10.1 : Concept of Alligations

1) Alligations
2) Problems on mixtures
3) Replacement or Removal Problems.

Alligations

Weighted Average :

$$W_A = \frac{x_1 n_1 + x_2 n_2 + x_3 n_3 + \cdots + x_n n_n}{n_1 + n_2 + n_3 + \cdots + n_n}$$

→ If two groups are considered

$$W_A = \frac{x_1 n_1 + x_2 n_2}{n_1 + n_2}$$

- Let x_1 = cheaper value
  n_1 = cheaper quantity
  x_2 = higher value
  n_2 = higher quantity

$$W_A(n_1 + n_2) = x_1 n_1 + x_2 n_2$$

$$n_1 W_A + n_2 W_A = x_1 n_1 + x_2 n_2$$

$$n_1 W_A - x_1 n_1 = x_2 n_2 - n_2 W_A$$

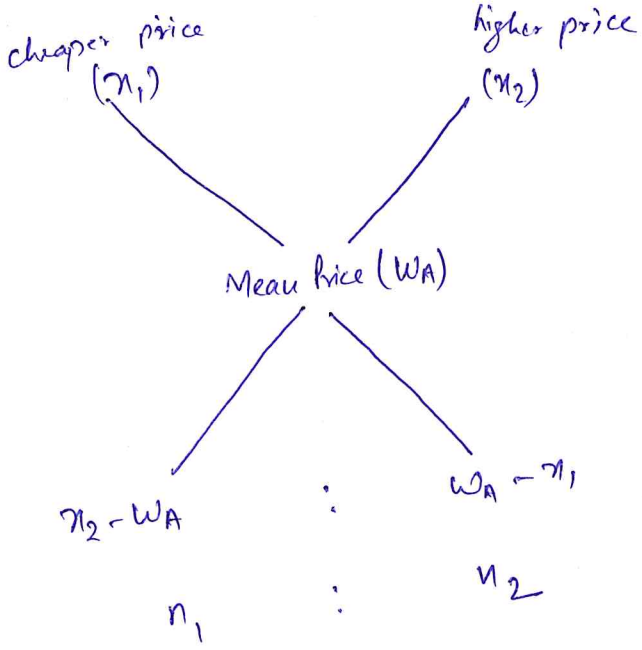
$$n_1(W_A - x_1) = n_2(x_2 - W_A)$$

$$\boxed{\frac{n_1}{n_2} = \frac{x_2 - W_A}{W_A - x_1}}$$ ← Alligation Rule

↓
Quantity Ratio.

x_2 = dearer value
W_A = mean value
x_1 = cheaper value

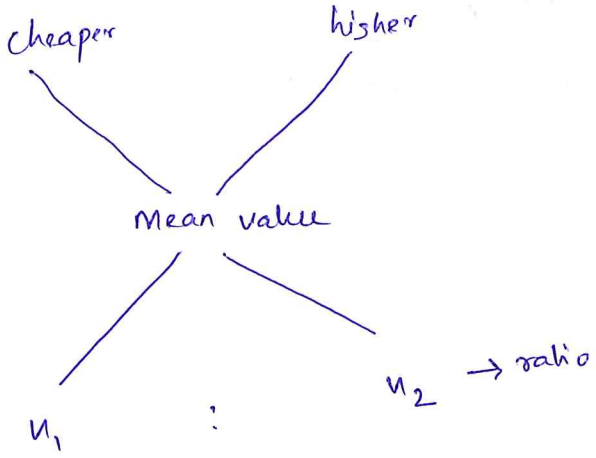
Alligation in Graphical Representation



$$\frac{n_1}{n_2} = \frac{x_2 - w_A}{w_A - x_1}$$

$A_1 = x_1$
$A_2 = x_2$

Given	Find
1) A_1, A_2, w_A	$n_1 : n_2$
2) A_1, A_2, w_A, n_1	$n_1 : n_2$ (or) n_2
3) A_1, A_2, n_1, n_2	w_A
4) A_2, A_w, n_1, n_2 or A_1	A_1 or A_2



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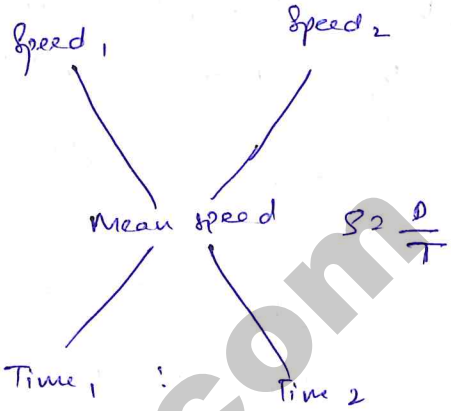
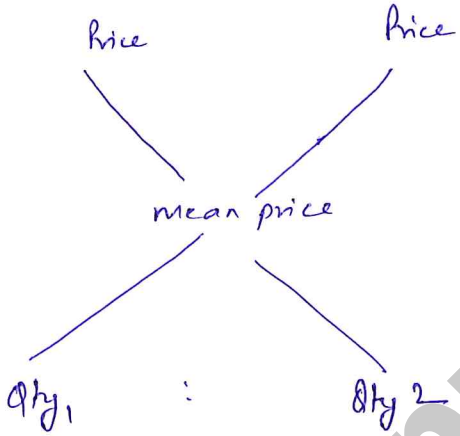


(1)



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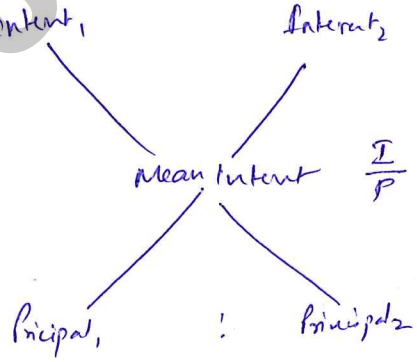

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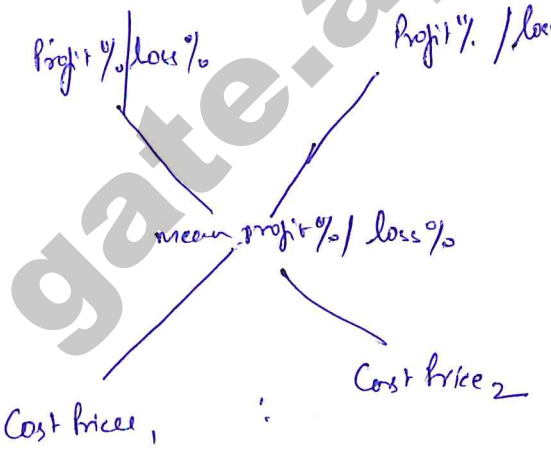


$\frac{P}{CP} \times 100$

10.2 : Problems on Alligation

Q.1) In what ratio must rice at ₹10.30 per kg mixed with rice at ₹11.80 per kg so that the mixture be worth ₹11 per kg ?

(a) 6:5 (b) 8:7 (c) 3:7 (d) 6:1

Mean Price = ₹11 per kg (W_A)

Type 1 Price = ₹10.30 per kg (A_1)

Type 2 Price = ₹11.80 per kg (A_2)

$$W_A = \frac{A_1 n_1 + A_2 n_2}{n_1 + n_2}$$

$$\Rightarrow 11 = \frac{(10.30) n_1 + (11.80) n_2}{n_1 + n_2}$$

$$\Rightarrow 11n_1 + 11n_2 = 10.30 n_1 + 11.80 n_2$$

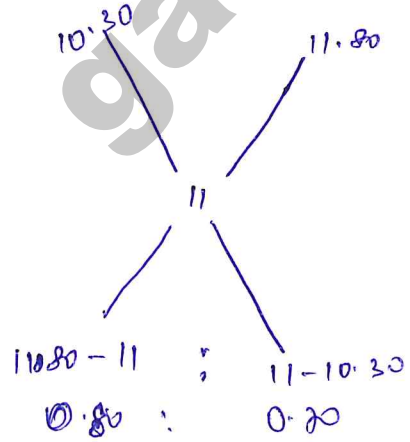
$$\Rightarrow 0.70 n_1 = 0.80 n_2$$

$$\Rightarrow \frac{n_1}{n_2} = \frac{8}{7}$$

(OR)

$$\frac{n_1}{n_2} = \frac{A_2 - W_A}{W_A - A_1} = \frac{11.80 - 11}{11 - 10.30} = \frac{0.80}{0.70} = \frac{8}{7}$$

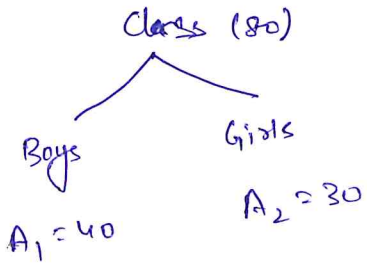
(OR)

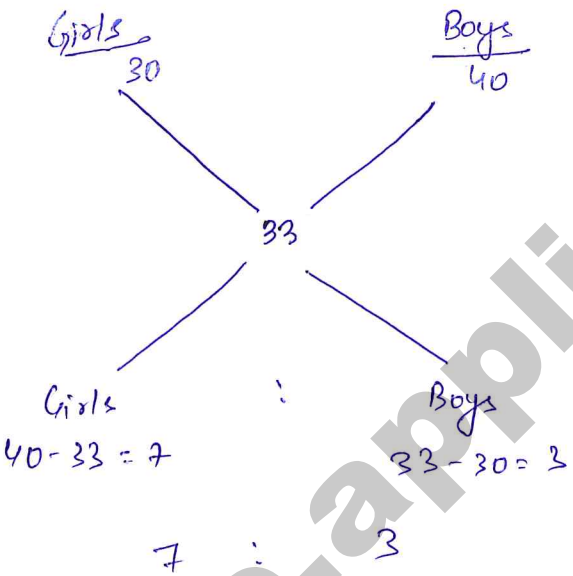


$= 8:7$

Q.2) The average weight of boys in a class is 40 kg and the average weight of girls in a class is 30 kg. If the average weight of the 80 students is 33 kg, the no. of boys in the class is :

(a) 24 (b) 56 (c) 34 (d) 46





$\therefore$ No. of boys in class = $80 \times \frac{3}{10}$

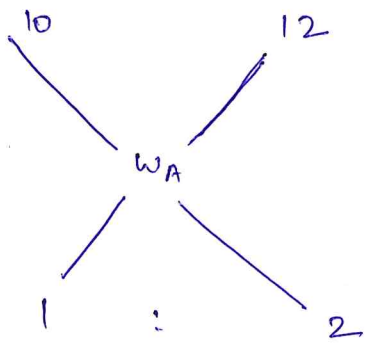
$= 24.$

Q.3) Two varities of rice at ₹10 per kg and ₹12 per kg are mixed together in the ratio 1:2. find the average price of resulting mixture.

Type 1 = ₹10/- per kg

Type 2 = ₹12/- per kg

$n_1 : n_2 = 1 : 2$



$$\frac{12 - w_A}{w_A - 10} = \frac{1}{2}$$

$$24 - 2w_A = w_A - 10$$

$$3w_A = 34$$

$$w_A = \frac{34}{3}$$

$$= ₹11.33 \text{ per kg.}$$

(OR)

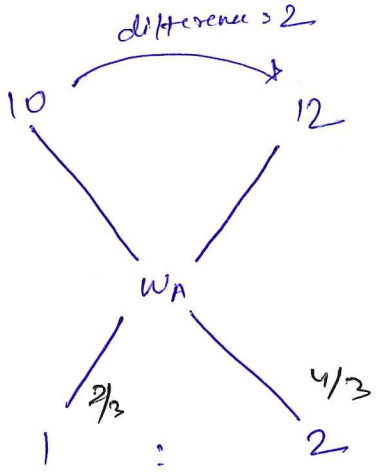
$$w_A = \frac{10 \times 1 + 12 \times 2}{3}$$

$$= \frac{10 + 24}{3}$$

$$= \frac{34}{3}$$

$$= ₹11.33 \text{ per kg.}$$

(Trick)

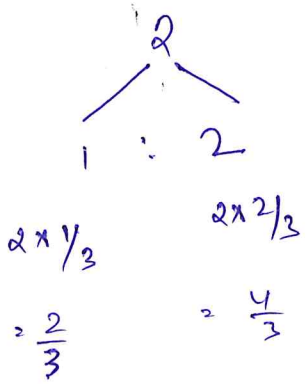


$W_A - \frac{4}{3} = 10$

$12 - W_A = \frac{2}{3}$

$W_A = 12 - \frac{2}{3} = \frac{34}{3}$

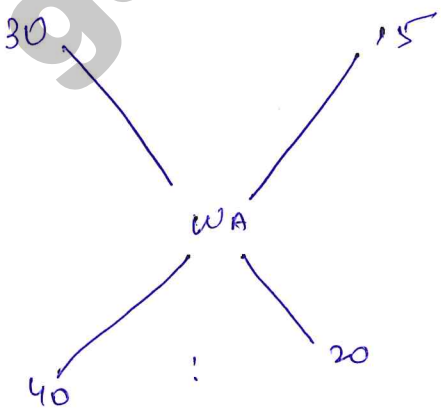
$W_A = 10 + \frac{4}{3} = \frac{34}{3}$



Q.4 The average weight of a class of 40 students is 30 and the average weight of a class of 20 students is 15. find the average weight of the combined class:

$W_A = \frac{40 \times 30 + 20 \times 15}{40 + 20} = \frac{1200 + 300}{60} = 25$

OR)



$\frac{30 - W_A}{W_A - 15} = \frac{20}{40} = \frac{1}{2}$

$\Rightarrow 60 - 2W_A = W_A - 15$

$\Rightarrow 3W_A = 75$

$\Rightarrow W_A = 25.$

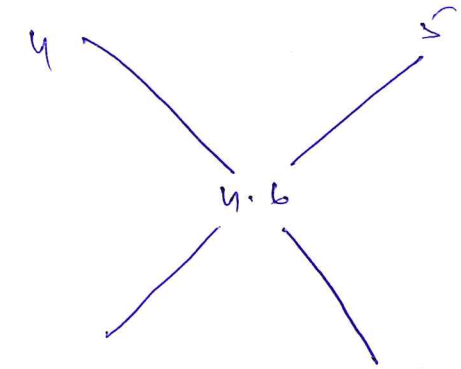
Q.57) Two types of oil having the rates ₹ 4 per kg and ₹ 5 per kg respectively are mixed in order to produce a mixture having the rate of ₹ 4.6 per kg. What should be the amount of the second type of oil, if the amount of the first type of oil in the mixture is 40 kg?

$A_1 = 4$

$A_2 = 5$

$W_A = 4.6$

$n_1 = 40\text{kg}$



$5 - 4.6 \quad : \quad 4.6 - 4$

$0.4 \quad : \quad 0.6$

$2 \quad : \quad 3$ (Ratio of Quantity)

$2 \longrightarrow 40$

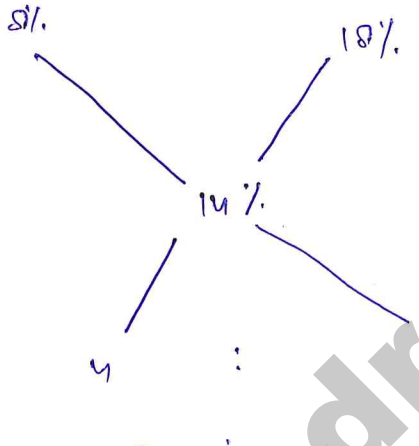
$3 \longrightarrow \frac{40}{2} \times 3 = 60\text{kg}.$

Q.6) A merchant has 50 kg of sugar, part of which he sells at 8% profit and the rest at 18% profit. He gains 14% on the whole. The quantity sold at 18% is?

$A_1 = 8\%$

$A_2 = 18\%$

$W_A = 14\%$

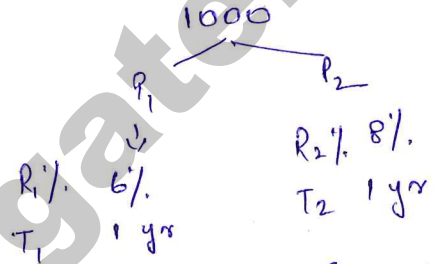


$= 2 : 3$

The quantity sold at 18% profit $= 50 \times \frac{3}{5}$

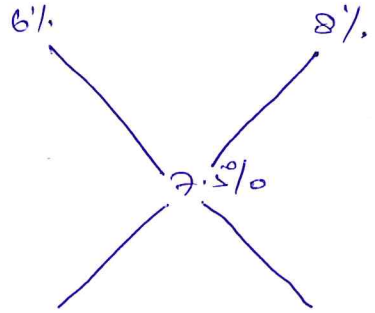
$= 30$ kg.

Q.7) ₹ 1000 is lent out in two parts, one at 6% simple interest and the other at 8% simple interest. The yearly income is ₹ 75. The sum lent at 8% is?



Income = ₹ 75

$\frac{75}{1000} \times 100 = 7.5\%$



0.5 : 1.5

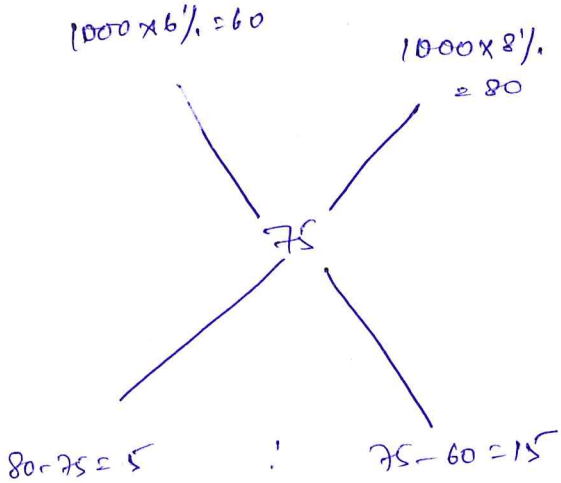
$= 5 : 15$

$= 1 : 3$ → Principal Ratio.

The sum lent at 8% $= \frac{3}{4} \times 1000$

$= ₹ 750/-$

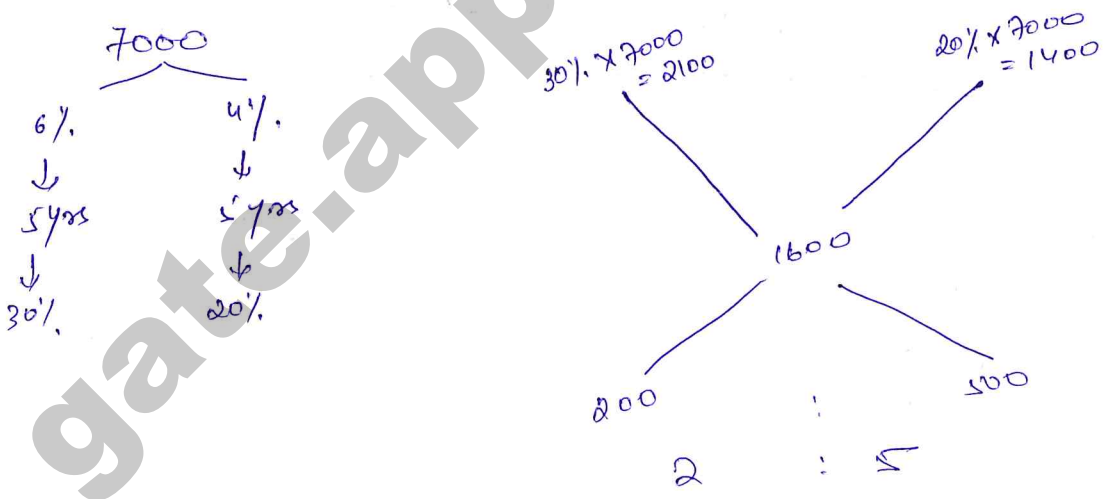
(ER)



The sum lent at 6% = $1000 \times \frac{3}{4}$

= ₹750/-

Q.8) Some amount out of ₹7,000 was lent at 6% p.a. and the remaining at 4% p.a. If the total simple interest from both the fraction in 5 year was ₹1600, the sum lent at 6% p.a. was.



Sum lent at 6% p.a: $\frac{2}{7} \times 7000$

= ₹2,000/-

Q.9) How many kgs of sugar costing ₹9 per kg must be mixed with 27kg of sugar costing ₹7 per kg so that there may be a gain of 10% by selling the mixture at ₹9.24/kg?

Type 1 = ₹9/- kg

Type-2 = ₹7/- kg

Quantity of Type 2 = 27kg.

Selling price of mixture = ₹9.24/kg @ 10% profit.

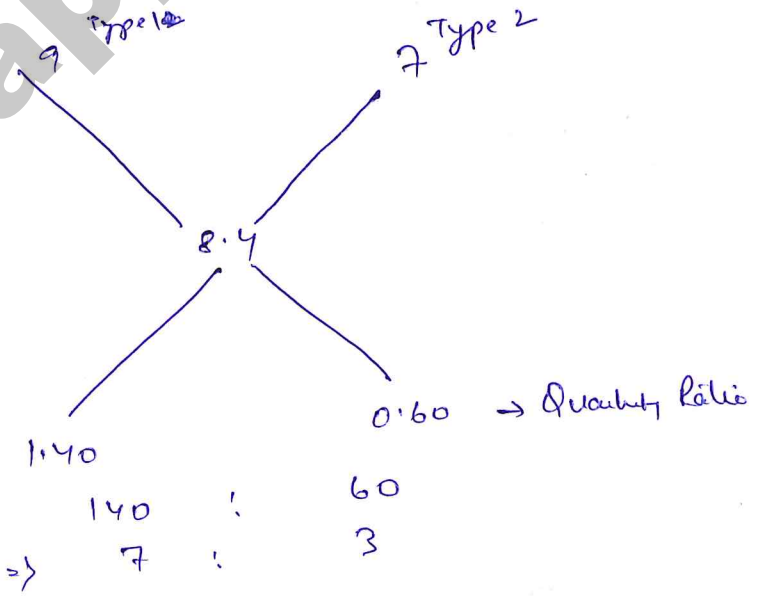
Profit % = 10% = $\frac{1}{10}$

CP	P	SP
10	1	11

11 ⟶ 9.24

10 ⟶ $\frac{9.24}{11} \times 10$

= ₹8.4/- per kg.



140 : 60

⇒ 7 : 3

3 ⟶ 27

7 ⟶ $\frac{27}{3} \times 7$ = 63kg ⟶ Type 1 quantity.

Q.10) Two vessels A and B contain milk and water mixed in the ratio 5 : 3 and 2 : 3. When these mixtures are mixed to form a new mixture containing half milk and half water, they must be taken in the ratio?

(a) 2 : 5 (b) 3 : 5 (c) 4 : 5 (d) 7 : 3

Vessel A

Milk : Water

5 : 3

$\frac{5}{8}$ $\frac{3}{8}$

Vessel B

Milk : Water

2 : 3

$\frac{2}{5}$ $\frac{3}{5}$

Vessel (C) → New

Milk : Water

$\frac{1}{2}$: $\frac{1}{2}$

1 : 1

A(milk) $\frac{5}{8}$ B(Milk) $\frac{2}{5}$

$\frac{1}{2}$ (milk)

$\frac{1}{2} - \frac{2}{5}$: $\frac{5}{8} - \frac{1}{2}$

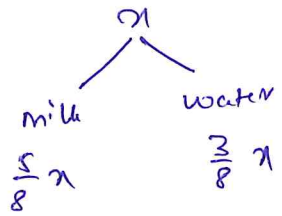
$\frac{1}{10}$: $\frac{1}{8}$

8 : 10

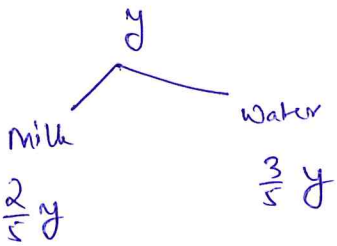
4 : 5

(OR)

Let A vessel be x litres



let B vessel be y litres



$$\frac{\frac{5}{8}x + \frac{2}{5}y}{\frac{3}{8}x + \frac{3}{5}y} = \frac{\frac{1}{2}}{\frac{1}{2}}$$

$$\Rightarrow \frac{5}{8}x + \frac{2}{5}y = \frac{3}{8}x + \frac{3}{5}y$$

$$\Rightarrow \frac{1}{4}x = \frac{1}{5}y$$

$$\Rightarrow x : y = 4 : 5$$

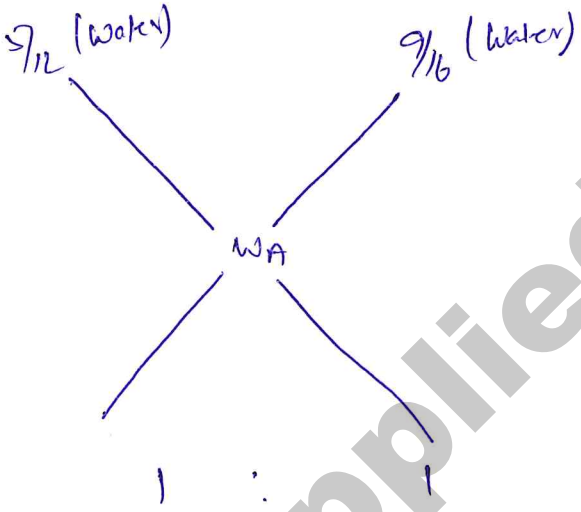
Q.11) Two vessels contain milk and water in the ratio 7:5 and 7:9. If both vessels are mixed in the ratio 1:1, find the ratio of milk and water in new mixture?

Vessel A

Milk	Water
7	5
7/12	5/12

Vessel B

Milk	Water
7	9
7/16	9/16



$$\frac{9/16 - W_A}{W_A - 5/12} = \frac{1}{1}$$

$$\Rightarrow 9/16 - W_A = W_A - 5/12$$

$$\Rightarrow 2W_A = 9/16 + 5/12$$

$$\Rightarrow 2W_A = \frac{47}{48}$$

$$\Rightarrow W_A = \frac{47}{96} \quad \text{(Water in new mixture)}$$

Milk in new mixture $= 1 - \frac{47}{96} = \frac{49}{96}$

$\therefore$ Milk : Water = 49 : 47.

(OR)

Milk	Water
7 : 5	12 × 4
7 : 9	16 × 3

) LCM = 48 litre

28	20
21	27
49	47

49 : 47. (Required Ratio)

10.3 Problems on Alligation and Mixture

Q.12) 15 litres of a mixture contains 20% alcohol and rest water. If 3 litres of water be mixed in it, the percentage of alcohol in the new mixture will be?

15 litre

Alcohol	Water
20%	80%
3 l	12 l
	+3 l water added
3 l	15 l

$3 : 15$

$= 1 : 5$

$\therefore$ % of alcohol $= \frac{3}{18} \times 100$

$= 16\frac{2}{3}\%$

Q.13) The ratio of milk and water in 66kg of adulterated milk is 5:1. Water is added to it to make the ratio 5:3. The quantity of water added is?

66

Milk	Water
5	1
$\frac{5}{6} \times 66$	$\frac{1}{6} \times 66$
$= 55$	$= 11$

After adding water ratio =

$5 : 3$

Let the water added be x

$\therefore \frac{55}{11+x} = \frac{5}{3}$

$\Rightarrow x = 22$

Q.14) 729 ml. of a mixture contains milk and water in the ratio 7 : 2. How much more water is to be added to get a new mixture containing milk and water in the ratio 7 : 3?

729

Milk : Water

7 : 2 → 7+2 = 9

7 : 3 → +1

9 → 729

1 → $\frac{729}{9}$ = 81 litre to be added.

Q.15) A mixture contains milk and water in the ratio 4 : 3. If 7 litres of water is added to it, the ratio of milk and water becomes 3 : 4. Find the quantity of milk in mixture.

Milk		Water	
4	:	3	7 litre of water
New 3	:	4	

4 : 3 × 3

3 : 4 × 4

12 : 9 → +7

12 : 16

7 → 7

12 → $\frac{7}{7} \times 12$ = 12 litres.

10.4: Replacement or Removal Problems

100 litres → Pure Milk

↓ Removed 20 l milk

↘ added 20 l water

100 litres (Mixture)

Milk — 80 l

Water — 20 l

8 : 2

⇒ 4 : 1

Removed again 20 litres of Mixture

4 : 1

16 l Milk

4 l water

80 l − 16 l = 64 l

20 l − 4 l = 16 l

Added 20 litre water: 16 l + 20 l = 36 l

Ratio = 64 : 36

= 16 : 9

Formula

Final amount of ingredient that is not replaced $= \text{Initial} \times \left(1 - \frac{\text{Replace}}{\text{Total}}\right)^{n}$ → no. of repetations

$$\text{Final (milk)} = 100 \times \left(1 - \frac{20}{100}\right)^{2}$$

$= 64$ litre.

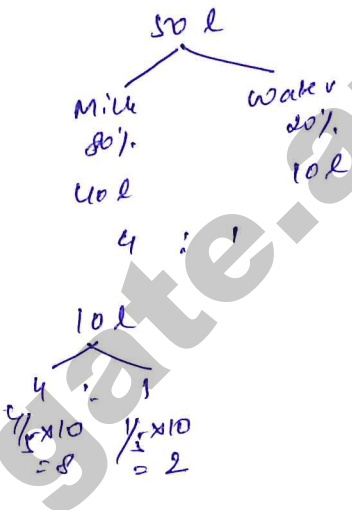
Water $= (100 - 64)$ litre

$= 36$ litre

Q.16) In a 50 litre mixture of water and milk, water is only 20%. The milkman gives 10 litres of this mixture to a customer and then he adds up 10 litres of pure water in the remaining mixtures. The % of water in the final mixture is :

(a) 84% (b) 74% (c) 26% (d) 36%

Total Quantity of mixture = 50 litre



$$\begin{array}{cc} 40 & 10 \\ -8 & -2 \\ \hline 32 & 8 \end{array}$$

add $\quad +10$

$$\begin{array}{cc} 32 & 18 \end{array}$$

Ratio 16 : 9 (Milk & Water)

% of water $= \frac{18}{50} \times 100 = 36\%$

(OR)

$$\text{final} = \text{Initial} \times \left(1 - \frac{\text{replace}}{\text{Total}}\right)^n$$

$$\text{final}_{(\text{Milk})} = 40 \times \left(1 - \frac{10}{50}\right)^1$$

$$= 40 \times \frac{4}{5}$$

$$= 32\ l$$

$$\text{Water} = 50 - 32$$

$$= 18\ l$$

$$\%\text{ of water in mixture} = \frac{18}{50} \times 100$$

$$= 36\%.$$

Q.17) From the 100 litre milk, 10 litre of milk is taken out and added the water. Again 10 litre of mixture is taken out and added the same amount of water. If this process is continued one more time, the quantity of milk left after the third replacement is :

	Milk	Water
Initial	100 ltr	0 ltr
1st time: Taken out milk	10 ltr	
	90 ltr	0 ltr
added water		10 ltr
	90 ltr	10 ltr ⇒ 9 : 1
2nd time: 10 ltr of mixture taken out (9 : 1)	−9	−1
	81 ltr	9 ltr
added water		10 ltr
	81 ltr	19 ltr ⇒ 81 : 19
3rd time: Taken out 10 ltr. (81 : 19)	−8.1	−1.9
	72.9 ltr	17.1 ltr
added water		10 ltr
	72.9 ltr	27.1 ltr

Q.18) A vessel of 24 litre is full of pure alcohol, for the first time 1/3 of alcohol is replaced by water and for the second time 3/4 of mixture is replaced by water. find the quantity of water at the end.

Total Quantity = 24 litre

Pure Alcohol = 24

	Alcohol	Water
1st time $\frac{1}{3} \times 24 = 8$	16	8 $\Rightarrow$ 2 : 1
2nd time $\frac{3}{4} \times 24 = 18$	-12	-6
2 : 1	4	2
12 6		
Added	4	18
		20 litre quantity of water

(OR)

$$\text{Final} = \text{Initial}\left(1 - \frac{R}{T}\right)^n$$

$$= 24\left(1 - \frac{1}{3}\right)\left(1 - \frac{3}{4}\right)$$

$$= 24 \times \frac{2}{3} \times \frac{1}{4}$$

$$= 4 \text{ (amount of alcohol)}$$

Quantity of Water = 24 − 4 = 20 ltr.

Q.19) The ratio of petrol and kerosene in the container 3:2 when 10 litres of the mixture is taken out and is replaced by the kerosene, the ratio becomes 2:3. The total quantity of the mixture in the container is :

(a) 25 (b) 30 (c) 45 (d) Cannot determine

Let Petrol be $3x$ and kerosene $2x$.

Total Quantity $= 5x$

Petrol	Kerosene
$3x$	$2x$
-6	-4
$3x-6$	$2x-4$
	$+10$
$3x-6$	$2x+6$

10

3 : 2

6 4

$$\rightarrow \frac{3x-6}{2x+6} = \frac{2}{3}$$

$$9x-18 = 4x+12$$

$$5x = 30$$

Total Quantity $= 30$ ltr.

(OR)

$$\text{Final} = \text{Initial} \times \left(1 - \frac{\text{Replace}}{\text{Total}}\right)^n$$

$$\Rightarrow \text{Final}_{(\text{Petrol})} = \text{Initial}_{(\text{Petrol})} \times \left(1 - \frac{R}{T}\right)^n$$

$$\Rightarrow \frac{2}{5} = \frac{3}{5} \times \left(1 - \frac{10}{T}\right)^1$$

$$\Rightarrow \frac{2}{3} = 1 - \frac{10}{T}$$

$$\Rightarrow T = 30 \text{ ltrs.}$$

	Petrol	Kerosene
Initial	3	2
	3/5	
Final	2	3
	2/5	

(Trick) (One time repetation then only)

	Petrol		Kerosene
Initial	3	:	2
final	2	:	3

-1 on the left, $+1$ on the right

difference of value should be same

$n = 1$, Replace $= 10$

10

3 : 2

6 4

$1 \longrightarrow 6$

$5 \longrightarrow \frac{6}{1} \times 5 = 30$ litres.

Q.20) A can contains a mixture of two liquids A and B in proportion 7 : 5. When 12 litres of mixture are drawn off and the can is filled with B, the proportion of A and B becomes 7 : 9. How many litres of liquid A was contained by the can initially?

Initial Ratio $= \frac{A}{7} : \frac{B}{5}$ $\quad R = 12$, $n = 1$

Final Ratio $= 7 : 9$

$$\text{final}_{(A)} = \text{Inital}_{(A)} \left(1 - \frac{R}{T}\right)^n$$

$$\frac{7}{16} = \frac{7}{12}\left(1 - \frac{12}{T}\right)^1$$

$$\Rightarrow \frac{12}{16} = 1 - \frac{12}{T}$$

$$\Rightarrow T = 48 \text{ litres.}$$

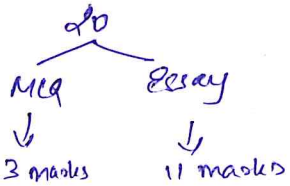
A Initially $= 48 \times \frac{7}{12} = 28$ litres.

10.5: Previous Year Gate Exam Questions on Alligations

Q.1) A test has twenty questions worth 100 marks. The test consists of MCQ's worth 3 marks each and Essay questions worth 11 marks each. The test consists of ____ MCQ's. (GATE 2017)

(a) 12 (b) 15 (c) 18 (d) 13

Solution



Let MCQ = x

Essay = y

$x + y = 20 \quad \times -3$

$3x + 11y = 100$

$-3x - 3y = -60$

$8y = 40$

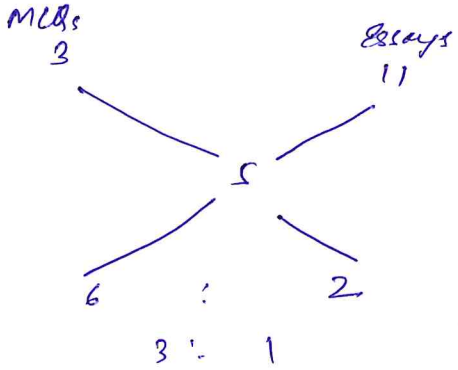
$y = 5$

$\therefore x = 15$

2nd method

20 questions → 100 marks

$\text{Avg} = \frac{100}{20} = 5$ (mean)



$\text{MCQs} = 20 \times \frac{3}{4}$

$= 15$

10.6 : Previous Year Gate Exam Questions on Alligations

Q. 2) A container originally contains 10 litres of pure spirit. From this container 1 litre of spirit is replaced with 1 litre of water. Subsequently, 1 litre of mixture is again replaced by with 1 litre of water and this process is repeated one more time. How much spirit is now left in container?

(a) 7.58 ltr (b) 7.84 ltr (c) 7 ltr (d) 7.29 ltr.

$$\text{final (spirit)} = \text{initial (spirit)} \left(1 - \frac{R}{T}\right)^n$$

$R = 1$ ltr

$n = 3$

$T = 10$ ltr.

$$\text{final} = 10\left(1 - \frac{1}{10}\right)^3$$

$$= 10 \times \frac{9}{10} \times \frac{9}{10} \times \frac{9}{10}$$

$= 7.29$ litre of spirit is left.
