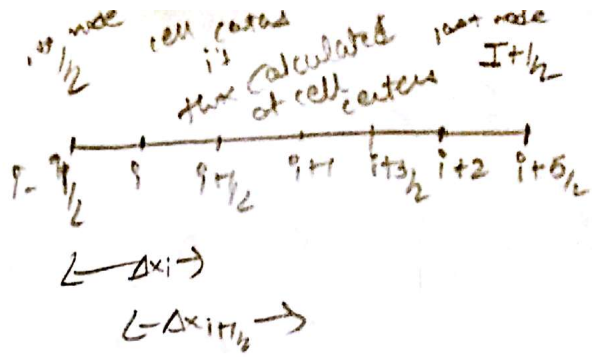
For non uniform media: Finite Difference



$$-\frac{1}{r^2}\frac{\partial}{\partial r} r^2 D \frac{\partial \phi}{\partial r} + \sigma_a \phi = Q$$

Calculate at cell centers: is

$$-\frac{1}{r^2}\frac{\partial}{\partial r} r^2 D \frac{\partial \phi}{\partial r}\bigg|_i + \sigma_a \phi\Big|_i = Q\Big|_i$$

$\boxed{1}$
$$-\frac{1}{r^2}\frac{\partial}{\partial r} r^2 D \frac{\partial \phi}{\partial r}\bigg|_i = \frac{1}{r_i^2}\left[\frac{r_{i+1/2}^2 D_{i+1/2} \frac{\partial \phi}{\partial r}\Big|_{i+1/2} - r_{i-1/2}^2 D_{i-1/2} \frac{\partial \phi}{\partial r}\Big|_{i-1/2}}{\Delta x_i}\right]$$

$$= \frac{-1}{r_i^2}\left[\frac{r_{i+1/2}^2 D_{i+1/2}\left(\frac{\phi_{i+1}-\phi_i}{\Delta x_{i+1/2}}\right) - r_{i-1/2}^2 D_{i-1/2}\left(\frac{\phi_i - \phi_{i-1}}{\Delta x_{i-1/2}}\right)}{\Delta x_i}\right]$$

$$= -\frac{r_{i+1/2}^2}{r_i^2}\frac{D_{i+1/2}[\phi_{i+1}]}{\Delta x_i \, \Delta x_{i+1/2}} + \frac{r_{i+1/2}^2}{r_i^2}\frac{D_{i+1/2}[\phi_i]}{\Delta x_i \, \Delta x_{i+1/2}}$$

$$+ \frac{r_{i-1/2}^2}{r_i^2}\frac{D_{i-1/2}[\phi_i]}{\Delta x_i \, \Delta x_{i-1/2}} - \frac{r_{i-1/2}^2}{r_i^2}\frac{D_{i-1/2}[\phi_{i-1}]}{\Delta x_i \, \Delta x_{i-1/2}}$$

$\boxed{2}$ $\sigma_a \phi_i = \sigma_{a,i}\, \phi_i$ $\quad$ $\boxed{3}$ $Q\big|_i = Q_i$

Then we have

$$[\phi_{i-1}]\left(-\frac{r_{i-1/2}^2}{r_i^2}\frac{D_{i-1/2}}{\Delta x_i \Delta x_{i-1/2}}\right) + [\phi_i]\left(\sigma_{a,i} + \frac{r_{i-1/2}^2}{r_i^2}\frac{D_{i-1/2}}{\Delta x_i \Delta x_{i-1/2}} + \frac{r_{i+1/2}^2}{r_i^2}\frac{D_{i+1/2}}{\Delta x_i \Delta x_{i+1/2}}\right)$$

$$+ [\phi_{i+1}]\left(-\frac{r_{i+1/2}^2}{r_i^2}\frac{D_{i+1/2}}{\Delta x_i \Delta x_{i+1/2}}\right) = Q_i .$$

Make sure material boundaries are at cell centers. So $D_{i-1/2}$ & $D_{i+1/2}$ can be defined naturally based on which material region $i+1/2$ & $i-1/2$ fall into.

Now for 0 boundaries, we have

$\phi_{1/2} = 0$ where $i = 1/2$ is left boundary

& $\phi_{I+1/2} = 0$ & $i = I+1/2$ is right boundary.

For left boundary:

we have $\phi_{i-1/2} = 0$.

So, eqn will be

$$[\phi_i]\left(\sigma_{a,i} + \frac{r_{i+1/2}^2}{r_i^2}\frac{D_{i+1/2}}{\Delta x_i \Delta x_{i+1/2}}\right) + [\phi_{i+1}]\left(-\frac{r_{i+1/2}^2}{r_i^2}\frac{D_{i+1/2}}{\Delta x_i \Delta x_{i+1/2}}\right)$$

$$= Q_i$$

Likewise we get eqn for right boundary.