

12.1: Time and Work Introduction

Ex

100 chairs in 5 days is the requirement.

Shop-1 *
10 chairs → per day
↓
5 days

Shop 2 ✓
20 chairs → per day
↓
5 days

Days →	1 st	2 nd	3 rd	4 th	5 th
Chairs →	20	20	20	20	20

$$\frac{20}{100} = \frac{1}{5} \text{ th of work}$$

$$Y_1 + Y_2 + Y_3 + Y_4 + Y_5 = \frac{5}{5} = 1 \Rightarrow \text{Total work has been completed.}$$

1) Basic Assumption

Amount of work done by the person should be equal on all days.

2) Total amount of work is considered as one unit.

$$W = M \times B \times D$$

Work → Total work / work assigned

Men → No. of person

Efficiency → capability of every individual

Time → Hrs / mins / days

$$\boxed{\begin{aligned} \text{Total work} &= \text{Efficiency} \times \text{No. of days} \\ &= \text{Efficiency} \times \text{Time} \end{aligned}}$$

Ex
 \downarrow 100 chains
A do work in 10 days
 \downarrow 100 chains
B do the same work in 30 days] (A+B) together do

$$\text{Work} = \text{Efficiency} \times \text{Time}$$

$$\text{Days} \propto \frac{1}{\text{Efficiency}}$$

More people, time reduced
Less people, time increases

$$\text{Days} \propto \frac{1}{\text{No. of people}}$$

$$\text{Work} > \frac{\text{Efficiency}}{(\text{W})} \times \frac{\text{Time}}{(\text{T})}$$

When work is constant

$$\text{E} \propto \frac{1}{T}$$

$$\text{Efficiency ratio} = a : b$$

$$\text{Time ratio} = b : a$$

When Efficiency is constant

$$W \propto T$$

More time	more work
less time	less work

When Time is constant

$$W \propto E$$

more people	more work
less people	less work

A	10 days
B	30 days

Find A+B.

$$1 \text{ day work of } A = \frac{1}{10}$$

$$1 \text{ day work of } B = \frac{1}{30}$$

We need to convert into 1 day work.

If a person 'A' can do work in 'n' days

$$1 \text{ day work} = \frac{1}{n}$$

Similarly, If a person 'A' one day work = $\frac{1}{n}$

To complete the total work = n days

Ex
A can do 25% work in one day. In how many days A can complete the work.

$$25\% = \frac{1}{4} \text{ th of work in 1 day}$$

Total work can be done in 4 days.

Combined 1 day's work

$$\begin{array}{r} A \quad 10 \text{ days} \\ B \quad 30 \text{ days} \\ \hline \end{array}$$

$$A \quad 1 \text{ day work} = \frac{1}{10} \quad] \quad \frac{1}{10} + \frac{1}{20}$$

$$B \quad 1 \text{ day work} = \frac{1}{20}$$

$$\text{Combined 1 day's work} = \frac{1}{10} + \frac{1}{20} = \frac{2+1}{20} = \frac{3}{20}$$

Total work is completed in $\frac{20}{3}$ days.

If A do job in ' n ' days.

B do the same job in ' m ' days.

(A+B) do job in ' t ' days.

$$A \text{ 1 day's work} = \frac{1}{n}$$

$$B \text{ 1 day's work} = \frac{1}{m}$$

$$\text{Combined 1 day's work} = \frac{1}{t}$$

$$\frac{1}{m} + \frac{1}{n} = \frac{1}{t}$$

$$\frac{M+N}{MN} = \frac{1}{t}$$

$$t = \frac{MN}{M+N}$$

$$\frac{\text{No. of days worked by A}}{\text{A alone days}} + \frac{\text{No. of days worked by B}}{\text{B alone days}} = 1$$

Chain Rule (Men Days)

M_1 work D_1 days in H_1 hours

The same work was done by M_2 in D_2 days in H_2 hours.

$$\frac{W_1}{W_2} = \frac{M_1 \times D_1 \times H_1 \times E_1}{M_2 \times D_2 \times H_2 \times E_2}$$

$$\Rightarrow \frac{M_1 D_1 \times H_1 \times E_1}{W_1} = \frac{M_2 D_2 \times H_2 \times E_2}{W_2}$$

Ex 12 men work 16 hours a day to make 12000 papers in 18 days. Now, 24 men would work how many hours per day to make 18000 papers in 36 days?

$$\frac{M_1 D_1 H_1}{W_1} = \frac{M_2 D_2 H_2}{W_2}$$

$$\Rightarrow \frac{12 \times 18 \times 16}{12000} = \frac{24 \times 36 \times H_2}{18000}$$

$$\Rightarrow H_2 = 6 \text{ hours}$$

Q.1) A completes the $\frac{4}{5}$ th of work and left. B joins and completes the remaining work. What part of the work was completed by B.

$$A's \text{ work} + B's \text{ work} = 1$$

$$\frac{4}{5} + B = 1$$

$$B = 1 - \frac{4}{5} = \frac{1}{5}$$

Q.2) A can do work in 6 days and B in 9 days. How many days will both take together to complete the work?

- (a) 7.5 (b) 5.4 (c) 3.6 (d) 3

Traditional Method
A's 1 day work = $\frac{1}{6}$

B's 1 day work = $\frac{1}{9}$

$$(A+B)'s \text{ 1 day work} = \frac{1}{6} + \frac{1}{9} \\ = \frac{5}{18}$$

$\therefore (A+B)$ can complete total work in $\frac{18}{5}$ days i.e. 3.6 days.

(OR)

$$\frac{mn}{(m+n)} = \frac{6 \times 9}{6+9} = \frac{54}{15} = 3.6 \text{ days.}$$

(OR)

$$\frac{\text{lcm}(m,n)}{\frac{\text{lcm}}{m} + \frac{\text{lcm}}{n}} = \frac{\text{lcm}(6,9)}{\frac{\text{lcm}}{6} + \frac{\text{lcm}}{9}} = \frac{18}{\frac{18}{6} + \frac{18}{9}} = 3.6 \text{ days.}$$

(OR)

LCM Method
Total work = $(\text{lcm}(6,9)) = 18$

$$A \rightarrow \frac{\text{Days}}{6} \rightarrow \frac{\text{Efficiency}}{18/6 = 3}$$

$$B \rightarrow 9 \rightarrow \frac{\text{Efficiency}}{18/9 = 2}$$

$$A+B \rightarrow$$

$$\frac{5}{18} \rightarrow 1 \\ 18 \rightarrow \frac{18}{5} = 3.6 \text{ days.}$$

Q.37 A alone can complete a work in 12 days. A and B together can complete it in 8 days. How long will B alone take to complete the work?

$$A = 12 \text{ days}$$

$$A+B = 8 \text{ days}$$

$$A's 1 \text{ day work} = \frac{1}{12}$$

$$(A+B)'s 1 \text{ day work} = \frac{1}{8}$$

$$B's 1 \text{ day work} = \frac{1}{n}$$

$$\frac{1}{12} + \frac{1}{n} = \frac{1}{8}$$

$$\frac{1}{n} = \frac{1}{8} - \frac{1}{12}$$

$$n = 24 \text{ days.}$$

Q.48 A, B and C can complete a piece of work in 24, 6 and 12 days respectively. Working together, they will complete the same work in

$$A's 1 \text{ day work} = \frac{1}{24}$$

$$B's 1 \text{ day work} = \frac{1}{6}$$

$$C's 1 \text{ day work} = \frac{1}{12}$$

$$(A+B+C)'s 1 \text{ day work} = \frac{1}{24} + \frac{1}{6} + \frac{1}{12}$$

$$= \frac{1+4+2}{24} = \frac{7}{24}$$

A+B+C can complete the work in $\frac{24}{7}$ days.

Q.5) A and B can do $\frac{1}{18}$ work in 18 and 24 days respectively. They worked together for 8 days and then A left. The remaining work was finished by B in:

$$\text{A's 1 day work} = \frac{1}{18}$$

$$\text{B's 1 day work} = \frac{1}{24}$$

$$(\text{A}+\text{B})'s 1 \text{ day work} = \frac{1}{18} + \frac{1}{24} \\ = \frac{7}{72}$$

$$\therefore 8 \text{ days work} = 8 \times \frac{7}{72} \\ = \frac{7}{9}$$

They completed $\frac{7}{9}$ th work.

$$\text{The remaining work} = 1 - \frac{7}{9} = \frac{2}{9} \quad (\text{done by B})$$

$$\begin{array}{ccc} 1 \text{ day} & \longrightarrow & \frac{1}{24} \\ ? & \longrightarrow & \frac{2}{9} \\ \frac{2}{9} \times 24 & = & \frac{16}{3} \text{ days.} \end{array}$$

(OR)

$$\frac{I}{A} + \frac{I}{B} = 1$$

$$\text{A worked days} = 8$$

$$\text{B worked days} = 8+n$$

$$\text{Remaining work} = 'n' \text{ days.}$$

$$\frac{8}{18} + \frac{8+n}{24} = 1$$

$$\frac{4}{9} + \frac{8+n}{24} = 1$$

$$\frac{32+24+3n}{72} = 1$$

$$3n = 16$$

$$n = \frac{16}{3} \text{ days.}$$

Q.6) A and B together can complete a piece of work in 12 days. B and C in 20 days, C and A in 15 days. A, B and C together can complete in _____ days?

$$\text{T/w LCM}(12, 20, 15) = 60.$$

$$\begin{aligned} A+B &\rightarrow 12 \rightarrow 5 \\ B+C &\rightarrow 20 \rightarrow 3 \\ C+A &\rightarrow 15 \rightarrow 4 \\ Q(A+B+C) &\rightarrow 12 \end{aligned}$$

$$A+B+C \rightarrow 6$$

A+B+C can complete in $\frac{60}{6} = 10$ days.

Q.7) A and B together can complete a work in 8 days and B and C together in 12 days. All of the three together can complete the work in 6 days. In how much time will A and C together complete the work?

$$A+B = 8 \text{ days}, B+C = 12 \text{ days}, A+B+C = 6 \text{ days.}$$

$$\text{LCM}(8, 12, 6) = 24 \rightarrow \text{Total work}$$

$$\begin{aligned} A+B &\rightarrow 8 \rightarrow 3 \\ B+C &\rightarrow 12 \rightarrow 2 \\ C+A &\rightarrow \rightarrow \frac{23}{8} \\ Q(A+B+C) &\rightarrow \rightarrow 8 \\ A+B+C &\rightarrow 6 \rightarrow 4 \end{aligned}$$

A+C can together work in $\frac{24}{3}$ days, i.e., 8 days.

Q.8} A can do a piece of work in 18 days and B in 12 days. They began the work together, but B left the work 3 days before its completion. In how many days, in all, was the work completed?

$$A \rightarrow 18 \text{ days}, B \rightarrow 12 \text{ days}$$

$$(A+B) \rightarrow n \text{ days}$$

A worked for last three days after B left the work.

$$(A+B) \rightarrow n \text{ days}$$

$$\downarrow \quad \downarrow$$

$$n \quad \frac{n}{2}$$

$$A \text{ worked days} = n+3$$

$$B \text{ worked days} = \frac{n}{2}$$

$$\frac{n+3}{18} + \frac{\frac{n}{2}}{12} = 1$$

$$\Rightarrow \frac{2n+6+3n}{36} = 1$$

$$\Rightarrow n = 6 \text{ days.}$$

$$A \text{ days} = n+3 = 6+3 = 9 \text{ days.}$$

(OR)

$$\text{Total Work} \Rightarrow \text{LCM}(12, 18, 3) = 36$$

$$A \rightarrow 18 \rightarrow 2$$

$$B \rightarrow 12 \rightarrow 3$$

$$\overline{A+B} \quad \overline{5}$$

$$A \text{ worked for last 3 days} = 2 \times 3 = 6 \text{ units}$$

$$\text{Remaining part} = 36 - 6 = 30 \quad \Downarrow (A+B)$$

$$(A+B) \text{ do remaining part in } \frac{30}{5} = 6 \text{ days}$$

$$\text{Total days} = 3+6 = 9 \text{ days.}$$

Q.9) A can do work in 15 days and B in 20 days. If they together work on it for 4 days, then the fraction of the work that is left is

$$A \rightarrow 15 \text{ days}$$

$$B \rightarrow 20 \text{ days}$$

$$(A+B) \rightarrow 4 \text{ days}$$

$$A's 1 \text{ day work} = \frac{1}{15}$$

$$B's 1 \text{ day work} = \frac{1}{20}$$

$$(A+B)'s 1 \text{ day work} = \frac{1}{15} + \frac{1}{20} \\ = \frac{7}{60}$$

$$4 \text{ days work} = 4 \times \frac{7}{60} = \frac{7}{15}$$

$$\text{Remaining Work} = 1 - \frac{7}{15} \\ = \frac{8}{15}.$$

Q.10) A and B working separately can do a piece of work in 6 and 9 days respectively. They work on alternate days starting with A on the first day. In how many days will the work be done?

$$\text{Total Work} = \text{LCM}(6, 9) = 18$$

$$A \rightarrow 6 \text{ days} \rightarrow 3$$

$$B \rightarrow 9 \text{ days} \rightarrow 2$$

$$1^{\text{st}} \text{ day} \rightarrow A \rightarrow 3$$

$$2^{\text{nd}} \text{ day} \rightarrow B \rightarrow 2$$

$$2 \text{ days} \\ \times 3$$

$$6 \text{ days}$$

$$7^{\text{th}} \text{ day} \quad A \quad \frac{3}{18}$$

$$7 \text{ days } \underline{\underline{\text{done}}}$$

Q.11) A, B and C can do a piece of work in 30, 20 and 10 days respectively. A is assisted by B on one day and by C on the next day, alternatively. How long would the work take to finish?

$$1^{\text{st}} \text{ day} \rightarrow A+B$$

$$2^{\text{nd}} \text{ day} \rightarrow A+C$$

$$\text{Total Work} \rightarrow \text{LCM}(10, 20, 30) = 60$$

$$A \rightarrow 30 \rightarrow 2$$

$$B \rightarrow 20 \rightarrow 3$$

$$C \rightarrow 10 \rightarrow 6$$

$$1^{\text{st}} \text{ day} \rightarrow A+B \rightarrow 5$$

$$2^{\text{nd}} \text{ day} \rightarrow A+C \rightarrow \frac{8}{13} \times 4$$

2 days

$$\times 4$$

3 days

$$7^{\text{th}} \text{ day} \quad \begin{array}{r} A+B \\ \hline 5 \end{array}$$

$$\begin{array}{r} 52 \\ 5 \end{array}$$

$$10^{\text{th}} \text{ day} \quad \begin{array}{r} A+C \\ (8) \end{array} \quad \begin{array}{r} +3 \\ \hline 60 \end{array}$$

$$9\frac{3}{8} \text{ days} \quad \underline{\text{(Ans)}}$$

2.3! Solved problems on time & work

Q.12) A can do a certain job in 12 days. B is 60% more efficient than A. How many days does B alone take to do the same job?

$$\text{Efficiency Ratio} = A : B$$

$$100 : 160$$

$$= 5 : 8$$

$$\text{Days ratio} = 8 : 5 \quad [Eff \propto \frac{1}{\text{days}}]$$

$$8 \rightarrow 12$$

$$5 \rightarrow \frac{12 \times 5}{8} = 7.5 \text{ days.}$$

Q.13) A is twice as good as workman B and together they can finish a piece of work in 18 days. In how many days will A alone finish the work?

$$\text{Total work} =$$

$$\text{Efficiency Ratio}$$

$$A : B$$

$$2 : 1$$

$$\begin{array}{rcl} A & \frac{2W}{2} \\ B & \frac{1}{3} \\ (A+B) & \frac{3}{3} \end{array}$$

$$\text{Total work} = \frac{3 \times 18}{54} \quad [Eff \times \text{Days}]$$

$$A \text{ days} = \frac{54}{2} = 27 \text{ days}$$

$$B \text{ days} = \frac{54}{1} = 54 \text{ days.}$$

Q.14) If 3 men or 6 boys can do piece of work in 20 days, then how many days with 6 men and 8 boys take to do the same work?

$$\frac{M_1 D_1}{W_1} = \frac{M_2 D_2}{W_2}$$

$$3m \times 20 = 6B \times 20$$

$$\therefore 3m = 6B$$

$$\Rightarrow 1m = 2B$$

$$\therefore 6m + 8B$$

$$= (6 \times 2) + 8B$$

$$= 20B$$

$$6B \times 20 = 20B \times n$$

$$n = 6 \text{ days.}$$

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Q.15) 3 men and 5 women can complete a work in 12 days, which 5 men and 12 women can complete in 6 days. In how many days can 4 men and 4 women complete the same work?

Equate the work.

$$\text{Total Work} = (3m + 5w) \times 12$$

$$\text{Total Work} = (5m + 12w) \times 6$$

$$\therefore (3m + 5w) \times 12 = (5m + 12w) \times 6$$

$$\Rightarrow 6m + 10w = 5m + 12w$$

$$\Rightarrow 1m = 2w$$

$$(3m + 5w) \times 12 = (4m + 4w) \times n$$

$$\Rightarrow (6w + 5w) \times 12 = (8w + 4w) \times n$$

$$\Rightarrow 11w \times 12 = 12w \times n$$

$$\Rightarrow n = 11 \text{ days}$$

Q.16) 2 men and 3 women finish a job in 100 days. For same job 3 women and a child takes 300 days to complete. How many days will 6 men, 18 women and 3 children will take to finish the same job if they all work together?

$$2m + 3w \rightarrow 100 \text{ days} \times 3$$

$$6m + 18w + 3c \rightarrow n \text{ days}$$

$$3w + 1c \rightarrow 300 \text{ days} \times 3$$

$$9w + 3c \rightarrow \frac{300}{3} \text{ days}$$

$$6m + 9w \rightarrow \frac{100}{3} \text{ days}$$

$$9w + 3c \rightarrow \frac{300}{3} \text{ days}$$

$$(6m + 9w) 1 \text{ day's work} = \frac{3}{100}$$

$$(9w + 3c) 1 \text{ day's work} = \frac{1}{100}$$

$$6m + 18w + 3c = \frac{3}{100} + \frac{1}{100} = \frac{4}{100} = \frac{1}{25}$$

They complete the work in 25 days.

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Q.17) 39 persons can repair a road in 12 days working 5 hours a day. In how many days will 30 persons working 6 hours a day complete the work?

$$M_1 D_1 H_1 = M_2 D_2 H_2$$

$$\Rightarrow 39 \times 12 \times 5 = 30 \times 6 \times D_2$$

$$\Rightarrow D_2 = 13 \text{ days.}$$

Q.18) A contractor undertook a contract to complete a part of a stadium in 9 months with a team of 560 persons. Later on, it was required to complete the job in 5 months. How many extra person should be employed to complete the work?

Work = to complete a part of stadium.

$$\frac{M_1 D_1}{W_1} = \frac{M_2 D_2}{W_2}$$

$$\Rightarrow 560 \times 9 = M_2 \times 5$$

$$\Rightarrow M_2 = 1008$$

$$\text{Ans: } 1008 - 560 = 448.$$

Q.19) A contractor undertook a contract for a construction work to be completed in 180 days. He engaged 120 men, but only one-sixth of the work was done in one-fifth of the scheduled time. The additional number of labours that will be required to complete the work in time, is:

Scheduled time = 180 days

Labour employed = 120 men.

$$\frac{1}{5} \text{th of scheduled time} = \frac{180}{5} = 36 \text{ days}$$

120 men completed $\frac{1}{6}$ th work in 36 days

$$\text{Work remained} = 1 - \frac{1}{6} = \frac{5}{6}$$

$$\text{No. of days left} = 180 - 36 = 144 \text{ days.}$$

$$\frac{M_1 D_1}{W_1} = \frac{M_2 D_2}{W_2} \Rightarrow \frac{120 \times 36}{120} = \frac{x \times 144}{5/6}$$

$$\Rightarrow x = 150 \text{ men.}$$

$$\text{Additional labour} = 150 - 120 = 30$$

Q.20) A team of 30 men is supposed to do a work in 38 days. After 25 days, 5 more men were employed and the work finished one day earlier. How many days would it have been delayed if 5 more men were not employed?

- (a) 2 days (b) $1\frac{1}{8}$ days (c) 1 day (d) 14 days

35 men do work in 12 days = 30 men do work in n days

$$35 \times 12 = 30 \times n$$

$$\Rightarrow n = 14 \text{ days}$$

Remaining work is to be completed in 14 days if 5 more men are not employed.

$$\text{Delay} = 14 - 13 = 1 \text{ day.}$$

Q.21) A and B can complete a piece of work in 15 days and 10 days respectively. They contracted to complete the work for ₹ 15,000. The share of A in the contracted money will be

④ Wages are always distributed based on efficiency.

$$\begin{aligned}\text{Day Ratio} &= A : B \\ &= 15 : 10 \\ &= 3 : 2\end{aligned}$$

$$\therefore \text{Efficiency Ratio} = 2 : 3$$

$$\begin{aligned}\text{A's share} &= \frac{2}{5} \times 15000 \\ &= ₹ 6,000/-\end{aligned}$$

2.4: Gate Previous Years Questions Time & Work

Q.1) 5 skilled workers can build a wall in 20 days; 8 semi skilled workers can build a wall in 25 days; 10 unskilled workers can build a wall in 30 days. If a team has 2 skilled, 6 semi skilled and 5 unskilled workers, how long will it take to build the wall?

(GATE 2010)

(a) 20 days

(b) 18 days

(c) 16 days

(d) 15 days

$$5 \text{ skilled} \rightarrow 20 \text{ days}$$

$$8 \text{ semi-skilled} \rightarrow 25 \text{ days}$$

$$10 \text{ unskilled} \rightarrow 30 \text{ days}$$

$$2 \text{ skilled} + 6 \text{ semi-skilled} + 5 \text{ unskilled} = ?$$

$$2 \text{ skilled} \times x \text{ days} = 5 \times 20$$

$$2 \text{ skilled} = 50 \text{ days}$$

$$2 \times x \text{ days} = 5 \times 20$$

$$6 \text{ semi-skilled} = \frac{100}{3} \text{ days}$$

$$10 \times 30 = 5 \times ?$$

$$5 \text{ unskilled} = 60 \text{ days}$$

$$2 \text{ skilled} + 6 \text{ semi-skilled} + 5 \text{ unskilled}$$

$$\frac{1}{20} + \frac{3}{100} + \frac{1}{60} = \frac{6+9+5}{300}$$

$$= \frac{20}{300} = \frac{1}{15}$$

The required answer is 15 days.

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Q.2) 1200 men and 500 women can build a bridge in 2 weeks; 900 men and 450 women will take 3 weeks to build the same bridge. How many men will be needed to build the bridge in one week? (CAT 2012)

(a) 3000

(b) 3300

(c) 3600

(d) 3900

1200 Men + 500 Women \rightarrow 2 weeks900 men + 450 women \rightarrow 3 weeks

$$(1200m + 500w) \times 2 = (900m + 450w) \times 3$$

$$2400m + 1000w = 2700m + 1350w$$

$$300m = 350w$$

$$\times 2$$

$$600m = 700w$$

$$(1200m + 500w) \times 2 = n \text{ men} \times 1$$

$$1800m \times 2 = n \times 1$$

$$n = 3600 \text{ men.}$$

Q.3) X bullocks and Y tractors takes 8 days to plough a field. If we half the number of bullocks and double the number of tractors, it takes 5 days to plough the same field. How many days will it take X bullocks alone to plough the field? (CAT 2012)

(a) 30

(b) 35

(c) 40

(d) 45

$$X + Y = 8$$

$$\frac{Y}{2} + 2Y = 5$$

$$(X+Y) \times 8 = \left(\frac{Y}{2} + 2Y\right) \times 5$$

$$\frac{11}{2}X = 2Y$$

$$Y = \frac{11}{4}X$$

$$\therefore (X+Y) \times 8 = n \times 5$$

$$\therefore \left(1 + \frac{11}{4}\right) \times 8 = 1 \times 5$$

$$\Rightarrow n = 30 \text{ days.}$$

Q.4) Seven machines takes 7 minutes to make 7 identical toys. At the same rate, how many minutes would it take for 100 machines to make 100 toys?

(CAT 2018)

- (a) 1 (b) 7 (c) 100 (d) 700

$$\frac{M_1 D_1 H_1}{W_1} = \frac{M_2 D_2 H_2}{W_2}$$

$$\frac{T \times 7}{7} = \frac{100 \times H_2}{100}$$

$$H_2 = 7 \text{ mins (ans)}$$

Q.5) A contract is to be completed in 52 days and 125 identical robots were employed each operational for 7 hours a day. After 39 days, $\frac{5}{7}$ th of the work was completed. How many additional robots would be required to complete the work on time, if each robot is now operational for 8 hours a day?

(CAT 2018)

$$\frac{N_1 D_1 H_1}{W_1} = \frac{N_2 D_2 H_2}{W_2}$$

$$\Rightarrow \frac{125 \times 39 \times 7}{7} = \frac{\alpha \times (52-39) \times 8}{(1-\frac{5}{7})}$$

$$\Rightarrow \alpha = \frac{525}{4} = 131.25$$

$$\begin{aligned} \text{Additional robots} &= 131.25 - 125 \\ &= 6.25 \text{ or } 7 \text{ robots.} \end{aligned}$$

P, Q, R and S are working on a project. Q can finish the task in 25 days, working alone for 10 hours a day. R can finish the task in 50 days, working alone for 12 hours per day. Q worked 12 hours a day but took sick leave in the beginning for two days. R worked for 18 hours a day on all days. What is the ratio of work done by Q and R after 7 days from the start of the project?

(GATE 2016)

(A) 10:11

(B) 11:10

(C) 20:21

(D) 21:20

$$Q \text{ alone work } 25 \times 10$$

$$R \text{ alone work } 50 \times 12$$

$$\text{But Q worked for 5 days} \rightarrow Q \text{ worked hours } = 5 \times 10$$

$$R \text{ worked for 7 days} \rightarrow R \text{ worked hours } = 7 \times 12$$

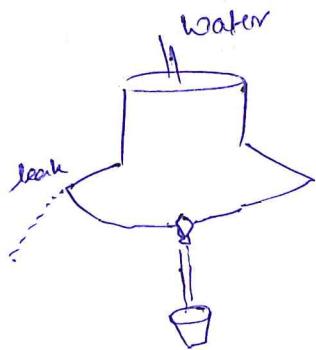
Work done

$$\frac{Q \text{ worked hours}}{Q \text{ Alone work}} : \frac{R \text{ worked hours}}{R \text{ Alone work}}$$

$$\frac{5 \times 10}{25 \times 10} : \frac{7 \times 12}{50 \times 12}$$

$$20 : 21$$

2.5 : Pipes & Cistern



Inflow/inlet Pipe \rightarrow water flowing into the tank

Outflow/outlet pipe \rightarrow water flowing out from the tank

leak \rightarrow outflow

Outflow \rightarrow negative work

$$\text{Efficiency} \propto \frac{1}{\text{Time}}$$

Efficiency ratio $a:b$

Time ratio $b:a$

Pipe A is filling tank in 10 hrs.

Pipe B can fill tank in 20 hrs.

less time \rightarrow more efficiency
more time \rightarrow less efficient

1 hr work

\Rightarrow If a pipe A fills the tank in (x) hours, part filled in 1 hr $= \frac{1}{x}$

\Rightarrow If a pipe B empties the tank in (y) hours, part emptied in 1 hr $= \frac{1}{y}$

\Rightarrow If a pipe B can empty the tank in (m) hours and pipe B can empty the tank

3) If a pipe A fills the tank in (n) hours and pipe B can empty by A and B

in $1/y$ hours, then the net part filled in 1 hr by A and B

\Rightarrow if $(n < y)$, then $\frac{1}{n} - \frac{1}{y}$

\Rightarrow if $(y > n)$, then $\frac{1}{y} - \frac{1}{n}$

4) Pipe A fills the tank in (n) hrs and pipe B fills the tank in y hours, net part filled by A & B is $= \frac{1}{n} + \frac{1}{y}$

12.6: Pipes & Cistern Problems

Q.1) Two pipes A and B can fill a tank in 36 hours and 45 hours respectively. If both the pipes are opened simultaneously, how much time will be taken to fill the tank?

Efficiency = Capacity of filling the tank

Part filled by pipe A in 1 hr = $\frac{1}{36}$

Part filled by pipe B in 1 hr = $\frac{1}{45}$

$$\begin{aligned} \text{Net Part filled by pipe A and pipe B in 1 hr} &= \frac{1}{36} + \frac{1}{45} \\ &= \frac{1}{20} \end{aligned}$$

$$1 \text{ hr} \quad \frac{1}{20}$$

$$? \quad 1$$

20 hrs. (Ans)

Q.2) Pipe A can empty the tank in 10 hrs and pipe B can empty a tank in 5 hrs. If both the pipes are opened simultaneously, find the time in which a full tank is emptied?

Part emptied by pipe A in 1 hr = $\frac{1}{10}$

Part emptied by pipe B in 1 hr = $\frac{1}{5}$

$$\begin{aligned} \text{Part emptied by A and B in 1 hr} &= \frac{1}{10} + \frac{1}{5} \\ &= \frac{3}{10} \end{aligned}$$

(A+B) can empty this full tank in $\frac{10}{3}$ hrs

$$= 3\frac{1}{3} \text{ hrs.}$$

Q.3) Pipe A can fill the tank in 10 hrs and Pipe B can empty it in 12 hrs. If both the pipes are opened, find the time in which tank is filled.

$$\text{Part filled by the pipe A} = \frac{1}{10}$$

$$\text{Part filled by the Pipe B} = \frac{1}{12}$$

$$\text{Next part filled by the A \& B} = \frac{1}{10} - \frac{1}{12}$$

$$= \frac{6-5}{60}$$

$$= \frac{1}{60}$$

Tank filled by A & B together in 60 hrs.

Q.4) Tap A can fill the empty tank in 12 hrs, but due to leak in the bottom it is filled in 15 hrs. If the tank is full and then tap A is closed then in how many hours the leak can empty it?

leak \rightarrow outflow

$$\text{Part filled by Tap A} = \frac{1}{12}$$

$$\text{Part emptied by leak (B)} = \frac{1}{n}$$

$$\text{Part filled combinedly (Tap A + leak)} = \frac{1}{15}$$

$$\text{Tap A + leak} = \frac{1}{12} - \frac{1}{n} = \frac{1}{15}$$

$$\Rightarrow \frac{1}{n} = \frac{1}{12} - \frac{1}{15}$$

$$= \frac{5-4}{60}$$

$$= \frac{1}{60}$$

leak can empty the tank in 60 hrs.

Q.5) Two pipes can full the tank in 10h and 16h respectively. A third pipe can empty the tank in 32h. If all the three pipes function simultaneously, then in how much time the tank will be full?

$$\text{1st pipe part filled in 1 hr} = \frac{1}{10}$$

$$\text{2nd pipe part filled in 1 hr} = \frac{1}{16}$$

$$\text{3rd pipe part emptied in 1 hr} = \frac{1}{32}$$

$$\text{Net part} = \frac{1}{10} + \frac{1}{16} - \frac{1}{32}$$

$$= \frac{2}{160}$$

$$\text{Three pipes can fill the tank in } \frac{160}{21} \text{ hrs} \\ = 7\frac{13}{21} \text{ hrs.}$$

Q.6) Two pipes can fill a cistern in 12 and 16 min respectively. Both pipes are opened together but 4 min before the cistern is full, pipe A is closed. How much time will the cistern take to fill?

$$A = 12, B = 16$$

$$\text{Part filled by Pipe A} = \frac{1}{12}$$

$$\text{Part filled by pipe B} = \frac{1}{16}$$

$$\text{Part filled by } (A+B) \text{ in 1 min} = \frac{1}{12} + \frac{1}{16} \\ = \frac{7}{48}$$

$$(A+B) \rightarrow x \text{ mins}$$

So B is alone for last 4 mins.

$$\text{for 1 min} \rightarrow \frac{1}{16} \\ 4 \text{ min} \rightarrow 4 \times \frac{1}{16} = \frac{1}{4} \text{ part is completed by B.}$$

$$\text{Remaining part} = 1 - \frac{1}{4} = \frac{3}{4} \Rightarrow (A+B)$$

$$1 \text{ min} \rightarrow \frac{7}{48}$$

$$? \rightarrow \frac{3}{4}$$

$$\frac{3}{4} \times \frac{48}{7} = \frac{36}{7} \text{ mins}$$

$$\therefore \text{Required answer} = 4 + \frac{36}{7} \\ = 9\frac{1}{7} \text{ mins.}$$

Q.7) Two pipes A and B can fill a tank in 24 min and 32 min respectively. If both the pipes are opened simultaneously, after how much time B should be closed so that the tank is filled in 18 mins?

A filled time = 18 mins

$$\frac{\text{No. of mins worked by A}}{\text{A alone mins}} + \frac{\text{No. of mins worked by B}}{\text{B alone mins}} = 1$$

$$\frac{18}{24} + \frac{n}{32} = 1$$

$$n = 8 \text{ mins}$$

$$\text{No. of mins worked by B} = 8 \text{ mins.} \\ \text{or} \\ (A+B)$$

Q.8) Two pipes A and B can alone fill a tank in 20 min and 30 min respectively. But due to leak at the bottom of tank, it took 3 more minutes to fill the tank. In how many hours, the leak can alone empty the full tank?

$$\text{Part filled by A} = \frac{1}{20}$$

$$\text{Part filled by B} = \frac{1}{30}$$

$$\text{Part filled by A and B} = \frac{1}{20} + \frac{1}{30} \\ = \frac{1}{12}$$

$$(A+B) \text{ take the time to fill the tank} = 12 \text{ mins.}$$

$$\text{Instead of 12 mins, the tank filled in 15 mins}$$

$$\frac{1}{12} - \frac{1}{\text{leak mins}} = \frac{1}{15}$$

$$\frac{1}{\text{leak mins}} = \frac{1}{60}$$

$$\text{leak can empty the tank in 60 mins.}$$

Q.9) It takes 30 mins to empty a half tank by draining it at a constant rate. It is decided to simultaneously pump water into the half tank while draining it. What is the rate at which water has to be pumped in so that it gets fully filled in 10 mins? (GATE 2014)

- (a) 4 times the draining rate
- (b) 3 times the draining rate
- (c) 2.5 times the draining rate
- (d) 2 times the draining rate

LCM

Tank Capacity 30 litre

Outlet = A

Inlet = B

$$A \rightarrow 30 \rightarrow -1$$

$$B \rightarrow \underline{\quad} \rightarrow \underline{4}$$

$$A+B \rightarrow 10 \rightarrow 3$$

Answer \Rightarrow 4 times the draining rate.