Homework Problems for Week 02

MATH 4665/4875/7140/7300, HKBU

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1. Let $y = f(x)$ be continuous on $[a, b]$ and sufficiently high order differentiable over (a, b). Suppose $\bar{\mathcal{D}}$ is the corresponding mesh defined. Use Taylor expansions to show the following estimate.

$$\delta f(x_i) = f'(x_i) + \begin{cases} \mathcal{O}(\tilde{h}_i) & \text{if } h_{i+1} \neq h_i \\ \mathcal{O}(\tilde{h}_i^2) & \text{if } h_{i+1} = h_i, \end{cases}$$

   where $\tilde{h}_i = \max\{h_{i+1}, h_i\}$ and $x_i \in \mathcal{D}$.

2. Read program sample002.m carefully. Comprise your new Matlab program to study the backward finite difference formula in the same fashion. You may use the same test function.

3. Let $\bar{\mathcal{D}}$ be a nonuniform mesh. Show mathematically that the following must be true.

$$\Delta \nabla f(x_i) \neq \nabla \Delta f(x_i) \quad \text{for some } x_i \in \mathcal{D}.$$