# CPSC 340:
# Machine Learning and Data Mining

Recommender Systems

Fall 2021

# Last Few Lectures: Latent-Factor Models

- We've been discussing latent-factor models of the form:

$$f(W,Z) = \sum_{i=1}^{n} \sum_{j=1}^{d} (\langle w^j, z_i \rangle - x_{ij})^2$$

- We get different models under different conditions:
  - K-means: each $z_i$ has one '1' and the rest are zero.
  - Least squares: we only have one variable (d=1) and the $z_i$ are fixed.
  - PCA: no restrictions on W or Z.
    - Orthogonal PCA (usual case): the rows $w_c$ have norm 1 and inner products of zero.
  - NMF: all elements of W and Z are non-negative.

# Variations on Latent-Factor Models

- We can use all our tricks for linear regression in this context:

$$f(W,Z) = \sum_{i=1}^{n}\sum_{j=1}^{d} |\langle w^j z_i \rangle - x_{ij}| + \frac{\lambda_1}{2}\sum_{i=1}^{n}\sum_{c=1}^{k} z_{ic}^2 + \frac{\lambda_2}{2}\sum_{j=1}^{d}\sum_{c=1}^{k} |w_{cj}|$$

- Absolute loss gives robust PCA that is less sensitive to outliers.
- We can use L2-regularization.
  - Though only reduces overfitting if we regularize both 'W' and 'Z'.
- We can use L1-regularization to give sparse latent factors/features.
- We can use logistic/softmax/Poisson losses for discrete $x_{ij}$.
- Can use change of basis to learn non-linear latent-factor models.

bonus!

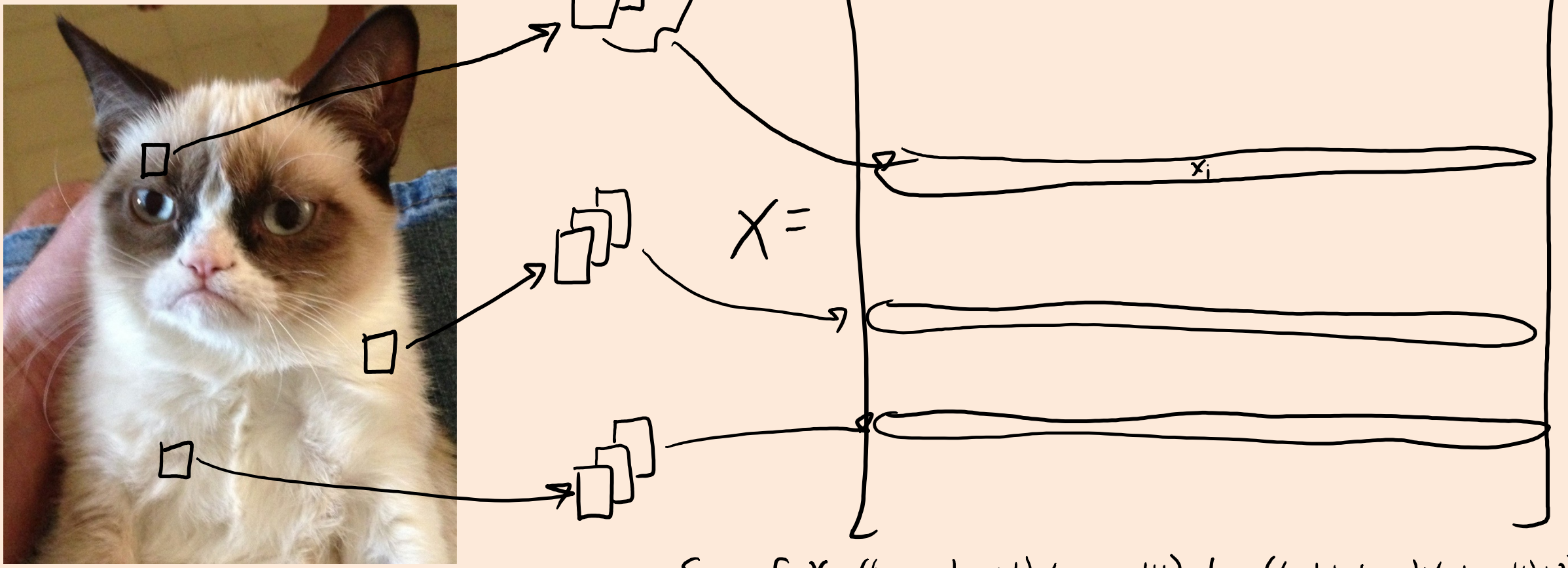
# Application: Image Restoration


http://www.jmlr.org/papers/volume11/mairal10a/mairal10a.pdf

bonus!

# Latent-Factor Models for Image Patches

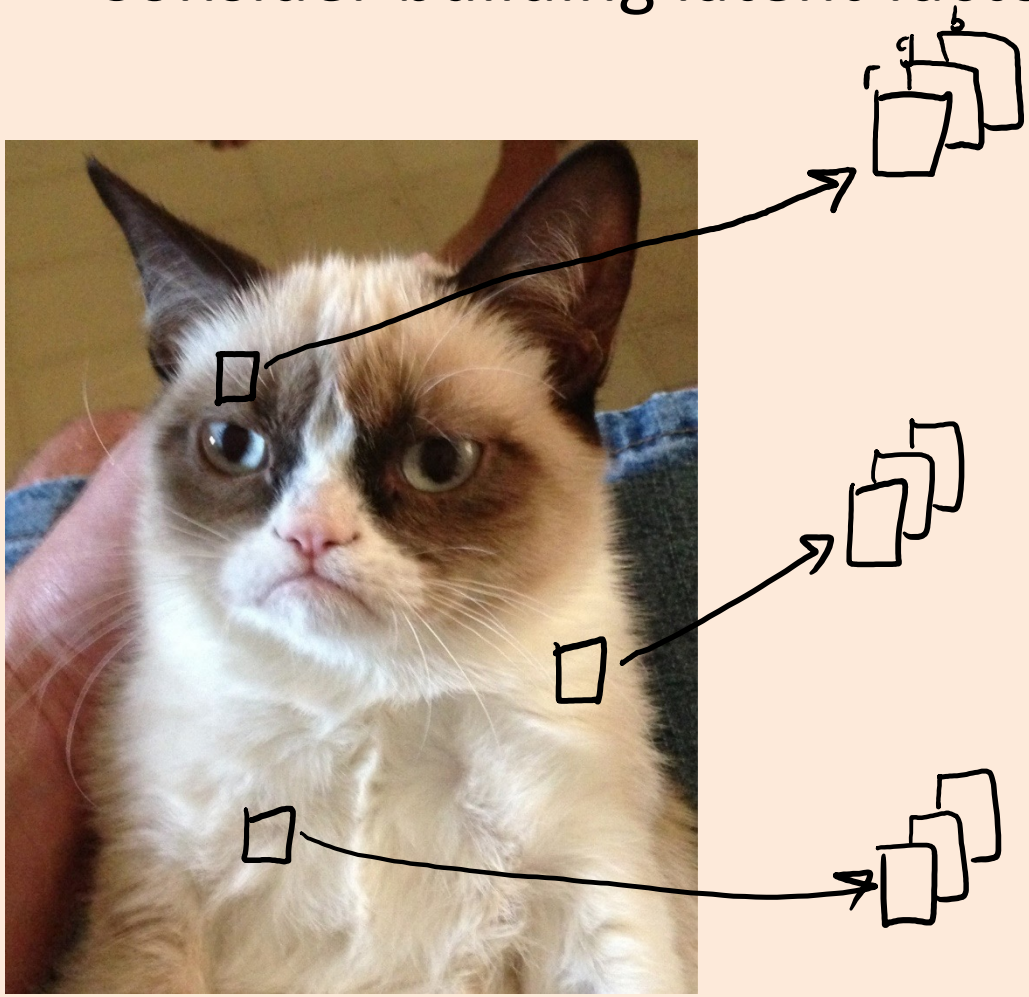
- Consider building latent-factors for general image patches:



bonus!

# Latent-Factor Models for Image Patches

- Consider building latent-factors for general image patches:



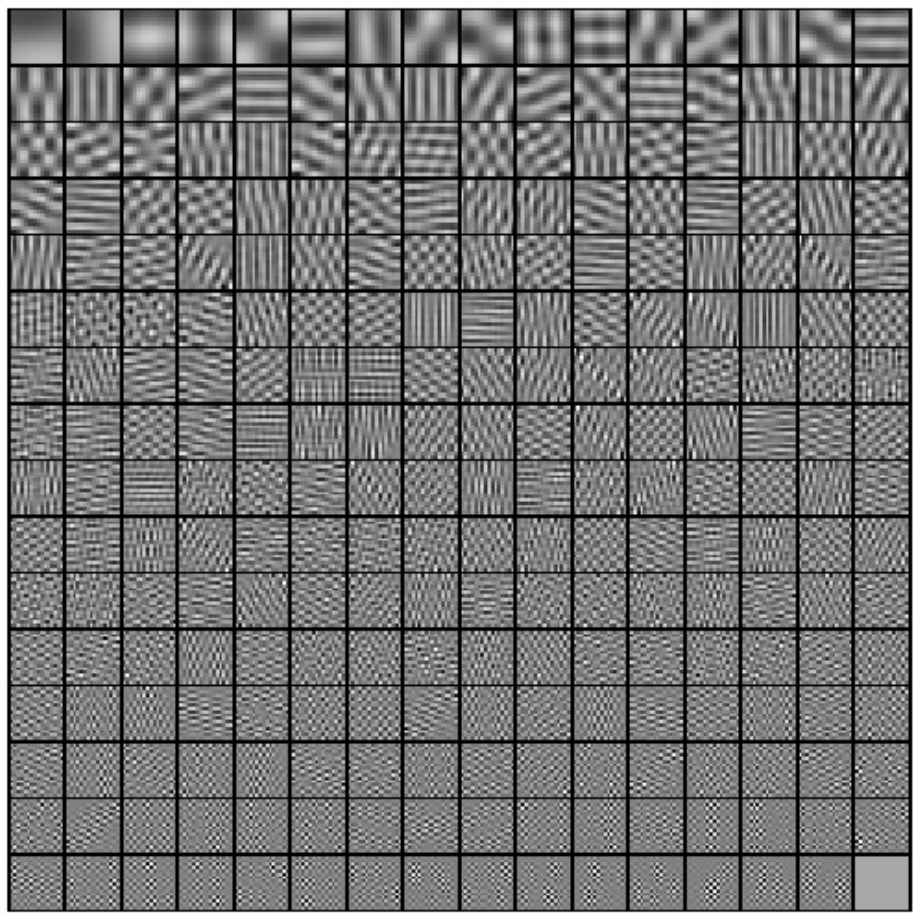
Typical pre-processing:

1. Usual variable centering
2. “Whiten” patches.

(remove correlations - bonus)

bonus!

# Latent-Factor Models for Image Patches

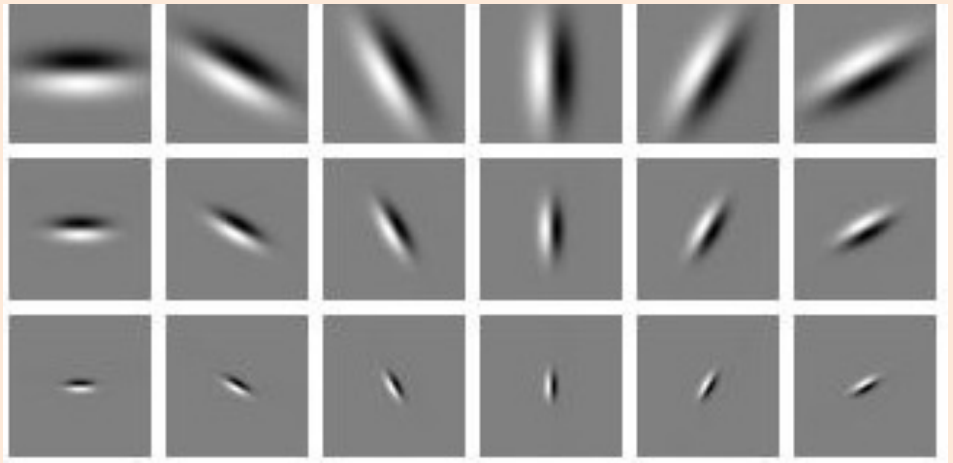


(b) Principal components.

Orthogonal bases don't seem right:

- Few PCs do almost everything.
- Most PCs do almost nothing.

We believe "simple cells" in visual cortex use:



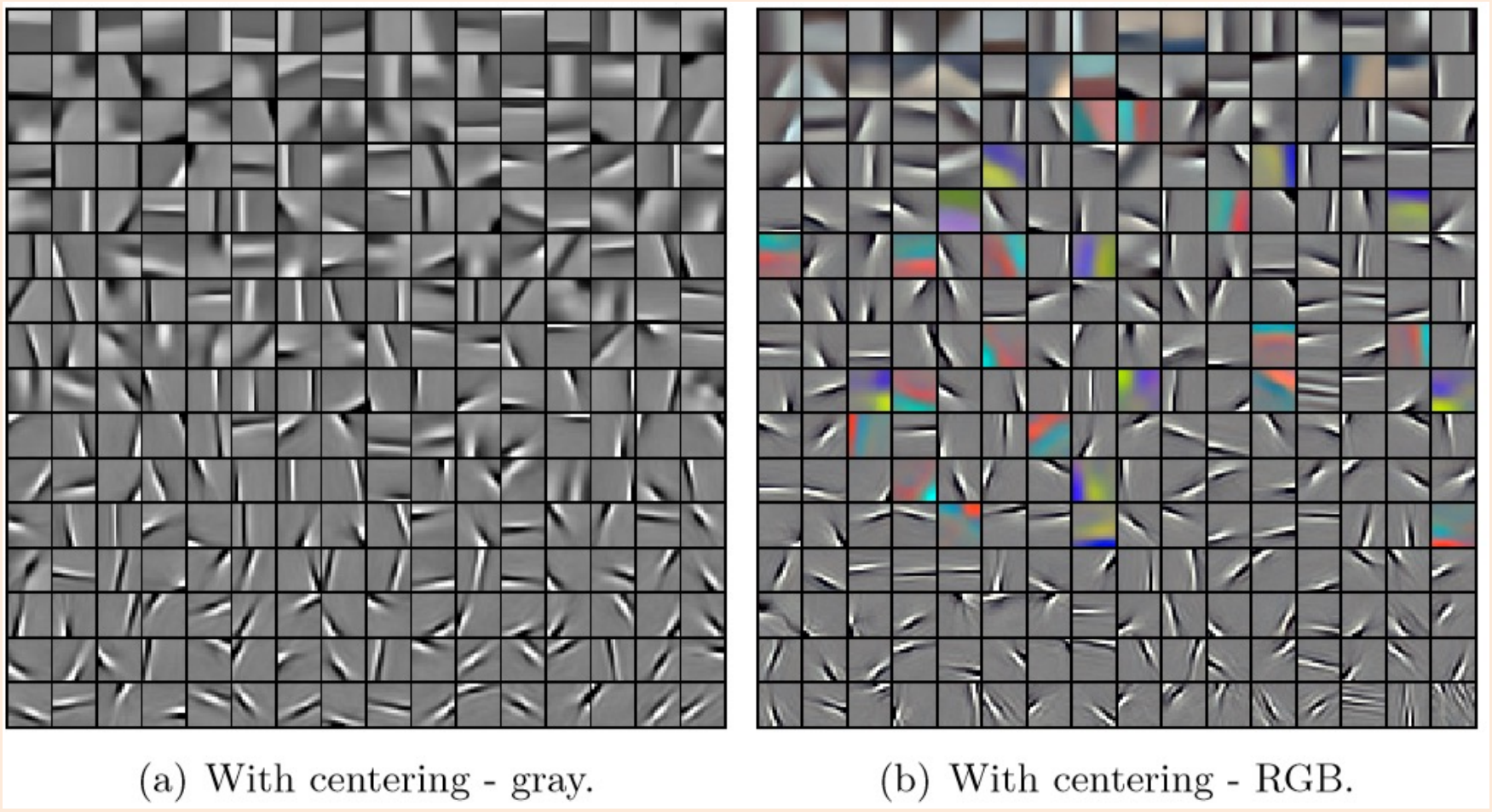
'Gabor' filters

http://lear.inrialpes.fr/people/mairal/resources/pdf/review_sparse_arxiv.pdf
http://stackoverflow.com/questions/16059462/comparing-textures-with-opencv-and-gabor-filters

bonus!

# Latent-Factor Models for Image Patches

- Results from a “sparse” (non-orthogonal) latent factor model:



(a) With centering - gray.

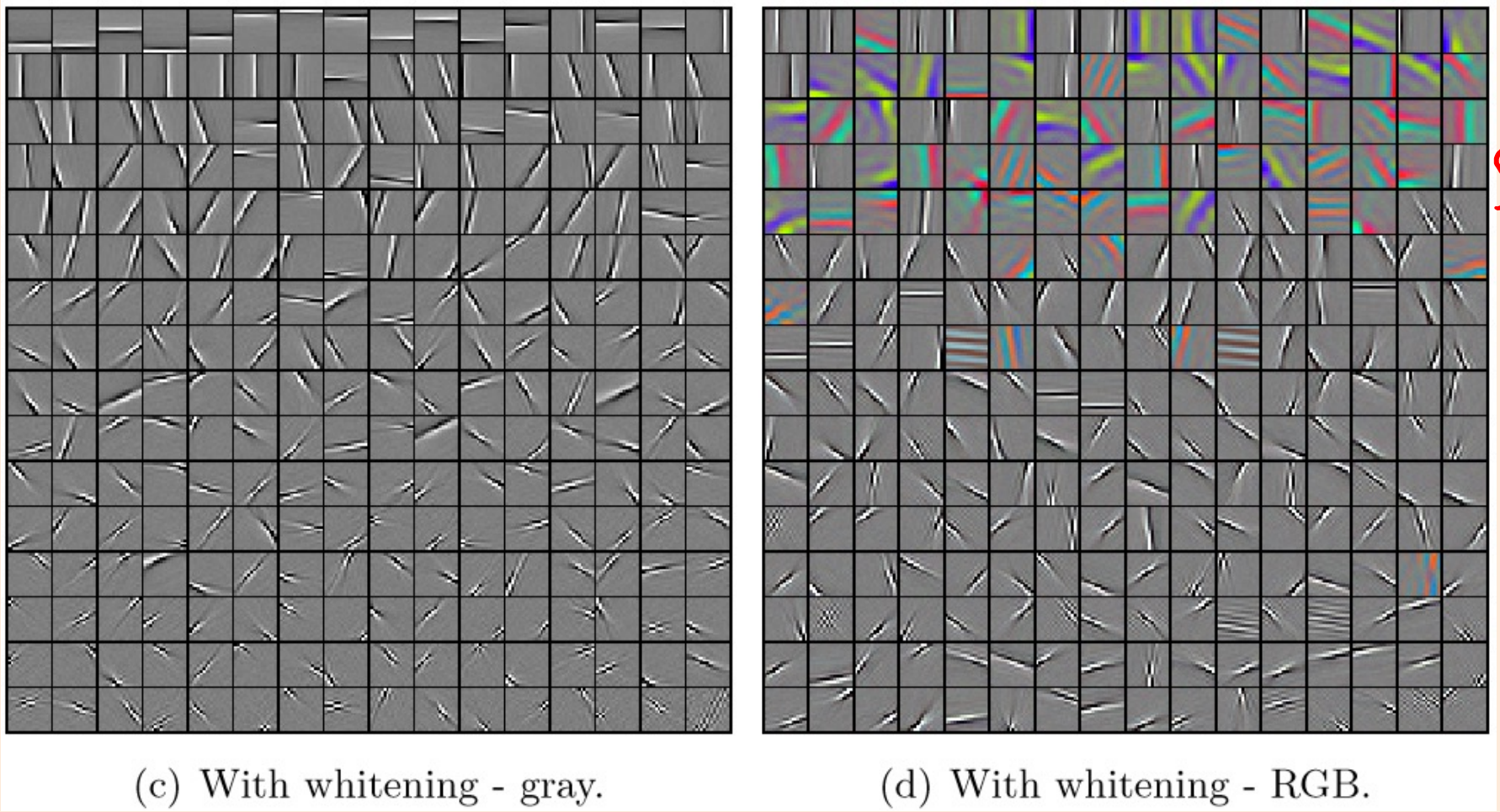
(b) With centering - RGB.

http://lear.inrialpes.fr/people/mairal/resources/pdf/review_sparse_arxiv.pdf

bonus!

# Latent-Factor Models for Image Patches

- Results from a “sparse” (non-orthogonal) latent-factor model:



(c) With whitening - gray.

(d) With whitening - RGB.

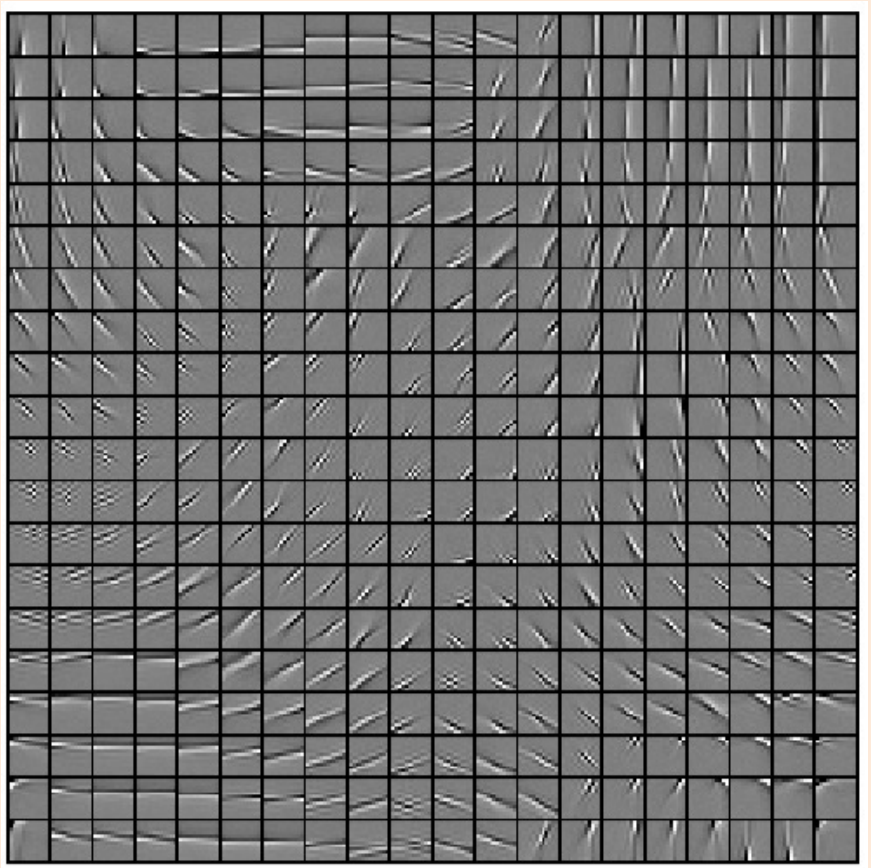
"colour opponency"

http://lear.inrialpes.fr/people/mairal/resources/pdf/review_sparse_arxiv.pdf

bonus!

# Recent Work: Structured Sparsity

- Basis learned with a variant of "structured sparsity":



(b) With $4 \times 4$ neighborhood.

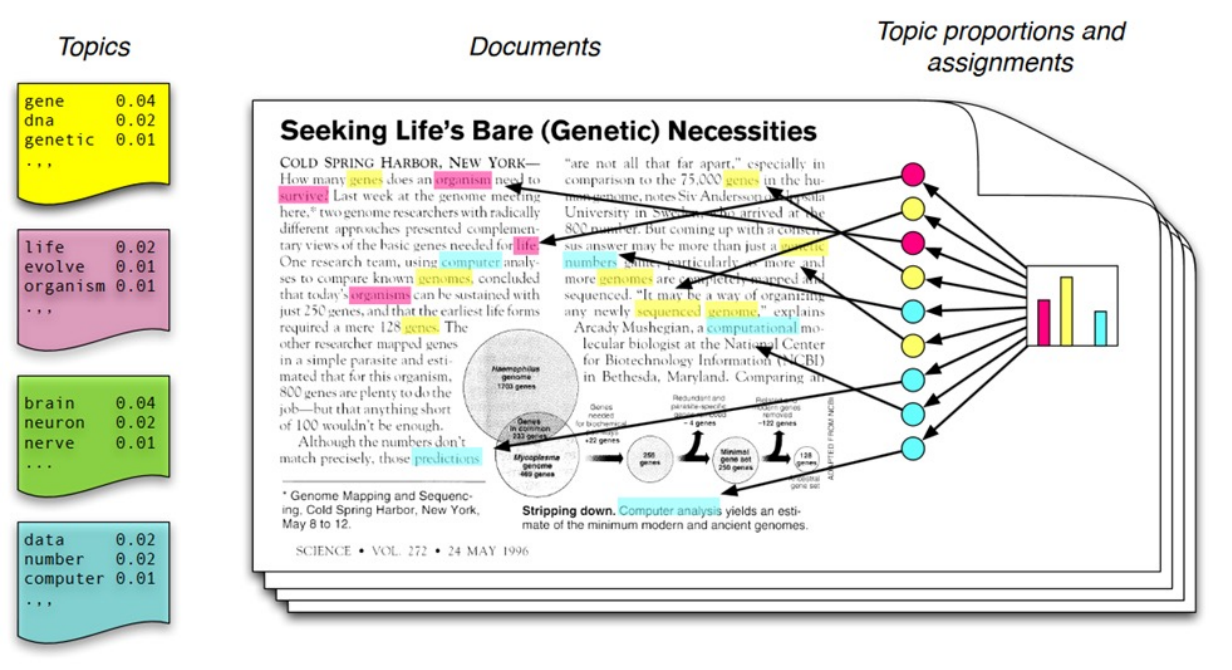
Similar to "cortical columns" theory in visual cortex.

http://lear.inrialpes.fr/people/mairal/resources/pdf/review_sparse_arxiv.pdf

bonus!

# Beyond NMF: Topic Models

- For modeling data as combinations of non-negative parts, NMF has largely been replaced by "topic models".
  - A "fully-Bayesian" model where sparsity arises naturally.
  - Most popular example is called "latent Dirichlet allocation" (CPSC 440).

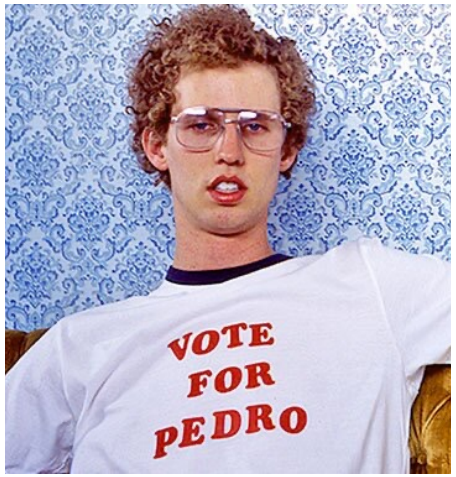



http://menome.com/wp/wp-content/uploads/2014/12/Blei2011.pdf

(pause)

# Recommender System Motivation: Netflix Prize

- Netflix Prize:
  - 100M ratings from 0.5M users on 18k movies.
  - Grand prize was $1M for first team to reduce squared error by 10%.
  - Started on October 2nd, 2006.
  - Netflix's system was first beat October 8th.
  - 1% error reduction achieved on October 15th.
  - Steady improvement after that.
    - ML methods soon dominated.
  - One obstacle was 'Napolean Dynamite' problem:
    - Some movie ratings seem very difficult to predict.
    - Should only be recommended to certain groups.



# Lessons Learned from Netflix Prize

- Prize awarded in 2009:
  - Ensemble method that averaged 107 models.
  - Increasing diversity of models more important than improving models.


- Winning entry (and most entries) used collaborative filtering:
  - Methods that only looks at ratings, not features of movies/users.
- A simple collaborative filtering method that does really well (7%):
  - "Regularized matrix factorization". Now adopted by many companies.

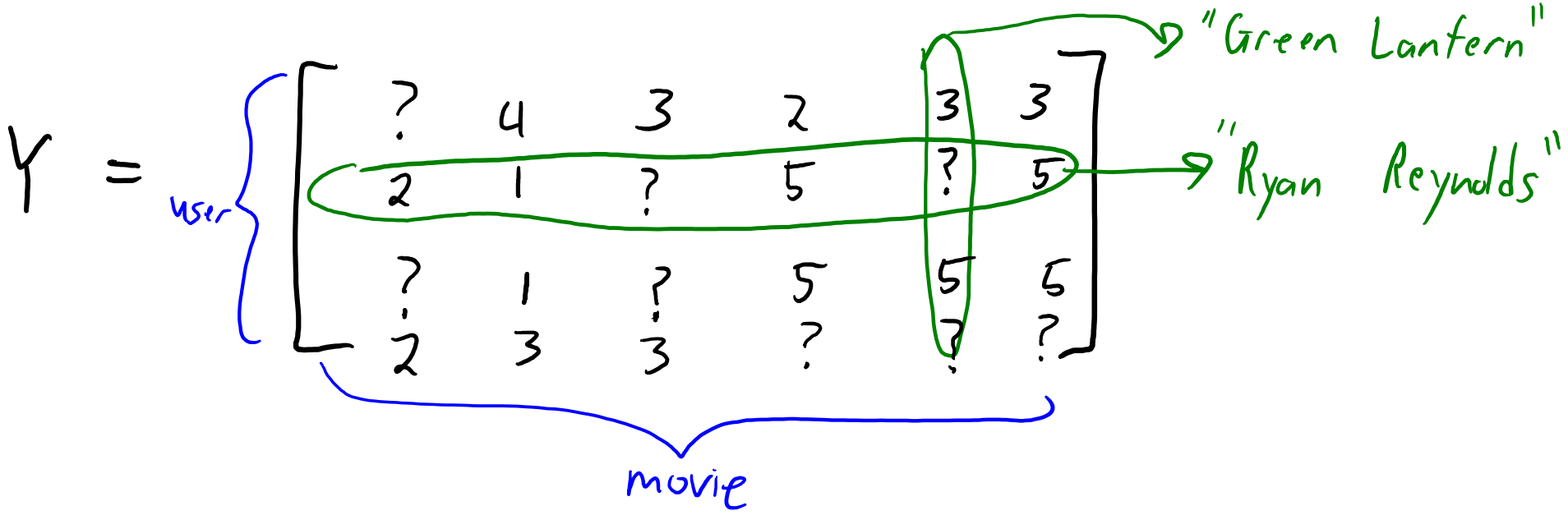
http://bits.blogs.nytimes.com/2009/09/21/netflix-awards-1-million-prize-and-starts-a-new-contest/?_r=0

# Motivation: Other Recommender Systems

- Recommender systems are now everywhere:
  - Music, news, books, jokes, experts, restaurants, friends, dates, etc.
- Main types of approaches:
  1. Content-based filtering.
     - Supervised learning:
       - Extract features $x_i$ of users and items, building model to predict rating $y_i$ given $x_i$.
       - Apply model to prediction for new users/items.
     - Example: G-mail's "important messages" (personalization with "local" features).
  2. Collaborative filtering.
     - "Unsupervised" learning (have label matrix 'Y' but no features):
       - We only have labels $y_{ij}$ (rating of user 'i' for movie 'j').
     - Example: Amazon recommendation algorithm.

# Collaborative Filtering Problem

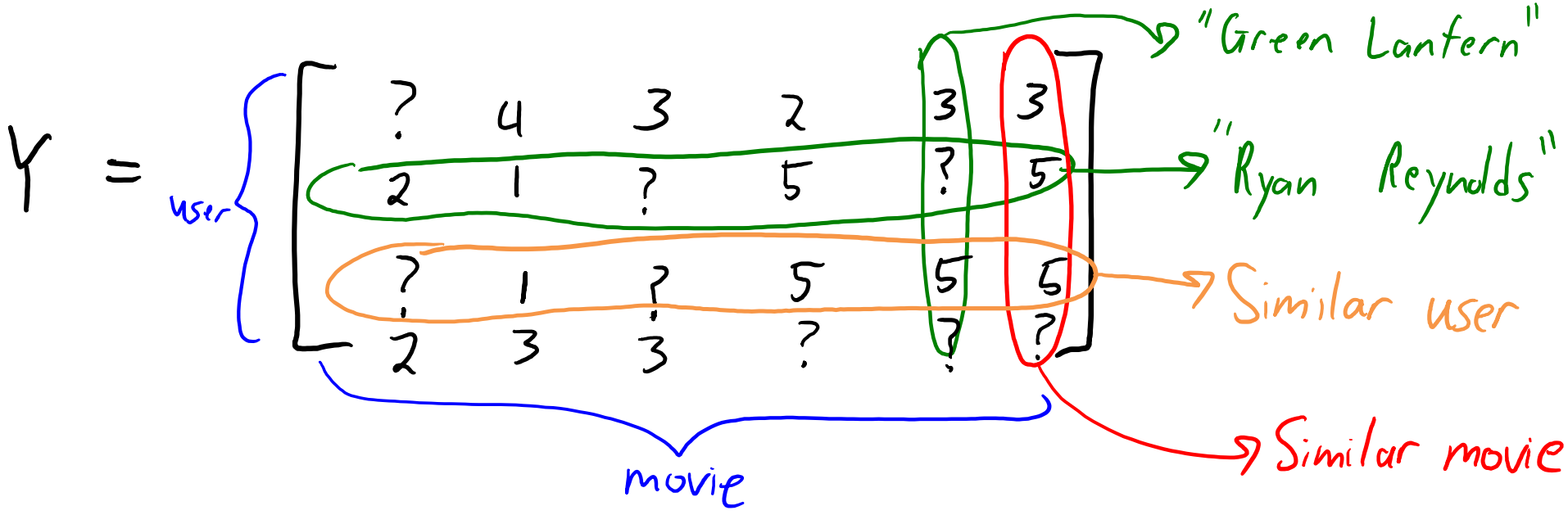
- Collaborative filtering is ‘filling in’ the user-item matrix:




- We have some ratings available with values {1, 2, 3, 4, 5}.
- We want to predict ratings “?” by looking at available ratings.

# Collaborative Filtering Problem

- Collaborative filtering is ‘filling in’ the user-item matrix:




- What rating would “Ryan Reynolds” give to “Green Lantern”?
  - Why is this not completely hopeless? It *could* be anything.
  - But we may have similar users and movies.

# Matrix Factorization for Collaborative Filtering

- Our standard latent-factor model for entries in matrix 'Y':

$$Y \approx ZW$$

$n \times d \quad n \times k \quad k \times d$

$$y_{ij} \approx \langle w^j, z_i \rangle$$

- User 'i' has latent features $z_i$.
- Movie 'j' has latent features $w^j$.

$z_{ic}$ could mean "likes Nicolas Caye"

$w_{jc}$ could mean "has Nicholas Cage"

- Our loss functions sums over available ratings 'R':

$$f(Z,W) = \sum_{(i,j) \in R} (\langle w^j, z_i \rangle - y_{ij})^2 + \frac{\lambda_1}{2} \|Z\|_F^2 + \frac{\lambda_2}{2} \|W\|_F^2$$

- And we add L2-regularization to both types of features.
  - Basically, this is regularized PCA on the available entries of Y.
  - Typically fit with SGD.
- This simple method gives you a 7% improvement on the Netflix problem.

bonus!

# Adding Global/User/Movie Biases

- Our standard latent-factor model for entries in matrix 'Y':

$$\hat{y}_{ij} = \langle w^j, z_i \rangle$$

- Sometimes we don't assume the $y_{ij}$ have a mean of zero:
  - We could add bias $\beta$ reflecting average overall rating:

$$\hat{y}_{ij} = \beta + \langle w^j, z_i \rangle$$

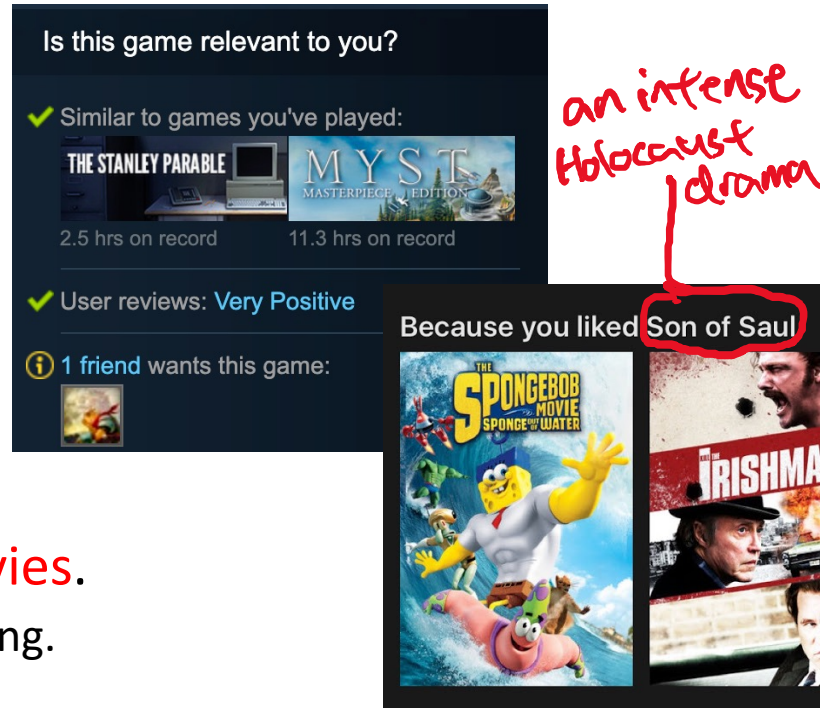
  - We could also add a user-specific bias $\beta_i$ and item-specific bias $\beta_j$.

$$\hat{y}_{ij} = \beta + \beta_i + \beta_j + \langle w^j, z_i \rangle$$

    - Some users are more generous, and some movies are just better.
    - These might also be regularized.

# Beyond Accuracy in Recommender Systems

- Winning system of Netflix Challenge was never adopted.
- Other issues important in recommender systems:
  - Diversity: how different are the recommendations?
    - If you like 'Battle of Five Armies Extended Edition', recommend 'Battle of Five Armies'?
    - Even if you really really like Star Wars, you might want non-Star-Wars suggestions.
  - Persistence: how long should recommendations last?
    - If you keep not clicking on 'Justice League', should it go away?
  - Trust: tell user *why* you made a recommendation.
  - Social recommendation: what did your friends watch?
  - Freshness: people tend to get more excited about *new/surprising* things.
    - Collaborative filtering does not predict well for new users/movies.
      - New movies don't yet have ratings, and new users haven't rated anything.



# Content-Based vs. Collaborative Filtering

- Our latent-factor approach to collaborative filtering (Part 4):

$$\hat{y}_{ij} = \langle w^j, z_i \rangle$$

"hidden" features of movie

"hidden" features of user

  - Learns about each user/movie, but can't predict on new users/movies.
- A linear model approach to content-based filtering (Part 3):

$$\hat{y}_{ij} = w^T x_{ij}$$

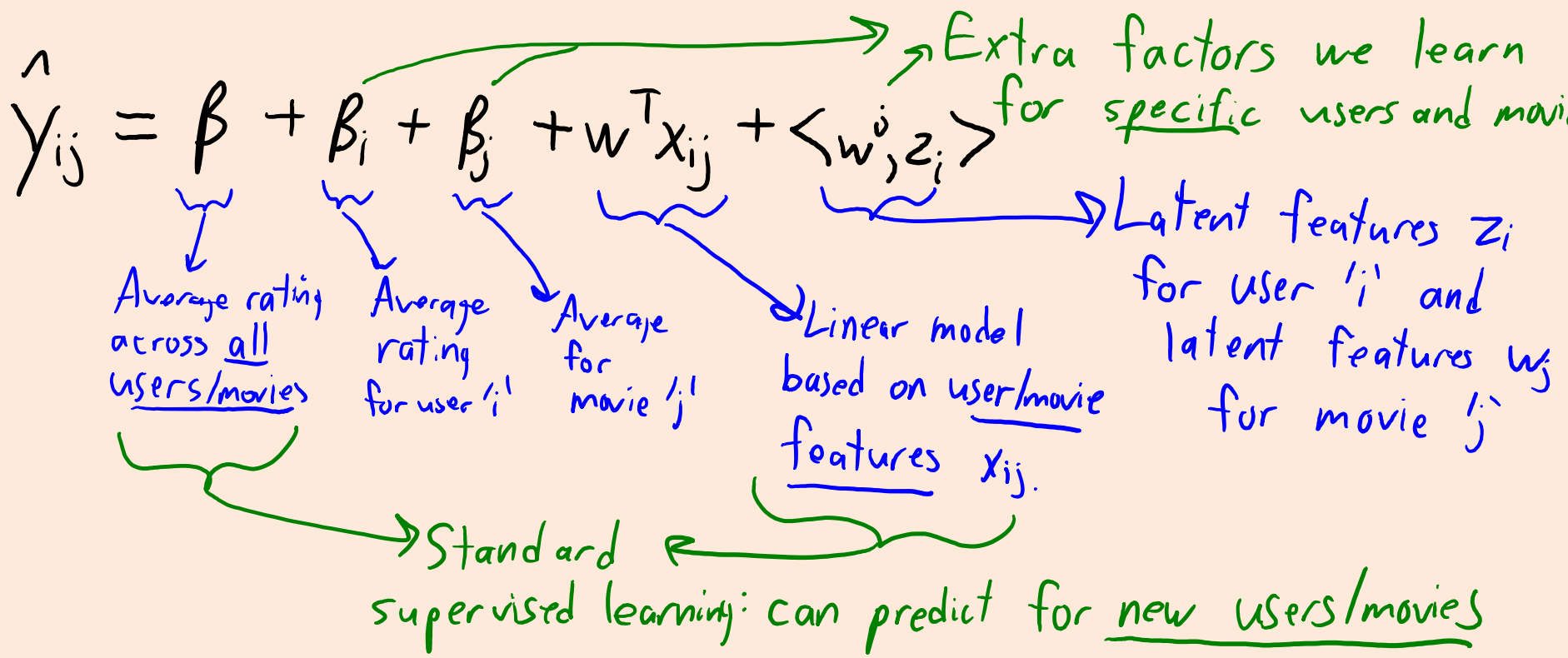
Our usual supervised learning setup. $y_i = w^T x_i$

  - Here $x_{ij}$ is a **vector of features** for the movie/user.
    - Usual supervised learning setup: 'y' would contain all the $y_{ij}$, X would have $x_{ij}$ as rows.
  - Can predict on new users/movies, but can't learn about each user/movie.

bonus!

# Hybrid Approaches

- Hybrid approaches combine content-based/collaborative filtering:
  - SVDfeature (won "KDD Cup" in 2011 and 2012).




  - Note that $x_{ij}$ is a feature *vector*. Also, 'w' and '$w^j$' are different parameters.

bonus!

# Stochastic Gradient for SVDfeature

- Common approach to fitting SVDfeature is stochastic gradient.
- Previously you saw stochastic gradient for supervised learning:
  - Choose a random example 'i'
  - Update parameters 'w' using gradient of example 'i'
- Stochastic gradient for SVDfeature (formulas as bonus):
  - Choose a random user 'i' and a random product 'j'
  - Update $\beta$, $\beta_i$, $\beta_j$, $w$, $z_i$, and $w^j$ based on their gradient for this user-product.

Updated every time

bonus!

# Social Regularization

- Many recommenders are now connected to social networks.
  - "Login using your Facebook account".

- Often, people like similar movies to their friends.

- Recent recommender systems use social regularization.
  - Add a "regularizer" encouraging friends' weights to be similar:

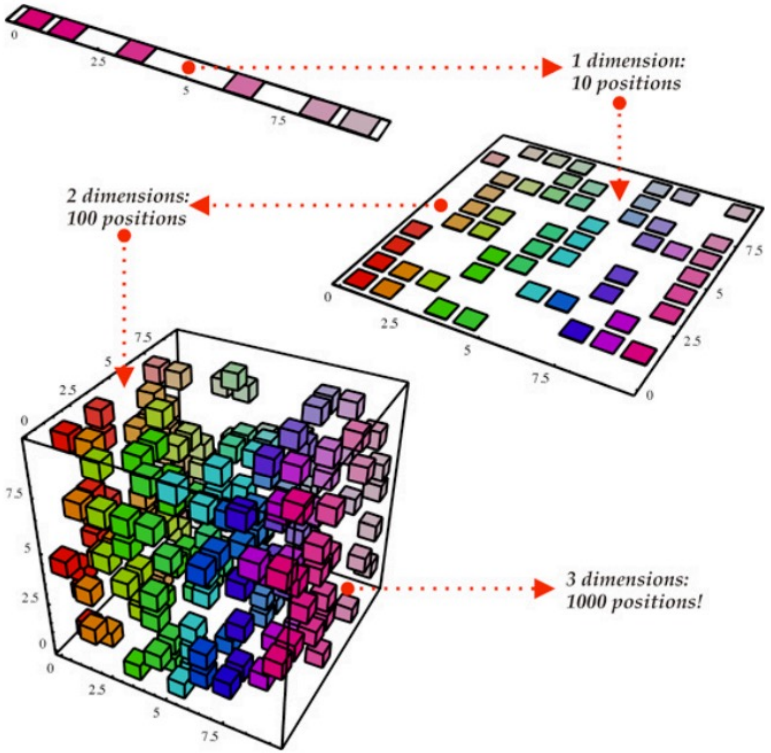
$$\frac{\lambda}{2} \sum_{(i,j) \in \text{"friends"}} \|w_i - w_j\|^2$$

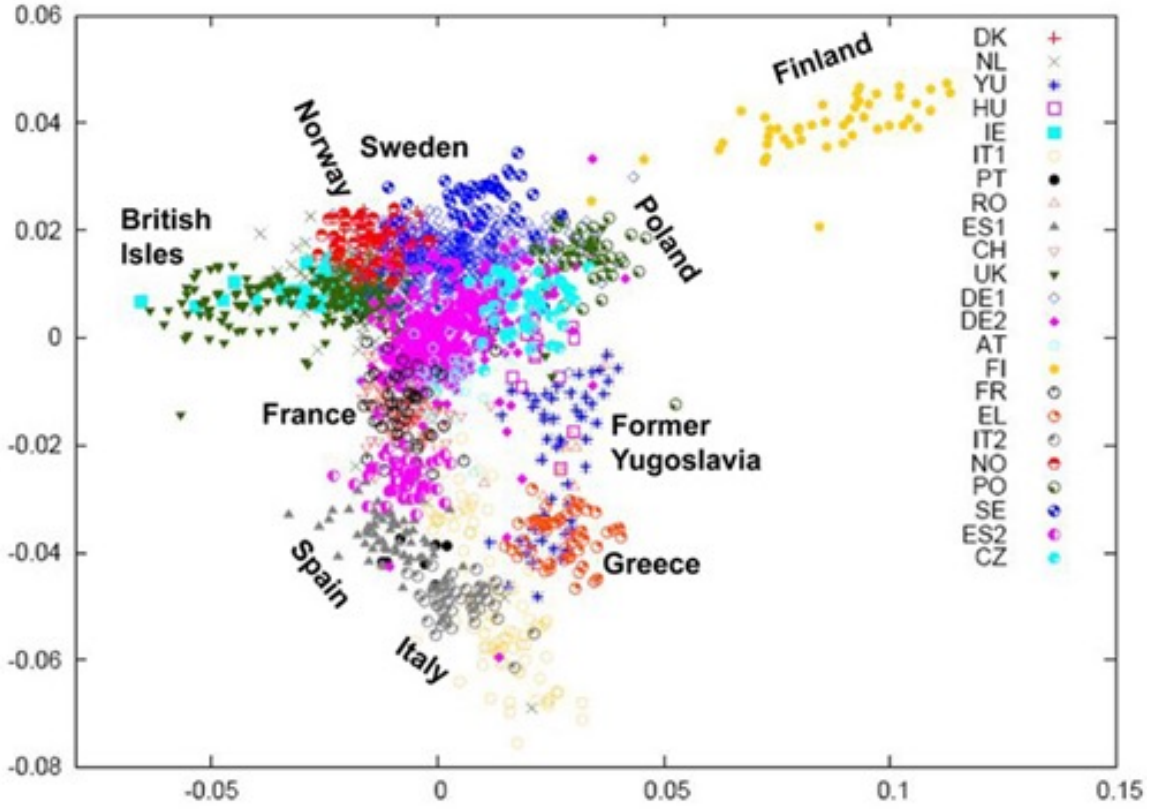
  - If we get a new user, recommendations are based on friend's preferences.

(pause)

# Latent-Factor Models for Visualization

- PCA takes features $x_i$ and gives k-dimensional approximation $z_i$.
- If k is small, we can use this to visualize high-dimensional data.

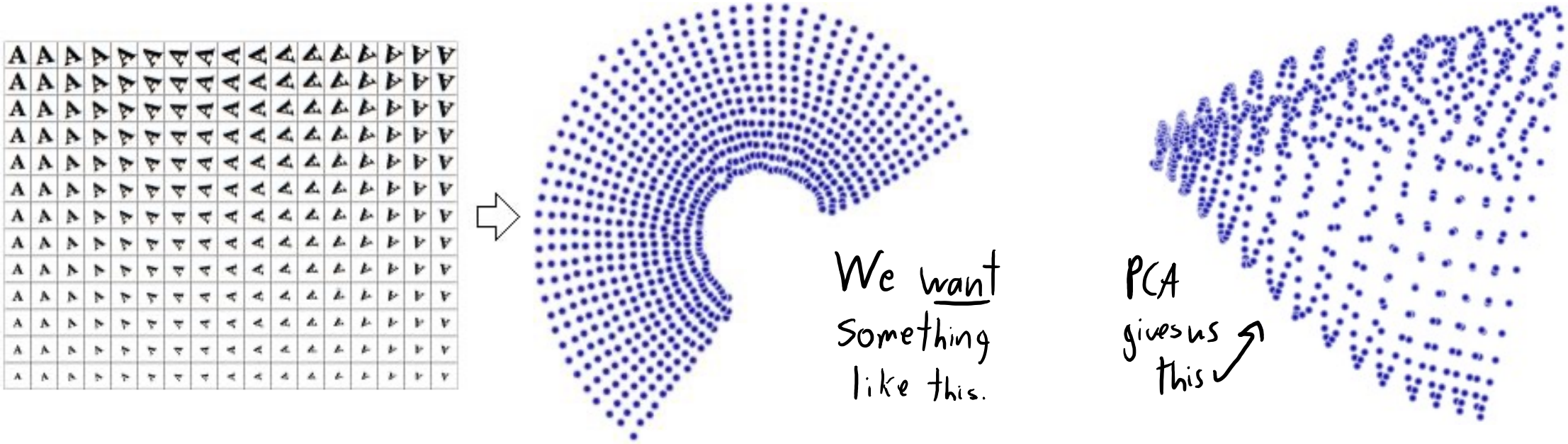



http://www.turingfinance.com/artificial-intelligence-and-statistics-principal-component-analysis-and-self-organizing-maps/
http://scienceblogs.com/gnxp/2008/08/14/the-genetic-map-of-europe/

# Motivation for Non-Linear Latent-Factor Models

- But PCA is a parametric linear model
- PCA may not find obvious low-dimensional structure.




- We could use change of basis or kernels: but still need to pick basis.

https://en.wikipedia.org/wiki/Nonlinear_dimensionality_reduction

# Multi-Dimensional Scaling

- PCA for visualization:
  - We're using PCA to get the location of the $z_i$ values.
  - We then plot the $z_i$ values as locations in a scatterplot.
- Multi-dimensional scaling (MDS) is a crazy idea:
  - Let's directly optimize the pixel locations of the $z_i$ values.
    - "Gradient descent on the points in a scatterplot".
  - Needs a "cost" function saying how "good" the $z_i$ locations are.
    - Traditional MDS cost function:

$$f(Z) = \sum_{i=1}^{n} \sum_{j=i+1}^{n} \left( \|z_i - z_j\| - \|x_i - x_j\| \right)^2$$

"Try to make scatterplot distances match high-dimensional distance"

sum over pairs of examples

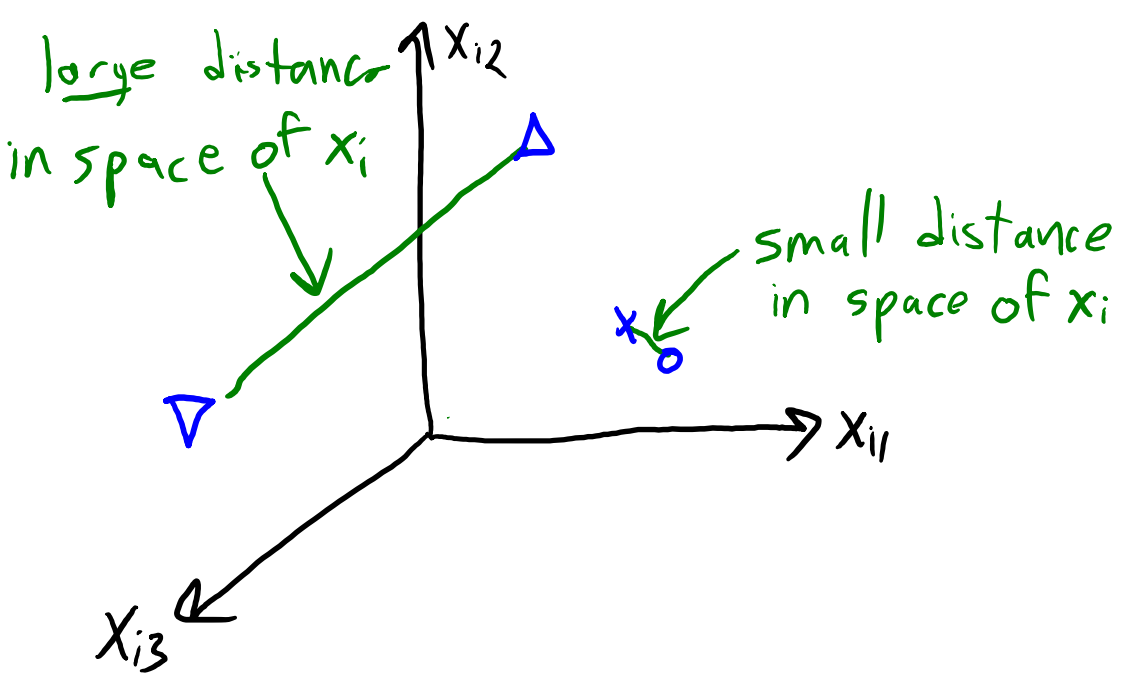
distance in scatterplot

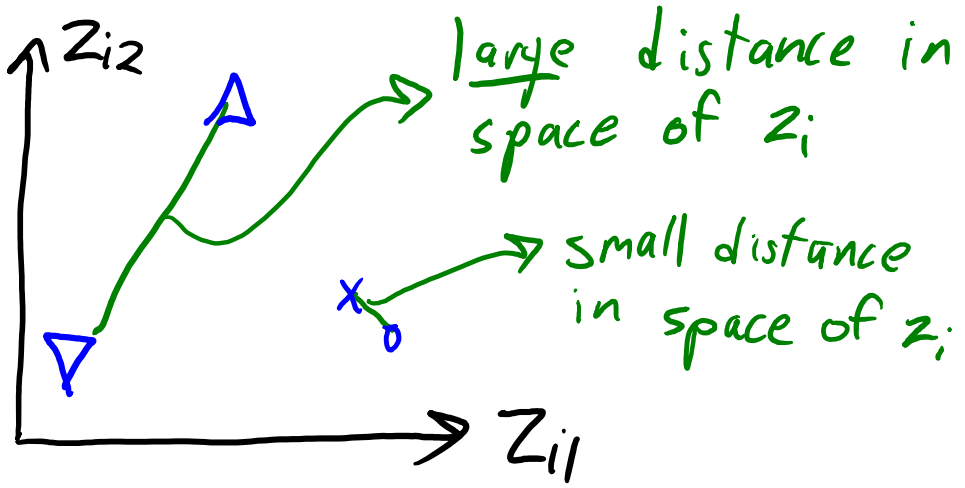
Distance between points in original 'd' dimensions

# Multi-Dimensional Scaling

- Multi-dimensional scaling (MDS):
  - Directly optimize the final locations of the $z_i$ values.

$$f(Z) = \sum_{i=1}^{n} \sum_{j=i+1}^{n} (\|z_i - z_j\| - \|x_i - x_j\|)^2$$

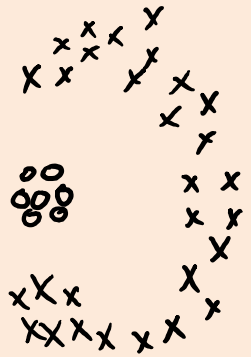


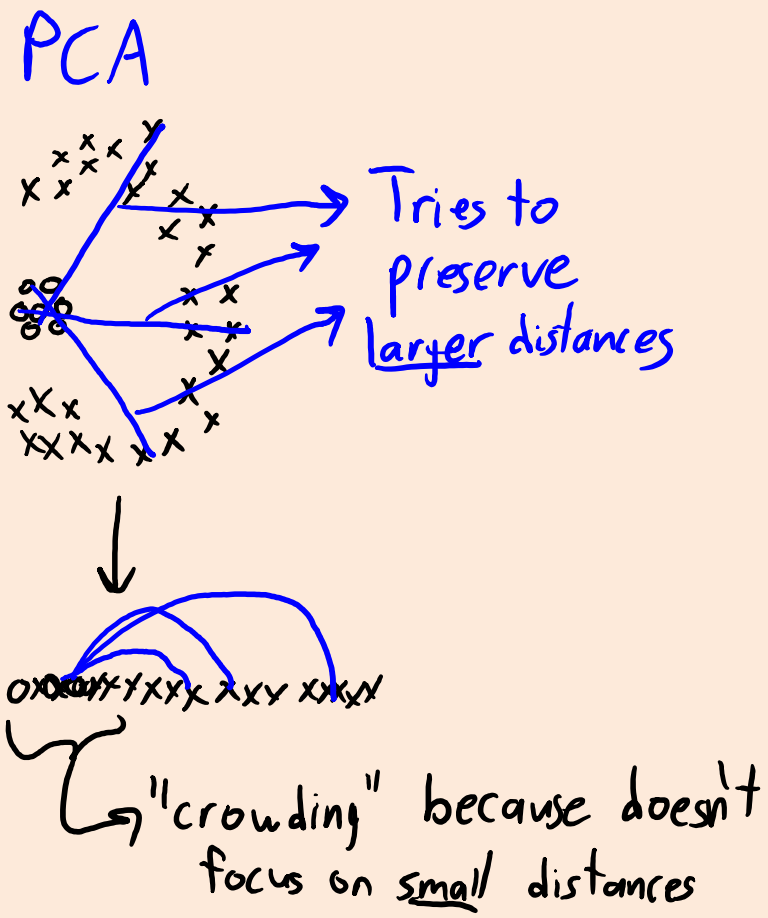



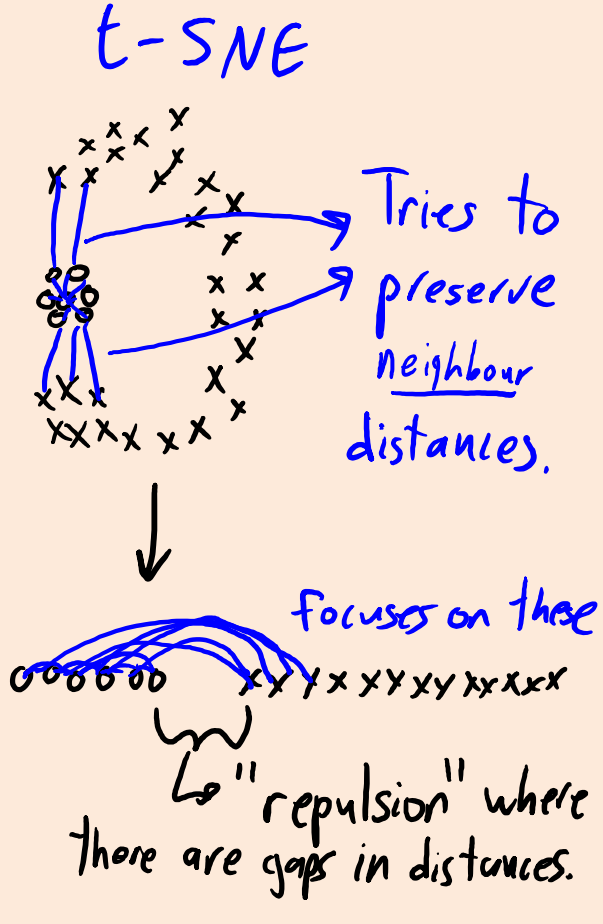
bonus!

# t-Distributed Stochastic Neighbour Embedding

- One key idea in t-SNE:
  - Focus on distance to “neighbours”(allow large variance in other distances)






Good demo: https://distill.pub/2016/misread-tsne

# Summary

- Recommender systems try to recommend products.
- Collaborative filtering tries to fill in missing values in a matrix.
  - Matrix factorization is a common approach.
- Multi-dimensional scaling is a non-parametric latent-factor model.
  - Big space of variants that we didn't have time to go into.

- Next time: the long-awaited start of deep learning.

bonus!

# Digression: "Whitening"

- With image data, features will be very redundant.
  - Neighbouring pixels tend to have similar values.
- A standard transformation in these settings is "whitening":
  - Rotate the data so features are uncorrelated.
  - Re-scale the rotated features so they have a variance of 1.

- Using SVD approach to PCA, we can do this with:
  - Get 'W' from SVD (usually with k=d).
  - $Z = XW^T$ (rotate to give uncorrelated features).
  - Divide columns of 'Z' by corresponding singular values (unit variance).

- Details/discussion here.

bonus!

# Motivation for Topic Models

- Want a model of the "factors" making up documents.
  - Instead of latent-factor models, they're called topic models.
  - The canonical topic model is latent Dirichlet allocation (LDA).

Suppose you have the following set of sentences:

- I like to eat broccoli and bananas.
- I ate a banana and spinach smoothie for breakfast.
- Chinchillas and kittens are cute.
- My sister adopted a kitten yesterday.
- Look at this cute hamster munching on a piece of broccoli.

What is latent Dirichlet allocation? It's a way of automatically discovering **topics** that these sentences contain. For example, given these sentences and asked for 2 topics, LDA might produce something like

- **Sentences 1 and 2**: 100% Topic A
- **Sentences 3 and 4**: 100% Topic B
- **Sentence 5**: 60% Topic A, 40% Topic B
- **Topic A**: 30% broccoli, 15% bananas, 10% breakfast, 10% munching, … (at which point, you could interpret topic A to be about food)
- **Topic B**: 20% chinchillas, 20% kittens, 20% cute, 15% hamster, … (at which point, you could interpret topic B to be about cute animals)

  - "Topics" could be useful for things like searching for relevant documents.

http://blog.echen.me/2011/08/22/introduction-to-latent-dirichlet-allocation/

# Term Frequency – Inverse Document Frequency

bonus!

- In information retrieval, classic word importance measure is TF-IDF.
- First part is the term frequency tf(t,d) of term ‘t’ for document ‘d’.
  - Number of times “word” ‘t’ occurs in document ‘d’, divided by total words.
  - E.g., 7% of words in document ‘d’ are “the” and 2% of the words are “Lebron”.
- Second part is document frequency df(t,D).
  - Compute number of documents that have ‘t’ at least once.
  - E.g., 100% of documents contain “the” and 0.01% have “LeBron”.
- TF-IDF is tf(t,d)*log(1/df(t,D)).

# Term Frequency – Inverse Document Frequency

bonus!

- The TF-IDF statistic is tf(t,d)*log(1/df(t,D)).
  - It's high if word 't' happens often in document 'd', but isn't common.
  - E.g., seeing "LeBron" a lot it tells you something about "topic" of article.
  - E.g., seeing "the" a lot tells you nothing.

- There are *many* variations on this statistic.
  - E.g., avoiding dividing by zero and all types of "frequencies".

- Summarizing 'n' documents into a matrix X:
  - Each row corresponds to a document.
  - Each column gives the TF-IDF value of a particular word in the document.

bonus!

# Latent Semantic Indexing

- TF-IDF features are very redundant.
  - Consider TF-IDFs of “LeBron”, “Durant”, “Harden”, and “Kobe”.
  - High values of these typically just indicate topic of “basketball”.

- We can probably compress this information quite a bit.

- Latent Semantic Indexing/Analysis:
  - Run latent-factor model (like PCA or NMF) on TF-IDF matrix X.
  - Treat the principal components as the “topics”.
  - Latent Dirichlet allocation is a variant that avoids weird df(t,D) heuristic.

bonus!

# SVDfeature with SGD: the gory details

Objective: $\frac{1}{2}\sum_{(i,j)\in R}(\hat{y}_{ij}-y_{ij})^2$ with $\hat{y}_{ij}=\beta+\beta_i+\beta_j+w^T x_{ij}+(w^j)^T z_i$

$r_{ij}$

Update based on random $(i,j)$:

$$\beta = \beta - \alpha r_{ij}$$
$$\beta_i = \beta_i - \alpha r_{ij}$$
$$\beta_j = \beta_j - \alpha r_{ij}$$

Updates are the same, but '$\beta$' is always update while $\beta_i$ and $\beta_j$ are only updated for the specific user + product

$$w = w - \alpha r_{ij} x_{ij} \leftarrow \text{Updated every time.}$$
$$z_i = z_i - \alpha r_{ij} w^j$$
$$w^j = w^j - \alpha r_{ij} z_i$$

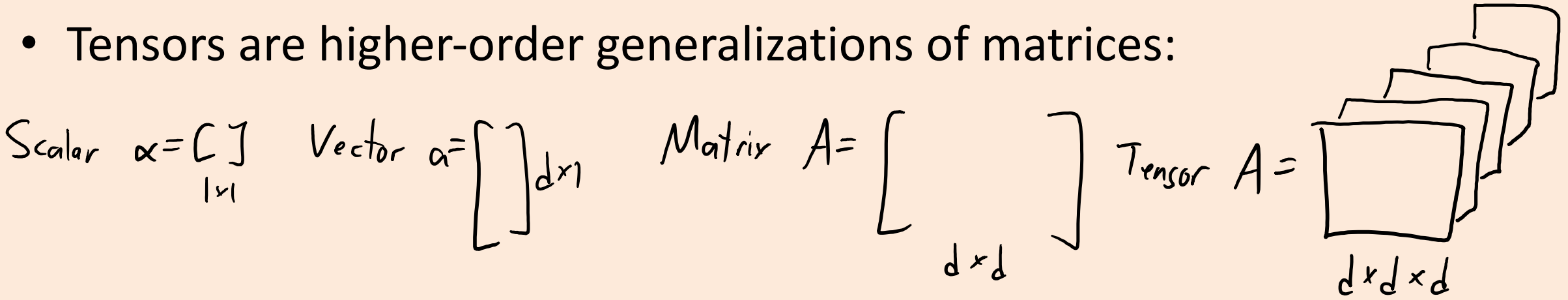
Updated for specific user and product.

(Adding regularization adds an extra term)

bonus!

# Tensor Factorization

- Tensors are higher-order generalizations of matrices:




- Generalization of matrix factorization is tensor factorization:

$$y_{ijm} \approx \sum_{c=1}^{k} w_{jc} z_{ic} v_{mc}$$

- Useful if there are other relevant variables:
  - Instead of ratings based on {user,movie}, ratings based {user,movie,group}.
  - Useful if you have groups of users, or if ratings change over time.

bonus!

# Field-Aware Matrix Factorization

- Field-aware factorization machines (FFMs):
  - Matrix factorization with multiple $z_i$ or $w_c$ for each example or part.
  - You choose which $z_i$ or $w_c$ to use based on the value of feature.

- Example from "click through rate" prediction:
  - E.g., predict whether "male" clicks on "nike" advertising on "espn" page.
  - A previous matrix factorization method for the 3 factors used:
  - FFMs could use:

$$w_{espn}\, w_{nike} + w_{espn}\, w_{male} + w_{nike}\, w_{male}$$

$$w^{A}_{espn}\, w^{P}_{nike} + w^{G}_{espn}\, w^{P}_{male} + w^{G}_{nike}\, w^{A}_{male}$$

    - wespnA is the factor we use when multiplying by a an advertiser's latent factor.
    - wespnG is the factor we use when multiplying by a group's latent factor.
- This approach has won some Kaggle competitions (link), and has shown to work well in production systems too (link).

bonus!

# Warm-Starting

- We've used data {X,y} to fit a model.
- We now have new training data and want to fit new and old data.

- Do we need to re-fit from scratch?

- This is the warm starting problem.
  - It's easier to warm start some models than others.

bonus!

# Easy Case: K-Nearest Neighbours and Counting

- K-nearest neighbours:
  - KNN just stores the training data, so just store the new data.

- Counting-based models:
  - Models that base predictions on frequencies of events.
  - E.g., naïve Bayes.
  - Just update the counts:

$$p(\text{"vicodin"} \mid \text{"spam"}) = \frac{\text{count of } \{\text{vicodin, spam}\} \text{ in new and old data}}{\text{count of "spam" in new and old data}}$$

  - Decision trees with fixed rules: just update counts at the leaves.

# Medium Case: L2-Regularized Least Squares

bonus!

- L2-regularized least squares is obtained from linear algebra:

$$w = (X^T X + \lambda I)^{-1}(X^T y)$$

  - Cost is $O(nd^2 + d^3)$ for 'n' training examples and 'd' features.
- Given one new point, we need to compute:
  - $X^T y$ with one row added, which costs $O(d)$.
  - Old $X^T X$ plus $x_i x_i^T$, which costs $O(d^2)$.
  - Solution of linear system, which costs $O(d^3)$.
  - So cost of adding 't' new data point is $O(td^3)$.

- With "matrix factorization updates", can reduce this to $O(td^2)$.
  - Cheaper than computing from scratch, particularly for large d.

bonus!

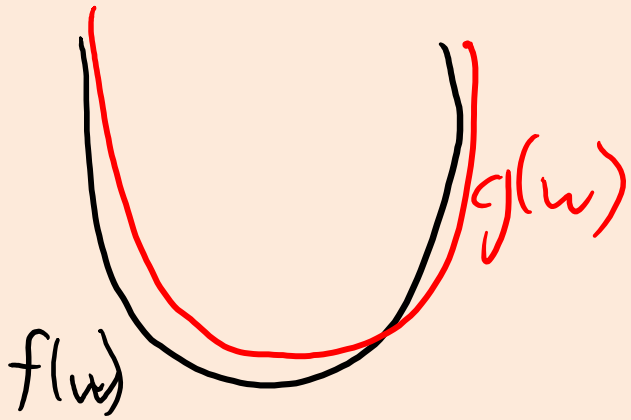
# Medium Case: Logistic Regression

- We fit logistic regression by gradient descent on a convex function.

- With new data, convex function f(w) changes to new function g(w).

$$f(w) = \sum_{i=1}^{n} f_i(w) \qquad g(w) = \sum_{i=1}^{n+1} f_i(w)$$

- If we don't have much more data, 'f' and 'g' will be "close".
  - Start gradient descent on 'g' with minimizer of 'f'.
  - You can show that it requires fewer iterations.



# Hard Cases: Non-Convex/Greedy Models

bonus!

- For decision trees:
  - "Warm start": continue splitting nodes that haven't already been split.
  - "Cold start": re-fit everything.

- Unlike previous cases, this won't in general give same result as re-fitting:
  - New data points might lead to different splits higher up in the tree.

- Intermediate: usually do warm start but occasionally do a cold start.

- Similar heuristics/conclusions for other non-convex/greedy models:
  - K-means clustering.
  - Matrix factorization (though you can continue PCA algorithms).