13 Solving Quadratic Equations

13.1 Using factorised quadratics

Consider the following equation:

$$(x-6)(x+3)=0$$

To solve this equation, we do not expand the brackets.
we can note that the equation is of the form:

$$A \times B = 0 \qquad \text{where } A=(x-6) \text{ and } B=(x+3)$$

And we can also note that if $A \times B = 0$, then either $A = 0$ or $B = 0$

So $(x-6)(x+3)=0$ has **two** solutions.

$$\begin{aligned} (x-6)(x+3) &= 0 \\ x-6=0 \quad &\text{or} \quad x+3=0 \\ \Rightarrow \quad x=6 \quad &\text{or} \quad x=-3 \end{aligned}$$

13.1.1 Exercise

Solve the following:

a) $(x-1)(x+7)=0$

b) $(x-5)(x+5)=0$

c) $(x+11)(x+9)=0$

d) $(x-3)(x-15)=0$

13.1.2 Exercise

Solve the following:

a) $(3x-15)(2x+8)=0$

b) $(2x-8)(4x+1)=0$

c) $(8x+10)(2x+9)=0$

d) $(2x-7)(2x-15)=0$

13.1.3 Exercise

Solve the following:

a) $3x(x+8)=0$

b) $x(x+4)=0$

c) $-3x(2x+9)=0$

d) $x(2x-15)=0$

13.2 Using un-factorised quadratics

To solve the equation:

$$x^2 + 3x - 88 = 0$$

$$\begin{aligned} & x^2 + 3x - 88 = 0 \\ \textbf{factorise} \qquad & (x-8)(x+11) = 0 \\ & x + 8 = 0 \quad \text{or} \quad x + 11 = 0 \\ & x = -8 \quad \text{or} \quad x = -11 \end{aligned}$$

13.2.1 Exercise

a) $x^2 + 7x + 6 = 0$

b) $x^2 + 12x + 32 = 0$

c) $x^2 - 5x + 6 = 0$

d) $x^2 + 3x - 10 = 0$

13.2.2 Exercise

a) $x^2 + 6x = 0$

b) $x^2 - 5x = 0$

c) $x^2 - 18 - 3x = 0$

d) $2x - 99 + x^2 = 0$

13.2.3 Exercise

a) $x^2 + 0x - 9 = 0$

b) $x^2 - 0x - 36 = 0$

c) $x^2 - 25 = 0$

d) $x^2 - 49 = 0$

13.2.4 Exercise

a) $x^2 - 6x + 9 = 0$

b) $x^2 + 10x + 25 = 0$

c) $x^2 - 2x + 1 = 0$

d) $x^2 + 2x + 1 = 0$

13.3 Re-arranging into standard form

If we get an equation with an x^2 as its highest power, we need to re-organise it into **standard form**

$$x^2 + bx + c = 0$$

Once this is done, we can factorise and solve.

(Example) Solve:

$$3 = 4x - x^2$$

rearrange	$3 = 4x - x^2$
	$3 + x^2 = 4x$
	$3 + x^2 - 4x = 0$
standard form	$x^2 - 4x + 3 = 0$
factorise	$(x - 3)(x - 1) = 0$
	$x - 3 = 0$ or $x - 1 = 0$
	$x = 3$ or $x = 1$

13.3.1 Exercise

a) $-26 = 11x - x^2$

b) $18x + x^2 = -45$

c) $42 = x^2 - 11x$

d) $x^2 + 225 = 30x$

13.3.2 Exercise

a) $x(x - 2) = 15$

b) $(x - 3)^2 = 25$

c) $(x - 4)(x - 6) = 8$

d) $(x - 3)(x + 2) = 4x$

e) $(x + 2)(x + 7) = (x + 5)(x + 7)$

f) $x + 3 = \dfrac{12}{x + 4}$

13.4 Word questions

13.4.1 Exercise

a) When a number is squared, and then 1 is subtracted, the result is 8.

   i) Write this information down as a quadratic equation.

   ii) Solve the equation to work out the two possible numbers.

b) A number is squared, and then added to the original number. The result is 20.

   i) Write this information down as a quadratic equation.

   ii) Solve the equation to work out the two possible numbers.

c) When a number x, is multiplied by a number 4 less than x, the result is 12.
What two numbers have this property?

d) Squaring a number gives the same result as multiplying the number by 8, and then subtracting 12.

   i) Write this information down as a quadratic equation.

   ii) Solve the equation to work out the two possible numbers.

e) When the result of adding 2 to a number is multiplied by the result of subtracting 2 from the same number, the answer is 21.
What two numbers have this property?
