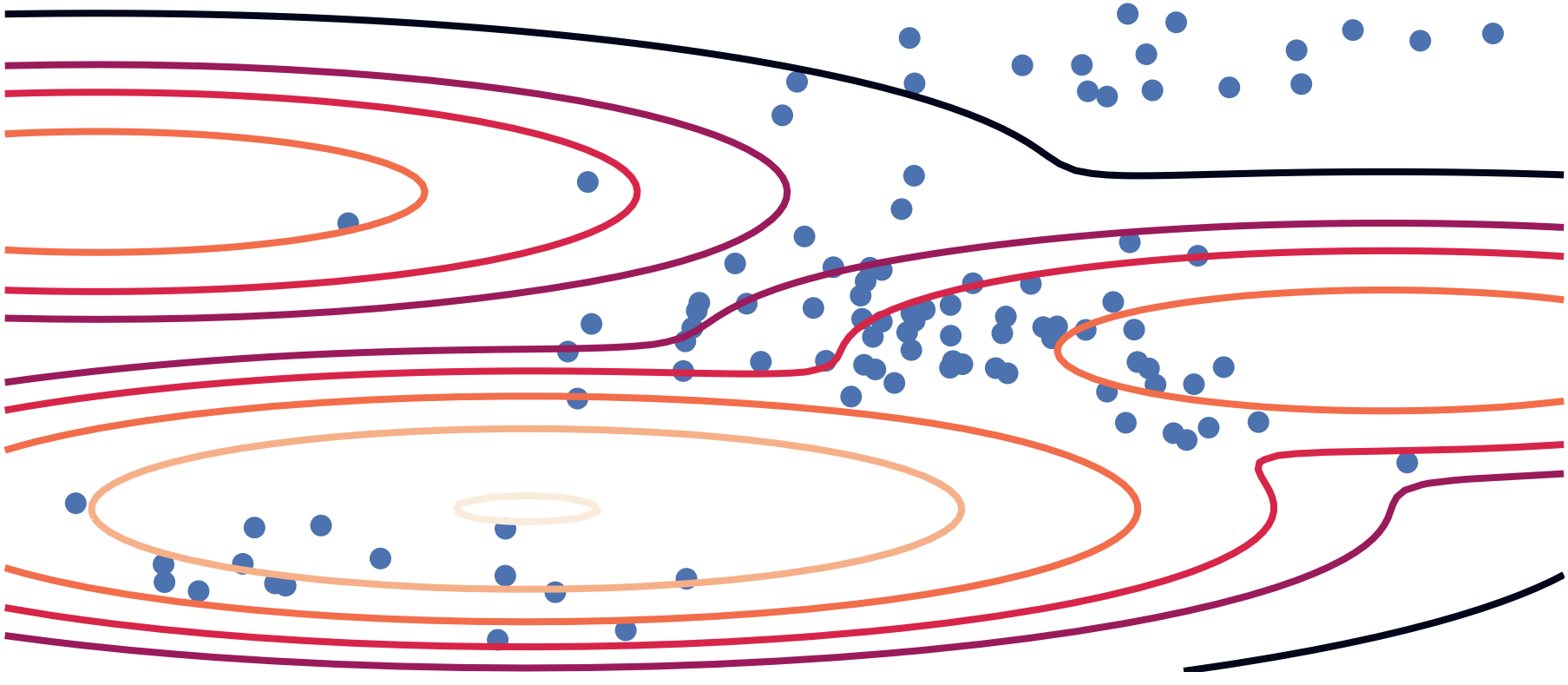

Maximum likelihood: $\max_\theta \mathbb{E}_{X \sim \mathbb{P}} [\log q_\theta(X)]$

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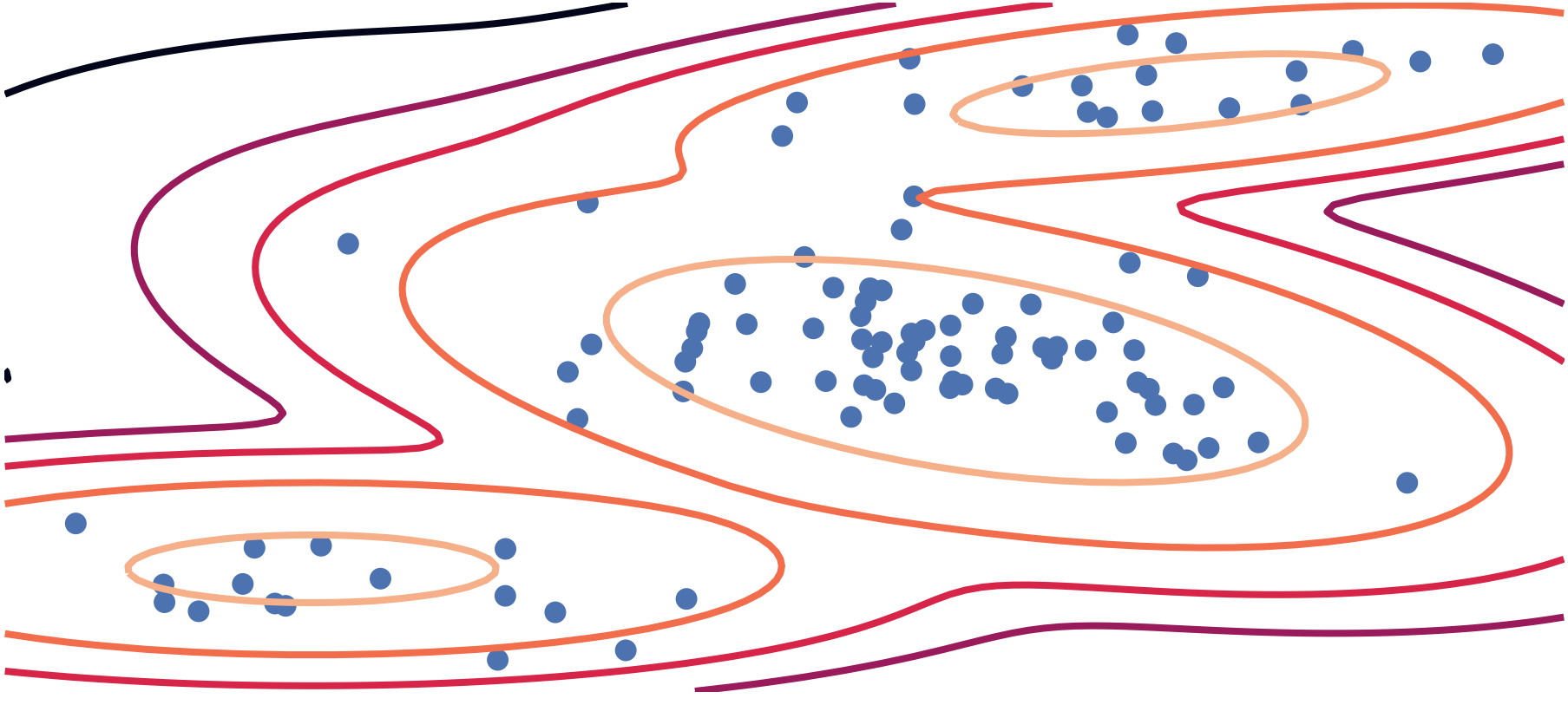

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Kullback-Liebler (KL) divergence

- $\mathrm{KL}(\mathbb{P}\|\mathbb{Q}) = \int p(x) \log \frac{p(x)}{q(x)} \mathrm{d}x$
- A measure of how different two distributions are
- Related to entropy, etc: average number of bits to encode a sample from $\mathbb{P}$ with a code for $\mathbb{Q}$

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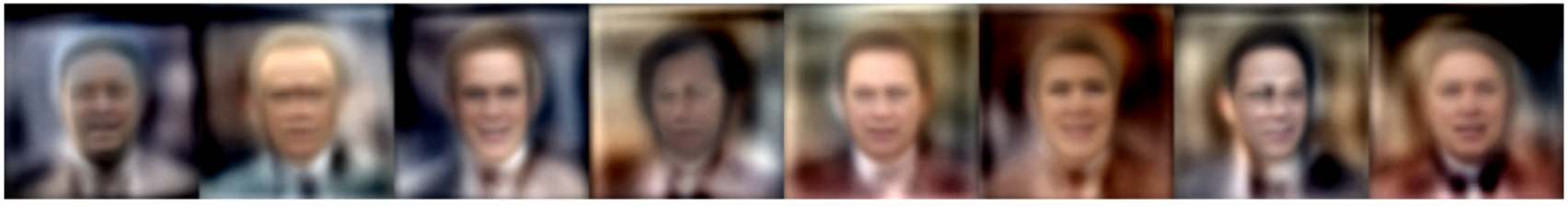
&= \arg\max_\theta \mathop{\mathbb{E}}_{X\sim\mathbb{P}}[\log q_\theta(X)]
\end{aligned}$$

Traditional models for images

- 1987-style generative model of faces (Eigenface via Alex Egg)



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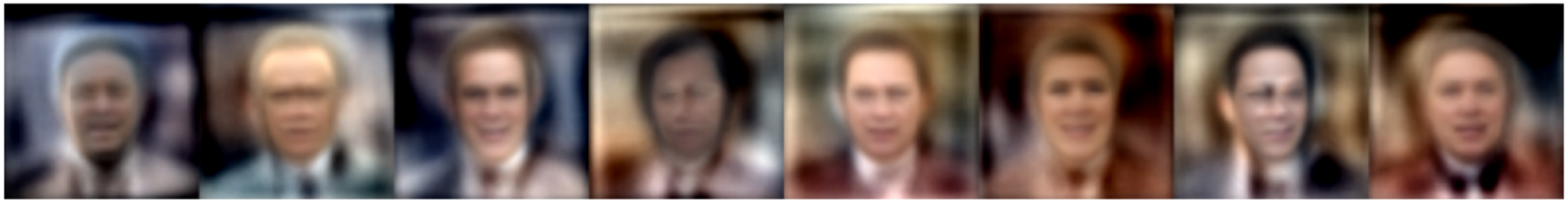

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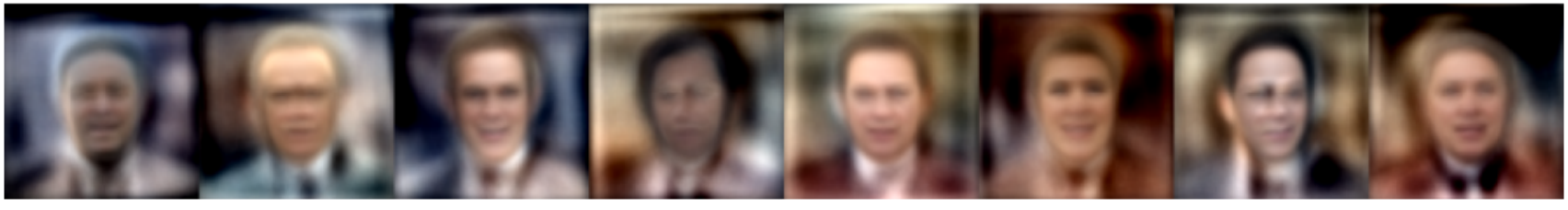

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- Usually based on Gaussian noise $\approx L_2$ loss

A hard case for traditional approaches

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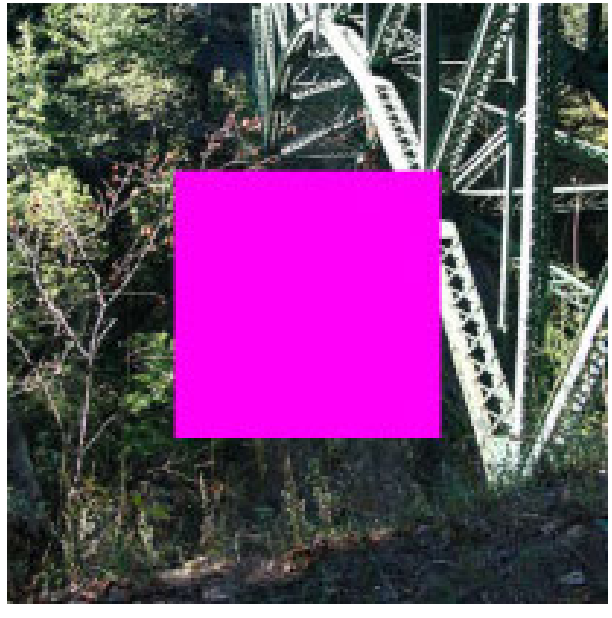

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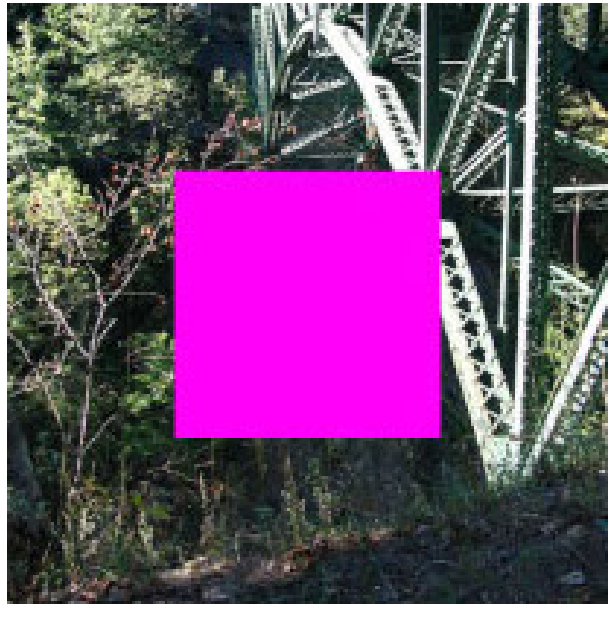

- L_2 loss / Gaussians will pick the *mean* of possibilities

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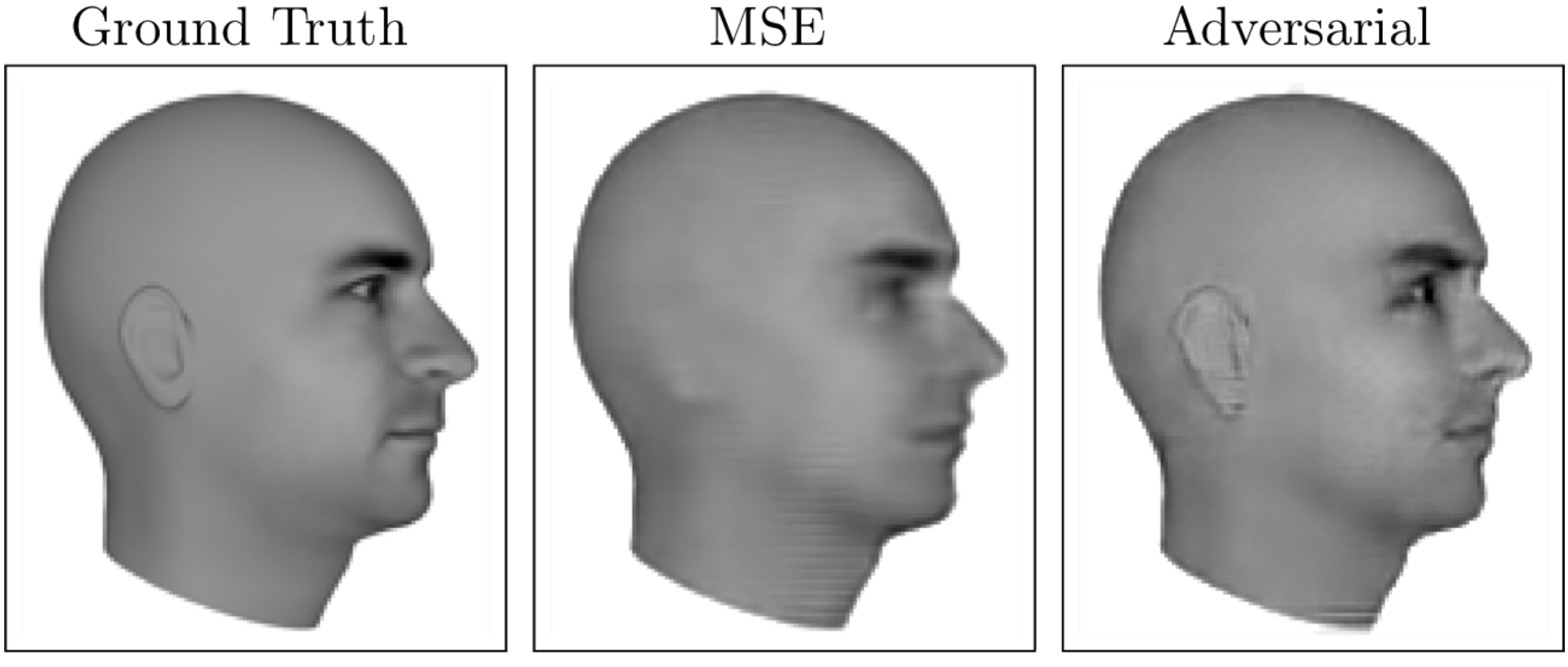
Next-frame video prediction



[Lotter+ 2016]

Trick a discriminator [Goodfellow+ NeurIPS-14]

Generator ($\mathbb{Q}_\theta$)

Discriminator

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Is this real?

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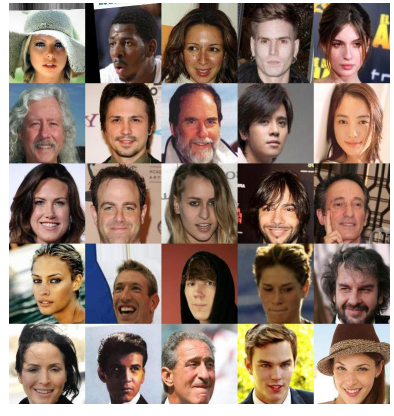

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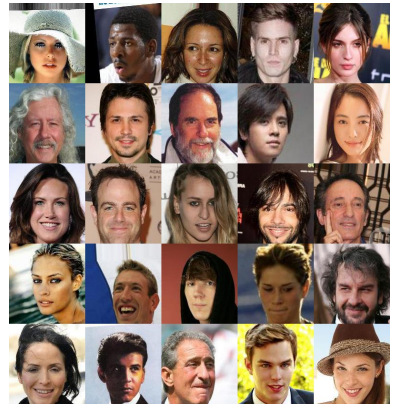

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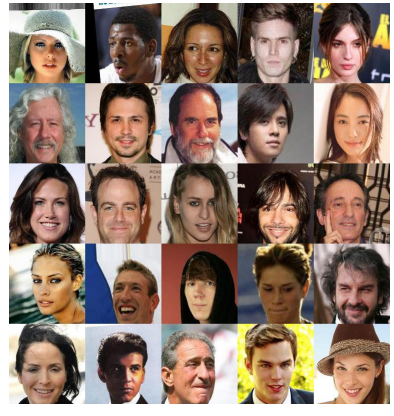

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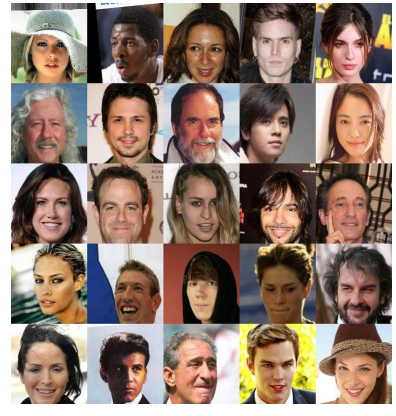

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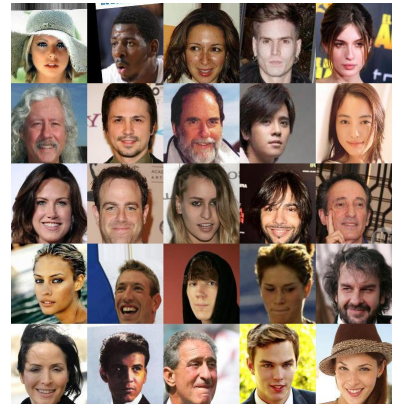

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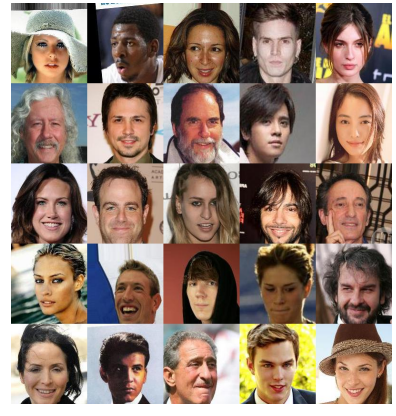

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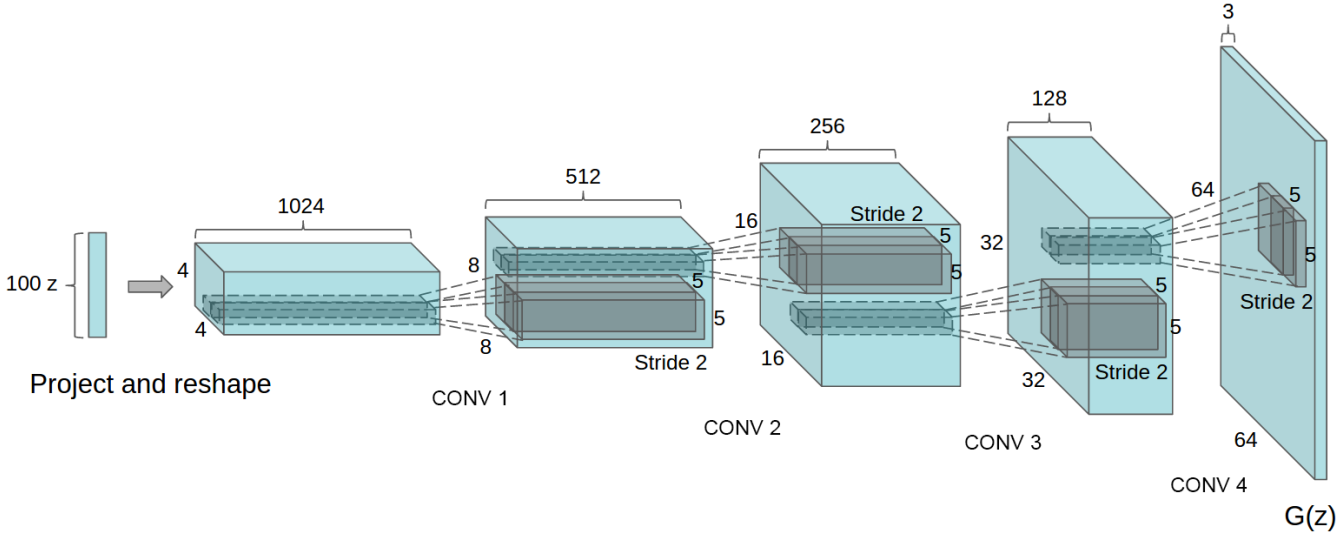
⋮

Is this real?

Umm... $\mathbf{Pr(real) = 0.48}$

Generator networks

- How to specify $\mathbb{Q}_\theta$?



[Radford+ ICLR-16]

- $Z \sim \mathbb{Z} = \mathbf{Uniform}\left([-1,1]^{100}\right)$
- $G_\theta : [-1,1]^{100} \to \mathcal{X}, G_\theta(Z) \sim \mathbb{Q}_\theta$

GANs in equations

- Tricking the discriminator:

$$\min_{\theta} \max_{\psi} \frac{1}{2} \mathop{\mathbb{E}}_{X \sim \mathbb{P}} [\log D_{\psi}(X)] + \frac{1}{2} \mathop{\mathbb{E}}_{Y \sim \mathbb{Q}_{\theta}} [\log(1 - D_{\psi}(Y))]$$

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- Using the generator network for $\mathbb{Q}_{\theta}$:

$$\min_{\theta} \max_{\psi} \frac{1}{2} \mathop{\mathbb{E}}_{X \sim \mathbb{P}} [\log D_{\psi}(X)] + \frac{1}{2} \mathop{\mathbb{E}}_{Z \sim \mathbb{Z}} [\log(1 - D_{\psi}(G_{\theta}(Z)))]$$

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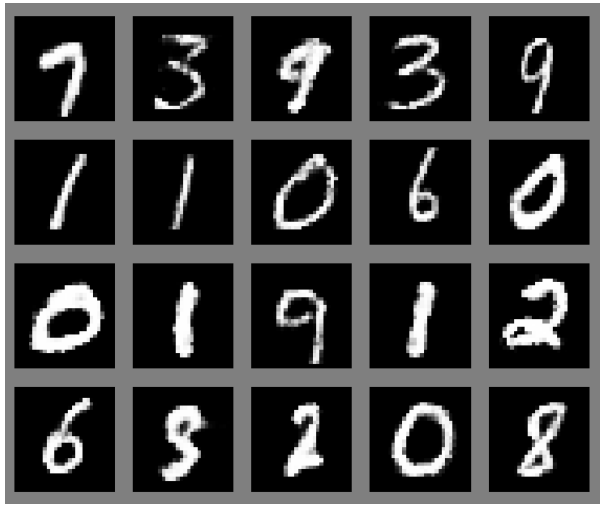
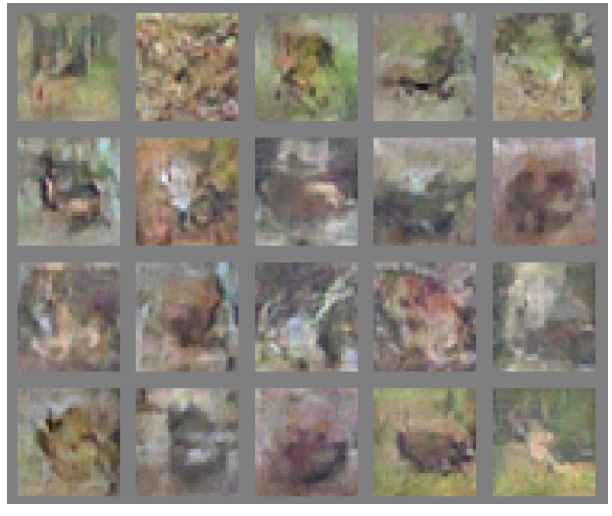
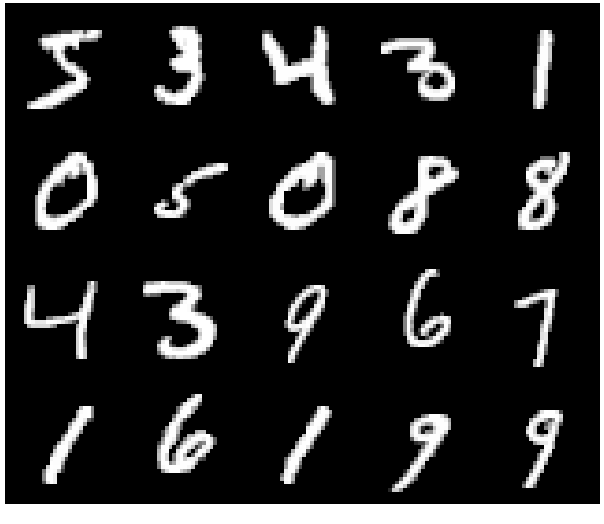
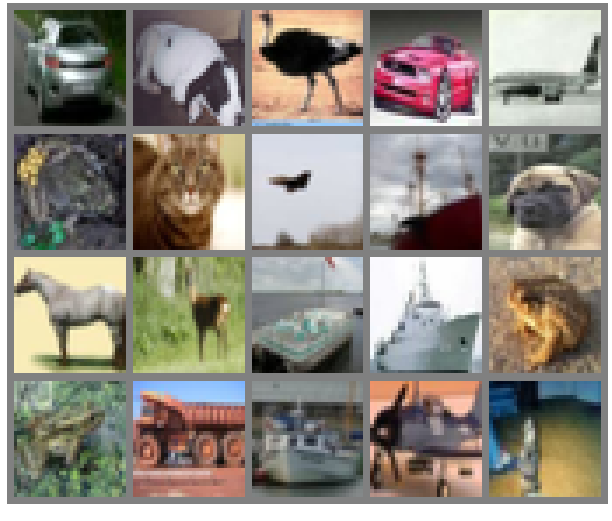
- Can do alternating gradient descent!

Original paper's results [Goodfellow+ NeurIPS-14]





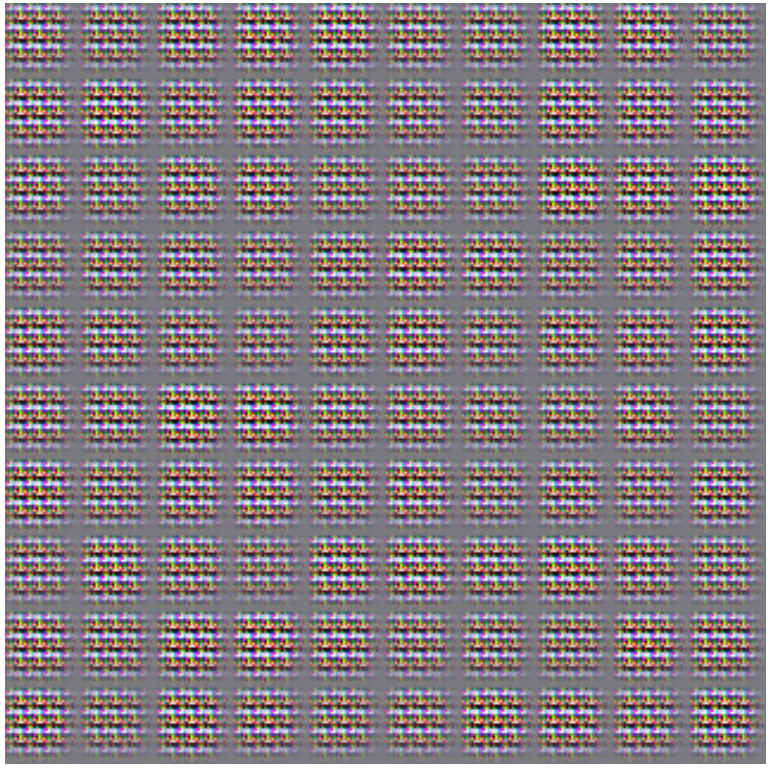




DCGAN results [Radford+ ICLR-16]

Training instability

Running code from [Salimans+ NeurIPS-16]:



Run 1, epoch 1

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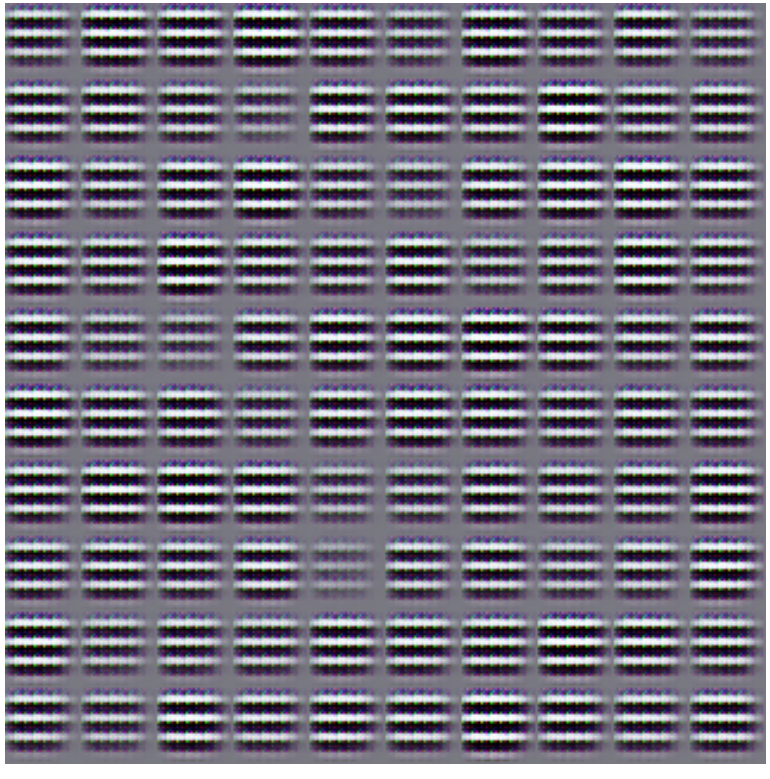
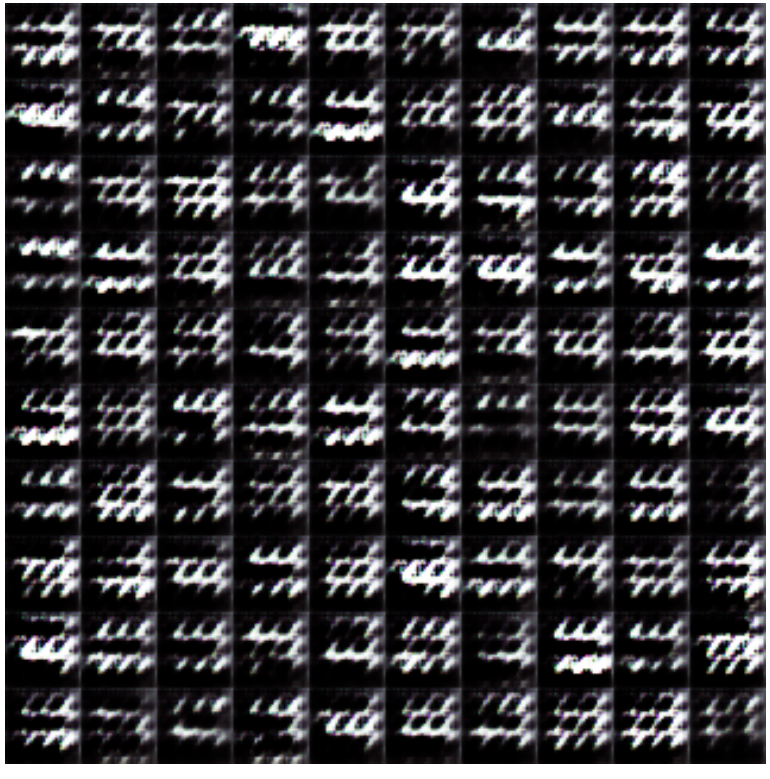


Run 1, epoch 2

Training instability

Running code from [Salimans+ NeurIPS-16]:



Run 1, epoch 3

Training instability

Running code from [Salimans+ NeurIPS-16]:

Run 1, epoch 4

Training instability

Running code from [Salimans+ NeurIPS-16]:

Run 1, epoch 5

Training instability

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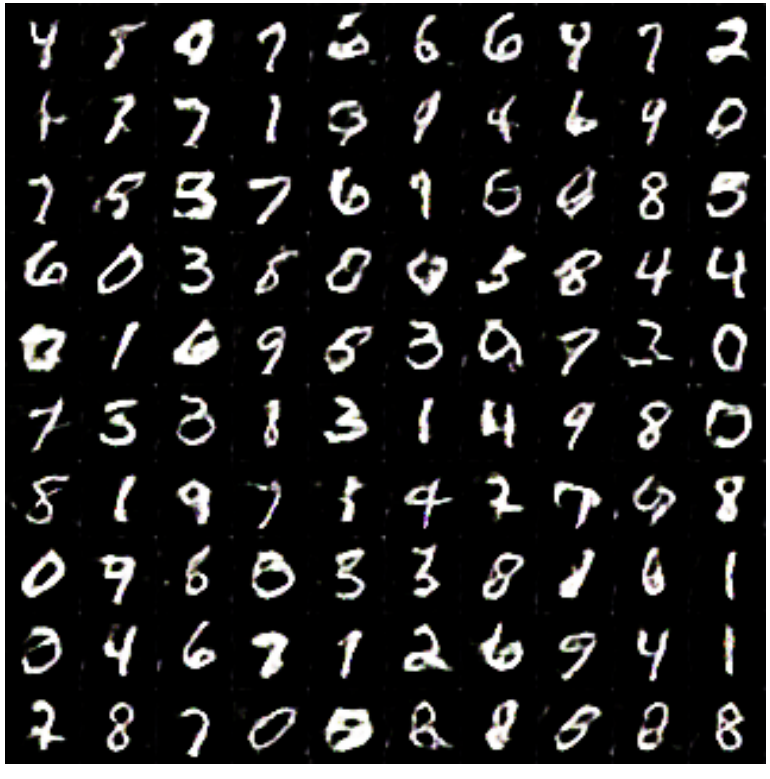

Run 1, epoch 6

Training instability

Running code from [Salimans+ NeurIPS-16]:

Run 1, epoch 11

Training instability

Running code from [Salimans+ NeurIPS-16]:

Run 1, epoch 501

Training instability

Running code from [Salimans+ NeurIPS-16]:

Run 1, epoch 900

Training instability

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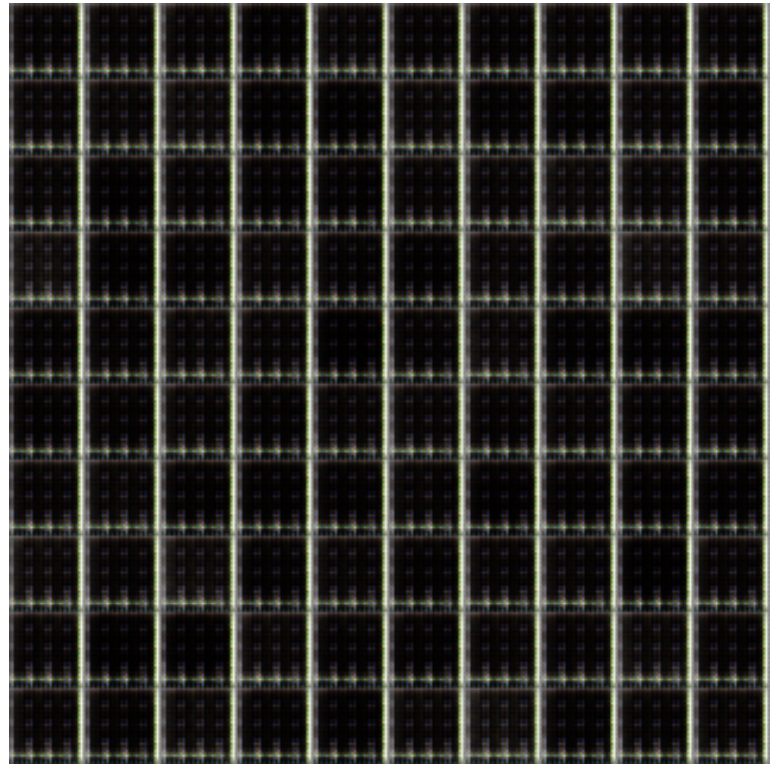
Run 1, epoch 900

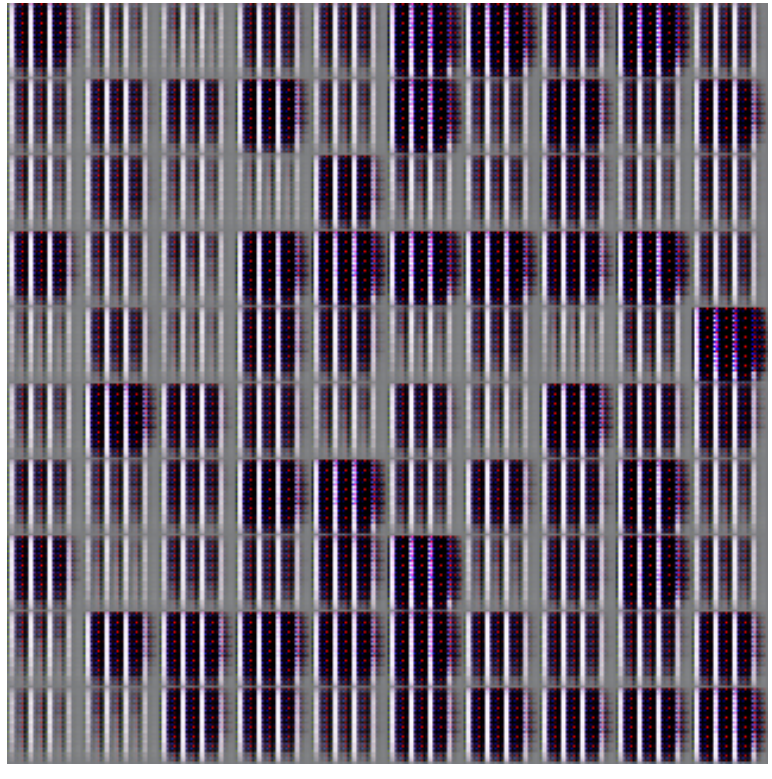


Run 2, epoch 1

Training instability

Running code from [Salimans+ NeurIPS-16]:

Run 1, epoch 900



Run 2, epoch 2

Training instability

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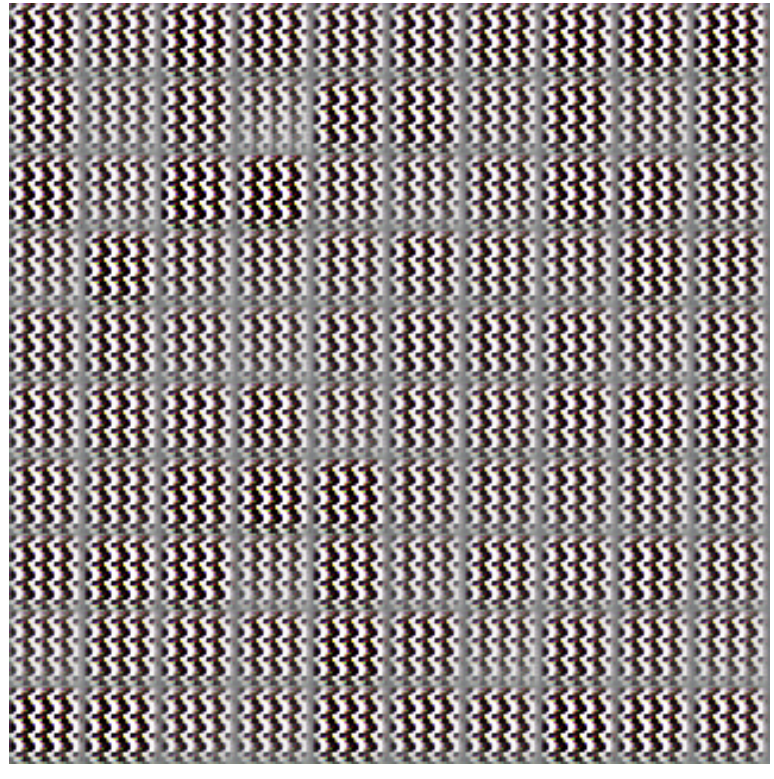

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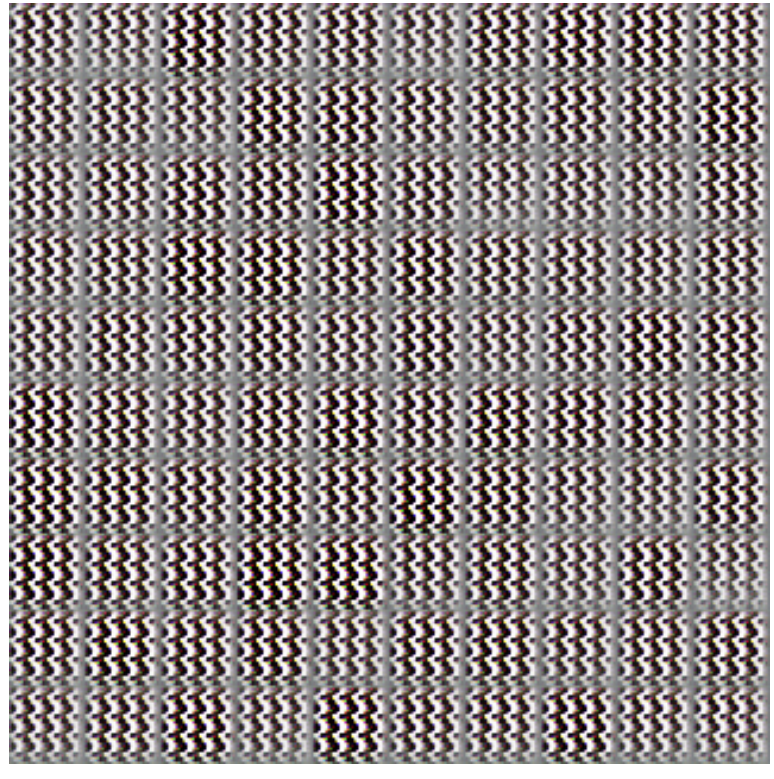
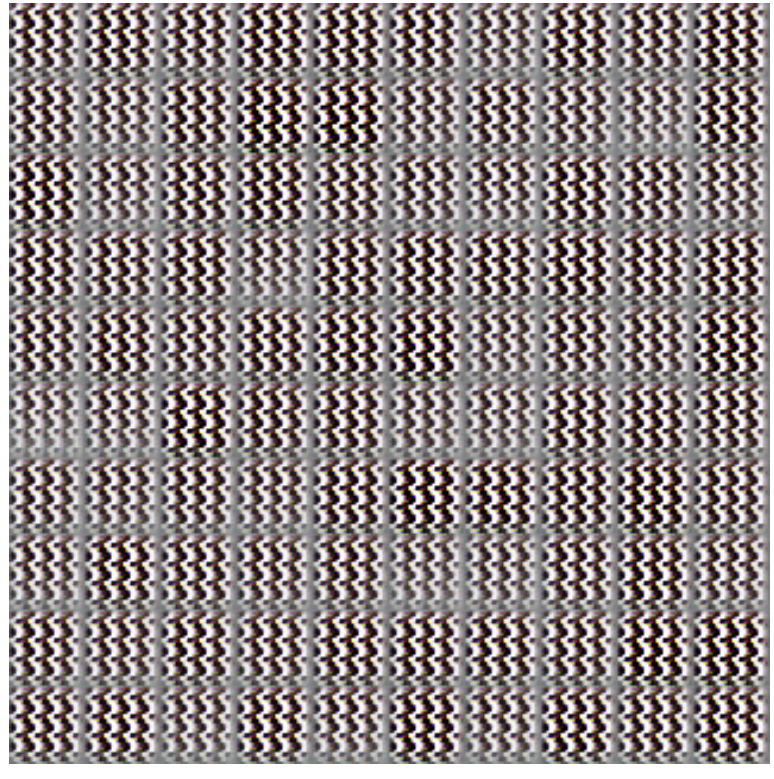


Run 2, epoch 4

Training instability

Running code from [Salimans+ NeurIPS-16]:

Run 1, epoch 900



Run 2, epoch 5

One view: distances between distributions

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One view: distances between distributions

- What happens when D_ψ is at its optimum?
- If distributions have densities, $D^*_\psi(x) = \frac{p(x)}{p(x)+q_\theta(x)}$
- If D_ψ stays optimal throughout, θ tries to minimize

$$\frac{1}{2}\mathop{\mathbb{E}}_{X\sim\mathbb{P}}\left[\log\frac{p(X)}{p(X)+q_\theta(X)}\right]+\frac{1}{2}\mathop{\mathbb{E}}_{Y\sim\mathbb{Q}_\theta}\left[\log\frac{q_\theta(X)}{p(X)+q_\theta(X)}\right]$$

which is $\mathrm{JS}(\mathbb{P},\mathbb{Q}_\theta)-\log 2$

Jensen-Shannon divergence

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JS with disjoint support [Arjovsky/Bottou ICLR-17]

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so $\mathrm{JS}(\mathbb{P}, \mathbb{Q}_\theta) = \log 2$

Discriminator point of view

Generator ($\mathbb{Q}_\theta$)

Discriminator

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Is this real?

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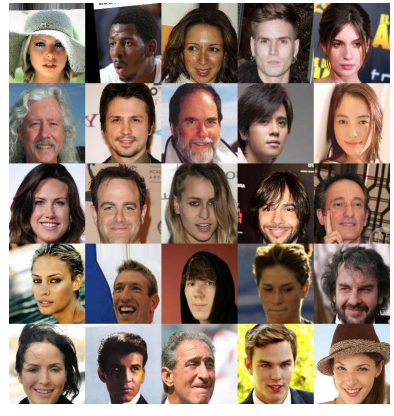

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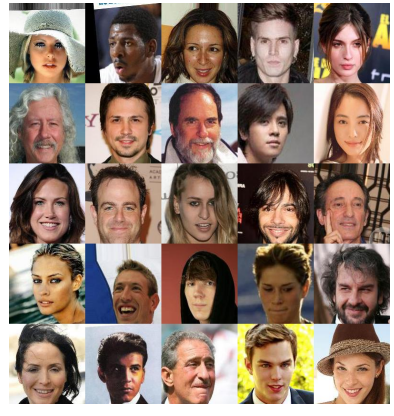

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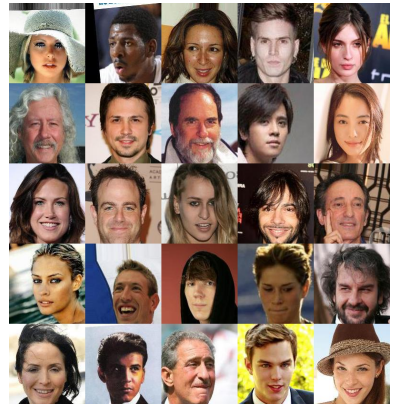

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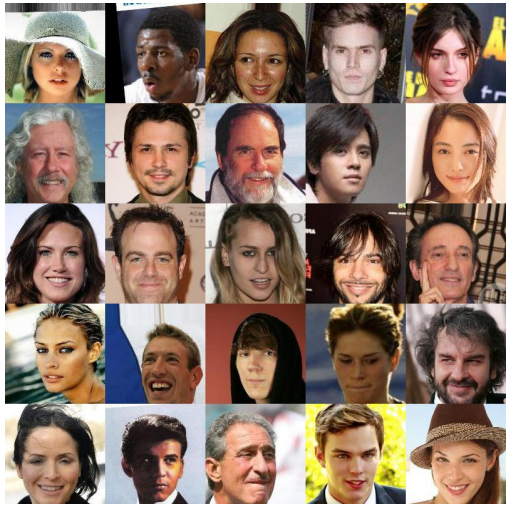
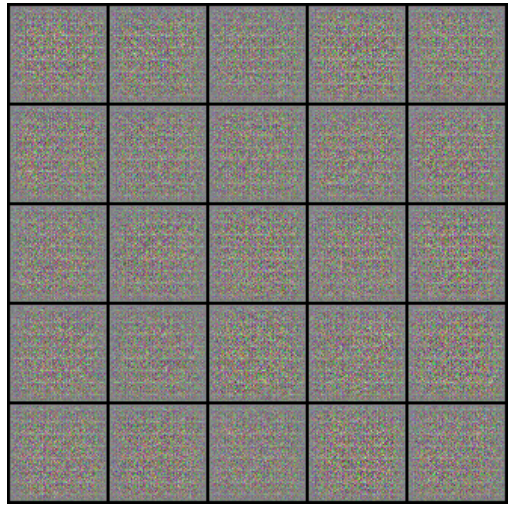
:(I don't know how to do any better...

How likely is disjoint support?

- At initialization, pretty reasonable:

$\mathbb{P}$: 

$\mathbb{Q}_\theta$: 

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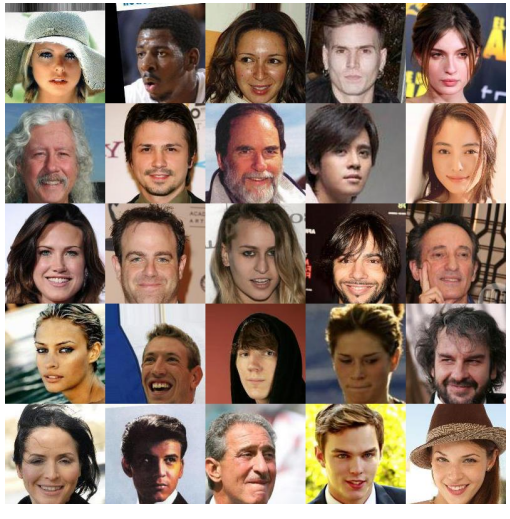
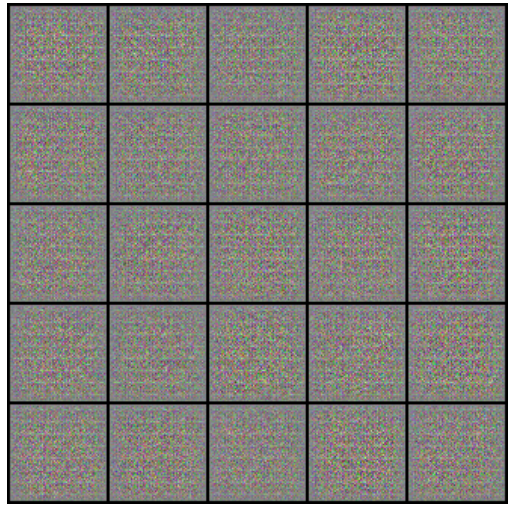
$\mathbb{P}$:  $\mathbb{Q}_\theta$: 

- Note that intersecting *almost nowhere* (with probability 0) doesn't change matters

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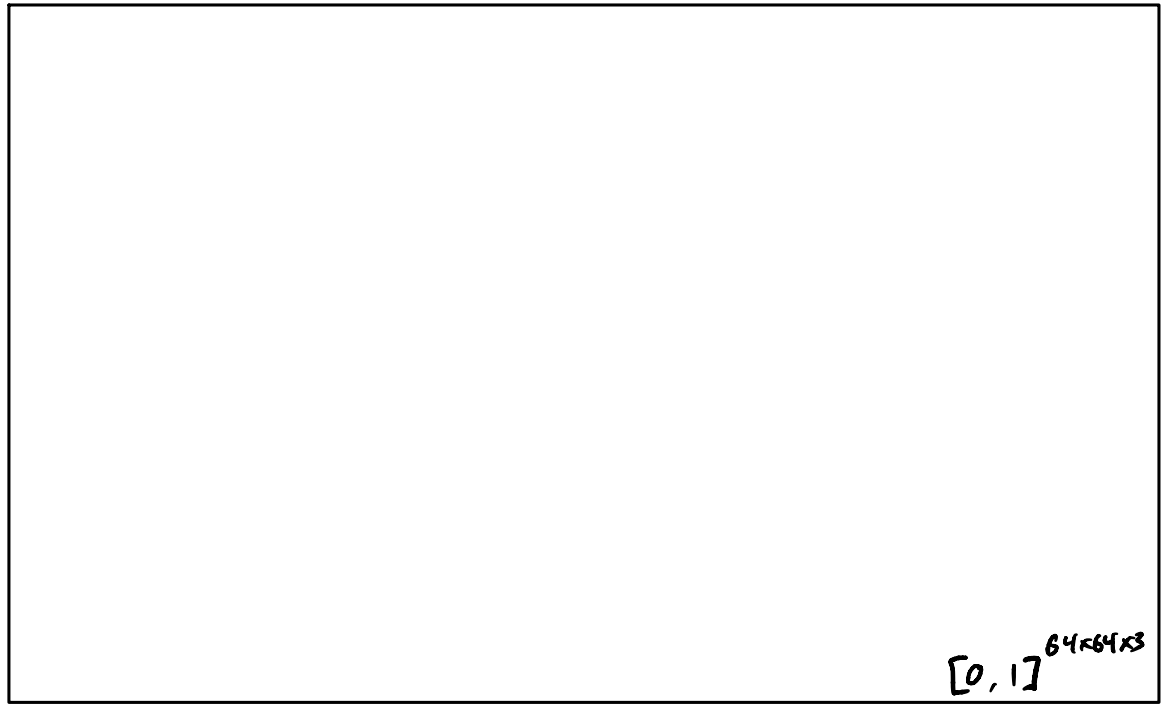

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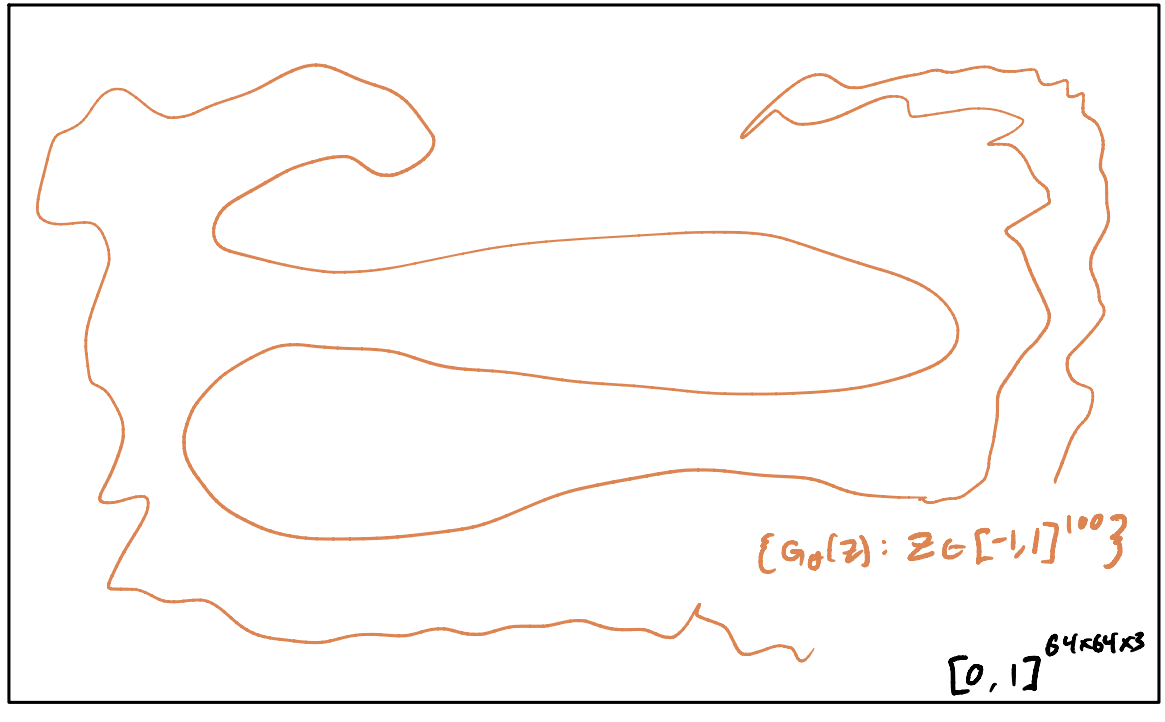

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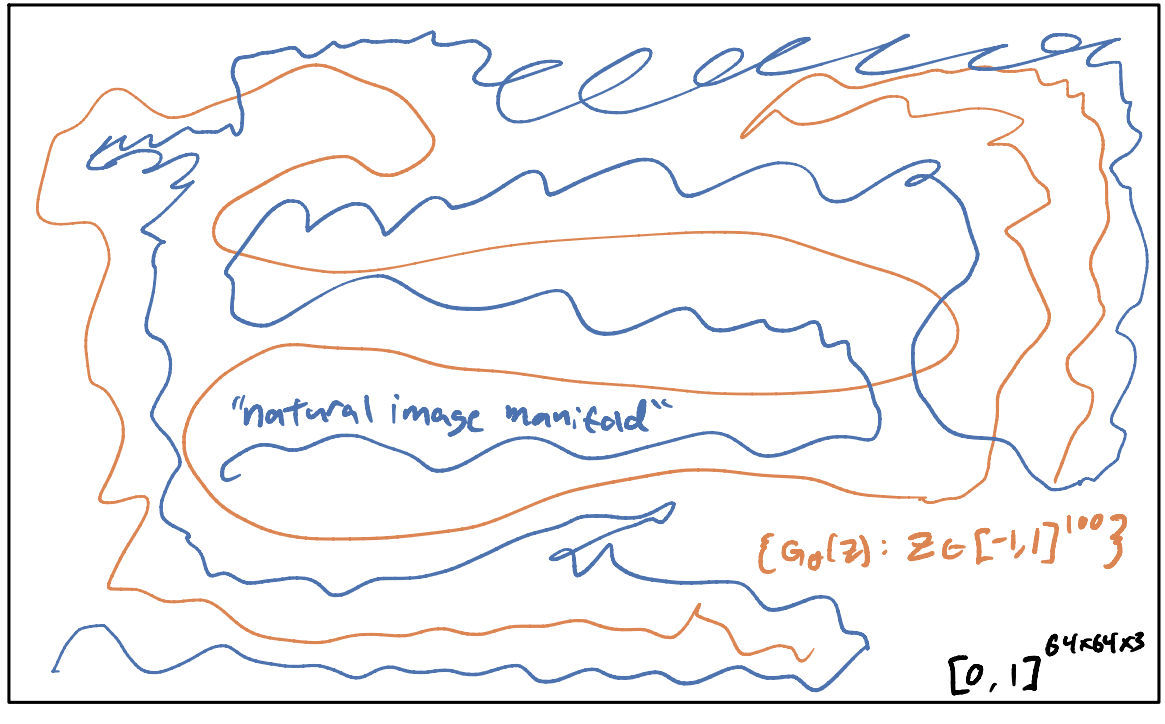

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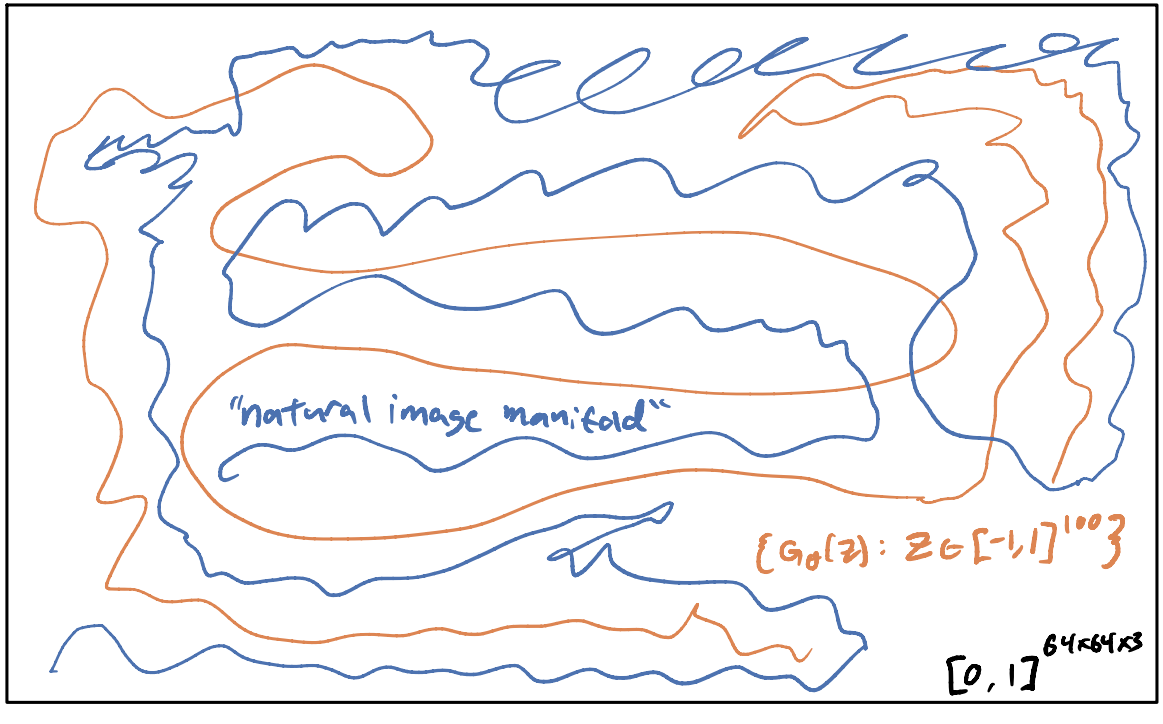
- No chance that they'd align at init, so $\mathrm{JS}(\mathbb{P}, \mathbb{Q}_\theta) = \log 2$



A heuristic partial workaround

- Original GANs almost never use the minimax game

$$\min_{\theta} \max_{\psi} \frac{1}{2} \mathop{\mathbb{E}}_{X \sim \mathbb{P}} [\log D_{\psi}(X)] + \frac{1}{2} \mathop{\mathbb{E}}_{Y \sim \mathbb{Q}_{\theta}} [\log(1 - D_{\psi}(Y))]$$

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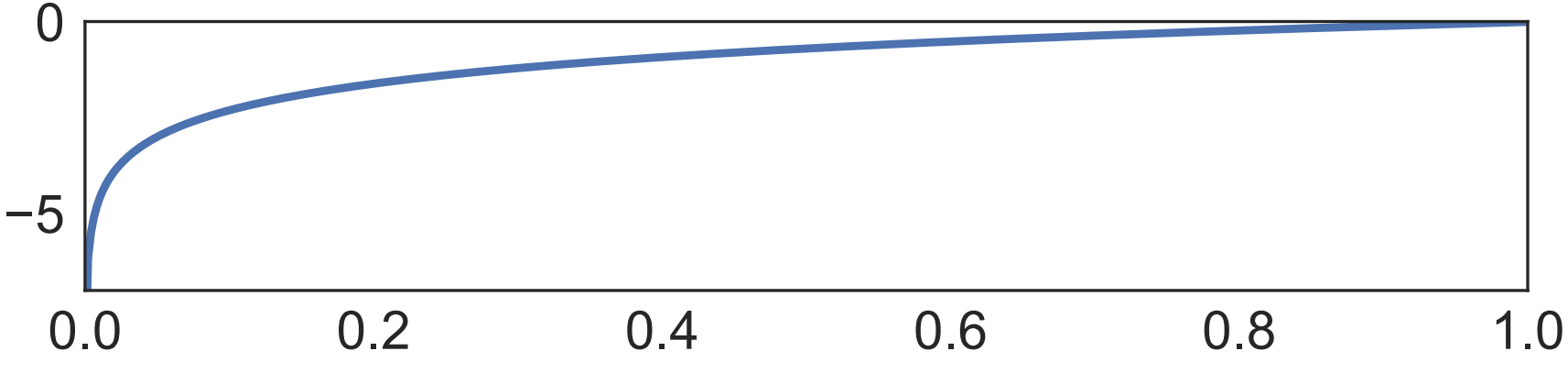

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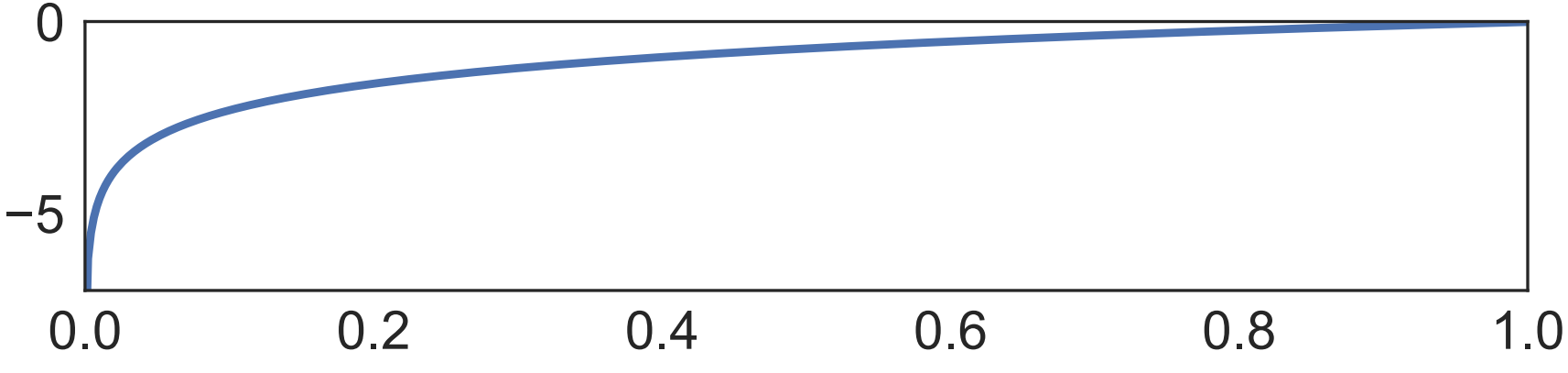

- When D_{ψ} is near-perfect, makes it unstable instead of stuck

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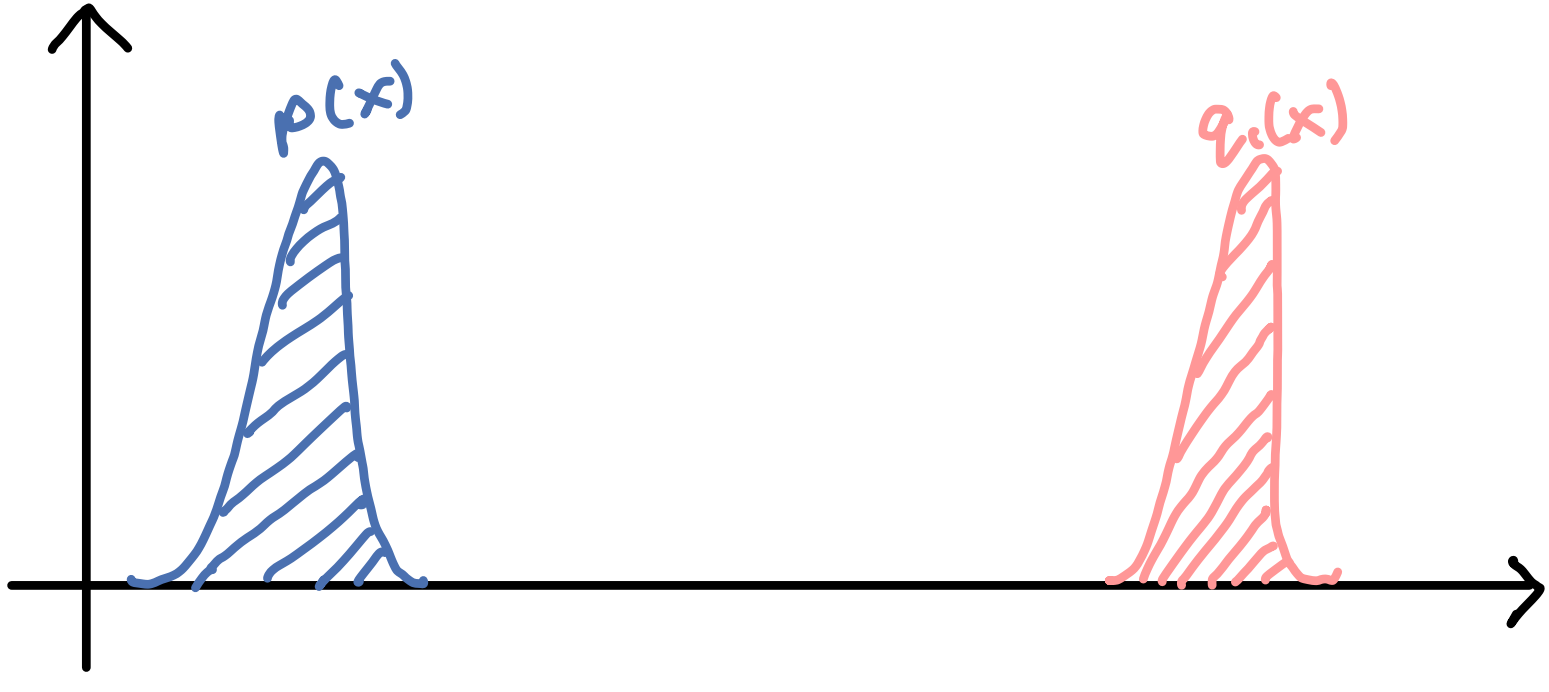

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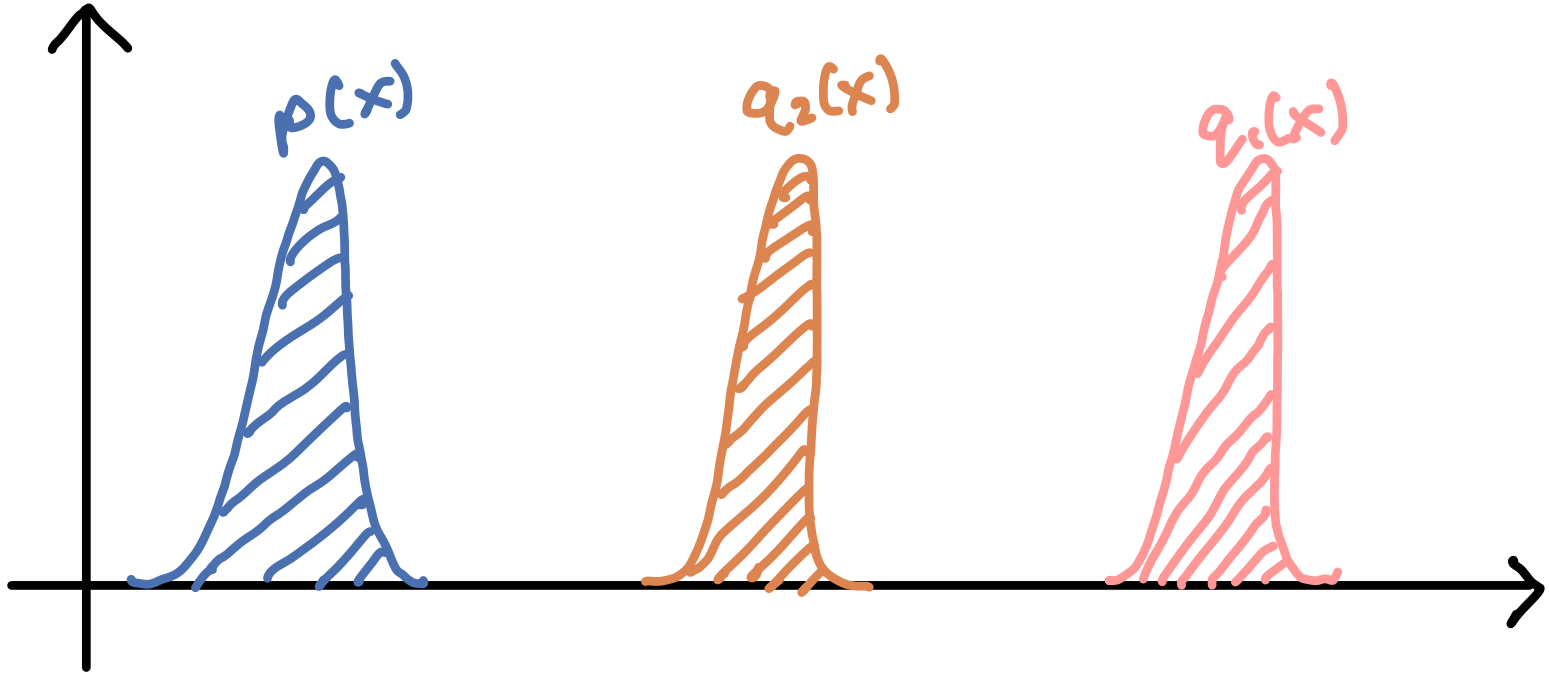

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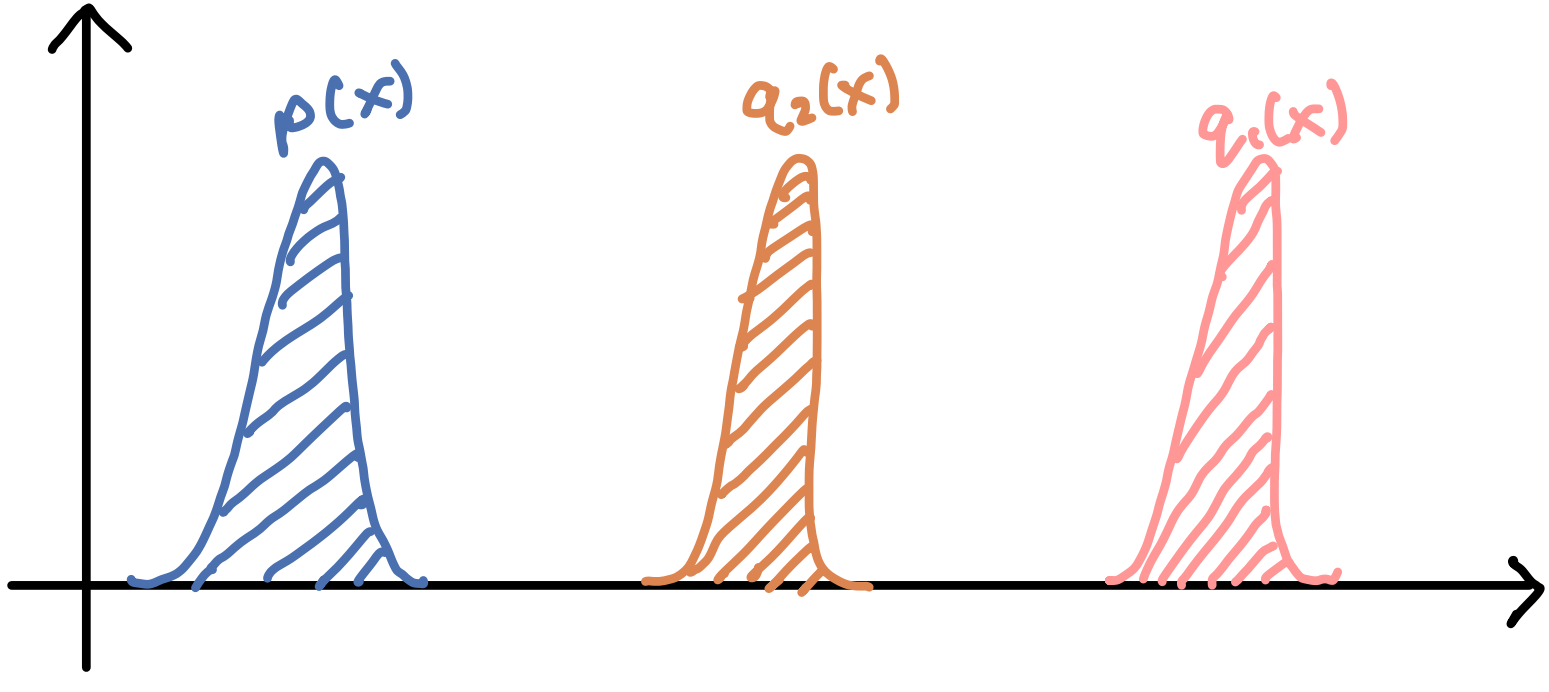

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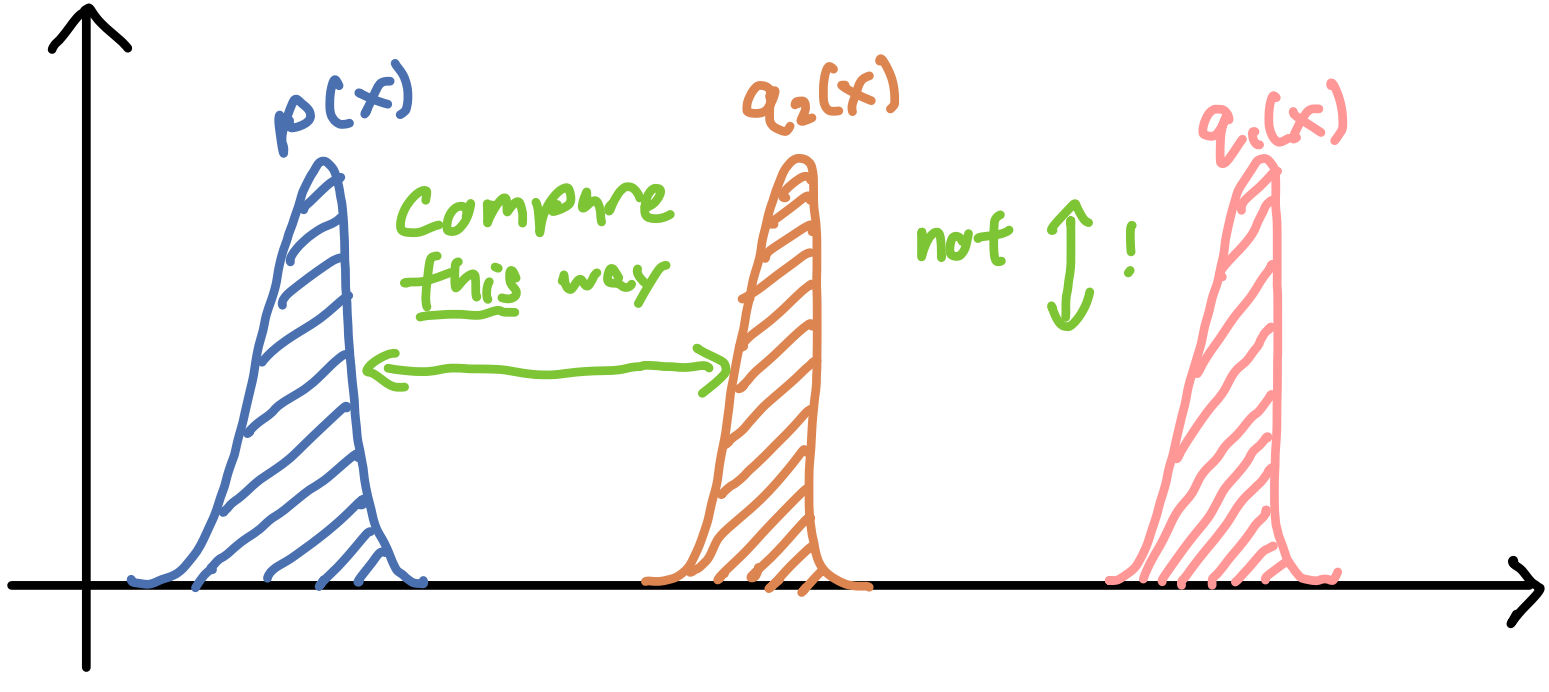

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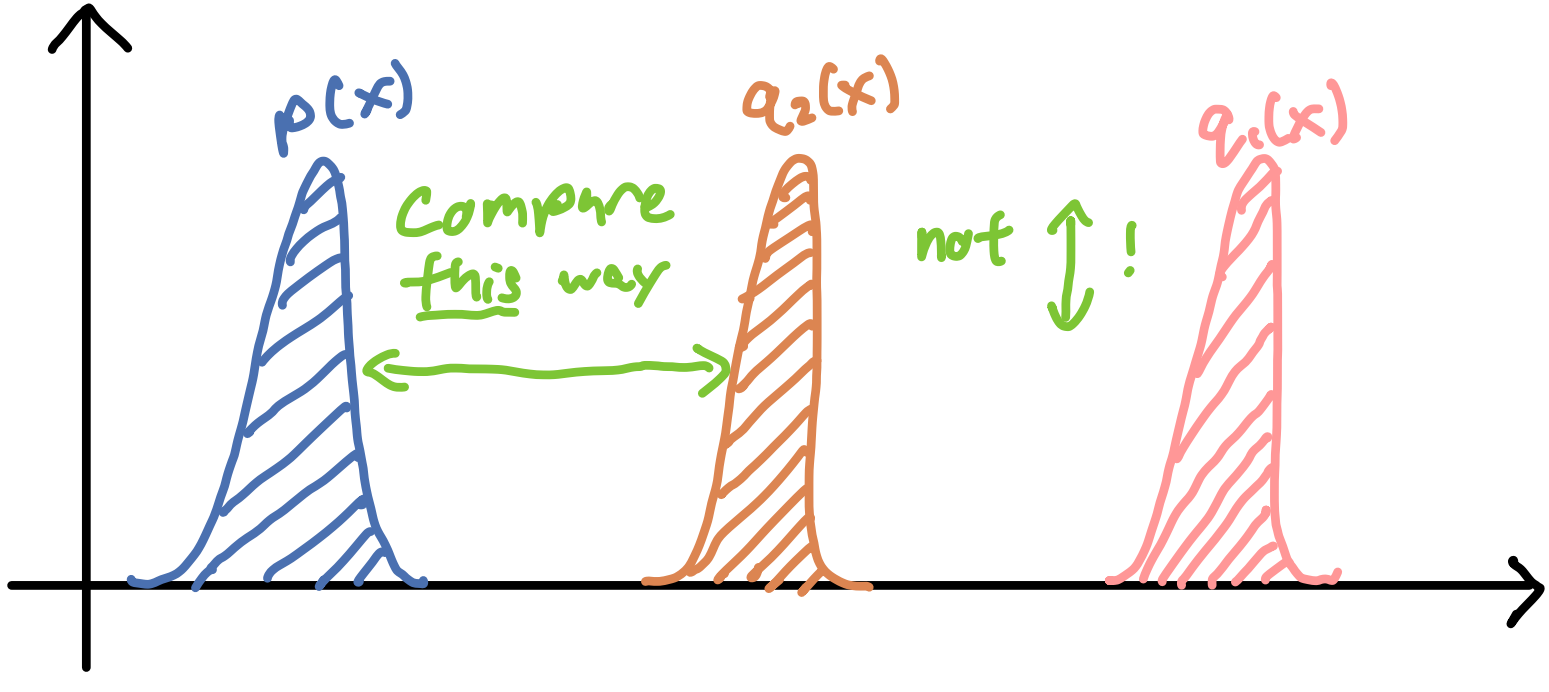
  - Should be *continuous in the weak topology*



Better Solution 1: Optimal Transport

Wasserstein, aka earth mover's distance:
How far do I have to move probability mass to turn $\mathbb{P}$ into $\mathbb{Q}$?

(from David Alvarez-Melis / Nicolo Fusi)

$$\mathcal{W}(\mathbb{P}, \mathbb{Q}) = \inf_{C} \mathop{\mathbb{E}}_{(X,Y)\sim C(\mathbb{P},\mathbb{Q})} d(X, Y)$$

(C is a joint distribution on (X, Y) with marginals $\mathbb{P}$ and $\mathbb{Q}$)

Wasserstein via Kantorovich-Rubinstein duality

$$\mathcal{W}(\mathbb{P}, \mathbb{Q}) = \sup_{f:\|f\|_{\text{Lip}} \leq 1} \mathop{\mathbb{E}}_{X \sim \mathbb{P}}[f(X)] - \mathop{\mathbb{E}}_{Y \sim \mathbb{Q}}[f(Y)]$$

$f : \mathcal{X} \to \mathbb{R}$ is a 1-Lipschitz *critic function*

$$\|f\|_{\text{Lip}} = \sup_{x,y \in \mathcal{X}} \frac{|f(x) - f(y)|}{\|x - y\|} = \sup_{x \in \mathcal{X}} \|\nabla f(x)\|$$

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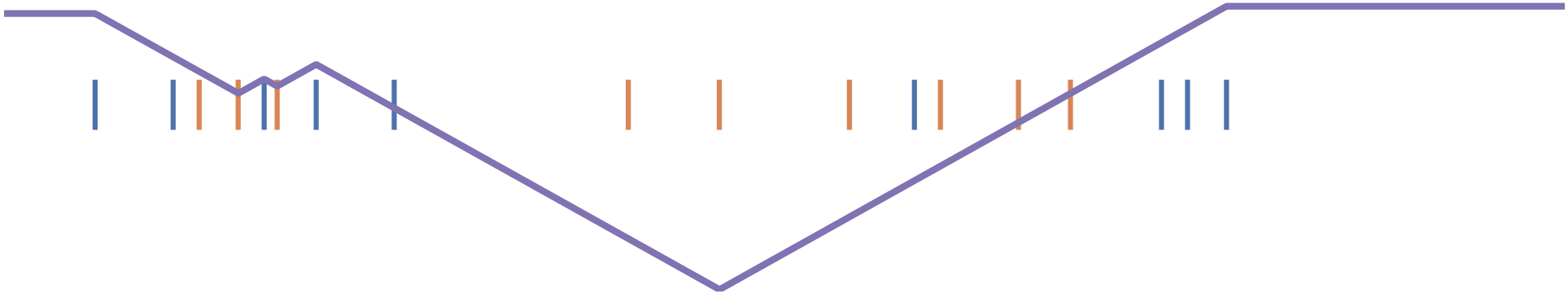

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- Idea: replace discriminator D_ψ with a critic f_ψ
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- h is fixed; often $\|h\|_{\text{Lip}} \leq 1$

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- ...this turns out not to be a great idea.

WGAN-GP [Gulrajani+ NeurIPS-17]

- Controlling $\|\nabla f(X)\|$ *everywhere* is hard

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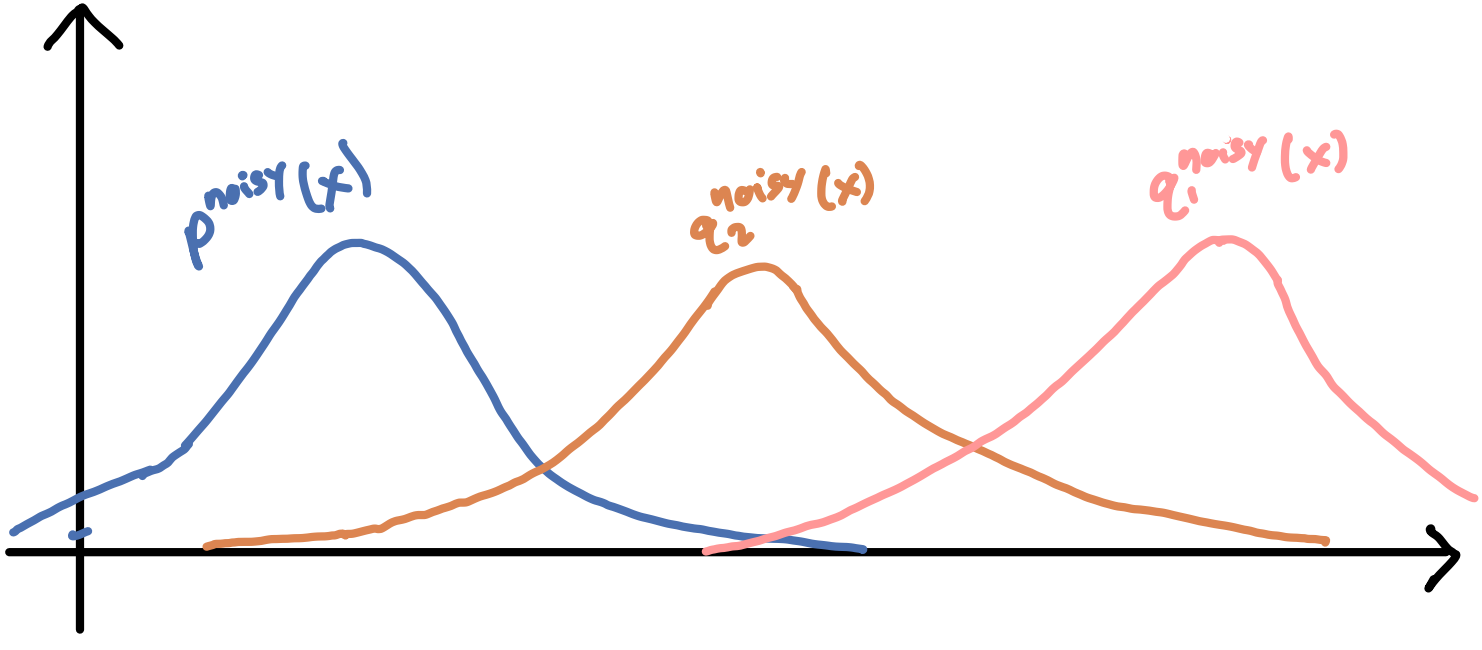
- Works well! But…does it really estimate Wasserstein?

Solution 2: add noise

- Can keep JS if we “blur” the data
- Use $X + \varepsilon$, $Y + \varepsilon'$ for some independent, full-dim noise ε



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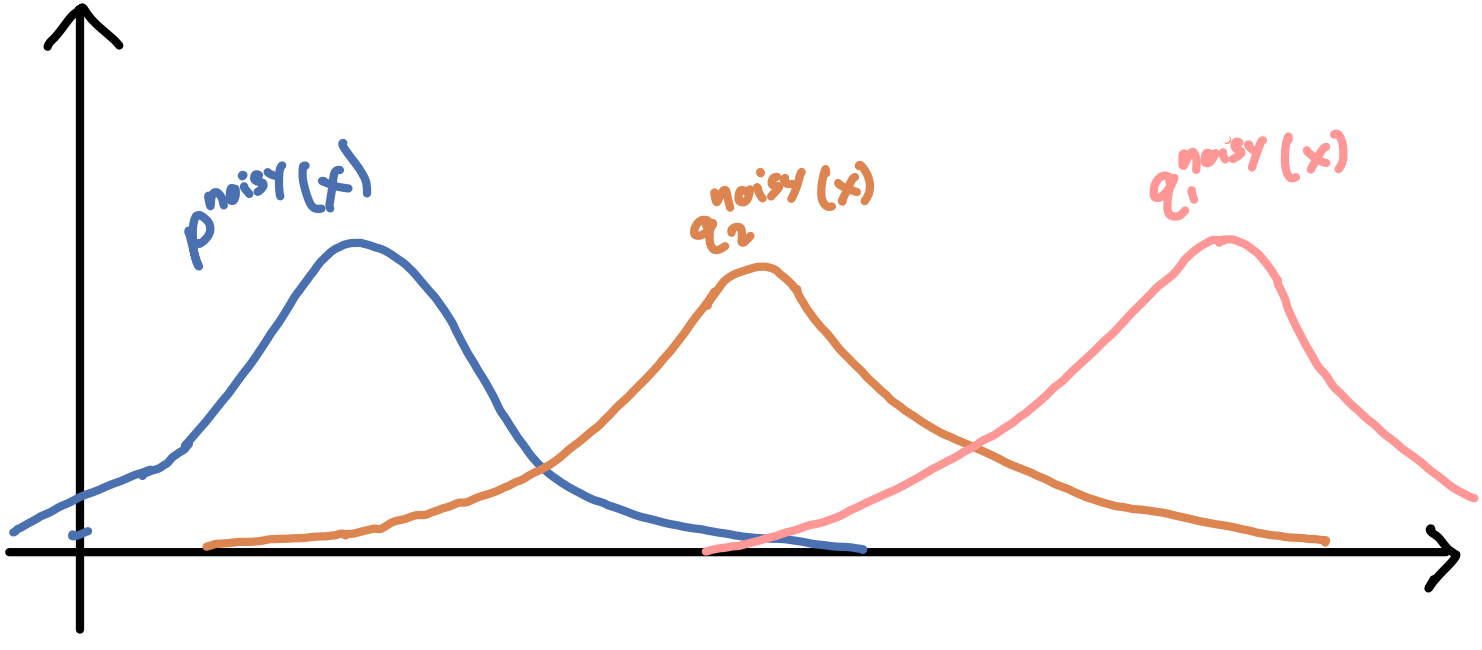

- But…how much noise ε to add? Also need more samples.

Solution 2': add noise in closed form

- Use $X + \varepsilon$, $Y + \varepsilon'$ for some independent, full-dim noise ε
- If $\varepsilon \sim \mathrm{N}(0, \gamma I)$ and $\gamma \to 0$, get [Mescheder+ NeurIPS-17]

$$\gamma \mathop{\mathbb{E}}_{\mathbb{P}}\left[(1 - D_\psi)^2 \|\nabla \log(D_\psi)\|^2\right] + \gamma \mathop{\mathbb{E}}_{\mathbb{Q}_\theta}\left[D_\psi^2 \|\nabla \log(D_\psi)\|^2\right]$$

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- Similar kind of gradient penalty!
- Can also simplify to e.g. [Mescheder+ ICML-18]

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Solution 3: Spectral norm [Miyato+ ICLR-18]

- Regular deep nets: $f_\ell = h_\ell(W_\ell f_{\ell-1}(x))$
- Spectral normalization: $f_\ell = h\left(\frac{1}{\|W_\ell\|_{\text{op}}} W_\ell f_{\ell-1}(x)\right)$
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New samples [Mescheder+ ICML-18]

How to evaluate?

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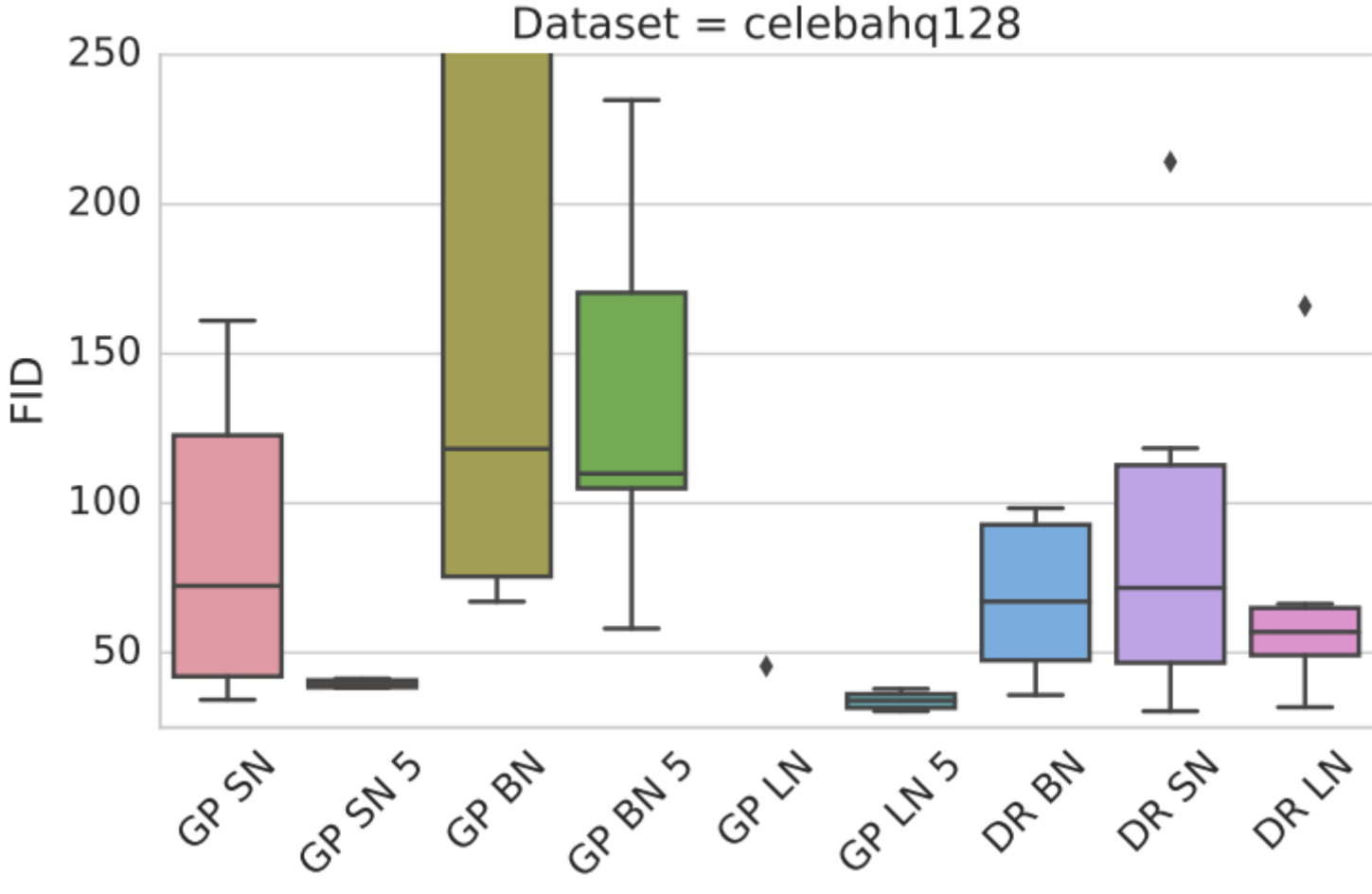
- KID: use Maximum Mean Discrepancy instead
  - Similar distance with unbiased, ~normal estimator!

Comparing approaches [Kurach+ ICML-19]



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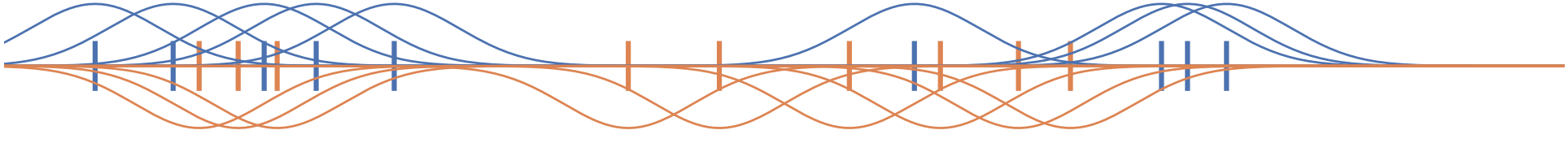

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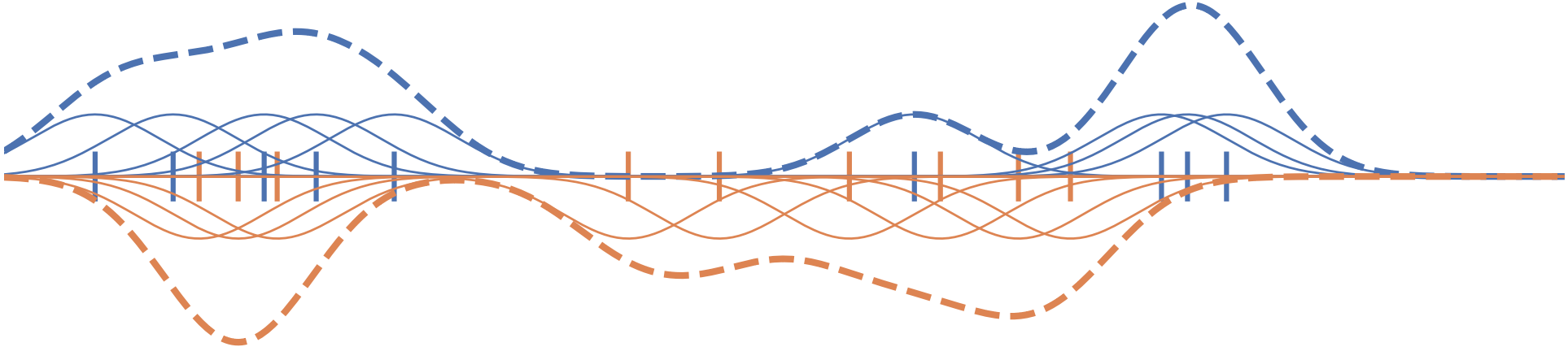

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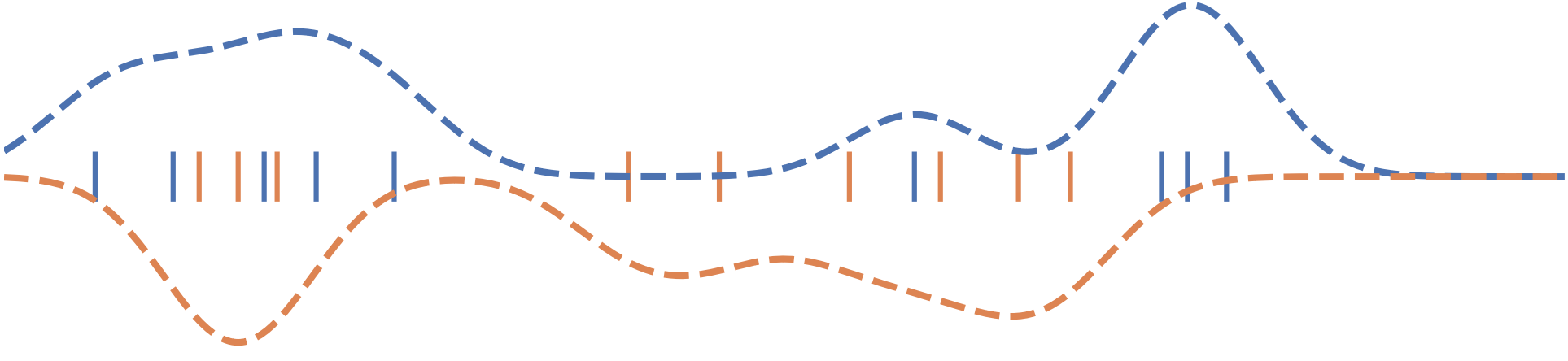

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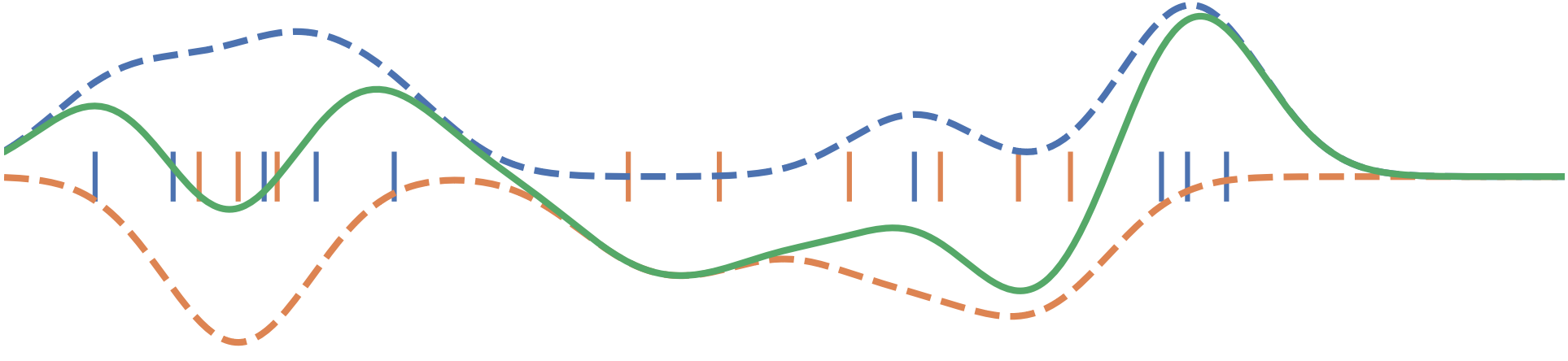

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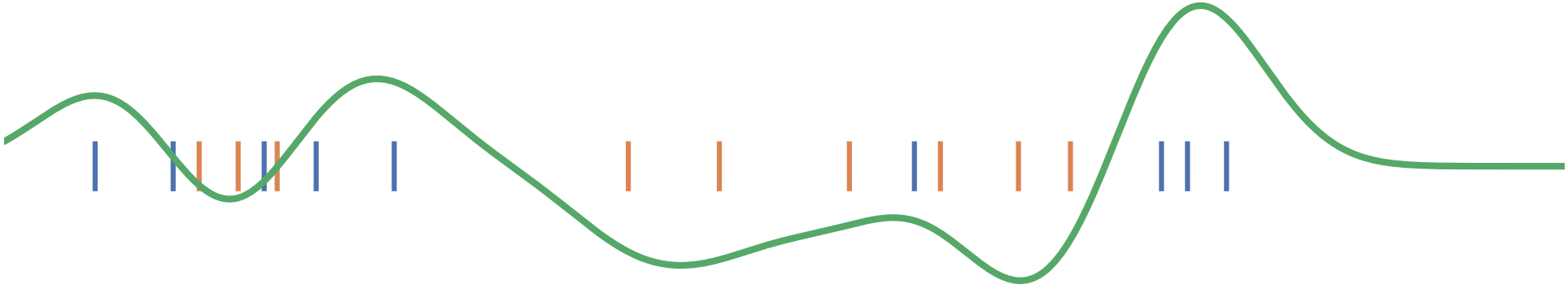

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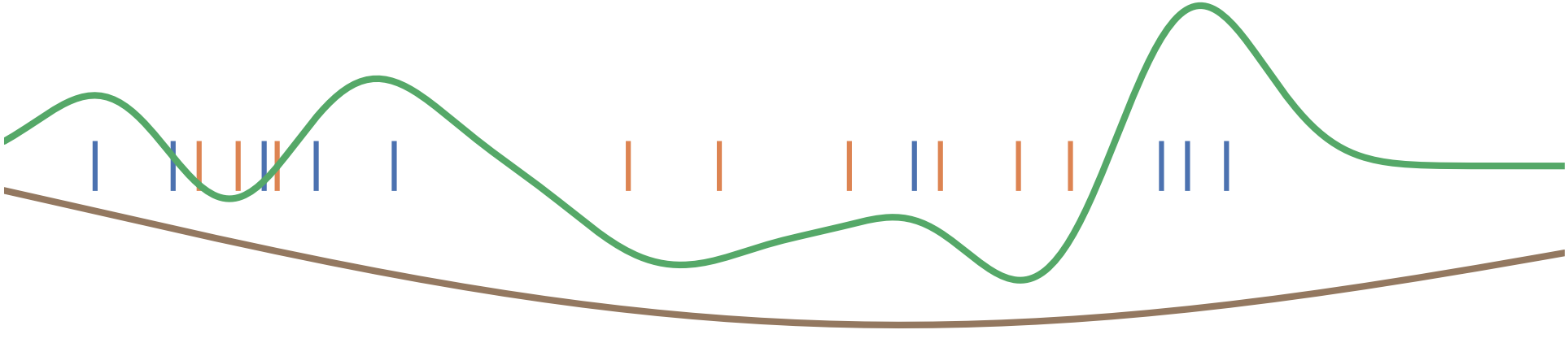

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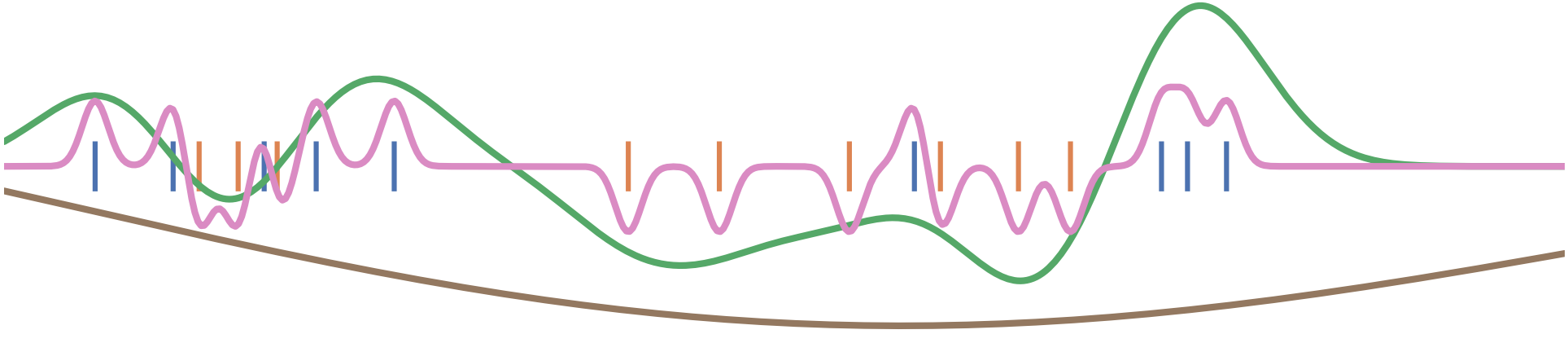

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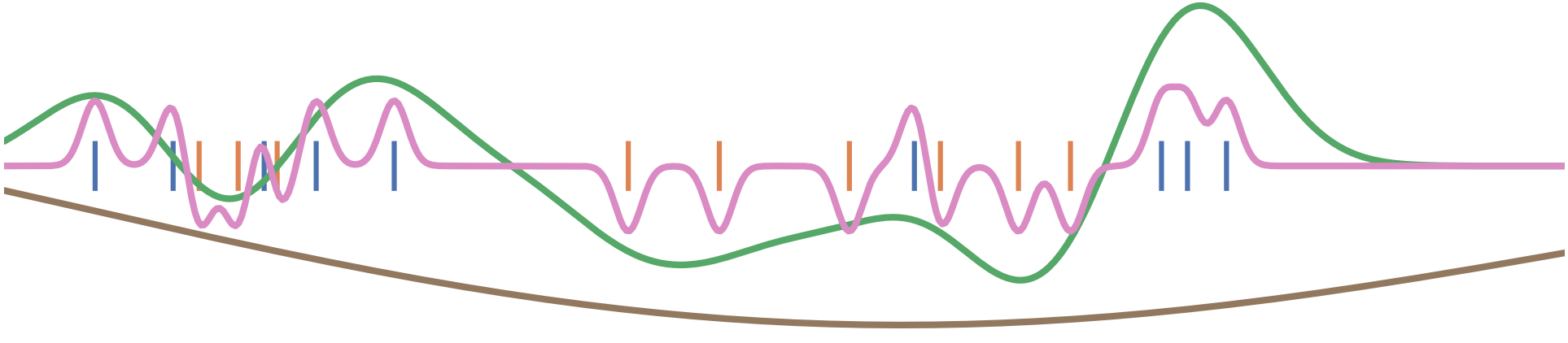
Optimal f *analytically*: $f^*(t) \propto \mathbb{E}_{X \sim \mathbb{P}}\, k(t, X) - \mathbb{E}_{Y \sim \mathbb{Q}}\, k(t, Y)$



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- No need for a discriminator – just minimize $\widehat{\mathrm{MMD}}_k$!
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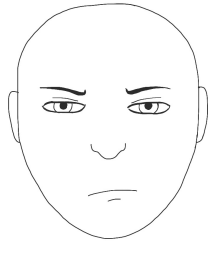

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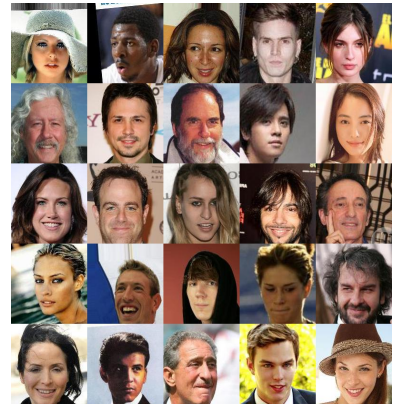

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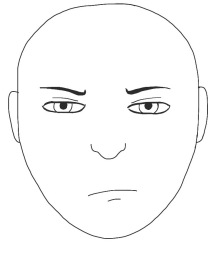

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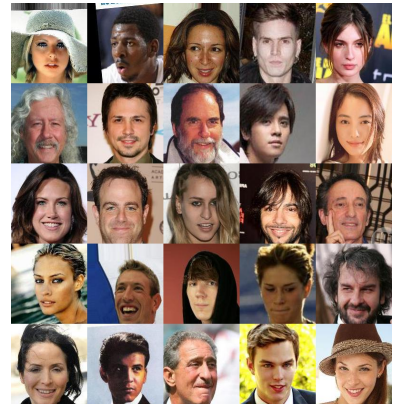

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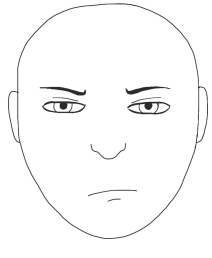

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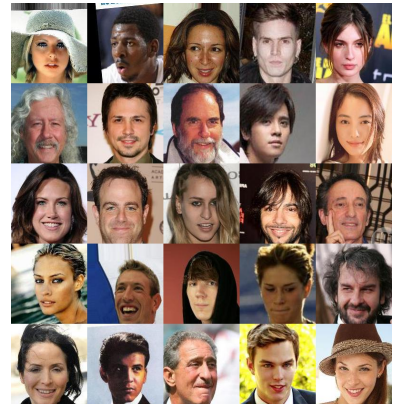

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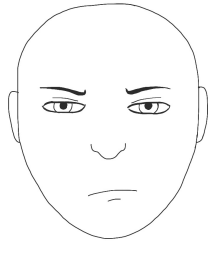

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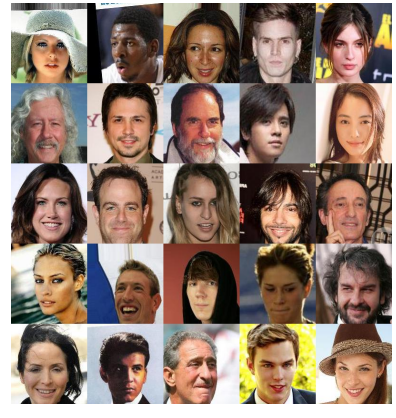

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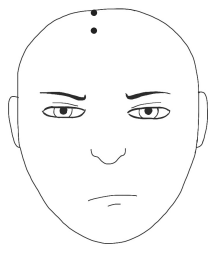

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$\mathbb{P}$

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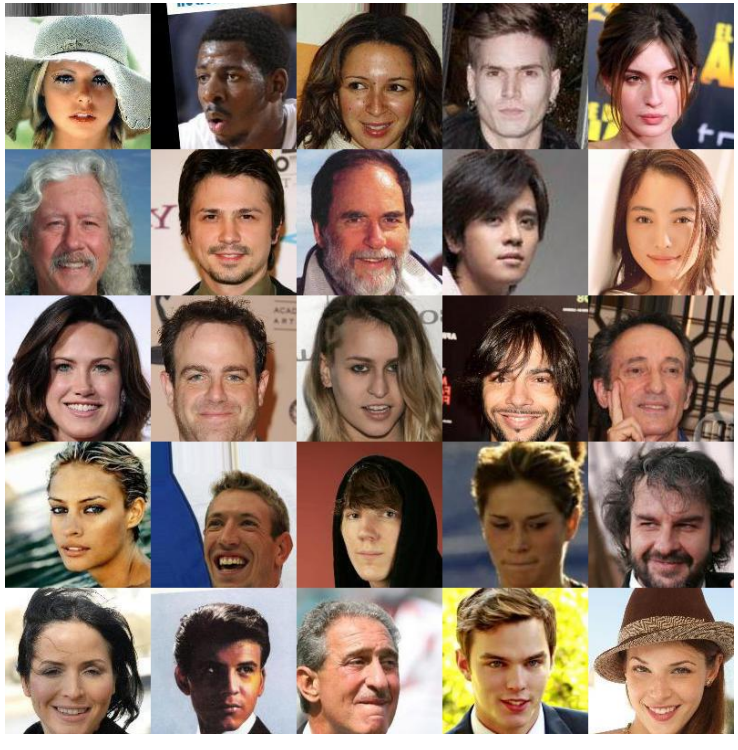
$\mathbb{Q}_\theta$

Celeb-A, mix of rational quadratic + linear kernels



$\mathbb{P}$

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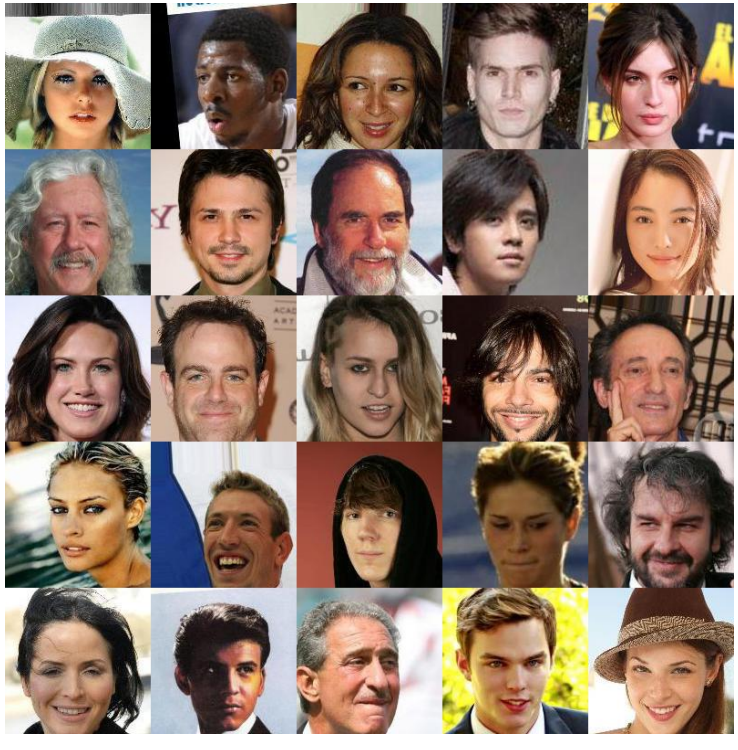
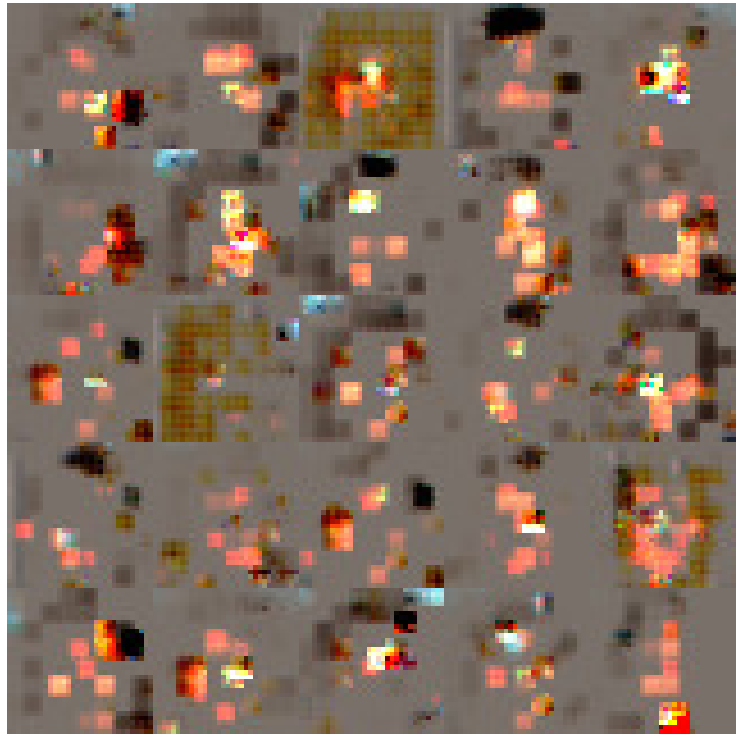


$\mathbb{P}$



$\mathbb{Q}_\theta$

MMD loss with a smarter kernel

$$k(x, y) = k_{\text{top}}(\phi(x), \phi(y))$$

- $\phi : \mathcal{X} \to \mathbb{R}^{2048}$ from pretrained Inception net
- k_{top} simple: exponentiated quadratic or polynomial

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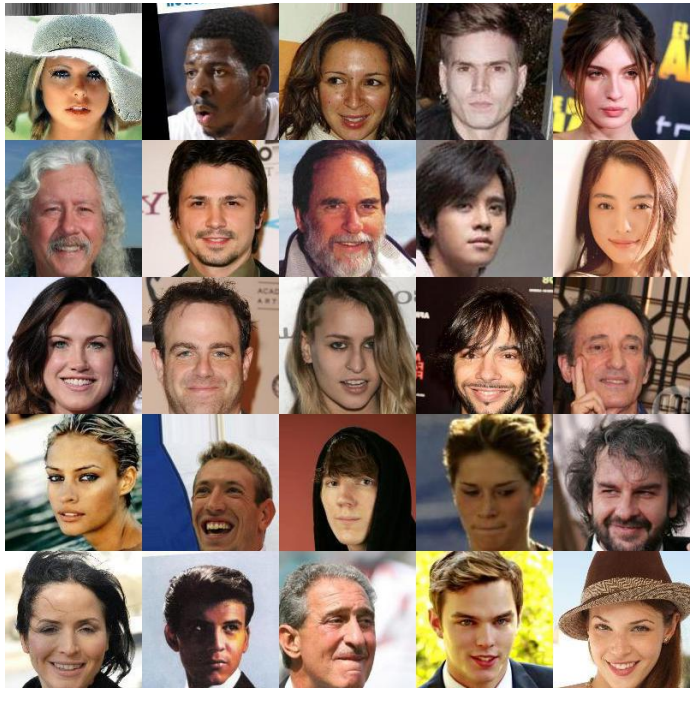

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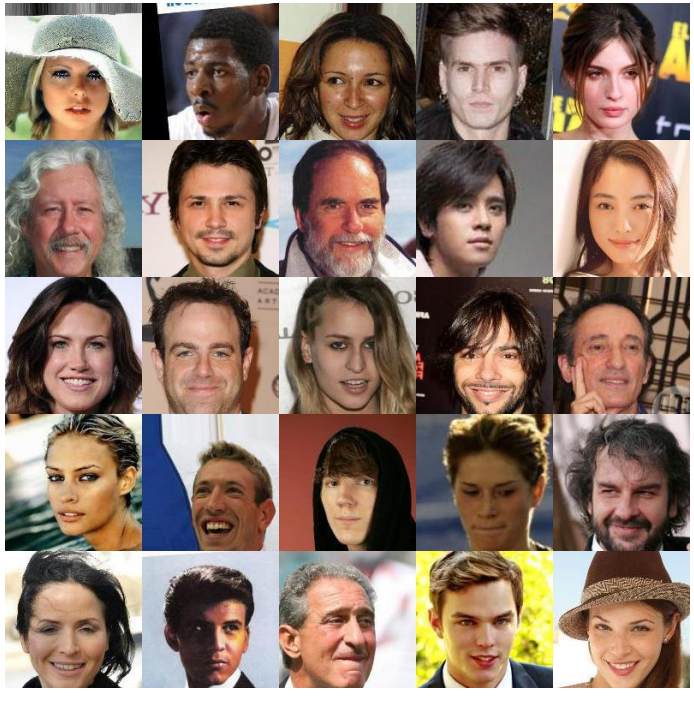
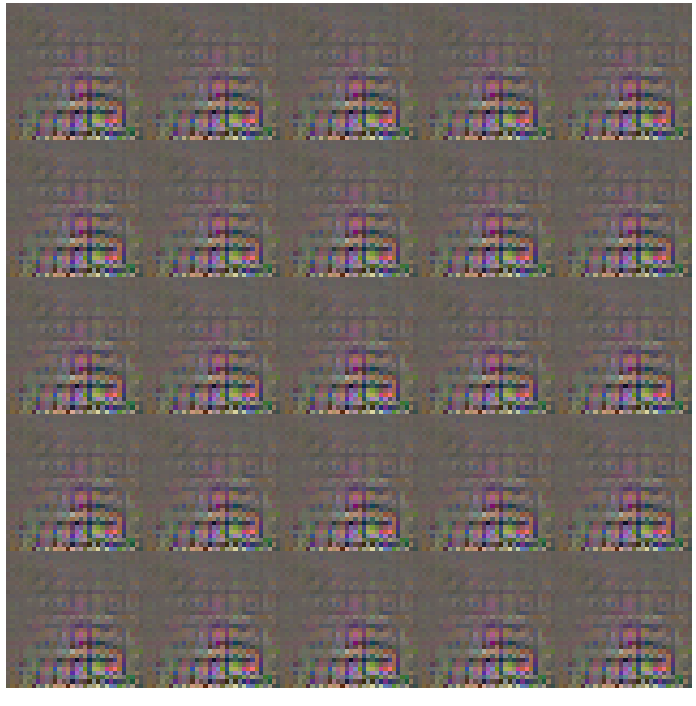


$\mathbb{P}$



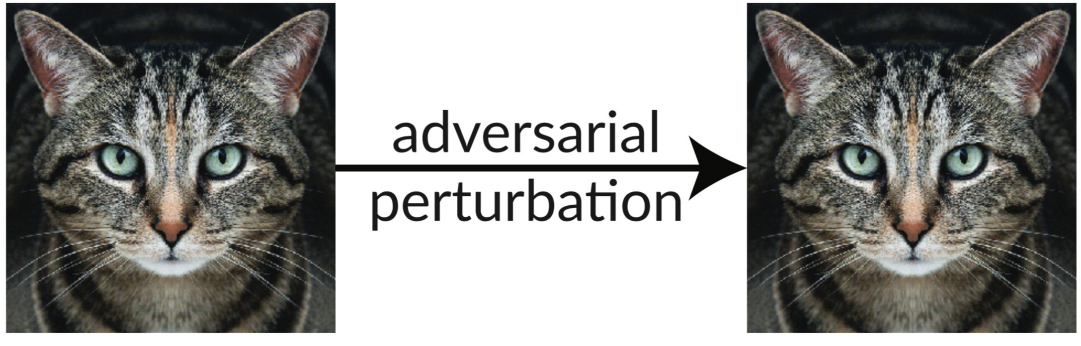
$\mathbb{Q}_\theta$

MMD loss with a smarter kernel

$$k(x, y) = k_{\text{top}}(\phi(x), \phi(y))$$

- $\phi : \mathcal{X} \to \mathbb{R}^d$
- k_{top} is a polynomial

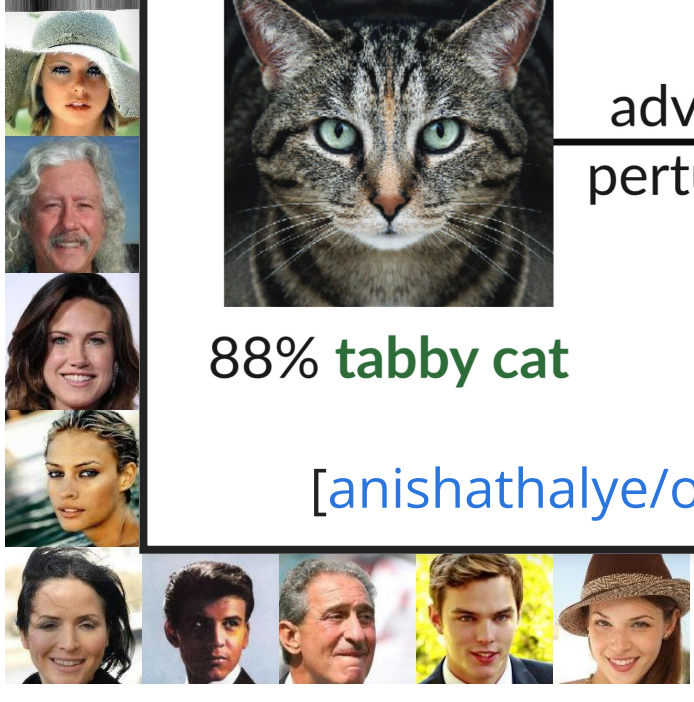
We just got adversarial examples!



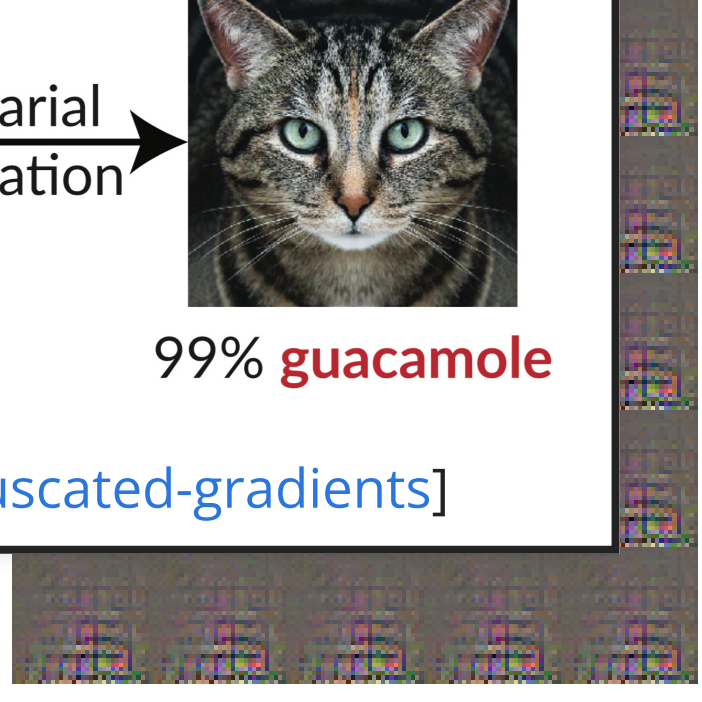
88% tabby cat

99% guacamole

[anishathalye/obfuscated-gradients]



$\mathbb{P}$



$\mathbb{Q}_\theta$

Optimized MMD: MMD GANs [Li+ NeurIPS-17]

- Don't just use one kernel, use a *class* parameterized by ψ:

$$k_\psi(x, y) = k_{\text{top}}(\phi_\psi(x), \phi_\psi(y))$$

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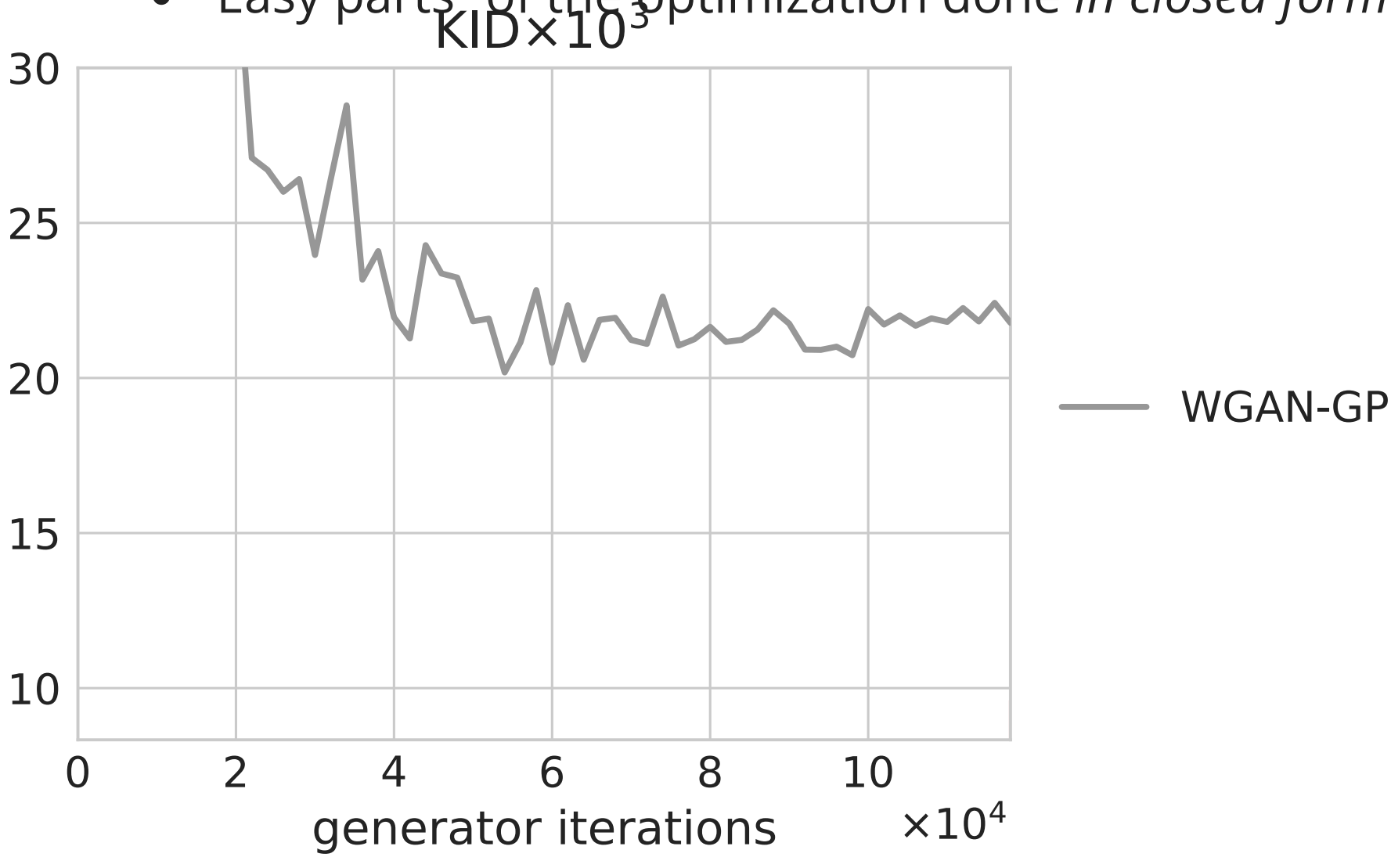
- Scaled MMD GANs [Arbel+ NeurIPS-18] correct $\mathcal{D}_{\text{MMD}}$ with a gradient penalty to make it continuous

Why MMD GANs?

- “Easy parts” of the optimization done *in closed form*



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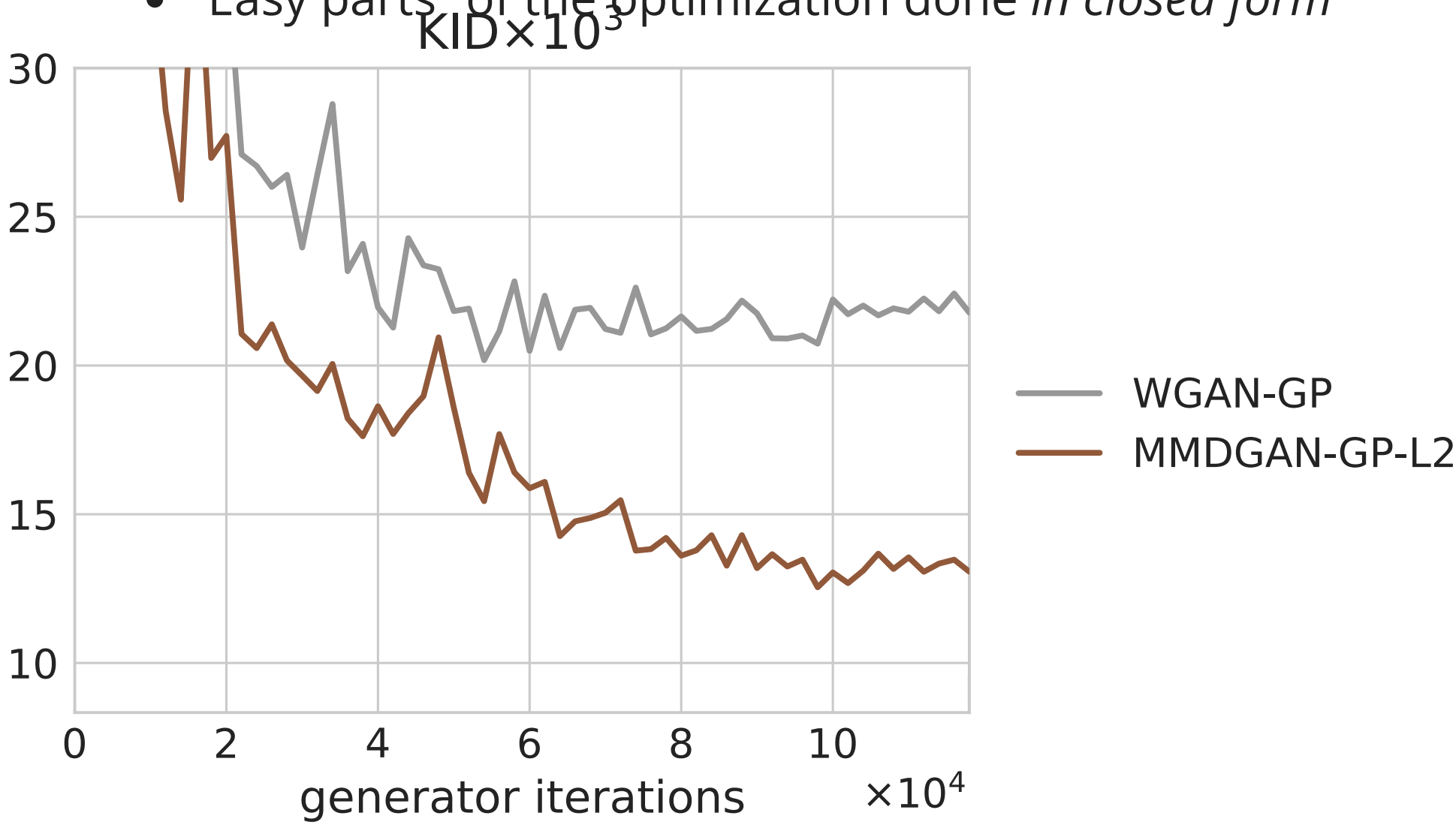

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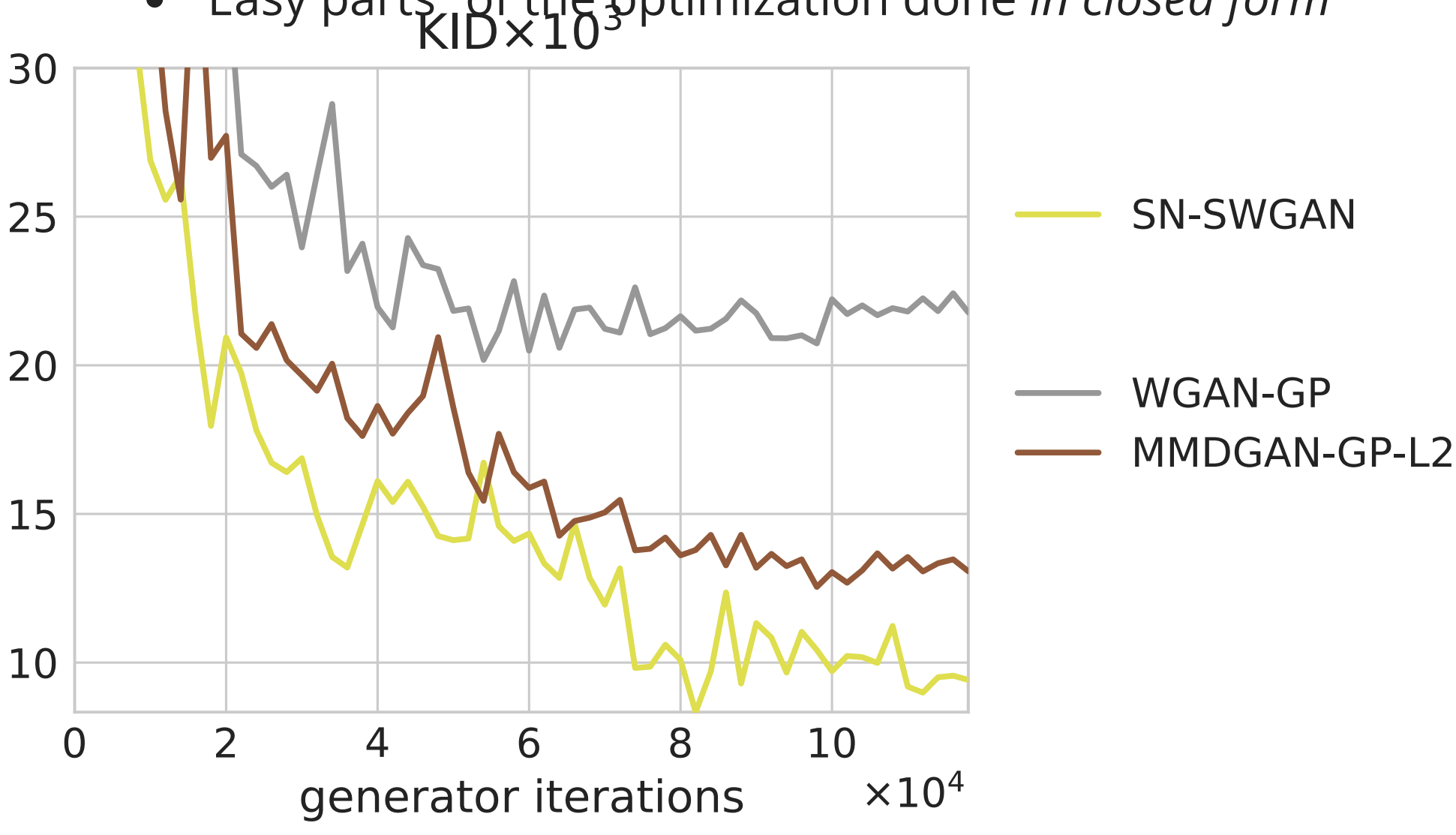

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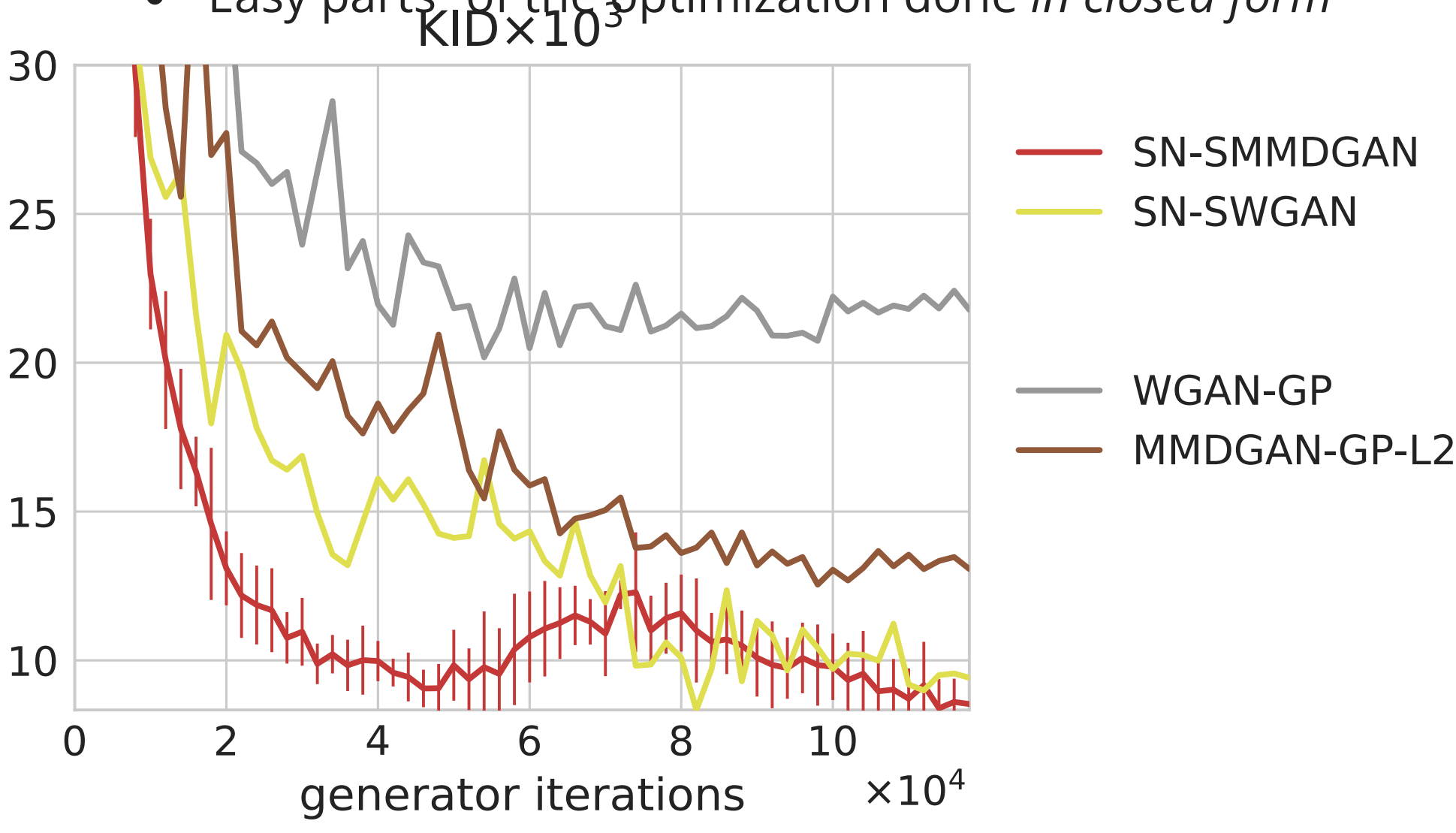

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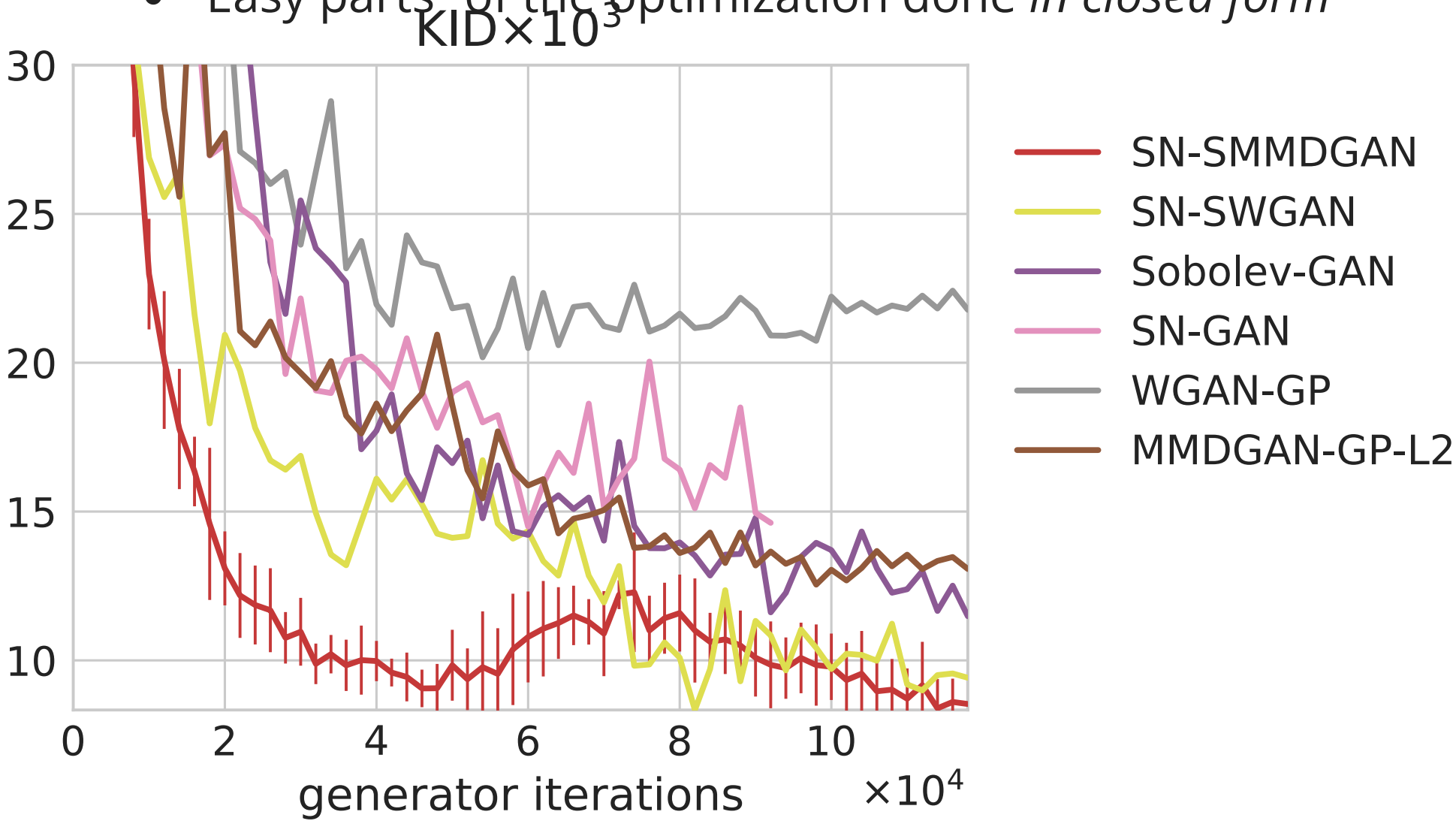
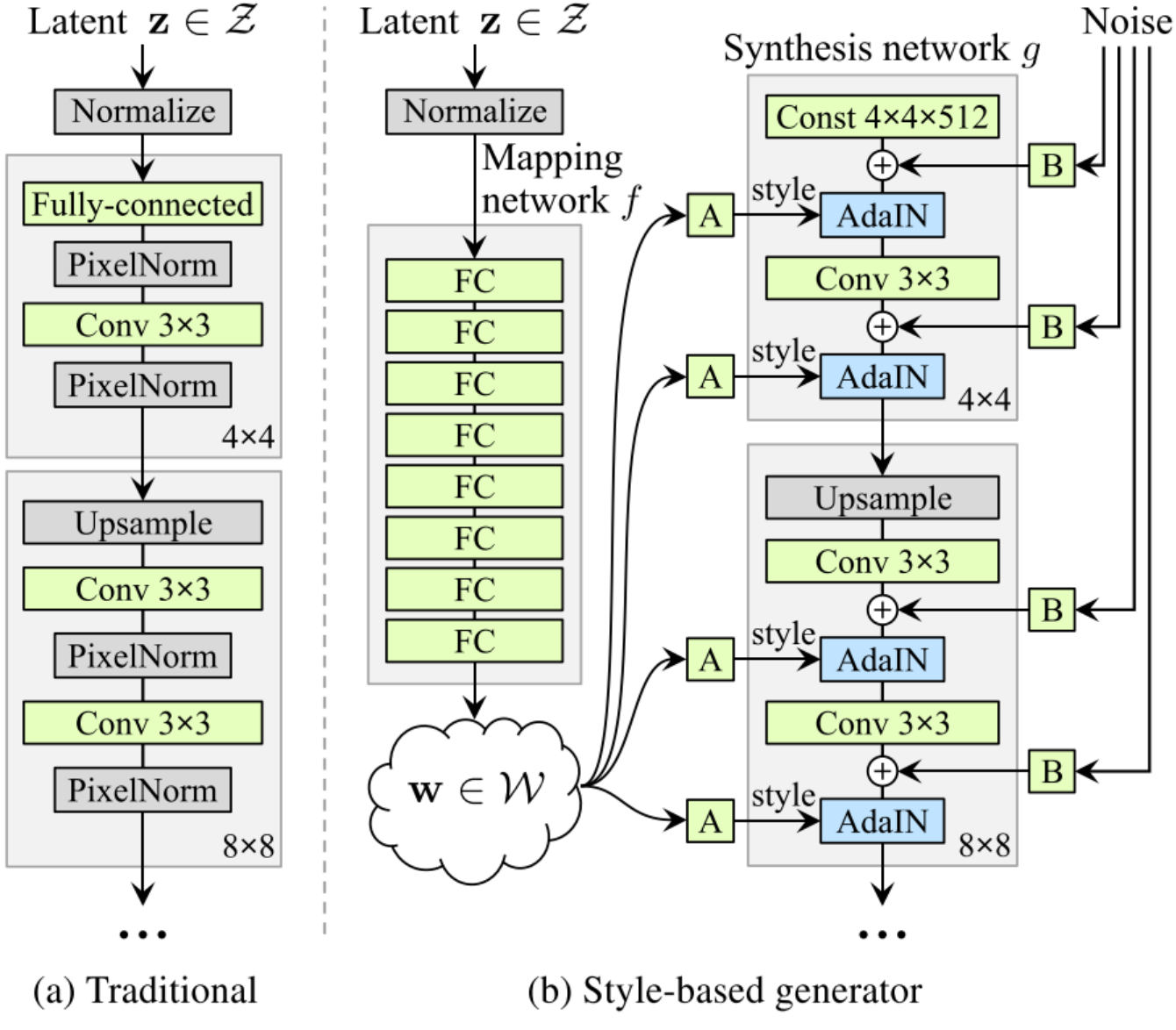


StyleGANs [Karras+ 2018]



(a) Traditional

(b) Style-based generator

StyleGAN: latent structure

StyleGAN: local noise

(a) Generated image (b) Stochastic variation (c) Standard deviation

StyleGANs on a different domain [@roadrunning01]

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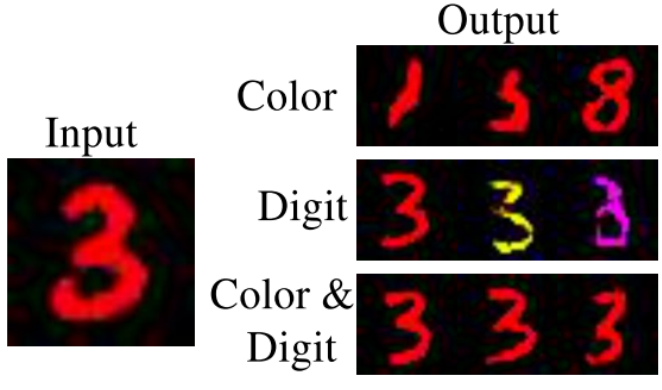
Finding samples you want [Jitkrittum+ ICML-19]

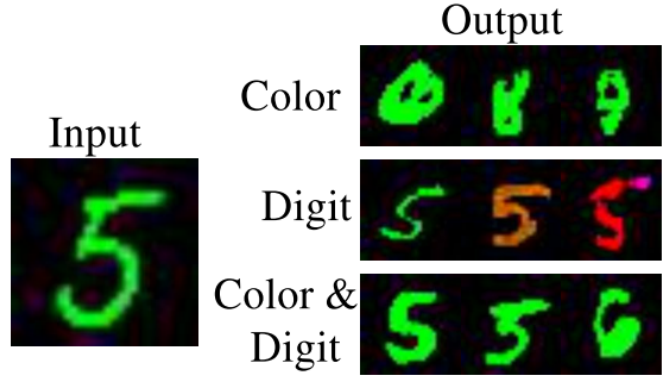
If we want to find "more samples like $\{X\}$":

$$\min_{\{Z_1,\ldots,Z_n\}} \widehat{\mathrm{MMD}}_k^2\left(\{X_i\}_{i=1}^m, \{G_\theta(Z_i)\}_{i=1}^n\right)$$

(a) Samples from DCGAN

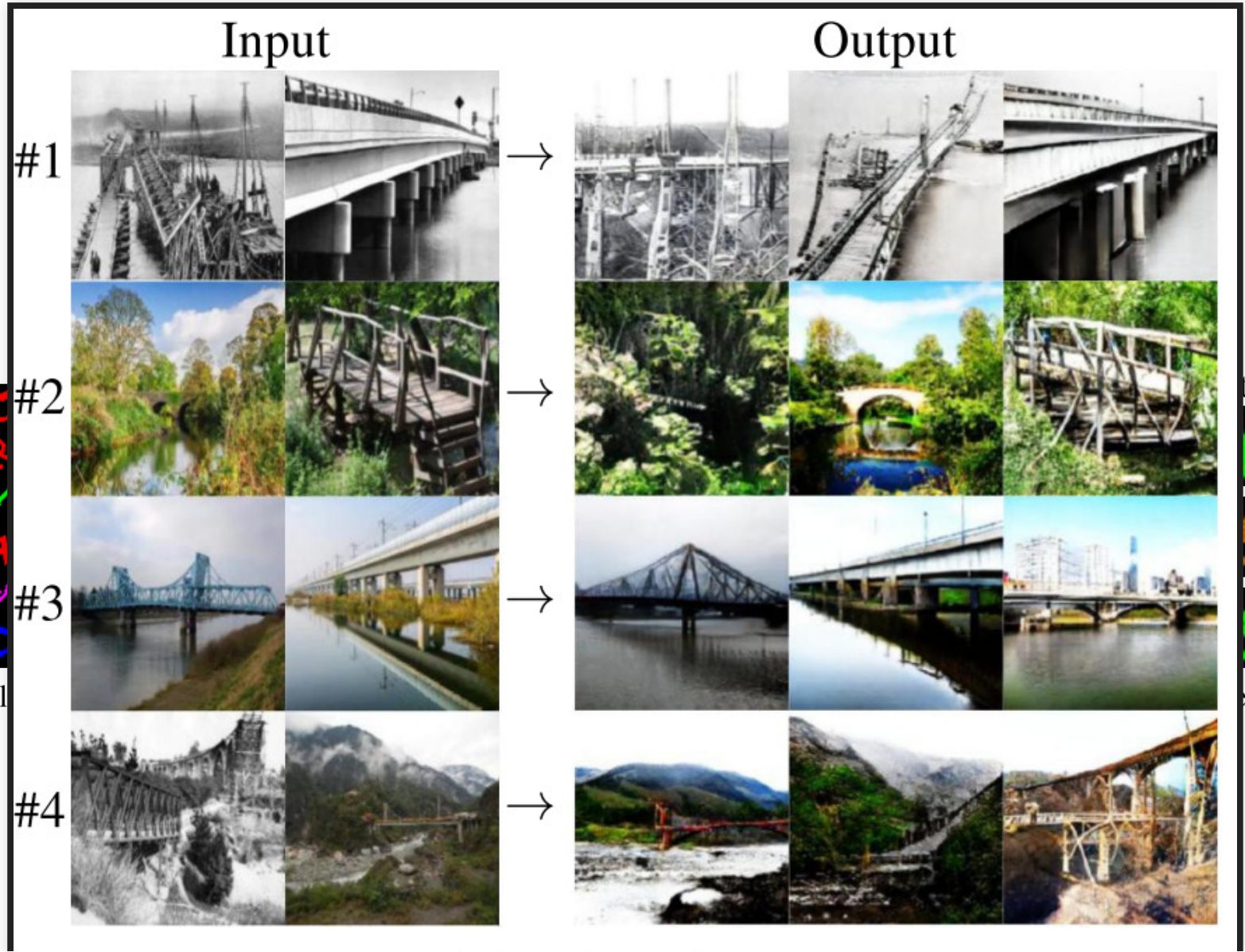


(b) Input: digit 3 in red



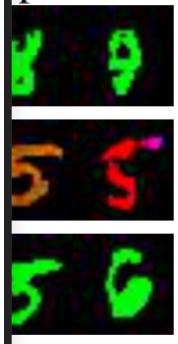
(c) Input: digit 5 in green

Finding samples you want [Jitkrittum+ ICML-19]



(a) Sampl

tput



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Conditional GANs and BigGAN

- Conditional GANs: [Mirza+ 2014]
  - Just add a class label as input to G_θ and D_ψ
- BigGAN [Brock+ ICLR-19]: a bunch of tricks to make it huge

Image-to-image translation [Isola+ CVPR-17]

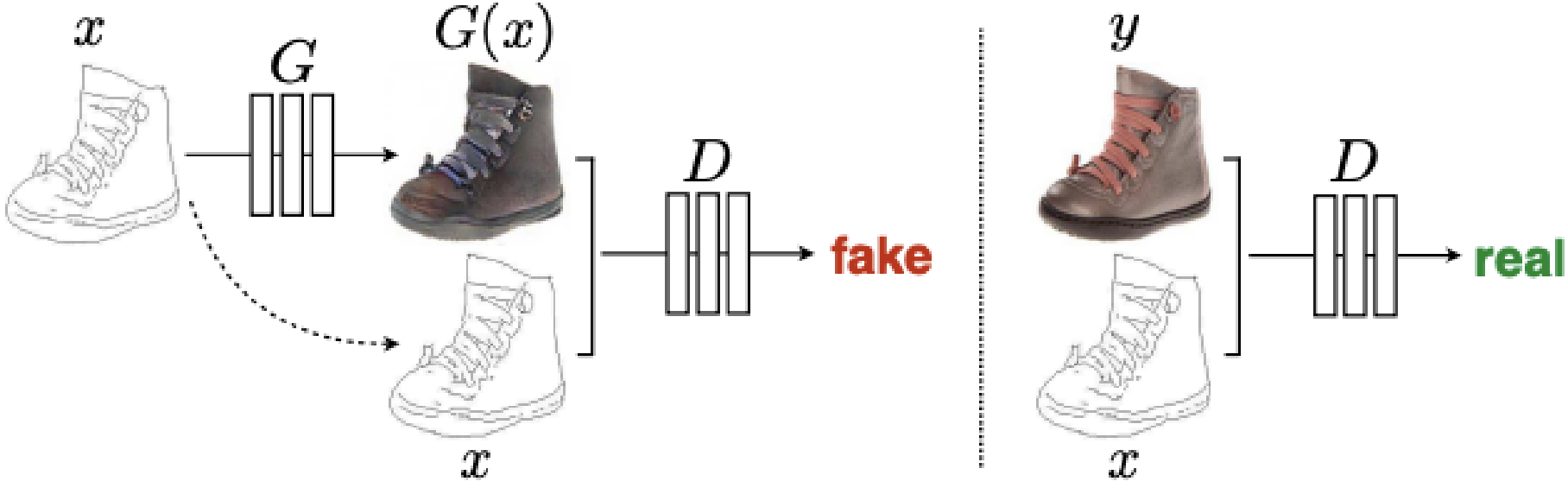
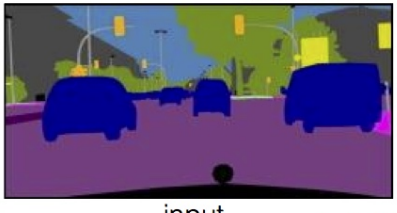


Figure 2: Training a conditional GAN to map edges→photo. The discriminator, D, learns to classify between fake (synthesized by the generator) and real {edge, photo} tuples. The generator, G, learns to fool the discriminator. Unlike an unconditional GAN, both the generator and discriminator observe the input edge map.

Image-to-image translation [Isola+ CVPR-17]

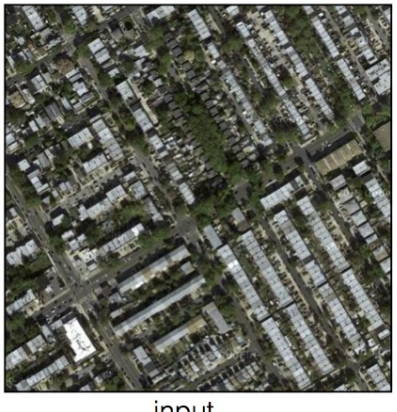
Labels to Street Scene



input

output

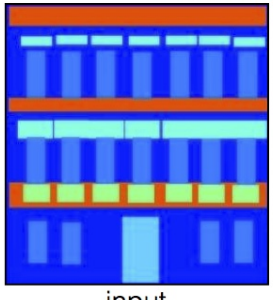
Aerial to Map



input

output

Labels to Facade



input

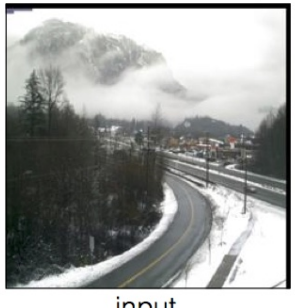
output

BW to Color

input

output

Day to Night



input

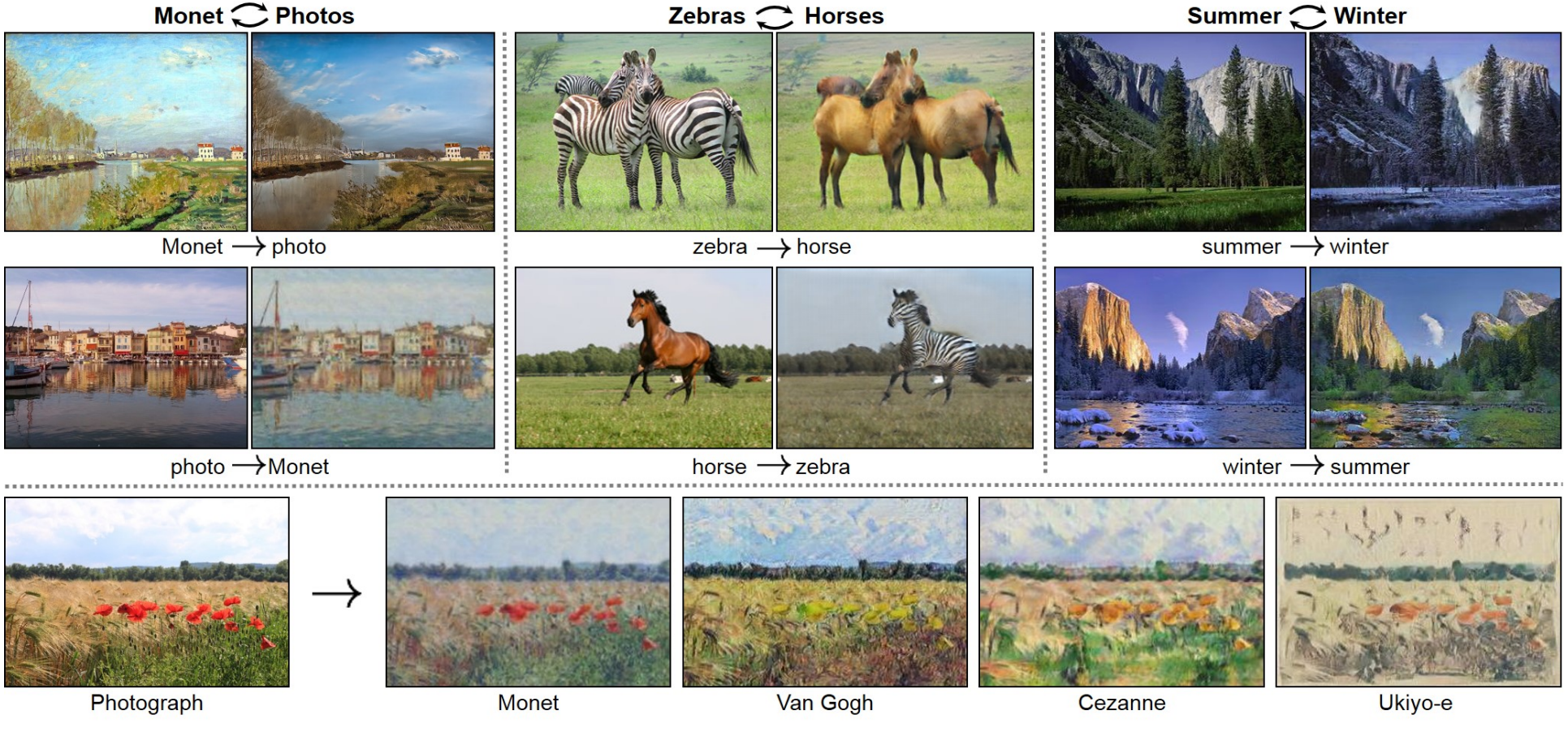
output

Edges to Photo

input

output

CycleGAN [Zhu+ ICCV-17]



Pose-to-image translation [Chan+ 2018]

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YouTube: Everybody Dance Now

DeepFakes

YouTube: Mark Zuckerberg 'deepfake' will remain online

More

- Optimal transport stuff:
  - Gabriel Peyré: *Optimal transport for machine learning* talk
  - Peyré and Cuturi, Computational Optimal Transport book
  - Kantorovich Initiative: `kantorovich.org`
  - Pacific Interdisciplinary Hub on Optimal Transport

- GANs / generative models…so much.