1. In the expansion of $(a–3b)^n$, the sum of 9th and 10th term is zero. Find the value of $\frac{a}{b}$ in terms of n in terms of n

$T_9 = {}^nC_8\, a^{n-8} b^8$, $T_{10} = {}^nC_9\, a^{n-9} b^9$

$T_9 + T_{10} = 0 \Rightarrow T_9 = -T_{10}$

$$\frac{n!}{8!\,(n-8)!} a^{n-8} b^8 = -\frac{n!}{9!\,(n-9)!} a^{n-9} b^9$$

$$\Rightarrow \frac{a^{n-8} b^8}{a^{n-9} b^9} = \frac{-8!\,(n-8)!}{9!\,(n-9)!}$$

${}^nC_n = \frac{n!}{n!\,(n-n)!} = \frac{n!}{n!\,0!} = 1$

${}^nC_0 = \frac{n!}{0!\,(n-0)!} = 1$

$$\Rightarrow \frac{a}{b} = \frac{-8\times7\times\ldots\times1 \times (n-8)(n-9)\ldots\times(n-n)}{9\times8\times\ldots\times1 \times (n-9)\times\ldots\times(n-n)}$$

$$\frac{a}{b} = \frac{-(n-8)}{9}$$

$$= \frac{8-n}{9}$$

2. If the coefficient of 5th, 6th and 7th terms in the expansion of $(1+x)^n$ are in arithmetic sequence, then find the value(s) of n

$T_6 - T_5 = T_7 - T_6$ Where T_n is the coefficient

$\Rightarrow T_7 - 2T_6 + T_5 = 0$

$T_7 = {}^nC_6 = \frac{n!}{6!\,(n-6)!}$ $T_6 = {}^nC_5 = \frac{n!}{5!\,(n-5)!}$ $T_5 = {}^nC_4 = \frac{n!}{4!\,(n-4)!}$

$$\Rightarrow \frac{n!}{6!\,(n-6)!} - \frac{2n!}{5!\,(n-5)!} + \frac{n!}{4!\,(n-4)!} = 0$$

$$\Rightarrow \frac{n!}{6!\,(n-4)!}\left[(n-4)(n-5) - 2.6(n-4) + 6\times5\right] = 0$$

$\Rightarrow (n-4)(n-5) - 12(n-4) + 30 = 0$

$n^2 - 9n + 20 - 12n + 48 + 30 = 0$

$n^2 - 21n + 98 = 0$

$(n-7)(n-14) = 0$

$n = 7$, $n = 14$

3. If $x_n - y_n\sqrt{2} = (1-\sqrt{2})^n$, then show that ${x_n}^2 - 2{y_n}^2 = (-1)^n$

conjugates: $(1-\sqrt{2})(1+\sqrt{2}) = 1-2 = -1$

$\Rightarrow \quad (1-\sqrt{2})^n(1+\sqrt{2})^n = [(1-\sqrt{2})(1+\sqrt{2})]^n = (-1)^n$

If $x_n - y_n\sqrt{2} = (1-\sqrt{2})^n$

then $(1+\sqrt{2})^n = x_n + y_n\sqrt{2}$

$\Rightarrow \quad (x_n - y_n\sqrt{2})(x_n + y_n\sqrt{2}) = (1-\sqrt{2})^n(1+\sqrt{2})^n$

$\Rightarrow \quad {x_n}^2 - {y_n}^2 . 2 = (-1)^n \quad \Rightarrow \quad {x_n}^2 - 2{y_n}^2 = (-1)^n$

4. Find the first three terms in the expansion, in ascending powers of x, of $(1–2x)^5$

$$(1-2x)^5 = \sum_{k=0}^{5} {}^5C_k (1)^{5-k}(-2x)^k$$

1 4 6 4 1

1 5 10 10 5 1

$$= \sum_{k=0}^{5} {}^5C_k (-2x)^k$$

$$= 1(-2x)^0 + 5(-2x)^1 + 10(-2x)^2 + 10(-2x)^3 + 5(-2x)^4 + 1(-2x)^5$$

$$= \underline{1 - 10x + 40x^2} - 80x^3 + 80x^4 - 32x^5$$

$$= 1, \ -10x, \ 40x^2$$

5. Given that the coefficient of x^2 in the expansion of $(1 + ax + 2x^2)(1–2x)^5$ is 12, find the value of the constant a.

$(1 + ax + 2x^2) \qquad 1\,T_3 + ax\,T_2 + 2x^2\,T_1$ is x^2 term of expansion

0 1 2 3 4

$$T_3 = {}^5C_2 (1)^3(-2x)^2 = 40x^2$$

$$T_2 = {}^5C_1 (1)^4(-2x)^1 = -10x$$

$$T_1 = {}^5C_0 (1)^5(-2x)^0 = 1$$

$$40x^2 + ax(-10x) + 2x^2(1)$$

$$= (42 - 10a)x^2 \qquad 42 - 10a = 12$$

$$a = 3$$

6.

(a) Use the binomial theorem to expand $(a+\sqrt{b})^4$

(b) Hence, deduce an expression in terms of a and b for $(a+\sqrt{b})^4+(a-\sqrt{b})^4$

$$\begin{matrix} & & 1 & 4 & 6 & 4 & 1 \end{matrix}$$

$$(a+\sqrt{b})^4 = a^4 + 4a^3(\sqrt{b})^1 + 6a^2(\sqrt{b})^2 + 4a(\sqrt{b})^3 + 1(\sqrt{b})^4$$

$$= a^4 + 4a^3\sqrt{b} + 6a^2b + 4ab\sqrt{b} + b^2$$

$$(a-\sqrt{b})^4 = a^4 - 4a^3\sqrt{b} + 6a^2b - 4ab\sqrt{b} + b^2$$

$$(a+\sqrt{b})^4 + (a-\sqrt{b})^4 = 2a^4 + 12a^2b + 2b^2$$

7.

(a) Write down and simplify the general term in the binomial expansion of $(2x^2 - \frac{d}{x^3})^7$, where d is a constant

(b) Given that the coefficient of $\frac{1}{x}$ is $-70,000$ find the value of d

$$\left(2x^2 - \frac{d}{x^3}\right)^7 = \sum_{k=0}^{7} {}^7C_k \, (2x^2)^{7-k} (-dx^{-3})^k$$

$$= \sum_{k=0}^{7} {}^7C_k \, 2^{7-k} x^{14-2k} \cdot (-d)^k x^{-3k}$$

$$= \sum_{k=0}^{7} {}^7C_k \, 2^{7-k} (-d)^k x^{14-5k}$$

$$\frac{1}{x} = x^{-1} \Rightarrow 14-5k = -1$$

$$-5k = -15 \Rightarrow k = 3$$

$${}^7C_3 \cdot 2^{7-3} (-d)^3 = -70000$$

$$35 \cdot 16 (-d)^3 = -70000$$

$$(-d)^3 = -125$$

$$-d = -5$$

$$d = 5$$