General Circulatory Models

Primitive Equations and what lies beyond

June 26, 2022

Books and resources

- Atmospheric Circulation Dynamics and General Circulation Models, Masaki Satoh
- Before 1955: Numerical Models and the Prehistory of AGCMs
- Invisible in the Storm, Roulstone and Norbury
- Differential Forms with Applications to the Physical Sciences, Flanders
- Numerical Methods for Fluid Dynamics: With Applications to Geophysics, Dale Durran

Resolution and choice of dynamical equations

GCMs work with a set of *dynamical equations*.The resolution we are operating at is an important factor in the choice of equations (and consequently, the method of solution).

- At a resolution of $100 : 1$ (horizontal to vertical), 'meteorogically important phenomenon' can be assumed to be in **hydrostatic** balance in the vertical direction. Thus, Primitive Equations [1] are chosen.
- With 21st century's ever-increasing computational power, ratios smaller than $10 : 1$ are feasible[2]. However, this means the hydrostatic assumption must be forsaken. Such models are called **non-hydrostatic**.

[1] First written down by Bjerknes. The very first AGCMs were based on the Primitive Equations.

[2] See, for instance, 'A new dynamical framework of nonhydrostatic global model using the icosahedral grid', Hirofumi Tomita and Masaki Satoh 2004.

Primitive Equations

They are fearsome! And working with PEs would require at least a computational fluency in differential forms and PDEs.

Overview of the Spectral Method I

Consider the differential equation

$$\frac{\partial}{\partial t}\psi(x,\, t) = F(\psi),$$

where F is an operator involving the derivatives of ψ. By Fourier theory, the function can be expanded as a series of orthogonal functions $\{\phi_k\}$

$$\psi(x,\, t) = \sum_{k=1}^{\infty} \psi_k(t)\phi_k(x).$$

To solve our differential equation numerically, we truncate this series to (finitely many) N terms. Denote this truncated sum by

$$\phi(x) = \sum_{k=1}^{N} \psi_k(t)\phi_k(x)$$

Overview of the Spectral Method II

and define its residual as

$$\mathcal{R}(\phi) = \frac{\partial}{\partial t}\phi - F(\phi).$$

We wish to minimize $\mathcal{R}$. One approach is the *Galerkin approximation* which requires that the residual be orthogonal to all expansion functions, i.e.

$$\int \mathcal{R}(\phi(x))\phi_k(x)dx = 0 \text{ for } k = 1, 2, \ldots, N,$$

which would imply

$$\frac{d\psi_k}{dt} = \frac{1}{\|\phi_k\|} \int F(\phi(x))\phi_k(x)dx.$$

Non-Hydrostatic Models

Dynamic equations are special cases of the Navier-Stokes equations, such as the Euler equations (inviscid and adiabatic flow).