

# HW - 3

3.4 For thermal  $\omega$  calculate  $\bar{\eta}$  as a function of  $\tilde{e}$ .

$$\epsilon = \frac{N^{235}}{N^{238} + N^{235}}$$

~~$$\epsilon = \frac{(N^{235})^{235}}{(N^{235})^{235} + (N^{238})^{238}}$$~~

$$\bar{\eta} = \tilde{e} \nu \sigma_f^{235} + (1-\tilde{e}) \nu \sigma_f^{238}$$

$$\tilde{e} \bar{\sigma}_a^{235} + (1-\tilde{e}) \bar{\sigma}_a^{238}$$

$$= \tilde{e} \nu^{235} \sigma_f^{235} + (1-\tilde{e}) \nu^{238} \sigma_f^{238}$$

$$\tilde{e} \bar{\sigma}_a^{235} + (1-\tilde{e}) \bar{\sigma}_a^{238}$$

$$\bar{\eta} = \frac{\tilde{e} \nu^{235} \sigma_f^{235}}{\tilde{e} \bar{\sigma}_a^{235} + (1-\tilde{e}) \bar{\sigma}_a^{238}}$$

← plot that

3.10 Coolant is now 99%  $D_2O$  & 1%  $H_2O$ .

Before it was 100%  $D_2O$ .

Determine percentage increase or decrease in the following characteristics of coolant:

(a) Slowing down decrement. (b) Slowing down power (c) Slowing down ratio.

Slowing down decrement  $\rightarrow \xi \leftarrow \xi_{D_2O} = 0.51$

Slowing down power  $\rightarrow \xi \xi_s \leftarrow \xi_s \xi_{D_2O} = 0.18$

Slowing down ratio  $\rightarrow \xi \xi_s / \xi_a \leftarrow \frac{\xi \xi_s}{\xi_a \text{ (measured)}} = 21000$

So will need  $\xi_s, \xi_a$  of  $H_2O$  &  $D_2O$ .

Note that  $\bar{\epsilon}_s = \frac{1}{3} \epsilon_s$

*↓ Slowing down power*

$\bar{\epsilon}_s & \text{ Slowing down decrement}$

$$\text{Then } \bar{\epsilon}_s^{D_{20}} = \frac{\bar{\epsilon}_s^{D_{20}} \epsilon_s^{D_{20}}}{\bar{\epsilon}_s} = \frac{0.18}{0.551} = 0.353.$$

$$\bar{\epsilon}_s^{H_{20}} = \frac{\bar{\epsilon}_s^{H_{20}} \epsilon_s^{H_{20}}}{\bar{\epsilon}_s} = \frac{1.28}{0.93} = 1.38.$$

Also note that

$$\bar{\epsilon}_a = \frac{\frac{1}{3} \epsilon_s}{\frac{1}{3} \epsilon_s} \leftarrow \begin{array}{l} \text{sloving down} \\ \text{power} \end{array}$$

$\bar{\epsilon}_a \leftarrow \text{Sloving down ratio.}$

$$\Rightarrow \bar{\epsilon}_a^{D_{20}} = \frac{\frac{1}{3} \bar{\epsilon}_s^{D_{20}}}{\frac{1}{3} \bar{\epsilon}_s^{D_{20}}} = \frac{0.18}{21000} = 8.57 \times 10^{-6}$$

$$\bar{\epsilon}_a^{H_{20}} = \frac{\frac{1}{3} \bar{\epsilon}_s^{H_{20}} \epsilon_s^{H_{20}}}{\frac{1}{3} \bar{\epsilon}_s^{H_{20}}} = \frac{1.28}{58} = 0.022.$$

(a) Slowing down decrement.

$$\bar{\epsilon}_s = \frac{1}{3} \sum_i \epsilon_s^{D_{20}} = \cancel{\frac{1}{3} \sum_i \bar{\epsilon}_s^{D_{20}}}$$

$$\bar{\epsilon}_s^{\min} = \text{frac}^{D_{20}} \bar{\epsilon}_s^{D_{20}} + \text{frac}^{H_{20}} \bar{\epsilon}_s^{H_{20}} = 0.295 \cancel{+ 0.022}$$

$$\bar{\epsilon}_a^{\min} = \text{frac}^{D_{20}} \bar{\epsilon}_a^{D_{20}} + \text{frac}^{H_{20}} \bar{\epsilon}_a^{H_{20}}$$

$$\bar{\varepsilon} = \frac{1}{\sum_{i} \varepsilon_i} \sum_{i} \varepsilon_i$$

in mix

i.e. atom densities for the mixture are diff now & can't use stand-alone  
 $\text{H}_2\text{O}$  &  $\text{D}_2\text{O}$   $\Sigma S R$  Eq.  
 Gotta incorporate <sup>atom</sup> fraction now.

$$(a) \bar{\varepsilon}_{\text{mix}} = \frac{\sum_{i} \varepsilon_i \text{frac}^{\text{D}_2\text{O}} \varepsilon_i^{\text{D}_2\text{O}} + \sum_{i} \varepsilon_i \text{frac}^{\text{H}_2\text{O}} \varepsilon_i^{\text{H}_2\text{O}}}{\text{frac}^{\text{D}_2\text{O}} \varepsilon_i^{\text{D}_2\text{O}} + \text{frac}^{\text{H}_2\text{O}} \varepsilon_i^{\text{H}_2\text{O}}}$$

$$= \frac{0.51 \cdot 0.91 \cdot 0.353 + 0.93 \cdot 0.01 \cdot 1.38}{0.99 \cdot 0.353 + 0.01 \cdot 1.38} = \frac{0.41}{0.363} = 0.53$$

(b) find slowing down power.

$$\bar{\varepsilon} \varepsilon_s = \bar{\varepsilon}_{\text{mix}} \varepsilon_{s,\text{mix}}$$

$$= 0.53 (\text{frac}^{\text{H}_2\text{O}} \varepsilon_s^{\text{H}_2\text{O}} + \text{frac}^{\text{D}_2\text{O}} \varepsilon_s^{\text{D}_2\text{O}})$$

$$= 0.53 (0.99 \cdot \cancel{0.353} + 0.01 \cdot 1.38)$$

$$= \underline{\underline{0.1925}}$$

$$(c) \frac{\bar{\varepsilon} \varepsilon_{s,\text{mix}}}{\varepsilon_{a,\text{mix}}} = \frac{\bar{\varepsilon} \varepsilon_s}{\varepsilon_{a,\text{mix}}} = \frac{\sum_{i} \varepsilon_i \varepsilon_s}{\varepsilon_{a,\text{mix}}} = \underline{\underline{0.191}}$$

$$= \frac{0.99 \cdot 8.57 \times 10^{-6} + 0.01 \cdot 0.022}{833} = \underline{\underline{833}}$$

Find percent decreases accordingly.

3.11 Using Appendix E-3, calculate absorption  $\chi_s$  of  $H_2O$ , averaged over a thermal  $\omega$  spectrum.

(a) At room temp.

$G_3$  already gives  $\bar{\sigma}_{at} = 0.5896 b$ .

(b) At  $300^\circ C$ ?

See page 80.

- $\chi_s$ 's are in table E-3 are averaged over thermal spectrum (torr @ energy).
- Now ambient media have their own properties that affect  $\chi_s$  based on temperature.
- See pg. 80 for more ~~details~~ details.

$$\bar{\sigma}_{at}(T) = \left(\frac{T_0}{T}\right)^{1/2} \bar{\sigma}_{at}(T_0) \quad \text{given in E-3.}$$

$$300^\circ C = 300 + 273^\circ K = 573^\circ K.$$

$$\Rightarrow \bar{\sigma}_{at}(573) = \left(\frac{293.6}{573}\right)^{1/2} 0.5896$$

$$= \underline{\underline{0.422 b}}.$$

3.12 Do as for 3.11 for  $D_{20}$ .

(a)  $0.001 b.$

(b)  $7.2 \times 10^{-4} b.$

(a) Obtain an expression for  $\Sigma$  p.

$\Sigma$  = fast fission constant factor.

= total fission @ produced

~~minus thermal fission @~~

total fission @ produced  $\rightarrow$  in fuel

$\hookrightarrow$  @ produced in thermal range  $\rightarrow \int_T^\infty \Sigma_f^+(E) \Phi_f(E) dE$

+ @ produced in epithermal range  $\rightarrow 0$

+ @ produced in fast range  $\rightarrow 0$

# produced fission  
# produced thermal fission  
# produced epithermal fission  
# produced fast fission

$$\int_T^\infty \Sigma_f^+(E) \Phi_f(E) dE$$

thermal fission @

$$\hookrightarrow \int_T^\infty \Sigma_f^+(E) \Phi_f(T) dE .$$

$$\Sigma = \frac{\int_T^\infty \Sigma_f^+(E) \Phi_f(E) dE + \int_F^\infty \Sigma_f^+(E) \Phi_f(E) dE}{\int_T^\infty \Sigma_f^+(E) \Phi_f(E) dE}$$

$$\Sigma = 1 + \frac{\int_F^\infty \Sigma_f^+(E) \Phi_f(E) dE}{\int_T^\infty \Sigma_f^+(E) \Phi_f(E) dE} .$$

## Resonance escape probability:

We want fast  $\alpha$  scatter down to thermal regions and so while slowing down,  $\alpha$  get absorbed in intermediate rays in resonances. We want to characterize the  $\alpha$  that bypass resonances - we do this through resonance escape probability.

$$\text{Resonance escape probability} = 1 - \text{resonance absorption probability.}$$

$$\text{Resonance absorption prob} = \frac{\text{total absorbed in resonances}}{\text{total } \alpha \text{ absorbed in thermal & intermediate energy ranges.}}$$

$\alpha$  System model

$$\text{② absorbed in resonances} = V_f \int \Sigma_a^r(E) \Phi_a(E) dE$$

Total @ ablated =  $V_f \left[ \int_T^{\infty} \varepsilon_a^f(\varepsilon) G_f(\varepsilon) d\varepsilon + \int_I^{\infty} \varepsilon_a^f(\varepsilon) G_f(\varepsilon) d\varepsilon \right]$

while slowing down  
in thermal & intermediate

$$+ V_m \left[ \int_T^{\infty} \varepsilon_a^m(\varepsilon) G_m(\varepsilon) d\varepsilon \right]$$

~~//~~

2)  $P = 1 - V_f \left[ \int_I^{\infty} \varepsilon_a^f(\varepsilon) G_f(\varepsilon) d\varepsilon \right]$

$$V_f \left[ \int_T^{\infty} \varepsilon_a^f(\varepsilon) G_f(\varepsilon) d\varepsilon + \int_I^{\infty} \varepsilon_a^f(\varepsilon) G_f(\varepsilon) d\varepsilon \right]$$

$$+ V_m \left[ \int_T^{\infty} \varepsilon_a^m(\varepsilon) G_m(\varepsilon) d\varepsilon \right]$$

↳ more simplifications in pg 103-105 of textbook.

$$\begin{aligned} P_0(u) &= 1 ; & \Phi_0(x) &= \int_{-1}^1 du P_0(u) \Psi(x, u) \\ P_1(u) &= u ; & \Phi_1(x) &= \int_{-1}^1 du P_1(u) \Psi(x, u) \end{aligned}$$

Q. Derive diffusion equation from transport eqn.

$$u \frac{\partial \Psi(x, u)}{\partial x} + \varepsilon_t \Psi(x, u) = \frac{1}{2} \left[ \varepsilon_s \Phi_0(x) + \frac{s(x)}{2} \right]$$

$P_0$  - moment

$$\int_{-1}^1 du P_0(u) \left[ u \frac{\partial \Psi(x, u)}{\partial x} + \varepsilon_t \Psi(x, u) \right] = \frac{1}{2} \varepsilon_s \Phi_0(x) + \frac{s(x)}{2}$$

$$(1) \quad \int_{-1}^1 du P_0(u) u \frac{\partial \Psi(x, u)}{\partial x} + \int_{-1}^1 du P_0(u) \varepsilon_t \Psi(x, u)$$

$$(2) \quad \int_{-1}^1 du P_0(u) \Phi_0(x) \frac{s(x)}{2} + \int_{-1}^1 du P_0(u) s(x).$$

$$(3) \quad \frac{\partial}{\partial x} \underbrace{\int_{-1}^1 du P_0(u) u \Psi(x, u)}_{\Phi_1(x)} + \varepsilon_t \int_{-1}^1 du P_0(u) \Psi(x, u) + \frac{\varepsilon_s \Phi_0}{2} \int_{-1}^1 du P_0(u) + \frac{s_0}{2} \int_{-1}^1 du P_0(u)$$

$$(4) \quad \boxed{\frac{\partial \Phi_1(x)}{\partial x} + \varepsilon_t \Phi_0(x) = \varepsilon_s \Phi_0 + \varepsilon s}$$

$P_1$ -moment.

$$\int_{-1}^1 du P_1(u) \left[ u \frac{\partial}{\partial x} \Psi(x, u) + \varepsilon_f \Psi(x, u) = \frac{1}{2} \sum_s \Phi_0^{(s)} + \frac{S(x)}{2} \right]$$

$$\hookrightarrow \int_{-1}^1 P_1(u) u \frac{\partial}{\partial x} \Psi(x, u) du + \int_{-1}^1 du P_1(u) \varepsilon_f \Psi(x, u)$$

$$= \int_{-1}^1 \frac{1}{2} \sum_s \Phi_0^{(s)}(u) P_1(u) du + \int_{-1}^1 du P_1(u) \frac{S(x)}{2}.$$

$$= \frac{1}{2} \sum_s \Phi_0^{(s)} \int_{-1}^0 du P_1(u) + \frac{S(x)}{2} \int_{-1}^0 du P_1(u)$$

$$\hookrightarrow \int_{-1}^1 \cancel{\int_{-1}^0 du} P_1(u) u \frac{\partial}{\partial x} \Psi(x, u) + \varepsilon_f \Phi_1(x) = 0$$

$$\hookrightarrow \int_{-1}^1 u \frac{\partial}{\partial x} \underbrace{\int_{-1}^1 du u P_1(u) \Psi(x, u)}_{=0} + \varepsilon_f \Phi_1(x) = 0$$

By Recurrence relation of Legendre polynomials,

Sub.

$$(m+1) P_{m+1}(u) = (2m+1) u P_m(u) - m P_{m-1}(u)$$

$$\hookrightarrow \frac{(m+1)}{(2m+1)} P_{m+1}(u) P_{m+1}(u) + \frac{m}{2m+1} P_{m-1}(u) = u P_m(u)$$

$$\hookrightarrow m P_1(u) = \frac{1}{2(1) \cdot 1} P_{11}(u) + \frac{1}{2(1) \cdot 1} P_{1-1}(u) = \frac{2}{3} P_1(u) + \frac{1}{3} P_0(u)$$

$$\textcircled{1} \quad \frac{\partial}{\partial x} \int_1^l du \left[ \frac{2}{3} P_2(u) + \frac{1}{3} P_0(u) \right] \Psi(x, u) + \varepsilon_F \Phi_0(x) = 0$$

$$\textcircled{1} \quad \frac{\partial}{\partial x} \left[ \frac{2}{3} \int_1^l du P_2(u) \Psi(x, u) \right] + \frac{\partial}{\partial x} \left[ \frac{1}{3} \int_1^l du P_0(u) \Psi(x, u) \right] \\ + \varepsilon_F \Phi_0(x) = 0.$$

$\rightarrow 0 \in \text{ansatz}$

$$\textcircled{1} \quad \frac{2}{3} \frac{\partial}{\partial x} \Phi_2(x) + \frac{1}{3} \frac{\partial}{\partial x} \Phi_0(x) + \varepsilon_F \Phi_0(x) = 0.$$

$$\textcircled{4} \quad \Phi_1(x) = \cancel{\frac{1}{3} \frac{\partial}{\partial x} \Phi_0(x)}$$

$$\textcircled{1} \quad \boxed{\Phi_1(x) = -\frac{1}{3\varepsilon_F} \frac{\partial}{\partial x} \Phi_0(x).} \quad \textcircled{2}$$

Sub \textcircled{2} in \textcircled{1} to get

$$\frac{\partial}{\partial x} \left[ -\frac{1}{3\varepsilon_F} \frac{\partial}{\partial x} \Phi_0(x) \right] + \varepsilon_F \Phi_0(x) = \varepsilon_S \Phi_0(x) + S_0$$

$$\textcircled{1} \quad -\frac{\partial}{\partial x} \frac{D}{Dx} \frac{\partial}{\partial x} \Phi_0(x) + (\varepsilon_F - \varepsilon_S) \Phi_0(x) = S(x)$$

$$\textcircled{1} \quad \boxed{-\frac{\partial}{\partial x} \frac{D}{Dx} \frac{\partial}{\partial x} \Phi_0(x) + \varepsilon_S \Phi_0(x) = S(x)} = D_i A_{\text{eff}}.$$

Define members as:

$$\int_{-1}^1 du P_e(u) \psi(x, u) = \beta_e \phi_e(x).$$

Now Derive  $P_e$  equations.

~~① Define eqn~~

① Multiply transp eqn by  $P_e(u)$  & integrate over ~~interval~~ -1 to 1. Then see what you get. ~~george~~

→ go from  $\ell=0$  to L.

Let,  $\frac{\partial \phi_{\ell+1}(x)}{\partial x} = 0$ .

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