Homework Problems 04

MATH 4665/4875/7140/7300, HKBU

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1. (35%) Extend the Averaging Formula (2.17) for $f(t) \Rightarrow f(t, u(t))$. Show mathematically that we may derive the following recursive algorithm for solving the initial value problem (2.18), (2.19):

$$
\begin{aligned}
u_0 &= \phi, \\
u_1 &= u_0 + \frac{h}{2}[f(t_0, u_0) + f(t_1, u_1)], \\
u_2 &= u_1 + \frac{h}{2}[f(t_1, u_1) + f(t_2, u_2)], \\
u_3 &= u_2 + \frac{h}{2}[f(t_2, u_2) + f(t_3, u_3)], \\
u_4 &= u_3 + \frac{h}{2}[f(t_3, u_3) + f(t_4, u_4)], \\
u_5 &= u_4 + \frac{h}{2}[f(t_4, u_4) + f(t_5, u_5)], \\
& \ldots\ldots
\end{aligned}
$$

2. (20%) What should be the truncation error for the Averaging algorithm?

3. (45%) Given positive parameters h and λ. Consider following error estimates

$$\epsilon_{i+1} \leq (1 + h\lambda)\epsilon_i + c_1 h^2, \quad i = 0, 1, 2, \ldots, M,$$

where c_1 is a positive constant. Suppose that there exists another positive constant c_2 such that

$$\epsilon_i \leq \frac{c_2}{\lambda} h \left[(1 + h\lambda)^i - 1\right], \quad i = 0, 1, 2, \ldots, M.$$

Mathematically prove that we must have

$$\epsilon_{i+1} \leq \frac{c}{\lambda} h \left[(1 + h\lambda)^{i+1} - 1\right], \quad i = 0, 1, 2, \ldots, M,$$

where $c = \max\{c_1, c_2\}$.