Solution references for project 2

MATH 4665/4875/7140/7300, Q. Sheng, HKBU, Fall 2019

We should have

$$\begin{aligned} u'(t) &= \frac{1}{h^2} A u(t), \quad t \in J, \\ u(0) &= u_0, \end{aligned}$$

where

$$A = \text{diag}\{1, \ -2, \ 1\}, \quad u_0 = \begin{bmatrix} \sin(\pi h) \\ \sin(2\pi h) \\ \vdots \\ \sin((N-2)\pi h) \\ \sin((N-1)\pi h) \end{bmatrix}.$$

A backward Euler gives

$$\left(I - \frac{\tau}{h^2} A\right) u_{i+1} = M u_{i+1} = u_i, \quad i = 0, 1, \ldots, N,$$

where $0 < \tau \ll 1$ is the temporal mesh step.

We use the backslash operation in Matlab.

1. Case with the CFL $= 0.25$.

   In the situation

   $$\tau = 0.25 \times h^2 = 0.25 \times (0.1)^2 = 0.0025.$$

   Thus $N = 1/\tau = 400$.

2. Case with the CFL $= 0.50$.

   In the situation

   $$\tau = 0.5 \times h^2 = 0.5 \times (0.1)^2 = 0.005.$$

   Thus $N = 1/\tau = 200$.

3. Case with the CFL $= 1.00$.

   In the situation

   $$\tau = 1 \times h^2 = (0.1)^2 = 0.01.$$

   Thus $N = 1/\tau = 100$.

See sample Matlab program.