Extension activities

Power of zero and negative powers

Evaluate the powers given to gain an intuitive understanding of what a power of zero means and what a negative power means.

2^4	2^3	2^2	2^1	2^0	2^{-1}	2^{-2}	2^{-3}
16	8	4	2	1	$\frac{1}{2}$	$\frac{1}{4}$	$\frac{1}{8}$

$\times\frac{1}{2}$ $\times\frac{1}{2}$ $\times\frac{1}{2}$

3^4	3^3	3^2	3^1	3^0	3^{-1}	3^{-2}	3^{-3}
81	27	9	3	1	$\frac{1}{3}$	$\frac{1}{9}$	$\frac{1}{27}$

$\times\frac{1}{3}$

From this we can deduce some rules for exponents:

$$x^0 = 1$$
$$x^{-n} = \frac{1}{x^n}$$

we also have:

$$x^m \times x^n = x^{m+n}$$
$$(x^m)^n = x^{mn}$$

and can show:

$$\frac{x^m}{x^n} = \frac{x^m}{1} \times \frac{1}{x^n}$$
$$= x^m \times x^{-n} = x^{m-n}$$

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1) a) $(2x)^0 = 2^0 x^0$
$= 1 \times 1 = 1$

c) $(-5)^0 = 1$

b) $3x^0 = 3 \times 1 = 3$

d) $x \times 4x^0 = x \times 4 \times 1 = 4x$

2) a) $x^{-2} = \frac{1}{x^2}$

d) $x^4 \times x^{-5} = x^{4-5}$
$= x^{-1} = \frac{1}{x}$

b) $3x^{-2} = 3 \times x^{-2}$
$= 3 \times \frac{1}{x^2}$
$= \frac{3}{x^2}$

e) $y \times y^{-5} = y^1 \times y^{-5}$
$= y^{-4} = \frac{1}{y^4}$

f) $3x^{-1} \times y = 3 \times \frac{1}{x} \times y$
$= \frac{3y}{x}$

c) $x^3 \times x^{-2} = x^3 \times \frac{1}{x^2}$
$= \frac{x^3}{x^2} = x$

g) $4y^{-2} \times 3y^{-1} = 12y^{-2+-1} = 12y^{-3} = \frac{12}{y^3}$

or
$x^3 \times x^{-2} = x^{3-2}$
$= x^1$
$= x$

h) $7x^{-1} \times y^{-3} = \frac{7}{xy^3}$

i) $3x^2y \times 2xy^{-3} = 6x^{2+1}y^{1-3} = \frac{6x^3}{y^2}$

3) a) $(3x)^{-2} = \frac{1}{(3x)^2} = \frac{1}{9x^2}$

b) $(3x)^{-2} \times x^4 = \frac{x^4}{9x^2} = \frac{x^2}{9}$

c) $(7x)^2 \times x^{-2} = 49\, x^2 \times x^{-2}$
$= 49$

d) $(3x)^{-2} \times x^{-2} = \frac{1}{9x^2 \times x^2} = \frac{1}{9x^4}$

e) $(4xy)^2 \times (xy)^{-3} = 16\, x^2 \times x^{-3} \times y^2 \times y^{-3}$
$= \frac{16}{xy}$

f) $9x^{-2} \times (3x)^{-1} = \frac{9}{x^2} \times \frac{1}{3x}$
$= \frac{3}{x^3}$

g) $4^2 x^{-3} \times 4^{-1} xy$
$= 4^2 \times 4^{-1} \times x^{-3} \times x \times y$
$= 4 \times x^{-2} \times y$
$= \frac{4y}{x^2}$

i) $(y^{-2})^{-3} \times x^{-3}$
$= y^6 \times x^{-3} = \frac{y^6}{x^3}$

h) $3x^{-1} \times (y^{-2})^{-1}$
$= 3x^{-1} \times y^2$
$= \frac{3y^2}{x}$

4) a) $\dfrac{1}{\frac{1}{2}} = 1 \div \frac{1}{2}$

$= 1 \times \dfrac{2}{1}$

$= 2$

Idea how many $\frac{1}{2}$'s fit into 1.

b) $\dfrac{3}{\frac{1}{2}} = 3 \div \frac{1}{2}$

$= 3 \times \dfrac{2}{1}$

$= 6$

how many $\frac{1}{2}$'s fit into 3

c) $\dfrac{1}{\frac{2}{3}} = 1 \times \dfrac{3}{2} = \dfrac{3}{2}$

d) $\dfrac{\frac{3}{2}}{\frac{2}{3}} = \dfrac{3}{2} \times \dfrac{3}{2} = \dfrac{9}{4}$

e) $\dfrac{1}{2^{-1}}$ we could see it as $\dfrac{1}{\frac{1}{2}} = 2$

but also consider $x^{-m} = \dfrac{1}{x^m} \Rightarrow x^m = \dfrac{1}{x^{-m}}$

$\dfrac{1}{2^{-1}} = 2^1 = 2$

f) $\dfrac{1}{2^{-2}} = 2^2 = 4$

g) $\left(\dfrac{1}{3}\right)^{-1} = \dfrac{1}{\left(\frac{1}{3}\right)^1} = 3$

or $\left(\dfrac{1}{3}\right) = \left(1 \times 3^{-1}\right)^{-1} = 1^{-1} \times 3^1$

$= \cancel{1} \times 3$

$= 3$

h) $\left(\dfrac{2}{3}\right)^{-2}$

$= \left(2^1 \times 3^{-1}\right)^{-2}$

$= 2^{-2} \times 3^2$

$= \dfrac{3^2}{2^2} = \dfrac{9}{4}$

Can now note

$\left(\dfrac{a}{b}\right)^{-n} = \left(\dfrac{b}{a}\right)^n$

5)

a) $\frac{1}{x^{-2}} = x^2$

b) $\left(\frac{1}{x}\right)^{-2} = \left(\frac{x}{1}\right)^2$

$= x^2$

c) $\left(\frac{y}{x}\right)^{-2} = \left(\frac{x}{y}\right)^2$

$= \frac{x^2}{y^2}$

d) $\left(\frac{y x^3}{x^2}\right)^{-2}$

simplify inside

$= \left(\frac{yx}{1}\right)^{-2}$

$= \left(\frac{1}{xy}\right)^2 = \frac{1}{x^2y^2}$

e) $\left(\frac{3x^4}{y}\right)^{-3}$

$= \left(\frac{y}{3x^4}\right)^3$

$= \frac{y^3}{3^3x^{12}} = \frac{y^3}{27x^{12}}$

f) $\left(\frac{6x^2}{3x^2y}\right)^{-1}$

$= \left(\frac{2}{y}\right)^{-1}$

$= \frac{y}{2}$