2/9/18
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notes.

1

Last time.

Reaction rate: $\int_0^\infty \Sigma_x(E)\varphi(E)dE$ = probable # of rxn. of type x /s / cc of all E

Balance Equation

$$\Sigma_t(E)\varphi(E) = \int \Sigma_s(E' \to E)\varphi(E')dE' + \chi(E)S_f'''$$

loss — gain thro scatter — gain thro fission.

Flux distribution in energy $\varphi(E)$

Fast: $\varphi(E) = \dfrac{\chi(E)\, S_f'''}{\Sigma_t(E)}$

Intermediate:

$q(E)$ ← slowing down density constant as
$\Sigma_a(E)$ small enough to ignore ~0.
$\chi(E)$ ~0 because epithermal
only elastic scattering.

Between resonances. $\boxed{\varphi(E) = q / \bar{\xi}\, \Sigma_s(E) E}$ ↳ "1/E" flux

$$\bar{\xi} = \frac{\xi^f \Sigma_s^f(E) + \Sigma_s^m(E)\xi}{\Sigma_s^f(E) + \Sigma_s^m(E)}$$

Thro resonances:

- Assume that only fuel & moderator are present
- only elastic scattering.

Balance equation: $\Sigma_t(E)\varphi(E) = \int p(E' \to E)\Sigma_s(E')\varphi(E')dE'$

$$= \int_E^{E/\alpha^f} \frac{1}{(1-\alpha^f)E'} \Sigma_s^f(E')\varphi(E')dE' + \int_E^{E/\alpha^m} \frac{1}{(1-\alpha^m)E'} \Sigma_s^m(E')\varphi(E')dE'$$

To incorporate resonance, we have
Γ = Full width half max of resonance spike. & most resonance takes place in $\pm\Gamma$.

Narrow resonance approximation: Area under integration of resonance << Area under scatter energy range.

as resonance energy range $(E-\Gamma \le E \le E+\Gamma)$
<< scatter energy range $(\alpha E \le E' \le E/\alpha)$

Today.

If we ~~assume~~ make narrow resonance approximation, most of the energy range is occupied by $\varphi(E) \propto \frac{1}{E}$.

Then we insert $\varphi(E) = \dfrac{q}{\bar{\xi}\, \Sigma_s(E)\, E}$ into the RHS of balance eqn.

$$\Sigma_t(E)\,\varphi(E) = \int_E^{E/\alpha^f} \cdots + \int_E^{E/\alpha^m} \cdots$$

Upon integrating, $\boxed{\varphi(E) = \dfrac{q}{\bar{\xi}\, \Sigma_t(E)\, E}}$.

Note that the only difference now is that we have $\Sigma_t(E)$ in the denominator instead of ~~just~~ $\Sigma_s(E)$.

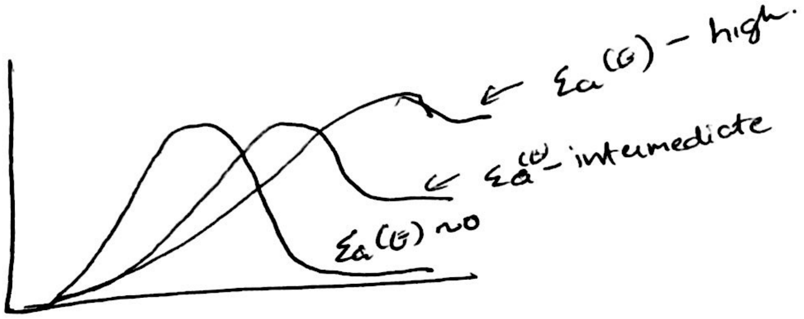
As $\Sigma_t(E)$ increases, $\varphi(E)$ decreases.
This flux depression is called energy self shielding.

Thermal ⓝ

Thermal ⓝ follow maxwell-Boltzmann distribution.
In absence of absorption, we obtain $\varphi(E)$ by multiplying ⓝ speed with $M(E)$:

$$\boxed{\varphi_m(E) = \frac{1}{(kT)^2} E e^{-E/kT}} \qquad \text{with} \int_0^\infty \varphi_m(E)\,dE = 1$$

In reality, some absorption exists & that pushes the spectrum up.
This is called spectral hardening.



HW: Summarize fast & thermal reactor spectra section (right before 3.5) from textbook.

ENERGY-AVERAGED REACTION RATES

- Ability to sustain chain reactions depends on the distribution of ⓝ in energy.
- ⓝ distribution in energy is determined by composition of non fissile materials in core & their effectiveness in slowing down ⓝ from fast to thermal range.
- In order to determine overall characteristics of reactor we must average XS & other data over energy spectrum of ⓝ.

Reaction rates are for ⓝ of energy $E \rightarrow \Sigma_x(E)\varphi(E)$
for all ⓝ $\rightarrow \int_0^\infty \Sigma_x(E)\varphi(E)\,dE$

Rxn rates are commonly expressed as product of energy averaged xs & ⓝ flux.

$$\int_0^\infty \Sigma_x(E)\varphi(E)\,dE = \bar{\Sigma}_x \Phi$$

$$\rightarrow \bar{\Sigma}_x = \frac{\int_0^\infty \Sigma_x(E)\varphi(E)\,dE}{\int_0^\infty \varphi(E)\,dE} \quad \& \quad \Phi = \int_0^\infty \varphi(E)\,dE.$$

If we recall $\Sigma_x = N\sigma_x$, & sub that above, we can get a relation for $\bar{\sigma}_x$

$$\int_0^\infty \sigma_x(E)\varphi(E)\,dE = \bar{\sigma}_x \Phi \quad \text{where} \quad \bar{\sigma}_x = \frac{\int_0^\infty \sigma_x(E)\varphi(E)\,dE}{\int_0^\infty \varphi(E)\,dE}.$$

- Now, ~~we know, the on~~ we know that $\varphi(E)$ does not behave the same way for all energy. i.e. No one function can describe $\varphi(E)$ over the entire energy spectrum of ⓝ.

- So we break the energy range into multiple parts where ~~even~~ $\varphi(E)$ can be determined to a fair amount of accuracy & calculate our average xs accordingly.

Soln.

We have looked at 3 specific energy ranges

- fast
- intermediate
- thermal.

if we break energy range down into 3 ~~segments~~ segments (groups) more appropriate as it refers to multigroup method to resolve energy dependence.

So, reaction rates are

$$\int \sigma_x(E)\varphi(E)dE = \int_T \sigma_x(E)\varphi(E)dE + \int_I \sigma_x(E)\varphi(E)dE + \int_F \sigma_x(E)\varphi(E)dE$$

sub

$T \rightarrow$ thermal $\quad 0 \le E \le 1\,eV$

$I \rightarrow$ Intermediate $\quad 1eV \le E \le 0.1 MeV$

$F \rightarrow$ fast $\quad 0.1MeV \le E \le \cancel{\infty}\ 10\,MeV$.

Now,

$$\int_0^\infty \sigma_x(E)\,\varphi(E)\,dE = \bar{\sigma}_x \phi$$

so substituting that above,

$$\bar{\sigma}_x \phi = \bar{\sigma}_{xT}\phi_T + \bar{\sigma}_{xI}\phi_I + \bar{\sigma}_{xF}\phi_F$$

$$\phi_T = \int_T \varphi(E)dE \ , \quad \phi_I = \int_I \varphi(E)dE \ , \quad \phi_F = \int_F \varphi(E)dE .$$

&

$$\bar{\sigma}_{xT} = \frac{\int_T \sigma_x(E)\varphi(E)dE}{\int_T \varphi(E)dE}, \quad \bar{\sigma}_{xI} = \frac{\int_I \sigma_x(E)\varphi(E)dE}{\int_I \varphi(E)dE}, \quad \bar{\sigma}_{xF} = \frac{\int_F \sigma_x(E)\varphi(E)dE}{\int_F \varphi(E)dE}$$

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Lets look at each group separately now.

Fast xs

$$\varphi(E) = \chi(E) \frac{S_f'''}{\Sigma_t(E)}.$$

If we assume $\Sigma_t(E) \sim \Sigma_t$ - energy independent (it is not but assuming this simplifies analysis)

& normalize $\frac{S_f'''}{\Sigma_t} \sim 1$.

we have $\varphi(E) \simeq \chi(E)$.

Then, $$\bar{\sigma}_{xF} = \int_{0.1\,\text{mev}}^{10\,\text{mev}} \sigma_x(E)\varphi(E)\,dE \Big/ \int_{0.1}^{10} dE\, \varphi(E).$$

$$\simeq \int_{0.1}^{10} \sigma_x(E)\, \chi(E)\, dE \Big/ \int_{0.1}^{10} dE\, \chi(E)$$

Note: we know that $\chi(E)$ is insignificant for $E < 0.1$ & $E > 10$ Me so extending limits to 0 & ∞ does not affect integral eval.

Using that fact,

$$\bar{\sigma}_{xF} = \int_0^{\infty} \sigma_x(E)\, \chi(E)\,dE \Big/ \cancel{\int_0^{\infty} dE\, \chi(E)}^{\,1\ \text{(normalization)}} = \int_0^{\infty} \sigma_x(E)\, \chi(E)\,dE.$$

$$\Rightarrow \boxed{\bar{\sigma}_{xF} = \int_0^{\infty} \sigma_x(E)\, \chi(E)\, dE}$$

Intermediate range:

$$\varphi(E) = \frac{q}{\bar{\xi}\,\Sigma_t(E)\,E} \qquad \text{Choose } \overset{\text{normalization}}{\frac{q}{\Sigma_t(E)\,\bar{\xi}}} \sim 1$$

$\rightarrow \quad \varphi(E) \sim 1/E.$

Then, $\bar{\sigma}_{xI} = \int_I \frac{\sigma_x(E)}{E}\,dE \Big/ \int_I dE/E$

Define

I_x = Resonance integral = $\int_I \frac{\sigma_x(E)\,dE}{E}$

~~Re~~ then

$$\bar{\sigma}_{xI} = I_x \Big/ \int_I dE/E = I_x \Big/ \int_{1eV}^{0.1MeV} \frac{dE}{E} = 0.0869 I_x$$

$$\Rightarrow \boxed{\bar{\sigma}_{xI} = \cancel{0.8}\ 0.0869\, I_x}.$$

- Note table 3.2, ~~[illegible]~~ has XS data & resonance integral data.

Thermal XS average.

$\varphi(E) \approx \varphi_M(E)$

We know, $\varphi_M(E) \approx 0$ if $E > 1\,\text{MeV}$

So upper limit of integration can be extended to ∞.

$$\bar{\sigma}_{xT} = \int_T \sigma_x(E)\varphi(E)\,dE \Big/ \int_T \varphi(E)\,dE$$

$$= \int_0^{1eV} \sigma_x(E)\varphi_M(E)\,dE \Big/ \int_0^{1eV} \varphi_M(E)\,dE.$$

$$= \int_0^{\infty} \sigma_x(E)\varphi_M(E)\,dE \Big/ \int_0^{\infty} \varphi_M(E)\,dE$$

1 (normalization)

=> $$\boxed{\bar{\sigma}_{xT} = \int_0^\infty \sigma_x(v)\, \phi_M(v)\, dv}$$

- ~~Next time, we look at talk more about XS & corrections to thermal XS averages.~~

- ~~We go over HW2 & review Ch 2.~~

If there is time, lets discretize diffusion eqn. & solve it in Matlab.

$$-\frac{\partial}{\partial x} D \frac{\partial \phi}{\partial x} + \Sigma_a \phi = S$$

$$S = \nu \Sigma_f \phi + q$$

q = some f^n that defines external source.
lets say this is constant.

<u>HW</u>: <u>3.4</u>, ~~3.9 (see page 74)~~, 3.10, $\underbrace{3.11,\ 3.12}_{\text{after Monday's class}}$.