13 E

3 b

Determine the shape of

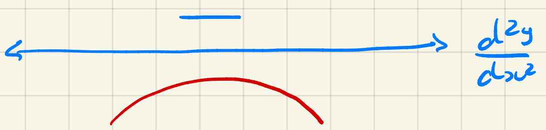
$y = -2(x-3)(x+1)$

$\frac{dy}{dx} = -2\left[(x-3)1 + 1(x+1)\right]$

$= -2(2x-2)$

$= -4x+4$

$\frac{d^2y}{dx^2} = -4 \qquad \frac{d^2y}{dx^2} < 0$ always



function is concave down always.

4 f

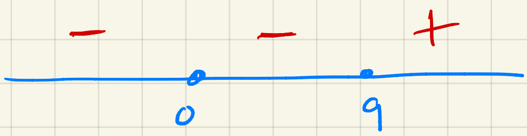
$f(x) = x^4 - 12x^3$

$f'(x) = 4x^3 - 36x^2$

$4x^3 - 36x^2 = 0$

$4x^2(x-9) = 0$

$x = 0 \qquad x = 9$



$f''(x) = 12x^2 - 72x$

$f''(x) = 0$

$12x^2 - 72x = 0$

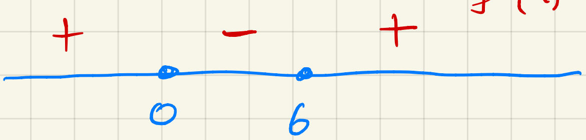
$12x(x-6) = 0$

$x = 0 \qquad x = 6$

$f''(-1) = 84 > 0$

$f''(3) = -108 < 0$

$f''(9) = 324 > 0$



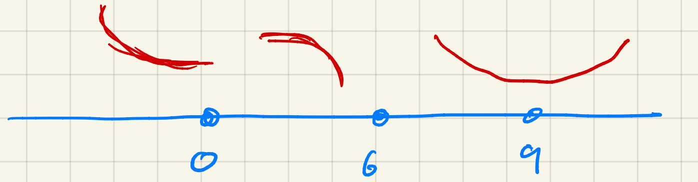
$f(x)$ is decreasing for $x < 9$

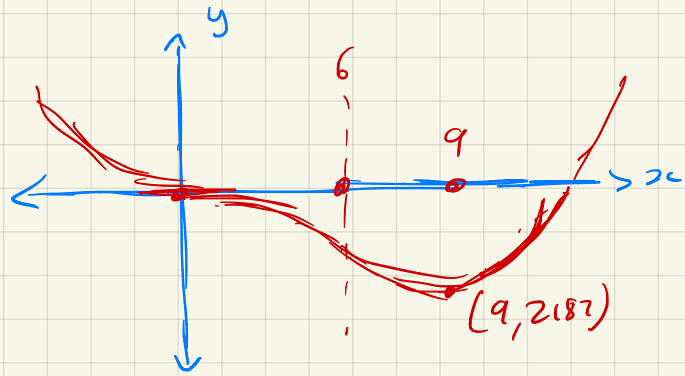
$f(x)$ is increasing for $x > 9$

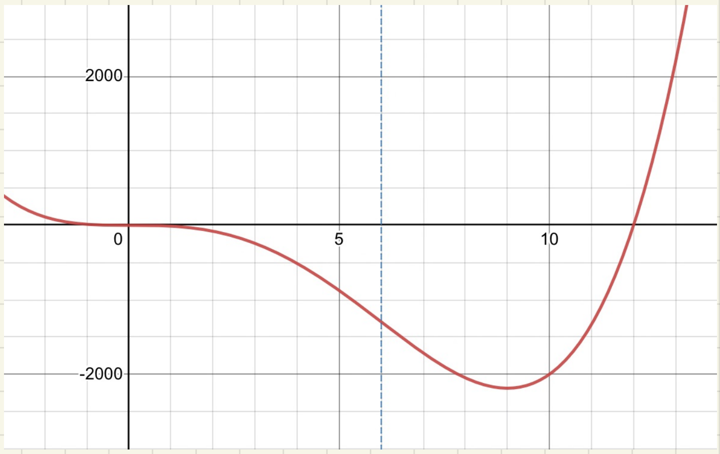
$f(x)$ is concave up $x < 0$

concave down $0 < x < 6$

concave up $x > 6$







7. $f(x)=\frac{e^x}{x}$

a) When $x=0$, $f(x)$ is undefined
$\Rightarrow$ y-intercept does not exist.

for x-intercepts $\frac{e^x}{x}=0$

but $e^x \neq 0$, so no x-intercepts

b) as $x \to \infty$

e^x gets very large

$\frac{1}{x}$ gets very small

but e^x is an exponential

$$\Rightarrow \lim_{x\to\infty}\frac{e^x}{x}=+\infty$$

and $$\lim_{x\to-\infty}\frac{e^x}{x}=0$$

c) $f(x)=\frac{e^x}{x}$

$$f'(x)=\frac{xe^x-e^x.1}{x^2}$$

$$=\frac{e^x(x-1)}{x^2}$$

$f'(x)=0$

$\Rightarrow e^x(x-1)=0$

$x=1$ is a stationary pt $(1,e)$

$-$ $\quad$ $-$ $\quad$ $+$

0 $\quad$ 1 $\quad$ $f'(x)$

$(1,e)$ is a stationary point and is a local minimum.

$$f'(x)=\frac{xe^x-e^x}{x^2}$$

$u=xe^x-e^x$

$u'=xe^x+e^x-e^x$

$=xe^x$

$v=x^2$

$v'=2x$

$$f''(x)=\frac{xe^x(x^2)-(xe^x-e^x)2x}{x^4}$$

$$=\frac{x^3e^x-2x^2e^x+2xe^x}{x^4}$$

$$=\frac{xe^x(x^2-2x+2)}{x^4}$$

$xe^x(x^2-2x+2)=0$

$x=0$ $\quad$ no sol.

so no inflection points.

but need to look at either side of undefined

$-$ $\quad$ $+$

0 $\quad$ $f''(x)$

Concave down $x<0$ $\quad$ Concave up $x>0$

tangent at $x=-1$

$f'(-1)=\frac{-2}{e}$ $\quad$ $f(-1)=\frac{-1}{e}$

$y+\frac{1}{e}=-\frac{2}{e}(x+1)$

$y=-\frac{2x}{e}-\frac{3}{e}$ or $2x+ey=-3$