Homework Problems 06

MATH 4665/4875/7140/7300, HKBU

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1. (30%) Let
$$A = \begin{bmatrix} -100 & 1 \\ 0 & -1/10 \end{bmatrix} \tag{1}$$
be a 2×2 real matrix given.
   (a) Evaluate $\text{Cond}_1(A)$ if the matrix is nonsingular.
   (b) Evaluate $\text{Cond}_2(A)$ if the matrix is nonsingular.
   (c) Evaluate $\text{Cond}_\infty(A)$ if the matrix is nonsingular.

2. (30%) Consider the matrix exponential function
$$F(tA) = \exp(tA), \quad 0 \le t \le 2,$$
where A is given by (1). Further, we consider a uniform mesh
$$\Omega_h = \{0, h, 2h, 3h, \ldots, 100h\},$$
where $h = 2/100$.
   (a) Use Matlab to calculate and plot a curve for $\text{Cond}_1(F(tA))$ over Ω_h.
   (b) Use Matlab to calculate and plot a curve for $\text{Cond}_2(F(tA))$ over Ω_h.
   (c) Use Matlab to calculate and plot a curve for $\text{Cond}_\infty(F(tA))$ over Ω_h.

3. (40%) Consider following system of linear differential equations together with an initial vector,
$$u'(t) = Au(t), \quad 0 < t \le 25; \quad u(0) = u_0, \tag{2}$$
where A is define in (1) and
$$u_0 = \begin{bmatrix} 1 \\ -1 \end{bmatrix}.$$
Suppose that a uniform mesh with $h = 1/10$ is used. Can you solve this initial-value price problem (2) via a backward Euler method on Matlab? A standard "backslash" technique (or better) can be used for solving the system of equation generated. Plot the absolute error in the Euclidean norm.