LCM and HCF

2.1: LCM & HCF Introduction

LCM

Least Common Multiple

HCF

Highest Common Factor

Ex: $12 = [1, 2, 3, 4, 6, 12]$ ← These numbers divide the given number.

↓ Products for 12

1×12

2×6

3×4

↓

Factors

Any number which divide the given number is called factors.

Ex: Factors of 15 = 5 and 3.

* 1 and number itself are always called the improper factors.

Ex: Factor of 18 = 1, 2, 3, 6, 9, 18.

Multiple

The given number is called the multiple.

A number divided by another number.

↓

Multiple

* Factors always divide the number.
* Multiple always divided by another number.

Ex: $\frac{18}{3}$ → 18 → Multiple, 3 → factor (or) Divisor.

Ex

Factors of 35 = 1, 5, 7, 35

35
↓
1 × 35
5 × 7

Ex

Multiples of 12 = 12, 24, 36, 48, - - -

Ex

Multiples of 15 = 15, 30, 45, 60, 75, 90, - - - -

Prime factorization

Any natural number is expressed as product of prime numbers.

Ex

$24 = 2^3 \times 3$

8×3

Ex

$35 = 5 \times 7$

Ex

$36 = 12 \times 3 = 3 \times 4 \times 3 = 3 \times 2 \times 2 \times 3$

12 → Non Prime; 3 → Prime

$= 2^2 \times 3^2$.

LCM - Least Comman Multipk

Ex

12 = 12, 24, 36, 48, (60), 72, 84, 96, 108, (120) .....

15 = 15, 30, 45, (60), 75, 90, 105, (120) 135, 150, ...

LCM of 12 & 15 is 60.

Ex

4 = 4, 8, (12), 16, 20, (24), 28, 32, (36), 40, 44, (48) .....

6 = 6, (12), 18, (24) 30, (36), 42, (48), 54, 60, ...

LCM of 4 & 6 is 12.

LCM can be done in 2 Methods

↳ Divison Method

↳ Prime factorization Method

Ex 12, 18

1) Divison Method

2	12, 18
3	6, 9
	2, 3

LCM = 2 x 3 x 2 x 3

= $2^2 \times 3^2$

= 36

2) Prime factorization

12 = 4 x 3 = $2^2 \times 3$

18 = 9 x 2 = $3^2 \times 2$

$2^2 \times 3^2$ = 4 x 9

= 36

Ex

Find the LCM of 48, 72.

1) Divison Method

2	48, 72
3	24, 36
2	8, 12
2	4, 6
	2, 3

LCM : $2 \times 3 \times 2 \times 2 \times 2 \times 3$

$= 2^4 \times 3^2$

$= 16 \times 9$

$= 144.$

2) Prime factorization

$48 = 8 \times 6 = 2^3 \times 2 \times 3$

$= 2^4 \times 3$

$72 = 36 \times 2 = 9 \times 4 \times 2$

$= 3^2 \times 2^3$

$2^4 \times 3^2$

$= 16 \times 9$

$= 144.$

Ex

Find the LCM of 24, 40, 42, 44.

1) Division Method

$$\begin{array}{r|l} 2 & 24, 40, 42, 44 \\ \hline 2 & 12, 20, 21, 22 \\ \hline 3 & 6, 10, 21, 11 \\ \hline 2 & 2, 10, 7, 11 \\ \hline & 1, 5, 7, 11 \end{array}$$

LCM $= 2 \times 2 \times 3 \times 2 \times 5 \times 7 \times 11$

$= 2^3 \times 3 \times 5 \times 7 \times 11$

$= 9240.$

2) Prime factorization

$24 = 8 \times 3 = 2^3 \times 3$

$40 = 8 \times 5 = 2^3 \times 5$

$42 = 6 \times 7 = 2 \times 3 \times 7$

$44 = 4 \times 11 = 2^2 \times 11$

$2^3 \times 3 \times 5 \times 7 \times 11$

$= 9240.$

Co-Prime

Any two numbers have no common factor other than 1.

Ex:

2, 7 are co-prime.

↓

they have common factor 1. [HCF of two numbers is 1] → said to be co-prime.

(15, 19)

2 = 1, 2

7 = 1, 7

1) Any two consecutive numbers are said to be coprime.

12, 13

13, 14

14, 15

2) Any two prime numbers are said to be co prime.

2, 3

13, 19

Ex:

Find LCM of 12, 18.

12, 18

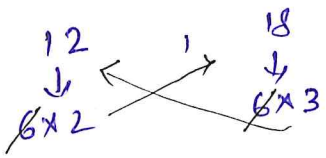
6 (2, 3) => 6 × 6 = 36.

Product of any two co-prime numbers = LCM.

Ex:

find LCM of 3, 7 = 21

Ex: Find the LCM of 12, 18.



LCM = 12×3 (or) 18×2

Ex: Find the LCM of 48, 72.

48, 72

12(4, 6)

24(2, 3)

24×6

= 144

48 , 72

24×2 24×3

LCM = 48×3 (or) 72×2

= 144.

Ex: Find the LCM of 24, 40, 42, 44.

24, 40, 42, 44

2(12, 20, 21, 22)

2(12, 20, 22, 21)

2(2(6, 10, 11), 21)

2(2(330), 21)

2(3(220, 7))

6×1540

= 9240

Ex

Find the LCM for 12, 72, 144

12, 72, 144

12 (1, 6, 12)

12 (12)

144

HCF - Highest Common Factor (OR) Greatest Common Divisor (GCD)
(or) Greatest Common Measure (GCM)

Two Methods

(a) Divison Method

(b) Prime factorisation Method

Ex

Find HCF of 12, 18.

12 = 1, (2), (3), 4, (6), 12

18 = 1, (2), (3), (6), 9, 18

2, 3, (6)

HCF = 6.

Division Method

```
12) 18 ( 1
    12
   ---
    6) 12 ( 2
       12
      ---
       0
```

HCF = 6.

Prime factorisation

$12 = 2^2 \times 3$

$18 = 2 \times 3^2$

$2^1 \times 3^1$

$= 6.$

Ex HCF of $2^2 \times 3^5 \times 5^2$, $5^3 \times 7^2 \times 2$

HCF = $2^1 \times 5^2$

Ex HCF of 24, 40, 42.

(1) Division Method

$$
\begin{array}{r|l}
24 & 40 \ (1 \\
24 & \\
\hline
16 & 24 \ (1 \\
16 & \\
\hline
8 & 16 \ (2 \\
16 & \\
\hline
0 &
\end{array}
$$

HCF is 8 for 24 and 40.

$$
\begin{array}{r|l}
8 & 42 \ (5 \\
40 & \\
\hline
2 & 8 \ (4 \\
8 & \\
\hline
0 &
\end{array}
$$

HCF is 2.

(2) Prime factorization

$24 = 8 \times 3 = 2^3 \times 3$

$40 = 8 \times 5 = 2^3 \times 5$

$42 = 6 \times 7 = 2 \times 3 \times 7$

2^1

HCF = 2.

Ex =

HCF of 12, 18.

12 , 18

6×2 6×3

HCF = 6.

Ex =

HCF of 48, 72

48 , 72

12×4 , 12×6

24×2, 24×3

HCF = 24

Ex =

HCF of 24, 40, 42

24, 40, 42

2×12, 2×20, 2×21

HCF = 2.

- LCM concept is useful to solve the problems on races, traffic signals
- HCF concept is useful to solve the problems finding largest size of tile to fit in a room.

2.2 Problems on LCM

Q.1) Find the least possible number which can be divided by 32, 36 and 40?

* LCM always divided by
* HCF always divide

LCM (32, 36, 40)

4 (8, 9, 10)

4 (2(4, 5), 9)

4 (40, 9)

LCM = 1440

(OR)

32, 36, 40

$32 = 2^5$

$36 = 2^2 \times 3^2$

$40 = 2^3 \times 5$

$2^5 \times 3^2 \times 5^1$

$= 32 \times 9 \times 5$

$= 1440.$

Q.2) Find the least number of 4 digits which is divisible by 4, 6, 8 and 10?

(a) 1050 (b) 1070 (c) 1080 (d) 1008

LCM of (4, 6, 8, 10)

2^2 2×3 2^3 2×5

$2^3 \times 3^1 \times 5^1 = 120$

120, 240, 360 ..... 960, 1080, 1200

Ans => 1080.

Alternative method

Least 4 digit = 1000

LCM of (4, 6, 8, 10) = 120

$$\begin{array}{r|l} 120 & 1000 \\ \hline & 960 \\ \hline & 40 \end{array}\quad 8$$

Required Number = 1000 + Difference (120 − 40)

= 1000 + 80

= 1080

(OR)

Required Number (least) = Dividend + Divisor − Remainder

Q.3) Find the greatest number of 4 digits which is divisible by 48, 60 and 64?

(a) 9600 (b) 1960 (c) 9620 (d) 9610

LCM (48, 60, 64)

$48 = 2^4 \times 3$

$60 = 2^2 \times 3 \times 5$

$64 = 2^6$

$2^6 \times 3 \times 5 = 64 \times 15$

$= 960.$

960 × 1 = 960

× 2 = 1920

× 3 = 2880

⋮

× 10 = 9600

× 11 = 10560.

∴ Ans = 9600

Alternative

$$\begin{array}{r|l} 960 & 9999 \\ \hline & 960 \\ \hline & 399 \end{array}\quad 1$$

Required Number (Greatest) = Dividend − Remainder

= 9999 − 399 = 9600.

LCM with Remainders

Q.4) What is the least possible number which when divided by 24, 32 or 42 in each case it leaves the remainder 5 ?

LCM (24, 32, 42)

$24 = 2^3 \times 3$

$32 = 2^5$

$42 = 2 \times 3 \times 7$

$2^5 \times 3 \times 7$

$= 96 \times 7$

$= 672$

$24)672(28$
$\underline{\quad}$
(0)

Required Number = $672 + 5 = 677$.

Q.5) What is the least possible number which must be added to 4722 so that it becomes divisible by 21, 25, 27 & 35?

LCM (21, 25, 27, 35)

$21 = 3 \times 7$

$25 = 5^2$

$27 = 3^3$

$35 = 5 \times 7$

$3^3 \times 5^2 \times 7$

$= 27 \times 25 \times 7$

$= 4725$

Required number = $4725 - 4722$

$= 3.$

Q. 6) What is the least possible number which when divided by 18, 35. or 42 it leaves the 2, 19, 26 as the remainder respectively!

18	35	42
− 2	− 19	− 26
16	16	16

LCM (18, 35, 42)

$18 = 3^2 \times 2$

$35 = 5 \times 7$

$42 = 2 \times 3 \times 7$

$2 \times 3^2 \times 5 \times 7$

$= 630$

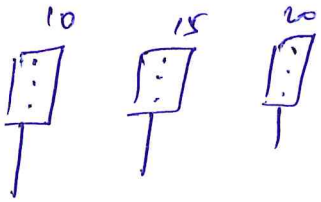
Required Number = 630 − 16

= 614.

18) 614 (34
54
74
72
(2)

35) 614 (17
35
264
245
(19)

Q.7) Three traffic signals change from Reds to Green in 10, 15 & 20 seconds respectively. After how much time will all the three signals together become green?



→ Same color

1st → 10, 20, 30, 40, 50, (60), 70, 80, 90, ...

2nd → 15, 30, 45, (60), 75, 90, 105, ...

3rd → 20, 40, (60), 80, 100, 120, ...

Ans ⇒ After 60 secs.

LCM (10, 15, 20)

5(2, 3, 4)

5 × 12 = 60 sec.

2.3: Problems on HCF

Q.1) What is the greatest number that will exactly divide 39, 299, 338?

(a) 13 (b) 39 (c) 17 (d) None

HCF (39, 299, 338)
↓ ↓ ↓
13×3 13×23 13×26

HCF = 13

Q.2) find the greatest possible number which when we divide 37 and 58, it leaves the respective remainder of 2 and 3?

N) 37 (2 N) 58 (3

HCF of [(37-2) & (58-3)]

HCF of (35, 55)

35 = 5×7
55 = 5×11
HCF = 5

Q.3) find the longest possible number with which when 60 and 98 are divided it leaves the remainder 3 in each case?

N) 60 (3 N) 98 (3

HCF of [(60-3), (98-3)]

HCF of (57, 95)

= 19.

Q.4) Find the largest possible number with which when 38, 66 and 80 are divided the remainder remains the same.

$N) \frac{38}{R}$ ($N) \frac{66}{R}$ ($N) \frac{80}{R}$ (

HCF of $[(66-38), (80-66), (80-38)]$

HCF of (28, 14, 42)

$28 = 14 \times 2$
$14 = 14 \times 1$
$42 = 14 \times 3$

HCF = 14.

Q.5) Find the largest scale size to measure accurately, the three equilateral triangles with sides measuring 76 cm, 114 cm and 152 cm.

(a) 19 cm (b) 21 cm (c) 38 cm (d) None

76, 114, 152

HCF (76, 114, 152)

$76 = 19 \times 4 = 38 \times 2$

$114 = 19 \times 6 = 38 \times 3$

$152 = 19 \times 8 = 38 \times 4$

HCF = 38.

38 cm equidateral triangle.

(X)

No. of equilateral triangles

$\frac{76}{38} = 2$, $\frac{114}{38} = 3$, $\frac{152}{38} = 4$

$2 + 3 + 4 = 9$.

Q.6) Find the number of pairs of two numbers whose HCF is 5 and their sum is 50?

HCF = 5

let the numbers be 5a, 5b

$5a + 5b = 50$

$a + b = 10$

(1, 9), (2, 8), (3, 7), (4, 6), (5, 5)

Pairs showed by co-prime.

2 possible pairs ⇒ (1, 9) and (3, 7)

Q.7) If the ratio of the two numbers is 2:3 and their LCM is 54, then the sum of two numbers is:

$2x, 3x$ LCM = 54

LCM of $(2x, 3x)$

$x(2, 3)$

$6x$

$6x = 54$

$x = 9$

Numbers = 18, 27 Sum = 18 + 27 = 45.

Q.8) If the ratio of two numbers is 2:3 and their HCF is 6, then the sum of two numbers is

$2x, 3x$

HCF of $(2x, 3x)$

HCF = x

$\therefore x = 6$

Numbers = 12, 18

Sum = 12 + 18 = 30.

2.14: LCM & HCF of decimals

Ex: Find the HCF and LCM of 1.08, 0.36 and 0.9.

Multiply with 100

$(1.08, 0.36, 0.9) \times 100$

HCF (108, 36, 90)

$108 = 18 \times 6$

$36 = 18 \times 2$

$90 = 18 \times 5$

HCF = 18

$\therefore$ HCF of $(1.08, 0.36, 0.9) = \frac{18}{100} = 0.18$

LCM (1.08, 0.36, 0.9) $\times 100$

LCM (108, 36, 9)

LCM $(18\times6, 18\times2, 18\times5)$

LCM $18(6, 2, 5)$

18×30

540

LCM of $(1.08, 0.36, 0.9) = \frac{540}{100} = 5.4$

HCF and LCM of fractions

$$\text{HCF (Fractions)} = \frac{\text{HCF (Numerators)}}{\text{LCM (Denominators)}}$$

$$\text{LCM (Fractions)} = \frac{\text{LCM (Numerators)}}{\text{HCF (Denominators)}}$$

Ex: Find HCF & LCM of $\frac{2}{3}, \frac{8}{9}, \frac{10}{27}$

$$\text{HCF} = \frac{\text{HCF}(2,8,10)}{\text{LCM}(3,9,27)}$$

$$= \frac{2}{27}$$

$$\text{LCM} = \frac{\text{LCM}(2,8,10)}{\text{HCF}(3,9,27)}$$

$$= \frac{40}{3}$$

(*) Let numbers be a, b

Product of (a, b) = HCF (a, b) × LCM (a, b)

$$a \times b = \text{HCF}(a,b) \times \text{LCM}(a,b)$$

Ex

LCM of two numbers is 225 and HCF is 5. If one number is 25, then the other number will be:

$$a \times b = \text{HCF} \times \text{LCM}$$

$$25 \times b = 5 \times 225$$

$$b = 45$$