

Homework Problems for Week 03

MATH 4665/4875/7140/7300, HKBU

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Instructor: Prof. Tim Sheng

1. (45%) Let $y = f(x)$ be continuous on $[a, b]$, and continuously differentiable on (a, b) . Suppose $\bar{\mathcal{D}}$ is the corresponding mesh defined. Use Taylor expansions to discuss the following formula:

$$\diamond f(x_i) = \frac{\Delta f(x_i) - \nabla f(x_i)}{(h_i + h_{i+1})/2}, \quad x_i \in \mathcal{D}.$$

- (a) Is the formula an approximation of $f''(x_i)$?
 - (b) If it is, how accurate it is when $h_i \neq h_{i+1}$?
 - (c) If it is, how accurate it is when $h_i = h_{i+1}$?
2. (10%) State the Intermediate Value Theorem (IVT) in calculus.
3. (45%) Given a function

$$g(x) = \sin^2(x), \quad -\infty < x < \infty.$$

- (a) Find the derivative value $g'(x_n)$, where $x_n = 5\pi/8$.
- (b) Find the equation of the tangent line of $g(x)$ at $(x_n, g(x_n))$.
- (c) Compute the x -intercept of the tangent line obtained.