Find stationary and inflection points for
$$y = x^3(5-x)$$

$$y = 5x^3 - x^4$$
$$\frac{dy}{dx} = 15x^2 - 4x^3$$
$$15x^2 - 4x^3 = 0$$
$$x^2(15-4x) = 0$$
$$x = 0 \qquad x = \frac{15}{4} = 3.75$$

Stationary points at $(0,0)$ and $(3.75, 65.918)$

$$\frac{d^2y}{dx^2} = 30x - 12x^2$$
$$= 6x(5-2x)$$

If $x = 0$

$$\frac{d^2y}{dx^2} = 0$$

unknown at this stage.

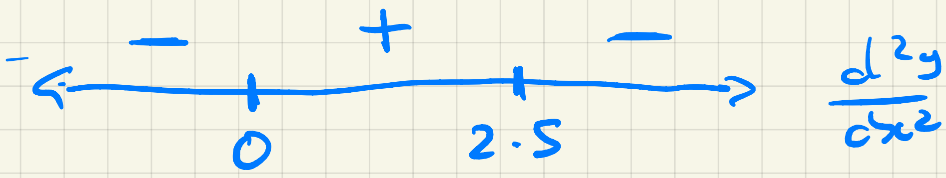
If $x = 3.75$

$$\frac{d^2y}{dx^2} = -56.25$$

$(3.75, 65.918)$ is a local max.

Sign diagram for $\frac{d^2y}{dx^2}$

$$6x(5-2x) = 0$$
$$x = 0 \qquad x = 2.5$$

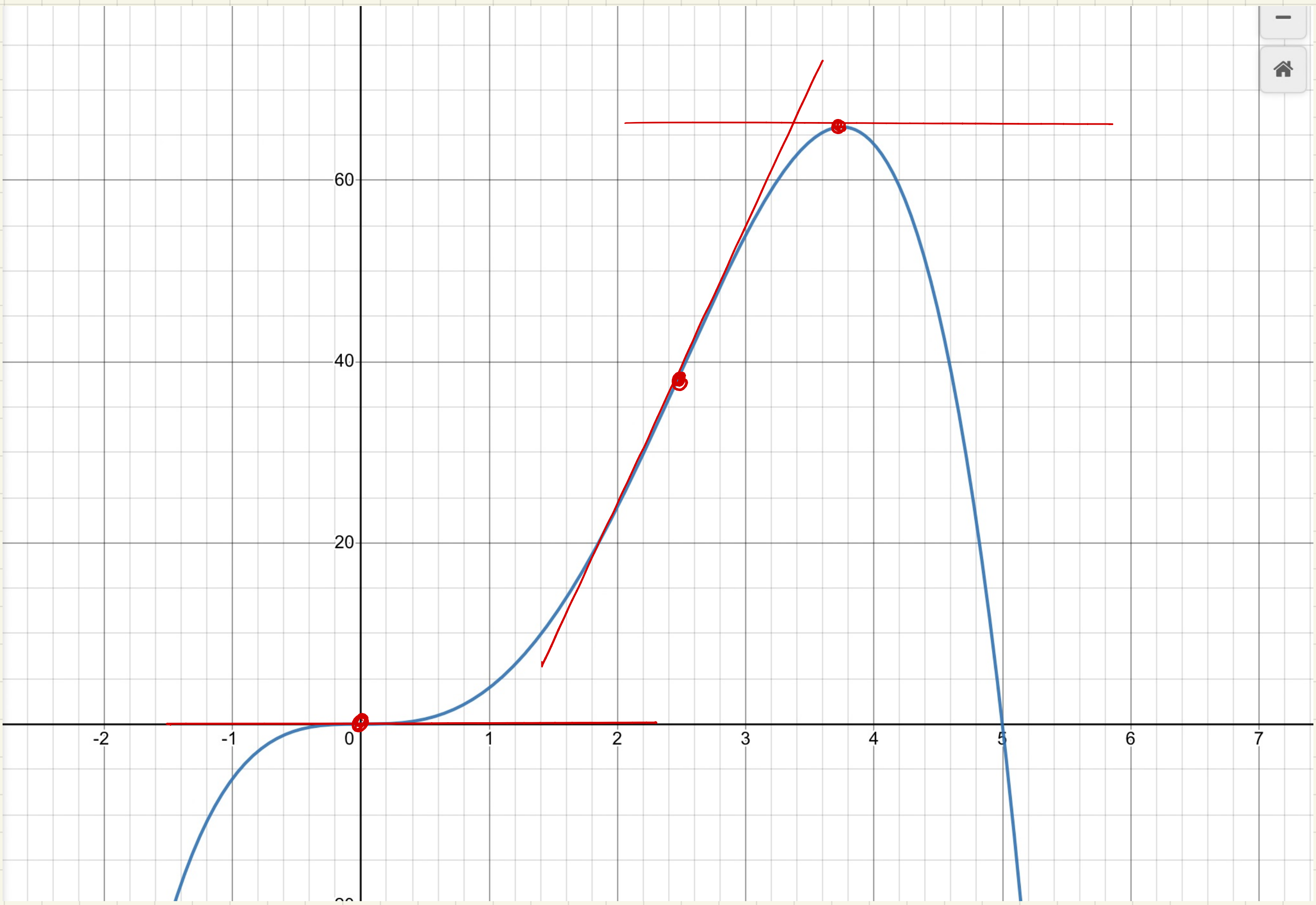


Change of sign at $x = 0$

So $(0,0)$ is a stationary inflection point

Change of sign at 2.5

so $(2.5, 39.0625)$ is a non-stationary inflection point



60
40
20
-2
-1
0
1
2
3
4
5
6
7

Find stationary and inflection points for:

$$f(x) = \frac{x^6}{30} + 6.4x$$

$$f'(x) = \frac{6x^5}{30} + 6.4 = \frac{x^5}{5} + 6.4$$

$$\frac{x^5}{5} + 6.4 = 0$$

$$x^5 = -32$$

$$x = \sqrt[5]{-32} = -2$$

Stationary point at $\left(-2, -10\frac{2}{3}\right)$

$$f''(x) = \frac{5x^4}{5} = x^4$$

Sign diagram not required.

$$f''(-2) = 16 > 0$$

Stationary point is a local minimum.

Set $f''(x) = 0$

$\Rightarrow x = 0$

could be an inflection point

but

+	0	+

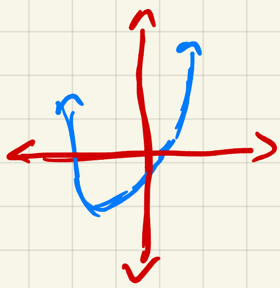
$f''(x)$

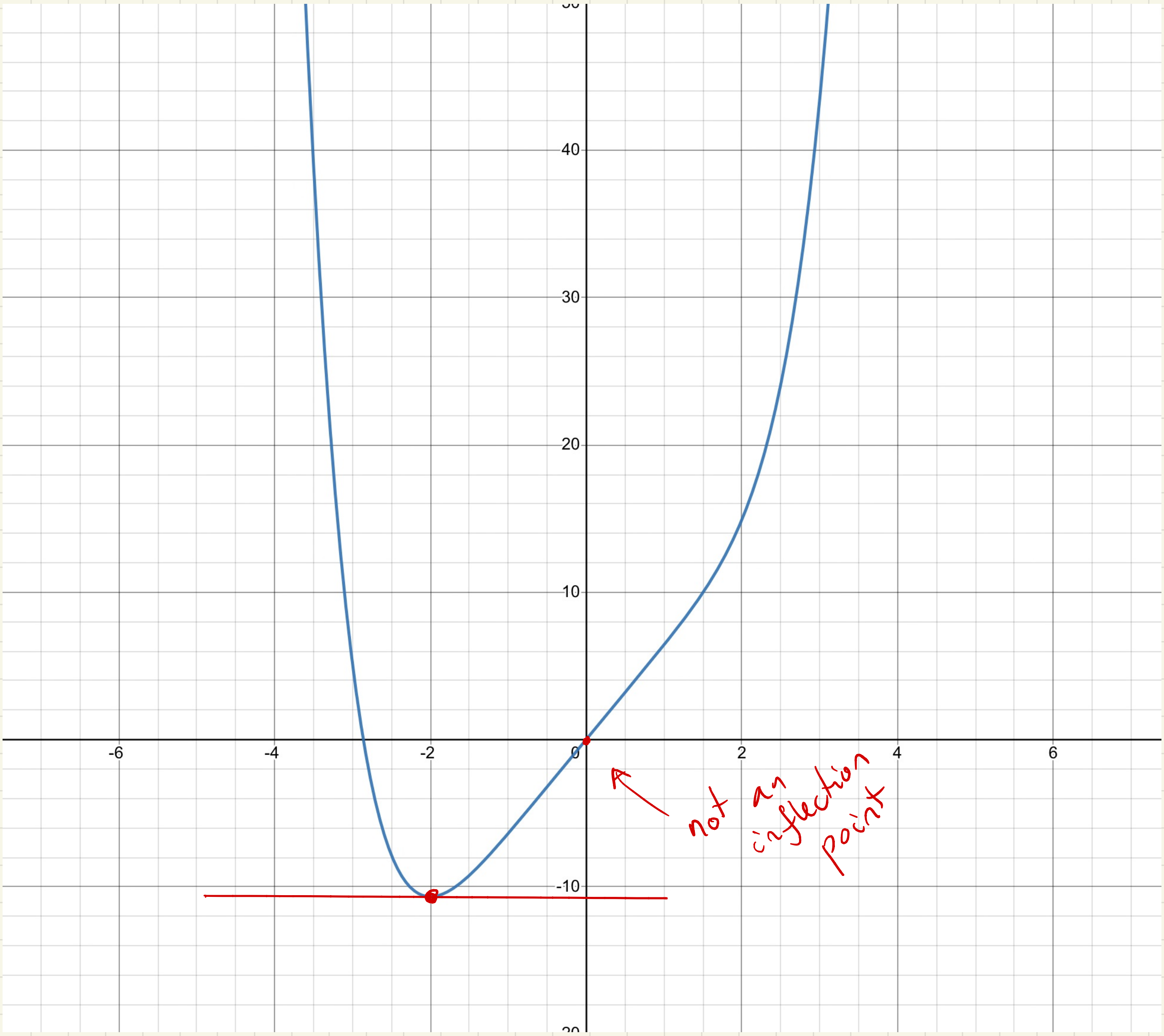
Does not change sign

So $(0,0)$ is <u>not</u> an inflection point.

A local minimum at $\left(-2, -10\frac{2}{3}\right)$

No inflection points.





40
30
20
10
-10
-6
-4
-2
0
2
4
6
not an inflection point