

13 Solving Quadratic Equations

13.1 Using factorised quadratics

Consider the following equation:

$$(x - 6)(x + 3) = 0$$

To solve this equation, we do not expand the brackets.

we can note that the equation is of the form:

$$A \times B = 0 \quad \text{where } A = (x - 6) \text{ and } B = (x + 3)$$

And we can also note that if $A \times B = 0$, then either $A = 0$ or $B = 0$

So $(x - 6)(x + 3) = 0$ has **two** solutions.

$$\begin{aligned} (x - 6)(x + 3) &= 0 \\ x - 6 = 0 &\quad \text{or} \quad x + 3 = 0 \\ \Rightarrow x = 6 &\quad \text{or} \quad x = -3 \end{aligned}$$

13.1.1 Exercise

Solve the following:

- a) $(x - 1)(x + 7) = 0$ c) $(x + 11)(x + 9) = 0$
b) $(x - 5)(x + 5) = 0$ d) $(x - 3)(x - 15) = 0$
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13.1.2 Exercise

Solve the following:

- a) $(3x - 15)(2x + 8) = 0$ c) $(8x + 10)(2x + 9) = 0$
b) $(2x - 8)(4x + 1) = 0$ d) $(2x - 7)(2x - 15) = 0$
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13.1.3 Exercise

Solve the following:

- a) $3x(x + 8) = 0$ c) $-3x(2x + 9) = 0$
b) $x(x + 4) = 0$ d) $x(2x - 15) = 0$
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13.1.1 Exercise

Solve the following:

a) $(x - 1)(x + 7) = 0$

c) $(x + 11)(x + 9) = 0$

b) $(x - 5)(x + 5) = 0$

d) $(x - 3)(x - 15) = 0$

a) $x - 1 = 0 \text{ or } x + 7 = 0$
 $x = 1 \text{ or } x = -7$

b) $x - 5 = 0 \text{ or } x + 5 = 0$
 $x = 5 \text{ or } x = -5$

c) $x + 11 = 0 \text{ or } x + 9 = 0$
 $x = -11 \text{ or } x = -9$

d) $x - 3 = 0 \text{ or } x - 15 = 0$
 $x = 3 \text{ or } x = 15$

13.1.2 Exercise

Solve the following:

a) $(3x - 15)(2x + 8) = 0$

c) $(8x + 10)(2x + 9) = 0$

b) $(2x - 8)(4x + 1) = 0$

d) $(2x - 7)(2x - 15) = 0$

a) $3x - 15 = 0 \text{ or } 2x + 8 = 0$
 $x = \frac{15}{3} = 5 \text{ or } x = -\frac{8}{2} = -4$

c) $8x + 10 = 0 \text{ or } 2x + 9 = 0$
 $x = -\frac{10}{8} = -\frac{5}{4} \text{ or } x = -\frac{9}{2}$

b) $2x - 8 = 0 \text{ or } 4x + 1 = 0$
 $x = 4 \text{ or } x = -\frac{1}{4}$

d) $2x - 7 = 0 \text{ or } 2x - 15 = 0$
 $x = \frac{7}{2} \text{ or } x = \frac{15}{2}$

13.1.3 Exercise

Solve the following:

a) $3x(x + 8) = 0$

c) $-3x(2x + 9) = 0$

b) $x(x + 4) = 0$

d) $x(2x - 15) = 0$

a) $3x = 0 \text{ or } x + 8 = 0$
 $x = 0 \text{ or } x = -8$

c) $-3x(2x + 9) = 0$
 $-3x = 0 \text{ or } 2x + 9 = 0$
 $x = \frac{0}{-3} = 0 \text{ or } x = -\frac{9}{2}$

b) $x(x + 4) = 0$
 $x = 0 \text{ or } x + 4 = 0$
 $x = -4$

d) $x(2x - 15) = 0$
 $x = 0 \text{ or } 2x - 15 = 0$
 $x = \frac{15}{2}$

13.2 Using un-factorised quadratics

To solve the equation:

$$x^2 + 3x - 88 = 0$$

$$\begin{array}{ll} x^2 + 3x - 88 = 0 & \\ \text{factorise} & (x - 8)(x + 11) = 0 \\ & x + 8 = 0 \quad \text{or} \quad x + 11 = 0 \\ & x = -8 \quad \text{or} \quad x = -11 \end{array}$$

13.2.1 Exercise

- a) $x^2 + 7x + 6 = 0$ c) $x^2 - 5x + 6 = 0$
b) $x^2 + 12x + 32 = 0$ d) $x^2 + 3x - 10 = 0$
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13.2.2 Exercise

- a) $x^2 + 6x = 0$ c) $x^2 - 18 - 3x = 0$
b) $x^2 - 5x = 0$ d) $2x - 99 + x^2 = 0$
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13.2.3 Exercise

- a) $x^2 + 0x - 9 = 0$ c) $x^2 - 25 = 0$
b) $x^2 - 0x - 36 = 0$ d) $x^2 - 49 = 0$
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13.2.4 Exercise

- a) $x^2 - 6x + 9 = 0$ c) $x^2 - 2x + 1 = 0$
b) $x^2 + 10x + 25 = 0$ d) $x^2 + 2x + 1 = 0$
-

13.2.1 Exercise

a) $x^2 + 7x + 6 = 0$

b) $x^2 + 12x + 32 = 0$

c) $x^2 - 5x + 6 = 0$

d) $x^2 + 3x - 10 = 0$

a) $x^2 + 7x + 6 = 0$
 $(x+6)(x+1) = 0$
 $x = -6 \quad x = -1$

c) $x^2 - 5x + 6 = 0$
 $(x-2)(x-3) = 0$
 $x = 2 \quad x = 3$

b) $x^2 + 12x + 32 = 0$
 $(x+4)(x+8) = 0$
 $x = -4 \quad x = -8$

d) $x^2 + 3x - 10 = 0$
 $(x+5)(x-2) = 0$
 $x = -5 \quad x = 2$

13.2.2 Exercise

a) $x^2 + 6x = 0$

b) $x^2 - 5x = 0$

c) $x^2 - 18 - 3x = 0$

d) $2x - 99 + x^2 = 0$

a) $x^2 + 6x = 0$
 $x(x+6) = 0$
 $x = 0 \quad x = -6$

b) $x^2 - 5x = 0$
 $x(x-5) = 0$
 $x = 0 \quad x = 5$

c) $x^2 - 18 - 3x = 0$
 $x^2 - 3x - 18 = 0$
 $(x-6)(x+3) = 0$
 $x = 6 \quad x = -3$

d) $2x - 99 + x^2 = 0$
 $x^2 + 2x - 99 = 0$
 $(x+11)(x-9) = 0$
 $x = -11 \quad x = 9$

13.2.3 Exercise

a) $x^2 + 0x - 9 = 0$

b) $x^2 - 0x - 36 = 0$

c) $x^2 - 25 = 0$

d) $x^2 - 49 = 0$

a) $x^2 + 0x - 9 = 0$
 $(x+3)(x-3) = 0$
 $x = -3 \quad x = 3$

c) $x^2 - 25 = 0$
 $(x-5)(x+5) = 0$
 $x = 5 \quad x = -5$

b) $x^2 - 0x - 36 = 0$
 $(x+6)(x-6) = 0$
 $x = -6 \quad x = 6$

d) $x^2 - 49 = 0$
 $(x-7)(x+7) = 0$
 $x = 7 \quad x = -7$

13.2.4 Exercise

a) $x^2 - 6x + 9 = 0$

b) $x^2 + 10x + 25 = 0$

c) $x^2 - 2x + 1 = 0$

d) $x^2 + 2x + 1 = 0$

a) $x^2 - 6x + 9 = 0$
 $(x-3)(x-3) = 0$
 $x = 3 \quad x = 3$

So we have 1 (repeated)
Solution, $x = 3$

c) $x^2 - 2x + 1 = 0$
 $(x-1)(x-1) = 0$
 $x = 1$

b) $x^2 + 10x + 25 = 0$
 $(x+5)(x+5) = 0$
 $x = -5$

d) $x^2 + 2x + 1 = 0$
 $(x+1)(x+1) = 0$
 $x = -1$

Could write as
a) $(x-3)^2 = 0$

b) $(x+5)^2 = 0$

c) $(x-1)^2$

d) $(x+1)^2$

13.3 Re-arranging into standard form

If we get an equation with an x^2 as its highest power, we need to re-organise it into **standard form**

$$x^2 + bx + c = 0$$

Once this is done, we can factorise and solve.

(Example) Solve:

$$3 = 4x - x^2$$

<u>rearrange</u>	$3 = 4x - x^2$
	$3 + x^2 = 4x$
	$3 + x^2 - 4x = 0$
<u>standard form</u>	$x^2 - 4x + 3 = 0$
<u>factorise</u>	$(x - 3)(x - 1) = 0$
	$x - 3 = 0 \quad \text{or} \quad x - 1 = 0$
	$x = 3 \quad \text{or} \quad x = 1$

13.3.1 Exercise

- a) $26 = 11x - x^2$ c) $42 = x^2 - 11x$
b) $18x + x^2 = -45$ d) $x^2 + 225 = 30x$

13.3.2 Exercise

- a) $x(x - 2) = 15$ d) $(x - 3)(x + 2) = 4x$
b) $(x - 3)^2 = 25$ e) $(x + 2)(x + 7) = (x + 5)(x + 7)$
c) $(x - 4)(x - 6) = 8$ f) $x + 3 = \frac{12}{x + 4}$

13.3.1 Exercise

a) $-26 = 11x - x^2$

c) $42 = x^2 - 11x$

b) $18x + x^2 = -45$

d) $x^2 + 225 = 30x$

$$\begin{aligned} a) -26 &= 11x - x^2 \\ x^2 - 11x - 26 &= 0 \\ (x-13)(x+2) &= 0 \quad -26 \\ x = 13 \text{ or } x = -2 &\quad +2 -13 \\ &\quad -11 \end{aligned}$$

$$\begin{aligned} b) 18x + x^2 &= -45 \\ x^2 + 18x + 45 &= 0 \\ (x+3)(x+15) &= 0 \quad +45 \\ x = -3 \text{ or } x = -15 &\quad +3 +15 \\ &\quad +18 \end{aligned}$$

$$\begin{aligned} c) 42 &= x^2 - 11x \\ 42 - x^2 + 11x &= 0 \\ -x^2 + 11x + 42 &= 0 \\ (x-1)x^2 - 11x - 42 &= 0 \quad (x-1) \\ (x-14)(x+3) &= 0 \quad -42 \\ x = 14 \text{ or } x = -3 &\quad -14 +3 \\ &\quad -11 \end{aligned}$$

$$\begin{aligned} d) x^2 + 225 &= 30x \\ x^2 - 30x + 225 &= 0 \\ (x-15)(x-15) &= 0 \quad +225 \quad 1,225 \\ x = 15 &\quad -15 -15 \quad 3, 75 \\ &\quad -30 \quad 5, 45 \\ &\quad 9, 25 \\ &\quad 15, 15 \end{aligned}$$

1, 42
2, 21
3, 14
6, 7

13.3.2 Exercise

a) $x(x-2) = 15$

d) $(x-3)(x+2) = 4x$

b) $(x-3)^2 = 25$

e) $(x+2)(x+7) = (x+5)(x+7)$

c) $(x-4)(x-6) = 8$

f) $x+3 = \frac{12}{x+4}$

$$\begin{aligned} a) x(x-2) &= 15 \\ x^2 - 2x &= 15 \\ x^2 - 2x - 15 &= 0 \\ (x-5)(x+3) &= 0 \\ x = 5 \text{ or } x = -3 & \end{aligned}$$

$$\begin{aligned} c) (x-4)(x-6) &= 8 \\ x^2 - 10x + 24 &= 8 \\ x^2 - 10x + 16 &= 0 \\ (x-2)(x-8) &= 0 \\ x = 2 \text{ or } x = 8 & \end{aligned}$$

$$\begin{aligned} b) x^2 - 6x + 9 &= 25 \\ x^2 - 6x - 16 &= 0 \\ (x-8)(x+2) &= 0 \\ x = 8 \text{ or } x = -2 & \end{aligned}$$

$$d) (x-3)(x+2) = 4x$$

$$x^2 - x - 6 = 4x$$

$$\begin{array}{rcl} x^2 - 5x - 6 = 0 & & -6 \\ (x-6)(x+1) = 0 & -6 + 1 & \\ x = 6 \quad x = -1 & & -5 \end{array}$$

$$e) (x+2)(x+7) = (x+5)(x+7)$$

$$x^2 + 9x + 14 = x^2 + 12x + 35$$

$$(-x^2) \quad 9x + 14 = 12x + 35 \quad (-x^2)$$

Reduces to a Linear Equation

$$-3x = 21$$

$$x = \frac{21}{-3} = -7$$

$$f) (x+3) = \frac{12}{(x+4)}$$

$$\frac{(x+3)(x+4)}{x+4} = \frac{12}{x+4}$$

$$(x+3)(x+4) = 12$$

$$x^2 + 7x + 12 = 12$$

$$x^2 + 7x = 0$$

$$x(x+7) = 0$$

$$x = 0 \quad x = -7$$

13.4 Word questions

13.4.1 Exercise

- a) When a number is squared, and then 1 is subtracted, the result is 8.
 - i) Write this information down as a quadratic equation.
 - ii) Solve the equation to work out the two possible numbers.
 - b) A number is squared, and then added to the original number. The result is 20.
 - i) Write this information down as a quadratic equation.
 - ii) Solve the equation to work out the two possible numbers.
 - c) When a number x , is multiplied by a number 4 less than x , the result is 12.
What two numbers have this property?
 - d) Squaring a number gives the same result as multiplying the number by 8, and then subtracting 12.
 - i) Write this information down as a quadratic equation.
 - ii) Solve the equation to work out the two possible numbers.
 - e) When the result of adding 2 to a number is multiplied by the result of subtracting 2 from the same number, the answer is 21.
What two numbers have this property?
-

a) When a number is squared, and then 1 is subtracted, the result is 8.

i) Write this information down as a quadratic equation.

ii) Solve the equation to work out the two possible numbers.

let the number be n we could call it x or y or p or q ...

i) $n^2 - 1 = 8$

ii) $n^2 - 9 = 0$

$$(n+3)(n-3) = 0$$
$$n = -3 \quad n = 3$$

b) A number is squared, and then added to the original number. The result is 20.

i) Write this information down as a quadratic equation.

ii) Solve the equation to work out the two possible numbers.

i) $n + n^2 = 20$

ii) $n^2 + n - 20 = 0$
 $(n+5)(n-4) = 0$

$$n = -5 \text{ or } n = 4$$

c) When a number x , is multiplied by a number 4 less than x , the result is 12.

What two numbers have this property?

$$x(x-4) = 12$$

$$x^2 - 4x - 12 = 0$$

$$(x-6)(x+2) = 0$$

$$x = 6 \quad x = -2$$

- d) Squaring a number gives the same result as multiplying the number by 8, and then subtracting 12.
- Write this information down as a quadratic equation.
 - Solve the equation to work out the two possible numbers.
- e) When the result of adding 2 to a number is multiplied by the result of subtracting 2 from the same number, the answer is 21.
What two numbers have this property?

d)

$$x^2 = 8x - 12$$

$$x^2 - 8x + 12 = 0$$

$$(x - 6)(x - 2) = 0$$

$$x = 6 \quad x = 2$$

e)

$$(x + 2)(x - 2) = 21$$

$$x^2 - 4 = 21$$

$$x^2 - 25 = 0$$

$$(x + 5)(x - 5) = 0$$

$$x = 5 \quad x = -5$$