Project 1

February 23, 2018

1 Problem: Find the best degree 5 polynomial that fits the data from the file datafile.npy.

Let

$$C(a_0, \ldots, a_5) = \sum_{i=0}^{N} (y_i - p(x_i))^2 =$$

where

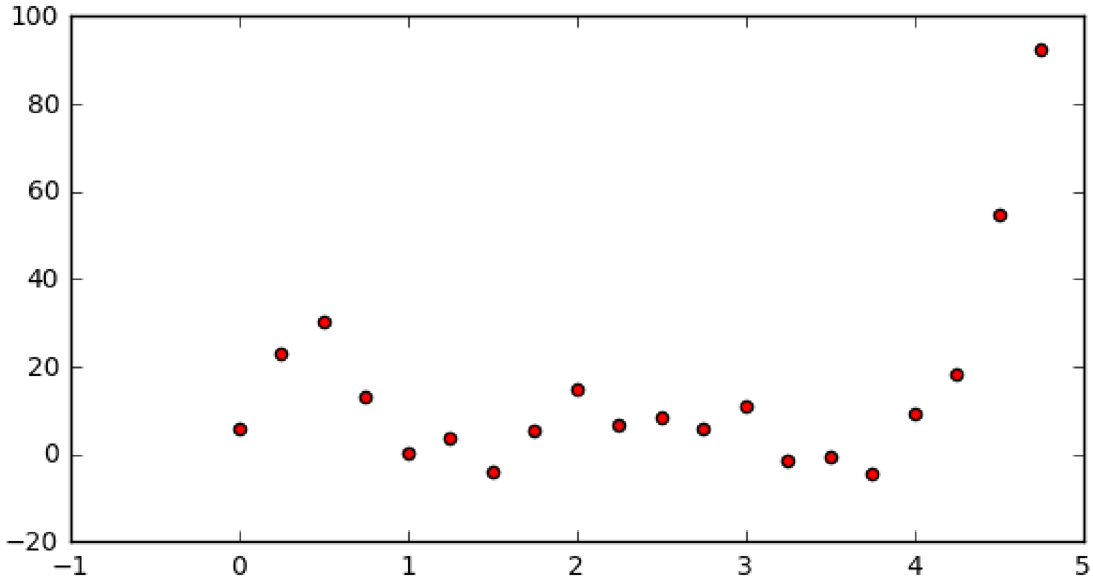
$$p(x_i) = a_0 + a_1 x_i + a_2 x_i^2 + a_3 x_i^3 + a_4 x_i^4 + a_5 x_i^5$$

In [2]:
```
# import modules
import numpy as np
import matplotlib
import matplotlib.pyplot as plt
```

In [3]:
```
%matplotlib inline
```

In [4]:
```
# load the data:
data=np.load('datafile.npy')
fig = plt.figure()
axes = fig.add_axes([0.1, 0.1, 0.9, 0.7])
axes.scatter(data[:,0],data[:,1],c="r");
```

Out[4]:




```
In [5]: def cost_function(x,data):

            cost = 0
            for j in data:
                cost += (j[1] - p(x,j[0]))**2
            return cost

In [6]: def p(x,const):

            value = 0
            for i in range(len(const)):
                value += const[i] * x**i
            return value
```

The partial derivatives of C is as follows:

$$\frac{\partial C}{\partial a_0} = \sum_{i=0}^{N} -2(y_i - p(x_i)) \cdot 1$$
$$\frac{\partial C}{\partial a_1} = \sum_{i=0}^{N} -2(y_i - p(x_i)) \cdot x_i$$
$$\frac{\partial C}{\partial a_2} = \sum_{i=0}^{N} -2(y_i - p(x_i)) \cdot x_i^2$$
$$\frac{\partial C}{\partial a_3} = \sum_{i=0}^{N} -2(y_i - p(x_i)) \cdot x_i^3$$
$$\frac{\partial C}{\partial a_4} = \sum_{i=0}^{N} -2(y_i - p(x_i)) \cdot x_i^4$$
$$\frac{\partial C}{\partial a_5} = \sum_{i=0}^{N} -2(y_i - p(x_i)) \cdot x_i^5$$

Or simply

$$\begin{aligned}\nabla C &= \Big(\frac{\partial C}{\partial a_0}, \frac{\partial C}{\partial a_1}, \frac{\partial C}{\partial a_2}, \frac{\partial C}{\partial a_3}, \frac{\partial C}{\partial a_4}, \frac{\partial C}{\partial a_5}\Big) \\ &= \Big(\sum_{i=0}^{N} -2 \cdot (y_i - p(x_i)), \sum_{i=0}^{N} -2x_i \cdot (y_i - p(x_i)), \sum_{i=0}^{N} -2x_i^2 \cdot (y_i - p(x_i)), \\ &\sum_{i=0}^{N} -2x_i^3 \cdot (y_i - p(x_i)), \sum_{i=0}^{N} -2x_i^4 \cdot (y_i - p(x_i)), \sum_{i=0}^{N} -2x_i^5 \cdot (y_i - p(x_i))\Big)\end{aligned}$$

In [7]:
```
def cost_gradient(constants):
    mygrad = np.zeros(len(constants))
    for j in range(len(constants)):
        summation = 0
        for point in data:
            summation += (p(point[0],constants)-point[1])*point[0]**j
        mygrad[j] = summation
    return mygrad
```

In [8]:
```
def norm(vec):

    length = 0
    for i in range(len(vec)):
        length += vec[i]**2
    return length**(1.0/2)
```

In [9]:
```
def normalized(vec):

    length = norm(vec)
    unitVec = np.zeros(len(vec))
    for i in range(len(unitVec)):
```

```
            unitVec[i] = vec[i] / length
        return unitVec
```

In [10]:
```
def multSeq(h,vec):

    newVec = np.zeros(6)
    for i in range(len(newVec)):
        newVec[i] = vec[i] * h
    return newVec
```

In [11]:
```
#x_old is the list of constants
x_old = np.zeros(6)
h = 0.01 # step size
tolerance = 0.001
maxIterations = 5000
t = 1
x_new = np.zeros(6)
```

In [12]:
```
for i in range(len(x_new)):
    x_new[i] = x_old[i] + multSeq(-h,normalized(cost_gradient(x_old)))[i]
```

In [13]:
```
while t < maxIterations and
(abs(x_new[0] - x_old[0])+abs(x_new[1] - x_old[1])+abs(x_new[2] - x_old[2])+
 abs(x_new[3] - x_old[3])+abs(x_new[4] - x_old[4])+abs(x_new[5] - x_old[5])
) > tolerance:

    for i in range(len(x_new)):
        x_old[i] = x_new[i]
    for j in range(len(x_new)):
        x_new[j] = x_old[j] - multSeq(h, normalized(cost_gradient(x_old)))[j]
    t += 1
print(x_new)
```

```
[ 0.08032254  0.06458722  0.04706228 -0.06684329 -0.38389691  0.11123817]
```

In [14]:
```
if i == maxIterations:
    print("Maximum iteration was reached.")

print("The best degree 5 polynomial has coefficients: ")
for i in range(len(x_new)):
    print("a" + str(i) + ": " + str(x_new[i]))

x = np.linspace(0,5,1001)
y = p(x, x_new)
fig = plt.figure()
axes = fig.add_axes([0.1, 0.1, 0.9, 0.7])
axes.plot(x,y)
axes.scatter(data[:,0],data[:,1],c="r");
```

```
The best degree 5 polynomial has coefficients:
a0: 0.0803225404923
a1: 0.0645872156651
a2: 0.0470622772964
a3: -0.0668432941729
a4: -0.383896909967
a5: 0.11123817405
```

Out[14]: