Computational Project 2

MATH 4665/4875/7140/7300, HKBU

– Due Date: Wednesday, November 20, 2019 –

There will no office hours on 13 and 14, Nov. Replacement office hours are 13:00-16:00 on Fri 8, 15 and 22 Nov. 2019

Recall homework 5, 6 and sample MATLAB program **matlab003.m**.

↪ Given a spatial interval $I = [0,1]$ and a temporal interval $J = [0,1]$. We may define following price-time domain:

$$\Omega = I \times J = \{(s,t) : \ s \in I, \ t \in J\}.$$

↪ Consider a much simplified Black-Scholes initial-boundary value problem (BSIVP):

$$\begin{aligned} u_t(s,t) &= u_{ss}(s,t), \quad 0 < s < 1, \ t \in J, & (1.1)\\ u(s,0) &= \sin(\pi s), \quad 0 < s < 1, & (1.2)\\ u(0,t) &= 0, \quad t \in J, & (1.3)\\ u(1,t) &= 0, \quad t \in J. & (1.4) \end{aligned}$$

1. (20%) Let $h = 1/10$. Complete a semi-discretization procedure. What is the condition number (under a norm of your choice) of the coefficient matrix?

2. (30%) Set your Courant number as 0.25, 0.5 and 1, respectively. Solve your semi-discretized linear system of ordinary differential equations by using a backward Euler method respectively (for solving linear systems of equations involved, you may try either, or both, Matlab "backslash" command or "linsolve(A,b)" command[1]). Plot the maximal prices at each time level within $J = [0,1]$.

3. (30%) Repeat the above numerical solution procedure by employing a forward Euler method with the three different Courant number if possible, respectively. Plot the maximal prices at each time level again. Can you continue your computations all the way to the end (that is, $T = 1$) with each of the Courant numbers? Any major differences between solutions obtained between backward and forward Euler methods (you may compare numerical solutions obtained by using different Courant numbers at the terminal time T or any meaningful time $\tilde{T} \in J$)?

4. (20%) Comment on your numerical errors obtained.

5. (bonus problem: 20%). Can you continue your backward Euler method based computations till a larger terminal time, say, $T = 5$. Any new observations? Can you plot a 3D simulation of your result?

[1] Use "help" command for more details in Matlab.

References

[1] K. in 't Hout, *Numerical Partial Differential Equations in Finance Explained,* Springer, Antwerp, Belgium, 2017.

[2] A. Iserles, *A First Course in the Numerical Analysis of Differential Equations,* Cambridge University Press, Cambridge and New York, 2004 (pages 139-146, 251-269, 349-380).

[3] R. Pratap, *Getting Started with Matlab: A Quick Introduction for Scientists and Engineers,* 7th Ed., Oxford Univ. Press, 2016.

updated November 6, 2019