# HW 4 Sol$^n$

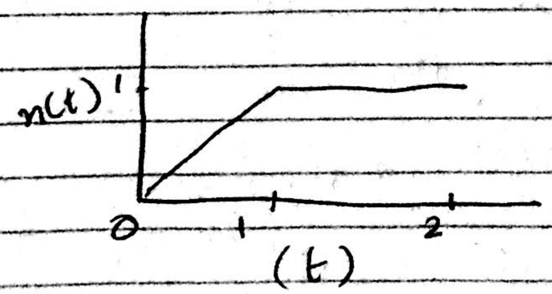
5.3 At $t=0$ there are no n in a reactor. n source is inserted into the reactor at $t=0$ & withdrawn at $t=1$ min. Sketch n population for $0 \le t \le 2$ min.

(a) Subcritical
(b) Critical.

(a) ~~Subcritical~~ We have a time dependent source. So recall section 5.3 & manipulation of Eq. 5.26 (see notes)

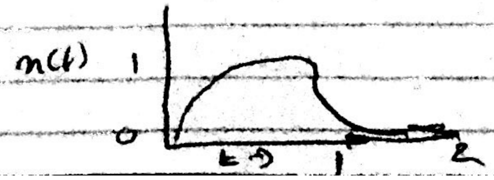
(b) Critical. $n(t) \sim \cancel{S(t)}\ S_0 t$.

So when src. is present, $n(t)$ will grow linearly with time. After src. is removed, ~~the~~ system is still critical & n population flatters out.




(a) Subcritical when there is a source term, we have Eq. 5.27, in absence, we have (5.24)

In presence of source, n population will increase first & then exponentially decrease away according to (5.24.)



(5.6) A thermal U reactor operates at 1 W. The operator is to operate power to 1 kW over a 2hr span.

(a) what should be the reactor period?
(b) How many cents of reactivity must be present to achieve part (a)?

→ Recall the definition of reactor period.

$n(t) \propto e^{t/T}$ where T is the reactor period.
~~If P(t) ∝ n(t) the~~ T is the amount of time it takes a reactor power to increase/~~decrease~~ by a factor of e.

Here $\Delta t = 2$ hrs.
factor of power increase = $10^3$

Now $n(t) \propto e^{t/T}$
$P(t) \propto n(t)$
$\Rightarrow P(t) \propto e^{t/T}$.
$\Rightarrow \frac{P_1(t)}{P_2(t)} = \frac{e^{t_1/T}}{e^{t_2/T}}$

$\Rightarrow \frac{1}{10^3} = e^{(t_1-t_2)/T}$

$\Delta t$

$\Rightarrow 10^3 = e^{t_2-t_1/T}$
$\Rightarrow 10^3 = e^{2/T}$
$\Rightarrow T = 2/\ln(10^3) = \underline{\underline{0.29 \text{ hrs}}}$

(b) How many cents of reactivity is required to get a ?

Assuming small, positive reactivity insertion, we can use Eq. 5.57.

$T \approx \beta / \rho\lambda$ ; Moreover $\$ = \rho/\beta$.

$$\Rightarrow T\lambda \approx \frac{\beta}{\rho} = \frac{1}{\$}$$

$$\Rightarrow \boxed{\$ = \frac{1}{T\lambda}}$$

So now, what is $\lambda$?

$\Rightarrow$ Use Eq. 5.34. $t_{1/2} = 0.693/\lambda_i$ $\Rightarrow \lambda_i = \frac{0.693}{t_{1/2}}$

$$\boxed{\frac{1}{\lambda} = \frac{1}{\beta} \sum_{i=1}^{6} \beta_i \frac{1}{\lambda_i}}$$

Use info in Table 5.1.

If you plug all those values in R use the above eqn.

$\lambda = 0.077 \; s^{-1}$

$$\Rightarrow \$ = \frac{1}{T\lambda} = \frac{1}{0.29 \times 60 \times 60 \times 0.077} = 0.0124 \$$$

$= \boxed{1.24 \text{¢}}$

5.13 A critical reactor operates at a power level of 80W. Dropping a control rod into the core causes flux to undergo a sudden decrease of 60W. How many \$ is control rod worth?

Use Eq 5.63

reactivity drop measured in \$

$$(\rho)/_{\beta} = \text{rod worth} = \frac{n_0}{n_1} - 1$$

$$= \frac{P_0}{P_1} - 1 \quad (\text{as } P \propto n)$$

$$= \frac{80}{60} - 1$$

$$= \boxed{0.33\$}$$

① Go to 5.8 from 5.7

$$5.7 \rightarrow \quad \frac{d}{dt} n(t) = S_0 - \frac{1}{l_\infty} n(t)$$

~~Integral~~ $$\frac{d}{dt} n(t) + \frac{1}{l_\infty} n(t) = S_0.$$

Integrating factor $= e^{\int \frac{1}{l_\infty} dt} = e^{t/l_\infty}$

multiply with eqn.

$$\cancel{e^{t/l}} \; e^{t/l_\infty} \left( \frac{d}{dt} n(t) + \frac{1}{l_\infty} n(t) \right) = e^{t/l_\infty} S_0.$$

$$\hookrightarrow \quad e^{t/l_\infty} \frac{d}{dt} n(t) + \frac{e^{t/l_\infty}}{l_\infty} n(t) = S_0 e^{t/l_\infty}$$

$$\hookrightarrow \quad \frac{d}{dt} \left( n(t) e^{t/l_\infty} \right) = S_0 e^{t/l_\infty}$$

Now integrate both sides from 0 to t

$$\int_0^t dt \left[ dt \left( n(t) e^{t/l_\infty} \right) = S_0 e^{t/l_\infty} \right]$$

$$\Rightarrow \quad \left[ n(t) e^{t/l_\infty} \right]_0^t = S_0 \int_0^t dt \; \cancel{s} \; e^{t/l_\infty}$$

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$$n(t)\, e^{t/l_\infty} - \cancel{n(0)}^{0} = S_0\left(l_\infty\left(e^{t/l_\infty} - 1\right)\right)$$

$$\Rightarrow \boxed{n(t)\, e^{t/l_\infty} = S_0 l_\infty \left(e^{t/l_\infty} - 1\right)}$$

$\hookrightarrow$ 5.8

(3) Calculate $\lambda$.

$$\frac{1}{\lambda} = \frac{1}{\beta}\sum_{i=1}^{6} \beta_i \frac{1}{\lambda_i} = \frac{1}{\beta}\sum_{i=1}^{6} \beta_i/\lambda_i$$

$$\lambda_i = \frac{0.693}{t_{1/2,i}} \Rightarrow \frac{1}{\lambda} = \frac{1}{\beta}\sum_{i=1}^{6}\left(\beta_i \Big/ \frac{0.693}{t_{1/2,i}}\right)$$

$$= \frac{1}{\beta}\sum_{i=1}^{6} \frac{\beta_i t_{1/2,i}}{0.693}$$

Using values in table 5.1

$$\frac{1}{\lambda} = \frac{1}{0.00650}\left(\frac{(56)(0.00021) + 23(0.00142) + 6.2\,(0.00128)\ \ldots}{0.693}\right)$$

Eventually

$$\Rightarrow \boxed{\lambda = 0.077\ s^{-1}}$$

(2) prove $k_\infty > 1$ super.

$k < 1$ sub.

$$n = n_0 e^{\frac{k_\infty - 1}{l_\infty} t}$$

if $k_\infty > 1$, exponential growth of $n(t) \Rightarrow$ super

$k_\infty < 1$, exp. decay of $n(t) \Rightarrow$ sub.

If $k_\infty = 1$, const. $n(t) \Rightarrow$ critical.

5.10 $\frac{d}{dt} n(t) = S(t) + \frac{(\rho-\beta)}{\Lambda} n(t) + \lambda C(t).$

$$\frac{d}{dt} C(t) = \frac{\beta}{\Lambda} n(t) - \lambda C(t).$$

Assume $\frac{1}{\lambda\Lambda}|\beta-\rho| \gg 1$ & $\frac{\beta}{\lambda\Lambda} \gg 1$

$S(t) = 0$, $n_0 = n(0)$ At time $t=0$, $\rho$ is inserted into the reactor.

(a) $S(t) = 0$, let $n(t) = n e^{\omega t}$

$C(t) = C e^{\omega t}$

Substitute in differential eqn.

$$\begin{cases} \frac{dn(t)}{dt} = \frac{\rho-\beta}{\Lambda} n(t) + \lambda C(t) \\ \frac{dC(t)}{dt} = \frac{\beta}{\Lambda} n(t) - \lambda C(t) \end{cases}$$

$$\begin{cases} \frac{d}{dt} n e^{\omega t} = \frac{\rho-\beta}{\Lambda} n e^{\omega t} + \lambda C e^{\omega t} \\ \frac{d}{dt} C e^{\omega t} = \frac{\beta}{\Lambda} n e^{\omega t} - \lambda C e^{\omega t} \end{cases}$$

$$\begin{cases} n\omega e^{\omega t} = \frac{\rho-\beta}{\Lambda} n e^{\omega t} + \lambda C e^{\omega t} \\ C\omega e^{\omega t} = \frac{\beta}{\Lambda} n e^{\omega t} - \lambda C e^{\omega t} \end{cases}$$

Cancelling $e^{\omega t}$ returns

$$\omega n = \frac{\rho-\beta}{\Lambda} n + \lambda C$$

$$\omega C = \frac{\beta}{\Lambda} n - \lambda C$$

$$+ \quad + \quad +$$

$$\omega n + \omega C = \frac{\rho-\beta}{\Lambda} n - \frac{\beta}{\Lambda} n$$

$$\Rightarrow \quad \omega C = \frac{\rho}{\Lambda} n - \omega n$$

$$\Rightarrow \quad C = \frac{1}{\omega}\left(\frac{\rho}{\Lambda} - \omega\right) n.$$

Substitute

we get, $\omega n = \frac{\rho-\beta}{\Lambda} n + \frac{\lambda}{\omega}\left(\frac{\rho}{\Lambda} - \omega\right) n$

Simplification returns a quadratic eqn.

$$\Lambda \omega^2 + (\lambda\Lambda + \beta - \rho)\omega - \lambda\rho = 0.$$

Now, $\frac{1}{\Lambda\lambda}|\beta-\rho| \gg 1 \Rightarrow |\beta-\rho| \gg \lambda\Lambda.$

$\Rightarrow$ In above eqn. Ignore $\lambda\Lambda$ in favor of $\beta-\rho$ to get

$$\boxed{\Lambda\omega^2 + (\beta-\rho)\omega - \lambda\rho = 0}$$

Solving this quadratic eqn. returns roots

$$\omega = \frac{1}{2\Lambda}\left[-(\beta-\rho) \pm \sqrt{(\beta-\rho)^2 + 4\lambda\Lambda\rho}\right]$$

$$= \frac{\beta-\rho}{2\Lambda}\left[-1 \pm \sqrt{1 + \frac{4\lambda\Lambda\rho}{(\beta-\rho)^2}}\right]$$

Now, $\frac{1}{\lambda\Lambda}(\beta-\rho) >> 1$

$\Rightarrow \quad \frac{\lambda\Lambda}{\beta-\rho} << 1$

so $\sqrt{1 + \frac{4\lambda\Lambda\rho}{(\beta-\rho)^2}} \sim 1 + 2\lambda\Lambda\rho(\beta-\rho)^2$

solutions will be

$$\omega_1 = \lambda\rho/\beta-\rho \qquad \omega_2 = \frac{\rho-\beta}{\Lambda}$$

Now, $n(t) = n_1 e^{\omega_1 t} + n_2 e^{\omega_2 t}$

$C(t) = C_1 e^{\omega_1 t} + C_2 e^{\omega_2 t}$

Next we find $n_1$ & $n_2$,

To find $n_1$ & $n_2$, for a critical initial condition

$C(0) = \frac{\beta}{\lambda\Lambda} n(0)$ (from 2nd kinetics equation)

Also, $n(0) = n_1 e^{\omega_1(0)} + n_2 e^{\omega_2(0)} = n_1 + n_2$.

Recall that

$$C = \omega\left(\frac{\rho}{\omega} - \omega\right)n = \left(\frac{\rho}{\Lambda\omega} - 1\right)n$$

then $C(0) = \frac{\beta}{\lambda\Lambda} n(0) = \left(\frac{\rho}{\Lambda\omega_1} - 1\right)n_1$

$$+ \left(\frac{\rho}{\Lambda\omega_2} - 1\right)n_2.$$

Adding equations for $n(0)$ & inserting values of $\omega_1$ & $\omega_2$ returns

$$\left(\frac{\beta}{\lambda\Lambda} + 1\right)n(0) = \frac{(\beta - \rho)}{\lambda\Lambda} n_1 + \frac{\beta}{\rho - \beta} n_2$$

Since $\frac{1}{\lambda\Lambda}|\beta - \rho| \gg 1$ & $\frac{\beta}{\lambda\Lambda} \gg 1$ a

$\frac{\beta}{\rho - \beta}$ cannot be large without violating constraint that $\beta \neq \rho$, we have $\frac{\beta}{\lambda\Lambda} n(0) = \frac{(\beta - \rho)}{\lambda\Lambda} n_1$

$\therefore$ $n_1 = \dfrac{\beta}{\beta-\rho} n(0)$ ; $n_2 = n(0) - n_1$

$= \dfrac{\rho}{\rho-\beta} n(0)$.

Combining $n_1, n_2, \omega_1, \omega_2$ & $n = n_1 e^{\omega_1 t} + n_2 e^{\omega_2 t}$,

we get,

$$n(t) = n(0)\left( \frac{\rho}{\rho-\beta} e^{\left(\frac{\rho-\beta}{\lambda}\right)t} + \frac{\beta}{\beta-\rho} e^{(\lambda\rho/\beta-\rho)t} \right),$$

(h) Show for long time that solution is independent of $\lambda$ when $\rho > \beta$ & independent of $\lambda$ when $0 < \rho < \beta$.

Note that when $\rho > \beta$, ~~the~~ $1^{st}$ exponential will grow & $2^{nd}$ one will decay away.

$\therefore$ ~~$n(t) = n(0)\frac{\rho}{\rho}$~~

$$\therefore n(t) = n(0) \frac{\rho}{\rho-\beta} e^{\left(\frac{\rho-\beta}{\lambda}\right)t}$$ (independent of $\lambda$.)

Similarly $1^{st}$ term decays away when $\rho < \beta$.

$$\Rightarrow n(t) = n(0) \frac{\beta}{\beta-\rho} e^{(\lambda\rho/\beta-\rho)t}$$

– independent of ~~at~~ $\lambda$.