Computational Project 1

MATH 4665/4875/7140/7300, HKBU

– Due Date: Wednesday, October 9, 2019 –

Prof. Tim Sheng's Additional Office Hours: 10:00-16:00, Mon 07 & Tue 08 Oct.

Recall sample MATLAB programs **matlab001.m** and **matlab002.m**.

↪ Given interval $I = [0, 1]$;

↪ Given function $f(x) = \sin(\pi x),\ x \in I$.

↪ Let $N = 10, 50, 100$, respectively. Prepare 3 sets of randomly distributed internal mesh points with $N = 10, 50, 100$, respectively:

$$\mathcal{D}_N = \{x_1, x_2, \ldots, x_N\} \subset (0, 1).$$

↪ Calculate corresponding 3 sets of mesh steps:

$$H_N = \{h_1, h_2, \ldots, h_{N+1}\}.$$

↪ Complete the following tasks:

1. (20%) Find the second derivative $y = f''(x),\ x \in (0, 1)$.
2. (30%) Calculate and plot curves corresponding to the forward-backward, backward-forward and corrected finite difference values

$$\Delta\nabla f(x_1), \Delta\nabla f(x_2), \ldots, \Delta\nabla f(x_N);$$

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where

$$\begin{aligned}
\Delta\nabla f(x_i)) &= \frac{\nabla f(x_{i+1}) - \nabla f(x_i)}{h_{i+1}} = \left[\frac{f(x_{i+1}) - f(x_i)}{h_{i+1}} - \frac{f(x_i) - f(x_{i-1})}{h_i}\right] / h_{i+1} \\
&= \frac{h_i f(x_{i+1}) - (h_{i+1} + h_i) f(x_i) + h_{i+1} f(x_{i-1})}{h_{i+1}^2 h_i}, \\
\nabla(\Delta f(x_i)) &= \frac{\Delta f(x_i) - \Delta f(x_{i-1})}{h_i} = \left[\frac{f(x_{i+1}) - f(x_i)}{h_{i+1}} - \frac{f(x_i) - f(x_{i-1})}{h_i}\right] / h_i \\
&= \frac{h_i f(x_{i+1}) - (h_{i+1} + h_i) f(x_i) + h_{i+1} f(x_{i-1})}{h_{i+1} h_i^2}, \\
\diamondsuit f(x_i) &= \frac{\Delta f(x_i) - \nabla f(x_i)}{(h_i + h_{i+1})/2} = \left[\frac{f(x_{i+1}) - f(x_i)}{h_{i+1}} - \frac{f(x_i) - f(x_{i-1})}{h_i}\right] / (h_i + h_{i+1})/2 \\
&= \frac{h_i f(x_{i+1}) - (h_{i+1} + h_i) f(x_i) + h_{i+1} f(x_{i-1})}{h_{i+1} h_i (h_i + h_{i+1})/2}.
\end{aligned}$$

3. (30%) Calculate and plot absolute error curves based on

(a) the forward-backward formula, that is,

$$error_{a,i} = \left|\Delta\nabla f(x_i) - f''(x_i)\right|,\ i = 1, 2, \ldots, N;$$

(b) the backward-forward formula, that is,

$$error_{b,i} = \left|\nabla\Delta f(x_i) - f''(x_i)\right|,\ i = 1, 2, \ldots, N;$$

(c) the corrected formula, that is,

$$error_{c,i} = \left|\diamondsuit f(x_i) - f''(x_i)\right|,\ i = 1, 2, \ldots, N,$$

respectively.

4. (20%) Comment on your numerical errors obtained.

5. (bonus problem: 20%). Recall our lecture notes: FINANCE_01. Let $f(t) = \cos(t)$, $\phi = 1$ and $T = 1$. Can you solve the initial value problem (2.10), (2.11) numerically by using formula (2.12)? You may choose a mesh (uniform or nonuniform) and an ideal number of mesh points for it. What is the absolutely error occurred? Your explanations and comments?

References

[1] K. in 't Hout, *Numerical Partial Differential Equations in Finance Explained,* Springer, Antwerp, Belgium, 2017.

[2] R. Pratap, *Getting Started with Matlab: A Quick Introduction for Scientists and Engineers,* 7th Ed., Oxford Univ. Press, 2016.

[3] T. Sheng, *Lecture Notes on 0905, 0910, finance intro, finance 01 & sample programs,* HKBU, 2019.

updated September 25, 2019