

Example Questions: 5B.1

7. A sequence is defined by

$$u_n = \frac{71 - 7n}{2}$$

a) prove the sequence is arithmetic

Show $u_{n+1} - u_n = d$

$$u_{n+1} - u_n = \frac{71 - 7(n+1)}{2} - \frac{71 - 7n}{2}$$

$$= \frac{71 - 7n - 7 - 71 + 7n}{2}$$

$$= \frac{-7}{2} = -3.5$$

So $u_{n+1} - u_n = -3.5$ for all $n \in \mathbb{Z}^+$

b) $u_n = \frac{71}{2} - \frac{7n}{2}$

$$u_1 = \frac{71}{2} - \frac{7}{2} = \frac{64}{2} = 32$$

$$d = -\frac{7}{2}$$

c) $u_{75} = \frac{71 - 7(75)}{2} = -227$

d) $u_n < -200$

$$\frac{71 - 7n}{2} < -200$$

$$71 - 7n < -400$$

$$7n > 71 + 400$$

$$7n > 471$$

$$n > 67 \frac{2}{7}$$

for $n \geq 68$ $u_n < -200$

check $u_{68} = \frac{71 - 7(68)}{2}$

$$= -202.5$$

12. f) Find k given the consecutive arithmetic terms:

$$2k+18, -2-k, 2k+2$$

$$u_1 \quad u_2 \quad u_3$$

$$u_2 - u_1 = u_3 - u_2$$

$$-2-k - (2k+18) = 2k+2 - (-2-k)$$

$$-2-k-2k-18 = 2k+2+2+k$$

$$-20-3k = 3k+4$$

$$6k = -24$$

$$k = -4$$

check:

$$10, 2, -6$$

$$d = -8$$

Other notes:

Could have:

$$\frac{2k+2+2k+18}{2} = -2-k$$

$$4k+20 = -4-2k$$

$$6k = -24$$

or

$$u_2 - u_1 = u_3 - u_2$$

$$2u_2 - u_1 = u_3$$

$$2(-2-k) - (2k+18) = 2k+2$$

$$-4-2k-2k-18 = 2k+2$$

$$-22-4k = 2k+2$$

$$6k = -24$$

13. Find the general term u_n for the arithmetic sequence
 $u_5 = -2$ and $u_{12} = -12\frac{1}{2}$

$$u_5 = a + 4d$$

$$u_{12} = a + 11d$$

$$a + 4d = -2 \quad (1)$$

$$a + 11d = -\frac{25}{2} \quad (2)$$

$$\frac{a + 4d = -2}{a + 11d = -\frac{25}{2}} \quad (1) - (2)$$

$$-7d = +\frac{21}{2}$$

$$d = \frac{+\frac{21}{2}}{-7} = -\frac{3}{2}$$

from (1) $a = -2 - 4d$
 $= -2 - 4(-\frac{3}{2})$
 $= -2 + 6 = 4$

$$u_n = 4 - (n-1)\frac{3}{2}$$

Test: $u_5 = 4 - (5-1)\frac{3}{2}$

$$= 4 - 6 = -2$$

$$u_{12} = 4 - (12-1)\frac{3}{2}$$

$$= 4 - \frac{33}{2}$$

$$= \frac{8-33}{2}$$

$$= -\frac{25}{2} = -12\frac{1}{2}$$

15. Insert six numbers between -1 and 32 so all 8 are in an arithmetic sequence

$$a = -1$$

$$u_8 = -1 + 7d$$

$$-1 + 7d = 32$$

$$7d = 33$$

$$d = \frac{33}{7}$$

$$u_n = -1 + (n-1)\frac{33}{7}$$

or $d = \frac{u_8 - u_1}{7} = \frac{33}{7}$