HONG KONG BAPTIST UNIVERSITY
SEMESTER 1 TAKE-HOME ASSESSMENT, 2019-2020

Course Code:	**MATH 4665/4875/7300**	Section Number:	**01**	Time Allowed:	**7** Days
Course Title:	**Special Topics in Applied Mathematics/Statistics I - Advanced Computational Finance**	Total No. of Pages:	**2**		

INSTRUCTIONS:

1. Answer ALL of the following questions.
2. The full mark for this examination is 100.
3. Calculators are allowed.

1. (30%) Let $y = f(x)$ be continuous on $[a, b]$ and continuously differentiable to sufficiently high order on (a, b). Given that

$$\Delta f(x_i) = \frac{f(x_{i+1}) - f(x_i)}{h_{i+1}}, \quad i = 1, 2, \ldots, N; \tag{1.1}$$

$$\nabla f(x_i) = \frac{f(x_i) - f(x_{i-1})}{h_i}, \quad i = 1, 2, \ldots, N, \tag{1.2}$$

where $\mathcal{D}_N = \{x_0, x_1, x_2, \ldots, x_{N+1}\},\ N \gg 1$, is a mesh defined on $[a, b]$ for which $x_0 = a, x_{N+1} = b$ with $0 < h_\ell = x_\ell - x_{\ell-1} \ll 1,\ \ell = 1, 2, \ldots, N+1$.

(a) (10%) Show that

$$h_{i+1}\Delta(\nabla f(x_i)) = h_i\nabla(\Delta f(x_i)), \quad 1 \le i \le N.$$

(b) (20%) Let $x_i \equiv c$ be fixed and $h_i \ne h_{i+1}$. Show that neither $\Delta(\nabla f(x_i))$ nor $\nabla(\Delta f(x_i))$ approximates the financial derivative value $f''(x_i),\ 1 \le i \le N$.

2. (30%) Consider the risk-free price equilibrium evolution index

$$u'(t) = f(u), \quad 0 < t \le T, \tag{2.1}$$

$$u(0) = \phi, \tag{2.2}$$

where $f(u)$ satisfies the Lipschitz condition with Lipschitz constant $\lambda > 0$ and ϕ is a constant. Let mesh $\mathcal{D}_M = \{t_0, t_1, t_2, \ldots, t_{M+1}\};\ t_i = ih,\ i = 0, 1, \ldots, M+1;\ (M+1)h = T$, be uniform over $[0, T]$.

(a) (10%) Derive the forward Euler's method for solving (2.1), (2.2).

(b) (20%) Suppose that

$$\epsilon_{i+1} \le \frac{c}{\lambda} h \left[(1 + h\lambda)^{i+1} - 1\right], \quad i = 0, 1, 2, \ldots, M,$$

where ϵ_{i+1} is the absolute error between the exact solution $u(t_{i+1})$ and numerical solution u_{i+1} at t_{i+1}. Show that the forward Euler's method is convergent on any uniform mesh.

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3. (40%) Consider following simplified Black-Scholes initial-boundary value problem:

$$
\begin{aligned}
u_t(s,t) &= \alpha u_{ss}(s,t), \quad S_{\min} < s < S_{\max},\ t > 0, && (3.1)\\
u(s,0) &= \phi(s), \quad S_{\min} < s < S_{\max}, && (3.2)\\
u(S_{\min},t) &= \beta, \quad t > 0, && (3.3)\\
u(S_{\max},t) &= \gamma, \quad t > 0, && (3.4)
\end{aligned}
$$

where u is the option price, ϕ is the initial price function given, $S_{\min}, S_{\max}$ are real values. Further, α, β, γ are positive constants.

(a) (30%) Derive a consistent fully discretized scheme for solving the Black-Scholes problem (3.1)-(3.4). A matrix form is preferred.

(b) (10%) What is the Courant-Friedrichs-Lewy (CFL) number for your fully discretized scheme?

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