

Algebraic fractions

EXERCISE 14.05

Simplify the following (that is, write as one fraction).

1 $\frac{4}{p} + \frac{2}{p}$

2 $\frac{13}{q} - \frac{9}{q}$

3 $\frac{2c}{7} + \frac{3c}{7}$

4 $\frac{4c}{5} + \frac{c}{5}$

5 $\frac{x}{2} + \frac{x}{2}$

6 $\frac{4x}{5} + \frac{8}{5}$

7 $\frac{x}{3} + \frac{x}{2}$

8 $\frac{4x}{7} - \frac{x}{2}$

9 $\frac{2x}{5} + 3$

10 $\frac{4x^2}{5} + 10$

11 $x + \frac{x}{2}$

12 $x - \frac{x}{3}$

13 $\frac{2x}{5} - x$

14 $\frac{3x^2}{5} - \frac{x}{2}$

15 $\frac{7x}{2} - x$

16 $\frac{x^2}{10} + \frac{3x}{5} + 2$

17 $\frac{2x}{7} + \frac{7}{3}$

18 $\frac{1}{x} + \frac{1}{y}$

19 $\frac{2}{x} - \frac{1}{y}$

20 $\frac{2}{x} + \frac{3}{y}$

21 $\frac{2}{xy} - \frac{4}{y}$

22 $\frac{a}{x^2} + \frac{b}{xy}$

23 $\frac{3}{xy} + \frac{2}{5}$

24 $\frac{1}{x} - \frac{3x}{y}$

25 $\frac{3}{2x} + \frac{2y}{3x}$

26 $\frac{x}{ay} + \frac{y}{bx} - \frac{a}{by}$

27 $\frac{2}{5y} - \frac{3}{2y} + \frac{4}{3xy}$

28 $\frac{2}{xy} + \frac{3}{xy^2} + \frac{4}{x^2y}$

29 $\frac{x}{y} + \frac{y}{z} + \frac{z}{x}$

30 $\frac{x}{2y} - \frac{y}{3}$

31 $\frac{x}{3a} + \frac{2}{5x}$

32 $\frac{2a}{3x} - \frac{4a}{5xy} + \frac{5a}{2xyz}$

33 $\frac{x+1}{4} + \frac{x}{3}$

34 $\frac{x}{3} - \frac{x+2}{5}$

35 $\frac{2x}{3} + \frac{x+1}{6}$

36 $\frac{x-2}{2} + \frac{2x+3}{6}$

37 $\frac{2x-3}{5} - \frac{x+9}{6}$

38 $\frac{x+2y}{6y} - \frac{3x-2y}{9x}$

39 $\frac{x+7}{x} + \frac{1-x}{x^2}$

40 $\frac{3-5x}{2x} - \frac{6}{x^2}$

Other examples have linear factors that, when multiplied, give a *quadratic expression* for the common denominator.

Example

Simplify $\frac{4}{x-3} - \frac{3}{x+1}$.

Answer

The common denominator is

$$(x-3)(x+1) = x^2 - 2x - 3.$$

4 times $(x+1)$ gives $4x+4$ as the first

numerator and 3 times $(x-3)$ gives $3x-9$ as the second numerator.

$$\begin{aligned}\frac{4}{x-3} - \frac{3}{x+1} &= \frac{4x+4-(3x-9)}{x^2-2x-3} \\ &= \frac{4x+4-3x+9}{x^2-2x-3} \\ &= \frac{x+13}{x^2-2x-3}\end{aligned}$$

EXERCISE 14.06

Simplify these sums and differences by writing as one fraction.

1 $\frac{1}{x+2} + \frac{1}{x+3}$

2 $\frac{3}{x-1} + \frac{2}{x+5}$

3 $\frac{2}{2x+3} + \frac{3}{3x-1}$

4 $\frac{2}{x} - \frac{5}{x+2}$

5 $\frac{3}{x-1} - \frac{2}{x}$

6 $\frac{4x}{x+2} + \frac{5}{x}$

7 $\frac{x}{x+1} + \frac{x}{x+4}$

8 $\frac{a}{x+2a} + \frac{b}{x-2b}$

9 $\frac{4}{2x+3} + \frac{1}{x+3}$

10 $\frac{x}{x-4} - \frac{3}{x+5}$

11 $\frac{4}{x-2} - \frac{x}{x+3}$

12 $\frac{5}{x+1} - \frac{x+1}{3}$

13 $\frac{3x}{3x-1} + \frac{2x}{x+2}$

14 $\frac{x}{2} + \frac{x-1}{x} + \frac{4}{x-2}$

15 $\frac{5}{x} + \frac{2}{x-1} + \frac{3}{x+3}$

16 $\frac{2}{x-1} - \frac{3}{x+2} + \frac{5}{x-3}$

17 $\frac{x}{x+2} + \frac{x}{x-1} + \frac{2x-1}{x}$

18 $\frac{2}{x+1} + \frac{3}{x-1} - \frac{4}{x^2-1}$

19 $\frac{x}{x+4} - \frac{3}{x^2-16}$

20 $\frac{5x}{x^2+5x+6} - \frac{2}{x+3} + \frac{4}{x+2}$

Answers

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EXERCISE 14.04 (page 227) —

1 $\frac{3y}{2}$

2 $\frac{7c}{10d}$

3 $\frac{12x^4}{y}$

4 $a^2b^2c^2$

5 $\frac{b}{2}$

6 $\frac{b^{10}}{a^2c^8}$

7 $9x^8y^{10}$

8 $72x^{27}$

9 $\frac{1}{y^{10}}$

10 $\frac{9}{5x^6}$

11 $\frac{x^6}{y^2}$

12 $4x^2$

13 $\frac{3}{4y^9}$

14 $\frac{4x^4y^6}{9}$

15 1

16 $2^6 = 6^4$

PUZZLE**The shoemaker's will**
(page 227)The executor shared
the shoes in the ratio
 $\frac{1}{2} : \frac{1}{3} : \frac{1}{9} = 9 : 6 : 2$,
 which is probably what the
shoemaker intended. The
problem with the will is that
the three given fractions
do not add to 1. In fact,
 $\frac{1}{2} + \frac{1}{3} + \frac{1}{9} = \frac{17}{18}$ so, if
there were a large number
of shoes, $\frac{1}{18}$ would be
unallocated.**EXERCISE 14.05** (page 229) —

1 $\frac{6}{p}$

2 $\frac{4}{q}$

3 $\frac{5c}{7}$

4 c

5 x

6 $\frac{4x+8}{5}$

7 $\frac{5x}{6}$

8 $\frac{x}{14}$

9 $\frac{2x+15}{5}$

10 $\frac{4x^2+50}{5}$

11 $\frac{3x}{2}$

12 $\frac{2x}{3}$

13 $\frac{-3x}{5}$

14 $\frac{6x^2-5x}{10}$

15 $\frac{5x}{2}$

16 $\frac{x^2+6x+20}{10}$

17 $\frac{6x+49}{21}$

18 $\frac{x+y}{xy}$

19 $\frac{2y-x}{xy}$

20 $\frac{2y+3x}{xy}$

21 $\frac{2-4x}{xy}$

22 $\frac{ay+bx}{x^2y}$

23 $\frac{15+2xy}{5xy}$

24 $\frac{y-3x^2}{xy}$

25 $\frac{9+4y}{6x}$

26 $\frac{bx^2+ay^2-a^2x}{abxy}$

27 $\frac{40-33x}{30xy}$

28 $\frac{2xy+3x+4y}{x^2y^2}$

29 $\frac{x^2z+xy^2+yz^2}{xyz}$

30 $\frac{3x-2y^2}{6y}$

31 $\frac{5x^2+6a}{15ax}$

32 $\frac{20ayz-24az+75a}{30xyz}$

33 $\frac{7x+3}{12}$

34 $\frac{2x-6}{15}$

35 $\frac{5x+1}{6}$

36 $\frac{5x-3}{6}$

37 $\frac{7x-63}{30}$

38 $\frac{3x^2+4y^2}{18xy}$

39 $\frac{x^2+6x+1}{x^2}$

40 $\frac{3x-5x^2-12}{2x^2}$

14

EXERCISE 14.06 (page 229) —

1 $\frac{2x+5}{x^2+5x+6}$

2 $\frac{5x+13}{x^2+4x-5}$

3 $\frac{12x+7}{6x^2+7x-3}$

4 $\frac{-3x+4}{x^2+2x}$

5 $\frac{x+2}{x^2-x}$

6 $\frac{4x^2+5x+10}{x^2+2x}$

7 $\frac{2x^2+5x}{x^2+5x+4}$

8 $\frac{ax+bx}{x^2+2ax-2bx-4ab}$

9 $\frac{6x+15}{2x^2+9x+9}$

10 $\frac{x^2+2x+12}{x^2+x-20}$

11 $\frac{12+6x-x^2}{x^2+x-6}$

12 $\frac{14-2x-x^2}{3x+3}$

13 $\frac{9x^2+4x}{3x^2+5x-2}$

14 $\frac{x^3+2x+4}{2x^2-4x}$

15 $\frac{10x^2+13x-15}{x^3+2x^2-3x}$

16 $\frac{4x^2+15x-31}{x^3-2x^2-5x+6}$

17 $\frac{4x^3+2x^2-5x+2}{x^3+x^2-2x}$

18 $\frac{5x-3}{x^2-1}$

19 $\frac{x^2-4x-3}{x^2-16}$

20 $\frac{7x+8}{x^2+5x+6}$