Kinetics

Lec-4

Kinetics equations (multiplying, finite media with delayed n)

$$\frac{d}{dt} n(t) = s(t) + \sum_i \lambda_i C_i(t) + \frac{(1-\beta)k-1}{l} n(t)$$

$$\frac{d}{dt} C_i(t) = \beta_i \frac{k}{l} n(t) - \lambda_i C_i(t).$$

Steady state solⁿ:

$$\bar{l} = l + \frac{\beta}{\lambda}.$$

$$\boxed{n = \frac{S_0 l}{1-k}}$$

Reactivity formulation

$\rho = \frac{k-1}{k}$ $\quad \Lambda = \frac{l}{k}$ → prompt generation time.

$$\frac{d}{dt} n(t) = s(t) + \frac{\rho-\beta}{\Lambda} n(t) + \sum_i \lambda_i C_i(t)$$

$$\frac{d}{dt} C_i(t) = \frac{\beta_i}{\Lambda} n(t) - \lambda_i C_i(t).$$

Steady state solⁿ:

$$\boxed{n = \frac{S_0 \Lambda}{|\rho|}} \quad ; \quad \boxed{C_i = \frac{\beta_i}{\lambda_i \Lambda} n}$$

Reactor Period (τ): $S(t) \to 0$ & solve kinetics equation.

Length of time required for reactor power to increase or decrease by a factor of e.

$n(t) = Ae^{\omega t} \rightarrow \quad T \approx \frac{1}{\omega}$ (~~Soln~~ of form)

$C_i(t) = \beta_i e^{\omega t} \quad i = 1..6.$

7 soln.

Inhour equation:

$$\boxed{\rho = \omega \left(\Lambda + \sum_i \frac{\beta_i}{\omega + \lambda_i} \right)}$$

graphical solution.

Plotting returns

7 ω's. $\quad \omega_1 > \omega_2 \; \ldots > \omega_7$

$n(t) = \sum_{i=1}^{7} A_i e^{\omega_i t}$ (because 7 ω's)

↳ over time since $\omega_1 > \ldots > \omega_7$.

$$\boxed{n(t) \approx A_1 e^{\omega_1 t}}$$

& $\boxed{T = \frac{1}{\omega_1}}$

Special cases:

$\rho = \beta$ → prompt critical. (delayed n not required for criticality)

Don't want reactors to be prompt critical.

\$ re-worth of reactivity → $\$ = \frac{\rho}{\beta}$.

$\rho < 0$. (-ve reactivity → subcritical reactor)

from graph of in-hour equation,

~~ρ ... $\rightarrow 0$~~ $\omega_1 \rightarrow \lambda_1$ (asymptotically)

~~ω_1 ...~~ $\Rightarrow \boxed{T = \frac{1}{\lambda_1}}$

$\rho > 0$ but small (usually the case)

$\rho = f(\omega) \rightarrow 0$

$\Rightarrow \omega_1 \rightarrow 0$.

& $\omega_1 << \lambda_i$ for all i.

$$\rho = \left(\Lambda + \sum_i \frac{\beta_i}{\omega + \lambda_i}\right)\omega$$

Eliminate ω_1 from

$$\Rightarrow \rho = \left(\Lambda + \sum \frac{\beta_i}{\lambda_i}\right)\omega_1 = \left(\Lambda + \frac{\beta}{\lambda}\right)\omega_1$$

$$\Rightarrow \boxed{\omega_1 = \frac{\rho}{\Lambda + \beta/\lambda}}$$

In practice $\Lambda \to$ prompt generation time is extremely small.

$$\Lambda << \beta/\lambda$$

$$\Rightarrow \quad \omega_1 \approx \frac{\rho}{\beta/\lambda} \Rightarrow \frac{\lambda\rho}{\beta}$$

$$\Rightarrow \boxed{\frac{1}{\omega_1} = T = \frac{\beta}{\lambda\rho}}$$

Above prompt critical $\rho > \beta$.

When reactor is prompt critical, exponential growth is rapid.

$T \to$ small

$\Rightarrow \frac{1}{T} \to \omega_1 \to$ large.

$\Rightarrow$ Say, $\omega_1 >> \lambda_i$

$$\Rightarrow \quad \rho = \left(\Lambda + \sum_i \frac{\beta_i}{\omega+\lambda_i}\right)\omega$$

$$= \left(\Lambda + \sum_i \frac{\beta_i}{\omega}\right)\omega .$$

$$\rho = \cancel{\boxed{\Lambda\omega + \beta}}$$

$$\rho = \Lambda\omega + \beta$$

$$\Rightarrow \boxed{\omega = \frac{\rho - \beta}{\Lambda}}$$

$$T = \frac{1}{\omega_1} = \boxed{\frac{\Lambda}{\rho - \beta}} \rightarrow 7$$

HW: Read Step reactivity changes (sec 5.5) & summarize it.

Prompt Jump approximation to solve kinetics equation.

As soon as a step reactivity (a bunch of it is inserted at once) is inserted in a reactor,

ⓝ population changes rapidly at first & then changes are slower.

Moreover, precursor ch concentration changes slowly over time of rapid initial change in ⓝ population .. "prompt jump"

This is ~~possible~~ true because, $t_{1/2} > \Lambda$

for delayed ⓝ

prompt ⓝ generation time.

This initial ~~reactivity~~ $n(t)$ jump due to prompt ⓝ generation.

This is prompt jump approximation.

Example: Sudden change in reactivity due to insertion of control rod in reactor.

Rod drop:

According to prompt jump approximation, concentration change is miniscule.

$$\Rightarrow \cancel{\frac{d}{dt} C_i(t)}^{\,0} = \frac{\beta_i}{\Lambda} \underbrace{n(t)}_{n_0} - \lambda_i C_i(t).$$

$n(t)$ (initial concentration)

$$\Rightarrow C_{i0}\lambda_i = \frac{\beta_i}{\Lambda} n_0$$

$$\Rightarrow \boxed{C_{i0} = \frac{\beta_i}{\Lambda\lambda_i} n_0}$$

Immediately after rod insertion, precursor concentration stays C_{i0} as long as $\lambda_i t << 1$.

Source free kinetics equation becomes

$$\frac{d}{dt} n(t) = \frac{\rho - \beta}{\Lambda} n(t) + \sum_i C_i(t)\lambda_i$$

ρ will be -ve coz rod drop

$$\Rightarrow \frac{d}{dt} n(t) = -\frac{(|\rho| + \beta)}{\Lambda} n(t) + \sum_i \lambda_i \frac{\beta_i}{\lambda_i \Lambda} n_0$$

$$= -\frac{(|\rho| + \beta)}{\Lambda} n(t) + \frac{n_0}{\Lambda} \sum_i \beta_i$$

$$\boxed{\frac{d}{dt} n(t) = -\frac{(\beta + |\rho|)}{\Lambda} n(t) + \frac{n_0}{\Lambda}\beta} \qquad t << 1/\lambda_i$$

Solution (integration factor method) returns:

$$n(t) = \frac{\beta}{|\rho|+\beta} n_0 + \frac{|\rho|}{|\rho|+\beta} n_0 e^{-1/\Lambda (|\rho|+\beta)t}$$

Λ tiny so ~~over time~~ ignore that term ~~decays away~~.

$$\Rightarrow n(t) \sim \frac{\beta}{|\rho|+\beta} n_0$$

Since β, $|\rho|$ & n_0 are constant $n(t) \to n_1$.

$$n_1 = \frac{\beta}{|\rho|+\beta} n_0$$

$$\Rightarrow \frac{n_1}{n_0} = \frac{\beta}{|\rho|+\beta} \Rightarrow \frac{n_0}{n_1} = \frac{|\rho|+\beta}{\beta}$$

$$\Rightarrow \boxed{\frac{n_0}{n_1} - 1 = \frac{|\rho|}{\beta}} = \text{\$ worth of reactivity insertion.}$$

See source jerk & rod oscillation if you want.

HW - 5.3, 5.6, 5.10, 5.13.