NOTES - 2/14/18

Ch-1 → Where fission energy comes from?
Ch-2 → How likely is fission?
Ch-3 → Neutron energy dependence for fission & XS averaging over energy.
Ch-4 → Basics for sustaining fission. - MORE on K.
- We look more at K & how that what parameters to look for in design.

Ref: Duderstadt & Hamilton Ch-3

Recall: K = multiplication factor = $\dfrac{\#\ n \text{ in one generation}}{\#\ n \text{ in previous generation}}$

System
Critical → # n is time independent → $K = 1$
Subcritical → # n decays over time → $K < 1$
Critical → # n increases over time → $K > 1$.

This definition of K is conceptually straightforward but not always practical except in cases of back-of-the-envelop calculations.
It is usually difficult to determine neutron-generation time.
i.e. what do you call one generation?
~~[illegible]~~

In a reactor, n may induce fission immediately after fission.
- or they might slow down and then induce a fission
- or they might not induce a fission at all.
- n may even decay (if we wait long enough) away & disappear
- or they might appear from radioactive decay of fission products.

All these phenomena have different time scales. Therefore determination of 'one generation' time is difficult.

A more practical definition for multiplication factor is

$$k = \frac{\text{Rate of production of } n \text{ in reactor}}{\text{Rate of loss of } n}$$

Let $P(t) \equiv n$ production rate
$L(t) \equiv n$ loss rate } ← could change over time

This n balance definition is more useful as we can ~~define~~ build more sophisticated methods to determine k around this definition.

e.g. we define $P(t)$ & $L(t)$ more accurately ~~as a func~~ and we get a more accurate estimate of k.

eg. diffusion / transport equations

we study that later.

~~Now, if we define~~

<u>Neutron lifetime</u>:

Let $N(t)$ = total n population in reactor at time t

then $l \equiv n \text{ lifetime} = \dfrac{N(t)}{L(t)} = \dfrac{\# n}{\text{loss rate}} =$ ~~avg time before n death~~

= Average time before n death.

SIMPLE CHAIN REACTION KINETICS

We don't, at this point, but assuming we did know $N(t)$, Then time rate of change of @ population in reactor is

$$\frac{dN}{dt} = \text{Production} - \text{loss rate} = P(t) - L(t).$$

$$\Rightarrow \frac{dN}{dt} = P(t) - L(t).$$ (want to write in terms of k & l)

Now divide eqn by $L(t)$.

$$\frac{1}{L(t)}\frac{dN}{dt} = \frac{P(t)}{L(t)} - \frac{L(t)}{L(t)} = k - 1$$

$$\Rightarrow \frac{dN}{dt} = (k-1)\,L(t).$$

further multiply & divide rhs by $N(t)$. so,

$$\frac{dN}{dt} = (k-1)\frac{L(t)}{N(t)}N(t).$$ (but $\frac{N(t)}{L(t)} = l$)

$$\Rightarrow \frac{dN}{dt} = \frac{(k-1)}{l}N(t).$$

$$\Rightarrow \frac{dN}{N(t)} = \frac{k-1}{l}dt$$

Obviously,

$$\boxed{N(t) = N_0 e^{(k-1)/l\, t}}$$

So now what does this simple model of reactor kinetics tell us?

$$N(t) = N_0 e^{(k-1)/\ell \, t} = N_0 e^{-(\ell/k-1)t}$$ time constant

very basic, naive

This model tells us that growth or decay of @ population ~~decay~~ obeys an exponential growth law.

We will find later that reactor power is proportional to its population.

The time behavior of reactor power level can then be ~~thought~~ thought of as an exponential with time constant or, $T = \frac{\ell}{k-1}$.

T is also called reactor period.

As $T \to \infty$, $N(t) \sim N_0$ $\Rightarrow$ stable population.

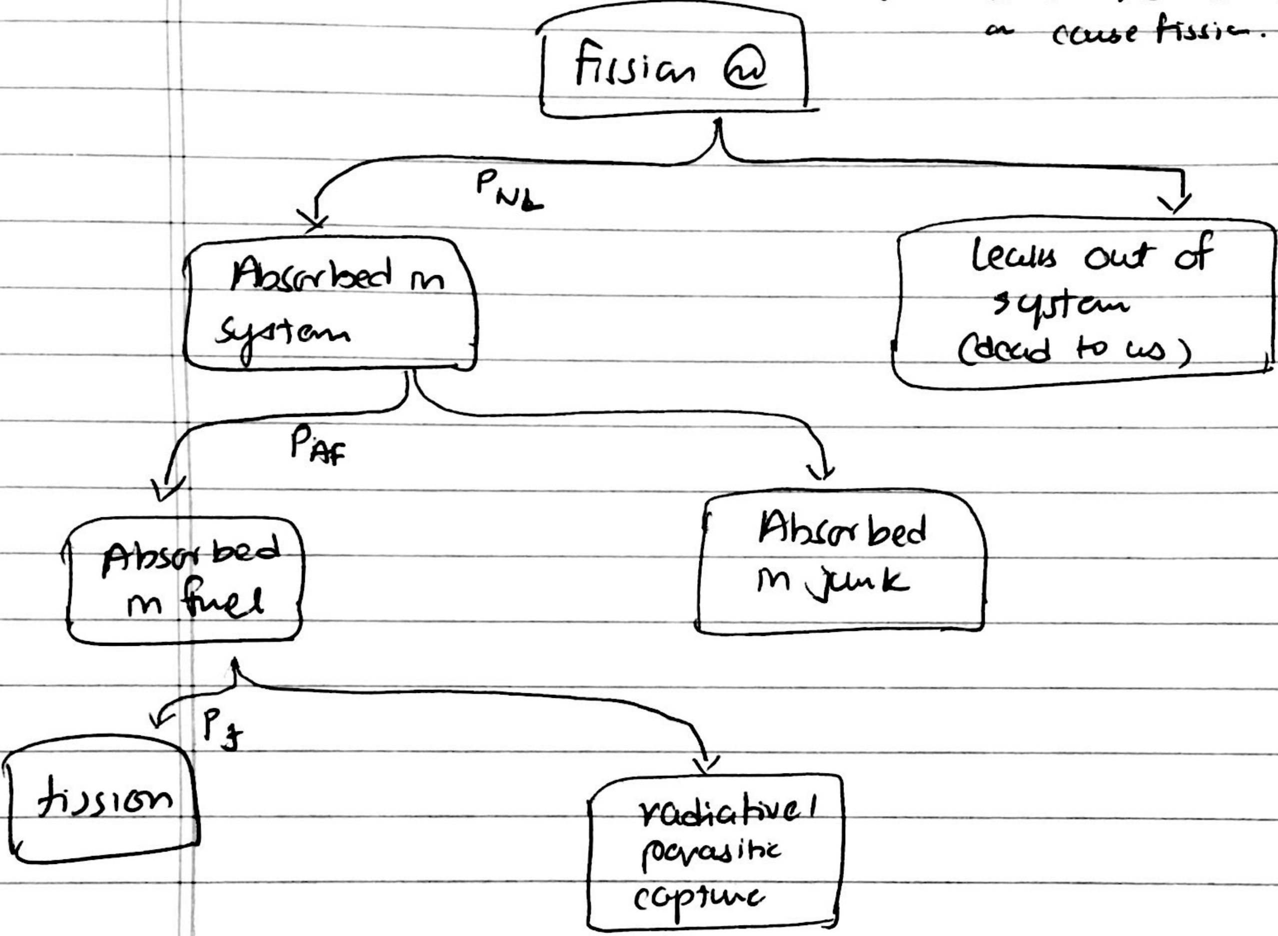
When $k \neq 1$, however this model predicts slow / rapid growth / decay based on size of ℓ ~~and~~ 2 k.

"BACK-OF-THE-ENVELOP" calculation for k - 4 factor formula

In order to ~~est~~ estimate what happens to k, we must ~~first~~ first think about what happens to a @.

Once n enters system,
It can either leak / absorbed in system.
If absorbed in system, it can be absorbed in junk / fuel.
If it is absorbed in fuel, it can either cause ~~fission~~ be ~~a~~ radiative capture or cause fission.

Once a n is born,



P_{NL} ≡ probability that a n will not leak before absorption.

P_{AF} ≡ conditional probability that if a n is absorbed, it will be absorbed in fuel.

P_f ≡ conditional probability that if a n is absorbed in fuel, it will induce fission.

P_{NL} is rather difficult to calculate ~~because would require sophisticated solving methodology gives us an idea of~~

P_{AF} & P_F can be calculated with relative ease.

P_{AF} ≡ fuel absorption probability = $\boxed{\dfrac{\Sigma_a^F}{\Sigma_a}}$ = ~~probability that ⓝ is absorbed unit~~

= probability that ⓝ is absorbed in fuel in the next unit length

~~probability that a ⓝ is absorbed in next unit length~~

In reactor physics, for thermal reactors, this probability P_{AF} is also called thermal utilization & is denoted by f.

so $P_{AF} \equiv f$.

Represents the effectiveness of a fuel in competing with other material in reactor for absorption of thermal ⓝ (in thermal reactors).

P_F = Conditional probability for inducing fission can also be given in terms of xs.

$$P_F = \frac{\Sigma_f^F}{\Sigma_a^F} = \frac{\sigma_f^F}{\sigma_a^F}$$

Note that, probable # of ⓝ produced from that fission will be

$$\nu P_F = \nu\left(\frac{\sigma_f^F}{\sigma_a^F}\right) = \eta$$

Now, assuming all probabilities are known, and following the chart & equations we got from before, if we follow ⓝ.

Start with N_1 ⓝ in the first generation, the # of ⓝ in subsequent generation will be N_2,

out of ~~...~~ N_1

N_2 will contain ⓝ that are born to fission in fuel, as opposed to capture

ⓝ absorbed in fuel as opposed to junk

ⓝ that are absorbed at all as opposed to leaky.

$\eta = \nu \frac{\sigma_f^F}{\sigma_a^F}$ f P_{NL}

Then $N_2 = \eta f P_{NL} N_1$

$\Rightarrow \frac{N_2}{N_1} = \eta f P_{NL}$

$\Rightarrow \boxed{k = \eta f P_{NL}}$.

For an infinite reactor, no ⓝ leaks so $P_{NL} = 1$.

$\Rightarrow$ we define $\boxed{k_\infty = \eta f}$ ← infinite medium multiplication factor.

For finite reactors $P_{NL} < 1$ as there is almost always some leakage in realistic reactors.

However, in order to have a sustained chain reaction, $k_\infty > 1$. ~~because that~~ as that is the max value any reactor can ~~achie~~ have theoretically.

THERMAL REACTORS

More realistically, for thermal reactors, there is always n slow down. Energy dependence ~~[illegible]~~ also becomes important. In such situations, we modify our diagram a little to incorporate more effects (or more details)

→ f → thermal utilization
& η → probable n produced
are now evaluated for thermal n.

So f → $\dfrac{\text{thermal n absorptions in fuel}}{\text{total thermal n absorptions}}$

η → avg # of fissions produced per absorption of thermal n in fuel.

To account for processes occurring while n slow down.

fission n
↓
Slowing down via scatter w/ light nuclei
↓
Resonance abs/ leakage
↓
thermal n leakage/ capture/ fission.

fast fission factor: Accounts for the fact that although most fissions will be induced in fissile material by thermal n, some fission will be induced in fissionable materials by fast n. We therefore scale up our previous expression for k by ε.

$$\varepsilon = \frac{\text{total \# of fission n}}{\text{total \# of n from thermal fission.}}$$

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For thermal reactors $\varepsilon \sim 1$. (ranges betn 1.03 to 1.15)

~~Second~~ Resonance escape probability:

Second factor characterizes possibility that ⓝ might be absorbed while slowing down from fission to thermal region.

$p \equiv$ fraction of fission ⓝ that manage to slow down from fission to thermal energies w/o being absorbed.

Finally, non leakage probability is ~~taken~~ considered.

- ⓝ mfp is relatively large for high energies so we consider them separately ~~# thats #~~ → ⓝ that are leaked while slowing down. This is characterized by P_{FNL}.
- Next we consider leakage of ⓝ that have slowed down thro P_{TNL}.

Then $P_{NL} = P_{FNL} P_{TNL}$.

probability that fast ⓝ will not leak out (fast non leakage)

probability that thermal ⓝ will not leak out. (thermal non leakage)

Inserting these new definitions in our earlier expression, & scaling the formulas for k_∞, we have

$k_\infty = \eta f p \varepsilon$

now only thermal

scaling for fast ⓝ

ⓝ from thermal fission per absorption in fuel

probability that fission ⓝ slow down past thermal energies without absorption.

This formula for k_∞ is called the ~~four~~ four factor formula.

$$\boxed{k_\infty = \eta f p \varepsilon}$$

moreover, incorporating leakage,

$$k = \cancel{\eta}\, \eta f p \varepsilon P_{NL} = \eta f p \varepsilon P_{TNL} P_{FNL}$$

– six factor formula.//

Notes

21/6/19

Review:

$$k = \frac{\#\ \text{n in one gen}}{\#\ \text{n in } \cancel{\text{one}}\ \text{gen}}$$

$$k = \frac{\text{Rate of production of n}}{\text{Rate of loss of n}} = \frac{P(t)}{L(t)}$$

both $P(t)$ & $L(t)$ come from more sophisticated theories which ~~take~~ account for average behavior of particles ~~gas~~ in systems ~~account~~.

One example is diffusion equation itself.

$$-\frac{\partial}{\partial x} D \frac{\partial}{\partial x}\phi + \Sigma_a \phi = \frac{\nu \Sigma_f \phi}{k}$$

~~K~~ Here LHS represents $L(t)$ ~~2~~.

RHS represents $P(t)$.

$k \rightarrow$ multiplication factor.

More accurate theories exist and are often used.

High fidelity calculations are ~~the rage~~ in demand due to HPC.

~~Neutron~~

Neutron lifetime:

$$l = \frac{N(t)}{L(t)} = \frac{\#\text{n}}{\text{loss rate}} = \text{"Average" time before n death.}$$

this is like $\frac{\text{distance}}{\text{speed}} = \text{time}$.

Simple chain reaction kinetics:

$$\boxed{N(t) = N_0\, e^{(k-1)/\ell\; t}}$$

$T = \dfrac{\ell}{k-1}$ = reactor period.

$T \to \infty$, $N(t) \sim N_0$ → stable ⓝ population.

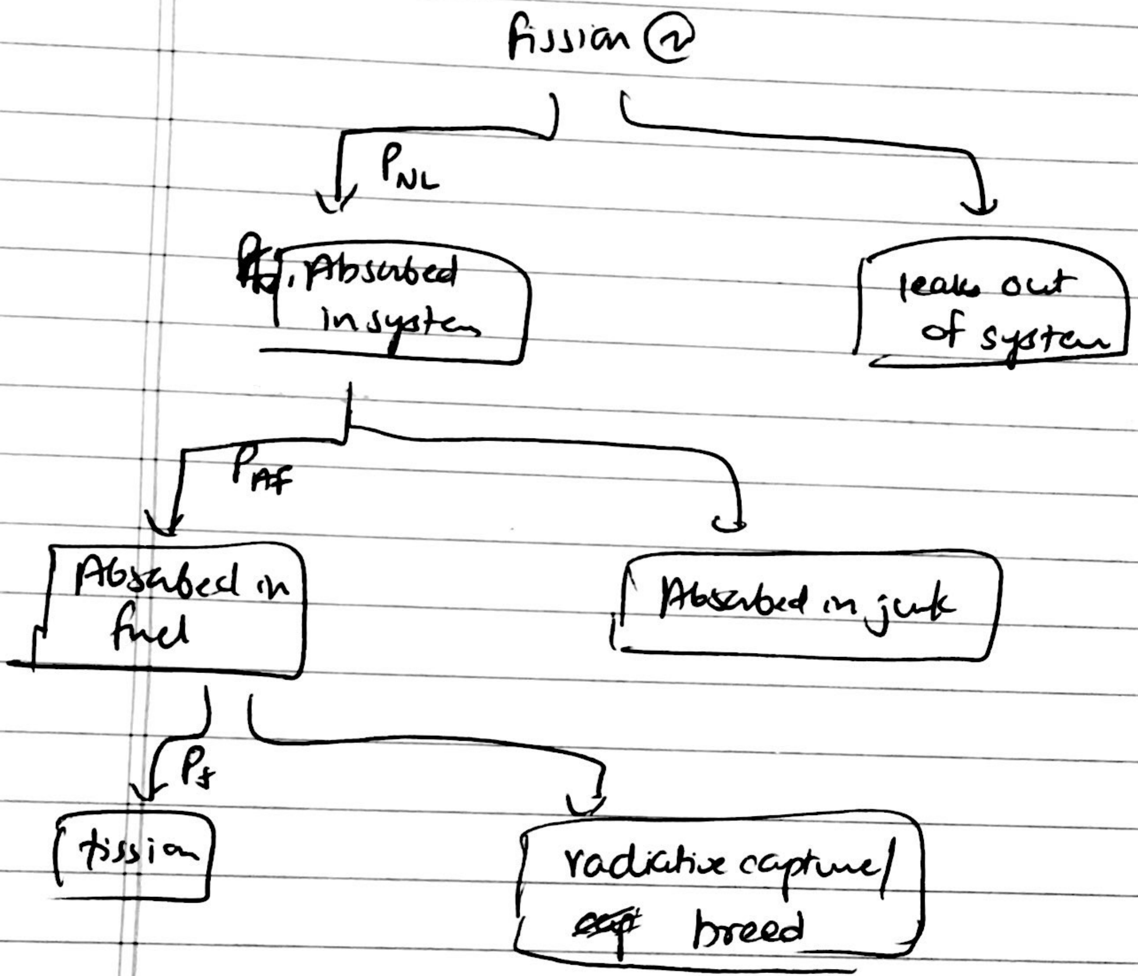
Size of ℓ & k will then determine how rapidly ~~the~~ ⓝ ~~population~~ reactor power grows.

"BACK-OF-THE-ENVELOPE" calculation of k – 4-factor formula.

ⓝ LIFE IN A NUTSHELL.

ONCE ⓝ enters system,

- It can either leak / absorbed in system (death is the only constant in life)
  ~~only fissile~~
- Absorbed in system → in fuel / junk.
- Absorbed in fuel → fission / radiative capture. (and breeding)
  - Breeding not taken into account here at all but we may club it with radiative capture for now
  - We are going to assume that breeding does not affect k ~~power~~ that much



P_{AF} = fuel absorption prob. = $\frac{\Sigma_a^F}{\Sigma_a}$

= ~~prob the~~

When we do specifically for thermal reactors $P_{AF} = f \equiv$ thermal utilization.

Represents effectiveness of fuel in competing w/ other material in reactor for absorption of thermal n for thermal reactor.

$$P_f = \frac{\Sigma_f^F}{\Sigma_a^F} = \frac{\sigma_f^F}{\sigma_a^F} = \text{conditional prob. of inducing fission}$$

$$\text{probable \# of } ⓝ \text{ produced} = \nu\left(\frac{\sigma_F^F}{\sigma_a^F}\right) = \eta.$$

<u>Follow the ⓝ.</u>

Start with N_1 ⓝ in 1st generation.
\# ⓝ in 2nd gen = N_2.

$$K = \frac{N_2}{N_1}.$$

However N_2 will contain ⓝ ~~that dont~~ that came from ⓝ that

- don't leak out — P_{NL}
- that upon absorption, ~~fa~~ ~~fuel~~ get absorbed in fuel & ~~upon~~ — P_{AF}
- upon absorption in fuel cause ν ⓝ to be born — η

η_p

So

$$N_2 = \eta f P_{NL} N_1$$

$$\& \quad \boxed{K = \frac{N_2}{N_1} = \eta f P_{NL}}$$

For an infinite reactor, no leakage $\Rightarrow P_{NL} = 1$. & $\boxed{k_\infty = \eta f.}$