HW-1 Sol^n

1. look up different models of atom.
→ (see notes slide share ~~see~~ link)

2. Calculate rest mass energy of e^-, p^+, n, photon.

$E_{rest} = m_0 c^2$

$\Rightarrow E_{rest,e} = m_e c^2$ J

$= (9.01954 \times 10^{-31})(2.9979 \times 10^8)^2$

$= 8.1062 \times 10^{-14}$ J

≈ 506 MeV

$c = 2.9979 \times 10^8$ m/s.
$m_e = 9.01954 \times 10^{-31}$ kg
$m_p = 1.67265 \times 10^{-27}$ kg
$m_n = 1.67495 \times 10^{-27}$ kg.
1 MeV $= 1.6022 \times 10^{-13}$ J.

If we take more accurate values of m_e & c, according to google,

$m_e = 9.10938356 \times 10^{-31}$ kg

$c = 2.99792458 \times 10^8$ m/s.

$E_{rest,e^-} = 0.510991489$ MeV

≈ 0.511 MeV.

Similarly $E_{rest,p} = 938.272$ MeV
$E_{rest,n} = 939.565$ MeV

If you get something close is fine. Values may vary slightly depending on precision chosen.

for photon $m_0 = 0$

$\Rightarrow$ rest mass energy of photon $= 0$.

) Calculate energy equivalent of 1 amu.

$1 \text{ amu} = 1.6606 \times 10^{-27}$ kg.

mass

Energy equivalent of $1e^- = 0.511$ MeV / e^-

mass of $e^- = 9.1095 \times 10^{-31}$ kg / e^-

So Energy equivalent of 1 amu $= \dfrac{\text{total mass}}{\text{mass of } e^-} \times$ restmass energy of

$$= \frac{1.6606 \times 10^{-27}}{9.1095 \times 10^{-31}} \times 0.511$$

$$= 931.5 \text{ MeV}.$$

Another way

$1 \text{ amu} = 1.6606 \times 10^{-27}$ kg

$$E = mc^2 = (1.6606 \times 10^{-27})(2.9979 \times 10^{8})^2$$

$$= 1.4924 \times 10^{-10} \text{ J}$$

$$= 931.5 \text{ meV}.$$

(4) Expand using binomial thm & prove for $v <<< c$ that $KE = \frac{1}{2} m_0 v^2$.

Binomial Expansion: $(1+x)^n = \dfrac{1}{0!} + \dfrac{n}{1!}x + \dfrac{n(n-1)}{2!}x^2 + \dfrac{n(n-1)(n-2)}{3!}x^3 + \dots \dfrac{n!}{(n-r)!\,r!} \dots$

binomial thm with $y = 1$.

Here, we want to expand

$$\frac{1}{\sqrt{1-v^2/c^2}} = (1-v^2/c^2)^{-1/2}$$

for $(1+x)^n \equiv (1-v^2/c^2)^{-1/2}$, $n = -1/2$ & $x = -v^2/c^2$.

$$(1+x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 \cdots$$

$$\Rightarrow \frac{1}{\sqrt{1-v^2/c^2}} = 1 + (-1/2)(-v^2/c^2) + \frac{(-1/2)(-3/2)}{2}(-v^2/c^2)^2$$

$$\Rightarrow \qquad = 1 + \frac{v^2}{2c^2} + \frac{3}{8}\left(\frac{v^2}{c^2}\right)^2 \cdots$$

If $v << c$, $\left(\frac{v^2}{c^2}\right)^2$ & higher order terms $\to 0$

$$\Rightarrow \boxed{\frac{1}{\sqrt{1-v^2/c^2}} \approx 1 + \frac{v^2}{2c^2}}$$

Now, $KE = E_{tot} - E_0 = m_0c^2\left(\frac{1}{\sqrt{1-v^2/c^2}} - 1\right)$

$$= m_0c^2\left(1 + \frac{v^2}{2c^2} - 1\right)$$

$$= m_0c^2 \frac{v^2}{2c^2}$$

$$\Rightarrow \boxed{KE = \frac{1}{2}m_0v^2}$$

(3) Rest mass energy of e^- is 0.511 MeV
" " (n) ≈ 1000 MeV.

~~At~~ what KE do we start using relativistic formula for both?

we said that if $E \leq 0.02\, E_{rest}$ we can do without ~~relativistic~~ non relativistic formulae.

So if $E > 0.02\, E_{rest}$, we use relativistic formulae.

for e^-, $0.02\, E_{rest} = (0.02)(0.511) = \underline{\underline{0.01022 \text{ MeV}}}$.

for ~~p~~ (n), $0.02\, E_{rest} = (0.02)(1000) = \underline{\underline{20 \text{ MeV}}}$

you can use also value you calculated in (2) for accuracy but for demonstration purpose, this should suffice.

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(6) Write conservation laws & calculate Q value Exo or Endothermic?

$$\Rightarrow \; {}^{3}_{1}H + {}^{2}_{1}H \rightarrow {}^{1}_{0}n + {}^{4}_{2}He$$

Nucleons: Atomic mass # conserved → $3+2 = 1+4$

Charge: Atomic # conserved → $1+1 = 0+2$.

Conservation of energy:

$$E_{tot,\,{}^3H} + E_{tot,\,{}^2H} = E_{tot,\,n} + E_{tot,\,{}^4He}$$

$$\Rightarrow \; Q = (m_{0,\,{}^3H} + m_{0,\,{}^2H} - m_{0,\,n} - m_{0,\,{}^4He})\, c^2$$

we know that $(1\,\text{amu})c^2 = 931.5$ MeV.

$\Rightarrow$ $1\,\text{amu} = 931.5 \text{ MeV}/c^2$.

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we will use this conversion factor.

$$Q = (m_{o,3H} + m_{o,2H} - m_{o,n} - m_{o,4He})\, c^2 \times 931.5\ \mathrm{MeV}/c^2$$

$$= (m_{o,3H} + m_{o,2H} - m_{o,n} - m_{o,4He}) \times 931.5\ \mathrm{MeV}.$$

$$= (3.016 + 2.014 - 1.009 - 4.003) \times 931.5$$

$$= (0.018)(931.5)$$

$$= \underline{\underline{16.767\ \mathrm{MeV}}}$$

(value will change based on how precise your m_o were) Only check method here

Since Q is +ve reaction is exothermic → produces energy.

<u>Note</u>: This is what Barry does. He tries to find the most precise answer possible to complicated problems. This is called getting benchmark results. Large codes are ~~checked for~~ verified against such benchmark results.

(7) calculate binding energy per nucleon for U-235 & U-238.

$$BE/\mathrm{nucleon} = \frac{\Delta c^2}{A} = \frac{(Z m_p + (A-Z)m_n - m_X)c^2}{A}$$

Choose m_p, m_X, m_n in amu to use the amu to MeV/c^2 conversion.

$$= \frac{(Z\, m_p + (A-Z)\, m_n - m_X)\ 931.5\ \mathrm{MeV}}{A}$$

for U^{235} $BE/\mathrm{nucleon} = \dfrac{(92(1.007) + (235-92)(1.008) - (235.044))\ 931.5}{235}$

again value depends on precision of # chosen. → $= \dfrac{1.744 \times 931.5}{235} = 6.912$ MeV/nucleon.

If you keep more decimals, eg. 7 decimals will give

$$\Delta c^2 = 1782.9 \text{ MeV}$$

& $\frac{\Delta c^2}{A} = 7.59$ MeV/nucleon //

Similar procedure for U-238 returns $\frac{BE}{\text{nucleon}}$ for U-238 ≈ 7.5690 MeV/nucleon

Note that BE/A does not need to be calculated for nuclides.
~~Be~~ BE/A for each nuclides has already been tabulated.

Go to atom.kaeri.re.kr/nuchart

Atomic mass, ~~[illegible]~~ BE/A

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(8) Look up thermonuclear fusion?
→ look up.

(9) Cold fusion?
→ look up.

(10) Youtube?
→ watch it.

(11) Nuclear decay?
Wikipedia — Radioactive decay.

(12) A reactor operates at 10^3 MW for 1 yr. (4)
calculate power from decay heat

(a) after 1 day. (b) one month.

$$P_d(t) = 0.0622\, P_0 \left(t^{-0.2} - (t_0+t)^{-0.2} \right) \qquad t \text{ in sec.}$$

1 day = 86400 s.
1 month = 2.64×10^6 s.
1 yr = 31.6×10^6 s.

(a) $P_d(1\text{ day}) = (0.0622 \times \underset{P_0}{10^3}) \left[86400^{-0.2} - (31.6 \times 10^6 + 86400)^{-0.2} \right]$

$= \underline{\underline{4.44 \text{ MW}}}$

(b) $P_d(1\text{ month}) = (0.0622 \times 10^3) \left((2.64 \times 10^6)^{-0.2} - (31.6 \times 10^6 + 2.64 \times 10^6)^{-0.2} \right)$

$= \underline{\underline{1.3 \text{ MW}}}$

Now, if reactor only operated for a month,

$t_0 = 2.64 \times 10^6$ s & not 31.6×10^6.

everything else stays the same.

(13) $t_{1/2}^{238} = 4.51 \times 10^{9}\ yr = 4.51\ B\ yr.$

$t_{1/2}^{235} = 7.13 \times 10^{8}\ yr = 0.713\ B\ yr.$

$$e(t) = \frac{N^{235}(t)}{N^{235}(t) + N^{238}(t)}$$

Current $e(t) = 0.7\% = 0.007.$

time since earth was born, $4.5\ B\ yr.$

(a) find enrichment at the begining of time.

$$e(t) = \cancel{\frac{N^{235}(t)}{N^{238}(t) + N^{235}(t)}} \qquad \frac{N^{235}(t)}{N^{238}(t) + N^{235}(t)}$$

$$\Rightarrow \boxed{e(t) = \frac{1}{1 + \frac{N^{238}(t)}{N^{235}(t)}}}$$

$\Downarrow$

$$1 + \frac{N^{238}(t)}{N^{235}(t)} = \frac{1}{e(t)} \Rightarrow \boxed{\frac{N^{238}(t)}{N^{235}(t)} = \frac{1}{e(t)} - 1}$$

Now, $N(t) = N(0)\, e^{-\lambda t}$

$$\Rightarrow \frac{N^{238}(t)}{N^{235}(t)} = \frac{N^{238}(0)\, e^{-\lambda_{238} t}}{N^{235}(0)\, e^{-\lambda_{235} t}} = \frac{1}{e(t)} - 1$$

$$\Rightarrow \frac{N^{238}(0)}{N^{235}(0)} = \left(\frac{1}{e(t)} - 1\right) e^{(+\lambda_{238} - \lambda_{235}) t}$$

$$\lambda_{238} = \frac{0.693}{t_{1/2}^{238}} = \frac{0.693}{4.51} = 0.153 \text{ Byr}^{-1}$$ (5)

$$\lambda_{235} = \frac{0.693}{\lambda_{1/2}^{235}} = \frac{0.693}{0.713} = 0.972 \text{ Byr}^{-1}$$

$$\Rightarrow \frac{N^{238}(0)}{N^{235}(0)} = \left(\frac{1}{e(t)} - 1\right) e^{(0.153-0.972)t}$$

$$= \left(\frac{1}{0.007} - 1\right) e^{-0.819 \times 4.5}$$

$$= \underline{\underline{3.56}}$$

$$e(t) = \frac{1}{1 + \frac{N^{238}(t)}{N^{235}(t)}}$$

$$\Rightarrow e(0) = \frac{1}{1 + \frac{N^{238}(0)}{N^{235}(0)}} = \frac{1}{1+3.56} = 0.219 = \underline{\underline{21.9\%}}.$$

(b) When was enrichment 4%?

Call time at 4% enrichment t_0

So $e = e(0)$ & $N = N(0)$

$$\frac{N^{238}(t)}{N^{235}(t)} = \frac{1}{e(t)} - 1.$$

Also, $$\frac{N^{238}(0)}{N^{235}(0)} = \left(\frac{1}{e(t)} - 1\right) e^{-0.819t}$$

$$\frac{1}{e(0)} - 1 = \left(\frac{1}{e(t)} - 1\right) e^{-0.819t}$$

Now, $e(0) = 4\%$
$e(t) = 0.7\%$.

$$\Rightarrow \frac{1}{0.04} - 1 = \left(\frac{1}{0.007} - 1\right) e^{-0.819t}$$

solving returns $t = \underline{2.17 \text{ Byrs.}}$

(14) Consider reaction given.
U-235 is produced at a constant rate.
How long will it take Pu-239 to reach $\frac{1}{2}$ saturation activity

we know that saturation activity is

$$\lambda N_\infty = A_0 .$$

we want to get to 1/2 of saturation activity so

$$\lambda N(t) = \frac{1}{2} A_0 .$$

Now, activity eqn. is

$$\lambda N(t) = A_0 (1 - e^{-\lambda t})$$

$$\Rightarrow \frac{1}{2} A_0 = A_0 (1 - e^{-\lambda t})$$

$$\Rightarrow \frac{1}{2} = 1 - e^{-\lambda t}$$

$$\Rightarrow \frac{1}{2} = e^{-\lambda t}$$

$$\Rightarrow 0.693 = \lambda t .$$

$$\Rightarrow \frac{0.693}{\lambda} = t .$$

Sub.

$t_{1/2}, NP = 2.36$ days.

$$\Rightarrow \lambda = \frac{0.693}{t_{1/2}} = \frac{0.693}{2.36} = 0.294 \text{ day}^{-1}$$

$$t = \frac{0.693}{0.294} = \underline{2.36 \text{ days}}$$

Now, think about this. Activity is reducing by half = half of nuclides decayed. So t = halflife so no need to do all this calc.

(15) Expand exponential in Taylor series. Prove initially activity increases linearly.

$$\lambda N(t) = A_0 (1 - e^{-\lambda t})$$

$$e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!} = 1 + x + \frac{x^2}{2} + \frac{x^3}{3!} \cdots$$

here we want to expand $e^{-\lambda t}$

$\Rightarrow x = -\lambda t$.

$$\Rightarrow e^{-\lambda t} = 1 + (-\lambda t) + \frac{(-\lambda t)^2}{2!} + \cdots$$

$$= 1 - \lambda t + \frac{\lambda^2 t^2}{2} + \cdots$$

Now, when $\lambda t << 1$, $t \to 0$, $\lambda^2 t^2 \to 0$

as long as λ is not a very large #.

so in that case, $\lambda^2 t^2$ is a very small # so we neglect that & higher order terms in t.

so $e^{-\lambda t} = 1 - \lambda t$.

$$\lambda N(t) = A_0 (1 - 1 - \lambda t) = \boxed{A_0 \lambda t = \lambda N(t) = A(t)}$$

$$\boxed{N(t) = A_0 t}$$

~~co...~~ linear.