Name / Ingoa _____

Year 10 Pāngarau (Mathematics)

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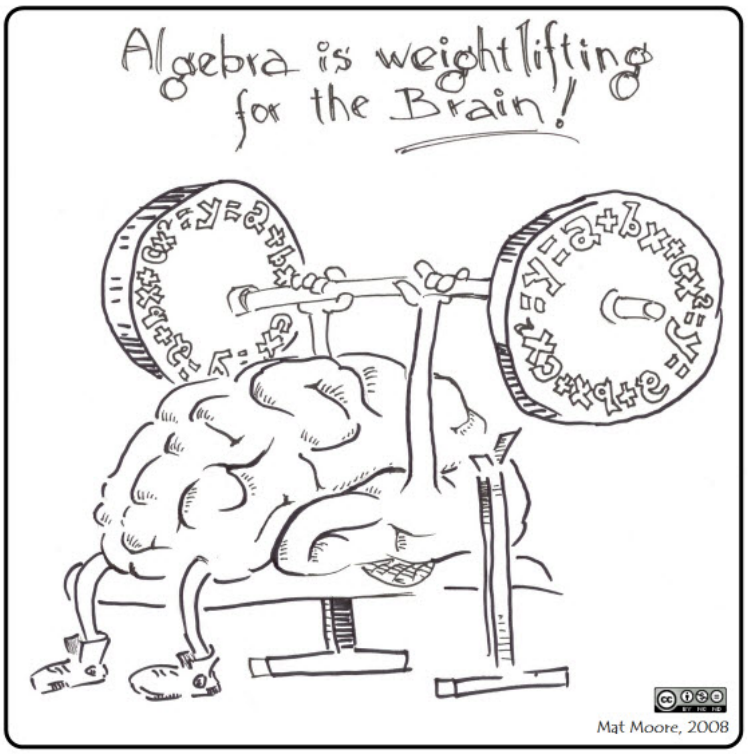
Algebra: Expressions and Equations



Contents

1 Substitution

1.0.1 Exercise

If $c = 5$ evaluate the following expressions

a) $c + c + c$

b) $4c$

c) c^2

d) c^3

e) $3c - 8$

f) $-4c^2$

1.0.2 Exercise

If $b = -3$ evaluate the following expressions

a) $2b$

b) $b + 9$

c) $-4b$

d) $b \times b$

e) b^2

f) b^3

g) b^4

h) $3b + 8$

1.0.3 Exercise

If $x = 7$ and $y = -2$ evaluate the following expressions

a) $x + y + x + y + x + y + x$

b) $4x + 3y$

c) $x + 4 + x$

d) $x^2 + 2y$

e) $x^2 + y + y + y + x^2$

f) $2x^2 + 3y$

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2 Adding and Subtracting with numbers

2.1 Addition

Addition is commutative.
The means that we can reorder an addition any way we like.
For example: $2 + 6 + 1 + 2 + 6 = 1 + 2 + 2 + 6 + 6$

For the questions below:

- Reorder the addition into order from smallest to largest numbers
- Evaluate each group of the same number
- Then complete the addition

Example: Re order and calculate: $9 + 2 + 3 + 9 + 9 + 2 + 9 + 3$

$$\begin{aligned} &= 2 + 2 + 3 + 3 + 9 + 9 + 9 \\ &= 2 \times 2 + 2 \times 3 + 3 \times 9 \\ &= 4 + 6 + 27 \\ &= 37 \end{aligned}$$

2.1.1 Exercise

Re order and calculate:

a) $4 + 7 + 7 + 2 + 7 + 4$

b) $12 + 6 + 4 + 12 + 4 + 6 + 4$

c) $11 + 2 + 11 + 2 + 11 + 2 + 2$

d) $13 + 10 + 3 + 10 + 13 + 3 + 3 + 3$

Example for the next set:
Re order and calculate: $4 + 2 \times 7 + 2 + 7 + 3 \times 4$

$$\begin{aligned} &= 2 + 4 + 3 \times 4 + 7 + 2 \times 7 \\ &= 2 + 4 \times 4 + 3 \times 7 \\ &= 4 + 16 + 21 \\ &= 41 \end{aligned}$$

2.1.2 Exercise

Re order and calculate:

a) $4+9+3\times 9+7+7+2\times 4$

b) $2\times 12+5+5+12+3\times 5+4$

c) $3\times 11+2+2\times 11+2+11+3\times 2+2$

d) $2\times 13+10+13+5\times 10+13+4+4$

2.2 Addition and Subtraction

We can do the reordering with subtractions as well , but we have to **think**.
The sign to the left of the number , moves with the number.

For example : $5-8=-8+5$ (there is an invisible + left of the 5)

For a longer sum, for example: $4-5+4+5+4-5$

- We can 'split' it as : $|\ +4|\ -5|\ +4|\ +5|\ +4|\ -5|$
- Then reorder: $+4+4+4+5-5-5=3\times 4-5=7$

2.2.1 Exercise

Re order and calculate:

a) $4-9+4-9+4+4$

b) $8-1-1+8-1+8$

c) $-7-2-2-7-7$

d) $-2\times 6-3\times 2-6-2$

e) $3\times 11+2-2\times 11+11-3\times 2$

f) $-2\times 13+5\times 10+13-2\times 10$

3 Adding and Subtracting Like Terms

When using algebra, the different letters represent (most likely) different numbers so we can re order and simplify algebra expressions , using the techniques used above.

Examples:

$$\begin{aligned} & x + y + x + y + y \\ & = x + x + y + y + y \\ & = 2x + 3y \end{aligned}$$

$$\begin{aligned} & 2y + x + 3y + x + x \\ & = x + x + x + 2y + 3y \\ & = 3x + 5y \end{aligned}$$

3.0.1 Exercise

Simplify the following expressions.

a) $x + 4 + x$

b) $y + y + y + y$

c) $x + y + x + x$

d) $2x + 3y + 4x + 5y + 3$

e) $5x + y + 3x + 6y$

f) $7y + y + x + 5x - 8$

Examples:

$$\begin{aligned} & -2x - 3x \\ & = -5x \end{aligned}$$

$$\begin{aligned} & 2y + 3x + 6y - 4x \\ & = 3x - 4x + 2y + 6y \\ & = -x + 8y \end{aligned}$$

3.0.2 Exercise

Simplify the following expressions.

a) $-7x - 4x$

b) $2y - 7x + 3y - 4x$

c) $2x - 3 + 3y - 4x + 5y + 1$

d) $-5x + y - 3x + 6y$

e) $-7y + y + x + 5x - 8$

f) $-8y + 3z - 4x - 8 + 5x - 7y$

If we have a power with an algebra letter (e.g. x^2) then we cannot 'add' this with the x terms (because it will evaluate to a different number than x).

This means that: $x + x + x^2 = 2x + x^2$

Or that $x^2 + y + 2y^2 + 2y + 3y^2 = x^2 + 3y + 5y^2$

3.0.3 Exercise

Simplify the following expressions.

a) $x^2 + x + x^2 + x^2 + x + x$

b) $y + y^2 + y^2 - 2y - 3y$

c) $3y^2 + 2y - 6y^2$

d) $2x^2 + 3y^2 - 4x + 5y^2$

e) $-5x^2 + y - 8x^2 + 6y$

f) $-7y^2 + y^2 + y + 5y - 3$

3.1 Mixed Questions

3.1.1 Exercise

Simplify the following expressions.

a) $2x + 4 + 5x$

b) $y + 9 - 6y + 2$

c) $x^2 + 2x + x^2 + 1$

d) $2x^2 - 5x + x^2 - x$

e) $-5x + y - 3x + x^2$

f) $x^2 - y^2 - x^2 + 2y^2$

3.1.2 Quiz

Farewell Words 2

Some accounts say that Te Arawa was the largest *waka*[1] to sail to Aotearoa from Hawaiki. War drove the *whanau*[2] onto the waka Te Arawa from Hawaiki. Tama Te Kapua was the *rangatira*[3] of the *waka*. Houmai, his elderly father, chose to stay on Hawaiki and he offered his son and *whanau* some good advice before the *waka* sailed to Aotearoa.
To discover some of Houmai's farewell words, simplify the algebraic expressions below. The letter beside each question and its answer will give the puzzle code.

a + a + a + a + a = A	5 + a + a = B	7 – (a + a) = D
2a + 5a = E	7a – 4a = F	9a – 11a = F
a + a + a + 9 = G	5a + 9 + 2a – 1 = H	
8a + 2 + a – 5 = H	5a + 4 – 2a – 2 = L	
9a – 2 – 12a – 1 = N	–3a – 4b – 2a – 2b = T	
2a + 3b + 4a + b = W	7a + 5b + 2a – b = O	
3a – 2 + 5a – 4 = L	2a + 5 + 4a + 2 = G	
a + 4b + 3b + 5a = R	2a + 7b – 5a + 2b = S	
– 7a + 5b + 7a – 5b = U	10a + 9b – 10a – 9b + 2 = Y	

7-2a	9a+4b	a	3a	9a+4b	8a-6	3a+2	9a+4b	6a+4b	b	6a+7b	9a+4b	-3a-3	3a+9	9a+4b

-5a-6b	7a+8	7a	2a	6a+7	9a+4b	7-2a	2a	9a+4b	-2a

9a-3	0	-3a+9b	5+2a	5a	-3a-3	7-2a	6a+7b	2

[1]*Waka* (canoe); [2]*Whanau* (people); [3]*Rangatira* (chief);

4 Multiplying Terms

4.1 Re-ordering multiplication

If we have the multiplication $2 \times 5 \times 3 \times 2 \times 3 \times 2 \times 3$ we can re-order the multiplication because multiplication is commutative.
So we can re-order the multiplication into groups of the same numbers: $2 \times 2 \times 2 \times 3 \times 3 \times 5$
For the questions below:

- re-order the multiplication into groups
- then evaluate each group
- then (use a calculator if you like) work out the value

For example: re order and calculate : $7 \times 2 \times 3 \times 3 \times 2 \times 7 \times 2$

$$\begin{aligned} &= 2 \times 2 \times 2 \times 3 \times 3 \times 7 \times 7 \\ &= 8 \times 9 \times 49 \\ &= 3528 \end{aligned}$$

4.1.1 Exercise

Re-order and calculate the following:

a) $5 \times 2 \times 5 \times 3 \times 5$

b) $7 \times 2 \times 2 \times 5 \times 7$

c) $5 \times 11 \times 2 \times 11 \times 2$

d) $13 \times 11 \times 2 \times 13 \times 11 \times 11 \times 2$

4.2 Multiplication and powers

We can write 3×3 as 3^2 (three to the power of 2), or $2 \times 2 \times 2 \times 2$ as 2^4 (two to the power of four).
For the questions below:

- re-order the multiplications and then
- write the expressions as multiplications of powers.

For example $2 \times 3 \times 5 \times 2 \times 2 \times 5 \times 2$

$$\begin{aligned} &= 2 \times 2 \times 2 \times 2 \times 3 \times 5 \times 5 \\ &= 2^3 \times 3^1 \times 5^2 \\ &= 600 \end{aligned}$$

4.2.1 Exercise

Write these as multiplications of powers , then calculate the answer:

a) $2 \times 5 \times 2 \times 2 \times 5 \times 2$

b) $7 \times 7 \times 7 \times 5 \times 2 \times 7 \times 5$

c) $3 \times 2 \times 3 \times 3 \times 5 \times 2$

d) $7 \times 3 \times 5 \times 3 \times 3 \times 3 \times 7$

4.3 Multiplying with algebra

We can now apply the same ideas when using algebra:

For example: $b \times a \times 3 \times a \times 5 \times a$

$$\begin{aligned} &= 3 \times 5 \times a \times a \times a \times b \\ &= 15 \times a^3 \times b \\ &= 15a^3b \end{aligned}$$

4.3.1 Exercise

Re-order and simplify the following expressions:

a) $2 \times x \times 3 \times y \times x$

b) $5 \times x \times 4 \times x$

c) $7 \times y \times 2 \times x \times x$

d) $3x \times 5x \times 2$

e) $6x \times 2x \times y$

f) $2 \times 5x \times 3y$

g) $3x^2 \times 2y^3$

4.4 Adding Powers

We can note now that :

$a^2 = a \times a$
and
$a^3 = a \times a \times a$

So: $a^2 \times a^3 = \underline{a \times a} \times \underline{a \times a \times a} = a^5$ (so the powers get <u>added</u>)

4.4.1 Exercise

Simplify the following:

a) $x^2 \times x$

b) $x \times x^3$

c) $y^2 \times y^3 \times x$

d) $x^2 \times y \times x^2 \times y$

e) $6x \times 2x \times y$

f) $2 \times 5x \times 3y \times x$

g) $3x^2 \times 2y^3$

h) $3x^2 \times 2y^3 \times 5 \times y$

4.5 Extension: powers of terms in brackets

4.5.1 Exercise

Work out the following powers (these will help you later on)

a) 2^2 e) 3^2 i) 4^3

b) 2^3 f) 3^3 j) 5^2

c) 2^4 g) 3^4 k) 5^3

d) 2^5 h) 4^2 l) 6^2

Example:

$$\begin{aligned}(5a^3)^2 &= 5a^3 \times 5a^3 \\ &= 5 \times 5 \times a^3 \times a^3 \\ &= 25a^6\end{aligned}$$

4.5.2 Exercise

Write these expressions without any brackets:

a) $(3a)^2$ d) $(4a^2)^3$

b) $(5x)^2$ e) $(a^2b)^3$

c) $(2a)^3$ f) $(3ab^3)^2$

Why Is A Lame Elephant Like Adding 10 And 3?

Find the simplest form for any expression below in the corresponding answer column. (Some of the expressions cannot be simplified.) The letter of the exercise goes in the box that contains the number of the answer. Keep working and you will get the answer to the title question.

(T) $x^2 \cdot x^4$	(8) $2x^7$	(W) $(4n^3t^2)(3n^2t^4)$	(28) $8n^5t^3$
(E) $x^3 \cdot x^7$	(2) $x^2 + x^5$	(D) $(^-2n^2t^5)(4nt)$	(9) $12n^5t^6$
(S) $x^2 \cdot x$	(11) x^6	(H) $(2n^4t^2)(nt^2)$	(21) $^-8n^2t^4$
(O) $2x^4 \cdot x^3$	(20) $6x^3$	(E) $(^-n^3t)(^-8n^2t^2)$	(10) $^-12n^7t^4$
(A) $3x^2 \cdot 2x$	(25) x^3	(N) $(4n^6t)(^-3nt^3)$	(5) $2nt^3$
(N) $x^2 \cdot y^3$	(17) x^2y^3	(T) $(t^2)(2nt)$	(18) $^-8n^3t^6$
(E) $x^2 + x^5$	(15) x^{10}	(R) $(^-n^2t)(8t^3)$ 21	(12) $2n^5t^4$
(U) $(3v^2)(4v^5)$	(26) $^-10v^5$	(E) $a^4 \cdot a^6$	(19) $6a^2b^4$
(O) $(^-2v^3)(5v^2)$	(16) $18v^2$	(N) $a^4 \cdot b^6$	(27) a^4b^6
(S) $(9v^4)(^-2v)$	(24) $2v^2 + 7v$	(D) $a^4 + a^6$	(23) $^-6a^3b^3$
(A) $(^-6v)(^-3v)$	(6) $^-18v^5$	(I) $(^-3ab^2)(2a^2b)$	(14) a^{10}
(R) $(2v^2)(7v)$	(4) $12v^7$	(P) $(3b)(^-2ab^3)$	(7) $a^4 + a^6$
(E) $2v^2 + 7v$	(1) $14v^2k$	(R) $(^-6a^2)(^-b)$	(3) $^-6ab^4$
(H) $(2v^2)(7k)$	(22) $14v^3$	(C) $(2a^2b)(3b^3)$	(13) $6a^2b$

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28

FIND A MATCH

Solve any equation in the top block and find the solution in the bottom block. Transfer the word from the top box to the corresponding bottom box. Keep working and you will get another joke.

①	$(u^2)^3$ AND	②	$(k^3)^4$ A	③	$(u^5)^3$ HER	④	$(^-k^2)^2$ BROTHER	⑤	$(2u^2)^2$ HE	⑥	$(^-6k^4)^2$ THE
⑦	$(^-3u^3)^3$ IS	⑧	$(7k)^2$ AND	⑨	$(^-4u^2)^3$ HER	⑩	$(^-k^2)^3$ A	⑪	$(2u^4)^5$ IF	⑫	$(^-2k)^5$ CANNOT
⑬	$(4u^3k)^2$ TOO	⑭	$(9u^2k^2)^2$ THAT	⑮	$(^-2u^2k^4)^3$ REASON	⑯	$(^-8u^5k^3)^2$ BE	⑰	$(5uk^6)^3$ FALLS	⑱	$(u^2k^3)^6$ TRIPS
⑲	$(^-3uk)^5$ HER	⑳	$(^-u^{10}k)^2$ CANNOT	㉑	$(^-2uk^4)^4$ HELP	㉒	$(10u^2k)^3$ BROTHER	㉓	$(^-2uk)^7$ LADY	㉔	$(^-uk)^9$ ASSIST

$32u^{20}$	k^{12}	$^-128u^7k^7$	$u^{12}k^{18}$	u^6	$125u^3k^{18}$
$36k^8$	$^-8u^6k^{12}$	u^{15}	$1000u^6k^3$	$^-32k^5$	$16u^4k^{16}$
$^-64u^6$	$^-27u^9$	$81u^4k^4$	$4u^4$	$u^{20}k^2$	$64u^{10}k^6$
$^-k^6$	k^4	$49k^2$	$^-u^9k^9$	$^-243u^5k^5$	$16u^6k^2$

DD-21

5 Dividing Terms

5.1 Cancelling with numbers

If we have a fraction and the numerator and denominator terms are all being multiplied, we can cancel terms (one for one) that appear on both the numerator and the denominator:
For example:

$$\frac{2 \times 3 \times 4}{3 \times 4} = \frac{2 \times \cancel{3} \times \cancel{4}}{\cancel{3} \times \cancel{4}} = \frac{4}{1} = 4$$

This is what we are doing when we are simplifying fractions:

$$\frac{20}{15} = \frac{4 \times 5}{3 \times 5} = \frac{4 \times \cancel{5}}{3 \times \cancel{5}} = \frac{4}{3}$$

5.1.1 Exercise

Simplify these fractions:

a) $\frac{20}{15}$ c) $\frac{4}{10}$ e) $\frac{30}{20}$

b) $\frac{6}{2}$ d) $\frac{2}{8}$ f) $\frac{16}{6}$

5.2 Cancelling with algebra

We can do the same thing with algebra as well

$$\begin{aligned} \frac{x^5}{x^2} &= \frac{x^2 \times x^3}{x^2} \\ &= \frac{\cancel{x^2} \times x^3}{\cancel{x^2}} \\ &= \frac{x^3}{1} = x^3 \end{aligned}$$

5.2.1 Exercise

Simplify these (algebraic) fractions:

a) $\frac{x^4}{x^3}$ c) $\frac{xy}{x^2y}$ e) $\frac{b^2}{b}$

b) $\frac{y}{y^2}$ d) $\frac{a^3}{ab^2}$ f) $\frac{b^2}{b^3}$

5.3 Bringing it together

We can now combine this to simplify fractions that contain both numbers and letters. Simplify:

$$\begin{aligned}\frac{12x^3y}{16x^2y} &= \frac{3\times 4\times x\times x^2\times y}{4\times 4\times x^2\times y}\\ &= \frac{3\times \cancel{4}\times x\times \cancel{x^2}\times \cancel{y}}{4\times \cancel{4}\times \cancel{x^2}\times \cancel{y}}\\ &= \frac{3\times x}{4\times 1}\\ &= \frac{3x}{4}\end{aligned}$$

5.3.1 Exercise

Simplify these fractions:

a) $\dfrac{20x^4}{15x^3}$

b) $\dfrac{6y}{2y^2}$

c) $\dfrac{4xy}{10x^2y}$

d) $\dfrac{2a^3}{8ab^2}$

e) $\dfrac{30b^2}{20b}$

f) $\dfrac{16b^2}{6b^3}$

5.3.2 Exercise

Simplify the fraction and the evaluate using the substitution given:

a) For $A = \dfrac{3b^2}{2b}$, find A if $b = 4$

b) For $B = \dfrac{18c^5}{12c^3}$, find B if $c = 5$

c) For $C = \dfrac{24d}{36d^3}$, find C if $d = 2$

d) For $D = \dfrac{18e^2f}{12ef^2}$, find D if $e = 1, f = -1$

e) For $G = \dfrac{27hj}{18h^2}$, find D if $h = 5, j = 2$

f) For $K = \dfrac{28l^2m^9}{26l^3m^5}$, find K if $l = 7, m = 2$

5.3.3 Quiz

Farewell Words 3

Some accounts say that Te Arawa was the largest *waka*[1] to sail to Aotearoa from Hawaiki. Tama Te Kapua was the *rangatira*[2] of the *waka*. Houmai, Tama Te Kapua's father was too old to sail the long journey to Aotearoa with his son on Te Arawa. He chose to stay behind in Hawaiki, where he died shortly after Te Arawa sailed. Before Te Arawa sailed, he had some wise words to share with his son.
To discover more of Houmai's farewell words to Tama Te Kapua and his *whanau*[3], simplify the expressions below using index notations. The letter beside each question and its answer will give the puzzle code.

$a^2 \times a^5 =$ A	$a^2 \times a^3 \times a^6 =$ E	$a^{12} \times a^3 =$ B
$2a^2 \times 4a^5 =$ E	$a^4 \times a^4 =$ A	$7a \times 2a =$ Y
$a^4 \times a =$ D	$2a \times 3a \times 4a =$ G	$a^5 \times a^4 =$ A
$5a^5 \times 4a^4 \times 3a^3 \times 2a^2 \times a =$ I	$\frac{a^5}{a^3} =$ L	$4a^5 \times 3a^7 =$ E
$a \times 5a^3 =$ E	$\frac{a^7}{a^3} =$ N	$\frac{a^7}{a} =$ O
$\frac{14a^2}{2a} =$ P	$\frac{24a^5}{3a^2} =$ R	$\frac{7a^3}{5a^2} =$ T
$\frac{6a^3}{24a} =$ U	$\frac{12a^5}{18a} =$ W	

a^{15}	$\frac{a^2}{4}$	$120a^{15}$	a^2	a^5	a	$\frac{a^2}{4}$	7a	a	a^7	a	$24a^3$	$8a^3$	a^{11}	a^8	$\frac{7a}{5}$	a	a^4	$8a^7$	$\frac{2a^4}{3}$

7a	$12a^{12}$	a^6	7a	a^2	$5a^4$	a	$120a^{15}$	a^4	a	$14a^2$	a^6	$\frac{a^2}{4}$	$8a^3$	a	a^4	a^{11}	$\frac{2a^4}{3}$	a	a^2	a^9	a^4	a^5

[1]*Waka* (canoe); [2]*Rangatira* (chief); [3]*Whanau* (people);

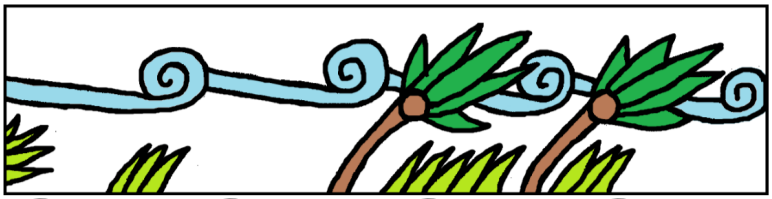
5.3.4 Quiz

Pungawere

Many *waka*[1] left Hawaiki at the time of the pungawere season to sail to Aotearoa. To find out what the pungawere season is, evaluate the expressions below. The letter beside each question and its answer will give the puzzle code.

Given x = 4 & y = 3.
Evaluate: $x + 2y =$

A



Given x = 2 & y = 5.
Evaluate: $2x - 3y =$

B

Given x = 4 & y = 5.
Evaluate: $3xy =$

D

Given x = 2 & y = –5.
Evaluate: $2(x - y) =$

E

Given x = 10 & y = 4.
Evaluate: $\frac{2x}{y} =$

F

Given x = 4 & y = 3.
Evaluate: $x^2 + y^2 =$

H

Given x = –3 & y = –1.
Evaluate: $(2x)^2 + y^2 =$

I

Given x = 2 & y = 4.
Evaluate: $(xy)^2 =$

L

Given x = –4 & y = –8.
Evaluate: $3x^2 + y =$

N

Given x = –2 & y = –3.
Evaluate: $2xy^2 =$

O

Given a = 2, b = 3 & c =5.
Evaluate: $abc =$

R

Given a = 3, b = 4 & c = –2
Evaluate: $2a + 3b - c =$

S

Given a = 5, b = –3
Evaluate: $2a - b =$

T

Given a = 5, b = 7
Evaluate: $a^2 - b^2 =$

U

Given a = 12, b = 5
Evaluate: $(a - b)^2 =$

V

Given a = 2, b = –8
Evaluate: $(a + b)^2 =$

W

13	25	14	19	20	14	10	20	-36	40	19	-36	5	19	13	25	14

5	10	49	-36	-24	30	10	-11	64	14	19	36	37	40	60

[1] *Waka* (canoe);

6 Multiplying Algebraic Fractions

When we **multiply** fractions, we multiply the numerators and multiply the denominators:

$$\frac{4}{5} \times \frac{2}{3} = \frac{4 \times 2}{5 \times 3} = \frac{8}{15}$$

$$\frac{2}{3} \times \frac{1}{2} = \frac{2 \times 1}{3 \times 2} = \frac{\cancel{2} \times 1}{3 \times \cancel{2}} = \frac{1}{3}$$

6.0.1 Exercise

Multiply and , where necessary, simplify these fractions:

a) $\frac{1}{2} \times \frac{1}{4}$

b) $\frac{3}{4} \times \frac{1}{3}$

c) $\frac{1}{12} \times \frac{6}{1}$

d) $\frac{7}{9} \times \frac{3}{5}$

e) $\frac{3}{8} \times \frac{1}{4}$

f) $\frac{2}{9} \times \frac{1}{2}$

Similarly for algebra

$$\frac{2a}{b} \times \frac{a^2}{b} = \frac{2a \times a^2}{b \times b} = \frac{2a^3}{b^2}$$

$$\frac{3a^2}{2b} \times \frac{ab^2}{a^2} = \frac{3a^2 \times ab^2}{2b \times a^3} = \frac{3a^3b^2}{2a^3b} = \frac{3}{2} \times \frac{\cancel{a^2}}{\cancel{a^2}} \times \frac{b \times \cancel{b}}{\cancel{b}} = \frac{3b}{2}$$

6.0.2 Exercise

Multiply and , where necessary, simplify these algebra fractions:

a) $\frac{3a^2}{4} \times \frac{1}{b}$

b) $\frac{2b^2}{a} \times \frac{b}{3}$

c) $\frac{12a}{b^2} \times \frac{a^3}{4b}$

d) $\frac{4a}{6b} \times \frac{a^3}{2b^2}$

e) $\frac{12a^2}{b^2} \times \frac{b}{2a^2}$

f) $\frac{4ab}{3} \times \frac{2a}{b^2}$

---

7 Adding Algebraic Fractions

The lowest common multiple of $4, 6$ is 12,
because 12 is a multiple of 4 and 12 is a multiple of 3, and it is the first number where this is the case.

7.0.1 Exercise

Find the lowest common multiple of the pair of numbers given below:

a) $2, 3$

b) $4, 5$

c) $6, 8$

d) $5, 15$

e) $4, 10$

f) $6, 9$

7.0.2 Exercise

Find these equivalent fractions:

a) $\frac{x}{3} = \frac{\square}{6}$

b) $\frac{y}{2} = \frac{\square}{6}$

c) $\frac{3x}{5} = \frac{\square}{20}$

d) $\frac{2y}{4} = \frac{\square}{20}$

e) $\frac{3x}{5} = \frac{\square}{35}$

f) $\frac{2y}{7} = \frac{\square}{35}$

7.0.3 Exercise

Add these fractions:

a) $\frac{x}{3} + \frac{x}{2}$
$= \frac{\square}{6} + \frac{\square}{6}$
$= \frac{\square}{6}$

b) $\frac{3x}{5} - \frac{2x}{4}$
$= \frac{\square}{20} - \frac{\square}{20}$
$= \frac{\square}{20}$

c) $\frac{3y}{5} - \frac{2y}{7}$
$= \frac{\square}{35} - \frac{\square}{35}$
$= \frac{\square}{35}$

7.0.4 Exercise

Add these fractions:

a) $\frac{x}{3}+\frac{y}{2}$

$=\frac{\square}{6}+\frac{\square}{6}$

$=\frac{\square+\square}{6}$

b) $\frac{3x}{5}-\frac{2y}{4}$

$=\frac{\square}{20}-\frac{\square}{20}$

$=\frac{\square-\square}{20}$

c) $\frac{3x}{5}+\frac{2y}{7}$

$=\frac{\square}{35}+\frac{\square}{35}$

$=\frac{\square+\square}{35}$

7.0.5 Exercise

Add these fractions:

a) $\frac{x}{6}+\frac{x}{4}$

b) $\frac{-8x}{3}+\frac{2x}{7}$

c) $\frac{11y}{2}-\frac{y}{8}$

d) $\frac{x}{3}+\frac{x}{2}$

e) $\frac{-3x}{5}-\frac{2x}{4}$

f) $\frac{-3y}{5}-\frac{-2y}{7}$

7.0.6 Exercise

Add these fractions:

a) $\frac{x}{5}+\frac{y}{2}$

b) $\frac{3y}{2}-\frac{8x}{3}$

c) $\frac{3x}{5}+\frac{2y}{7}$

d) $\frac{x}{5}+\frac{y}{2}+\frac{x}{1}$

e) $\frac{3y}{2}-\frac{8x}{3}-\frac{x}{2}$

f) $\frac{-3x}{5}-\frac{2y}{7}-\frac{-5y}{7}$

8 Linear Equations

8.1 Basic Linear Equations

Examples:

Solve: $x + 8 = 4$	Solve: $\frac{x}{5} = 3$
$x + 8 = 4$ $x + 8 - 8 = 4 - 8 \quad (-8)$ $x = -4$	$\frac{x}{5} = 3$ $\frac{x}{5} \times 5 = 3 \times 5 \quad (\times 3)$ $x = 15$
Solve: $x - 5 = 9$	Solve: $6x = 48$
$x - 5 = 9$ $x - 5 + 5 = 9 + 5 \quad (+5)$ $x = 14$	$6x = 48$ $\frac{6x}{6} = \frac{48}{6} \quad (\ /6)$ $x = 8$

8.1.1 Exercise

Find x:
Make sure you write at least one line of working:

a) $x + 6 = 30$

b) $x - 9 = 15$

c) $x + 8 = -4$

d) $x - 15 = -33$

e) $8x = 40$

f) $-12x = -132$

g) $9x = -117$

h) $-4x = 136$

i) $\frac{x}{9} = -14$

j) $\frac{-x}{5} = 13$

k) $x - 4 = 26$

l) $\frac{x}{8} = 13$

m) $4x - 13 = 15$

n) $-3x - 16 = 35$

o) $16 - 7x = 107$

p) $-38 + 5x = 22$

Never Say Die!

YOU MAY HAVE HEARD THAT OLD MATH TEACHERS NEVER DIE; THEY JUST REDUCE TO LOWEST TERMS. TO FIND OUT WHAT HAPPENS TO SOME OTHER OLD FOLKS, FOLLOW THESE DIRECTIONS:

The missing words in each sentence are written in code. Solve any equation below and find the solution in the code. Each time it appears, write the letter of that exercise above it. Keep working and you will discover the words to complete each sentence.

Old | $\frac{-4}{9}$ | $6\frac{2}{3}$ | $7\frac{1}{5}$ | $\frac{-1}{15}$ | $\frac{-4}{9}$ | $\frac{2}{5}$ | $3\frac{1}{3}$ | Never Die, They Just | $\frac{-1}{15}$ | $-4\frac{1}{3}$ | -4 | $\frac{2}{5}$ | $\frac{3}{5}$ | $1\frac{3}{4}$ | $\frac{2}{5}$ | $7\frac{1}{5}$ | -8 | $\frac{6}{7}$ | $7\frac{1}{5}$ | -10

Old | $1\frac{5}{7}$ | $1\frac{3}{4}$ | $\frac{2}{5}$ | $\frac{-4}{9}$ | $7\frac{1}{5}$ | -4 | $\frac{3}{5}$ | -4 | Never Die, They Just | $\frac{-3}{4}$ | $6\frac{2}{3}$ | $7\frac{1}{5}$ | $\frac{-1}{15}$ | $\frac{3}{5}$ | $-4\frac{1}{3}$ | -8 | $\frac{2}{5}$ | $6\frac{2}{3}$ | $1\frac{5}{7}$ | $\frac{3}{5}$

Old | $1\frac{5}{6}$ | $-5\frac{5}{7}$ | -8 | -20 | $\frac{-1}{15}$ | $6\frac{2}{3}$ | -8 | -4 | Never Die, They Just | -4 | $\frac{3}{5}$ | $\frac{2}{5}$ | $6\frac{2}{3}$ | $\frac{-1}{15}$ | $6\frac{2}{3}$ | $10\frac{1}{2}$ | $6\frac{2}{3}$ | $-2\frac{1}{2}$

Ⓗ $9x - 5x = 7$

Ⓔ $\frac{5}{4}x = \frac{1}{2}$

Ⓑ $3x = \frac{11}{2}$

Ⓩ $\frac{-3}{5} = \frac{-7}{10}x$

Ⓞ $4x - 7x = 13$

Ⓤ $\frac{-7}{2}w = 20$

Ⓟ $150 = -9x - 6x$

Ⓒ $\frac{7}{12}k = 1$

Ⓨ $-2u + 8u = -15$

Ⓖ $\frac{3}{5}t = -12$

Ⓘ $\frac{5}{6}y = 4 + 2$

Ⓐ $-11 - 4 = \frac{-9}{4}t$

Ⓣ $-4y - y = -3$

Ⓜ $8 = -18y$

Ⓕ $\frac{8}{3}s = -9 + 7$

Ⓛ $-10z = \frac{2}{3}$

Ⓦ $\frac{2}{3}m = 7$

Ⓝ $-30 = 3n - 12n$

Ⓡ $\frac{1}{12}m = \frac{-2}{3}$

Ⓢ $8y - 9y = 4$

8.2 Linear Equations with the unknown on both sides

8.2.1 Exercise

Find x:

a) $5x = 2x - 15$

b) $4x - 5 = -2x - 23$

c) $-5x + 3 = 4x - 69$

d) $-8x + 40 = -4x + 136$

e) $-5x + 5 = 3x - 43$

f) $-4x + 40 = 4x - 64$

g) $14x + 20 = 21x - 43$

h) $-3x - 15 = -5x + 73$

i) $6x - 4 = 2x + 6$

j) $-7x - 23 = -10x - 15$

k) $3x + 12 = 7x + 9$

l) $14 - 3x = -7x - 8$

8.3 Linear Equations with Words

8.3.1 Exercise

Form an equation involving 'x' for each question below and then solve it. Show your working.

a) Four times a number minus seven gives the same result as two times the same number plus fifteen.
What is the number?

b) Jason and his friend earn the same hourly rate for a job.
Jason works 15 hours while his friend works 12 hours, but gets an extra payment of $45.
What is Jason's and his work mate's hourly rate if their pay is the same for the job?

c) Tabitha can purchase seven ink cartridges from a store in town or order six cartridges online and pay a $15 courier charge.
If the cost is to be the same, what is the price of a single ink cartridge?

d) Steve and his neighbour get their cars repaired at the same garage.
Steve's account shows six hours labour and parts costing $125.
His neighbour's account shows three hours labour and $290 for parts.
If their accounts are for the same amount, what hourly labour rate does the garage charge?

e) The cost of hiring a motor home for seven days and paying $800 for insurance is exactly the same as a special deal of twelve days hireage with reduced insurance of $240.
What is the cost per day of hiring the motor home?

f) Two guests stay in the same hotel.
Guest A stays for 12 nights and spends an additional $490 on meals etc, while guest B stays for 6 nights and spends an additional $844 on meals etc.
If both guests end up paying the same amount for their respective stay, what is the hotels's room rate per night?

g) Eight times a number plus eleven gives the same result as four times the same number minus thirteen.
What is the number?

h) A boy cycles for three hours at a certain speed and then at double that speed for the next two hours.
If he travels a total of 126 km altogether, what is his initial speed in km/h?

8.3.2 Quiz

Books Never Written

Tragedy on the Cliff by 

Mystery of the Creaking Door by 

P.S. by 

ABOVE ARE THE TITLES OF THREE "BOOKS NEVER WRITTEN." TO DECODE THE NAMES OF THEIR AUTHORS, FOLLOW THESE DIRECTIONS:

Solve any problem below and find your answer in the code. Each time it appears, write the letter of that problem above it. Keep working and you will decode the names of all three authors.

L Eight less than 7 times a number is the same as 4 more than 3 times the number. Find the number.

V Four more than 6 times a number is the same as 9 times the number increased by 10. Find the number.

I Four times a number is the same as 14 less than twice the number. Find the number.

Y One more than 8 times a number is the same as 12 times the number decreased by 3. Find the number.

G Twelve less than a number is the same as 5 times the number increased by 4. Find the number.

D Twenty decreased by 2 times a number is the same as 10 less than 3 times the number. Find the number.

M Two more than a number is the same as 16 decreased by 6 times the number. Find the number.

R Four times a number decreased by 25 is the same as 9 times the number. Find the number.

T Twice a number plus 6 times the number is −72. Find the number.

H A number plus 5 more than 3 times the number is 37. Find the number.

A Eleven diminished by 5 times a number is the same as 4 times the number increased by 20. Find the number.

S Eight times a number is the same as 90 decreased by the number. Find the number.

U Nine more than 3 times a number is the same as 6 less than twice the number. Find the number.

O One increased by 7 times a number is the same as 5 times the number plus 25. Find the number.

E Twenty-eight decreased by 6 times a number is the same as the number. Find the number.

N Sixteen diminished by 8 times a number is the same as 5 diminished by 9 times the number. Find the number.

DD-47

8.4 Linear Equations with Brackets

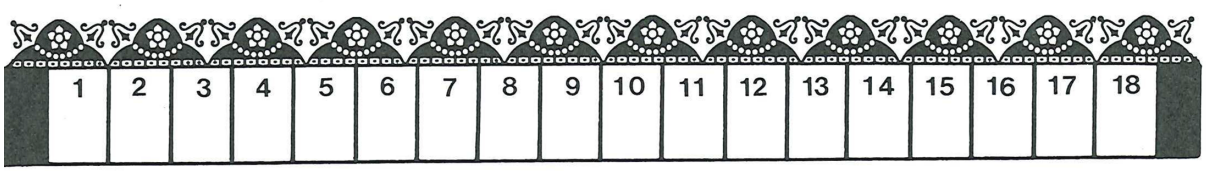
8.4.1 Quiz

What Happened To The Owl Who Swallowed A Watch?

TO ANSWER THIS QUESTION:

Simplify each expression below. Draw a staight line connecting each expression with its simplified form. Each line will cross a number and a letter. The number tells you where to put the letter in the row of boxes at the bottom of the page.

$^{-}5x - (7x + 2)$	■	■ $^{-}6x + 3$
$4 - (9x - 3)$	■	■ $^{-}x - 5$
$^{-}8x - (^{-}2x + 4)$	■	■ $^{-}2x - 2$
$^{-}(5 + 6x) + 8$	■	■ $6x - 3$
$^{-}4(^{-}9 + x) + 5x$	■	■ $^{-}12x - 2$
$6x - (3x - 7) + 1$	■	■ $8x + 6$
$^{-}9 + (2 - x) + 8x$	■	■ $4x - 12$
$(2x - 6) - (^{-}9x - 1)$	■	■ $^{-}9x + 7$
$^{-}(5x + 8) + 3 + 4x$	■	■ $x + 36$
$7x - (^{-}6 - 2x) - x$	■	■ $7x - 7$
$^{-}9 + x + 3\,(^{-}8 + 5x)$	■	■ $^{-}8x - 1$
$^{-}(6x - 6) + (4x - 8)$	■	■ $^{-}6x - 4$
$(3x + 1) - (^{-}3x - 7) - 9x$	■	■ $16x - 33$
$^{-}5 + (4 - 2x) - 8x + 5$	■	■ $^{-}10x + 4$
$^{-}(^{-}8x - 4) + x - 7 - 3x$	■	■ $3x + 8$
$(4 - 6x) - (^{-}4x + 5) - 6x$	■	■ $^{-}4x + 5$
$9x + 9 + 2x - 7(3 + x)$	■	■ $11x - 5$
$^{-}1 - (^{-}3x + 2) - 7x + 8$	■	■ $^{-}3x + 8$



DD-18

8.4.2 Exercise

Expand brackets and re-arrange, to find x

a) $4(x+7)=64$

b) $5(x-8)=40$

c) $7(x-2)=-42$

d) $6(x+4)=-54$

e) $4(3x+1)=76$

f) $-6(x-1)=-61$

g) $3(7+x)=20$

h) $7(2x-3)=-14$

i) $9(1-5x)=324$

j) $-3(2+5x)=99$

k) $2(10-x)=17$

l) $5(7-6x)=40$

m) $3(2x+5)+x=-20$

n) $2(1-4x)-2x=27$

o) $-4(2x-6)+3x=20$

p) $2(3x+5)-2(x+1)=20$

8.5 Linear Equations with Words

8.5.1 Exercise

Form an equation involving 'x' for each question below and then solve.

a) Jake thinks of a number and adds eight to it.
He then multiplies his answer by four and gets 68.
What was his original number?

b) Taylor thinks of a number and subtracts six from it.
She then multiplies it by seven and gets 98.
What was her original number?

c) Jacob has \$19 and Peter \$33.
How much must Jacob give Peter so that Peter has three times as much as Jacob?

d) Find the two consecutive numbers that when the smaller is multiplied by five and the large by three the total is 75?

e) Emily's father is 8 times as old as her.
In ten years' time Emily's father will be only three times older than her.
What are Emily and her father's present ages?

f) Find the two consecutive numbers that seven times the smaller is four less than six times the greater.

Why Did The Banana Go Out With The Prune?

TO ANSWER THIS QUESTION: Cross out each box that contains the solution of one of the equations. When you finish, write the letters in order from the boxes that are not crossed out in the boxes at the bottom of the page.

① $5(2x - 3) + 8 = 9$

② $4(9 + 3t) - 12 = -6$

③ $7y - 2(8y + 1) = 4$

④ $3 = 7(4 - 2u) - 6u$

⑤ $50 = 15 - 6(2x - 5)$

⑥ $3(-6x + 9) - 10x = 1$

⑦ $-9(8 - m) - 13 = 5$

⑧ $-8x + 6(3x + 5) = -25$

⑨ $12(4 + n) + 5(-2n - 9) = 18$

⑩ $-2 = -4(-7y + 1) + 5(8 + 2y)$

⑪ $18(-x - 2) - 4(-9 + 3x) = -14$

⑫ $3(6s + 12) - (10s - 6) = 0$

⑬ $-6(4x + 1) + 7x + 9(x - 3) = 4$

⑭ $10(-3 - 2t) + 10 - 2(6t - 13) = 0$

⑮ $-7 = -5y + 4(-y + 9) - 7(7 + 3y)$

⑯ $-(15p - 1) + 24 + 2(5 + 5p) = 0$

LO	BE	HE	CA	US	HA	EH	DT	EH	RO	VE	UB	IT	LE
$-\frac{2}{3}$	$\frac{3}{16}$	$2\frac{3}{11}$	$-\frac{1}{5}$	$\frac{7}{15}$	$-1\frac{5}{7}$	$-2\frac{1}{2}$	$-\frac{1}{13}$	$-4\frac{5}{8}$	$6\frac{1}{8}$	$1\frac{3}{5}$	-15	$-\frac{5}{12}$	$4\frac{1}{3}$
GE	ET	TT	RY	IN	ST	GA	ME	AN	DA	RK	FA	TE	LL
$-\frac{8}{17}$	-1	$11\frac{1}{2}$	$1\frac{1}{4}$	-4	10	$\frac{1}{6}$	$7\frac{1}{2}$	$\frac{13}{14}$	$-3\frac{11}{16}$	7	$-5\frac{1}{4}$	$2\frac{3}{7}$	$-5\frac{1}{2}$

8.6 Linear Equations with Fractions

8.6.1 Exercise

Solve these equations, for x:

a) $\dfrac{3x+1}{7} = 4$

b) $\dfrac{5x-3}{4} = 8$

c) $\dfrac{x}{3} + 14 = 2$

d) $\dfrac{2x}{3} - 12 = 40$

e) $\dfrac{x}{3} - \dfrac{x}{5} = 4$

f) $\dfrac{3x}{2} + \dfrac{x}{4} = 14$

g) $\dfrac{5x}{3} + 1 = \dfrac{x}{6} + 3$

h) $\dfrac{4x+3}{5} = \dfrac{x+3}{2}$

i) $\dfrac{x+6}{4} = \dfrac{x}{2}$

j) $1 - x = \dfrac{x+6}{9}$

8.7 Linear Equations with Words

8.7.1 Exercise

Form an equation involving 'x' for each question below and then solve. Show your working.

a) Sue thinks of a number and adds 55 to it then divides the result by three.
If the answer is four, what is the original number?

b) Leanne adds 42 to a number and then halves the result.
If the answer is four times the original number, find the number?

c) Half of John's age three years ago was fifteen.
How old is John now?

d) Half of Paul's age five years ago is equal to one quarter of what it will be in three years time.
What is Paul's present age?

e) One piece of timber beading is 9 cm longer than another and two-fifths of the longer piece is equal to half the shorter one.
Find the length of the longer piece of timber beading.

f) Tina buys 36 savouries for a morning tea shout at x cents each and 12 others which each cost 25 cents more.
If the morning tea shout costs her a total of $24.60, how much are each of the two different savouries?

g) A number divided by four and then subtracted from twenty-five gives the answer nine.
What is the number?

h) A number has 15 added to it and the result is multiplied by three to give 18.
What is the number?

i) A man has a large box of chocolates If he eats 12 a day they will last him 4 days longer than if he eats 14 a day.
How many chocolates in the box?

j) The sum of two-thirds of one number and three-fifths of the next consecutive number is 31.
Find the two numbers.

8.8 Extension: Solving with other algebra terms

For all questions in this section, **solve for** x

8.8.1 Exercise

i) $x - m = 9$

ii) $x + n = 0$

iii) $x - a + b = -5$

iv) $x - a = b - a$

v) $-n + x = -7$

vi) $-14 + x = a$

8.8.2 Exercise

i) $5x = c$

ii) $7x = 21b$

iii) $-3x = b + 1$

iv) $6x = 4b$

v) $2x = 6a + 4b$

vi) $-5x = 15a - 20c$

8.8.3 Exercise

i) $\frac{x}{4} = a$

ii) $\frac{x}{-3} = 5a$

iii) $\frac{x}{b} = 6$

iv) $\frac{x}{c} = a + b$

v) $\frac{x}{-2c} = a$

vi) $\frac{1}{4}x = 4a$

8.8.4 Exercise

i) $\frac{7x}{4} = 35a$

ii) $\frac{4x}{-3} = 16a$

iii) $\frac{4}{b}x = 100$

iv) $\frac{5x}{a} = 5 + b$

v) $\frac{-5}{3x} = m$

vi) $-\frac{3}{4}x = a + b$

8.8.5 Exercise

i) $2x + 1 = c$

ii) $-3x + 5 = 2b$

iii) $c + 3x = 10c$

iv) $a - 6x = b$

v) $7 + 2x = 6a + 4$

vi) $5 - 5x = 15a - 20$

8.8.6 Exercise

i) $2x + 1 = x + c$

ii) $5 - 3x = 2b - x$

iii) $c + 3x = 10c - 6x$

iv) $4a - 6x = 10x + 8a$

v) $4a + 2x = -6x + 4c$

vi) $ax + b = cx + d$

9 Expanding single brackets

9.0.1 Exercise

Expand the following

a) $4(x–1)$

b) $2(a + 2b–c)$

c) $x(x + 5)$

d) $2(3y–5x)$

---

10 Factorising to single brackets

Algebraic expressions can have common factors as well.

For example:

$$HCF(x, x^2) = x$$
$$HCF(x^3, x^2) = x^2$$
$$HCF(y, y^2) = y$$

Examples.

1. Factorise $6x^2 + 15x$.

$$\begin{aligned} 6x^2 + 15x &= 3x \times 2x + 3x \times 5 \\ &\quad \textit{3 is the HCF of 6 and 15} \\ &\quad \textit{x is the HCF of x and } x^2 \\ 6x^2 + 15x &= 3x(2x + 5) \end{aligned}$$

   It is generally easiest to look at each letter separately if there is more than one.

2. Factorise $18a^2y + 24a^3y^2$.

$$\begin{aligned} 18a^2y + 24a^3y^2 &= 6a^2y \times 3 + 6a^2y \times 4ay \\ &\quad \textit{6 is the HCF of 18 and 24} \\ &\quad a^2 \textit{ is the HCF of } a^2 \textit{ and } a^3 \\ &\quad y \textit{ is the HCF of } y^2 \textit{ and } y \\ 18a^2y + 24a^3y^2 &= 6a^2y(3 + 4ay) \end{aligned}$$

10.0.1 Exercise

Factorise the following

a) $3a–4ab$

b) $5xy + 25y$

c) $6x–12y$

d) $4pq–12q^2$

e) $b^2–a^2b^2$

f) $x^2 + 4x$

g) $18x–30$

h) $12x^2–6x$

i) $4p^2q–12pq^2 + 16pq$

j) $16g + 14gh^2$

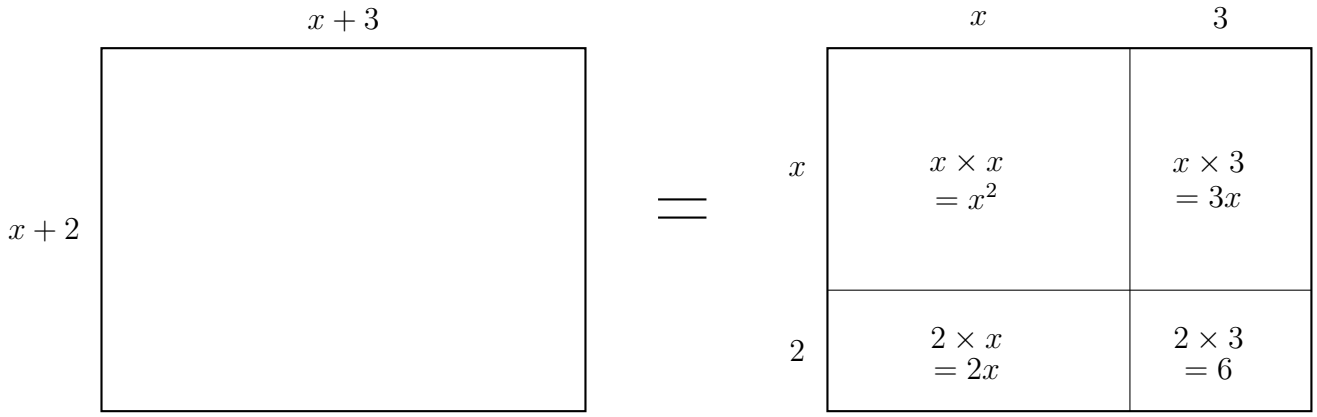
k) $36q^5 + 24q^3$

l) $10ab^3 + 15a^2b^2$

11 Expanding double brackets

Example: how do we expand $(x + 3)(x + 2)$?

Let's look at this as an **area problem**:

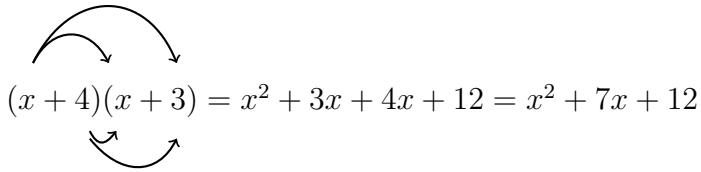


The area of the biggest rectangle is the sum of the areas of the smaller rectangles:

$$(x + 2)(x + 3) = x^2 + 3x + 2x + 6 = x^2 + 5x + 6$$

We can see that each term in the first bracket (with its sign) is multiplied against each term in the second bracket (with its sign).

For example:



11.0.1 Exercise

Expand and simplify

a) $(x+3)(x-1)$

b) $(x+7)(x-4)$

c) $(x-4)(x+2)$

d) $(x-3)(x-6)$

11.0.2 Exercise

Expand and simplify

a) $(x-3)(5-x)$

b) $(2-x)(4+x)$

11.0.3 Exercise

Expand and simplify

a) $(2x+3)(x-1)$

b) $(4x+7)(2x+3)$

c) $(7x-1)(2x-1)$

d) $(1-4x)(1-5x)$

11.0.4 Exercise

Expand and simplify

a) $(x+3)(x+3)$

b) $(x+5)^2$

c) $(x-1)^2$

d) $(x-2)^2$

11.0.5 Exercise

Expand and simplify

a) $(x-1)(x+1)$

b) $(x-5)(x+5)$

c) $(2x-1)(2x+1)$

d) $(3x+7)(3x-7)$

11.0.6 Exercise

Extension : Expand and simplify

a) $(x-1)(x^2+x+1)$

b) $(x+y-5)(x+y+5)$

c) $(x-y^3+1)(x-y^3-1)$

d) $(3x-2y^5-7)(3x-2y^5+7)$

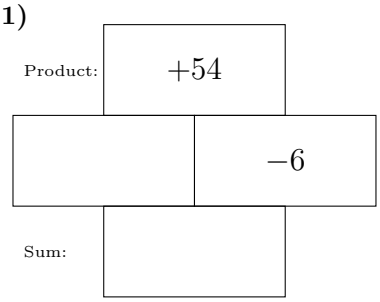
12 Factorising to double brackets (quadratic factorising)

To help us develop techniques to factorise quadratics, it is useful to do the number activity below.

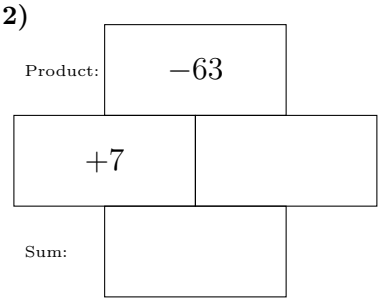
12.1 Sum Products

The top box is the product of the two middle boxes and the bottom box is the sum.
Find the empty boxes.

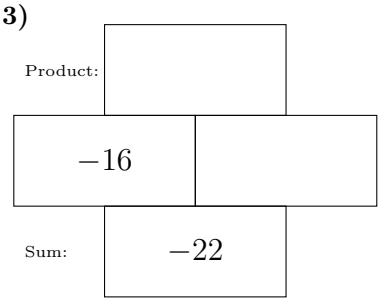
1)



2)

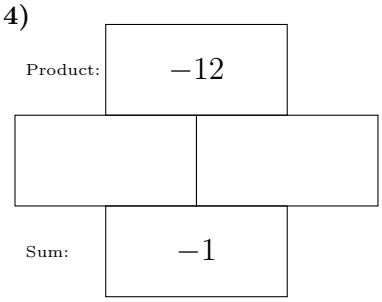


3)

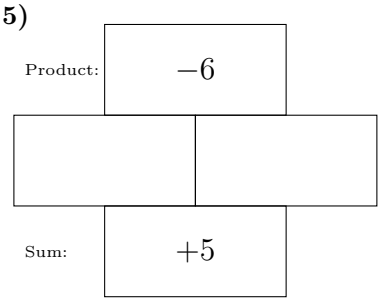


The ones below can go either way round in the boxes

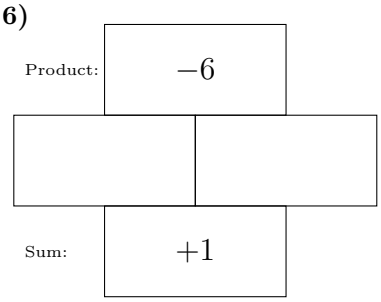
4)



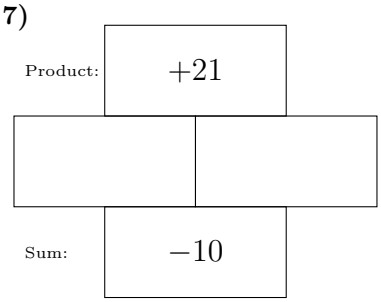
5)



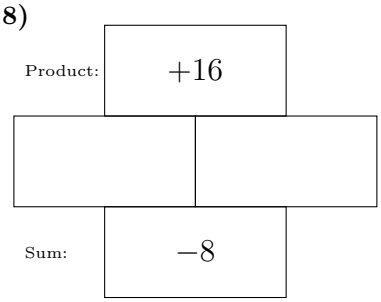
6)



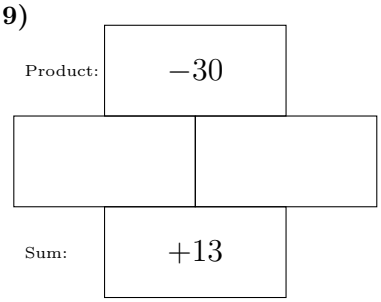
7)



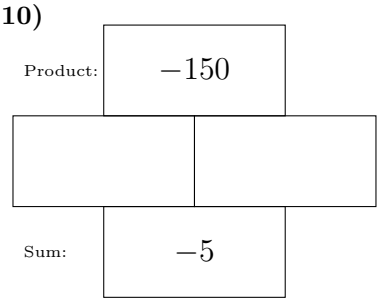
8)



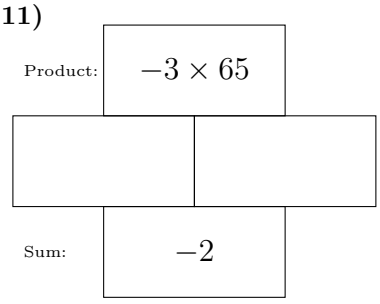
9)



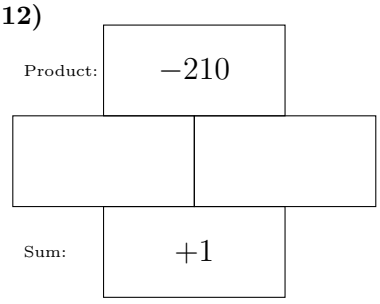
10)



11)



12)



12.2 Factorising

So for an expression of the same structure, e.g $x^2 + 10x + 21$
The a and b are two numbers that:

- *Add to=10*
- *Multiply to=21*

Finding the factors of 21 , we get 3 and 7.

$$x^2 + 10x + 21 = (x + 3)(x + 7)$$

12.2.1 Exercise

Factorise the following:

a) $x^2 + 7x + 6$

b) $x^2 + 12x + 32$

c) $x^2 - 5x + 6$

d) $x^2 + 3x - 10$

e) $x^2 + 8x - 20$

f) $x^2 + 11x - 26$

g) $x^2 - 10x + 21$

h) $x^2 - 3x + 2$

i) $x^2 - 14x + 48$

j) $x^2 - 5x - 84$

k) $x^2 - 6x - 16$

l) $x^2 - 4x - 32$

m) $x^2 - 30 + x$

n) $x^2 - 30 - x$

o) $x^2 - 9x$

p) $x^2 - 22x + 40$

q) $x^2 + 12x + 27$

r) $x^2 - 13x + 36$

s) $x^2 - 18 - 3x$

t) $30x + x^2 + 225$

u) $2x - 99 + x^2$

---

13 Solving Quadratic Equations

13.1 Using factorised quadratics

Consider the following equation:

$$(x-6)(x+3)=0$$

To solve this equation, we do not expand the brackets.
we can note that the equation is of the form:

$$A \times B = 0 \qquad \text{where } A=(x-6) \text{ and } B=(x+3)$$

And we can also note that if $A \times B = 0$, then either $A = 0$ or $B = 0$

So $(x-6)(x+3)=0$ has **two** solutions.

$$\begin{aligned} (x-6)(x+3) &= 0 \\ x-6=0 \quad &\text{or} \quad x+3=0 \\ \Rightarrow \quad x=6 \quad &\text{or} \quad x=-3 \end{aligned}$$

13.1.1 Exercise

Solve the following:

a) $(x-1)(x+7)=0$

b) $(x-5)(x+5)=0$

c) $(x+11)(x+9)=0$

d) $(x-3)(x-15)=0$

13.1.2 Exercise

Solve the following:

a) $(3x-15)(2x+8)=0$

b) $(2x-8)(4x+1)=0$

c) $(8x+10)(2x+9)=0$

d) $(2x-7)(2x-15)=0$

13.1.3 Exercise

Solve the following:

a) $3x(x+8)=0$

b) $x(x+4)=0$

c) $-3x(2x+9)=0$

d) $x(2x-15)=0$

13.2 Using un-factorised quadratics

To solve the equation:

$$x^2 + 3x - 88 = 0$$

$$\begin{aligned} & x^2 + 3x - 88 = 0 \\ \textbf{factorise} \qquad & (x-8)(x+11) = 0 \\ & x + 8 = 0 \quad \text{or} \quad x + 11 = 0 \\ & x = -8 \quad \text{or} \quad x = -11 \end{aligned}$$

13.2.1 Exercise

a) $x^2 + 7x + 6 = 0$

b) $x^2 + 12x + 32 = 0$

c) $x^2 - 5x + 6 = 0$

d) $x^2 + 3x - 10 = 0$

13.2.2 Exercise

a) $x^2 + 6x = 0$

b) $x^2 - 5x = 0$

c) $x^2 - 18 - 3x = 0$

d) $2x - 99 + x^2 = 0$

13.2.3 Exercise

a) $x^2 + 0x - 9 = 0$

b) $x^2 - 0x - 36 = 0$

c) $x^2 - 25 = 0$

d) $x^2 - 49 = 0$

13.2.4 Exercise

a) $x^2 - 6x + 9 = 0$

b) $x^2 + 10x + 25 = 0$

c) $x^2 - 2x + 1 = 0$

d) $x^2 + 2x + 1 = 0$

13.3 Re-arranging into standard form

If we get an equation with an x^2 as its highest power, we need to re-organise it into **standard form**

$$x^2 + bx + c = 0$$

Once this is done, we can factorise and solve.

(Example) Solve:

$$3 = 4x - x^2$$

rearrange	$3 = 4x - x^2$
	$3 + x^2 = 4x$
	$3 + x^2 - 4x = 0$
standard form	$x^2 - 4x + 3 = 0$
factorise	$(x - 3)(x - 1) = 0$
	$x - 3 = 0$ or $x - 1 = 0$
	$x = 3$ or $x = 1$

13.3.1 Exercise

a) $-26 = 11x - x^2$

b) $18x + x^2 = -45$

c) $42 = x^2 - 11x$

d) $x^2 + 225 = 30x$

13.3.2 Exercise

a) $x(x - 2) = 15$

b) $(x - 3)^2 = 25$

c) $(x - 4)(x - 6) = 8$

d) $(x - 3)(x + 2) = 4x$

e) $(x + 2)(x + 7) = (x + 5)(x + 7)$

f) $x + 3 = \dfrac{12}{x + 4}$

13.4 Word questions

13.4.1 Exercise

a) When a number is squared, and then 1 is subtracted, the result is 8.

   i) Write this information down as a quadratic equation.

   ii) Solve the equation to work out the two possible numbers.

b) A number is squared, and then added to the original number. The result is 20.

   i) Write this information down as a quadratic equation.

   ii) Solve the equation to work out the two possible numbers.

c) When a number x, is multiplied by a number 4 less than x, the result is 12.
What two numbers have this property?

d) Squaring a number gives the same result as multiplying the number by 8, and then subtracting 12.

   i) Write this information down as a quadratic equation.

   ii) Solve the equation to work out the two possible numbers.

e) When the result of adding 2 to a number is multiplied by the result of subtracting 2 from the same number, the answer is 21.
What two numbers have this property?

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14 Revision

1. Simplify

   a) $4x + x$

   b) $10x + y - 4 - 5x$

   c) $4x^2 + x - 2x^2 + 3x$

   d) $2abc + 4a - 5b + c + abc - 2a + 3b$

   e) $\dfrac{32x^6}{24x^2}$

   f) $\dfrac{4x^2y^3}{2xy^4}$

   g) $(2x^2)^3$

   h) $(4x^2y^3)^2$

2. Expand

   a) $3x(x + 2)$

   b) $5(2p - q)$

   c) $x(x - 3) + 8(x - 1)$

   d) $x(2x - 3) - 2(x + 2)$

   e) $(8x - 3)(x + 1)$

   f) $(2x + 3)(5 - 3x)$

   g) $(2x - 1)(2x + 1)$

3. Factorise (one bracket)

   a) $4a + 4b$

   b) $5x + 20y$

   c) $7x - 3x^2$

   d) $24xy^2z - 18x^3yz$

4. Factorise (two brackets)

   a) $x^2 + 8x + 15$

   b) $x^2 - 2x - 15$

   c) $x^2 - 15x + 56$

   d) $x^2 + 5x - 36$

5. Factorise (two brackets)

   a) $x^2 - 16$

   b) $x^2 - 121$

   c) $x^2 - 1$

6. Solve (Linear equations)

   a) $x + 3 = 14$

   b) $\dfrac{x}{5} = -2$

   c) $\dfrac{x - 4}{6} = -3$

   d) $12 - 3x = 36 + 5x$

7. Solve (Quadratic equations)

   a) $(x - 6)(x - 3) = 0$

   b) $3x(x + 2) = 0$

   c) $x^2 + 8x - 5 = 4$

   d) $2x(4 - x) = 6$

   e) $(x - 3)(x + 2) = 4x$

15 Practice Test

1. Simplify

   a) $2x^5 + 3x - 6x + 5x^5 + 2$

   b) $3pq + 5qp + 7pr - 2qr$

   c) $8x^2 \times 4x^4$

   d) $2abc \times 3a^2b \times 5a^2c^6$

   e) $(6t^4)^3$

   f) $\dfrac{125d^{10}e^4}{25d^5e^6}$

   g) $\dfrac{32j^{11}k^8}{24j^{14}k^{10}l}$

2. Expand and simplify if possible

   a) $3(2x + 9)$

   b) $4x(8y - 3) - 2(4x + 7)$

   c) $(x + 5)(x - 3)$

   d) $(x - 8)^2$

3. Factorise

   a) $14c + 21d$

   b) $36ab^2c^5 - 18b^4c^3$

   c) $x^2 + 10x + 21$

   d) $x^2 - 5x - 14$

   e) $x^2 - 4x - 165$

   f) $3x^2 - 75$

4. Solve

   a) $4x - 7 = 21$

   b) $\dfrac{25x - 5}{8} = 15$

   c) $5(x - 11) = 35$

   d) $2x + 3 = 6x - 33$

   e) $(x - 3)(x + 5) = 0$

   f) $x^2 + 8x - 33 = 0$

   g) $x^2 - 23x + 102 = 0$

5. The sum of two consecutive odd numbers is 88. Form an equation and use it to find the numbers.

6. A piece of land is a rectangle and the length is eight metres longer than twice its width.

   (a) Draw a diagram

   (b) Find expressions for the area and the perimeter.

   (c) The area of the land is $640m^2$, find the length and width of the land.

   (d) Find the perimeter of the land.