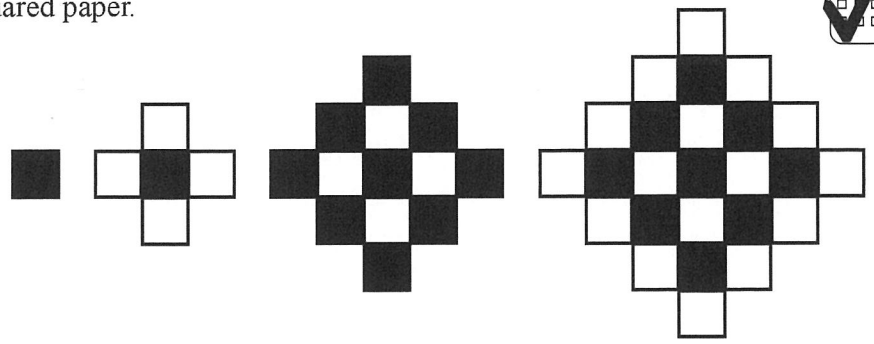
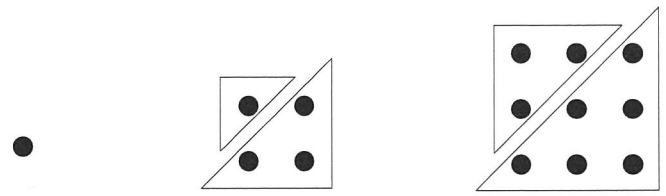
Investigations Using Indices

1). a). Copy this pattern on squared paper.
b). Continue the pattern by drawing the next three diagrams.
c). Investigate the number of shaded and unshaded squares in each diagram.



2). A chessboard is made up of 8 squares by 8 squares.
a). How many small squares are on this board?
b). How many squares of size 2×2 are on this board?
c). How many squares of **any size** are on this board?
d). How many squares of any size are on a board of dimensions
i). 4×4 ii). 6×6
iii). 10×10 iv). 20×20?

3).



1 $1 + 3 = \mathbf{4}$ $3 + 6 = \mathbf{9}$

a). Copy the pattern. Draw the next 3 in this sequence and write in the pattern of numbers.
b). What type of numbers are i). in bold, ii). the other numbers?
c). Write an explanation of how you can make square numbers from triangular numbers.

4). Write the first **30** square numbers.
a). Look at the last digit of these numbers. What pattern do you notice?
b). Which numbers do not appear as a last digit of a square number?
c). Could these numbers be a square number i). 723 ii). 1412?

5). Some numbers can be represented as the difference between two square numbers.
(One square number take away another square number).

$21 = 5^2 - 2^2$ $65 = 9^2 - 4^2$

a). There are **22** numbers up to **30** that can be written in this way. Find them.
b). Which whole numbers up to 30 **cannot** be represented this way?
c). Which numbers up to 30 can be expressed as the difference of two squares in more than one way?

6). a). Some numbers are equal to the sum of 2 square numbers. E.g. $\mathbf{34} = 3^2 + 5^2$.
Which numbers up to 100 can be written as the sum of two square numbers?
b). Every whole number is the sum of **not more than five** square numbers.

E.g. $1 = 1^2$ $2 = 1^2 + 1^2$ $3 = 1^2 + 1^2 + 1^2$
$4 = 2^2$ $5 = 2^2 + 1^2$ Investigate whether this is true or not.

7). a). Pick a number! **Say 23**

b). Square each digit and **add** the results together. $2^2 + 3^2 = 4 + 9 = 13$

c). Use the new number and do the same again.... 13 $1^2 + 3^2 = 1 + 9 = 10$

d). If you reach 1 you have a **happy number.** 10 $1^2 + 0^2 = 1 + 0 = 1$

The 23 chain is **23 → 13 → 10 → 1.**

e). **Sad numbers** are those that do not go to 1.

f). Put all the numbers up to 50 into your chains! Which are happy? Which are sad?

8). a). Pick any 2 digit number! **Say 95**

b). Square each digit and **subtract** the smaller from the larger. $9^2 - 5^2 = 81 - 25 = 56$

c). Use the new number and do the same again.... 56 $6^2 - 5^2 = 36 - 25 = 11$

d). If the answer goes to 0 it is a **sneaky number.** 11 $1^2 - 1^2 = 0$

The 95 chain is **95 → 56 → 11 → 0**

e). **Loopy numbers** are those that do not go to 0.

f). Find all the numbers up to 50 which are **sneaky** and which are **loopy.**

9). a). Pick a two digit number! **Say 42**

b). **Cube** each digit and **add** the results together. $4^3 + 2^3 = 64 + 8 = 72$

c). Use the new number and do the same again.... 72 $7^3 + 2^3 = 343 + 8 = 351$

351 $3^3 + 5^3 + 1^3 = 27 + 125 + 1 = 153$

The 42 chain so far is

42 → 72 → 351 → 153

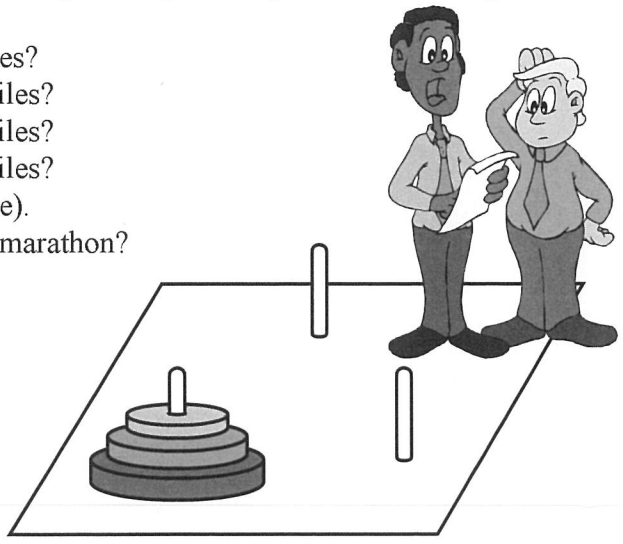
d). Keep repeating this process. What happens?

e). Try it with numbers that are one **less** than the three times table. What happens?

f). Try it with numbers that are one **more** than the three times table. What happens?

10). Draw Pascal's triangle and add each row. What connection with powers does it have?

11). You are given distance targets as sponsorship for a marathon. If you complete one mile you earn 1p, if you complete two miles 2p, if you complete three miles 4p and if you complete four miles 8p. This pattern is continued.



a). How much will you earn if you complete 5 miles?

b). How much will you earn if you complete 10 miles?

c). How much will you earn if you complete 15 miles?

d). How much will you earn if you complete 20 miles?

e). A marathon is 26 miles long (to the nearest mile). How much would you earn for completing the marathon?

f). Can you find a formula that would work out earnings for any length of race?

12). The Tower of Bramah. The idea is to move the tower, as it is, from one needle to another needle. You can only move one disc at a time and there must be no smaller discs underneath the disc moved.

a). How many moves will it take to transfer the three discs to recreate the tower?

b). How many moves will it take to transfer four discs?

c). How many moves will it take to transfer five discs?

d). Can you find a pattern between the number of discs and the number of moves?