HW-2

2.1 $\Sigma = 0.8\ cm^{-1}$

(a) How thick must a material be if no more than 5% @ are to penetrate material w/o making a collision?

→ 5% penetrate the material
→ So at thickness x, $I(x) = 5\%$ of I_0.

Now, using Eq.(2.4) we know that

$$I(x) = I_0 e^{-N\sigma x} = I_0 e^{-\Sigma x}$$

$$\Rightarrow \quad 0.05\, I_0 = I_0\, e^{-\Sigma x}$$

$$\Rightarrow \quad 0.05 = e^{-x\Sigma} = e^{-0.08\,x}$$

$$\Rightarrow \quad \frac{-\ln(0.05)}{0.08} = \boxed{3.74\ cm}$$

(b) What fraction of @ will make their first collision within first ~~two~~ 2.0 cm of the material?

In order to find what fraction of @ will make their first collision in 2 cm, we look at what fraction will not.

The fraction that will not make its first collision (survive) are

$$\cancel{\frac{I(x)}{I(0)}} \quad \frac{I(x)}{I_0} = f_1 = e^{-\Sigma x} = e^{-(0.8)(2)}$$

The fraction that makes a collision is then $1 - f_1 = 1 - e^{-(0.8)2}$

$= 0.798$

2.2 Uncollided flux at r is

$$\Phi_u(r) = \frac{e^{-\Sigma r}}{4\pi r^2} S_p.$$

(a) we are 1m away from 1 Ci source. What is the flux in n/cm²s neglecting scattering & absorption in air?

$r = 1m = 100$ cm.
1Ci $= 3.7\times10^{10}$ n/s.

If we neglect scatter & absorption, $\Sigma = 0$.

$$\Phi_u(1m) = \frac{e^{-\Sigma r}}{4\pi r^2} S_p$$

$$= \frac{e^{-0}}{4\pi(100)^2} 3.7\times10^{10}$$

$$= \boxed{2.94\times10^5 \text{ n/cm}^2\text{s}}$$

(b) If a sheild is placed betn you & src. what absorption xs would be required to reduce flux by a factor of 10?

Now, we have absorption ⇒ $\Sigma \neq 0$ ← we find it.
Thickness of absorber is given 100 cm = r.
we are reducing flux by a factor of 10.

⇒ $I(x) = \frac{1}{10} I(0)$ use $I(x) = I_0 e^{-\Sigma r}$

⇒ $\frac{1}{10} = e^{-\Sigma r}$

⇒ $\ln(1/10) = -\Sigma r$

⇒ $-\frac{\ln(\frac{1}{10})}{r} = -\Sigma \Rightarrow \Sigma = \frac{\ln 10}{100} = \boxed{0.023 \text{ cm}^{-1}}$

(c) Suppose shield thickness is now 0.5 cm thick. How far must we be ~~far~~ from the source so that flux is reduced by ~~a factor of 10?~~ the same factor ~~as compared to~~ ~~shielding with~~ a

$$\Phi_u = \frac{e^{-\Sigma r}}{4\pi r^2} S_p$$

$e^{-\Sigma r}$ ← attenuation due to absorber.

S_p ← ~~what you~~ src.

$4\pi r^2$ ← attenuation due to distance.

So absorber has a thickness ~~100~~ 50 cm ~~r~~ = r

$\Sigma = 0.023$.

Total flux atten attenuation with 100 cm absorber.

$$\Phi_{100} = \frac{e^{-\Sigma(100)}}{4\pi(100)^2} S_p.$$

For same attenuation, with 50 cm absorber.

$$\Phi_{50} = \frac{e^{-\Sigma(50)}}{4\pi(R)^2} S$$

Now, $\Phi_{100} = \Phi_{50}$

$$\Rightarrow \frac{e^{-\Sigma(100)}}{4\pi(100)^2} S_p = \frac{e^{-\Sigma(50)}}{4\pi R^2} S_p$$

$$\Rightarrow R^2 = e^{-\Sigma(50-100)} (100)^2$$

Σ ← 0.023

$$\Rightarrow \boxed{R = \cancel{179}\ 179\ cm}$$

~~2.3~~ $N = 4.2 \times 10^{23}$ nuclei/cc.
$\sigma = 3.5 \times 10^{-24}$ cm²/nuclei

(a) what is the macroscopic xs?

$\Sigma = N\sigma = \cancel{3.5 \times} \cancel{0.35 \times 4.2}$
$= 3.5 \times 10^{-24} \times 4.2 \times 10^{23}$
$= \boxed{1.47\ \text{cm}^{-1}}$

(b) what is mean free path?

$\lambda = 1/\Sigma = 1/1.47 = \boxed{0.68\ \text{cm}}$

(c) Slab is 3 cm thick.
what fraction penetrate without making a collision?

fraction that penetrate $= \frac{I(t)}{I(0)} = e^{-\Sigma x} = e^{-1.47 \cdot 3}$

$= \boxed{0.0122}$

(d) what fraction will collide in ⓒ before penetrating 1.5 cm?

fraction that collide $= 1 - \frac{I(t)}{I(0)} = 1 - e^{-\Sigma(x)}$ (x = 1.5) $= 1 - e^{-1.47 \cdot 1.5} = \underline{\underline{0.890}}$

2.4 $P = 1000$ psi.

$\rho_{H_2O,\,stm} = 0.036$ g/cc. $\quad \sigma_{\cancel{steam}} = 3.8\,b. = \sigma_O$

$\rho_{H_2O} = 0.74$ g/cc. $\quad \sigma_{\cancel{H_2O}} = 21.8\,b. = \sigma_H$

(a) what is macroscopic xs for H_2O?
(b) what is macroscopic xs for steam?

~~$\sigma_{H_2O} = 2\sigma_H + \sigma_O = 2(21.8 \times 10^{-24}) + \dots$~~

$$\sigma_{H_2O} = 2\sigma_H + \sigma_O$$
$$= 2(21.8) + 3.8$$
$$= 47.4\ b.$$
$$= 47.4 \times 10^{-24}\ cm^2$$

$$\Sigma_{H_2O} = \frac{\rho N_A}{A_{H_2O}}\sigma = \rho \frac{N_A}{2A_H + A_O}\sigma_{H_2O}$$

$$= \rho\left(\frac{0.6022 \times 10^{24}}{18} \times 47.4 \times 10^{-24}\right)$$

$$= \rho\,(1.586)$$

$$\Sigma^{steam}_{H_2O} = \rho^{steam}(1.586)$$
$$= (0.036)(1.586)$$
$$= \underline{0.057}\ cm^{-1}$$

$$\Sigma^{water}_{H_2O} = \rho^{H_2O}(1.586)$$
$$= (0.74)(1.586)$$
$$= \underline{1.174}.\ cm^{-1}$$

(c) On avg, 40% volume is occupied by steam, what is Σ for mixture?

Mix = 40% steam, 60% water

$$\Sigma = \Sigma_i \frac{V_i}{V} \Sigma^i$$

$$= \underbrace{(0.4)}_{\text{vol frac for steam}} \Sigma_{H_2O}^{steam} + \underbrace{(0.6)}_{\text{vol frac for liq. water}} \Sigma_{H_2O}^{water}$$

$$= (0.4)(0.057) + (0.6)(1.174)$$

$$= \underline{0.7272}$$

2.8 What is macroscopic thermal xs of UO_2?

$\tilde{e} = 4\%$

$\sigma^{25} = 607.5\text{ b}$

$\sigma^{28} = 11.8\text{ b}$

$\sigma^{0} = 3.8\text{ b}$

$\rho_{UO_2} = 10.5\text{ g/cc}$

layers of complexity

→ σ^u ← thru atom fraction

→ σ^{UO_2} ← thru molecule

→ Σ^{UO_2}

First calculate $\sigma^u = \tilde{e}\,\sigma^{25} + (1-\tilde{e})\,\sigma^{28}$

$= (0.04)(607.5) + (0.96)(11.8)$

$= \cancel{17.1\text{b}}\ 35.62\text{ b}$

Now find $\sigma^{UO_2} = \sigma_u + 2\sigma_o = 17.1 + 2(3.8)$

$= \cancel{24.7}\ 43.23$

Now find Σ^{UO_2}

$$\Sigma^{UO_2} = \frac{\rho^{UO_2} N_A}{A^{UO_2}} \sigma^{UO_2}$$

$$= \frac{(10.5)(0.6022 \times 10^{24})(25.3 \times 10^{-24})}{A^{U\cancel{238}} + 2(16)}.$$

round to 238 for enriched U for simplicity

but actually it will be $(0.04)(235) + (0.96)(238)$

$= 237.88 \approx 238$.

$$= \frac{(10.5)(0.6022)(\cancel{25.3}\ 43.23)}{238 + 2(16)} = \cancel{0.593}\ 1.01 \text{ cm}^{-1}$$

2-15 What is min # of elastic scattering collisions required to slow a n down from 1 MeV to 1 eV in

(a) ${}^{2}_{1}H$ (b) ${}^{12}_{6}C$ (c) ${}^{56}Fe$ (d) ${}^{238}U$

$E = \alpha E'$ for max energy loss in one collision.

final ↑ $\left(\frac{A-1}{A+1}\right)^2$ in one collision ↑ initial

Then assuming max energy loss for each collision.

$E_1 = \alpha E_0$

$E_2 = \alpha E_1 = \alpha(\alpha E_0) = \alpha^2 E_0$

$E_3 = \alpha E_2 = \alpha(\alpha^2 E_0) = \alpha^3 E_0$

$\vdots$

$\Rightarrow E_N = \alpha^N E_0.$

Then letting $E_N = 1\,eV$, $E_0 = 10^6\,eV$, we have,

~~$L = \alpha^N$~~ $E_N = \alpha^N E_0$

$\Rightarrow 1 = \alpha^N 10^6$

$\Rightarrow 10^{-6} = \alpha^N$

$\Rightarrow \log(10^{-6}) = \log(\alpha^N)$

$\Rightarrow N = \dfrac{\log(10^{-6})}{\log \alpha} = \dfrac{-6}{\log \alpha}$ //

(a) ^2H $\alpha = \left(\dfrac{A-1}{A+1}\right)^2 = \left(\dfrac{1}{3}\right)^2 = 0.11\overline{1}$

then $N = \dfrac{-6}{\log(1/9)} = 6.28 \rightarrow \underline{\underline{7}}$.

(b) Similarly for other nuclides, find α based on A & then take $\log_{10}$ & divide by -6.

b $\rightarrow$ $N = 41.3 \rightarrow 42$

c $\rightarrow$ $N = 193.2 \rightarrow 194$

d $\rightarrow$ $N = 821.8 \rightarrow 822$ //

Scanned by CamScanner

2.16

Find ξ_{UO_2}

Natural U. $\rightarrow$ 0.7% U^{235} & ~~98~~.99.3% U^{238}

$$\bar{\xi} = \frac{1}{\Sigma_s} \sum_i \xi_i \Sigma_{s_i}$$

$$= \frac{1}{N^{UO_2}\sigma_U + 2N^{UO_2}\sigma_O} \left[\xi^U \Sigma^U + 2\xi^O \Sigma^O \right]$$

$N^{UO_2}\sigma^U$ $N^{UO_2}\sigma^O$

$$\boxed{\bar{\xi}_{UO_2} = \frac{\xi^U \sigma^U + 2\xi^O \sigma^O}{\sigma^U + 2\sigma^O}}$$

Now, lets find ξ^U & ξ^O.

$$\alpha^{235} = \left(\frac{(235-1)}{(235+1)}\right)^2 = 0.9831$$

$$\alpha^{238} = \left(\frac{238-1}{238+1}\right)^2 = 0.9833.$$

Since most of natural U is U^{238}, lets just say $\alpha^U = \alpha^{238} = 0.9833$.

$$\xi_U = 1 + \frac{\alpha}{(1-\alpha)} \ln\alpha = 1 + \frac{0.9833}{1-0.9833} \ln(0.9833)$$

$$= \underline{0.00840}$$

Similaly for O,

$$\alpha^O = \left(\frac{16-1}{16+1}\right)^2 = 0.8823.$$

$$\xi_O = 1 + \frac{0.8823}{1-0.8823} \ln(0.8823) = \underline{0.0613}$$

Now, $\sigma_s^O = 3.761$

$\sigma_s^U = 9.146.$

$$\bar{\xi}_{UO_2} = \frac{\xi^U \sigma_s^U + 2\xi^O \sigma_s^O}{\sigma_s^U + 2\sigma_s^O}$$

$$= \frac{(0.0084)(9.146) + 2 \times 0.0613 \times 3.761}{9.146 + 2\ 3.761}$$

$$= \boxed{0.0322}$$

Q) US-Russia deal

look up Megatons to mega watts program

or just google US-Russia HEU agreement.

Scanned by CamScanner

(a) Show that $\tilde{e}_a = \dfrac{1.1027\,\tilde{e}_\omega}{(1+0.01276\,\tilde{e}_\omega)}$

$\tilde{e}_a = \dfrac{N^{235}}{N^{235}+N^{238}}$ $\qquad \tilde{e}_\omega = \dfrac{m^{235}}{m^{235}+m^{238}}$ $\qquad N = \dfrac{\rho N_A}{A}$ $\qquad m = \rho V$

$$\tilde{e}_a = \frac{N^{235}}{N^{238}+N^{235}} = \frac{\rho^{235} N_A / A^{235}}{\frac{\rho^{235} N_A}{A^{235}} + \frac{\rho^{238} N_A}{A^{238}}} = \frac{\frac{m^{235}}{\cancel{V}} \frac{N_A}{A^{235}}}{\frac{m^{238}}{V}\frac{N_A}{A^{238}} + \frac{m^{235}}{V}\frac{N_A}{A^{235}}}$$

$$= \frac{\frac{m^{235}}{A^{235}}}{\frac{m^{238}}{A^{238}} + \frac{m^{235}}{A^{235}}} = \frac{m^{235}}{A^{235}\frac{m^{238}}{A^{238}} + m^{235}\frac{A^{235}}{A^{235}}} = \frac{m^{235}}{\frac{m^{238}}{A^{238}/A^{235}} + m^{235}}$$

Now, $\cancel{A^{235}}\, A^{238}/A^{235} = 1.01276$

$$\Rightarrow \tilde{e}_a = \frac{m^{235}}{\frac{m^{238}}{1.01276} + m^{235}} = \frac{1.01276\, m^{235}}{m^{238} + 1.01276\, m^{235}}$$

Now, multiply & divide by $m^{235}+m^{238}$ so

$$\tilde{e}_a = \frac{1.01276\, m^{235} / \cancel{m^{235}+m^{238}}\,(m^{235}+m^{238})}{\underbrace{\frac{m^{238}}{m^{235}+m^{238}}}_{1-\tilde{e}_\omega} + \underbrace{\frac{m^{235}}{m^{235}+m^{238}}}_{\tilde{e}_\omega} 1.01276}$$

Then,

$$\tilde{e}_a = \frac{\cancel{1.01}\ 1.01276\,\tilde{e}_\omega}{(1-\tilde{e}_\omega) + (1.01276)\tilde{e}_\omega} \cancel{=} \cancel{1.01276}$$

$$\boxed{\tilde{e}_a = \frac{1.01276\,\tilde{e}_\omega}{1 + 0.01276\,\tilde{e}_\omega}}$$

(b) mass enrichment is 20%. $\Rightarrow \tilde{e}_\omega = 0.2$.
$\tilde{e}_a = ?$

$$\tilde{e}_a = \frac{1.01276\,\tilde{e}_\omega}{1+0.01276\,\tilde{e}_\omega} = \frac{(1.01276)(0.2)}{1+(0.01276)(0.2)}$$

$$\approx 0.205$$

$$\Rightarrow \tilde{e}_a \sim \underline{20.5\%}$$