Homework Problems for Week 01

MATH 4665/4875/7140/7300, HKBU

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1. For what values of a, m, b does the function

$$f(x) = \begin{cases} 3, & x = 0, \\ -x^2 + 3x + a, & 0 < x < 1, \\ mx + b, & 1 \le x \le 2, \end{cases}$$

safisfy the hypothese of the Mean Value Theorem on the interval $[0, 2]$?

2. Evaluate the integral

$$I = \int_0^{\pi/6} \tan(2x) dx.$$

3. Find the antiderivative of the function

$$y = \phi(t) = \sqrt{\frac{1}{t(t+1)}}$$

within its domain.

4. Let $0 < \Delta x \ll 1$ be a mesh step to use. Read Definition 2.3 in Lecture Notes 0905 carefully. Use a Taylor expansion formula to show that

   (a) The backward finite difference formula,

$$\nabla f(x) = \frac{f(x) - f(x - \Delta x)}{\Delta x},$$

   is a first order approximation of the derivative $f'(x)$ (that is, $p = 1$).

   (b) Use the same method to show that the central difference formula,

$$\delta f(x) = \frac{f(x + \Delta x) - f(x - \Delta x)}{2\Delta x},$$

   is a second order approximation of $f'(x)$ (that is, $p = 2$).