

Extension activities

Power of zero and negative powers

Evaluate the powers given to gain an intuitive understanding of what a power of zero means and what a negative power means.

2^4	2^3	2^2	2^1	2^0	2^{-1}	2^{-2}	2^{-3}

3^4	3^3	3^2	3^1	3^0	3^{-1}	3^{-2}	3^{-3}

From this we can deduce some rules for exponents:

$$x^0 = 1$$

$$x^{-n} = \frac{1}{x^n}$$

we also have:

$$x^m \times x^n = x^{m+n}$$

$$(x^m)^n = x^{mn}$$

and can show:

$$\begin{aligned}\frac{x^m}{x^n} &= \frac{x^m}{1} \times \frac{1}{x^n} \\ &= x^m \times x^{-n} = x^{m-n}\end{aligned}$$

Questions

1) Simplify

a) $(2x)^0$

c) $(-5)^0$

b) $3x^0$

d) $x \times 4x^0$

2) Simplify and write with positive exponents:

a) x^{-2}

d) $x^4 \times x^{-5}$

g) $4y^{-2} \times 3y^{-1}$

b) $3x^{-2}$

e) $y \times y^{-5}$

h) $7x^{-1} \times y^{-3}$

c) $x^3 \times x^{-2}$

f) $3x^{-1} \times y$

i) $3x^2y \times 2xy^{-3}$

3) Simplify and write with positive exponents:

a) $(3x)^{-2}$

d) $(3x)^{-2} \times x^{-2}$

g) $4^2x^{-3} \times 4^{-1}xy$

b) $(3x)^{-2} \times x^4$

e) $(4xy)^2 \times (xy)^{-3}$

h) $3x^{-1} \times (y^{-2})^{-1}$

c) $(7x)^2 \times x^{-2}$

f) $9x^{-2} \times (3x)^{-1}$

i) $(y^{-2})^{-3} \times x^{-3}$

4) Evaluate these fractions:

a) $\frac{1}{\frac{1}{2}}$

c) $\frac{1}{\frac{2}{3}}$

e) $\frac{1}{2^{-1}}$

g) $\left(\frac{1}{3}\right)^{-1}$

b) $\frac{3}{\frac{1}{2}}$

d) $\frac{\frac{3}{2}}{\frac{2}{3}}$

f) $\frac{1}{2^{-2}}$

h) $\left(\frac{2}{3}\right)^{-2}$

5) Simplify and write with positive exponents:

a) $\frac{1}{x^{-2}}$

c) $\left(\frac{y}{x}\right)^{-2}$

e) $\left(\frac{3x^4}{y}\right)^{-3}$

b) $\left(\frac{1}{x}\right)^{-2}$

d) $\left(\frac{yx^3}{x^2}\right)^{-2}$

f) $\left(\frac{6x^2}{3x^2y}\right)^{-1}$

Fractional powers

The square root of the number x is the (positive) number y such that $y^2 = x$.

For example: $\sqrt{25} = 5$ because $5^2 = 25$.

Please note that $-5 \times -5 = 25$ but the square root function $\sqrt{25}$ only returns a positive value.

1) Evaluate the following

a) $\sqrt{4}$

e) $\sqrt{\frac{121}{169}}$

i) $\sqrt{a^2 b^6}$

b) $\sqrt{64}$

f) $\sqrt{a^2}$

j) $\sqrt{\frac{9x^4}{y^8}}$

c) $\sqrt{49}$

g) $\sqrt{a^4}$

k) $\sqrt{\frac{16x^3y^2}{xy^4}}$

d) $\sqrt{\frac{25}{9}}$

h) $\sqrt{a^6}$

2) Evaluate the following

a) $\sqrt{4} \times \sqrt{4}$

c) $(\sqrt{a^2})^3$

e) $\sqrt{xy} \times \sqrt{xy}$

b) $\sqrt{a} \times \sqrt{a}$

d) $(3b\sqrt{a})^2$

f) $(4\sqrt{a^2})^3$

We can now see that:

$$\begin{aligned}\sqrt{x} \times \sqrt{x} &= x \\ x^{\frac{1}{2}} \times x^{\frac{1}{2}} &= x^1 = x\end{aligned}$$

Which tells us that:

$$x^{\frac{1}{2}} = \sqrt{x}$$

And it is probably good to note that the addition, subtraction and power rules still hold:

$$x^{\frac{1}{2}} \times x^2 = x^{\frac{1}{2}+2} = x^{\frac{5}{2}}$$

$$\frac{x^{\frac{7}{2}}}{x^{\frac{1}{2}}} = x^{\frac{7}{2}-\frac{1}{2}} = x^3$$

$$(9b)^{\frac{1}{2}} = 9^{\frac{1}{2}} \times b^{\frac{1}{2}} = 3\sqrt{b}$$

3) Simplify:

a) $(16b^4)^{\frac{1}{2}}$

c) $\sqrt{100a^2b^{10}}$

e) $\left(\frac{108x^2y^6}{3x^4}\right)^{\frac{1}{2}}$

b) $\left(81^{\frac{1}{2}}b^4\right)^{\frac{1}{2}}$

d) $\left(\frac{1}{16x^4}\right)^{\frac{1}{2}}$

f) $\sqrt{\frac{256y^6}{x^{12}}}$

Looking at the power rule we can note:

$$9^{\frac{3}{2}} = \left(9^{\frac{1}{2}}\right)^3 = 3^3 = 27$$

4) Evaluate without a calculator (use a calculator afterwards to check your work and make sure you can enter the powers correctly)

a) $4^{\frac{5}{2}}$

b) $16^{\frac{3}{2}}$

c) $100^{\frac{7}{2}}$

d) $36^{\frac{3}{2}}$

5) Simplify, by writing with no brackets, some powers will stay as fractions

a) $(3^4xy^2)^{\frac{1}{2}} \times 4y^2$

e) $\left(5^{\frac{3}{2}}x^{\frac{5}{2}}y^3\right)^2$

b) $\frac{5^2a}{b^4} \times \frac{b^{\frac{3}{2}}}{a}$

f) $\left(\sqrt{\frac{x}{2}}\right)^3$

c) $\left(\frac{x}{x^{\frac{1}{2}}}\right)^3$

g) $\sqrt{\left(\frac{x}{2}\right)^3}$

d) $(xy)^{\frac{1}{2}} \times x^{-\frac{5}{2}}$

6) Bit of review, simplify:

a) $\sqrt{16x^{16}}$

d) $\frac{\sqrt{9n^4}}{3n^2}$

g) $\frac{(4a^2)^3}{(8a^5)^2}$

b) $\sqrt{36x^{36}}$

e) $\frac{(3a^2b^2)^4}{27a^8b^9}$

h) $\sqrt{\frac{256a^{16}}{b^{12}}}$

c) $\frac{4m^8}{\sqrt{36m^{20}}}$

f) $(0.25x^3)^{\frac{1}{2}}$

Algebraic fractions

EXERCISE 14.05

Simplify the following (that is, write as one fraction).

1 $\frac{4}{p} + \frac{2}{p}$

2 $\frac{13}{q} - \frac{9}{q}$

3 $\frac{2c}{7} + \frac{3c}{7}$

4 $\frac{4c}{5} + \frac{c}{5}$

5 $\frac{x}{2} + \frac{x}{2}$

6 $\frac{4x}{5} + \frac{8}{5}$

7 $\frac{x}{3} + \frac{x}{2}$

8 $\frac{4x}{7} - \frac{x}{2}$

9 $\frac{2x}{5} + 3$

10 $\frac{4x^2}{5} + 10$

11 $x + \frac{x}{2}$

12 $x - \frac{x}{3}$

13 $\frac{2x}{5} - x$

14 $\frac{3x^2}{5} - \frac{x}{2}$

15 $\frac{7x}{2} - x$

16 $\frac{x^2}{10} + \frac{3x}{5} + 2$

17 $\frac{2x}{7} + \frac{7}{3}$

18 $\frac{1}{x} + \frac{1}{y}$

19 $\frac{2}{x} - \frac{1}{y}$

20 $\frac{2}{x} + \frac{3}{y}$

21 $\frac{2}{xy} - \frac{4}{y}$

22 $\frac{a}{x^2} + \frac{b}{xy}$

23 $\frac{3}{xy} + \frac{2}{5}$

24 $\frac{1}{x} - \frac{3x}{y}$

25 $\frac{3}{2x} + \frac{2y}{3x}$

26 $\frac{x}{ay} + \frac{y}{bx} - \frac{a}{by}$

27 $\frac{2}{5y} - \frac{3}{2y} + \frac{4}{3xy}$

28 $\frac{2}{xy} + \frac{3}{xy^2} + \frac{4}{x^2y}$

29 $\frac{x}{y} + \frac{y}{z} + \frac{z}{x}$

30 $\frac{x}{2y} - \frac{y}{3}$

31 $\frac{x}{3a} + \frac{2}{5x}$

32 $\frac{2a}{3x} - \frac{4a}{5xy} + \frac{5a}{2xyz}$

33 $\frac{x+1}{4} + \frac{x}{3}$

34 $\frac{x}{3} - \frac{x+2}{5}$

35 $\frac{2x}{3} + \frac{x+1}{6}$

36 $\frac{x-2}{2} + \frac{2x+3}{6}$

37 $\frac{2x-3}{5} - \frac{x+9}{6}$

38 $\frac{x+2y}{6y} - \frac{3x-2y}{9x}$

39 $\frac{x+7}{x} + \frac{1-x}{x^2}$

40 $\frac{3-5x}{2x} - \frac{6}{x^2}$

Other examples have linear factors that, when multiplied, give a *quadratic expression* for the common denominator.

Example

Simplify $\frac{4}{x-3} - \frac{3}{x+1}$.

Answer

The common denominator is

$$(x-3)(x+1) = x^2 - 2x - 3.$$

4 times $(x+1)$ gives $4x+4$ as the first

numerator and 3 times $(x-3)$ gives $3x-9$ as the second numerator.

$$\begin{aligned}\frac{4}{x-3} - \frac{3}{x+1} &= \frac{4x+4-(3x-9)}{x^2-2x-3} \\ &= \frac{4x+4-3x+9}{x^2-2x-3} \\ &= \frac{x+13}{x^2-2x-3}\end{aligned}$$

EXERCISE 14.06

Simplify these sums and differences by writing as one fraction.

1 $\frac{1}{x+2} + \frac{1}{x+3}$

2 $\frac{3}{x-1} + \frac{2}{x+5}$

3 $\frac{2}{2x+3} + \frac{3}{3x-1}$

4 $\frac{2}{x} - \frac{5}{x+2}$

5 $\frac{3}{x-1} - \frac{2}{x}$

6 $\frac{4x}{x+2} + \frac{5}{x}$

7 $\frac{x}{x+1} + \frac{x}{x+4}$

8 $\frac{a}{x+2a} + \frac{b}{x-2b}$

9 $\frac{4}{2x+3} + \frac{1}{x+3}$

10 $\frac{x}{x-4} - \frac{3}{x+5}$

11 $\frac{4}{x-2} - \frac{x}{x+3}$

12 $\frac{5}{x+1} - \frac{x+1}{3}$

13 $\frac{3x}{3x-1} + \frac{2x}{x+2}$

14 $\frac{x}{2} + \frac{x-1}{x} + \frac{4}{x-2}$

15 $\frac{5}{x} + \frac{2}{x-1} + \frac{3}{x+3}$

16 $\frac{2}{x-1} - \frac{3}{x+2} + \frac{5}{x-3}$

17 $\frac{x}{x+2} + \frac{x}{x-1} + \frac{2x-1}{x}$

18 $\frac{2}{x+1} + \frac{3}{x-1} - \frac{4}{x^2-1}$

19 $\frac{x}{x+4} - \frac{3}{x^2-16}$

20 $\frac{5x}{x^2+5x+6} - \frac{2}{x+3} + \frac{4}{x+2}$

Answers

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EXERCISE 14.04 (page 227) —

1 $\frac{3y}{2}$

2 $\frac{7c}{10d}$

3 $\frac{12x^4}{y}$

4 $a^2b^2c^2$

5 $\frac{b}{2}$

6 $\frac{b^{10}}{a^2c^8}$

7 $9x^8y^{10}$

8 $72x^{27}$

9 $\frac{1}{y^{10}}$

10 $\frac{9}{5x^6}$

11 $\frac{x^6}{y^2}$

12 $4x^2$

13 $\frac{3}{4y^9}$

14 $\frac{4x^4y^6}{9}$

15 1

16 $2^6 = 6^4$

PUZZLE**The shoemaker's will**
(page 227)The executor shared
the shoes in the ratio
 $\frac{1}{2} : \frac{1}{3} : \frac{1}{9} = 9 : 6 : 2$,
 which is probably what the
shoemaker intended. The
problem with the will is that
the three given fractions
do not add to 1. In fact,
 $\frac{1}{2} + \frac{1}{3} + \frac{1}{9} = \frac{17}{18}$ so, if
there were a large number
of shoes, $\frac{1}{18}$ would be
unallocated.
EXERCISE 14.05 (page 229) —

1 $\frac{6}{p}$

2 $\frac{4}{q}$

3 $\frac{5c}{7}$

4 c

5 x

6 $\frac{4x+8}{5}$

7 $\frac{5x}{6}$

8 $\frac{x}{14}$

9 $\frac{2x+15}{5}$

10 $\frac{4x^2+50}{5}$

11 $\frac{3x}{2}$

12 $\frac{2x}{3}$

13 $\frac{-3x}{5}$

14 $\frac{6x^2-5x}{10}$

15 $\frac{5x}{2}$

16 $\frac{x^2+6x+20}{10}$

17 $\frac{6x+49}{21}$

18 $\frac{x+y}{xy}$

19 $\frac{2y-x}{xy}$

20 $\frac{2y+3x}{xy}$

21 $\frac{2-4x}{xy}$

22 $\frac{ay+bx}{x^2y}$

23 $\frac{15+2xy}{5xy}$

24 $\frac{y-3x^2}{xy}$

25 $\frac{9+4y}{6x}$

26 $\frac{bx^2+ay^2-a^2x}{abxy}$

27 $\frac{40-33x}{30xy}$

28 $\frac{2xy+3x+4y}{x^2y^2}$

29 $\frac{x^2z+xy^2+yz^2}{xyz}$

30 $\frac{3x-2y^2}{6y}$

31 $\frac{5x^2+6a}{15ax}$

32 $\frac{20ayz-24az+75a}{30xyz}$

33 $\frac{7x+3}{12}$

34 $\frac{2x-6}{15}$

35 $\frac{5x+1}{6}$

36 $\frac{5x-3}{6}$

37 $\frac{7x-63}{30}$

38 $\frac{3x^2+4y^2}{18xy}$

39 $\frac{x^2+6x+1}{x^2}$

40 $\frac{3x-5x^2-12}{2x^2}$

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EXERCISE 14.06 (page 229) —

1 $\frac{2x+5}{x^2+5x+6}$

2 $\frac{5x+13}{x^2+4x-5}$

3 $\frac{12x+7}{6x^2+7x-3}$

4 $\frac{-3x+4}{x^2+2x}$

5 $\frac{x+2}{x^2-x}$

6 $\frac{4x^2+5x+10}{x^2+2x}$

7 $\frac{2x^2+5x}{x^2+5x+4}$

8 $\frac{ax+bx}{x^2+2ax-2bx-4ab}$

9 $\frac{6x+15}{2x^2+9x+9}$

10 $\frac{x^2+2x+12}{x^2+x-20}$

11 $\frac{12+6x-x^2}{x^2+x-6}$

12 $\frac{14-2x-x^2}{3x+3}$

13 $\frac{9x^2+4x}{3x^2+5x-2}$

14 $\frac{x^3+2x+4}{2x^2-4x}$

15 $\frac{10x^2+13x-15}{x^3+2x^2-3x}$

16 $\frac{4x^2+15x-31}{x^3-2x^2-5x+6}$

17 $\frac{4x^3+2x^2-5x+2}{x^3+x^2-2x}$

18 $\frac{5x-3}{x^2-1}$

19 $\frac{x^2-4x-3}{x^2-16}$

20 $\frac{7x+8}{x^2+5x+6}$