

Fractional powers

The square root of the number x is the (positive) number y such that $y^2 = x$.

For example: $\sqrt{25} = 5$ because $5^2 = 25$.

Please note that $-5 \times -5 = 25$ but the square root function $\sqrt{25}$ only returns a positive value.

1) Evaluate the following

a) $\sqrt{4}$

b) $\sqrt{64}$

c) $\sqrt{49}$

d) $\sqrt{\frac{25}{9}}$

e) $\sqrt{\frac{132}{169}}$

f) $\sqrt{a^2}$

g) $\sqrt{a^4}$

h) $\sqrt{a^6}$

i) $\sqrt{a^2b^6}$

j) $\sqrt{\frac{9x^4}{y^8}}$

k) $\sqrt{\frac{16x^3y^2}{xy^4}}$

2) Evaluate the following

a) $\sqrt{4} \times \sqrt{4}$

b) $\sqrt{a} \times \sqrt{a}$

c) $(\sqrt{a^2})^3$

d) $(3b\sqrt{a})^2$

e) $\sqrt{xy} \times \sqrt{xy}$

f) $(4\sqrt{a^2})^3$

We can now see that:

$$\begin{aligned}\sqrt{x} \times \sqrt{x} &= x \\ x^{\frac{1}{2}} \times x^{\frac{1}{2}} &= x^1 = x\end{aligned}$$

Which tells us that

$$x^{\frac{1}{2}} = \sqrt{x}$$

And it is probably good to note that the addition, subtraction and power rules still hold:

$$\begin{aligned}x^{\frac{1}{2}} \times x^2 &= x^{\frac{1}{2}+2} = x^{\frac{5}{2}} \\ \frac{x^{\frac{7}{2}}}{x^{\frac{1}{2}}} &= x^{\frac{7}{2}-\frac{1}{2}} = x^3 \\ (9b)^{\frac{1}{2}} &= 9^{\frac{1}{2}} \times b^{\frac{1}{2}} = 3\sqrt{b}\end{aligned}$$

1) Evaluate the following

a) $\sqrt{4}$

b) $\sqrt{64}$

c) $\sqrt{49}$

d) $\sqrt{\frac{25}{9}}$

e) $\sqrt{\frac{121}{169}}$

f) $\sqrt{a^2}$

g) $\sqrt{a^4}$

h) $\sqrt{a^6}$

i) $\sqrt{a^2b^6}$

j) $\sqrt{\frac{9x^4}{y^8}}$

k) $\sqrt{\frac{16x^3y^2}{xy^4}}$

a) $\sqrt{4} = 2$

b) $\sqrt{64} = \sqrt{8 \times 8} = 8$

c) $\sqrt{49} = 7$

d) $\sqrt{\frac{25}{9}} = \sqrt{\frac{5}{3} \times \frac{5}{3}} = \frac{5}{3}$

e) $\sqrt{\frac{121}{169}} = \frac{\sqrt{121}}{\sqrt{169}} = \frac{11}{13}$

f) $\sqrt{a^2} = \sqrt{a \times a} = a$

g) $\sqrt{a^4} = \sqrt{a^2 \times a^2} = a^2$

h) $\sqrt{a^6} = \sqrt{a^3 \times a^3} = a^3$

i) $\sqrt{a^2b^6} = \sqrt{a^2} \sqrt{b^6} = ab^3$

j) $\sqrt{\frac{9x^4}{y^8}} = \frac{3x^2}{y^4}$

k) $\sqrt{\frac{16x^3y^2}{xy^4}} = \sqrt{\frac{16x^2}{y^2}} = \frac{4x}{y}$

2) Evaluate the following

a) $\sqrt{4} \times \sqrt{4}$

b) $\sqrt{a} \times \sqrt{a}$

c) $(\sqrt{a^2})^3$

d) $(3b\sqrt{a})^2$

e) $\sqrt{xy} \times \sqrt{xy}$

f) $(4\sqrt{a^2})^3$

a) $2 \times 2 = 4$

b) a

c) a^3

d) $9b^2a$

e) xy

f) $64a^3$

3) Simplify:

a) $(16b^4)^{\frac{1}{2}}$	c) $\sqrt{100a^2b^{10}}$	e) $\left(\frac{108x^2y^6}{3x^4}\right)^{\frac{1}{2}}$
b) $\left(81^{\frac{1}{2}}b^4\right)^{\frac{1}{2}}$	d) $\left(\frac{1}{16x^4}\right)^{\frac{1}{2}}$	f) $\sqrt{\frac{256y^6}{x^{12}}}$

Looking at the power rule we can note:

$$9^{\frac{3}{2}} = \left(9^{\frac{1}{2}}\right)^3 = 3^3 = 27$$

4) Evaluate without a calculator (use a calculator afterwards to check your work and make sure you can enter the powers correctly)

a) $4^{\frac{5}{2}}$	b) $16^{\frac{3}{2}}$	c) $100^{\frac{7}{2}}$	d) $36^{\frac{3}{2}}$
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5) Simplify, by writing with no brackets, some powers will stay as fractions

a) $(3^3xy^2)^{\frac{1}{2}} \times 4y^2$	e) $\left(5^{\frac{3}{2}}x^{\frac{5}{2}}y^3\right)^2$
b) $\frac{5^2a}{b^4} \times \frac{b^{\frac{3}{2}}}{a}$	f) $\left(\sqrt{\frac{x}{2}}\right)^3$
c) $\left(\frac{x}{x^{\frac{1}{2}}}\right)^3$	g) $\sqrt{\left(\frac{x}{2}\right)^3}$
d) $(xy)^{\frac{1}{2}} \times x^{-\frac{5}{2}}$	

6) Bit of review, simplify:

a) $\sqrt{16x^{16}}$	d) $\frac{\sqrt{9n^4}}{3n^2}$	g) $\frac{(4a^2)^3}{(8a^5)^2}$
b) $\sqrt{36x^{36}}$	e) $\frac{(3a^2b^2)^4}{27a^8b^9}$	h) $\sqrt{\frac{256a^{16}}{b^{12}}}$
c) $\frac{4m^8}{\sqrt{36m^{20}}}$	f) $(0.25x^3)^{\frac{1}{2}}$	

3) Simplify:

a) $(16b^4)^{\frac{1}{2}}$

b) $(81^{\frac{1}{2}}b^4)^{\frac{1}{2}}$

c) $\sqrt{100a^2b^{10}}$

d) $\left(\frac{1}{16x^4}\right)^{\frac{1}{2}}$

e) $\left(\frac{108x^2y^6}{3x^4}\right)^{\frac{1}{2}}$

f) $\sqrt{\frac{256y^6}{x^{12}}}$

$$\begin{aligned} \text{a)} \quad & (16b^4)^{\frac{1}{2}} \\ &= 16^{\frac{1}{2}} b^{4 \times \frac{1}{2}} \\ &= 4b^2 \end{aligned}$$

$$\begin{aligned} \text{b)} \quad & (81^{\frac{1}{2}}b^4)^{\frac{1}{2}} \\ &= (9b^4)^{\frac{1}{2}} \\ &= 3b^2 \end{aligned}$$

$$\begin{aligned} \text{c)} \quad & \sqrt{100a^2b^{10}} \\ &= (100a^2b^{10})^{\frac{1}{2}} \\ &= 100^{\frac{1}{2}} a^1 b^5 \\ &= 10ab^5 \end{aligned}$$

$$\begin{aligned} \text{d)} \quad & \left(\frac{1}{16x^4}\right)^{\frac{1}{2}} \\ &= \frac{1^{\frac{1}{2}}}{16^{\frac{1}{2}} x^{4 \times \frac{1}{2}}} \\ &= \frac{1}{4x^2} \end{aligned}$$

$$\begin{aligned} \text{e)} \quad & \left(\frac{108x^2y^6}{3x^4}\right)^{\frac{1}{2}} \\ &= \left(\frac{36y^6}{x^2}\right)^{\frac{1}{2}} \\ &= \frac{36^{\frac{1}{2}} y^3}{x^1} \\ &= \frac{6y^3}{x} \end{aligned}$$

$$\begin{aligned} \text{f)} \quad & \sqrt{\frac{256y^6}{x^{12}}} \\ &= \left(\frac{256y^6}{x^{12}}\right)^{\frac{1}{2}} \\ &= \frac{256^{\frac{1}{2}} y^3}{x^6} \\ &= \frac{16y^3}{x^6} \end{aligned}$$

4) Evaluate without a calculator (use a calculator afterwards to check your work and make sure you can enter the powers correctly)

a) $4^{\frac{5}{2}}$

b) $16^{\frac{3}{2}}$

c) $100^{\frac{7}{2}}$

d) $36^{\frac{3}{2}}$

$$\begin{aligned} \text{a)} \quad & 4^{\frac{5}{2}} \\ &= \left(4^{\frac{1}{2}}\right)^5 \\ &= 2^5 \\ &= 32 \end{aligned}$$

$$\begin{aligned} \text{b)} \quad & 16^{\frac{3}{2}} \\ &= \left(16^{\frac{1}{2}}\right)^3 \\ &= (4)^3 \\ &= 64 \end{aligned}$$

$$\begin{aligned} \text{c)} \quad & 100^{\frac{7}{2}} \\ &= \left(100^{\frac{1}{2}}\right)^7 \\ &= 10^7 \\ &= 10,000,000 \end{aligned}$$

$$\begin{aligned} \text{d)} \quad & 36^{\frac{3}{2}} \\ &= \left(36^{\frac{1}{2}}\right)^3 \\ &= 6^3 \\ &= 216 \end{aligned}$$
$$\begin{array}{r} 3 \\ 36 \\ \underline{6} \\ 216 \end{array}$$

5) Simplify, by writing with no brackets, some powers will stay as fractions

a) $(3^4 xy^2)^{\frac{1}{2}} \times 4y^2$

b) $\frac{5^2 a}{b^4} \times \frac{b^{\frac{3}{2}}}{a}$

c) $\left(\frac{x}{x^{\frac{1}{2}}}\right)^3$

d) $(xy)^{\frac{1}{2}} \times x^{-\frac{5}{2}}$

e) $\left(5^{\frac{3}{2}} x^{\frac{5}{2}} y^3\right)^2$

f) $\left(\sqrt{\frac{x}{2}}\right)^3$

g) $\sqrt{\left(\frac{x}{2}\right)^3}$

$$\begin{aligned} \text{a) } & (3^4 xy^2)^{\frac{1}{2}} \times 4y^2 \\ &= 3^2 x^{\frac{1}{2}} y^1 \times 4 \times y^2 \\ &= 36 x^{\frac{1}{2}} y^3 \end{aligned}$$

$$\begin{aligned} \text{d) } & (xy)^{\frac{1}{2}} \times x^{-\frac{5}{2}} \\ &= x^{\frac{1}{2}} y^{\frac{1}{2}} x^{-\frac{5}{2}} \\ &= x^{-\frac{4}{2}} y^{\frac{1}{2}} \\ &= \frac{y^{\frac{1}{2}}}{x^2} \end{aligned}$$

$$\begin{aligned} \text{b) } & \frac{5^2 a}{b^4} \times \frac{b^{\frac{3}{2}}}{a} \\ &= 25 b^{\frac{3}{2}-4} \\ &= 25 b^{-\frac{5}{2}} \\ &= \frac{25}{b^{\frac{5}{2}}} \end{aligned}$$

$$\begin{aligned} \text{e) } & \left(5^{\frac{3}{2}} x^{\frac{5}{2}} y^3\right)^2 \\ &= 5^3 x^5 y^6 \\ &= 125 x^5 y^6 \end{aligned}$$

$$\begin{aligned} \text{c) } & \left(\frac{x}{x^{\frac{1}{2}}}\right)^3 \\ &= \left(x^{\frac{1}{2}}\right)^3 \\ &= x^{\frac{3}{2}} \end{aligned}$$

$$\begin{aligned} \text{f) } & \left(\sqrt{\frac{x}{2}}\right)^3 \\ &= \left(\frac{x^{\frac{1}{2}}}{2^{\frac{1}{2}}}\right)^3 \\ &= \frac{x^{\frac{3}{2}}}{2^{\frac{3}{2}}} \end{aligned}$$

$$\begin{aligned} \text{g) } & \sqrt{\left(\frac{x}{2}\right)^3} \\ &= \left(\frac{x^3}{2^3}\right)^{\frac{1}{2}} \\ &= \frac{x^{\frac{3}{2}}}{2^{\frac{3}{2}}} \end{aligned}$$

6) Bit of review, simplify:

a) $\sqrt{16x^{16}}$

d) $\frac{\sqrt{9n^4}}{3n^2}$

g) $\frac{(4a^2)^3}{(8a^5)^2}$

b) $\sqrt{36x^{36}}$

e) $\frac{(3a^2b^2)^4}{27a^8b^9}$

h) $\sqrt{\frac{256a^{16}}{b^{12}}}$

c) $\frac{4m^8}{\sqrt{36m^{20}}}$

f) $(0.25x^3)^{\frac{1}{2}}$

$$\begin{aligned} \text{a) } & \sqrt{16x^{16}} \\ &= (16x^{16})^{\frac{1}{2}} \\ &= 4x^8 \end{aligned}$$

$$\begin{aligned} \text{d) } & \frac{\sqrt{9n^4}}{3n^2} \\ &= \frac{3n^2}{3n^2} \\ &= 1 \end{aligned}$$

$$\begin{aligned} \text{g) } & \frac{(4a^2)^3}{(8a^5)^2} \\ &= \frac{64a^6}{64a^{10}} \end{aligned}$$

$$\begin{aligned} \text{b) } & \sqrt{36x^{36}} \\ &= 6x^{18} \end{aligned}$$

$$\begin{aligned} \text{e) } & \frac{(3a^2b^2)^4}{27a^8b^9} \\ &= \frac{81a^8b^8}{27a^8b^9} \\ &= \frac{3}{b} \end{aligned}$$

$$\text{h) } \sqrt{\frac{256a^{16}}{b^{12}}}$$

$$\begin{aligned} \text{c) } & \frac{4m^8}{\sqrt{36m^{20}}} \\ &= \frac{4m^8}{6m^{10}} \\ &= \frac{2}{3m^2} \end{aligned}$$

$$\begin{aligned} \text{f) } & (0.25x^3)^{\frac{1}{2}} \\ &= \left(\frac{x^3}{4}\right)^{\frac{1}{2}} \\ &= \frac{x^{3/2}}{2} \end{aligned}$$

$$\begin{aligned} &= \left(\frac{16a^{16}}{b^{12}}\right)^{\frac{1}{2}} \\ &= \frac{16a^8}{b^6} \end{aligned}$$