Properties of curves and applications of differentiation

Kh notes

Contents

Chapter: 1 Properties of curves

1.1 Review Questions

1.1.1 Some fundamental derivatives:

Function	Derivative
$f(x) = x^n$	$f'(x) = nx^{n-1} \quad (n \in \mathbb{R})$
$f(x) = e^x$	$f'(x) = e^x$
$f(x) = \ln x$	$f'(x) = \dfrac{1}{x}$
$f(x) = \sqrt{x}$	$f'(x) = \dfrac{1}{2\sqrt{x}}$
$f(x) = \sin x$	$f'(x) = \cos x$
$f(x) = \cos x$	$f'(x) = -\sin x$
$f(x) = \tan x$	$f'(x) = \sec^2 x$

1.1.2 Rules of differentiation:

Chain Rule:

$$y = g(u_{(x)})$$
$$\frac{dy}{dx} = g'(u_{(x)})u'_{(x)}$$

Product Rule:

$$y = u_{(x)}v_{(x)}$$
$$\frac{dy}{dx} = u_{(x)}v'_{(x)} + u'_{(x)}v_{(x)}$$

Quotient Rule:

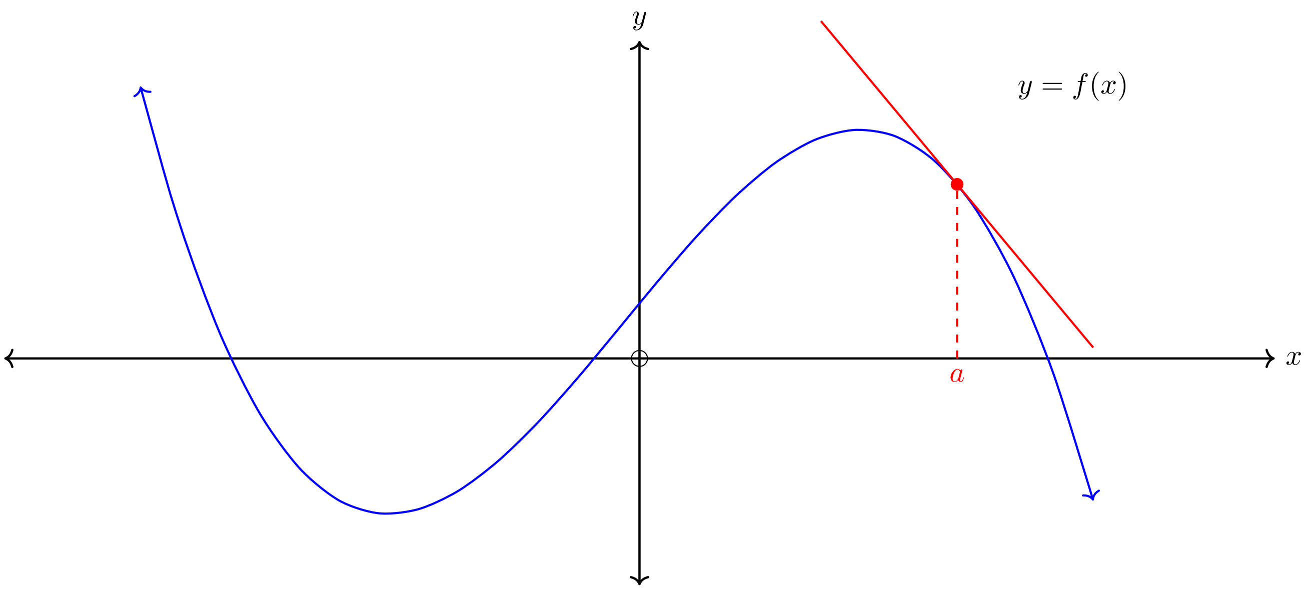
$$y = \frac{u_{(x)}}{v_{(x)}}$$
$$\frac{dy}{dx} = \frac{u'_{(x)}v_{(x)} - u_{(x)}v'_{(x)}}{[v_{(x)}]^2}$$

1.2 Tangents

- The tangent to a curve at a point A is the best approximating straight line to the curve at point A.
- (Leibniz definition) Tangent to the curve $y = f(x)$ at the point $(a, f(a))$ is the line through the infinitely close pair of points either side of $f(a)$

$$\frac{y - f(a)}{x - a} = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h}$$

- It is a single point of contact with the curve (although it may intersect the curve at some other point)



For the function $y = f(x)$, and some $x = a$

- $(a, f(a))$ is on the curve
- $f'(a)$ is the gradient of the curve at $x = a$

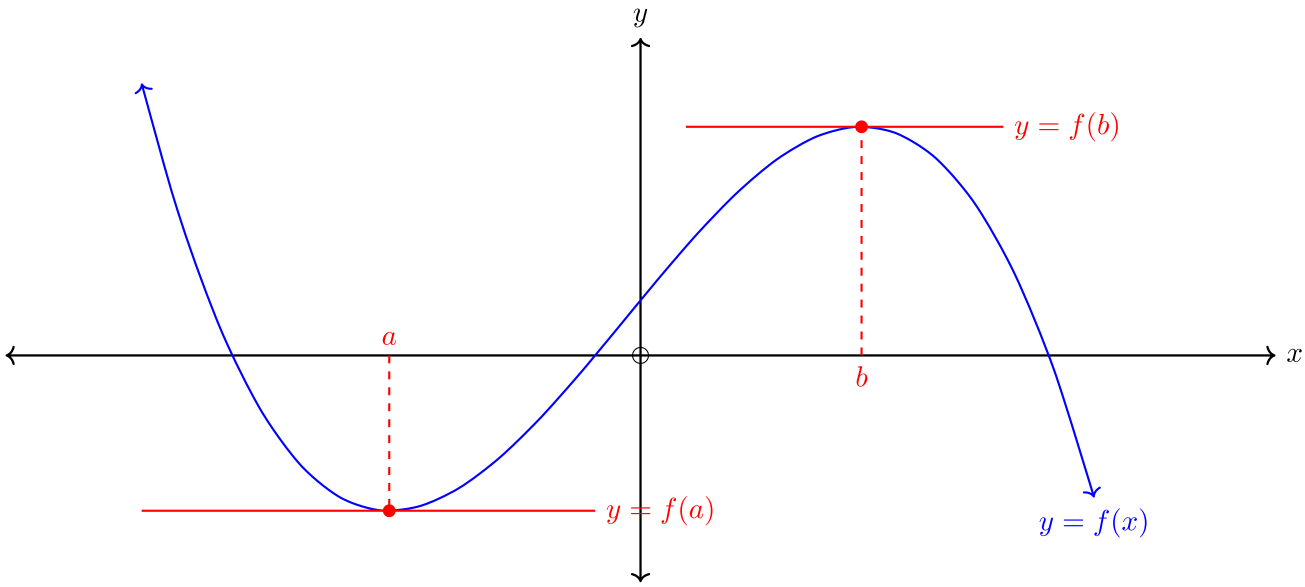
$$\frac{y - f(a)}{x - a} = f'(a)$$

$$\Rightarrow \qquad y = f'(a)(x-a) + f(a)$$

Is the equation of the tangent line

1.2.1 Horizontal Tangents

Horizontal tangents have a gradient of 0.
These will be very important later, when we investigate stationary points in more detail.



In general, we will need to find where the stationary points are:

- Find $f'(x)$
- Solve $f'(x) = 0$
- Substitue the solution(s) into $f(x)$ to find the constant terms for the horizontal line equation

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1.2.2 Ex13A

- Tangent equations from root and polynomial functions
- Horzontal tangents
- Natural log and exponent questions
- Concept questions

4 worked examples

Find the equation of the tangent to:

2.

a) $y = x - 2x^2 + 3$ at $x = 2$

$\frac{dy}{dx} = 1 - 4x$

when $x = 2$

$\frac{dy}{dx} = 1 - 4(2) = -7$

and $y = (2) - 2(2)^2 + 3$
$= -3$

$\frac{dy}{dx} = -7$ Point $(2, -3)$

$y - -3 = -7(x - 2)$

$y + 3 = -7x + 14$

$y = -7x + 11$

d) $y = \frac{4}{\sqrt{x}}$ at $(1, 4)$

$y = 4x^{-\frac{1}{2}}$

$\frac{dy}{dx} = 4 \cdot -\frac{1}{2} x^{-\frac{3}{2}}$

$= -2x^{-\frac{3}{2}} = \frac{-2}{x^{3/2}}$

$x = 1$

$\frac{dy}{dx} = \frac{-2}{1} = -2$

Point $(1, 4)$

$y - 4 = -2(x - 1)$

$y - 4 = -2x + 2$

$y = -2x + 6$

(Horizontal Tangents)

3. c)
Find the equation of horizontal tangents to

$y = \sqrt{x} + \frac{1}{\sqrt{x}}$

$\Rightarrow \quad y = x^{\frac{1}{2}} + x^{-\frac{1}{2}}$

$\frac{dy}{dx} = \frac{1}{2} x^{-\frac{1}{2}} + -\frac{1}{2} x^{-\frac{3}{2}}$

(factorise using lowest power)

$\frac{dy}{dx} = \frac{1}{2} x^{-\frac{3}{2}} (x^1 - 1)$

Set $\frac{dy}{dx} = 0$

$\frac{1}{2} x^{-\frac{3}{2}} (x - 1) = 0$

$x^{-\frac{3}{2}} \neq 0 \qquad x - 1 = 0$
$x = 1$

Find y

$y = \sqrt{1} + \frac{1}{\sqrt{1}} = 1 + \frac{1}{1}$

$= 2$

Horizontal tangent at $(1, 2)$

Equation: $y = 2$

$\frac{dy}{dx} = \frac{1}{2x^{1/2}} - \frac{1}{2x^{3/2}}$

$= \frac{x - 1}{2x^{3/2}}$

$\frac{x-1}{2x^{3/2}} = 0 \quad \Rightarrow \quad x - 1 = 0$
$x = 1$

5. Find another tangent to

$y = 1 - 3x + 12x^2 - 8x^3$

which is parallel to the tangent at $(1,2)$

$\frac{dy}{dx} = -3 + 24x - 24x^2$

when $x = 1$

$\frac{dy}{dx} = -3 + 24(1) - 24(1)^2$

$= -3$

so gradient is -3

Find all x values where gradient $= -3$

$-3 + 24x - 24x^2 = -3$

$24x - 24x^2 = 0$

$x - x^2 = 0$

$x(1-x) = 0$

$x = 0 \qquad x = 1$ already have

Find equation of tangent at $x = 0$

$y = 1$

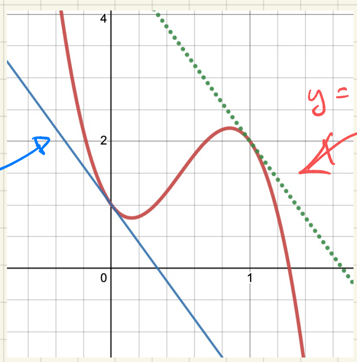
$y - 1 = -3(x - 0)$

$y - 1 = -3x, \quad y = -3x + 1$

other tangent $y - 2 = -3(x - 1)$

$y = -3x + 5$

$y = -3x + 1$



$y = -3x + 5$

6. $y = x^2 + ax + b$

when $x = 1$ tangent is $2x + y = 6$

$y = -2x + 6$

$\frac{dy}{dx} = -2 \qquad y = 4$

$(1,4)$ point on curve.

-2 gradient at that point.

$\frac{dy}{dx} = 2x + a$

$-2 = 2(1) + a$

$a = -4$

$y = x^2 - 4x + b$

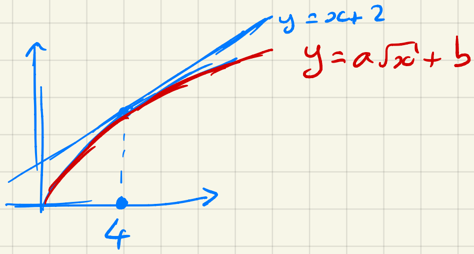
$4 = (1)^2 - 4(1) + b$

$4 = -3 + b$

$b = 7$

$y = x^2 - 4x + 7$

7. Find a and b



What do we know?

- when $x = 4$ $\frac{dy}{dx} = 1$
- $(4, 4+2) = (4, 6)$ is on $y = a\sqrt{x} + bx$

① using $(4, 6)$

$$6 = a\sqrt{4} + b(4)$$
$$6 = 2a + 4b$$
$$\Rightarrow 3 = a + 2b$$

② $\frac{dy}{dx} = \frac{a}{2\sqrt{x}} + b$

$$1 = \frac{a}{2\sqrt{4}} + b$$

$$1 = \frac{a}{4} + b$$

$$4 = a + 4b$$

Equations

$$3 = a + 2b \quad ①$$
$$4 = a + 4b \quad ②$$

② − ① $\quad 1 = 2b$

$$b = \frac{1}{2}$$

using ① $\quad a = 3 - 2b$

$$= 3 - 2\left(\frac{1}{2}\right)$$

$$= 2$$

$$y = 2\sqrt{x} + \frac{x}{2}$$

8. Show equation of tangent to $y = 2x^2 - 1$ at $x = a$ is

$$4ax - y = 2a^2 + 1$$

Find point:

$$x = a$$
$$y = 2a^2 - 1$$
$$(a, 2a^2 - 1)$$

$$\frac{dy}{dx} = 4x$$

$$= 4a$$

$$y - (2a^2 - 1) = 4a(x - a)$$

$$y - 2a^2 + 1 = 4ax - 4a^2$$

$(-y)$ $\quad -2a^2 + 1 = 4ax - 4a^2 - y$

$(+4a^2)$ $\quad 2a^2 + 1 = 4ax - y$

10 d

$y = \ln\sqrt{x}$

$y = \ln(x^{\frac{1}{2}})$

$y = \frac{1}{2}\ln(x)$

$\frac{dy}{dx} = \frac{1}{2x}$

$y = -1$

$\frac{1}{2}\ln(x) = -1$

$\ln(x) = -2$

$x = e^{-2}$

$(e^{-2}, -1)$

$\frac{dy}{dx} = \frac{1}{2e^{-2}}$

$y + 1 = \frac{1}{2e^{-2}}(x - e^{-2})$

$y = \frac{x}{2e^{-2}} - \frac{1}{2} - 1$

$y = \frac{e^{2}}{2}x - \frac{3}{2}$

Using $\ln x$ and e^x

Find the equation of the tangent to $y = \ln x$ where $y = -1$

- Find x

$\ln x = -1 \quad (\log_e x = 1)$

$x = e^{-1}$

- $\frac{dy}{dx} = \frac{1}{x} = x^{-1}$

$x = e^{-1}, \quad \frac{dy}{dx} = (e^{-1})^{-1} = e^{1}$

$= e$

Point $(e^{-1}, -1)$

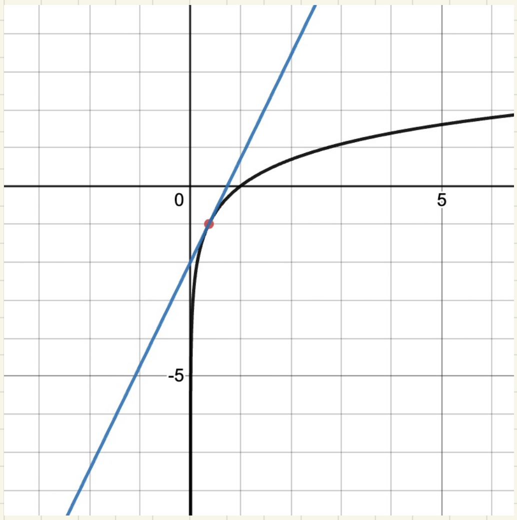
$\frac{dy}{dx} = e$

$y - -1 = e(x - e^{-1})$

$y + 1 = ex - e^{1}e^{-1}$

$y + 1 = ex - 1$

$y = ex - 2$

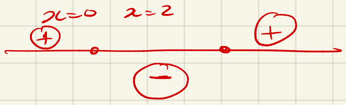


12. $f(x) = \ln\left(x(x-2)\right)$

Domain of log function > 0

$\Rightarrow \quad x(x-2) > 0$

solve $\quad x(x-2) = 0$



Domain $\quad x < 0 \ \cup \ x > 2$

$\{x \mid x < 0 \ \cup \ x > 2\}$

$$f'(x) = \frac{1}{x(x-2)} \cdot \frac{d}{dx}\left(x^2 - 2x\right)$$

$$= \frac{1}{x(x-2)} \cdot 2x-2$$

$$= \frac{2(x-1)}{x(x-2)}$$

$$= \frac{2x-2}{x^2-2x}$$

$$= \frac{2x-2}{x(x-2)}$$

$$= \frac{x + x-2}{x(x-2)}$$

$$= \frac{1}{x} + \frac{1}{x-2}$$

$x=3$, $f'(x) = \frac{1}{3} + \frac{1}{1} = \frac{4}{3}$

$x=3$, $f(3) = \ln\left(3(3-2)\right) = \ln 3$

$(3, \ln 3)$

$$y - \ln 3 = \frac{4}{3}(x-3)$$

$$y = \frac{4}{3}x - 4 + \ln 3$$

13. $y = x^2 e^x$

$y = uv \qquad \frac{dy}{dx} = uv' + u'v$ (product rule)

$$\frac{dy}{dx} = x^2e^x + 2xe^x$$

$$= xe^x(x+2)$$

$x = 1$

$$\frac{dy}{dx} = e(3) = 3e$$

$x = 1$, $y = e$

Gradient: $3e$

Point: $(1, e)$

$$y - e = 3e(x-1)$$

$$y = 3ex - 2e \quad \text{equation of tangent}$$

Find intercepts.

$x = 0 \qquad y = -2e$

$y = 0 \qquad x = \frac{2e}{3e}$

$x = \frac{2}{3}$

Intercepts

$(0, -2e)$ y-intercept

$\left(\frac{2}{3}, 0\right)$ x-intercept.

14. $y = 3xe^{\frac{x}{2}}$

$$\frac{dy}{dx} = 3x\left(e^{\frac{x}{2}}\right)' + (3x)'e^{\frac{x}{2}}$$

$$= 3x\, e^{\frac{x}{2}} \cdot \frac{1}{2} + 3e^{\frac{x}{2}}$$

$$= 3e^{\frac{x}{2}}\left(\frac{x}{2} + 1\right)$$

When $x = -1$

$$\frac{dy}{dx} = 3e^{-\frac{1}{2}}\left(\frac{-1}{2} + 1\right)$$

$$= \frac{3}{2}e^{-\frac{1}{2}}$$

$$x = -1, \quad y = -3e^{-\frac{1}{2}}$$

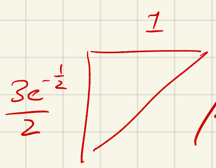
$$y + 3e^{-\frac{1}{2}} = \frac{3e^{-\frac{1}{2}}}{2}(x+1)$$

$$y = \frac{3e^{-\frac{1}{2}}}{2}x + \frac{3e^{-\frac{1}{2}}}{2} - 3e^{-\frac{1}{2}}$$

$$= \frac{3e^{-\frac{1}{2}}}{2}x - \frac{3e^{-\frac{1}{2}}}{2}$$

if $x = 0$ $\quad y = -\frac{3e^{-\frac{1}{2}}}{2}$

if $y = 0$ $\quad x = 1$



$$A = \frac{3e^{-\frac{1}{2}}}{4}$$

$$A = \frac{3}{4\sqrt{e}} \text{ units}^2$$

15. Find the equation of the tangent to $y = \frac{1}{\sin 2x}$ at the point where $x = \frac{\pi}{4}$

$u = 1,\ u' = 0$

$v = \sin 2x \quad v' = 2\cos(2x)$

$$\left(\frac{u}{v}\right)' = \frac{u'v - uv'}{v^2}$$

$$\frac{dy}{dx} = -\frac{1 \cdot 2\cos(2x)}{\sin^2(2x)} = \frac{-2\cos(2x)}{\sin^2(2x)}$$

$$= -2\cot(x)\operatorname{cosec}(x)$$

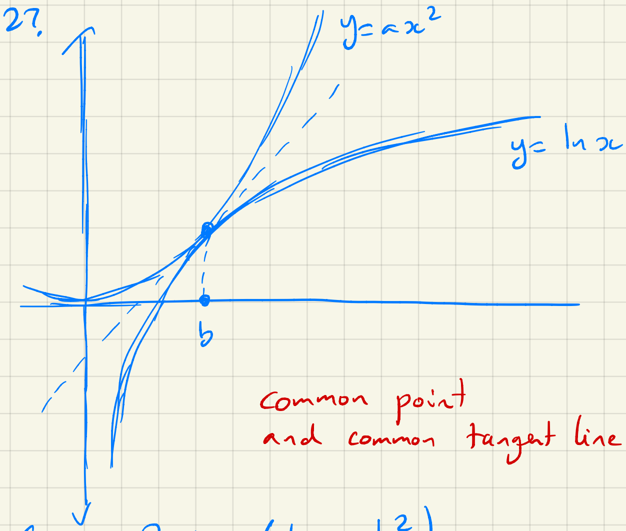
when $x = \frac{\pi}{4}$ $\quad \frac{dy}{dx} = \frac{-2\cos\left(\frac{\pi}{2}\right)}{\sin^2\left(\frac{\pi}{2}\right)} = 0$

when $x = \frac{\pi}{4}$, $\quad y = \frac{1}{\sin\left(\frac{\pi}{2}\right)} = 1$

Horizontal tangent

$y = 1$

27.



Common Point (b, ab^2)

a) $(b, \ln b)$

$\Rightarrow \quad ab^2 = \ln b$

Common Tangent

$\frac{dy}{dx} = 2ax \quad , \quad \frac{dy}{dx} = \frac{1}{x}$

$x = b \qquad 2ab = \frac{1}{b} \quad \Rightarrow \quad 2ab^2 = 1$

b) $ab^2 = \ln b \quad ①$

$2ab^2 = 1 \quad ②$

$2ab^2 = 2\ln b$

$2ab^2 = 1$

$2\ln b = 1$

$\ln b = \frac{1}{2}$

$b = e^{\frac{1}{2}} = \sqrt{e}$

Point $(e^{\frac{1}{2}}, \ln e^{\frac{1}{2}}) = (e^{\frac{1}{2}}, \frac{1}{2})$

c) $a = \frac{1}{2b^2}$

$a = \frac{1}{2e}$

d) $a = \frac{1}{2e} \quad , \quad b = \sqrt{e}$

Point $(e^{\frac{1}{2}}, \frac{1}{2})$

Gradient $\frac{dy}{dx} = \frac{1}{b} = \frac{1}{\sqrt{e}} = e^{-\frac{1}{2}}$

$y - \frac{1}{2} = e^{-\frac{1}{2}}(x - e^{\frac{1}{2}})$

$y - \frac{1}{2} = e^{-\frac{1}{2}}x - 1$

$y = x\sqrt{e} - \frac{1}{2}$

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28. $P(x) = ax^2$

$p'(x) = 2ax$

$x = s \qquad (s, as^2) \qquad p'(s) = 2as$

$y - as^2 = 2as(x - s)$

$y = 2asx - 2as^2 + as^2$

$y = 2asx - as^2$

$x = t$

$y = 2atx - at^2$

$2asx - as^2 = 2atx - at^2$

$x(2as - 2at) = as^2 - at^2$

$x = \frac{a(s^2 - t^2)}{2a(s-t)}$

$= \frac{(s+t)(s-t)}{2(s-t)}$

$= \frac{s+t}{2}$

If perpendicular

$2as \cdot 2at = -1$

$y = 2atx - at^2$

$x = \frac{s+t}{2}$

$y = \frac{2at(s+t)}{2} - at^2$

$= ast + at^2 - at^2$

$= ast$

$t = \frac{-1}{4a^2 s}$

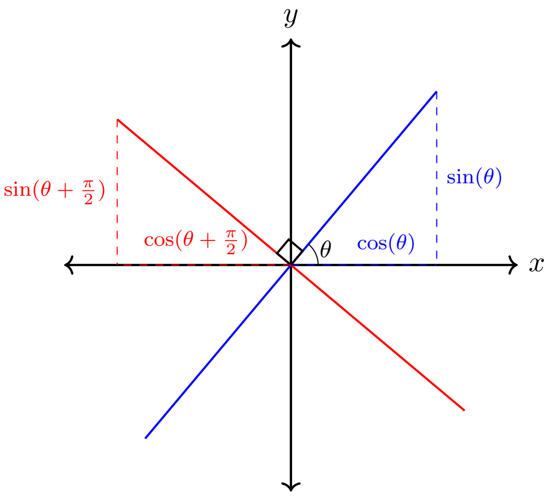
$y = ast$

$= as \cdot \frac{-1}{4a^2 s}$

$= \frac{-1}{4a}$

1.3 Normals

The product of the gradients of perpedicular lines = -1
There are various proofs, one example below.



$$m = \frac{\sin(\theta)}{\cos(\theta)}$$

$$m_{\perp} = \frac{\sin(\theta+\frac{\pi}{2})}{\cos(\theta+\frac{\pi}{2})} = -\frac{\cos(\theta)}{\sin(\theta)}$$

$$m \times m_{\perp} = -1$$

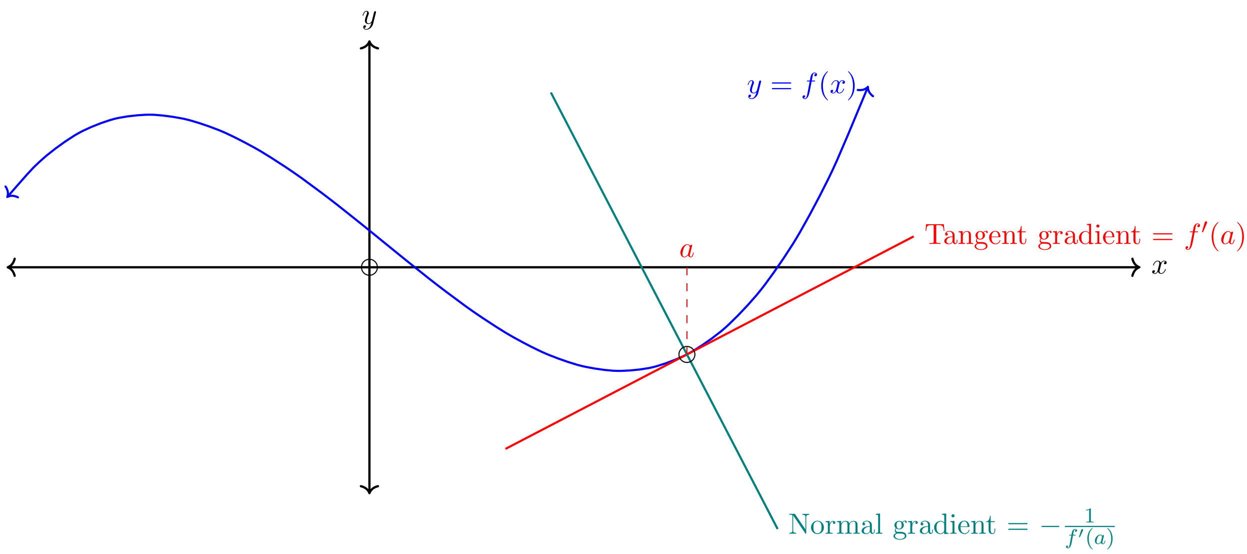
For the function $y = f(x)$, and some $x = a$

- $(a, f(a))$ is on the curve
- $-\dfrac{1}{f'(a)}$ is the gradient of the **normal to the curve** at $x = a$

$$\frac{y - f(a)}{x - a} = -\frac{1}{f'(a)}$$

$$\Rightarrow \qquad y = -\frac{1}{f'(a)}(x - a) + f(a)$$

Is the equation of the tangent line



1.3.1 Ex 13B

13B

1. d)

Find the equation of the normal to:

$$y = 8\sqrt{x} - \frac{1}{x^2} \quad \text{at } x = 1$$

Point $x = 1$

$$y = 8\sqrt{1} - \frac{1}{1^2} = 7$$

$(1, 7)$

Gradient:

$$y = 8x^{\frac{1}{2}} - x^{-2}$$

$$\frac{dy}{dx} = 8 \cdot \frac{1}{2} x^{-\frac{1}{2}} - -2x^{-3}$$

$$= \frac{4}{x^{1/2}} + \frac{2}{x^3}$$

$$= \frac{4x^{\frac{5}{2}} + 2}{x^3}$$

$$\frac{dy}{dx}_\perp = \frac{-x^3}{4x^{5/2} + 2}$$

$x = 1$

$$\frac{dy}{dx}_\perp = \frac{-1}{4+2} = \frac{-1}{6}$$

$(1, 7)$ and $-\frac{1}{6}$

$$y - 7 = -\frac{1}{6}(x - 1)$$

$$y - 7 = -\frac{x}{6} + \frac{1}{6}$$

$$y = -\frac{x}{6} + \frac{43}{6}$$

or

$$6y + x = 43$$

or $x + 6y - 43 = 0$:)

3. Find the equation of the

c) the normal to

$$y = e^{2x-1} \quad \text{at } x = 1$$

Point $(1, e^1)$

$$\frac{dy}{dx} = e^{2x-1} \cdot 2$$

$$= 2e^{2x-1}$$

$$\frac{dy}{dx}_\perp = \frac{-1}{2e^{2x-1}} = -\frac{e^{1-2x}}{2}$$

$x = 1$

$$\frac{dy}{dx}_\perp = \frac{-1}{2e}$$

$$y - e = \frac{-1}{2e}(x - 1)$$

$$y = \frac{-x}{2e} + \frac{1}{2e} + e$$

$$y = \frac{-xe^{-1}}{2} + \frac{1 + 2e^2}{2e}$$

or

$$2ey - 2e^2 = -x + 1$$

$$2ey + x = 1 + 2e^2$$

$$x + 2ey = 1 + 2e^2$$

5. Find the points where the normal to

$y = x^3 - 2x^2 + 1$

at $x = 1$ meets the curve again.

Find equation of normal at $x=1$ and then set equal to y, find intersection points.

$\frac{dy}{dx} = 3x^2 - 4x$

$\frac{dy}{dx}_{\perp} = \frac{-1}{3x^2 - 4x}$

$x = 1 \quad \frac{dy}{dx}_{\perp} = \frac{-1}{3-4} = 1$

$x = 1, \quad y = 1 - 2 + 1$
$\quad = 0$

$y - 0 = 1(x-1)$

$y = x - 1$ (normal)

set $x^3 - 2x^2 + 1 = x - 1$

$\Rightarrow x^3 - 2x^2 - x + 2 = 0$

We already have one solution $x = 1 \Rightarrow x - 1$ is a factor

$(x-1)(x^2 - x - 2) = 0$

← deduce

or

$$\begin{array}{r|l} & x^2 - x - 2 \\ x - 1 & x^3 - 2x^2 - x + 2 \\ & x^3 - x^2 \\ \hline & -x^2 - x \\ & -x^2 + x \\ \hline & -2x + 2 \\ & -2x + 2 \\ \hline & 0 \end{array}$$

$(x-1)(x^2 - x - 2)$

$\Rightarrow (x-1)(x-2)(x+1) = 0$

Solution $x = 1, x = 2 \quad x = -1$

so new points

$x = 2$

$y = (2)^3 - 2(2)^2 + 1$

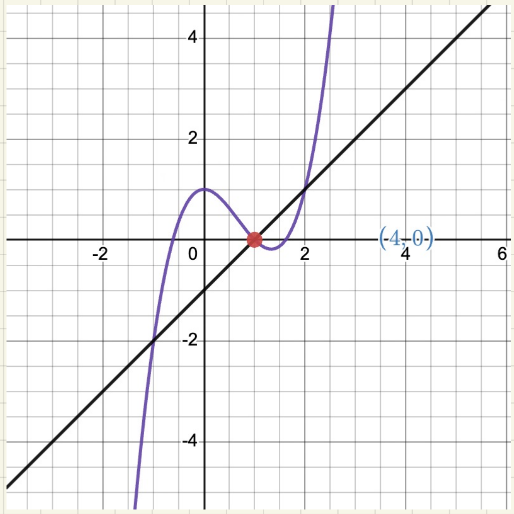
$= 1 \qquad (2, 1)$

$x = -1$

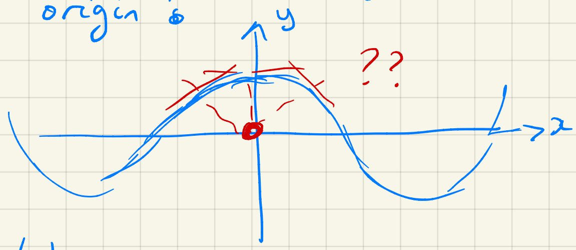
$y = (-1)^3 - 2(-1)^2 + 1$

$= -2 \qquad (-1, -2)$

points are $(2, 1), (-1, -2)$



6. Find the equation of the normal to $f(x) = \cos(x)$ which passes through the origin.



let $x = a$

$\Rightarrow$ $(a, \cos(a))$ is on the curve

$$\frac{dy}{dx} = -\sin(x)$$

$$\frac{dy}{dx}_{\perp} = \frac{1}{\sin(x)}$$

$$\frac{dy}{dx}_{\perp} = \frac{1}{\sin(a)}$$

$$\frac{\cos(a) - 0}{a - 0} = \frac{1}{\sin(a)}$$

$$\sin(a)\cos(a) = a$$
$$2\sin(a)\cos(a) = 2a$$

$$\sin(2a) = 2a$$

Solve $\sin(x) = x$?

We know $\lim_{x \to 0} \frac{\sin(x)}{x} = 1$

$\Rightarrow$ $\sin(x) = x$
when $x = 0$
$a = 0$

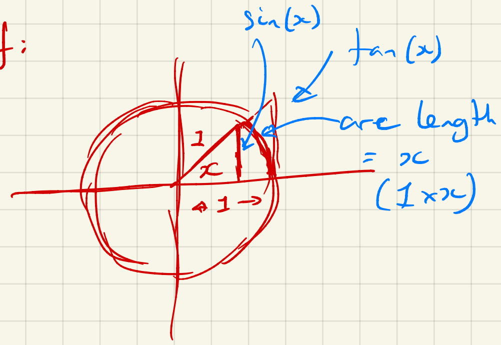
Point $(0, 1)$

Normal line is vertical

$\Rightarrow$ $x = 0$

$$\lim_{x \to 0} \frac{\sin(x)}{x} = 1$$

Proof:



$$\sin(x) < x < \tan(x)$$

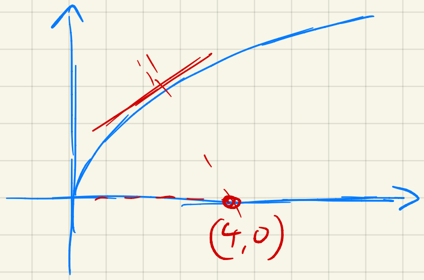
$$\Rightarrow \frac{1}{\sin(x)} > \frac{1}{x} > \frac{\cos(x)}{\sin(x)}$$

$$1 > \frac{\sin(x)}{x} > \cos(x)$$

$$\lim_{x \to 0} \Rightarrow 1 > \frac{\sin(x)}{x} > 1$$

$$\Rightarrow \lim_{x \to 0} \frac{\sin(x)}{x} = 1$$

7. Find the equation to $y=\sqrt{x}$ from the external point $(4,0)$



$y=\sqrt{x}$, $\frac{dy}{dx}=\frac{1}{2\sqrt{x}}$, $\frac{dy}{dx}_{\perp}=-2\sqrt{x}$

let the coordinate on the curve be $(a,\sqrt{a})$

so $\frac{dy}{dx}_{\perp}=-2\sqrt{a}$

and $\frac{\sqrt{a}-0}{a-4}=-2\sqrt{a}$

$\sqrt{a}=-2\sqrt{a}(a-4)$

$\sqrt{a}=-2a\sqrt{a}+8\sqrt{a}$

$2a\sqrt{a}-7\sqrt{a}=0$

$\sqrt{a}(2a-7)=0$

$\sqrt{a}=0$ $\quad 2a-7=0$

$a=\frac{7}{2}$

reject, because derivative is not defined at the end point of a curve

$a=\frac{7}{2}$

Point $\left(\frac{7}{2},\sqrt{\frac{7}{2}}\right)$

$\frac{dy}{dx}_{\perp}=-2\sqrt{\frac{7}{2}}$

$y-\sqrt{\frac{7}{2}}=-2\sqrt{\frac{7}{2}}\left(x-\frac{7}{2}\right)$

$y=-2\sqrt{\frac{7}{2}}x+7\sqrt{\frac{7}{2}}+\sqrt{\frac{7}{2}}$

$y=-2\sqrt{\frac{7}{2}}x+8\sqrt{\frac{7}{2}}$

$y=-\sqrt{\frac{4\times7}{2}}x+\sqrt{\frac{7\times64}{2}}$

$=-\sqrt{14}x+\sqrt{7\times32}$

$=-\sqrt{14}x+\sqrt{16\times7\times2}$

$=-\sqrt{14}x+4\sqrt{14}$

1.4 Increasing and Decreasing

When analysing functions, we are often interested in the intervals across which the function is increasing or decreasing.

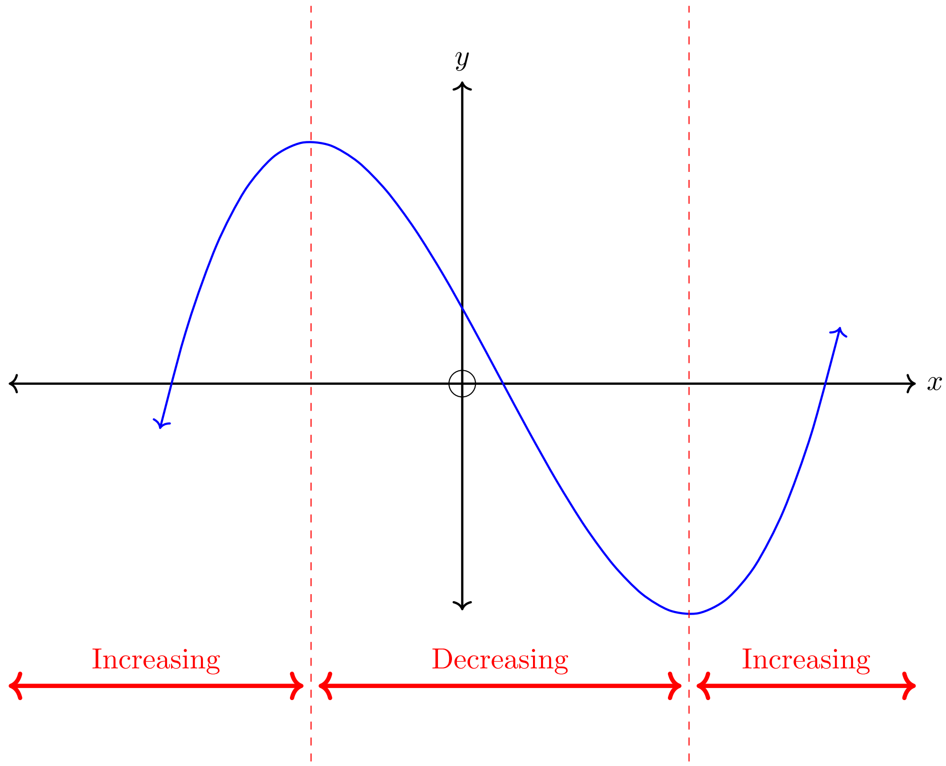
- If the gradient is entirely positive on an interval, the function is increasing on that interval.
- If the gradient is entirely negative on an interval, the function is decreasing on that interval.

If a function changes from increasing to decreasing, it must pass through a stationary point (or cross a region where the function is not differentiable)

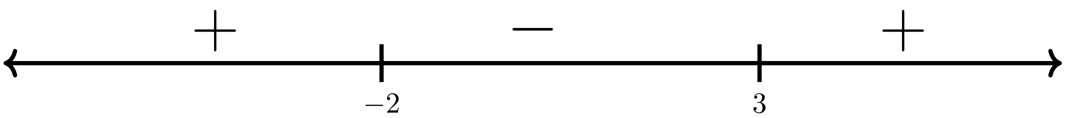
Hnece we need to divide up the domain into intervals bounded by stationary points and points where the function is not differentiable.
Finding a single gradient on each interval will tell us whether the function is increasing or decreasing.

We can communicate all of this using a **sign diagram**.

1.4.1 Basic Sign Diagram example



Sign diagram: $f'(x)$ either calculated or observed from a graph



1.4.2 Sign Diagram example with discontinuites

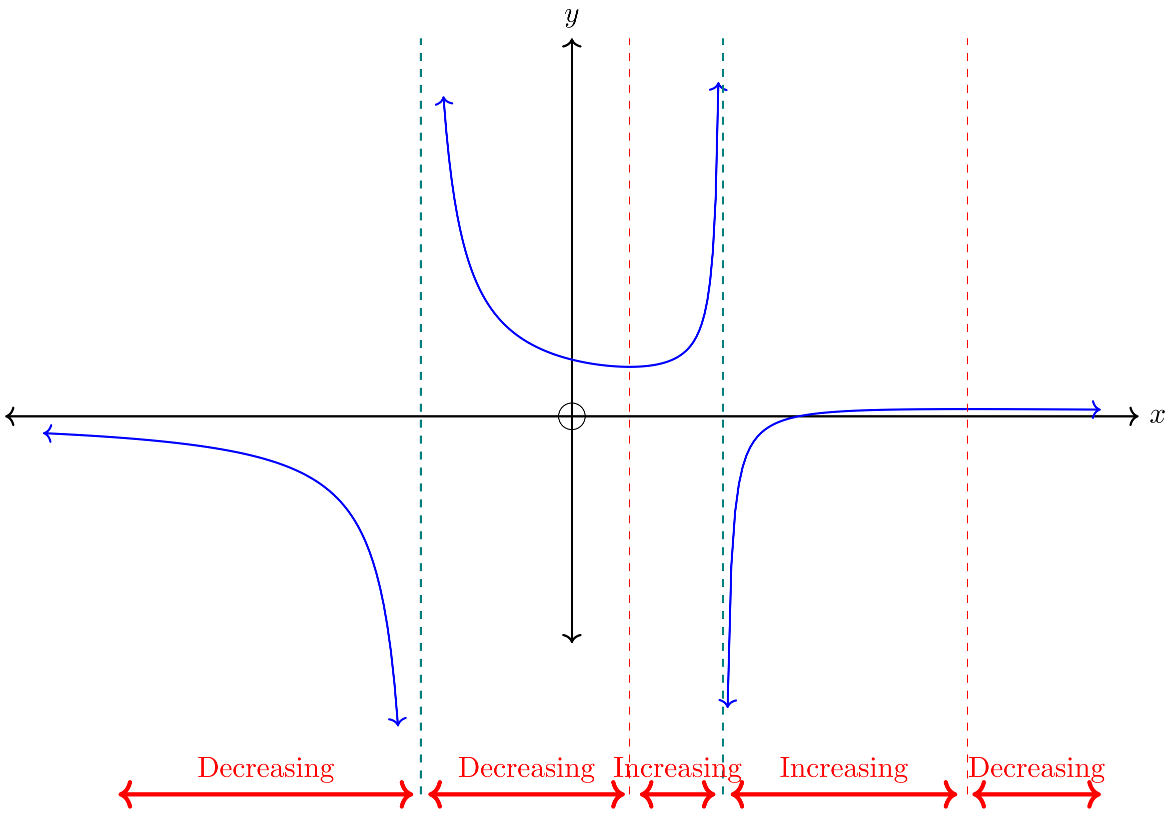
Below is the graph of

$$y = \frac{x-3}{x^2-4}$$

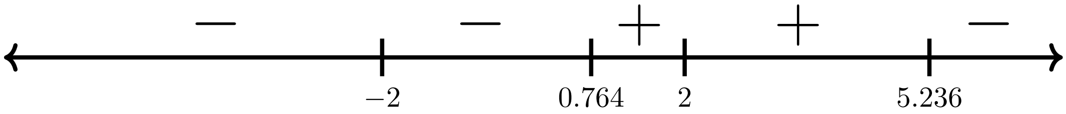
Because division by 0 is undefined, the graph has vertical asymptotes when $x^2 - 4 = 0$
(i.e when $x = 2$ and $x = -2$)

The derivative is $\frac{dy}{dx} = \frac{-x^2+6x-4}{(x^2-4)^2}$ and using a graphics calulator to solve $-x^2 + 6x - 4 = 0$,

we have stationary points at $x \approx 0.764$ and $x \approx 5.236$



Sign diagram: $f'(x)$ either calculated or observed from a graph



1.5 Stationary Points

1.5.1 Turning points (minima, maxima)

1.5.2 Stationary points of inflection

1.6 Shape

1.7 Inflection Points

1.8 Understanding functions and their derivatives

Chapter: 2 Applications of differentiation

$$
\begin{aligned}
\text{The equation is:} &\qquad 9a - 4 = 14 + 3a \\
\text{Subtract } 3a\text{:} &\qquad 6a - 4 = 14 \\
\text{Subtract 4:} &\qquad 6a = 18 \\
\text{Divide by 6:} &\qquad a = 3
\end{aligned}
$$

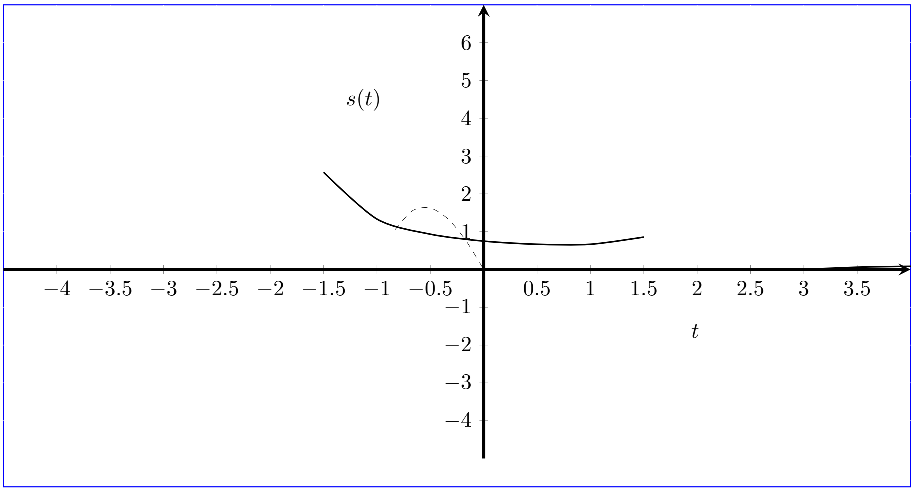
$$A\widehat{B}C$$

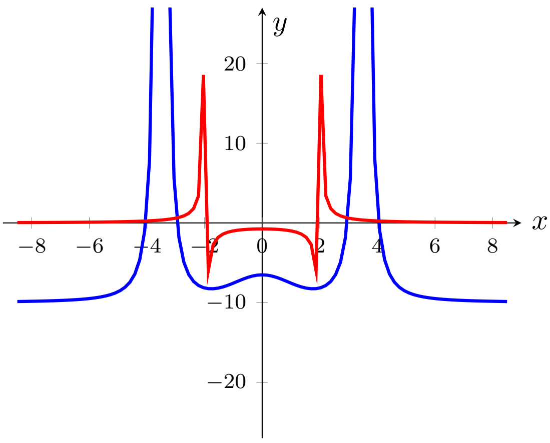
$$A\widehat{BC}C$$

$$A\hat{B}C$$

$$N\tilde{a}$$

$$X \sim \mathcal{N}(\mu, \sigma^2)$$


$s(t)$
t
6
5
4
3
2
1
−1
−2
−3
−4
−4 −3.5 −3 −2.5 −2 −1.5 −1 −0.5 0.5 1 1.5 2 2.5 3 3.5


y
x
20
10
−10
−20
−8 −6 −4 −2 0 2 4 6 8