10. $\left(x^2 + \dfrac{1}{ax}\right)^{10} = \displaystyle\sum_{r=0}^{10} \binom{10}{r} (x^2)^{10-r} \cdot \left(\dfrac{1}{ax}\right)^r$

$\qquad\qquad\qquad = \displaystyle\sum_{r=0}^{10} \binom{10}{r} x^{20-2r} \cdot \dfrac{1}{a^r x^r}$

$\qquad\qquad\qquad = \displaystyle\sum_{r=0}^{10} \binom{10}{r} \dfrac{x^{20-3r}}{a^r}$

$x^{11} = x^{20-3r} \implies 20 - 3r = 11$
$\qquad\qquad\qquad\qquad\qquad r = 3$

x^{11} is the fourth term

$T_{3+1} = \binom{10}{3} \dfrac{x^{11}}{a^3}$

$\qquad = \dfrac{120 x^{11}}{a^3}$

$\dfrac{120}{a^3} = 15$

$\implies a^3 = \dfrac{120}{15} = 8$

$a = 2$

11. a) $(x+4)(x-3)^6$

$x \cdot x^3 = x^4$
$4 \cdot x^4 = 4x^4$

want x^3 and x^4 terms
from $(x-3)^6$

$T_{r+1} = \binom{6}{r} x^{6-r}(-3)^r$

$r = 2, \; r = 3$

$Ax^4 = x T_{3+1} + 4 T_{2+1}$

$\qquad = x \binom{6}{3} x^3 (-3)^3 + 4 \binom{6}{2} x^4 (-3)^2$

$\qquad = 20 \times -27 \, x^4 + 4 \times 15 \times 9 \, x^4$

$\qquad = -540 x^4 + 540 x^4$

$\qquad = 0 x^4$

12. $(1+kx)^n = 1 - 12x + 60x^2 - \ldots, \quad n \in \mathbb{Z}^+$

$(1+kx)^n = 1 + \binom{n}{1}kx + \binom{n}{2}k^2x^2 + \ldots$

$\binom{n}{1}k = -12$

$\dfrac{n!}{1!(n-1)!}k = -12$

$nk = -12$

$k = -\dfrac{12}{n}$

$\binom{n}{2}k^2 = 60$

$\dfrac{n!}{2!(n-2)!}k^2 = 60$

$\dfrac{n(n-1)k^2}{2} = 60$

$n(n-1)k^2 = 120$

$n(n-1)\dfrac{144}{n^2} = 120$

$\boxed{k = -\dfrac{12}{n}}$

$144n - 144 = 120n$

$24n = 144$

$n = 6$

and $\quad k = -2$

13.

								Sum
			1		1			2
		1		2		1		4
	1		3		3		1	8
		1		4		6	4	1	16
1		5		10		10		5	1	32
1		6		15		20	15	6	1	64

The sum of the numbers in row n
of Pascal's Triangle is 2^n

d) $(1+x)^n = \displaystyle\sum_{r=0}^{n} \binom{n}{r} 1^{n-r} x^r$

$= 1 + \binom{n}{1}x + \binom{n}{2}x^2 + \ldots + x^n$

$\Rightarrow (1+1)^n = 1 + \binom{n}{1} + \binom{n}{2} + \ldots + 1$

$= 2^n$

$(1 + -1)^n = 0^n = 0$

$= 1 - \binom{n}{1} + \binom{n}{2} - \binom{n}{3} + \ldots (-1)^n$

$= 0$

f) $\displaystyle\sum_{r=0}^{n} \binom{n}{r} 2^r = \sum_{r=0}^{n} 1^{n-r} 2^r = (1+2)^n$

$= 3^n$

14. $(3+x)^n = 3^n + \binom{n}{1}3^{n-1}x + \binom{n}{2}3^{n-2}x^2$

$$+ \binom{n}{3}3^{n-3}x^3 + \ldots + \binom{n}{n-1}3^1 x^{n-1}$$

$$+ x^n$$

$(3+1)^n = 3^n + \binom{n}{1}3^{n-1} + \binom{n}{2}3^{n-2} + \binom{n}{3}3^{n-3}$

$$+ 3n + 1$$

$$= 4^n$$