Quiz 1

Let $y = f(x)$ be continuous on $[a, b]$ and continuously differentiable to sufficiently high order on (a, b). We define

$$\Delta f(x_i) = \frac{f(x_{i+1}) - f(x_i)}{h_{i+1}}, \tag{1}$$

$$\nabla f(x_i) = \frac{f(x_i) - f(x_{i-1})}{h_i}, \tag{2}$$

$$\diamondsuit f(x_i) = \frac{2}{h_i + h_{i+1}} \left(\Delta f(x_i) - \nabla f(x_i) \right), \tag{3}$$

where $x_{i-1}, x_i, x_{i+1} \in \mathcal{D}_N = \{x_0, x_1, x_2, \ldots, x_{N+1}\}$ is a mesh defined on $[a, b]$ for which $x_0 = a$, $x_{N+1} = b$ with $0 < h_\ell = x_\ell - x_{\ell-1} \ll 1$, $\ell = 1, 2, \ldots, N + 1$. Show that

$$h_{i+1}\Delta(\nabla f(x_i)) = h_i \nabla(\Delta f(x_i)) = \frac{h_i + h_{i+1}}{2} \diamondsuit f(x_i).$$

Most of project reports are fabulous!

Here are some minor concerns and suggestions.

1. To avoid ambitious conclusions.
2. How to evaluate an approximation? Mathematical correctness comes the first. If this is affirmative, then a large N may be deployed. However, keep in mind that a large N may not lead to ideal results since random number based meshes are utilized.
3. There is no "easy" job in mathematics.
4. More comments and remarks are needed in addition to observations.
5. To improve professional English writing.

6. Why are nonuniform meshes needed, since they perform poorly in our experiments? They are for future adaptive data approximations.

7. Average values of errors were calculated by several folks. How about the medians? We use them frequently in statistical calculations.

8. Nevertheless, norms will be used instead of absolute values...