Fractional powers

The square root of the number x is the (positive) number y such that $y^2 = x$.
For example: $\sqrt{25} = 5$ because $5^2 = 25$.
Please note that $-5 \times -5 = 25$ but the square root function $\sqrt{25}$ only returns a positive value.

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1) Evaluate the following

a) $\sqrt{4}$

b) $\sqrt{64}$

c) $\sqrt{49}$

d) $\sqrt{\frac{25}{9}}$

e) $\sqrt{\frac{132}{169}}$

f) $\sqrt{a^2}$

g) $\sqrt{a^4}$

h) $\sqrt{a^6}$

i) $\sqrt{a^2b^6}$

j) $\sqrt{\frac{9x^4}{y^8}}$

k) $\sqrt{\frac{16x^3y^2}{xy^4}}$

2) Evaluate the following

a) $\sqrt{4} \times \sqrt{4}$

b) $\sqrt{a} \times \sqrt{a}$

c) $(\sqrt{a^2})^3$

d) $(3b\sqrt{a})^2$

e) $\sqrt{xy} \times \sqrt{xy}$

f) $(4\sqrt{a^2})^3$

We can now see that:

$$\sqrt{x} \times \sqrt{x} = x$$
$$x^{\frac{1}{2}} \times x^{\frac{1}{2}} = x^1 = x$$

Which tells us that

$$x^{\frac{1}{2}} = \sqrt{x}$$

And it is probably good to note that the addition, subtraction and power rules still hold:

$$x^{\frac{1}{2}} \times x^2 = x^{\frac{1}{2}+2} = x^{\frac{5}{2}}$$
$$\frac{x^{\frac{7}{2}}}{x^{\frac{1}{2}}} = x^{\frac{7}{2}-\frac{1}{2}} = x^3$$
$$(9b)^{\frac{1}{2}} = 9^{\frac{1}{2}} \times b^{\frac{1}{2}} = 3\sqrt{b}$$

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3) Simplify:

a) $\left(16b^4\right)^{\frac{1}{2}}$

b) $\left(81^{\frac{1}{2}}b^4\right)^{\frac{1}{2}}$

c) $\sqrt{100a^2b^{10}}$

d) $\left(\dfrac{1}{16x^4}\right)^{\frac{1}{2}}$

e) $\left(\dfrac{108x^2y^6}{3x^4}\right)^{\frac{1}{2}}$

f) $\sqrt{\dfrac{256y^6}{x^{12}}}$

Looking at the power rule we can note:

$$9^{\frac{3}{2}} = \left(9^{\frac{1}{2}}\right)^3 = 3^3 = 27$$

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4) Evaluate without a calculator (use a calculator afterwards to check your work and make sure you can enter the powers correctly)

a) $4^{\frac{5}{2}}$ b) $16^{\frac{3}{2}}$ c) $100^{\frac{7}{2}}$ d) $36^{\frac{3}{2}}$

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5) Simplify, by writing with no brackets, some powers will stay as fractions

a) $\left(3^3xy^2\right)^{\frac{1}{2}} \times 4y^2$

b) $\dfrac{5^2a}{b^4} \times \dfrac{b^{\frac{3}{2}}}{a}$

c) $\left(\dfrac{x}{x^{\frac{1}{2}}}\right)^3$

d) $(xy)^{\frac{1}{2}} \times x^{-\frac{5}{2}}$

e) $\left(5^{\frac{3}{2}}x^{\frac{5}{2}}y^3\right)^2$

f) $\left(\sqrt{\dfrac{x}{2}}\right)^3$

g) $\sqrt{\left(\dfrac{x}{2}\right)^3}$

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6) Bit of review, simplify:

a) $\sqrt{16x^{16}}$

b) $\sqrt{36x^{36}}$

c) $\dfrac{4m^8}{\sqrt{36m^{20}}}$

d) $\dfrac{\sqrt{9n^4}}{3n^2}$

e) $\dfrac{(3a^2b^2)^4}{27a^8b^9}$

f) $(0.25x^3)^{\frac{1}{2}}$

g) $\dfrac{(4a^2)^3}{(8a^5)^2}$

h) $\sqrt{\dfrac{256a^{16}}{b^{12}}}$