CH-5 REACTOR KINETICS

IN this chapter, we examine time-dependent behavior of n chain reactions.

- We consider simple n balance equations for

1. Infinite medium nonmultiplying systems
2. Infinite medium multiplying systems
3. Finite multiplying systems
4.

- We assume ~~averaged~~ energy-averaged n distributions & xs
- ~~xs~~ We also assume that spatial effects ~~can be~~ are characterized by nonescape probability approximations.

- then we look at Delayed n kinetics
- derive kinetics equation & introduce reactivity.

* Neutron balance equation.

Define:

$n(t)$ = total # of n at time t

$\bar{v}$ = avg. n speed

Σ_x = energy-averaged xs of type x.

Infinite medium nonmultiplying systems.

Determine rate of change of # of n in nonmultiplying systems.

Assume - no fission

- infinite medium so no n leakage.

The @ balance equation then is

$$\frac{d}{dt} n(t) = \# \text{ @ produced / s} - \# \text{ @ lost / s.}$$

Let # @ produced / s = $S(t)$

@ lost is determined ~~dy~~ by @ absorption.

Recall Σ_a = macroscopic absorption xs
= probability that @ will be absorbed per cm of travel.

$\bar{v}$ = velocity.

=> $\bar{v}\,\Sigma_a$ = probability per second that a @ will be absorbed.

Dimensions:

$[\cancel{\Sigma\bar{v}}]\ [cm/s][cm^{-1}] \rightarrow [1/s]$

Then # @ lost / s = #@ absorbed per s
= (probability per s that @ will be absorbed) × (@ population at ~~that~~ time t)

$$= \bar{v}\,\Sigma_a\, n(t).$$

Then balance equation is:

$$\boxed{\frac{d}{dt} n(t) = S(t) - \Sigma_a \bar{v}\, n(t)}$$

We rewrite the balance equation in terms of avg. ⓝ lifetime, l_∞.

We can calculate l_∞ using ~~solution of~~ balance equation. Assume $S(t)=0$, the balance eqn becomes

$$\frac{d}{dt} n(t) = -\bar{v}\,\Sigma_a\, n(t)$$

↳ $n(t) = n_0 e^{-t v \Sigma_a}$ ← compare to decay, whatever multiplies t is a "decay" constant. ~~Different~~ inverse of that decay constant returns characteristic lifetime

Now, the exponential has to be unitless.

$t v \Sigma_a$ is unitless.

If t = time elapsed.

then whatever multiplies t, has ~~inverse~~ inverse unit of time. ~~This is lifetime~~ & inverse of that "whatever" has units of time. This is neutron lifetime, l_∞.

~~If you recall, radioactive decay.~~

↳ $\boxed{n(t) = n_0 e^{-t/l_\infty}}$; $\boxed{l_\infty = \frac{1}{\bar{v}\,\Sigma_a}}$ ← Different reactor types work with different l_∞.

Using this definition, we rewrite the balance equation.

$$\frac{d}{dt} n(t) = S(t) - \frac{1}{l_\infty} n(t)$$

Assuming there are no ⓝ in system before insertion of source, & $S(t) \to S_0$ (constant src over time)

We have

IVP
$$\begin{cases} \dfrac{d}{dt} n(t) = S_0 - \dfrac{1}{l_\infty} n(t) . \\ n(0) = 0 . \end{cases}$$

~~then~~ Solving the IVP returns

$$\boxed{n(t) = l_\infty S_0 \left[1 - e^{-t/l_\infty}\right]}$$

HW4.1 prove the above equation.
Also find $n(\infty)$.

Infinite <u>medium multiplying</u> systems.

- Now we incorporate fission (prompt fission only)
- we still ignore spatial effects - infinite medium & no leakage.

The balance equation is still the same.

$$\frac{d}{dt} n(t) = \# \text{ⓝ produced/s} - \# \text{ⓝ lost/s}.$$

$\#$ ⓝ lost/s $\rightarrow v \Sigma_a n(t)$ (from before)

$\#$ ⓝ produced then is $\rightarrow$ ⓝ from src
+ ⓝ from fission.

\# @ produced from src /s = $S(t)$ ← total @ at time t.

\#@ produced thro fissions /s = $\bar{v}\,\nu\,\Sigma_f\, n(t)$.

@ speed   # @ produced per fission   macroscopic fission xs

Then, balance equation is

$$\frac{d}{dt} n(t) = S(t) + \nu\Sigma_f \bar{v}\, n(t) - \Sigma_a \bar{v}\, n(t)$$

$$= S(t) + n(t)\bar{v}\left[\nu\Sigma_f - \Sigma_a\right]$$

$$= S(t) + n(t)\bar{v}\,\Sigma_a\left[\frac{\nu\Sigma_f - \Sigma_a}{\Sigma_a}\right]$$

$$= S(t) + n(t)\underbrace{\bar{v}\,\Sigma_a}_{\frac{1}{l_\infty}}\left[\underbrace{\frac{\nu\Sigma_f}{\Sigma_a}}_{k_\infty} - 1\right]$$

$$= S(t) + n(t)\left(\frac{k_\infty - 1}{l_\infty}\right)$$

$$\Rightarrow \boxed{\frac{d}{dt} n(t) = S(t) + \left(\frac{k_\infty - 1}{l_\infty}\right) n(t)}$$

An understanding of criticality can be obtained by setting $S(t) = 0$.

$$\frac{d}{dt} n(t) = \left(\frac{k_\infty - 1}{l_\infty}\right) n(t). \quad //$$

For criticality, we want $\frac{d}{dt} n(t) = 0$.

This only happens if $k_\infty = 1$ or $l_\infty = \infty$

l_∞ can't be ∞ (it is physically not possible)
- why?
→ even if ⓝ bounce around & never escape, they will decay away.

$\Rightarrow k_\infty = 1$. ⇐ This is criticality.

If ⓝ population decreases with time in absence of source, it is sub critical.
If ⓝ population increases with time in absence of src, it is super critical.

<u>HW 4.2</u> Show that if $k_\infty > 1$, system is super critical & if $k_\infty < 1$, system is sub critical using the above equation.

Finite multiplying systems.

Now we incorporate ~~spectral~~ @ leakage. We still make an approximation tho.

@ Balance equation now becomes

$$\frac{d}{dt} n(t) = \left.\begin{array}{l}\# \text{ source @ produced/s} \\ + \# \text{ fission @ produced/s}\end{array}\right\} \leftarrow \# \text{@ produced/s}$$

$$\left.\begin{array}{l}- \# \text{ @ absorbed/s} \\ - \# \text{ @ leaking from system/s}\end{array}\right\} \leftarrow \# \text{@ lost/s}$$

We've ~~known~~ seen every term except # @ leaking from system/s.

We make an assumption here (we will look at more refined calculation later)

Assume:

& @ leakage/s $\propto$ @ absorbed/s.

$\Rightarrow$ @ leakage/s $= \Gamma \Sigma_a \bar{v}\, n(t)$

$\uparrow$ proportionality constant.

Substituting that in the balance equation returns

$$\boxed{\frac{d}{dt} n(t) = S(t) + \nu \Sigma_f \bar{v}\, n(t) - \Sigma_a \bar{v}\, n(t) - \Gamma \Sigma_a \bar{v}\, n(t)}$$

we can express Γ in terms of leakage & non leakage probability.

In order to do that, we say a ⓝ born thru fission or source. Then the scatter & die either by - absorption or
- leakage

Leakage prob = P_L = $\dfrac{\text{ⓝ lost to leakage/s}}{\text{total ⓝ lost/s}}$

$$= \frac{\Gamma \Sigma_a \bar{v}\, n(t)}{\Sigma_a \bar{v}\, n(t) + \Gamma \Sigma_a \bar{v}\, n(t)}$$

$$\Rightarrow P_L = \frac{\Gamma}{1+\Gamma}$$

$$\Rightarrow P_{NL} = 1 - P_L = 1 - \frac{\Gamma}{1+\Gamma} = \frac{1}{1+\Gamma}.$$

~~What~~ happ~~ens~~ ~~to~~ ~~P_{NL}~~ as E ...

P_{NL} ~~gross~~ goes to 1 as reactor becomes large.
$\Rightarrow \Gamma \to 0$ as reactor becomes large.

$$P_{NL} = \frac{1}{1+\Gamma} \Rightarrow 1+\Gamma = \frac{1}{P_{NL}} \Rightarrow \Gamma = \frac{1}{P_{NL}} - 1$$

$$\Rightarrow \boxed{\Gamma = \frac{1-P_{NL}}{P_{NL}}}$$

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We can rewrite the balance equation:

$$\frac{d}{dt} n(t) = S(t) + \nu \Sigma_f \bar{v} n(t) - \Sigma_a \bar{v} n(t) - \Gamma \Sigma_a \bar{v} n(t)$$

$$\Rightarrow \frac{d}{dt} n(t) = S(t) + \nu \Sigma_f \bar{v} n(t) - \Sigma_a \bar{v} n(t) - \left(\frac{1-P_{NL}}{P_{NL}}\right) \Sigma_a \bar{v} n(t)$$

$$\Rightarrow \boxed{P_{NL} \frac{d}{dt} n(t) = P_{NL} S(t) + P_{NL} \nu \Sigma_f \bar{v} n(t) - \Sigma_a \bar{v} n(t)}$$

$$\Rightarrow P_{NL} \frac{d n(t)}{dt} = P_{NL} S(t) + (P_{NL} \nu \Sigma_f - \Sigma_a) \bar{v} \Sigma n(t)$$

$$\Rightarrow P_{NL} \frac{d}{dt} n(t) = P_{NL} S(t) + \left(P_{NL}\left(\frac{\nu \Sigma_f}{\Sigma a}\right) - \frac{\Sigma a}{\Sigma a}\right) \Sigma_a \bar{v} n(t)$$

$$\boxed{P_{NL} \frac{d}{dt} n(t) = P_{NL} S(t) + \left(\frac{P_{NL} k_\infty - 1}{l_\infty}\right) n(t)}$$

We know that

~~Keff~~ $k = P_{NL} k_\infty$ (~~seen~~ from ch. 4)

Similarly,

$l = P_{NL} l_\infty$

Then, $$\frac{d}{dt} n(t) = S(t) + \frac{(P_{NL} k_\infty - 1)}{P_{NL} l_\infty} n(t)$$

$$\Rightarrow \boxed{\frac{d}{dt} n(t) = S(t) + \left(\frac{k-1}{l}\right) n(t)}$$

This is identical to the eqn we got for infinite media except we have k, ℓ instead of k_∞ & ℓ_∞.

k & ℓ are now multiplied by nonleakage prob. which is < 1 usually so $k, \ell \leq k_\infty, \ell_\infty$

Multiplying Systems behavior:

We start by setting $S(t) \to 0$.

$$\frac{d}{dt} n(t) = \left(\frac{k-1}{\ell}\right) n(t).$$

$$\Rightarrow n(t) = n_0 e^{\left(\frac{k-1}{\ell}\right)t}$$

Obviously for a system to be critical,
$k = 1$.

Subcritical if $k < 1$
super if $k > 1$ } just like before.

Now, let us incorporate effects of source.
let $S(t) = S_0$ & $n_0 = 0$.
then we get the following IVP

$$\boxed{\begin{aligned} &\frac{d}{dt} n(t) = S_0 + \left(\frac{k-1}{\ell}\right) n(t) \\ &n_0 = 0 \end{aligned}}$$

– HW 4.3
Solve this IVP.
& show

$$\boxed{n(t) = \frac{l S_0}{k-1}\left(\exp\left(\frac{k-1}{l}\right)^t - 1\right)}$$

Supercritical $(k>1)$

- (a) population rises at an increasing rate,
- exponential at long times.

Subcritical $(k<1)$

$$n(t) = \frac{l S_0}{(k-1)}\left(e^{\left(\frac{k-1}{l}\right)t} - 1\right)$$

$$= \frac{l S_0}{1-k}\left(1 - e^{-(1-k)/l\, t}\right)$$

The exponential term decays away at long times &

$$n(\infty)\ \boxed{n_\infty = \frac{l S_0}{1-k}}$$

Critical $(k=1)$

$$n(t) = \frac{l S_0}{k-1}\left(\exp\left(\frac{k-1}{l}\right) - 1\right)$$

Hmm: what do we do?

Expand the exponential in Taylor series

$$\exp(x) = 1 + x + \frac{1}{2}x^2 + \dots$$

$$\Rightarrow \exp\left(\frac{k-1}{\ell}t\right) = 1 + \frac{k-1}{\ell}t + \frac{1}{2}\left(\frac{k-1}{\ell}\right)^2 t^2 + \dots$$

Then,

$$n(t) = \frac{\ell s_0}{\cancel{k-1}}\left[\cancel{1} + \frac{\cancel{k-1}}{\ell}t + \left(\frac{k-1}{\ell}\right)^2 t^2 + \dots \cancel{-1}\right]$$

$$= \ell s_0 \left[\frac{t}{\ell} + \frac{k-1}{\ell^2}t^2 + \dots\right]$$

Now, we don't have $k-1 \to 1-1 \to 0$ in denominator

& $\boxed{n(t) = s_0 t}$

Then @ population increases linearly in time in presence of a ~~time dependent~~ constant-in-time source.