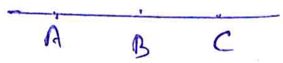
GEOMETRY (1)

4.1 : Geometry Basics

Basic Concepts / definitions

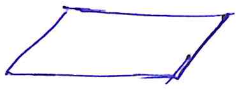
1) Point : • has no dimensions

2) line :

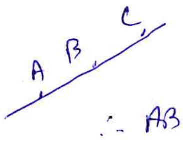


3) line segment :

4) Plane :



5) Collinear Points :

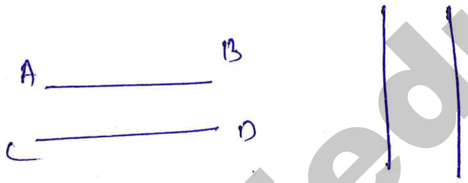


∴ ABC are collinear Points.

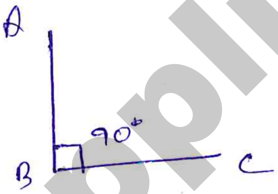
∴ PQR are non collinear Points

6) Parallel lines :

AB || CD

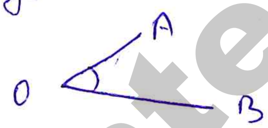


7) Perpendicular lines :



AB ⊥ BC

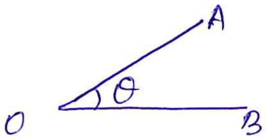
8) Angle



∠AOB

14.2: Types of Angles

(i) Acute Angle

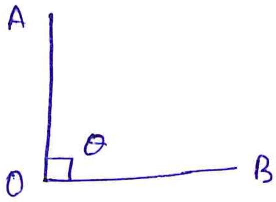


$\theta = 45^\circ, 60^\circ, \ldots$

$\theta < 90^\circ$

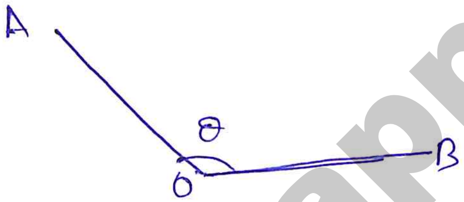
$0^\circ < \theta < 90^\circ$

(ii) Right Angle



$\theta = 90^\circ$

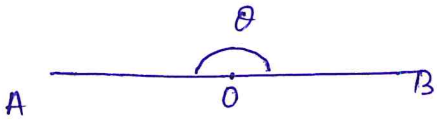
(iii) Obtuse Angle



$\theta = 125^\circ, 155^\circ, \ldots$

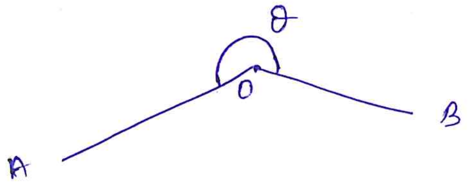
$90^\circ < \theta < 180^\circ$

(iv) Straight Angle



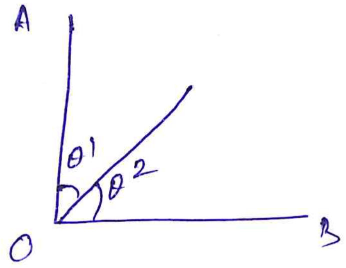
$\theta = 180^\circ$

v) Reflex angle



$180^\circ < \theta < 360^\circ$

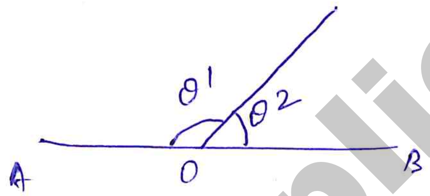
VI) Complimentary angle



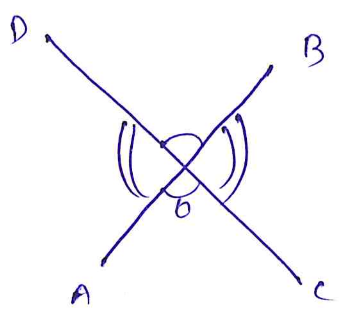
$\theta_1 + \theta_2 = 90^\circ$

VII) Supplementary angle

$\theta_1 + \theta_2 = 180^\circ$



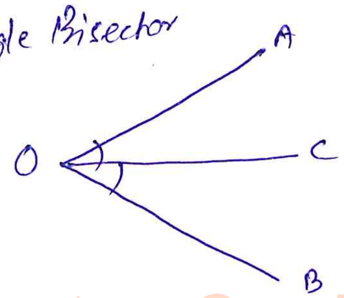
(VIII) Vertically opposite angle



$\angle AOC = \angle BOD$

$\angle BOC = \angle AOD$

ix) Angle Bisector



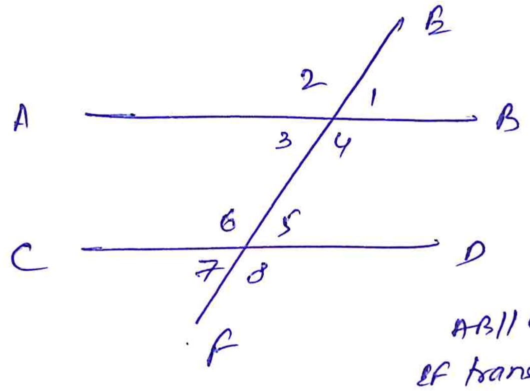
$\angle AOB$

OC Angle bisector

$\angle AOC = \angle COB = \frac{1}{2} \angle AOB$

Transversal

A line intersects the two or more parallel lines then that line is called the transversal.



$AB \parallel CD$

EF transversal

3, 4, 5, 6 → Interior angles

1, 2, 7, 8 → Exterior angles

(1,5), (2,6), (3,7), (4,8) → complimentary angles

(4,6), (3,5) → Interior alternate angles

(1,7), (2,8) → Exterior alternate angles

(1,3), (2,4), (5,7), (6,8) → vertically opposite angles

Sum of interior corresponding angles = 180°

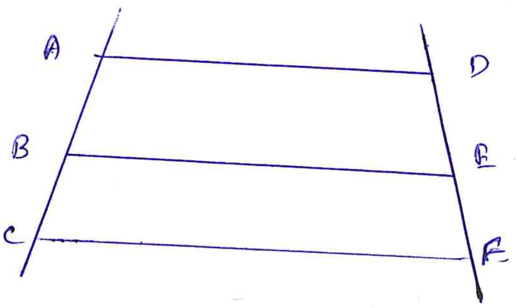
$\angle 4 + \angle 5 = 180^\circ$

$\angle 3 + \angle 6 = 180^\circ$

Sum of exterior corresponding angles = 180°

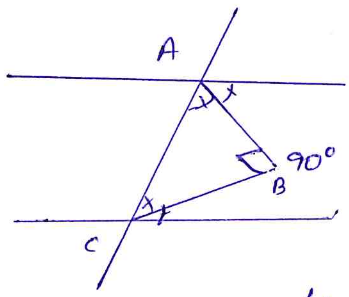
Other concepts of transversal

①



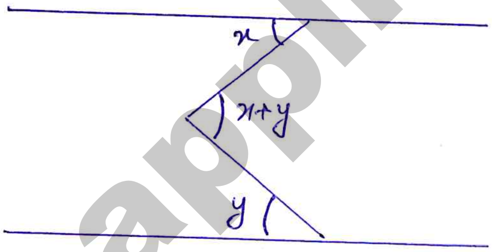
$$\frac{AB}{BC} = \frac{DE}{EF}$$

② Bisector of interior angles intersect at 90°

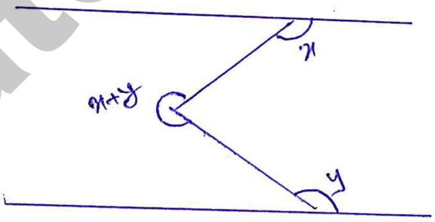


AB & BC are angle bisectors

③



④



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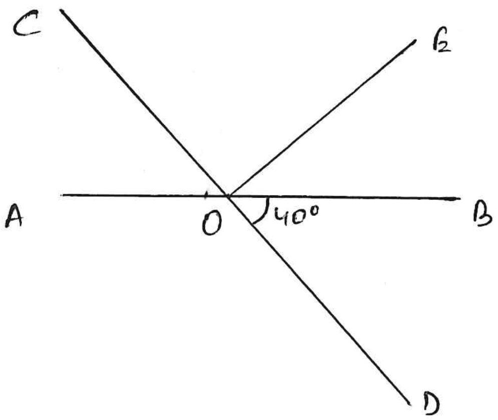
4.3 : Basic Problems on Geometry

Q.1) In the given figure, lines AB and CD intersect at O. If $\angle AOC + \angle BOE = 70^\circ$ and $\angle BOD = 40^\circ$, find $\angle COE$?

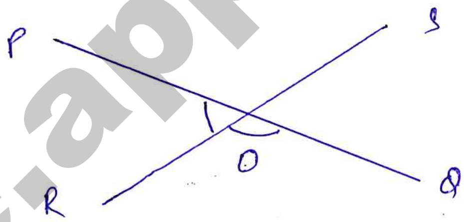


$\angle AOC + \angle COE + \angle BOE = 180^\circ$

$70^\circ + \angle COE = 180^\circ$

$\therefore \angle COE = 110^\circ$

Q.2) Lines PQ and RS intersect each other at point O. If $\angle POR : \angle ROQ = 5:7$, find $\angle SOQ$?



$\angle POR : \angle ROQ = 5:7$

$\angle POR + \angle ROQ = 180^\circ$

$5 : 7$

$5x + 7x = 180^\circ$

$12x = 180^\circ$

$x = 15$

$\angle POR = \angle SOQ$ (Vertically Opposite angles)

$5x = \angle SOQ$

$5 \times 15 = \angle SOQ$

$\therefore \angle SOQ = 75^\circ$

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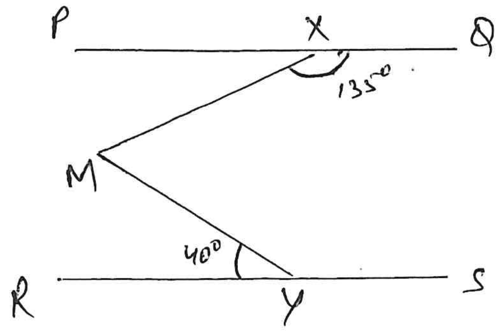
Q.3) In the given figure, if $PQ \parallel RS$, $\angle MXQ = 135°$ and $\angle MYR = 40°$, find $\angle XMY$.



$\angle PXM = 180° - 135° = 45°$

$\angle XMY = \angle PXM + \angle RYM$

$= 45° + 40°$

$= 85°.$

Q.4) If $(2x+17)°$ and $(x+4)°$ are complimentary, find x:

$(2x+17)° + (x+4)° = 90°$

$\Rightarrow 2x + 17 + x + 4 = 90°$

$\Rightarrow 3x = 90 - 17 - 4$

$\Rightarrow x = 69/3$

$\Rightarrow x = 23°$

Q.5) If $(5y+62)°$, $(22+y)°$ are supplementary, find y?

$5y + 62 + 22 + y = 180°$

$y = 16°.$

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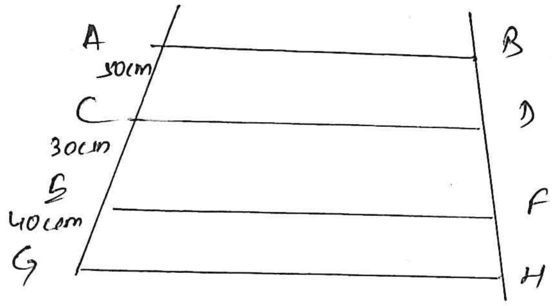
Q.6) In the given figure AB || CD || EF || GH and BH = 100cm. find the value of DF:

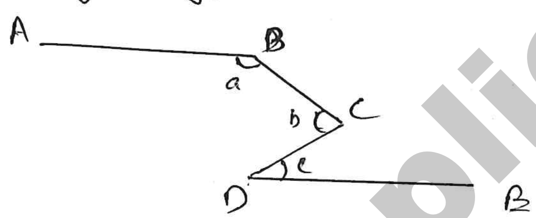


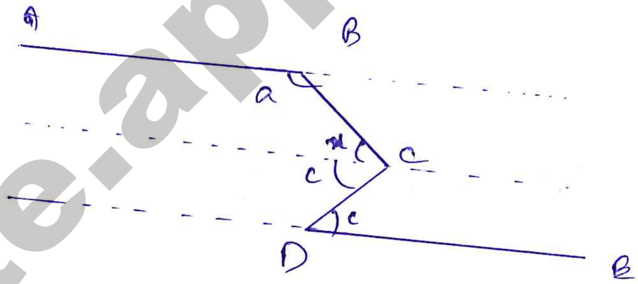
$$\frac{AG}{CE} = \frac{BH}{DF}$$

$$\frac{120}{30} = \frac{100}{DF}$$

DF = 25cm.

Q.7) In the given figure AB || DE. Find a+b−c.





$a + x = 180^0$

$b = x + c$

$x = b - c$

$a + b - c = 180^0$

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14.4: Triangles

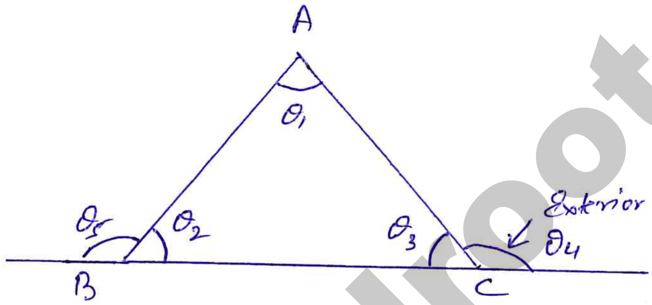
1) Triangle is a polygon having three sides.

2) Sum of all the angles of a triangle = 180

3) Sum of exterior angles = 360

4) Condition to form a triangle is "sum of any two sides is always is greater than the third side".

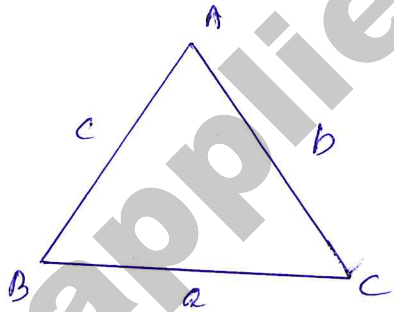
5) An exterior angle is equal to the sum of the opposite interior angles.



$\theta_1 + \theta_2 + \theta_3 = 180°$

$\theta_1 + \theta_2 = \theta_4$

$\theta_1 + \theta_3 = \theta_5$



$a + b > c$

$b + c > a$

$a + c > b$

Ex: 6, 4, 5

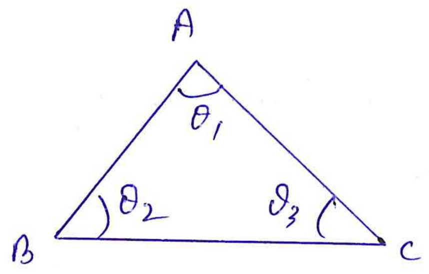
$6 + 4 = 10 > 5$

$9 > 6$

$11 > 4$

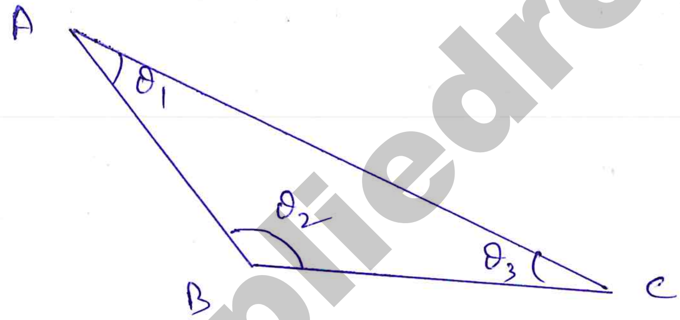
Types of Triangle

1) Acute Angle Triangle



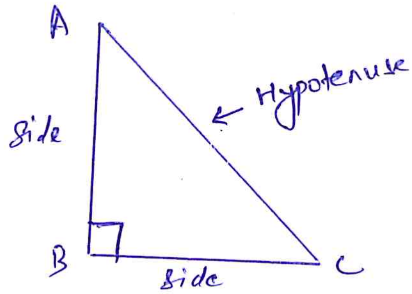
All the angles in the triangle are less than 90°.

2) Obtuse Angle Triangle



One of the angle should be greater than 90°.

3) Right Angle Triangle



$\theta = 90^\circ$
$\angle B = 90^\circ$ } largest angle 90°

$AC^2 = AB^2 + BC^2$

$\angle A + \angle B + \angle C = 180^\circ$

$\angle A + \angle C = 90^\circ$

$\angle A \; \& \; \angle C < 90^\circ$

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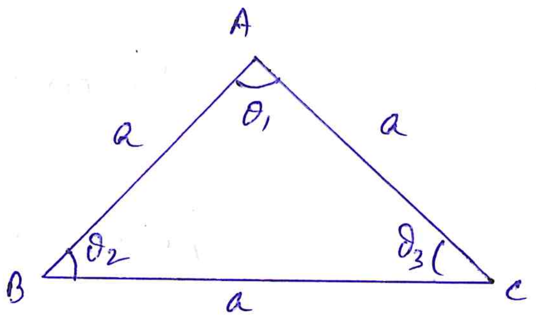
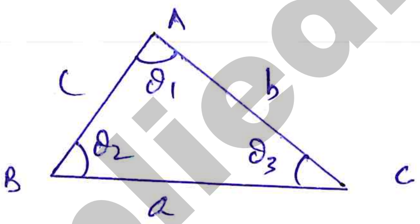
4) Equilateral Triangle



All sides are equal
All angles are equal

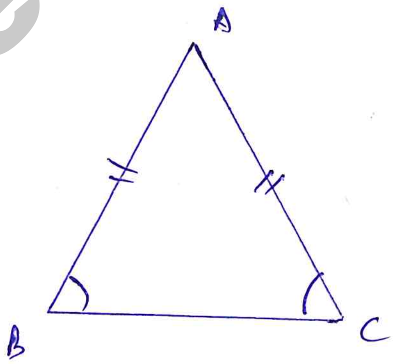
$$\theta_1 = \theta_2 = \theta_3 = \frac{180^\circ}{3} = 60^\circ$$

5) Scalene Triangle



All sides are not equal, ie, $a \neq b \neq c$
All angles are not equal, ie, $\theta_1 \neq \theta_2 \neq \theta_3$

6) Isosceles Triangle



$AB = AC$
$\angle B = \angle C$

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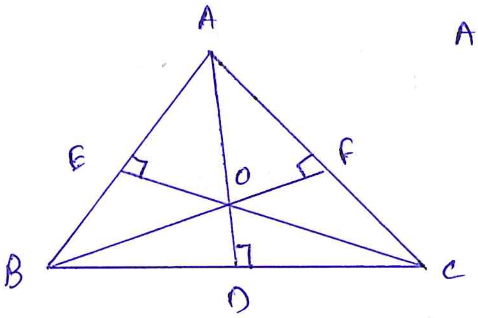
14.5: Important terms in triangle

Important terms

1) **Altitude / Height / Perpendicular**

A perpendicular drawn from any vertex to the opposite is called the altitude.



AD, BF, CE are altitudes.

Property: The point of intersection of the three attributes of the triangle is called orthocentre.

$\angle A + \angle BOC = 180^\circ$

$\angle B + \angle AOC = 180^\circ$

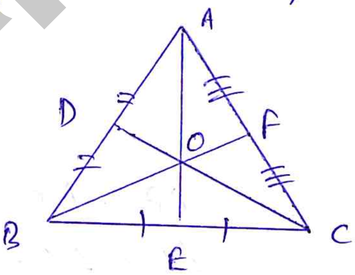
$\angle C + \angle AOB = 180^\circ$

2) **Median**

A line joining a vertex to the midpoint of opposite side is called median.

Median divides a triangle into two equal parts.

AE, BF & CD are Medians



centroid

$$\frac{AO}{OE} = \frac{CO}{OD} = \frac{BO}{OF} = \frac{2}{1}$$

Property: The point of intersection of the three medians of a triangle is called the centroid. (G)

Centroid divides the median in the ratio 2:1.

$$OE = \frac{1}{3} AE$$

$$OA = \frac{2}{3} AE$$

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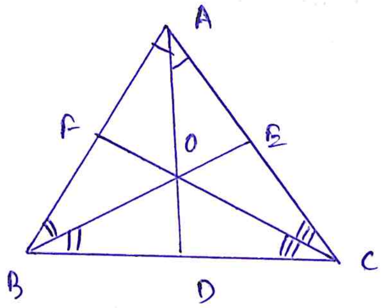
3) Angle bisectors

The lines bisecting the interior angles of a triangle are called angle bisector. The angle bisector divides the angle into two equal parts.



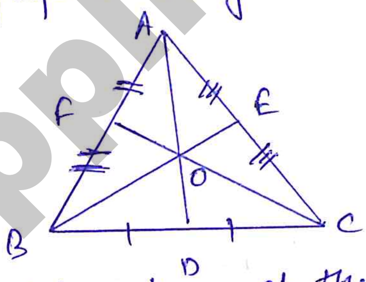
Property: The point of intersection of the angle bisectors of a triangle is called incentre.

The incentre is equidistant from all sides of the triangle [OD = OE = OF].

$\angle BOC = 90 + \frac{1}{2}\angle A$ / $\angle AOC = 90 + \frac{1}{2}\angle B$ / $\angle AOB = 90 + \frac{1}{2}\angle C$.

4) Perpendicular Bisectors

A line bisects a side perpendicularly is called a perpendicular bisector.



Property: The point of intersection of the perpendicular bisectors of the side of a triangle is called the circumcentre.

Circumcentre is equidistance from each vertex (OA = OB = OC)

$\angle BOC = 2\angle A$

$\angle AOB = 2\angle C$

$\angle AOC = 2\angle B$

A line form 90° and bisects.

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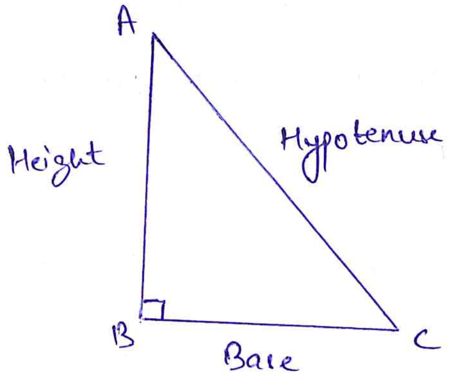
14.6: Important theorems in triangles

1) Pythagoras Theorem



Square of hypotenuse is equal to the sum of the squares of other two sides

i.e, $(AC)^2 = (AB)^2 + (BC)^2$

The numbers which satisfy the above relation, are called the pythagorean triplets.

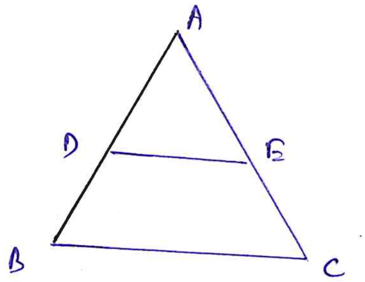
Ex, 3, 4, 5
5, 12, 13
7, 24, 25
8, 15, 17
9, 40, 41
11, 60, 61

Note:- All the multiples or sub-multiples of pythagorean triplets also satisfy the relation.

3, 4, 5
×2
6, 8, 10

3, 4, 5
÷2
1.5, 2, 2.5

2) Basic Proportionality Theorem



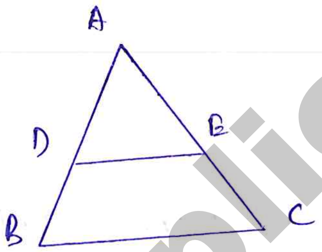
$DE \parallel BC$

If a line is drawn parallel to one side of a triangle divides the other two sides proportionally

i.e, $\frac{AD}{DB} = \frac{AE}{EC}$ (or) $\frac{AD}{AB} = \frac{AE}{AC}$

(or) $\frac{AD}{DE} = \frac{AB}{BC} = \frac{AB}{DE} = \frac{AC}{BC}$

3) Mid-Point Theorem



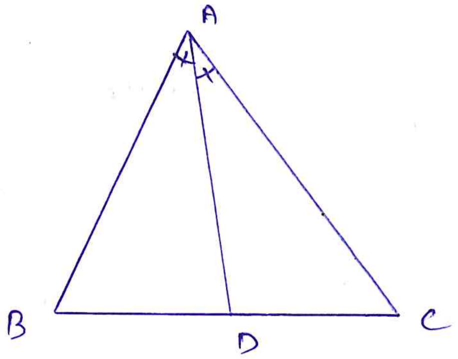
The line segment joining the midpoints of two sides of a triangle is parallel to the third side and equal to half of it.

D and E are mid points, then $DE \parallel BC$ and $DE = \frac{1}{2} BC$.

$AD = DB = \frac{1}{2} AB$

$AE = EC = \frac{1}{2} AC$

4) Interior Angle Bisector Theorem



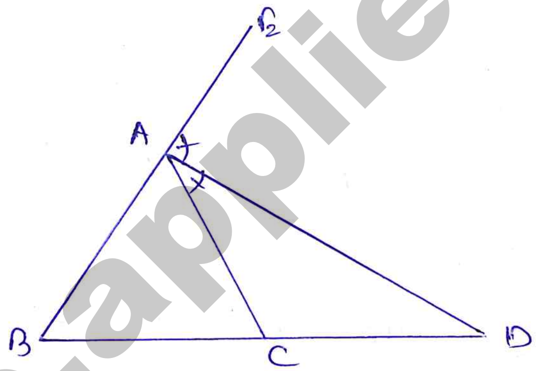
AD is angle bisector
$\angle BAD = \angle DAC$

In a triangle, the angle bisector of an angle divides the opposite side to the angle in the ratio of the remaining two sides, i.e.,

$$\frac{BD}{CD} = \frac{AB}{AC}$$

$$\boxed{BD \times AC - DC \times AB = AD^2}$$

5) Exterior Angle Bisector Theorem



$\angle CAD = \angle EAD$

In a triangle, the external bisector of an angle divides the opposite sides externally in the ratio of the remaining two sides

i.e.,
$$\frac{BD}{CD} = \frac{AB}{AC}$$

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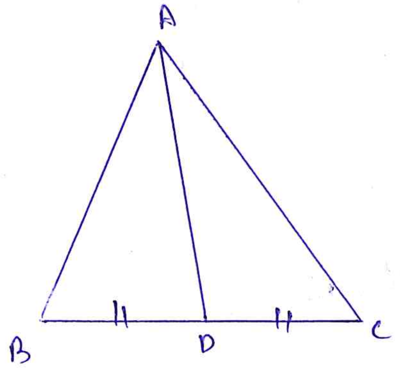
6) Apollonius Theorem



AD divides BC
BD = DC
AD is the median

The sum of the squares of any two sides of a triangle is equal to twice the sum of the square of the median and square of half the third side

ie, $AB^2 + AC^2 = 2AD^2 + (\frac{1}{2}BC)^2$ $\quad [BD = CD = \frac{1}{2}BC]$

$= 2(AD^2 + BD^2)$

$= 2(AD^2 + CD^2)$

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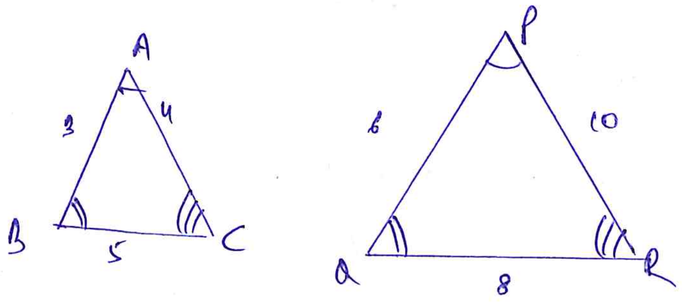
14.7: Similarity of triangles

Two triangles are said to be similar if the corresponding angles are equal and their corresponding sides are in proportion.

The symbol of similarity is $\sim$

If $\triangle ABC \sim \triangle PQR$



Results

① $\angle A = \angle P$

$\angle B = \angle Q$

$\angle C = \angle R$

② $\dfrac{AB}{PQ} = \dfrac{AC}{PR} = \dfrac{BC}{QR}$

$\dfrac{3}{6} = \dfrac{1}{2} \qquad \dfrac{AC}{PR} = \dfrac{1}{2} = \dfrac{5}{10}$

Tests of Similarity

1) AAA = Angle Angle Angle

2) SAS = Side Angle Side

3) SSS = Side Side Side

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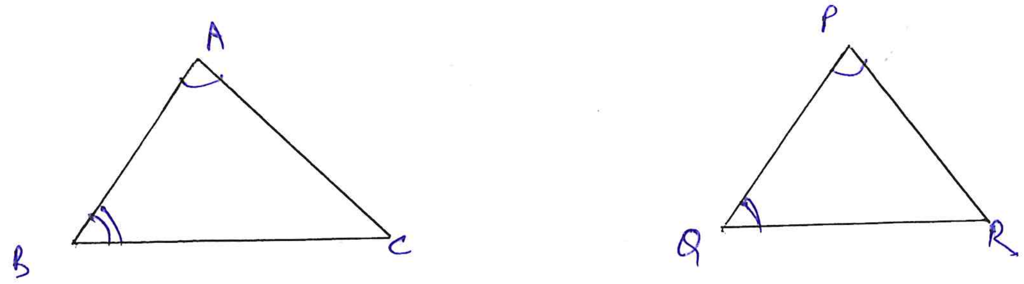
Angle Angle (AA)

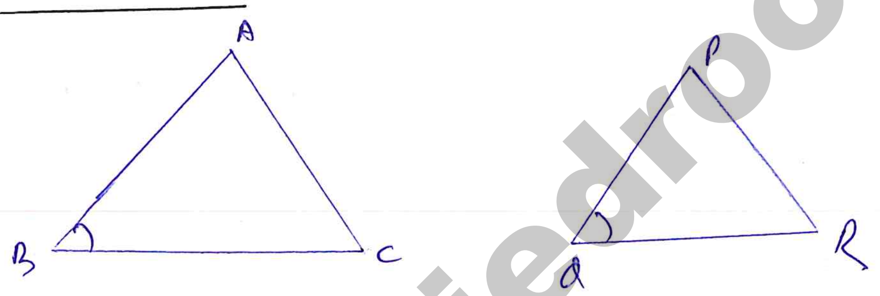


$\triangle ABC \sim \triangle PQR$

when $\angle A = \angle P$

$\angle B = \angle Q$

Side Angle Side (SAS)



$$\frac{AB}{PQ} = \frac{BC}{QR}$$

included angle is equal

$\angle B = \angle Q$

$\triangle ABC \sim \triangle PQR$

If two sides of triangles ABC and PQR are proportional and included angle is equal, then the triangles are similar.

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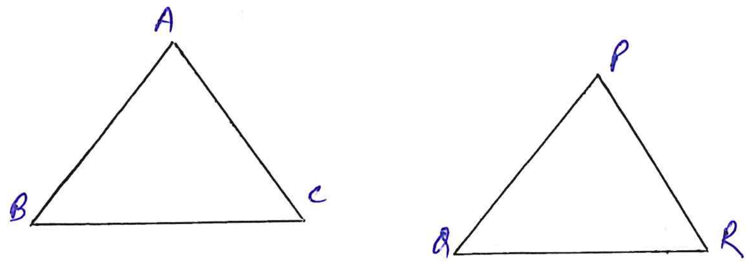
Side - Side - Side



If all three sides of triangles ABC and PQR are proportional, then the triangles are said to be similar.

$$\frac{AB}{PQ} = \frac{BC}{QR} = \frac{AC}{PR}$$ (in same ratio)

Properties

1) If the two triangles are similar, then for the proportional / corresponding sides we have following results

Ratio of sides = Ratio of heights
= Ratio of medians
= Ratio of angle bisectors
= Ratio of inradii
= Ratio of circumradii

2) Ratio of Areas = Ratio of squares of corresponding side

If $\Delta ABC \sim \Delta PQR$ then

$$\frac{Area(\Delta ABC)}{Area(\Delta PQR)} = \frac{(AB)^2}{(PQ)^2} = \frac{(BC)^2}{(QR)^2} = \frac{(AC)^2}{(PR)^2}$$

3) Ratio of Perimeter = Ratio of corresponding sides

If $\Delta ABC \sim \Delta PQR$ then

$$\frac{P(\Delta ABC)}{P(\Delta PQR)} = \frac{AB}{PQ} = \frac{BC}{QR} = \frac{AC}{PR}$$

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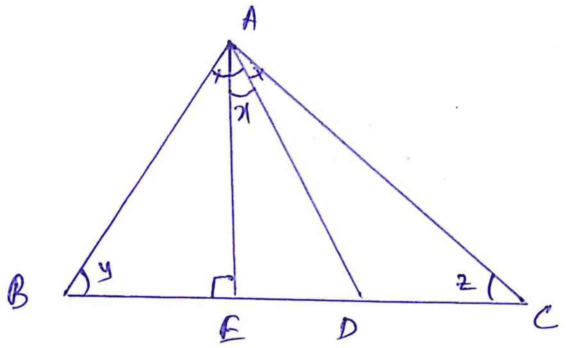
4.8: Useful results on triangles

1) In a $\triangle ABC$, if AD is the angle bisector of $\angle BAC$ and $AE \perp BC$ then

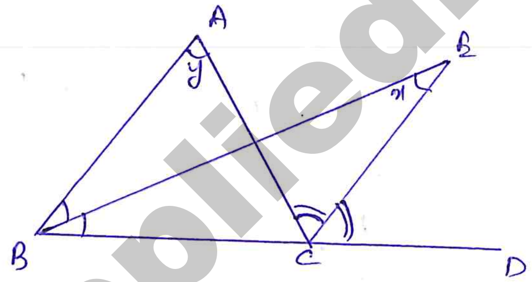
$\angle DAE = \frac{1}{2}(\angle ABC - \angle ACB)$



$\angle BAD = \angle DAC$

$x = \frac{1}{2}(y - z)$

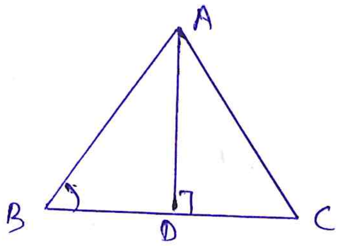
2) In $\triangle ABC$, if side BC is produced to D and bisectors of $\angle ABC$ and $\angle ACD$ meet at E, then $\angle BEC = \frac{1}{2}\angle BAC$.



$x = \frac{1}{2} y$

3) In an acute angle $\triangle ABC$, AD is a perpendicular dropped on the opposite side of $\angle A$, then

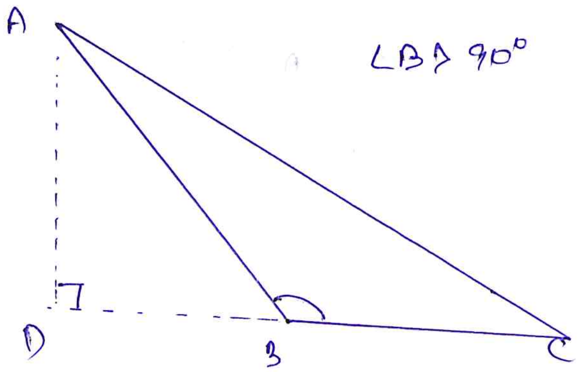
$AC^2 = AB^2 + BC^2 - 2\,BD \cdot BC$.



$\angle B < 90^\circ$

4) In an obtuse angle $\triangle ABC$, AD is perpendicular dropped on BC. BC is produced to D to meet AD, then

$AC^2 = AB^2 + BC^2 + 2BD \cdot BC$



$\angle B > 90^\circ$

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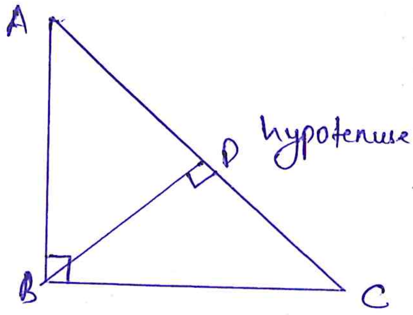
5) In a right angle ΔABC, $\angle B = 90°$ and AC is hypotenuse. The perpendicu BD is dropped on hypotenues AC from right angle vertex B, then



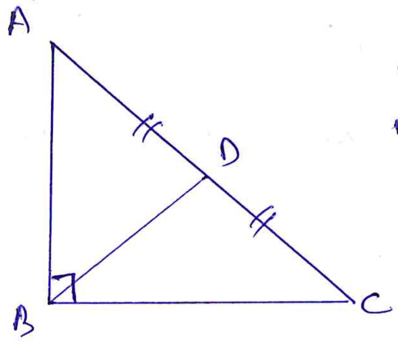
(i) $BD = \frac{AB \times BC}{AC}$

(ii) $AD = \frac{AB^2}{AC}$

(iii) $CD = \frac{BC^2}{AC}$

(iv) $\frac{1}{BD^2} = \frac{1}{AB^2} + \frac{1}{BC^2}$

6) In a right angled triangle, the median to the hypotenuse is half the length of the hypotenuse.



BD is median
$AD = DC$
Median = ½ hypotenuse

ie, $BD = \frac{1}{2} AC$

$AD = DC = \frac{1}{2} AC$

$AD = DC = BD$.

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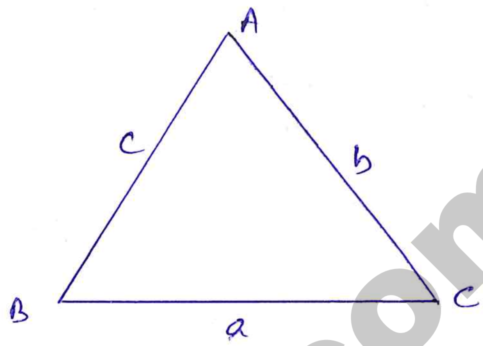
7) Sine Rule

In ΔABC, if a, b, c be the three sides opposite to the angles A, B, C, respectively, then



$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$\text{Area} = \frac{1}{2} ab \sin C = \frac{1}{2} bc \sin A = \frac{1}{2} ac \sin B$$

8) Cosine Rule

$$\cos A = \frac{b^2+c^2-a^2}{2bc}$$

$$\cos B = \frac{c^2+a^2-b^2}{2ac}$$

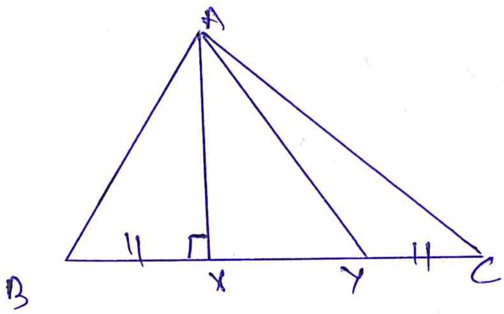
$$\cos C = \frac{a^2+b^2-c^2}{2ab}$$

14.9: Solved Problems triangles

Q.1) In $\triangle ABC$, $AX \perp BC$ and AY is the median. Find the value of $AB^2 + BC^2$?

(a) $2(AY^2 + BC^2)$ (b) $2(AX^2 + XC^2)$ (c) $(AY^2 + BX^2 + XC^2)$

(d) $2\left(AY^2 + \frac{BC^2}{4}\right)$



Given AY is median $\rightarrow BY = YC = \frac{1}{2} BC$

$AX \perp BC$

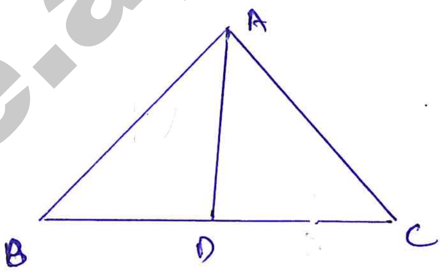
By Appolinius theorem

$AB^2 + BC^2 = 2(AY^2 + BY^2)$

$= 2\left[AY^2 + \left(\frac{1}{2} BC\right)^2\right]$

$= 2\left[AY^2 + \frac{BC^2}{4}\right]$

Q.2) In $\triangle ABC$, AD is a median. Find the ratio of $(AB^2 + AC^2) : (AD^2 + BD^2)$?



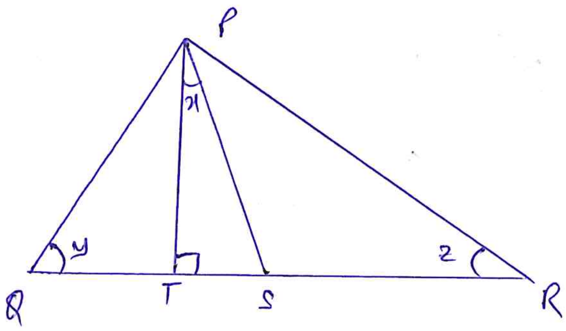
$BD = DC = \frac{1}{2} BC$

By Appolinius theorem,

$AB^2 + AC^2 = 2(AD^2 + BD^2)$

$\frac{AB^2 + AC^2}{AD^2 + BD^2} = \frac{2}{1} = 2:1$.

Q.3) In the given ΔPQR, PS is the bisector of $\angle QPR$ and $PT \perp QR$. If $\angle Q - \angle R = 30^\circ$ then find the value of $\angle TPS$.



$y - z = 30^\circ$

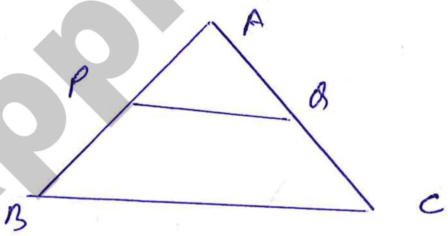
$x = \frac{1}{2}(y - z)$

$x = \frac{1}{2} \times 30^\circ$

$= 15^\circ$

Q.4) Inside a triangle ABC, a straight line parallel to BC intersects AB and AC at the points P and Q respectively. If $AB = 3PB$, then $PQ : BC$ is

(a) 1 : 3 (b) 3 : 4 (c) 1 : 2 (d) 2 : 3



$AB = 3PB$

$\frac{AB}{PB} = \frac{3}{1}$

Basic proportionality theorem.

$\frac{AP}{PQ} = \frac{AB}{BC}$

$\Rightarrow \frac{AP}{AB} = \frac{PQ}{BC} \Rightarrow \frac{2}{3} = \frac{PQ}{BC}$

2 : 3.

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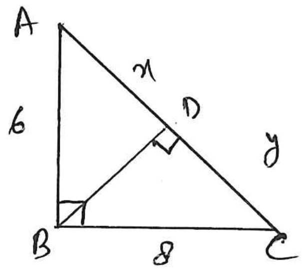
Q.5) In the adjoining right angled triangle, find the value of x and y



$AC^2 = AB^2 + BC^2$

$AC^2 = 6^2 + 8^2$

$AC = 10$

$AD = \frac{AB^2}{AC}$

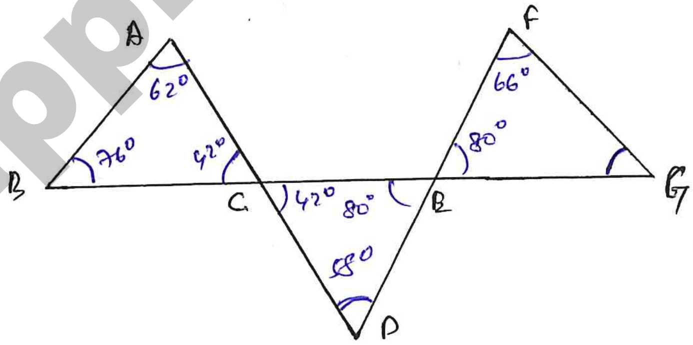
$= \frac{36}{10}$

$= 3.6$

$CD = 10 - 3.6$

$= 6.4.$

Q.6) In the adjoining figure $\angle CAB = 62^\circ$, $\angle CBA = 76^\circ$, $\angle ADB = 58^\circ$ & $\angle DFG = 66^\circ$. find $\angle FGE$?



In $\triangle ABC$

$62^\circ + 76^\circ + \angle C = 180^\circ$

$\angle C = 180^\circ - 138^\circ$

$= 42^\circ$

In $\triangle CDE$

$58^\circ + 42^\circ + \angle E = 180^\circ$

$\angle E = 180^\circ - 100^\circ$

$= 80^\circ$

In $\triangle FGE$

$66^\circ + 80^\circ + \angle G = 180^\circ$

$\angle G = 180^\circ - 146^\circ$

$= 34^\circ$

$\therefore \angle FGE = 34^\circ.$

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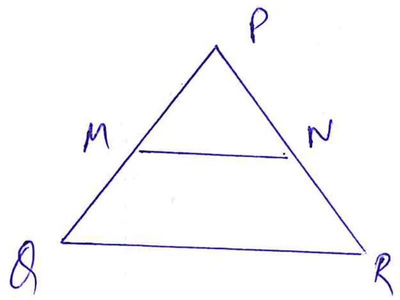
Q.7) In a ΔPQR, points M and N are on the sides PQ and PR respectively such that $PM = 0.6PQ$ and $NR = 0.4PR$, what percentage of the area of the ΔPQR does that ΔPMN form?

(a) 60% (b) 50% (c) 36% (d) 55%



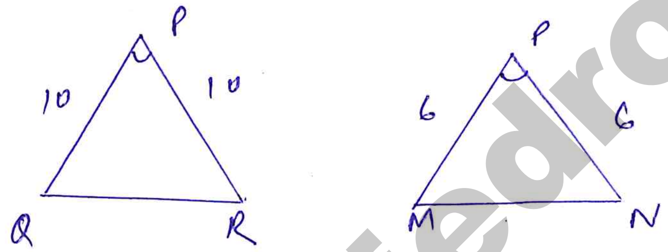
MN || QR

$PM = 0.6PQ$

$\frac{PM}{PQ} = \frac{6}{10}$

$NR = 0.4PR$

$\frac{NR}{PR} = \frac{4}{10}$



$\angle P = \angle P$

$\frac{PQ}{PM} = \frac{PR}{PN}$ } SAS $\therefore \Delta PQR \sim \Delta PMN$

$$\text{Ratio of } \frac{\text{Area } (\Delta PQR)}{\text{Area } (\Delta PMN)} : \frac{(PQ)^2}{(PM)^2}$$

$$\therefore \frac{A(\Delta PQR)}{A(\Delta PMR)} = \frac{(PQ)^2}{(PM)^2}$$

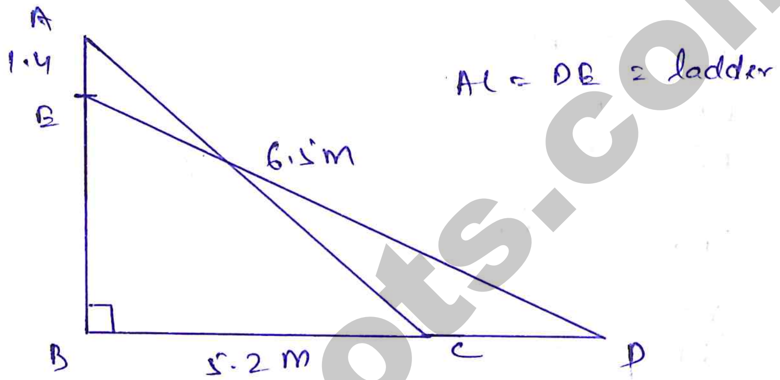
$$= \left(\frac{10}{6}\right)^2$$

$$= 25 : 9.$$

$\therefore$ Percentage of area of $\Delta PMN = \frac{9}{25} \times 100 = 36\%$

Q.8) A ladder 6.5 m long is standing against a wall and the difference between the base of the ladder and the wall is 5.2 m. If the top of the ladder now slips by 1.4 m, then by how much time will the foot of the ladder slip?

(A) 1.2 M (b) 0.2 m (c) 0.75 m (d) None.



In $\triangle ABC$

$AB^2 = AC^2 - BC^2$

$= (6.5)^2 - (5.2)^2$

$= (6.5 - 5.2)(6.5 + 5.2)$

$= 1.3 \times 11.7$

$\therefore AB = 3.9$

In $\triangle BED$

$BE^2 = DE^2 - BD^2$

$(2.5)^2 = (6.5)^2 - (BD)^2$

$(BD)^2 = (6.5)^2 - (2.5)^2$

$= 36$

$BD = 6$

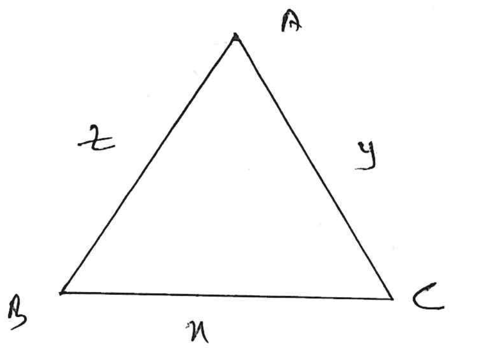
$BC + CD = 6$

$5.2 + x = 6$

$x = 0.8$ M (ans)

Q.7) If $x^2+y^2+z^2 = xy+yz+zx$ then the triangle is

(a) Isosceles
(b) Right angled
(c) Equilateral
(d) Scalene



$$x^2+y^2+z^2 = xy+yz+zx$$

multiplying 2 on both sides

$$2x^2+2y^2+2z^2 = 2xy+2yz+2zx$$

$$x^2+x^2+y^2+y^2+z^2+z^2-2xy-2yz-2zx$$

$$(x-y)^2+(y-z)^2+(z-x)^2 = 0$$

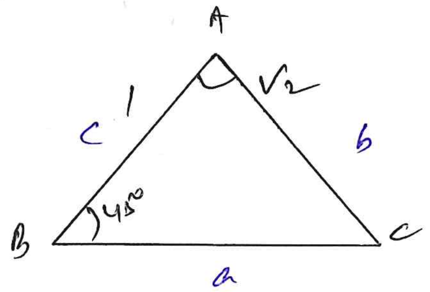
$x-y=0$
$y-z=0$
$z-x=0$

$x=y$
$y=z$
$z=x$

$\therefore x=y=z$

$\therefore$ The triangle is equilateral.

Q.10) $\angle ABC$ is equal to $45°$ as shown in figure. If $AC : AB = \sqrt{2} : 1$ then $\angle BAC$ is equal to



$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$\frac{a}{\sin A} = \frac{\sqrt{2}}{\sin 45°}$$

$$\frac{a}{\sin A} = \frac{\sqrt{2}}{\frac{1}{\sqrt{2}}} = 2$$

$$\frac{b}{\sin B} = \frac{c}{\sin C}$$

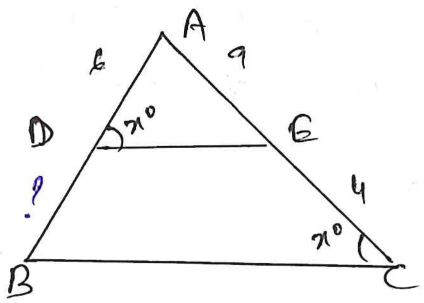
$$\frac{\sqrt{2}}{\sin 45°} = \frac{1}{\sin C}$$

$$\sin C = \frac{1}{2}$$

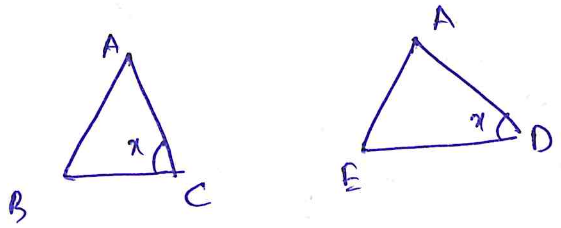
$$C = 30°$$

$$\angle A = 180 - 75$$
$$= 105°.$$

Q.11) In the given figure, find the value of BD?



ΔABC & ΔADE



$\angle A = \angle A$

$\angle C = \angle D$

By AA $\Delta ABC \sim \Delta AED$

$$\frac{AD}{AC} = \frac{AE}{AB}$$

$$\frac{6}{13} = \frac{9}{6+x}$$

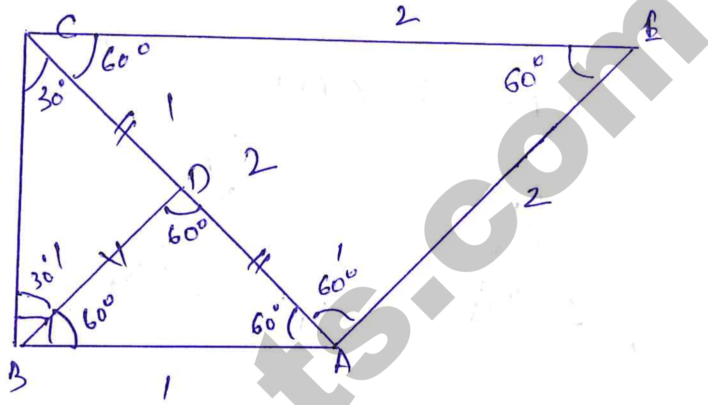
$12 + 2x = 39$

$x = 13.5$

$\therefore BD = 13.5$.

Q.12) In a right angled triangle ABC, ∠B is right angle, side AB is half of the hypotenuse. AE is parallel to median BD and CE is parallel to AB. What is the ratio of length of BC to that of EC?

(a) $\sqrt{2}:1$ (b) $\sqrt{3}:2$ (c) $\sqrt{5}:\sqrt{3}$ (d) Can't be determined



BD is median

$DC = AD = \frac{1}{2} AC$

$BD = \frac{1}{2} \times AC = 1$

$\triangle ADB$ form an equilateral triangle

$AB \parallel EC$

$BD \parallel AE$

$\angle D = \angle B = \angle A = 60^\circ$

$\triangle BDC$ is isoceles triangle

$\therefore \angle BCD = 30^\circ$

$\because AB \parallel EC$

$\angle BAC = \angle ACE$ (alternative angles)

$BD \parallel AE$

$\angle BDA = \angle DAE$ (alternative)

$AE = CE = AE$

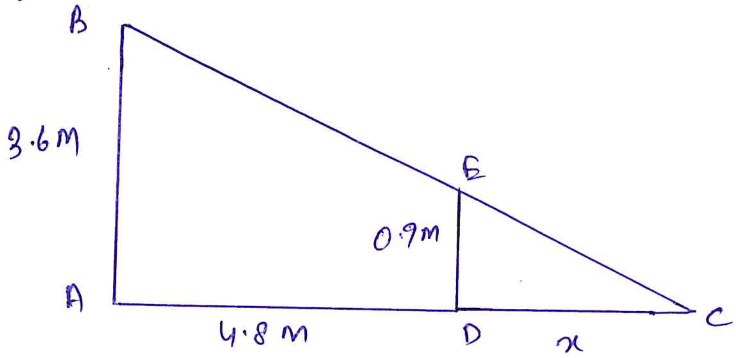
$BC^2 = AC^2 - AB^2$

$BC = \sqrt{3}$

$\therefore BC : EC = \sqrt{3} : 2$ (ans)

14.10: Solved Problems triangles

Q.13) A girl of height 90 cm is walking away from the base of a lamp post at the speed of 1.2 m/s. If the lamp is 3.6 m above the ground, find the length of her shadow after 4 sec?

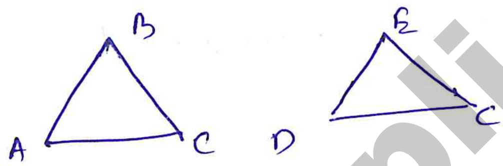


1 sec = 1.2 M

4 sec = 1.2 × 4 = 4.8 M

Let DE be the girl standing

DC be the shadow of the girl.



$\angle C = \angle C$

$\angle A = \angle D = 90^\circ$

$\triangle ABC \sim \triangle DEC$

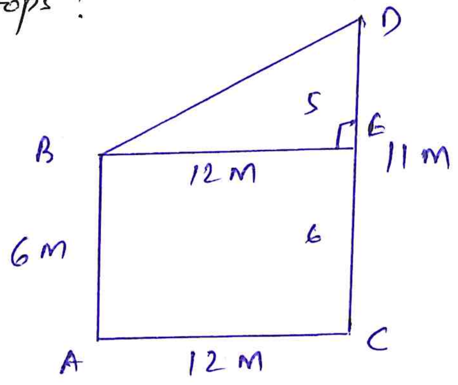
$$\frac{AB}{DE} = \frac{AC}{CD}$$

$$\frac{3.6}{0.9} = \frac{4.8 + x}{x}$$

$4x = 4.8 + x$

$x = 1.6$ M.

Q.14) Two poles of heights 6m and 11m stand on a plane ground. If the distance between the feet of the poles is 12 m, find the distance b/w their tops?



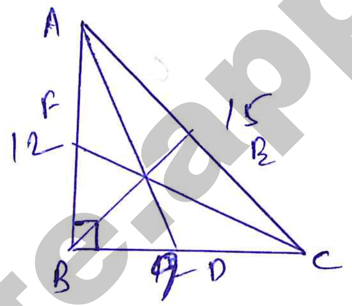
$AB = EC$

$BD^2 = BE^2 + ED^2$

$= 12^2 + 5^2$

$\therefore BD = 13$ M

Q.15) The sides of a right angle triangle are 9, 12 & 15 cm long. Find the sum of the squares of the medians.



$AD^2 + BE^2 + CF^2 = ?$

Sum of squares of the medians in a right angled triangle $= \frac{3}{2} \times (\text{Hypotenuse})^2$

$= \frac{3}{2} \times 15^2$

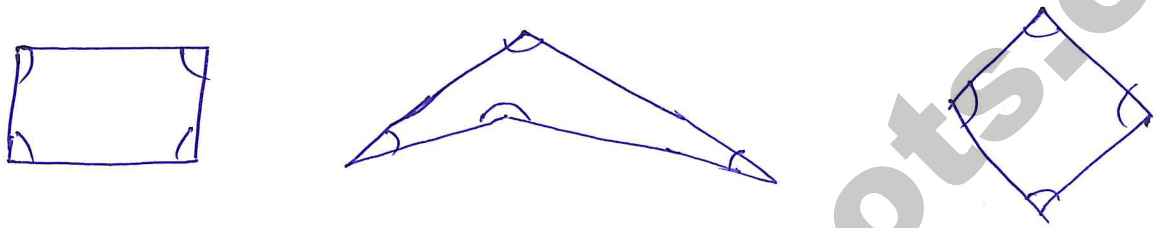
$= 337.5$ cm.

14.11: Quadilaterals

A quadilateral is a plane figure bounded by four straight lines.

Properties

1) Every quadilateral has four sides.
2) Every quadilateral consists of four corners or vertex or angles.
3) The interior angle of quadilateral adds up to 360° in every quadilateral

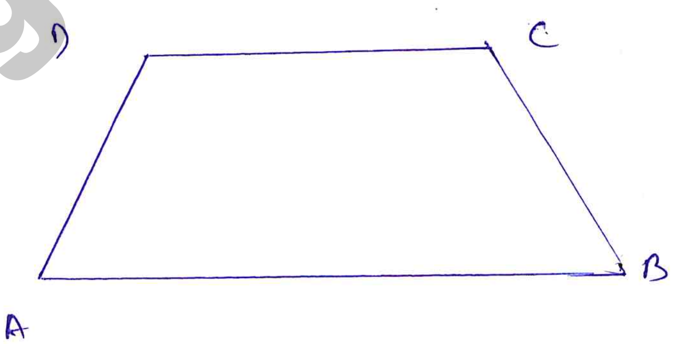


Types

1) Trapezium
2) Parallelogram
   - Rectangle
   - Square
   - Rhombus
3) Kite

Trapezium

1) Trapezium is a quadilateral whose only one pair of sides is parallel



$AB \parallel CD$

AD and BC are not parallel sides

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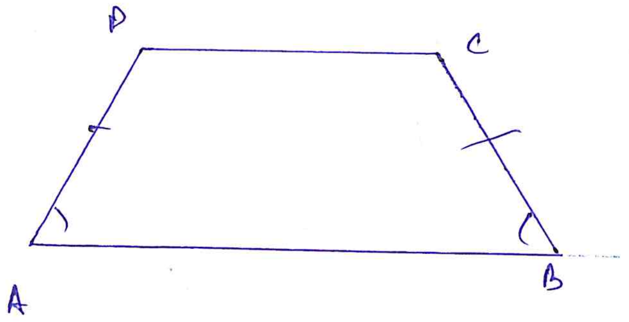
2) The non parallel sides of trapezium are called lateral sides. If these sides are equal in length it is called an isoscelese trapezium.



(a) AD = BC

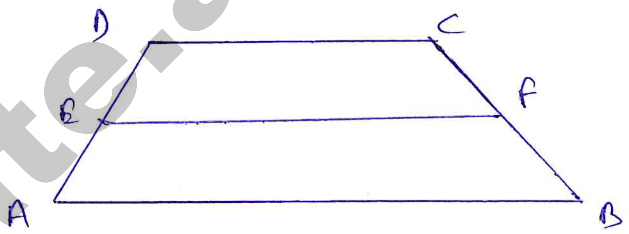
(b) AB || DC

(c) Base angles are equal ($\angle A = \angle B$)

(d) Diagonals are equal ($AC = BD$)

(e) It is also called cyclic quadilateral.

3) The line joining the mid-point of the non parallel sides is half the sum of the parallel sides and is called median.



AB || CD.

E and F are midpoints

EF is called median.

$EF = \frac{1}{2}(AB + CD)$

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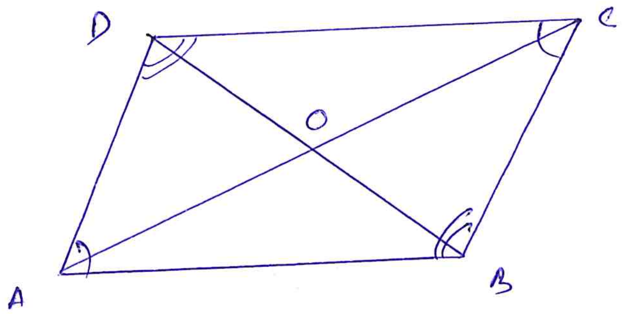
Parallelogram

It is a quadilateral whose opposite sides are parallel.



$AB \parallel DC$
$AD \parallel BC$

$AB = CD$ &
$AD = BC$

Properties

1) The opposite sides are parallel and equal.
2) Opposite angles are equal. $\angle A = \angle C$, $\angle B = \angle D$
3) Sum of any two adjacent angle is $180°$. $\angle B + \angle C = 180°$, $\angle C + \angle D = 180°$, $\angle D + \angle A = 180°$, $\angle A + \angle B = 180°$.
4) Diagonals bisect each other. $OA = OC$, $OD = OB$
5) Diagonals need not be equal in length. $AC \neq BD$
6) Each diagonal divides a parallelogram into two equal triangles. $\triangle ABD = \triangle BCD$, $\triangle ABC = \triangle ADB$
7) The sum of the squares of the diagonals is equal to the sum of the squares of the four sides.

$$(AC)^2 + (BD)^2 = (AB)^2 + (BC)^2 + (CD)^2 + (AD)^2$$
$$= 2\left[(AB)^2 + (BC)^2\right]$$
OR
$$= 2\left[(CD)^2 + (AD)^2\right]$$

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1) Parallelogram having all angles 90° then it is called a rectangle.

2) Parallelogram having all sides equal are called a rhombus.

3) Parallelogram having all sides equal and all angles 90°, then it is called a square.

Rectangle

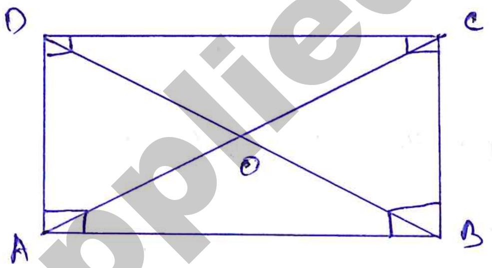
A parallelogram is said to be rectangle if all interior angles are equal to 90.

Properties

1) Opposite sides are parallel and equal. $AB \parallel CD$ & $AD \parallel BC$

2) Diagonals are equal and bisect each other. $AC = BD$ & $AO = OC$, $OD = OB$

3) All rectangles are parallelograms.



$AB = CD$
$AD = BC$

Rhombus

A parallelogram is said to be rhombus if all sides are equal.

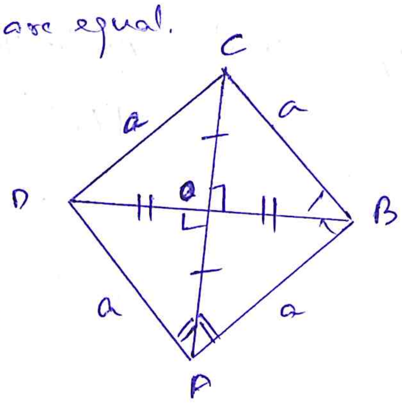
Properties

1) Opposite sides are parallel & equal.

2) Opposite angles are equal. $\angle A = \angle C$ & $\angle B = \angle D$

3) Diagonals bisect each other at right angle, but they are not necessarily equal.

4) Diagonals bisect the vertex angles.



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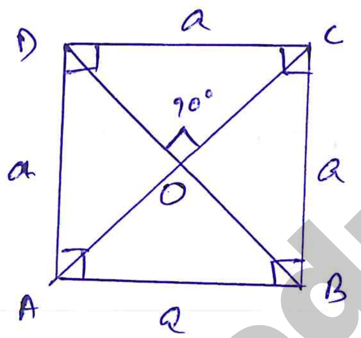
Square

A parallelogram is said to be square if all sides are equal and all angles are equal.

Properties

1) All sides are equal & parallel.
2) All angles are right angles.
3) Diagonals are equal and bisect each other at right angle.

AC = BD



Kite

A quadrilateral is said to be kite if two pairs of adjacent sides are equal.

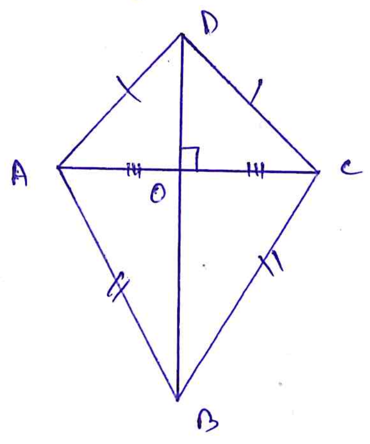
Properties

1) Adjacent sides are equal.
2) Diagonals intersect at right angles.
3) Shorter diagonal is bisected by the longer diagonal.
4) Shorter diagonal divides the kite into two isosceles triangle.

AD = DC
AB = BC

AC is shorter diagonal
OA = OC

△ADC and △ABC are isosceles triangle.



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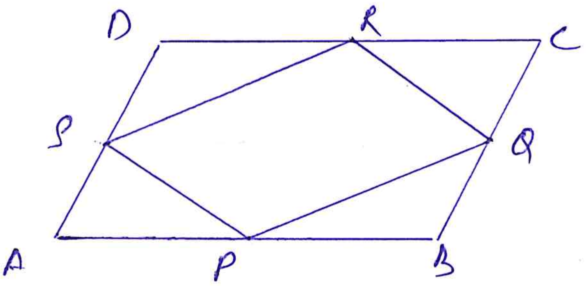
14.12 : Important concepts on quadilateral

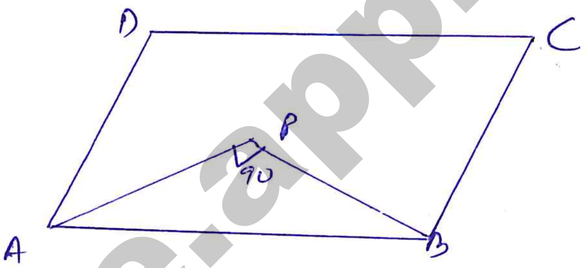
1) Joining the midpoints of quadilateral :

The figure formed by joining the Mid-points of the adjacent sides of a	Qudilateral	forms into	Parallelogram
	Isosceles Triangle		Rhombus
	Parallelogram		Parallelogram
	Rectangle		Rhombus
	Rhombus		Rectangle
	Square		Square



P, Q, R, S are the midpoints

2) The angle between the internal bisectors of two consecutive angles of a parallelogram is 90°.

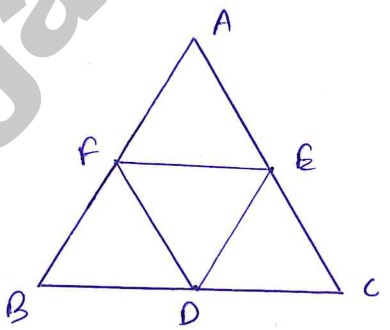


ABCD is a parallelogram

AP & BP are internal bisectors

$\angle APB = 90°$.

3) D, E and F are the midpoints of the sides BC, CA & AB of a triangle ABC, then



D, E, F of $\triangle ABC$

(i) BDEF is parallelogram

(ii) $ar(\triangle DEF) = \frac{1}{4}\, ar(\triangle ABC)$

(iii) $ar(\| gm\ BDEF) = \frac{1}{2}\, ar(\triangle ABC)$

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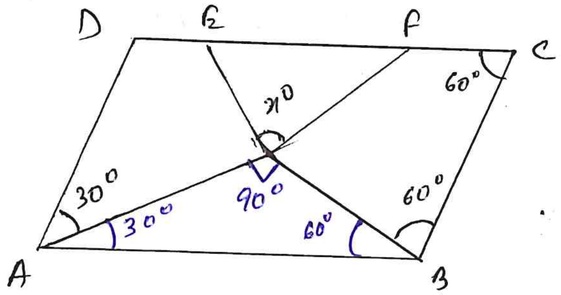
14.13: Problems on Quadrilateral

Q.1) In the adjoining figure of parallelogram find the measure of $\angle x$.



ABCD is a ||gm

$\angle C = 60^\circ$

$\angle B + \angle C = 180^\circ$

$\angle B = 180^\circ - 60^\circ$

$= 120^\circ$

$\angle A + \angle B = 180^\circ$

$\angle A = 180^\circ - 120^\circ$

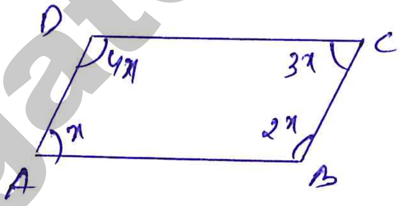
$= 60^\circ$

$\angle AOB = \angle EOF$

$\angle x = 90^\circ$.

Q.2) The measures of the angles of a quadrilateral ABCD are in the ratio 1:2:3:4 respectively. Then which of the following is true:

(a) AC = BD (b) ABCD is cyclic quadrilateral (c) AB || DC (d) AD || BC



1:2:3:4

$x + 2x + 3x + 4x = 360^\circ$

$10x = 360^\circ$

$x = 36^\circ$

$\angle A = 36^\circ$

$\angle B = 72^\circ$

$\angle C = 108^\circ$

$\angle D = 144^\circ$

(a) AC = BD ✗

(b) ABCD is cyclic quadrilateral ✗

(c) AB || DC $\because \angle A + \angle D = 180^\circ$, $\angle B + \angle C = 180^\circ$ ✓

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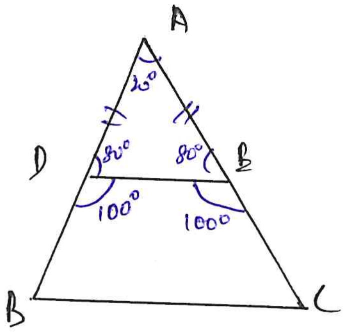
Q.3) In the given figure AD = AE, ∠BDE = 100°, then what is the value of ∠DBC + ∠BCE ?

(a) 200° (b) 160° (c) 80° (d) Can't be determined



△ADE is isosceles triangle

BCDE is a quadilateral

$\angle B + \angle C + \angle E + \angle D = 360°$

$\angle B + \angle C = 360° - 200°$

$= 160°$.

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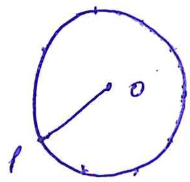
14014 Circles

Circles

A circle is a set of points on a plane lie at a fixed distance from a centre point.



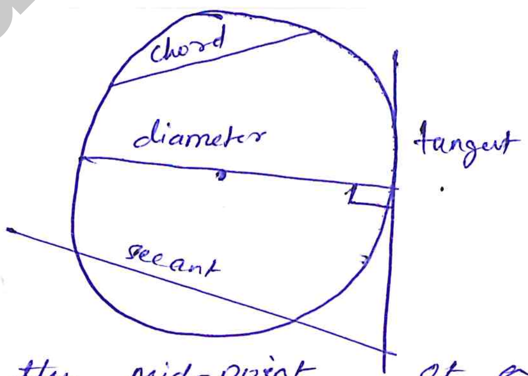
$d = 2r$
↓
diameter

fixed distance = radius. (r)

Chord: A line segment whose end points lie on the circle.

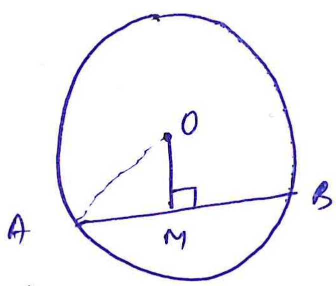
Secant: A line segment which intersects the circle in two distinct points.

Tangent: A line that touches the circumference of the circle, i.e, touches a circle at only one point.



Properties

1) The line joining the centre of a circle to the mid-point of a chord is perpendicular to the chord.



O is the centre of the circle

$AB = MB = \frac{1}{2} AB$

OM is a perpendicular bisector

AB is chord

OA is radius

OMA right angled triangle

$OA^2 = OM^2 + AM^2$

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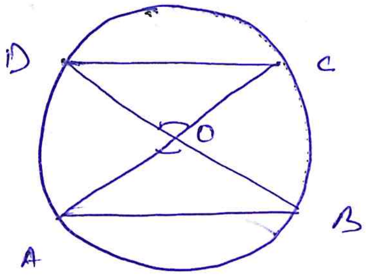
2) Equal chords of a circle subtend equal angles at the centre.



AB & CD are the equal chords.

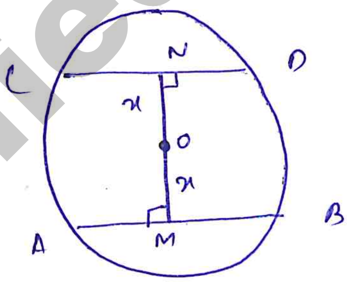
$AB = CD$

$\angle DOC = \angle AOB$

Converse is true.

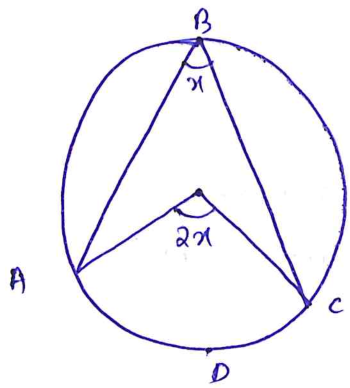
If $AB = CD$
then $\angle DOC = \angle AOB$.

3) Equal chords of a circle are equidistant from the centre.



If $AB = CD$
then $ON = OM = x$.

4) Inscribed angle theorem: The measure of an inscribed angle is half the measure of its intercepted arc.



$\angle ABC$ = inscribed angle

$\overset{\frown}{ADC}$ = intercepted arc

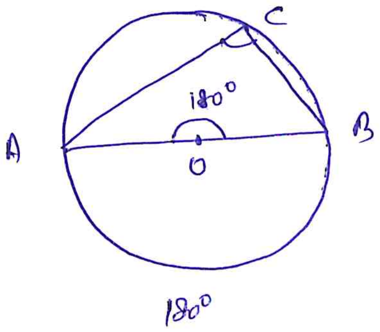
$\angle ABC = \frac{1}{2} \angle AOC = \frac{1}{2}$ arc ADC

Intercepted arc = centre angle

5) The angle in a semicircle is 90°.

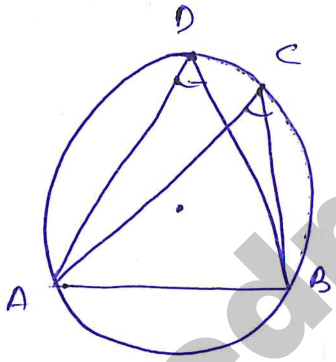
$\angle ACB = \frac{1}{2} \times 180$

$= 90°$



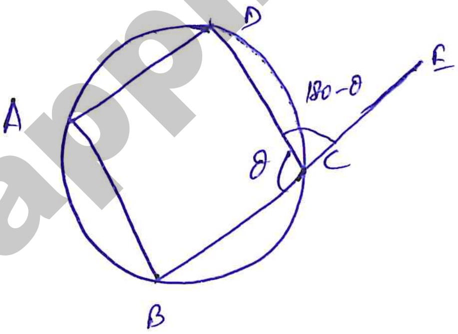
6) Angles in the same segment are equal.

$\angle ADB = \angle ACB$



7) Cyclic Quadilateral.

ABCD is called cyclic quadilateral



a) $\angle ABC + \angle ADC = 180°$

$\angle DAB + \angle BCD = 180°$

b) $\angle BAD + \angle BCD = 180$

$\angle BAD + \theta = 180$

$\angle BAD = 180 - \theta$

$\therefore \angle BAD = \angle DCE$

$\angle DCE$ (Exterior angle)

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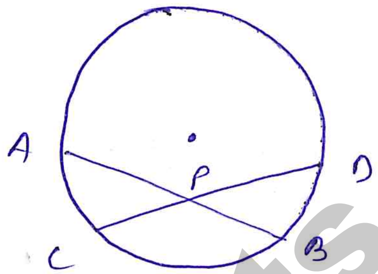
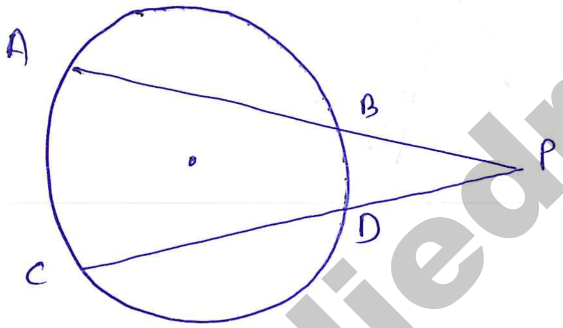
8 Two chords AB and CD intersect at any point P.

i) inside the circle

ii) Outside the circle

then $AP \times BP = PD \times PC$





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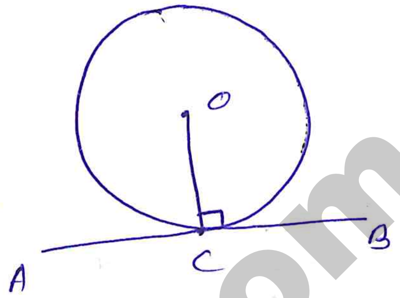
9) The tangent at any point of circle is perpendicular to the radius through that point.



O is the centre of circle

AB is the tangent

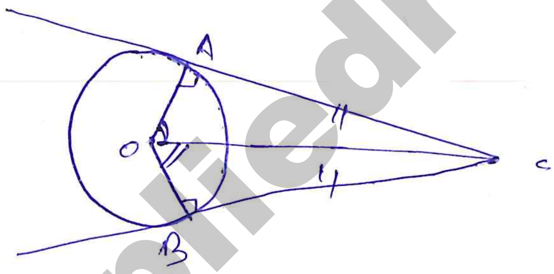
C is the point of contact of the tangent AB.

OC is radius.

$\therefore$ OC is perpendicular to AB

$OC \perp AB$

10) The lengths of two tangents drawn from an external point to a circle are equal.



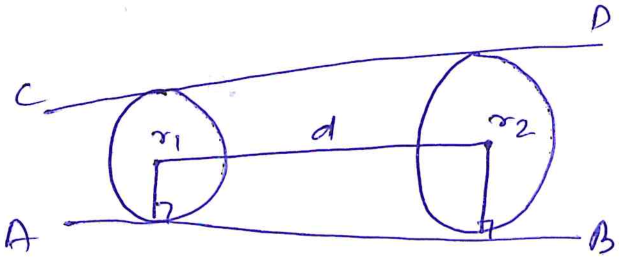
(i) ie, $AC = BC$

(ii) $\angle BCO = \angle OCA = \frac{1}{2} \angle ACB$

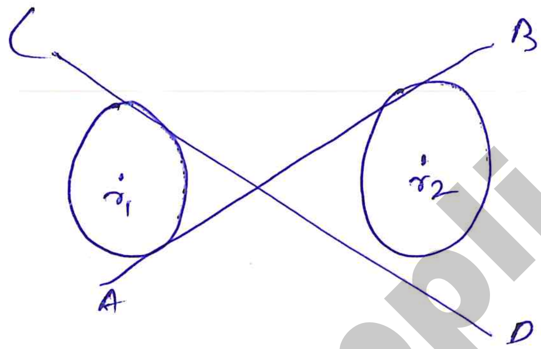
$\angle COB = \angle COA = \frac{1}{2} \angle AOB$

14.16 Circles Part 3

11) Direct Common Tangent (DCT) and Transverse Common Tangent (TCT)

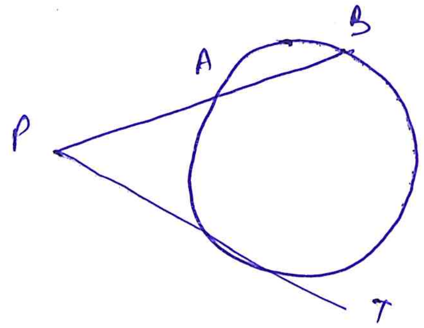


length of DCT $= \sqrt{d^2-(r_1-r_2)^2}$



length of TCT $= \sqrt{d^2-(r_1+r_2)^2}$

12) If PB be a secant which intersects the circle at A and B and PT be a tangent at T then $PA \cdot PB = (PT)^2$.



PB secant
PT Tangent

$PA \cdot PB = (PT)^2$

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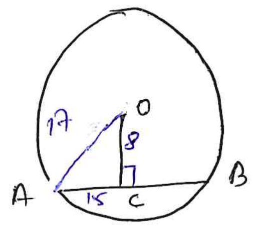
4.17: Circles Problems

Q.1) In the given figure, O is the centre of the circle. Radius of the circle is 17cm. If OC = 8cm then the length of ~~the~~ cord AB is:



OC $\perp^{lar}$ bisector of AB

$AC = CB = \frac{1}{2} AB$

$\triangle OAC$ is right angle triangle.

8, 15, 17

$\therefore AC = 15$

$OA^2 = AC^2 + OC^2$

$17^2 = AC^2 + 8^2$

$AC^2 = 17^2 - 8^2$

$AC = 15$

$AB = 15 \times 2 = 30cm$

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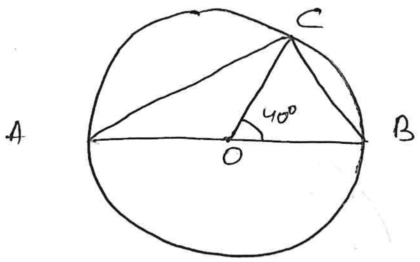
Q.2) In the given figure, $\angle COB = 40°$, AB is the diameter of the circle. find $\angle CAB$.



In $\triangle COB$

$OC = OB =$ radius

$40° + \angle OBC + \angle OCB = 180°$

$\angle OBC + \angle OCB = 140°$

$\angle OBC = \frac{140°}{2}$

$= 70°$

Angle in semi circle $= 90°$

$\angle ACB = 90°$

$\angle ACO = 90° - 70° = 20°$

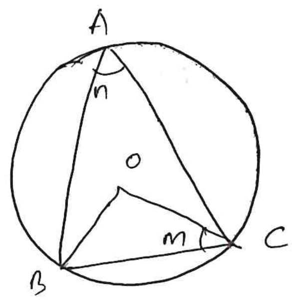
In $\triangle AOC$

$AO = OC =$ Radius

$\angle OCA = \angle OAC = 20°$

Q.3) In the following figure, 'O' is the centre of circle and $\angle BAC = n$, $\angle OCB = m$, then,

(a) $m + n = 90$ (b) $m + n = 180$ (c) $m + n = 120$ (d) $m + n = 150$



O is circumcentre.

$\therefore \angle BOC = 2\angle BAC$

$\angle BOC = 2n$

In $\triangle BOC$

$OB = OC =$ radius

$\angle OBC = \angle OCB = m$

$2n + m + m = 180$

$m + n = 90°$

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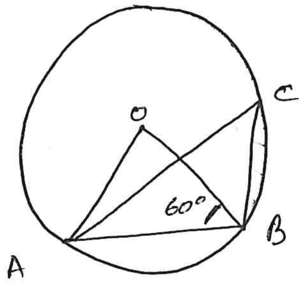
Q.4) In the given figure, O is the centre of the circle, $\angle ABO = 60°$, find the value of $\angle ACB$?



$\angle ACB = \frac{1}{2} \angle AOB$

In $\triangle AOB$

$OA = OB$

$\angle OAB = \angle OBA$

$\angle OAB = 60°$

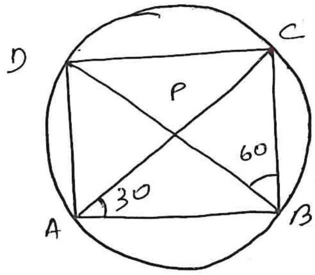
In $\triangle OAB$

$\angle AOB + 60 + 60 = 180°$

$\angle AOB = 60°$

$\angle ACB = \frac{1}{2} \times 60° = 30°$

Q.5) In the given figure, ABCD is cyclic quadilateral and diagonals bisect each other at P. If $\angle DCB = 60$ and $\angle BAC = 30$, then $\angle BCD$ is:



$\angle BAC = 30°$, $\angle DBC = 60°$

$\angle BAC = \angle BDC$ (angles in same segment are equal)

$\angle BDC = 30°$

In $\triangle BDC$

$\angle BDC + \angle DBC + \angle BCD = 180°$

$30 + 60° + \angle BCD = 180°$

$\angle BCD = 90°$

14.18: Circle Problems Part-2

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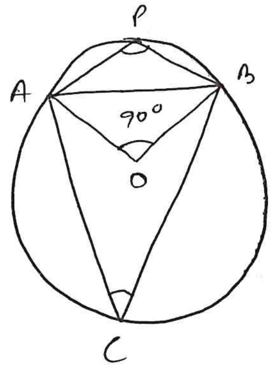
Q.6) In the given figure, O is the centre of circle, $\angle AOB = 90°$, find $\angle APB$.



$\angle ACB = \frac{1}{2} \angle AOB$

$\angle ACB = 45°$

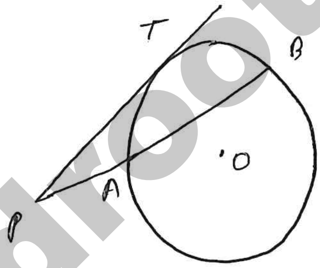
APBC is cyclic quadilaterd.

$\angle APB + \angle ACB = 180°$

$\angle APB = 180 - 45$

$= 135°$

Q.7) In the given figure, tangent $PT = 5cm$, $PA = 4cm$, find AB.



PB is secant

PB is tangent

$PT^2 = PA \times PB$

$(5)^2 = 4 \times (PA + AB)$

$25 = 4 \times (4 + AB)$

$\frac{25}{4} = 4 + AB$

$4 + AB = \frac{25}{4}$

$AB = \frac{25}{4} - 4$

$= \frac{9}{4}$ cm

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Q.8) The distance between the centres of equal circles each of radius 3 cm is 10 cm. The length of a transverse tangent is:

$d = 10, \quad r_1 = 3, \quad r_2 = 3$

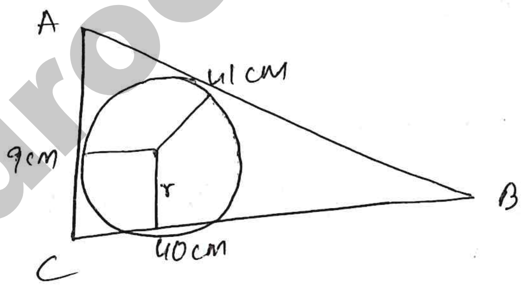
length of transverse tangent $= \sqrt{d^2 - (r_1 + r_2)^2}$

$= \sqrt{(10)^2 - (3+3)^2}$

$= \sqrt{(10)^2 - (6)^2}$

$= 8$ cm

Q.9) What is the inradius of the incircle shown in the figure?



9, 40, 41

$9^2 + 40^2 = 41^2$

Inradius for right angle triangle

$= \dfrac{\text{Base} + \text{Altitude} - \text{hypotenuse}}{2}$

$= \dfrac{40 + 9 - 41}{2} = \dfrac{8}{2} = 4$

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14.19 : Previous Year Gate Questions

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Q.1) The smallest angle of a triangle is equal to the $2/3$rd of the smallest angle of quadilateral. The ratio between of the angle of quadilateral is 3:4:5:6. The largest angle of the triangle is twice its smallest angle. What is the sum of the second largest angle of the triangle and largest angle of quadilateral? (GATE 2014)

Smallest angle of triangle = $2/3$rd of smallest angle of quadilateral

angles in quadilateral = 3:4:5:6

Sum of angles of a quadilateral = 360

$3x + 4x + 5x + 6x = 360$

$18x = 360$

$x = 20$

$3x = 3(20) = 60$

Smallest angle of triangle $= 2/3 \times 60 = 40$

largest $= 2 \times 40 = 80$

Second largest $= 180 - (80+40)$

$= 60$

Required Answer $= 60 + (6 \times 20)$

$= 60 + 120$

$= 180^\circ$

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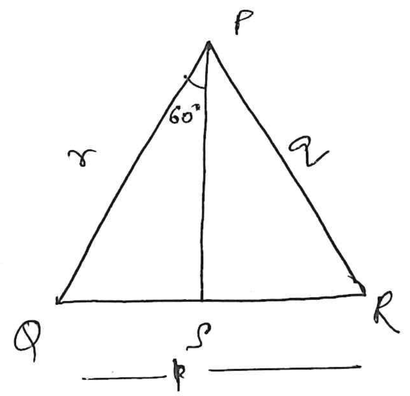
Q27 In a triangle PQR, PS is a angle bisector of $\angle QPR$ and $\angle QPS = 60°$. What is the length of PS?

(GATE 2015)

(a) $\frac{(q+r)}{qr}$ (b) $\frac{qr}{q+r}$ (c) $\sqrt{q^2+r^2}$ (d) $\frac{(q+r)^2}{qr}$



$\angle QPS = 60°$

$\therefore \angle SPR = 60°$ as PS in angle bisector.

Area of $\triangle PQS = \frac{1}{2} \times PS \times PQ \times \sin 60$

Area of $\triangle PSR = \frac{1}{2} \times PS \times PR \times \sin 60$

Area of $\triangle PQR = \frac{1}{2} \times PQ \times PR \times \sin 120$

Area of $\triangle PQR$ = Area $\triangle PQS$ + Area of $\triangle PSR$

$$\frac{1}{2} \times r \times q \times \frac{\sqrt{3}}{2} = \left(\frac{1}{2} \times PS \times r \times \frac{\sqrt{3}}{2}\right) + \left(\frac{1}{2} \times PS \times q \times \frac{\sqrt{3}}{2}\right)$$

$$r \times q = PS(r+q)$$

$$PS = \frac{qr}{r+q} \quad \underline{(ans)}$$

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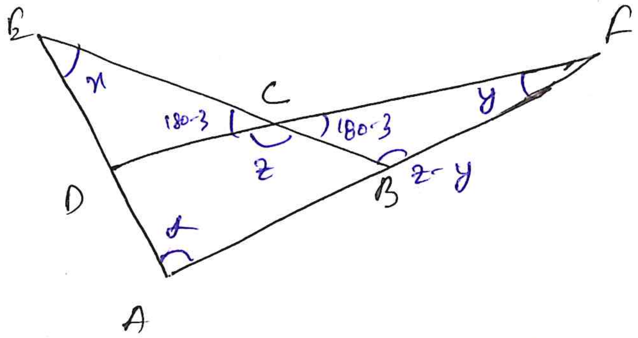
Q.3) In the given figure, $\angle DEC + \angle BFC$ is equal to:

(a) $\angle BCD - \angle BAD$ (b) $\angle BAD + \angle CFF$ (c) $\angle BAD + \angle BCD$ (d) $\angle CBA + \angle ADC$

(GATE 2018)



$$\angle CBF = 180 - (180 - 3 + y)$$
$$= 3 - y$$
$$\angle CBA = 180 - (3 - y)$$
$$= 180 - 3 + y$$
$$\angle CDE = 180 - (180 - 3 + x)$$
$$= 3 - x$$
$$\angle CDA = 180 - (3 - x)$$
$$= 180 - 3 + x$$

In quadilatral ABCD

$$\angle A + \angle B + \angle C + \angle D = 360^\circ$$
$$\alpha + 180 - 3 + y + 3 + 180 - 3 + x = 360^\circ$$
$$x + y = 3 - \alpha$$
$$x + y = \angle BCD - \angle BAD \quad \text{(ans)}$$

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