Properties of curves and applications of differentiation

Kh notes

Contents

Chapter 1

Properties of curves

1.1 Review Questions

1.1.1 Some fundamental derivatives:

Function	Derivative
$f(x) = x^n$	$f'(x) = nx^{n-1} \quad (n \in \mathbb{R})$
$f(x) = e^x$	$f'(x) = e^x$
$f(x) = \ln x$	$f'(x) = \frac{1}{x}$
$f(x) = \sqrt{x}$	$f'(x) = \frac{1}{2\sqrt{x}}$
$f(x) = \sin x$	$f'(x) = \cos x$
$f(x) = \cos x$	$f'(x) = -\sin x$
$f(x) = \tan x$	$f'(x) = \sec^2 x$

1.1.2 Rules of differentiation:

Chain Rule:

$$y = g(u_{(x)})$$
$$\frac{dy}{dx} = g'(u_{(x)})u'_{(x)}$$

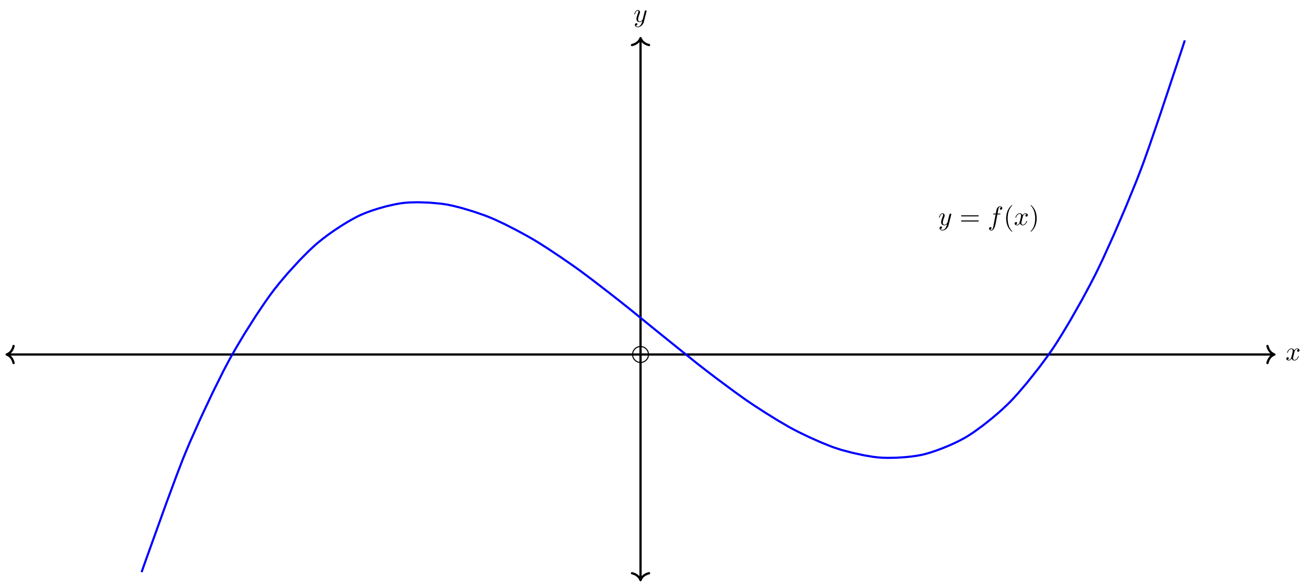
Product Rule:

$$y = u_{(x)}v_{(x)}$$
$$\frac{dy}{dx} = u_{(x)}v'_{(x)} + u'_{(x)}v_{(x)}$$

Quotient Rule:

$$y = \frac{u_{(x)}}{v_{(x)}}$$
$$\frac{dy}{dx} = \frac{u'_{(x)}v_{(x)} - u_{(x)}v'_{(x)}}{[v_{(x)}]^2}$$

1.2 Start Q and A



1.3 Tangents

The tangent to a curve at a point A is the best approximating straight line to the curve at point A.

(Leibniz definition) Tangent to the curve $y = f(x)$ at the point $(a, f(a))$ is the line through the infinitely close pair of points either side of $f(a)$

$$\frac{y - f(a)}{x - a} = \lim_{h \to 0} \frac{f(a + h) - f(a)}{h}$$

It is a single point of contact with the curve (although it may intersect the curve at some other point)

For the function $y = f(x)$, and some $x = a$
$(a, f(a))$ is on the curve
$f'(a)$ is the gradient of the curve at $x = a$

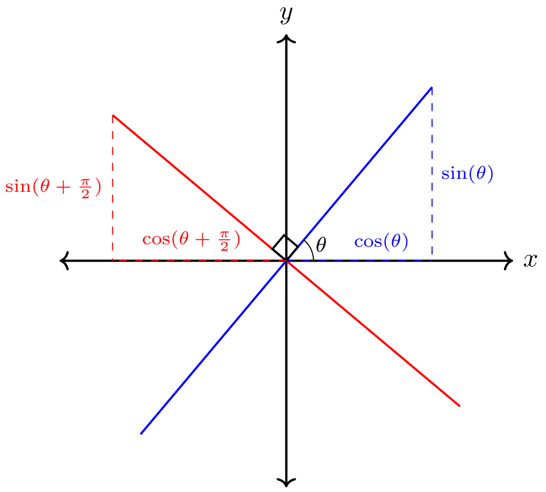
$\dfrac{y - f(a)}{x - a} = f'(a)$

$\Rightarrow y = f'(a)(x - a) + f(a)$ is the equation of the tangent line

4 worked examples

1.4 Normals

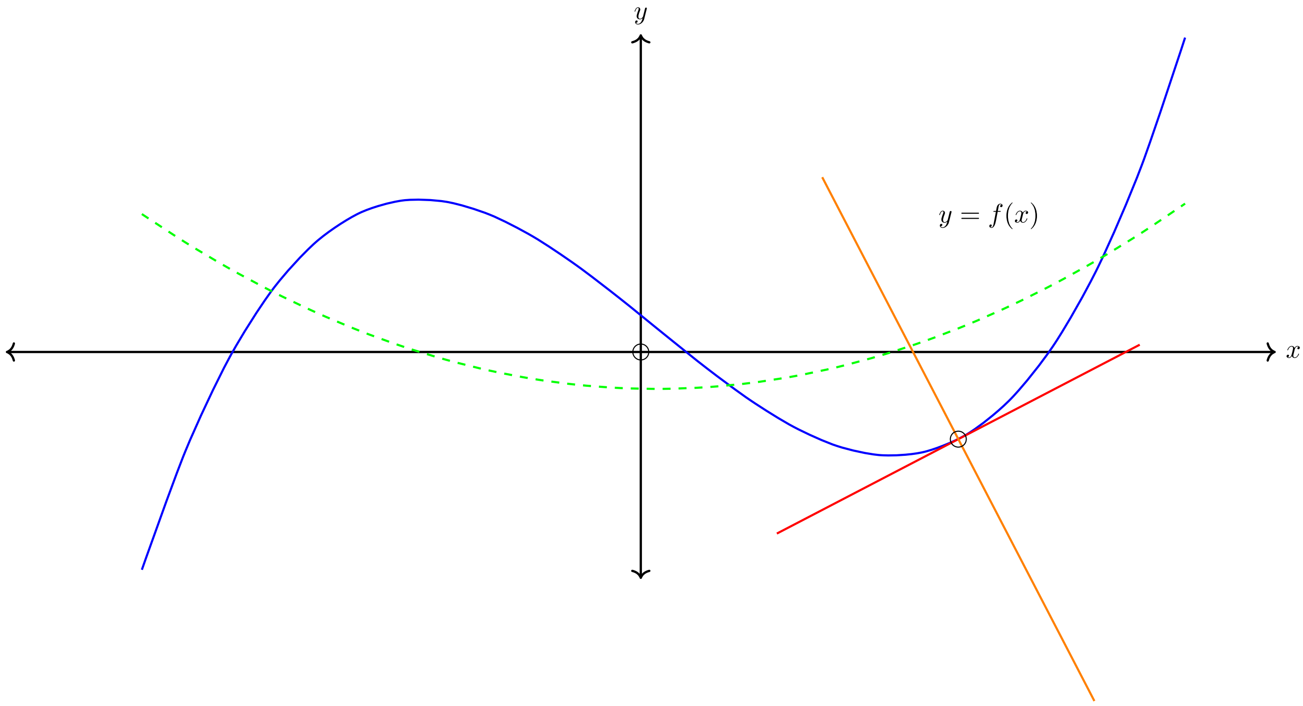
The product of the gradients of perpedicular lines = -1



$$m = \frac{\sin(\theta)}{\cos(\theta)}$$

$$m_{\perp} = \frac{\sin(\theta+\frac{\pi}{2})}{\cos(\theta+\frac{\pi}{2})} = -\frac{\cos(\theta)}{\sin(\theta)}$$

$$m \times m_{\perp} = -1$$



1.5 Increasing and Decreasing

1.6 Stationary Points

1.6.1 Turning points (minima, maxima)

1.6.2 Stationary points of inflection

1.7 Shape

1.8 Inflection Points

1.9 Understanding functions and their derivatives

Chapter 2

Applications of differentiation

$$
\begin{aligned}
\text{The equation is:} \quad && 9a - 4 &= 14 + 3a \\
\text{Subtract } 3a\text{:} \quad && 6a - 4 &= 14 \\
\text{Subtract 4:} \quad && 6a &= 18 \\
\text{Divide by 6:} \quad && a &= 3
\end{aligned}
$$

$$A\widehat{B}C$$

$$A\widehat{BC}C$$

$$A\hat{B}C$$

$$N\tilde{a}$$

$$X \sim \mathcal{N}(\mu,\, \sigma^2)$$