Homework Problems 05

MATH 4665/4875/7140/7300, HKBU

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1. (30%) Given following matrix

$$A = \begin{bmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 2 \end{bmatrix}. \tag{1}$$

   (a) By using our definition, show that the matrix is positive definite.

   (b) Calculate $\|A\|_1,\ \|A\|_2,\ \|A\|_\infty$. Are the values different?

   (c) Evaluate $w \in \mathbb{R}^3$:

$$w = Au + v, \quad \text{where } u = \begin{bmatrix} -2 \\ 3 \\ \kappa \end{bmatrix},\ v = \begin{bmatrix} 1 \\ 0 \\ 3 \end{bmatrix},\ \kappa \in \mathbb{R}.$$

2. (30%) Given following matrix

$$B = \begin{bmatrix} 1 & 0 & 3 \\ 2 & -2 & 1 \\ 1 & 3 & -1 \end{bmatrix}. \tag{2}$$

   (a) Evaluate $\|BB^\intercal\|_\infty,\ \|B^\intercal B\|_\infty$. Are the values different?

   (b) Define the Frobenius norm:

$$\|A\|_F = \sqrt{\sum_{i=1}^{n}\sum_{j=1}^{n} |a_{i,j}|^2}, \quad A \in \mathbb{R}^{n\times n}.$$

   Evaluate $\|BB^\intercal\|_F,\ \|B^\intercal B\|_F$. Are the values different?

   (c) Calculate the Lie bracket of $A,\ B$. Do matrices $A,\ B$ commute?

3. (40%) Consider following system of linear differential equations together with an initial vector,

$$u'(t) = \Lambda u(t), \quad u(0) = u_0, \tag{3}$$

   where

$$\Lambda = \begin{bmatrix} -100 & 1 \\ 0 & -1/10 \end{bmatrix},\ u_0 = \begin{bmatrix} 1 \\ -1 \end{bmatrix}.$$

   (a) Find the true solution of the above initial value problem.

   (b) Let $\{t_0, t_1, \ldots, t_N, t_{N+1}\}$ be a uniform mesh with $t_0 = 0, t_{N+1} = 25,\ h = 1/10$. Use a forward Euler method for solving the initial value problem (3) on Matlab and plot the Euclidean norm of the solution vectors u.

   (c) Comment on your results and graph.