Dragon Lord.

Midterm 2 Sol^n

② Derive SS diffusion equation.

2 ways:

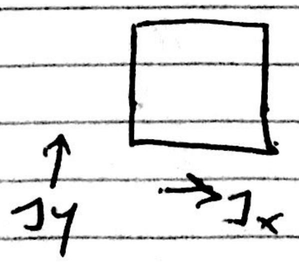
① Derive equation in general geometry ~~&~~ & reduce to 2D.

So do as in notes to get $-\underline{\nabla}.D\underline{\nabla}\phi + \Sigma_a\phi = S + \nu\Sigma_f\phi$
then

$$\underline{\nabla} = \frac{\partial}{\partial x}\hat{i} + \frac{\partial}{\partial y}\hat{j}$$

∴ In 2D $\cancel{\frac{\partial^2\phi}{\partial x^2}} \cancel{+ \frac{\partial^2\phi}{\partial}} - \left(\frac{\partial}{\partial x} D\frac{\partial}{\partial x}\phi + \frac{\partial}{\partial y} D\frac{\partial}{\partial x}\phi\right)$
$+ \Sigma_a\phi = \nu\Sigma_f\phi + S.$

② Do balance like in notes but for a rectangle/ sq. elem. volume



③ Derive kinetics eqn.

Do everything that you would do for a 7 group problem but now instead of 7 grp. you'll have 4.

So $i = 1, 2, 3, 4$ and not $i = 1, 2, 3, 4, 5, 6, 7$.

Material properties like λ, β etc are now averaged & lumped into 4 groups instead of 7.

④ See notes.

⑤ 5.7

Thermal reactor fueled with U operates for at 1W. Operator puts in 15 min period.

How long will it take reactor to reach 1 MW?

$$n(t) = n(0)\, e^{t/T}$$

$\hookrightarrow$ $P(t) = P(0)\, e^{t/T}$ (as $P(t) \propto n(t)$)

$\hookrightarrow$ $t = T \ln\left(\frac{P(t)}{P(0)}\right) = 15 \cdot \ln\left(\frac{10^6}{1}\right) = 207 \text{ min}$

$= 3.45 \text{ hrs}$

(6) 5.9 Find reactor period for reactor fuel by U-235, Pu-239, U-233.

~~(a)~~(a) 1 ¢ reactivity added.
(~~a~~ (b) 1¢ reactivity removed.

Reactivity small ⇒ $\omega_i << \lambda_i$

Eventually use $T = \beta/(\lambda\rho)$ for both a & b.

$T = \frac{\beta}{\rho\lambda}$; ~~β ⇒ Constant for all Isotopes.~~

$\beta/\lambda_{U233} = 0.0494\cancel{27}$	Use table 5.1 & eqns. on pg 123.
$\beta/\lambda_{U235} = 0.076625$	
$\beta/\lambda_{Pu239} = 0.063676$.	

(a) $\rho = 0.01$

∴ U-233 → $T = \frac{100}{0.0494} = 33.7$.

U235 → 21.8 mm

Pu-239 → T = 26.2 mm.

(b) negative of a values

(7) 9.12 fuel: 2% U-235
1% Pu-239
97% U-238

find η_T =

from U^{235} & Pu^{239}

$$\eta_T = \frac{\nu \Sigma_f^F}{\Sigma_a^F} = \frac{0.02N\,\nu^{235}\sigma_f^{235} + 0.01N\,\nu^{239}\sigma_f^{239}}{0.02N\,\sigma_a^{235} + 0.01N\,\sigma_a^{239} + 0.97N\,\sigma_a^{238}}$$

from U235, U-238, Pu-239

= 1.87

(8) 1.17 Suppose radioactive Co & Sr are allowed to decay for 10 yrs.

After 10 yrs 1 Ci of Co-60 remain. How many Ci of Sr will remain?

let $C \to Co$, $S \to Sr$.

at $t = 0$, $\lambda_c N_c(0) = \lambda_s N_s(0)$ α # Ci

at $t = 10$ yr, $1\,Ci = \lambda_c N_c(0) e^{-\lambda_c t}$

$\therefore$ $\lambda_c N_c(0) = 1\,Ci\ e^{\lambda_c t}$

# of Ci of Sr after 10 yrs = $\lambda_s N_s(0) e^{-\lambda_s t}$

but $\lambda_c N_c(0) = \lambda_s N_s(0)$ ∴

# of Ci of Sr = 1 Ci $e^{(\lambda_c - \lambda_s) t}$

$= \underline{4.73 \text{ Ci}}$

(9) 2.14 from textbook.
(n) scatter elastically at 1 MeV.
After one scattering collision. determine fraction of (n) that will have energy of less than 0.5 MeV. if they scatter from

(a) H (b) H^2 (c) C-12 (d) U-238

You calculate this using:

$$P(E \to E')\,dE' = \begin{cases} \dfrac{1}{(1-\alpha)E}\,dE' & \alpha E \le E' \le E \\ 0 & \text{otherwise} \end{cases}$$

This returns probability that after elastic scatter, (n) will have E' after collision.

Then fraction that will have energy less than a particular value E_1 will be

$$\int_{\alpha E}^{E_1} P(E \to E')\,dE'$$

$\therefore$ fraction:

$$\begin{cases} \displaystyle\int_{\alpha E}^{E_1} \frac{1}{(1-\alpha)E}\, dE' = \frac{E_1 - \alpha E}{1-\alpha} = \frac{1}{(1-\alpha)}\left(\frac{E_1}{E} - \alpha\right) & \text{if } \alpha < E_1/E \\ 0 & \text{otherwise.} \end{cases}$$

Here $E_1 = 0.5\,\text{MeV}$, $E = 1\,\text{MeV}$.

$$\alpha = \left(\frac{A-1}{A+1}\right)^2$$

$\therefore$ (a) H

$$\alpha = \left(\frac{1-1}{1+1}\right)^2 = 0$$

$\therefore$ fraction $= \dfrac{1}{1-0}\left(\dfrac{0.5}{1} - 0\right)$

$= \underline{\underline{0.5}}$

(b) H^2 $\quad \alpha = \left(\dfrac{2-1}{2+1}\right)^2 = \dfrac{1}{9}$

fraction $= \dfrac{1}{1-\alpha}\left(\dfrac{E_1}{E} - \alpha\right)$

$= \dfrac{1}{1-1/9}\left(\dfrac{0.5}{1} - 1/9\right) = \dfrac{9}{8}\left(\dfrac{3.5}{9}\right)$

$= 3.5/8 = \underline{0.4375}$

(c) C-12.

$$\alpha = \left(\frac{A-1}{A+1}\right)^2 = \left(\frac{11}{13}\right)^2 = 0.716 > 0.5$$

$\Rightarrow$ fraction = 0.

(d) U-238 $\alpha > \frac{E_1}{E} \Rightarrow$ fraction = 0.

(b) 3.10 from textbook but with 5% enrichment

(a) find slowing down decrement, $\cdot\, \xi$
(b) slowing down power - $\xi\Sigma_s$
(c) slowing down ratio - $\xi\, \Sigma_s / \Sigma_a$. (thermal)

use table 3.1

$$\Sigma_{s_i} = \xi_i\, \Sigma_{Ji} / \xi_j$$

$$\xi_J^{D_2O} = 0.353 \qquad \xi_J^{H_2O} = 1.38.$$

$$\Sigma a_i = \xi_i \Sigma_{Ji} / \xi_J \Sigma_{Ji} / \Sigma a_i$$

$$\Sigma a^{D_2O} = 857.10^{-6} \qquad \Sigma a^{H_2O} = 0.022.$$

(a) $\bar{\xi} = \frac{1}{\Sigma_J} \sum_1 \xi_i \Sigma_{Ji}$

for 5% Contamination,

$$\bar{\xi} = \frac{\xi_{D_2O}\, 0.95\, \Sigma_s^{D_2O} + \xi_{H_2O}\, 0.05\, \Sigma_s^{H_2O}}{0.95\, \Sigma_s^{D_2O} + 0.05\, \Sigma_s^{H_2O}}$$

$$= \frac{0.51 \cdot 0.95 \cdot 0.353 + 0.93 \cdot 0.05 \cdot 1.38}{0.95 \cdot 0.353 + 0.05 \cdot 1.38}$$

$$= \underline{0.5817}$$

(c) ~~$\bar{\xi}\, \Sigma_s / \Sigma_a = \frac{\xi\ \xi_s\ \Sigma_s'}{(0.95\, \Sigma_a^{D_2O} + 0.01\, \Sigma_a^{H_2O})}$~~

=

(b) Slowing down power

$$\xi \Sigma_s = \xi_{mix}\, \Sigma_{s,mix}$$

$$= (0.5817)(0.95\, \Sigma_s^{D_2O} + 0.05\, \Sigma_s^{H_2O})$$

$$= (0.5817)(0.95\,(0.353) + 0.05 \cdot 1.38)$$

$$= \underline{0.2352}$$

(c) $\frac{\bar{\xi}\, \Sigma_{s\,mix}}{\Sigma_{a\,mix}} = \frac{\bar{\xi}\, \Sigma_s}{\Sigma_{a\,mix}} = \frac{0.2352}{(0.95\ 8.57\times10^{-6} + 0.05\ 0.022)}$

$= \underline{212.24}$

(11) 3D diff. eq.

$-\nabla \cdot D \nabla \phi + \Sigma_a \phi = \frac{\nu \Sigma_f}{k} \phi + S.$

$= -\left(\frac{\partial}{\partial x} D \frac{\partial}{\partial x} \phi + \frac{\partial}{\partial y} D \frac{\partial}{\partial y} \phi + \frac{\partial}{\partial z} D \frac{\partial}{\partial z} \phi\right) + \Sigma_a \phi$

$= \frac{\nu \Sigma_f}{k} \phi + S.$

(a) strong form: same eq.

$-\left(\frac{\partial}{\partial x} D \frac{\partial}{\partial x} \phi + \frac{\partial}{\partial y} D \frac{\partial}{\partial y} \phi + \frac{\partial}{\partial z} D \frac{\partial}{\partial z} \phi\right) + \Sigma_a \phi = \frac{\nu \Sigma_f \phi}{k} + S$

(a) WR form.

$\int_V dV \left(-\frac{\partial}{\partial x} D \frac{\partial}{\partial x} \phi - \frac{\partial}{\partial y} D \frac{\partial}{\partial y} \phi - \frac{\partial}{\partial z} D \frac{\partial}{\partial z} \phi + \Sigma_a \phi - \frac{\nu \Sigma_f \phi}{k} - S\right) u$

(b) Weak form.

wr.

$dV \to dx\,dy\,dz$

$V \to$ some volume element

$$\int_V dx\,dy\,dz\ \omega\left[-D\left(\frac{\partial^2\phi}{\partial x^2}+\frac{\partial^2\phi}{\partial y^2}+\frac{\partial^2\phi}{\partial z^2}\right)+\Sigma_a\phi-\frac{\nu\Sigma_f\phi}{k}-S\right]=0$$

$\Downarrow$ To get ~~voltage~~ weak form we must distribute derivative equally on ϕ & ω so we integrate by parts.

the last three terms are simply

abs $\int_V dx\,dy\,dz\ \omega\,\Sigma_a\phi$; fiss $\int_V dx\,dy\,dz\ \omega\frac{\nu\Sigma_f}{k}\phi$;

& src $\int_V dx\,dy\,dz\ \omega S$

Now, lets look at the first three term.

we have:

stream
$$-\int_V dx\,dy\,dz\ \omega\left[\frac{\partial}{\partial x}\left(D\frac{\partial}{\partial x}\phi\right)+\frac{\partial}{\partial y}\left(D\frac{\partial}{\partial y}\phi\right)+\frac{\partial}{\partial z}\left(D\frac{\partial}{\partial z}\phi\right)\right]$$

let $F_1 = D\frac{\partial}{\partial x}\phi$; $F_2 = D\frac{\partial}{\partial y}\phi$; $F_3 = D\frac{\partial}{\partial z}\phi$.

$\therefore$ we have ~~streaming~~ streaming ~~three~~ term:

$$\int dxdydz\ \omega\left[-\frac{\partial}{\partial x}F_1 + -\frac{\partial}{\partial y}F_2 - \frac{\partial}{\partial z}F_3\right]$$

substitute

Now, note that

$$\frac{\partial}{\partial x}(\omega F_1) = \frac{\partial \omega}{\partial x}F_1 + \omega\frac{\partial F_1}{\partial x} \Rightarrow -\omega\frac{\partial}{\partial x}F_1 = \frac{\partial \omega}{\partial x}F_1 - \frac{\partial}{\partial x}(\omega F_1)$$

Sim.

$$\frac{\partial}{\partial y}(\omega F_2) = \frac{\partial \omega}{\partial y}F_2 + \omega\frac{\partial F_2}{\partial y} \Rightarrow -\omega\frac{\partial F_2}{\partial y} = \frac{\partial \omega}{\partial y}F_2 - \frac{\partial}{\partial y}(\omega F_2)$$

$$\frac{\partial}{\partial z}(\omega F_3) = \frac{\partial \omega}{\partial z}F_3 + \omega\frac{\partial F_3}{\partial z} \Rightarrow -\omega\frac{\partial F_3}{\partial y} = \frac{\partial \omega}{\partial y}F_3 - \frac{\partial}{\partial y}(\omega F_3)$$

$$\int dxdydz\ \omega\left[-\frac{\partial}{\partial x}F_1 - \frac{\partial}{\partial y}F_2 - \frac{\partial}{\partial z}F_3\right] \text{ becomes}$$

$$\int dxdydz\left[\frac{\partial \omega}{\partial x}F_1 + \frac{\partial \omega}{\partial y}F_2 + \frac{\partial \omega}{\partial z}F_3 - \frac{\partial}{\partial x}(\omega F_1) - \frac{\partial}{\partial y}(\omega F_2) - \frac{\partial}{\partial z}(\omega F_3)\right]$$

Now use divergence theorem:

$$\int_{\Omega} \frac{\partial}{\partial x}(\omega F_1)\,\partial V = \int_{\Gamma} \omega F_1 n_x \, ds$$

$$\int_{\Omega} dv \frac{\partial}{\partial y}(\omega F_2) = \int_{\Gamma} \omega F_2 n_y \, ds$$

$$\int_{\Omega} dv \frac{\partial}{\partial z}(\omega F_3) = \int_{\Gamma} \omega F_3 n_z \, ds.$$

n_x, n_y, n_z are components of unit normal to the surface of element.

Substitute that ~~a~~ other relations for abs, fiss src, we have weak form:

$$0 = \int_{V} dx\,dy\,dz \left[\left(\frac{\partial \omega}{\partial x}\right)\left(D\frac{\partial}{\partial x}\phi\right) + \left(\frac{\partial \omega}{\partial y}\right)\left(D\frac{\partial}{\partial y}\phi\right) + \left(\frac{\partial \omega}{\partial z}\right)\left(D\frac{\partial \phi}{\partial z}\right) + \omega \Sigma_a \phi - \frac{\omega \nu \Sigma_f \phi}{k} - \omega S \right]$$

$$- \int_{\Gamma} ds \, \omega \left[n_x\left(D\frac{\partial}{\partial x}\phi\right) + n_y\left(D\frac{\partial}{\partial y}\phi\right) + n_z\left(D\frac{\partial}{\partial z}\phi\right) \right]$$

volume of element

surface ~~of~~ of element