finite difference:

TAYLOR

$$u(x+h) = u(x) + hu'(x) + \frac{h^2}{2}u''(x) + \cdots$$

$$u(x-h) = u(x) - hu'(x) + \frac{h^2}{2}u''(x) + \cdots$$

FORWARD

Truncate

$$u(x+h) = u(x) + hu'(x) \,\vdots + \frac{h^2}{2}u''(x) + \cdots$$

$$u(x+h) \approx u(x) + h\,u'(x)$$

$$\Rightarrow \boxed{u'(x) \approx \frac{u(x+h) - u(x)}{h}}$$ for small h.

BACKWARD

Truncate

$$u(x-h) = u(x) - hu'(x) \,\vdots + \frac{h^2}{2}u''(x)$$

$$\Rightarrow u(x-h) \approx u(x) - hu'(x)$$

$$\Rightarrow \boxed{u'(x) \approx \frac{u(x) - u(x-h)}{h}}$$

CENTRAL

Truncate

$$u(x+h) = u(x) + hu'(x) + \frac{h^2}{2}u''(x) + \cdots$$

$$- \; u(x-h) \quad u(x) - hu'(x) + \frac{h^2}{2}u''(x) + \cdots$$

$$- \qquad - \quad + \qquad -$$

$$u(x+h) - u(x-h) \approx 2h\,u'(x)$$

$$\Rightarrow \boxed{u'(x) = \frac{u(x+h) - u(x-h)}{2h}}$$

2nd order

Truncate

$$u(x+h) = u(x) + h u'(x) + \frac{h^2}{2} u''(x) + \dots$$

$$u(x-h) = u(x) - h u'(x) + \frac{h^2}{2} u''(x) + \dots$$

$+ \quad + \quad - \quad + \quad !$

$$u(x+h) + u(x-h) \approx 2u(x) + h^2 u''(x)$$

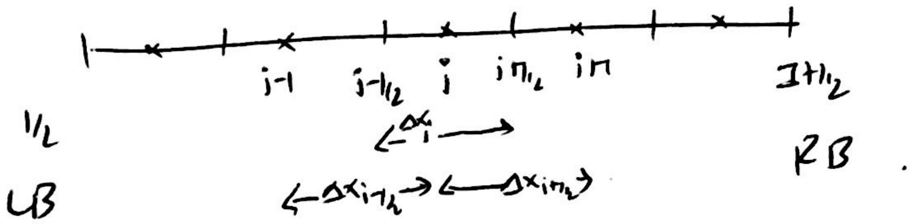
2) $$\boxed{u''(x) \approx \frac{u(x+h) - 2u(x) + u(x-h)}{h^2}}$$

Now discretize diff eq.

$$-\frac{\partial}{\partial x} D \frac{\partial \phi}{\partial x} + \Sigma_a \phi = S.$$

LB $\rightarrow \frac{\partial \phi}{\partial x} = 0$

RB $\rightarrow \phi = 0$.



we discretize into N nodes (cell centers) so N+1 edges.

Evaluate everything at cell centers (i)

$$\left[-\frac{\partial}{\partial x} D \frac{\partial \phi}{\partial x} + \Sigma_a \phi = S\right]_i$$

central diff

$$\left[-\frac{\partial}{\partial x} D \frac{\partial \phi}{\partial x}\right]_i = \frac{D \frac{\partial \phi}{\partial x}\Big|_{i+1/2} - D\frac{\partial \phi}{\partial x}\Big|_{i-1/2}}{\Delta x_i} = \frac{D_{i+1/2}\left[\frac{\phi_{i+1} - \phi_i}{\Delta x_{i+1/2}}\right] - D_{i-1/2}\left[\frac{\phi_i - \phi_{i-1}}{\Delta x_{i-1/2}}\right]}{\Delta x_i}$$

$$\Rightarrow \quad \frac{\partial}{\partial x} D \frac{\partial \phi}{\partial x}\bigg|_i = \frac{D_{i+1/2}}{\Delta x_i \Delta x_{i+1/2}} [\phi_{i+1}] - \left[\frac{D_{i+1/2}}{\Delta x_i \Delta x_{i+1/2}} + \frac{D_{i-1/2}}{\Delta x_i \Delta x_{i-1/2}} \right] [\phi_i] \qquad (2)$$

$$+ \frac{D_{i-1/2}}{\Delta x_i \Delta x_{i-1/2}} [\phi_{i-1}]$$

$$\Rightarrow \quad -\frac{\partial}{\partial x} D \frac{\partial}{\partial x} \phi \bigg|_i = [\phi_{i+1}] \left(- \frac{D_{i+1/2}}{\Delta x_i \Delta x_{i+1/2}} \right) + [\phi_i] \left(\frac{D_{i+1/2}}{\Delta x_i \Delta x_{i+1/2}} + \frac{D_{i-1/2}}{\Delta x_i \Delta x_{i-1/2}} \right)$$

$$+ [\phi_{i-1}] \left(- \frac{D_{i-1/2}}{\Delta x_i \Delta x_{i-1/2}} \right)$$

$$[\Sigma_a \phi]_i = \Sigma_{a\,i}\, \phi_i$$

$$[S]_i = S_i$$

$\Rightarrow$ Eqn evaluated at i $\leftarrow$ for arbitrary interior nodes.

$$\boxed{[\phi_{i-1}] \left(\frac{-D_{i-1/2}}{\Delta x_i \Delta x_{i-1/2}} \right) + [\phi_i] \left(\frac{D_{i-1/2}}{\Delta x_i \Delta x_{i-1/2}} + \frac{D_{i+1/2}}{\Delta x_i \Delta x_{i+1/2}} + \Sigma_{a\,i} \right) + [\phi_{i+1}] \left(\frac{-D_{i+1/2}}{\Delta x_i \Delta x_{i-1/2}} \right) = S_i}$$

Boundary nodes -

Left - Boundary → Neumann $\frac{\partial \phi}{\partial x} = 0$

$$[\Sigma_a \phi]_i = \Sigma_{a_1} \phi_1$$

$$[S]_i = S_1$$

$$\left[\frac{\partial}{\partial x} D \frac{\partial \phi}{\partial x}\right]_i = \frac{D \frac{\partial \phi}{\partial x}\Big|_{i+1/2} - D\frac{\partial \phi}{\partial x}\Big|_{i-1/2}}{\Delta x_i}$$

we do forward difference here

$$= \frac{D_{i+1/2}\left[\frac{\phi_{i+1} - \phi_i}{\Delta x_{i+1/2}}\right] - \cancel{D_{i-1/2}\left[\frac{\phi_i - \phi_{i-1/2}}{\frac{\Delta x_{i-1/2}}{2}}\right]} \to 0}{\Delta x_i}$$

$$= \frac{D_{i+1/2}[\phi_{i+1} - \phi_i]}{\Delta x_{i+1/2}\, \Delta x_i}$$

$\Rightarrow$ we have, $[\phi_i]\left(\frac{D_{i+1/2}}{\Delta x_i\, \Delta x_{i+1/2}} + \Sigma_{a_i}\right) + [\phi_{i+1}]\left(-\frac{D_{i+1/2}}{\Delta x_i\, \Delta x_{i+1/2}}\right) = S_i$

<u>for right boundry</u> - Drichelet $\phi_{I+1/2} = 0$

$$[\Sigma_a \phi]_i = \phi_I \Sigma_{a_I}$$

$$[S]_i = S_I$$

$$\left(\frac{\partial}{\partial x} D \frac{\partial}{\partial x}\phi\right)_i = \frac{D\frac{\partial}{\partial x}\phi\Big|_{i+1/2} - D\frac{\partial \phi}{\partial x}\Big|_{i-1/2}}{\Delta x_i}$$

$$= \frac{D_{i+1/2}\left[\frac{\cancel{\phi_{i+1/2}}^{0=BC} - \phi_i}{\frac{\Delta x_{i+1/2}}{2}}\right] - D_{i-1/2}\left(\frac{\phi_i - \phi_{i-1}}{\Delta x_{i-1/2}}\right)}{\Delta x_i}$$

$$= \frac{\frac{2D_{i+1/2}(-\phi_i)}{\Delta x_{i+1/2}} - D_{i-1/2}\frac{(\phi_i - \phi_{i-1})}{\Delta x_{i-1/2}}}{\Delta x_i}$$

$$\Rightarrow \boxed{[\phi_{i-1}]\left(\frac{-D_{i-1/2}}{\Delta x_{i-1/2}\Delta x_i}\right) + [\phi_i]\left(\frac{2D_{i+1/2}}{\Delta x_{i+1/2}\Delta x_i} + \frac{D_{i-1/2}}{\Delta x_{i-1/2}\Delta x_i} + \Sigma_{a_i}\right) = S_i}$$

Then form a matrix.

$$\begin{bmatrix} i=1 & & & \\ i=2 & \cdots & & \\ & & & \\ & & i=I & \end{bmatrix} \begin{bmatrix} \phi_1 \\ \vdots \\ \phi_I \end{bmatrix} = \begin{bmatrix} S_1 \\ \vdots \\ S_I \end{bmatrix}$$

Solving matrix eqn return soln.