BUBBA DOES SLOWING DOWN

Class 11

Recall Balance equation:

$$\Sigma_t(E)\varphi(E) = \int \Sigma_s(E' \to E)\varphi(E')dE' + \chi(E)S_f'''$$

loss — gain thro scatter (inscatter) — gain thro fission.

We want to know what distribution of ⓝ flux looks like in energy for specifically three energy ranges:

fast range (fission) - $0.1\,MeV \leq E \leq 10\,MeV$
Intermediate range (epithermal / slowing down) - $1\,eV \leq E \leq 0.1$
thermal range $E < 1\,eV$

~~Note:~~
No fission ⓝ are born in thermal & intermediate region.
$\Rightarrow$ in these regions $\chi(E) = 0$.

FAST NEUTRONS

what? ⓝ with $\underset{\text{fission}}{0.1\,MeV} \leq \underset{\chi(E)}{E} \leq 10\,MeV$.

- In this region, fission spectrum dominates.
- This is the range in which fission ⓝ are primarily born.

In order to find the flux distribution ~~in~~ for ⓝ in fast range we consider the balance equation.

$$\Sigma_t(E)\varphi(E) = \int \Sigma_s(E' \to E)\varphi(E')dE' + \chi(E)S_f'''$$

(4)

- We note that every one collision removes ⓝ from energy E to a lower energy.
- Therefore we ignore the inscatter (ⓝ scattering in to energy E) term for fast range.
- Our balance equation then becomes:

$$\Sigma_t(E)\varphi(E) = \chi(E)S_f'''$$

$$\Rightarrow \boxed{\varphi(E) = \frac{\chi(E)S_f'''}{\Sigma_t(E)}} \quad \text{for fast ⓝ.}$$

For fast reactors

The distribution of fast ⓝ can be degraded by:

- Inelastic scattering with heavy nuclides
- Elastic scattering with light nuclei

Note:

- We want most ⓝ to be absorbed before scattering collisions slow them down below fast range.

- Therefore fast reactor cores must
  - avoid use of light nuclei in design
  - moreover, enrichment of U, ε must be increased on natural levels (typically ~10%)

INTERMEDIATE RANGE: $1\,eV \le E \le 0.1\,MeV$.

Energy range below fission spectrum but higher than thermal region (so still no upscatter).

* In this section, we want to find out what $\phi(E)$ looks like for the epithermal range.
* We want to relate $\phi(E)$ to slowing down density.
* As we know epithermal range has resonances.
* We break our study in two parts
  - ~~Energy spectrum~~ Between resonances
  - Through resonances.

Between resonances:

- We assume bet^n resonances $\Sigma_a(E)$ is small enough to be ignored so $\Sigma_a(E) \sim 0$.
- $\chi(E) \sim 0$ in epithermal range.
- Inelastic scattering has high threshold and we are in epithermal range so we ignore inelastic scattering.

- At first we only ~~an~~ look at slowing down through the moderator & then incorporate slowing down thro. fuel material also.

lets begin.

Define: Slowing down density - $q(E)$ = # of @ slowing down past energy E /s / cc.

Now lets think about this.

$q(E)$ = # of @ slowing past E /cc /s.

↳ which @ are slowing down? → fission @ above E $= \int_{E}^{\infty} \chi(E') dE' S_f'''$
(gain)

↳ which @ are not slowing down? @ that were absorbed $= \int_{E}^{\infty} \Sigma_a(E') \varphi(E') dE'$
(loss)

Therefore,
$$\boxed{q(E) = \int_{E}^{\infty} dE' \chi(E') S_f''' - \int_{E}^{\infty} dE' \Sigma_a(E') \varphi(E')}$$

Now, If we choose E such that $\chi(E') = 0$ if $E' < E$, then $\int_{E}^{\infty} \chi(E') dE' = 1$. (by normalization for fission spectrum)

for epithermal range this is true.

$\therefore$
$$\int_{E}^{\infty} dE' \chi(E') S_f''' = S_f''' \int_{E}^{\infty} \chi(E') dE' = S_f'''.$$

Then,

$$q(E) = S_f''' - \int_E^\infty \Sigma_a(E')\,\varphi(E')\,dE'$$

If we differentiate the equation,

$$\boxed{\frac{d}{dE} q(E) = \Sigma_a(E)\varphi(E)}$$

Note how if $\Sigma_a(E) \sim 0$, $\frac{d}{dE} q(E) = 0 \Rightarrow q(E) = \text{const}$.

- So when there is no absorption (eg. betn resonances) the slowing down density is constant. Moreover, it is ~~it~~ independent of E (because it is a constant).

-

Now, lets go back to our balance equation:

$$\Sigma_t(E)\varphi(E) = \int p(E' \to E)\,\Sigma_s(E')\,\varphi(E')\,dE' + \chi(E)\,S_f'''$$

Recall our assumptions → $\chi(E) = 0$ for ~~[illegible]~~ epithermal range
- betn resonances $\Sigma_a(E) \approx 0$

Then $\Sigma_t(E) = \Sigma_s(E) + \Sigma_a(E) \approx \Sigma_s(E)$
& $\chi(E) = 0$.

$$\Rightarrow \boxed{\Sigma_s(E)\varphi(E) = \int p(E' \to E)\,\Sigma_s(E')\,\varphi(E')\,dE'}$$

If our system ~~is~~ predominantly has elastic scatter (we assumed minimal inelastic scattering)

For elastic scattering we know that

$$p(E \to E')\,dE' = \begin{cases} \frac{1}{(1-\alpha)E}\,dE' & \alpha E \le E' \le E \\ 0 & \text{otherwise} \end{cases}$$

~~Using these substitute that~~ ~~in~~ ~~balance equation~~

max energy loss $\to \alpha E$

~~$\Sigma_s(E)$~~ min energy loss $\to 0$

So if something is slowing down past E, we set our limits as lower $\to E$, upper $\to E/\alpha$.

(think of this as starting with an energy E/α which could lose α fraction ~~to~~ to get to E)

So now we have,

$$\boxed{\Sigma_s(E)\varphi(E) = \int_{E}^{E/\alpha} \frac{1}{(1-\alpha)E'}\,\Sigma_s(E')\varphi(E')\,dE'}$$

For now we are going to say $\boxed{\Sigma_s(E)\varphi(E) = C/E}$ where

$C \propto q$ and is a normalization constant. We ~~say~~ say

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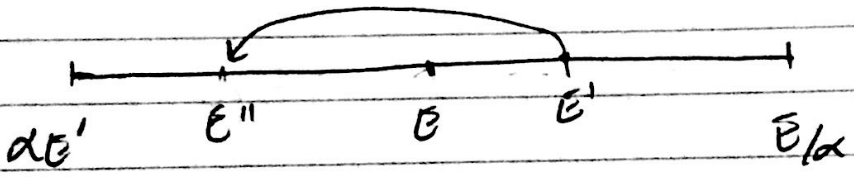
This because we want to obtain a relation betw $\varphi(E)$ & $q(E)$

keep that in mind for now.

Lets look at what $q(E)$ looks like.



- We want to calculate # Ⓝ slowing down past E is/c
- They are going to slow down from energy range with

$E \le E' \le E/\alpha$ (as anything higher energy than E/α can't slow past E)

Moreover, Ⓝ slows from there to an energy E'' where $\alpha E' \le E'' \le E'$ (as it is slowing down from energy E' & we dont want it to slow past $\alpha E'$ - it can't)

Accounting for all that, we get the following double integral:

$$\boxed{q(E) = \int_{\alpha E'}^{E} \int_{E}^{E/\alpha} \frac{1}{(1-\alpha)E'} \Sigma_s(E')\varphi(E')\,dE'\,dE''}$$

- Inner integral ~~det~~. integrates over energies @ can slow down ~~to~~ from (above $\bar{E}$ to somewhere below it)
- Outer integral integrates over energies @ slow down to (range below $\bar{E}$)

- Must account for ~~all~~ ~~n~~. @ with all these energies so double integral.

Now, recall $\Sigma_s(E)\,\varphi(E) = C/E$ was our sol^n.

Upon substitution & integration in previous eqn, we get

$$q = \left[1 + \frac{\alpha}{1-\alpha}\ln\alpha\right] C.$$

slowing down decrement

$$q = \xi C$$

$$\Rightarrow \boxed{C = q/\xi}$$

Therefore, $\Sigma_s(E)\varphi(E) = \frac{C}{E} = \frac{q}{\xi E} \Rightarrow \boxed{\varphi(E) = \frac{q}{\Sigma_s(E)\,\xi E}}$

Now in case we have fuel also in the mix, slowing down decrement (ξ) becomes weighted sum

$$\bar{\xi} = \frac{\xi^f \Sigma_s^f(E) + \xi^m \Sigma_s^m(E)}{\Sigma_s^f(E) + \Sigma_s^m(E)}$$

& $\varphi(E) = \dfrac{q}{\bar{\xi}\,\Sigma_s(E)\,E}$

composite xs.

The flux here is $\propto 1/E$ & is called the "one-over-E" flux.

Since $\xi^f << \xi^m$, contribution from fuel is much less than that from moderator.

Now incorporate resonance Energy self shielding.

Now,

Obtain an estimate of $\varphi(E)$ in presence of a resonance absorber.

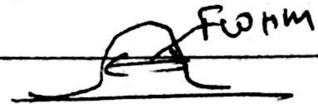
- Assume that only fuel & moderator are present.
- Assume only elastic scattering.

Balance equation:

$$\Sigma_t(E)\,\varphi(E) = \int P(E'\to E)\,\Sigma_s(E')\,\varphi(E')\,dE'$$

$$= \int_E^{E/\alpha_f} \frac{1}{(1-\alpha^f)E'}\,\Sigma_s^f(E')\,\varphi(E')\,dE'$$

$$+ \int_{E}^{E/\alpha_m} \frac{1}{(1-\alpha^m)E'} \Sigma_s^m(E') \phi(E') dE' .$$



- Now, to incorporate resonance,
- recall resonances are characterized by their full width half max, Γ
- Bulk of resonance takes place within $\pm \Gamma$
- Outside interval (→ b/w resonances), absorption is ignored & $\phi(E) \propto 1/E$.

We use narrow resonance approximation to estimate $\phi(E)$ in presence of resonance absorption.

<u>Narrow resonance approximation</u>: Resonance energy range is much smaller than scatter energy range

⇒ Area under integration of resonance << area under scattering energy ra

Area under scattering energy range is occupied by $\phi \propto 1/E$.

Then we insert $\phi(E) = \frac{q}{\xi \Sigma_s(E) E}$

Into the balance equation.

$$\Sigma_t(E)\varphi(E) = \int_E^{E/\alpha^+} \ldots + \int_E^{E/\alpha m} \ldots$$

Upon integrating, we get $\varphi(E) = \frac{q}{\bar{\xi}\,\Sigma_t(E)\,E}$

q is the slowing down density above resonance.

~~Not~~ Note how as ~~the~~ $\Sigma_t(E)$ increases, $\varphi(E)$ decreases. Such flux depression is called energy self sheilding.

Moreover as ⓝ slow down thro a resonance, slowing down density is reduced by

$$\int \varphi(E)\,\Sigma_a(E)\,dE \approx \int \frac{\Sigma_a(E)}{\Sigma_t(E)\,E}\,\frac{q}{\xi}\,dE$$

Thermal ⓝ

Thermal ⓝ follow Maxwell Boltzmann distribution. In absence of absorption. We obtain $\varphi(E)$ by multiplying ⓝ speed with $M(E)$

So,
$$\boxed{\varphi_m(E) = \frac{1}{(KT)^2} E e^{-E/kT_0}}$$

with normalization
$$\int_0^{\infty} \varphi_m(E)\,dE = 1 .$$

In reality some absorption exist and it pushes spectrum upward. This is spectral hardening