Properties of Curves

Kh notes

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1 Review Questions

1.1 Some fundamental derivatives:

Function	Derivative
$f(x) = x^n$	$f'(x) = nx^{n-1} \quad (n \in \mathbb{R})$
$f(x) = e^x$	$f'(x) = e^x$
$f(x) = \ln x$	$f'(x) = \dfrac{1}{x}$
$f(x) = \sqrt{x}$	$f'(x) = \dfrac{1}{2\sqrt{x}}$
$f(x) = \sin x$	$f'(x) = \cos x$
$f(x) = \cos x$	$f'(x) = -\sin x$
$f(x) = \tan x$	$f'(x) = \sec^2 x$

1.2 Rules of differentiation:

Chain Rule:

$$y = g(u_{(x)})$$
$$\frac{dy}{dx} = g'(u_{(x)})u'_{(x)}$$

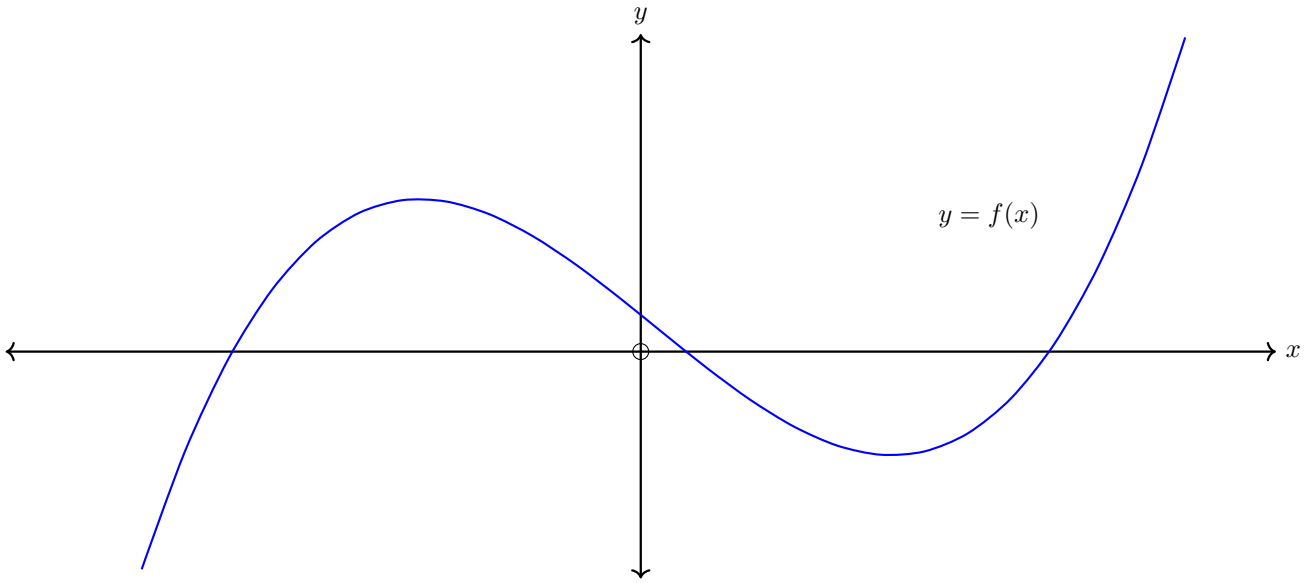
Product Rule:

$$y = u_{(x)}v_{(x)}$$
$$\frac{dy}{dx} = u_{(x)}v'_{(x)} + u'_{(x)}v_{(x)}$$

Quotient Rule:

$$y = \frac{u_{(x)}}{v_{(x)}}$$
$$\frac{dy}{dx} = \frac{u'_{(x)}v_{(x)} - u_{(x)}v'_{(x)}}{[v_{(x)}]^2}$$

2 Start Q and A



3 Tangents

The tangent to a curve at a point A is the best approximating straight line to the curve at point A.

(Leibniz definition) Tangent to the curve $y = f(x)$ at the point $(a, f(a))$ is the line through the infinitely close pair of points either side of $f(a)$

$$\frac{y - f(a)}{x - a} = \lim_{h \to 0} \frac{f(a + h) - f(a)}{h}$$

It is a single point of contact with the curve (although it may intersect the curve at some other point)

For the function $y = f(x)$, and some $x = a$
$(a, f(a))$ is on the curve
$f'(a)$ is the gradient of the curve at $x = a$

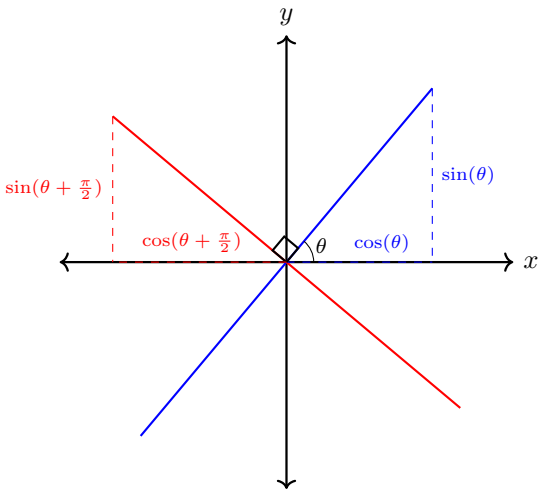
$$\frac{y - f(a)}{x - a} = f'(a)$$

$\Rightarrow y = f'(a)(x - a) + f(a)$ is the equation of the tangent line

4 worked examples

4 Normals

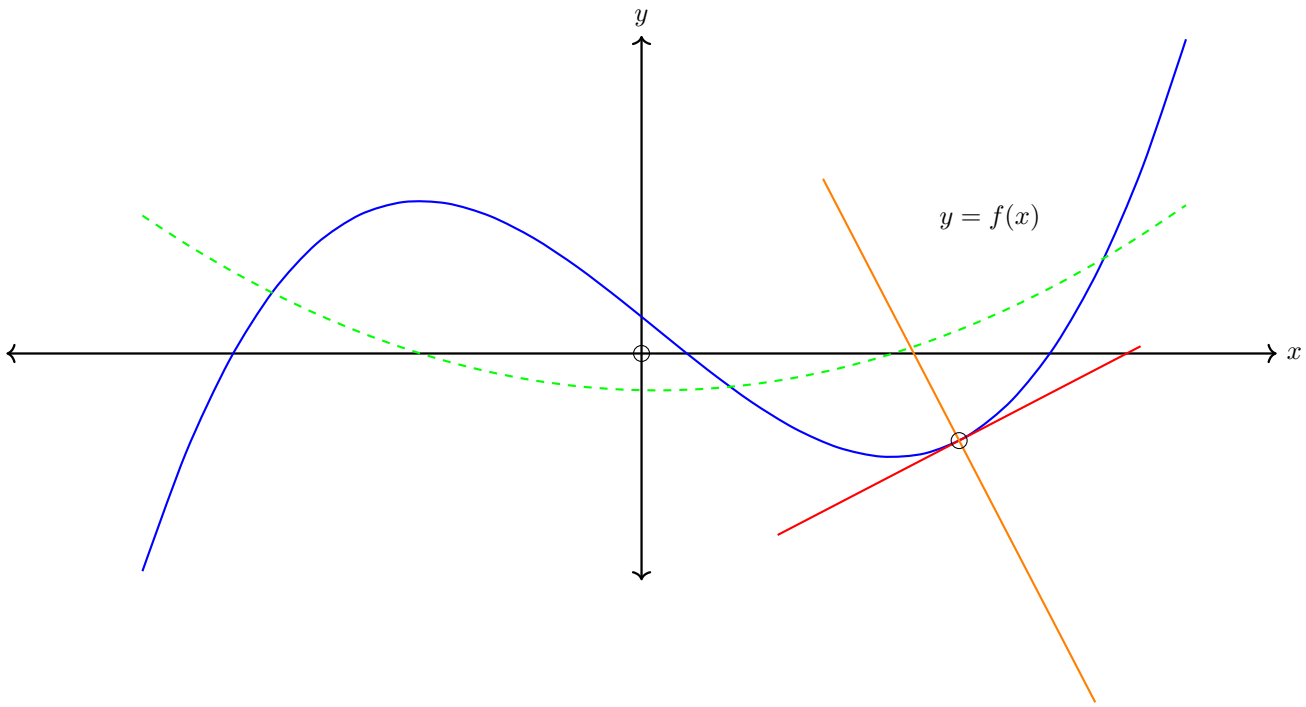
The product of the gradients of perpedicular lines = -1



$$m = \frac{\sin(\theta)}{\cos(\theta)}$$

$$m_{\perp} = \frac{\sin(\theta+\frac{\pi}{2})}{\cos(\theta+\frac{\pi}{2})} = -\frac{\cos(\theta)}{\sin(\theta)}$$

$$m \times m_{\perp} = -1$$



5 Increasing and Decreasing

6 Stationary Points

6.1 Turning points (minima, maxima)

6.2 Stationary points of inflection

7 Shape

8 Inflection Points

9 Understanding functions and their derivatives

The equation is:	$9a - 4 = 14 + 3a$
Subtract $3a$:	$6a - 4 = 14$
Subtract 4:	$6a = 18$
Divide by 6:	$a = 3$

$$A\widehat{B}C$$

$$\widehat{ABC}C$$

$$A\hat{B}C$$

$$N\tilde{a}$$

$$X \sim \mathcal{N}(\mu, \sigma^2)$$