BOOLEAN ALGEBRA

CLASS XII

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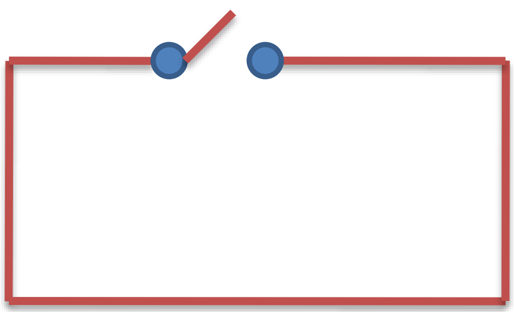
Introduction

- Boolean Algebra is a set of rules and regulation which is suitable for Digital Circuits, whose answer is either True or False



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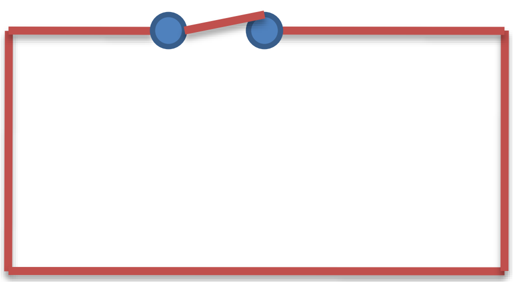

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History

- Mr. Aristotal constructed a compete system of formal logic to organize man's reasoning. Only Mathematician George Boole become able to manipulate these symbols to arrive the solution with mathematical system of Logic and produce new system the Algebra of logic .i.e. Boolean Algebra.

Truth Table

- Present all the possible values and result of logical variable with given combinations of values.

No. of combination = 2^n

[n is no. of variables / Options]

Eg. (1) I want to have tea

(2) Tea is available

I want to have tea	Tea is available	Result
T	T	T
T	F	F
F	T	F
F	F	F

OR

I want to have tea	Tea is available	Result
1	1	1
1	0	0
0	1	0
0	0	0

- TAUTOLOGY : If result of any logical statement or expression if always **TRUE** or **1** called Tautology.

- FALLACY : If result of any logical statement or expression if always **FALSE** or **0** called Fallacy.

Logical Operators

- NOT
- OR
- AND

NOT Operator

eg. I want to have tea.

NOT [I want to have tea]

means

I don’t want to have tea

Truth table for NOT operator

I want to have tea X	Result X’
1	0
0	1

OR Operator

OR operator being denoted as Logical Addition and symbol used for it is '**+**'

Truth Table for OR operator

X	Y	X + Y
1	1	1
1	0	1
0	1	1
0	0	0

AND Operator

AND operator being denoted as Logical Multiplication and symbol used for it is ‘ . ’

Truth Table for OR operator

X	Y	X . Y
1	1	1
1	0	0
0	1	0
0	0	0

NOT, OR, AND Operator

Truth Table for NOT, OR, AND operator

X	Y	X’	Y’	X + Y	X . Y
1	1	0	0	1	1
1	0	0	1	1	0
0	1	1	0	1	0
0	0	1	1	0	0

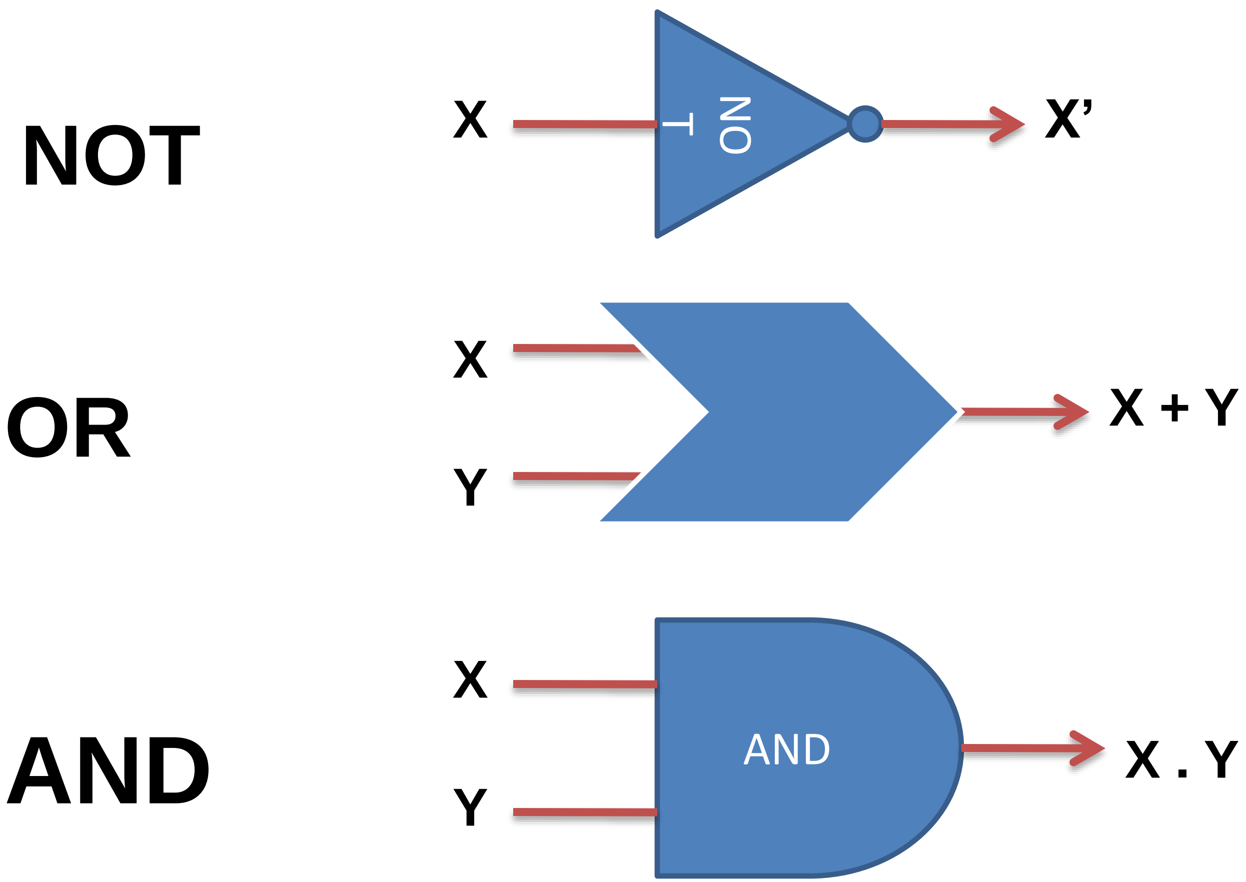
Truth Table for
X.Z, Y.Z’ and (YZ)’

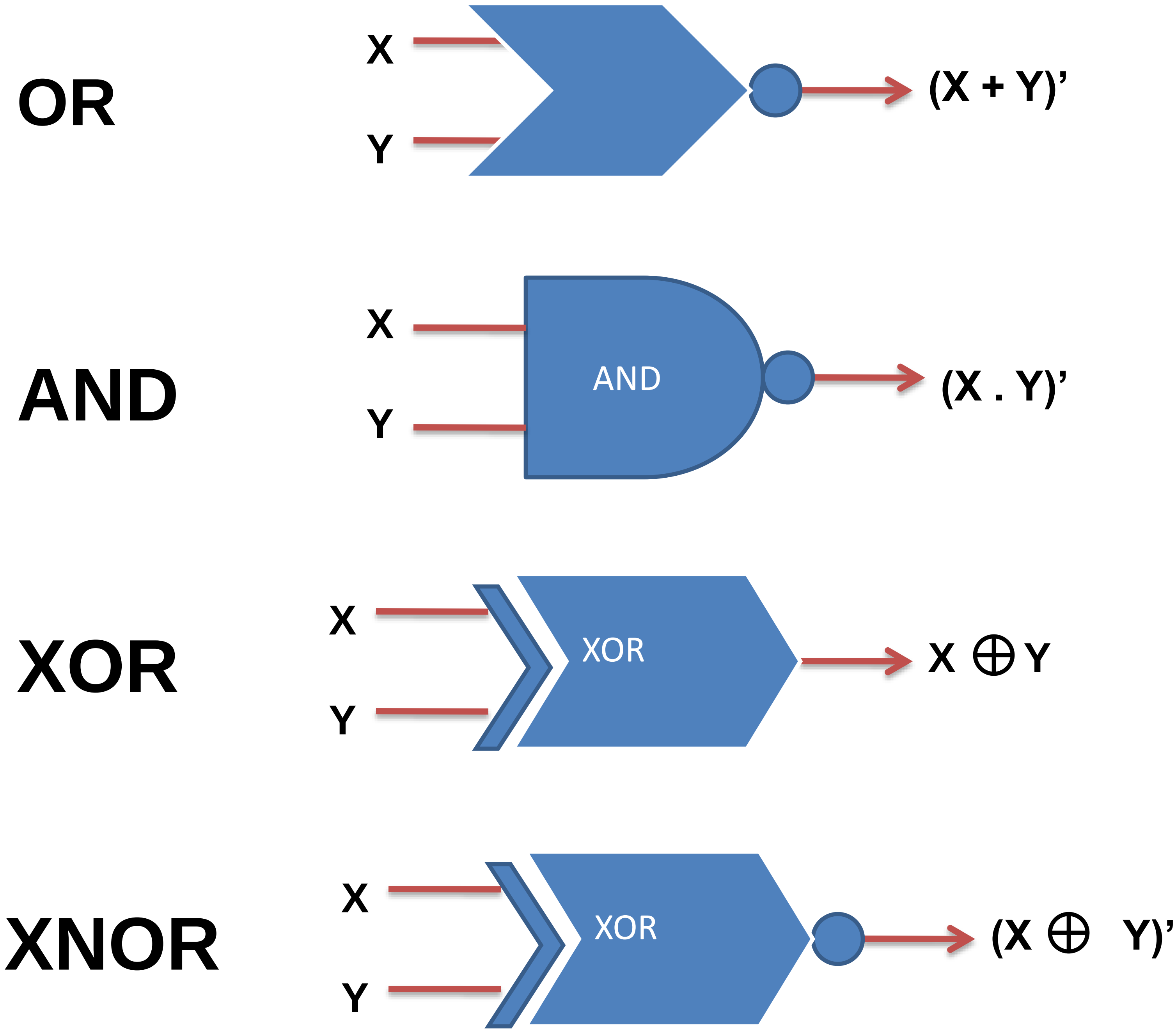
Truth Table for NOT, OR, AND operator

X	Y	Z	Z’	X . Y	Y . Z’	YZ	(YZ)’
1	1	1	0	1	0	1	0
1	1	0	1	1	0	0	1
1	0	1	0	0	0	0	1
1	0	0	1	0	0	0	1
0	1	1	0	0	1	1	0
0	1	0	1	0	0	0	1
0	0	1	0	0	0	0	1
0	0	0	1	0	0	0	1

Basic Logic Gates

- Gate is an electronic circuit being operates on one or more signals to produce output signals.




OR
X
Y
(X + Y)'
AND
X
Y
AND
(X . Y)'
XOR
X
Y
XOR
X ⊕ Y
XNOR
X
Y
XOR
(X ⊕ Y)'

Principal of Duality

- Using this Principal Dual relation can be obtained by :
  - Changing each OR sign (+) with AND sign (.)
  - Changing each AND sign (.) with OR sign (+)
  - Replacing each 0 by 1 and 1 by 0.

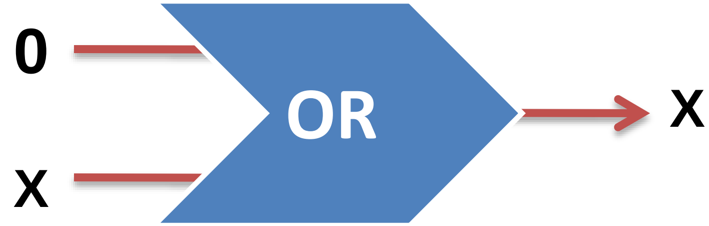
eg . $1 + 0 = 1$

$0 . 1 = 0$ [Dual relation]

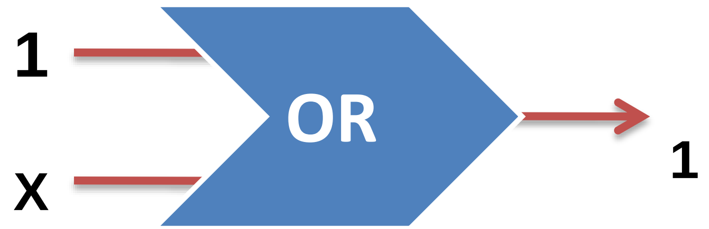
BASIC THEOREMS OF BOOLEAN ALGEBRA

Property of 0 and 1

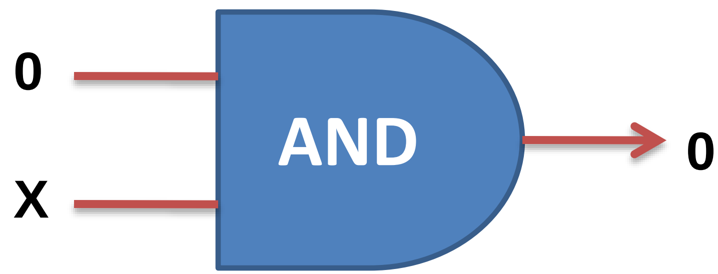
$0 + X = X$



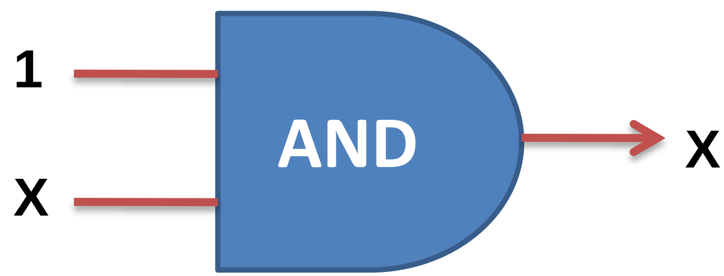
$1 + X = 1$



$0 \,.\, X = 0$

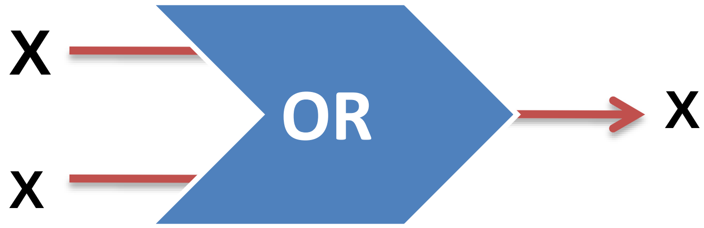


$1 \,.\, X = X$

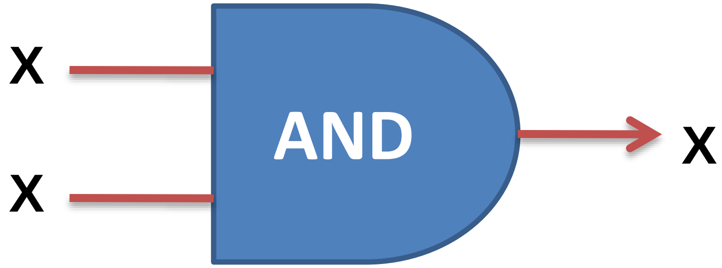


Indempotence Law

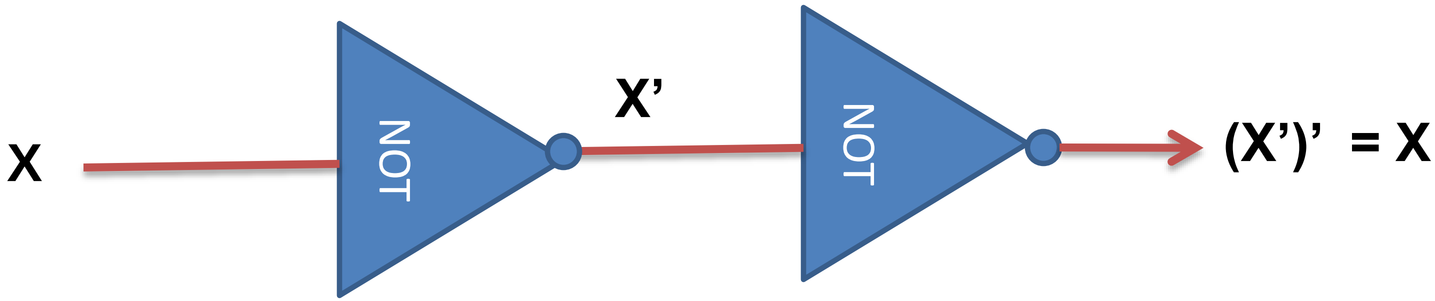
$X + X = X$



$X . X = X$



Involution Law

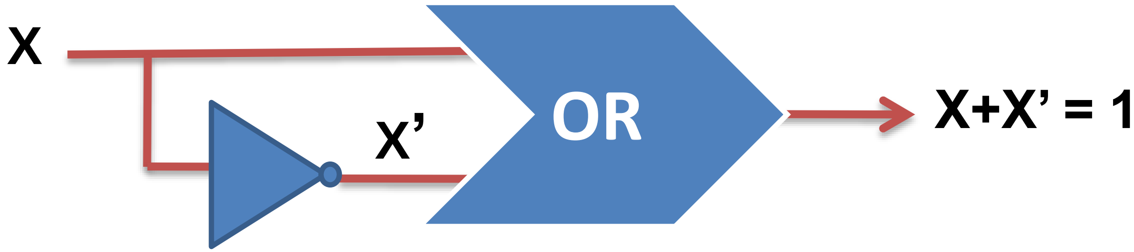


Truth Table of Involution Law

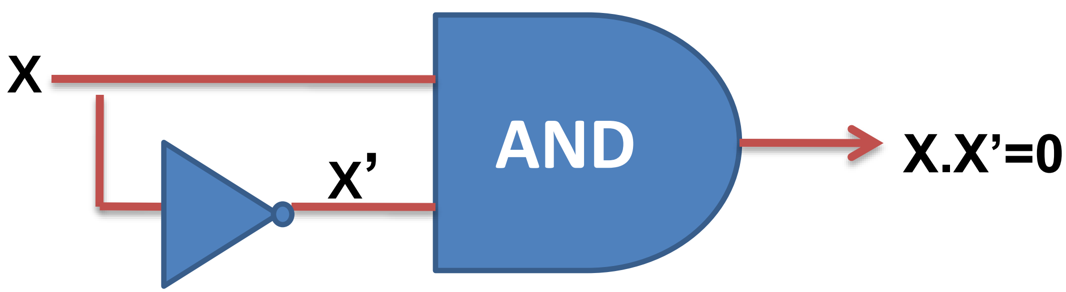
X	X’	(X’)’
1	0	1
0	1	0

Complementarity Law

$X + X' = 1$

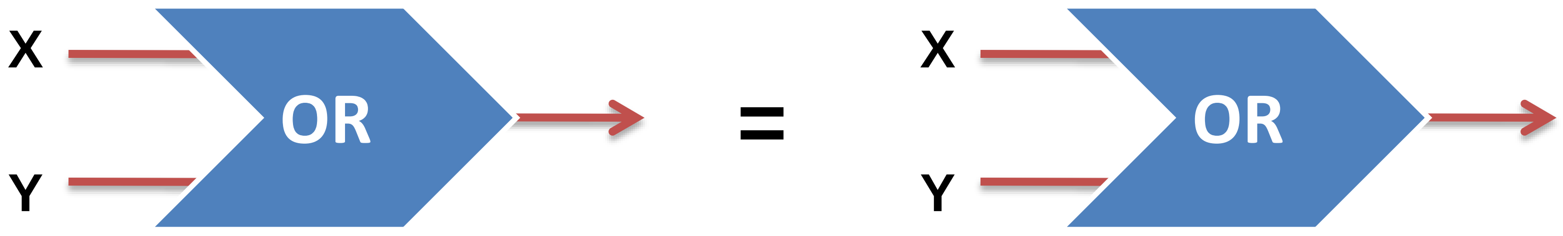


$X \cdot X' = 0$

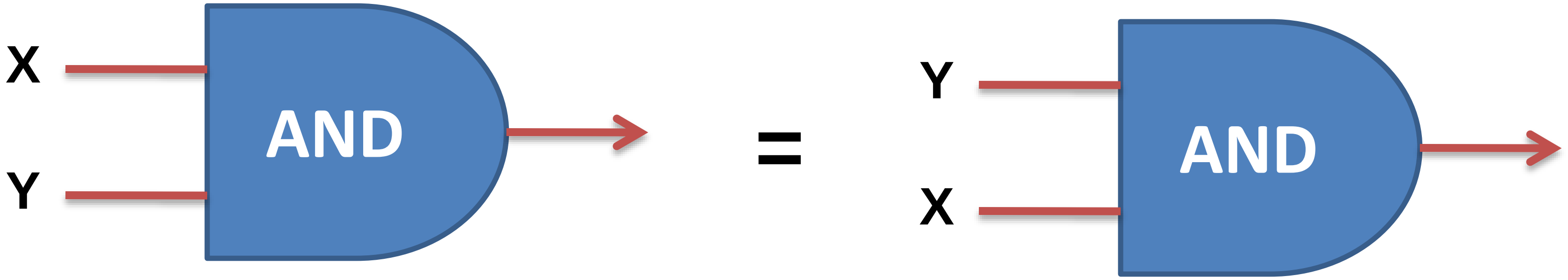


Commutative Law

$X + Y = Y + X$

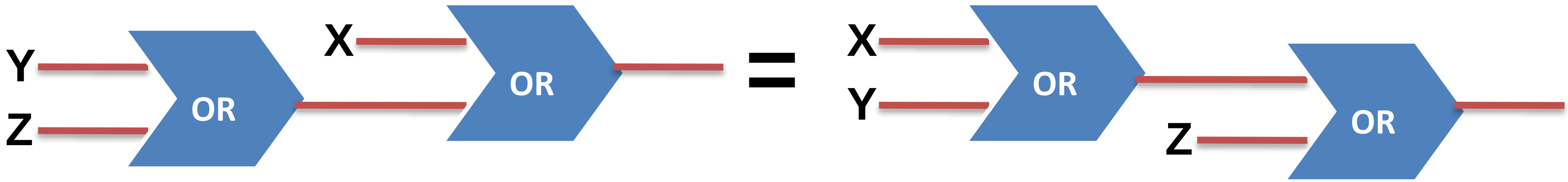


$X \cdot Y = Y \cdot X$

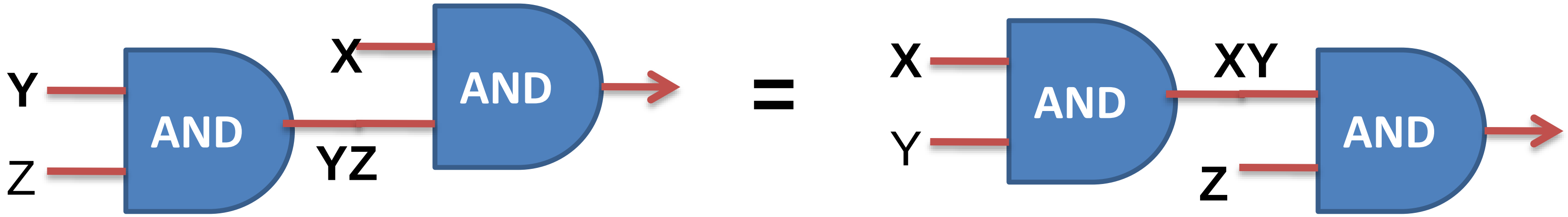


Associative Law

$$X + (Y+Z) = (X+Y) + Z$$



$$X\,(YZ) = (XY)\,Z$$



Truth Table of Associative Law

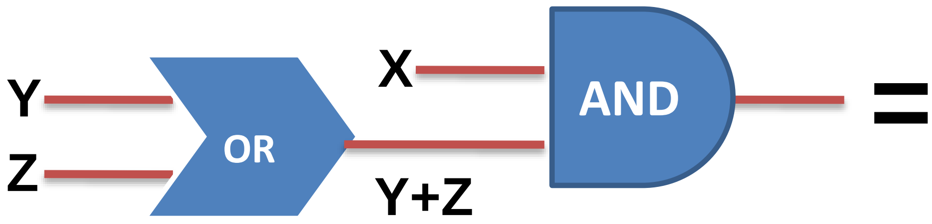
$X + (Y+Z) = (X+Y) + Z$

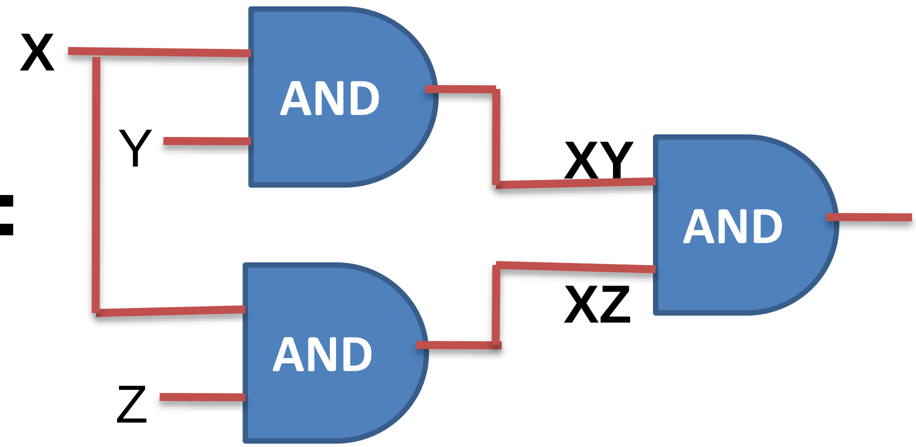
X	Y	Z	Y + Z	X + Y	X + (Y + Z)	(X + Y) + Z
1	1	1	1	1	1	1
1	1	0	1	1	1	1
1	0	1	1	1	1	1
1	0	0	0	1	1	1
0	1	1	1	1	1	1
0	1	0	1	1	1	1
0	0	1	1	0	1	1
0	0	0	0	0	0	0

$X (YZ) = (XY) Z$ **can be obtain as above**

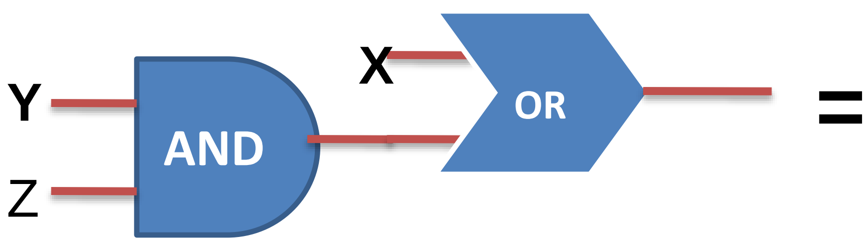
Distributive Law

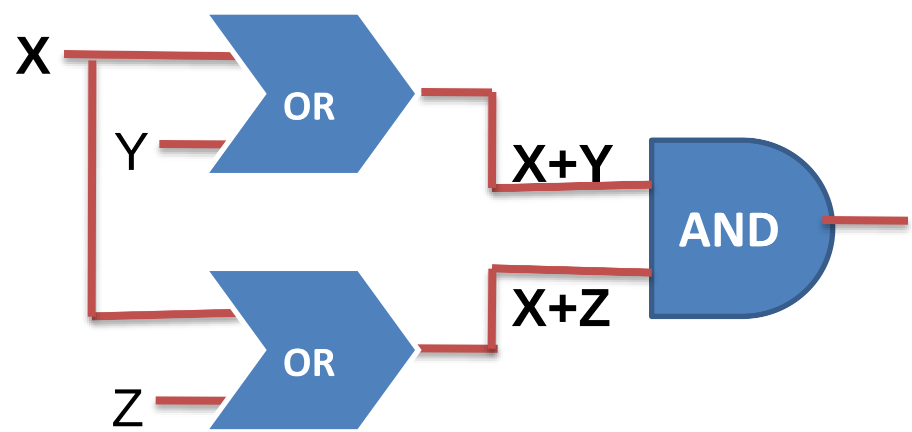
$$X\,(Y+Z) = XY + XZ$$





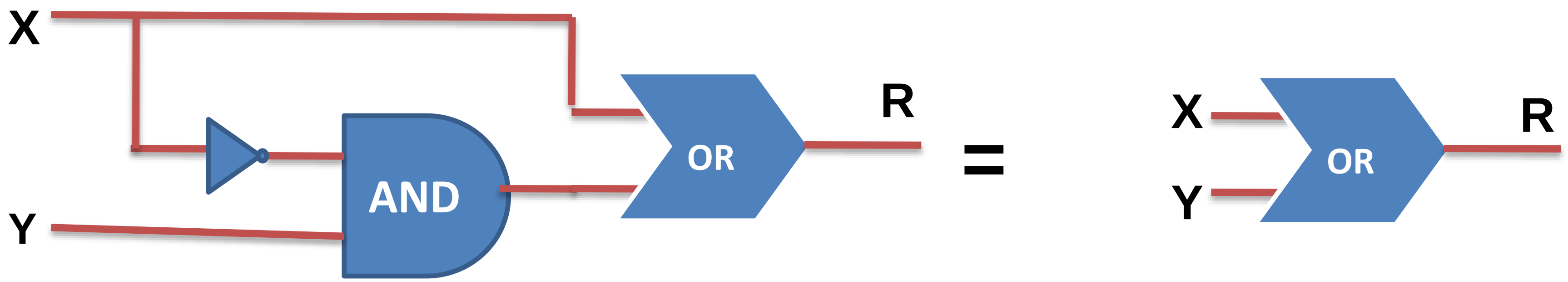
$$X+(YZ) = (X+Y)\,(X+Z)$$





$X+(YZ) = (X+Y)\ (X+Z)$

3rd Law



Truth Table of Distributive Law

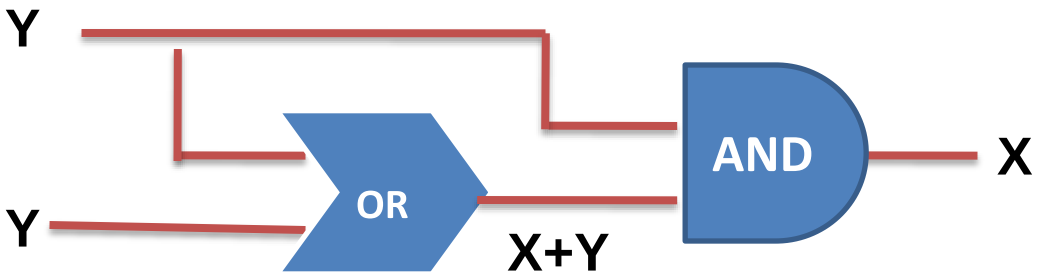
X (Y+Z) = XY + XZ

X	Y	Z	Y + Z	X Y	X Z	X (Y + Z)	X Y + X Z
1	1	1	1	1	1	1	1
1	1	0	1	1	1	1	1
1	0	1	1	0	1	1	1
1	0	0	0	0	0	0	0
0	1	1	1	0	0	0	0
0	1	0	1	0	0	0	1
0	0	1	1	0	0	0	1
0	0	0	0	0	0	0	1

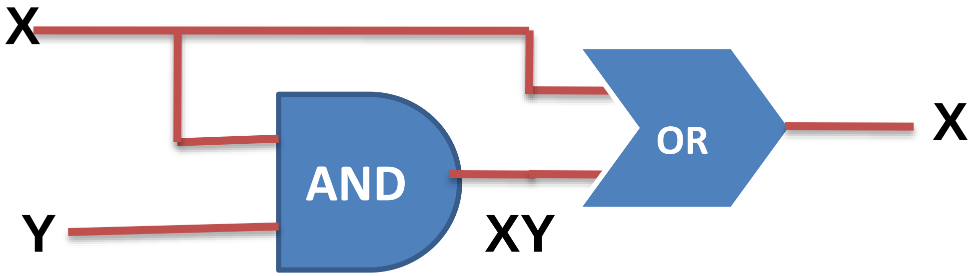
X+(YZ) = (X+Y) (X+Z) can be obtain as above

Absorption Law

$X(X+Y) = X$



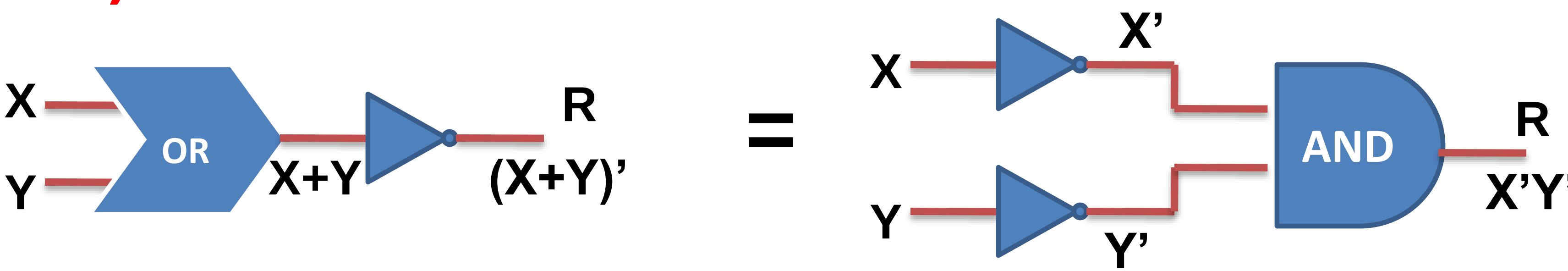
$X + XY = X$



De Morgan's Theorems

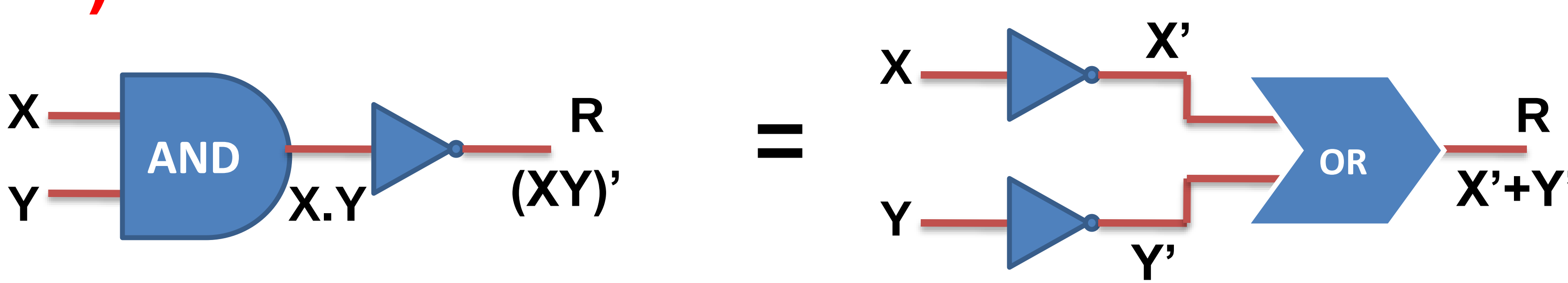
$(X+Y)' = X'Y'$

1st Theorem



$(X.Y)' = X'+Y'$

2nd Theorem



Prove of 1st De Morgan's Theorem

$(X + Y)' = X' Y'$

Suppose : X'Y' = 1 then (X + Y)' = 1 so (X+Y) = 0

So (X + Y) + X'Y' = 1 [0 + 1 = 1]

= ((X + Y) + X') . ((X + Y) + Y') [X+YZ=(X+Y)(X+Z)]

= (X + X' + Y) . (X + Y + Y')

= (1 + Y) . (X + 1)

= 1 . 1

= 1 **PROVED**

Distributive Law

Suppose : X'Y' = 0 then (X + Y)' = 0 so (X+Y) = 1

So (X + Y) . X'Y' = 0 [0 . 1 = 0]

= X' Y' . (X + Y) [X (YZ)=(XY).Z)]

= (X' + Y' + X) . (X' + Y' + Y) [X+YZ=(X+Y)(X+Z)]

= (0 . Y') + (X'. 0)

= 0 + 0

= 0 **PROVED**

Associative Law

Distributive Law

Minterms

It is a product of all the literals (with or without bar) within the logic system.

Comple mentry Law

Find the minterms of X + Y.

Sol. $X + Y = X.1 + Y.1$

$= X.(Y+Y') + Y.(X+X')$ $\quad X+X'=1$

$= XY + XY' + YX + YX'$

$= XY + XY + XY' + X'Y$

$= \mathbf{XY + XY' + X'Y}$ $\quad X + X = X$

Find the minterms of AB + C

Insert X to make equal to variables

Sol. Write the terms	AB + C
Insert X's where letter is missing	ABX + XXC
Write all the combinations of 1st term i.e. ABX :	ABC, ABC'
Write all the combinations of 2nd term i.e. XXC :	ABC, A'BC, A'B'C, AB'C

Add all the terms

AB + C = ABC + ABC' + ABC + A'BC+ A'B'C + AB'C

Remove the duplicates

AB + C = ABC + ABC' + A'BC + A'B'C + AB'C

Minterm Designation Shorthand

Find the minterms designation of XYZ'

Sol. Write the terms X Y Z'

Substitute 1 and 0 1 1 0

Find the Decimal equivalent $1 \times 2^2 + 1 \times 2^1 + 0 \times 2^0$

$= 4 + 2 + 0 = 6$

Express decimal subscript of m $= m_6$

Thus **X Y Z' = m_6**

Find the minterms designation of AB'CD'

Sol. Write the terms A B' C D'

Substitute 1 and 0 1 0 1 0

Find the Decimal equivalent $1 \times 2^3 + 0 \times 2^2 + 1 \times 2^1 + 0 \times 2^0$

$= 8 + 0 + 2 + 0 = 10$

Express decimal subscript of m $= m_{10}$

Thus **AB'CD' = m_{10}**

Maxterm

This is the sum of all literals (with or without bar) within the logic system.

It is the opposite of Minterm, here Bar represent to 1 and non-bar to 0 means X =0 X' = 1

Find the maxterms designation of AB'CD'

Sol. Write the terms		A B' C D'
Substitute 1 and 0		0 1 0 1
Find the Decimal equivalent		$0 \times 2^3 + 1 \times 2^2 + 0 \times 2^1 + 1 \times 2^0$
	=	$0 + 4 + 0 + 1 = 5$
Express decimal subscript of m	=	m_5
Thus	**AB'CD' = m_5**	

Canonical Expression

Boolean expression composed of either **Minterm** or **Maxterms** called **Canonical Expression.**
It can be represented in two forms :
(i) Sum-of-Product **(ii) Product-of-Sum**

Sum-of-Product

SOP of Two variables X and Y and output is Z :

X	Y	Z	Product
1	1	**1**	XY
1	0	**1**	XY'
0	1	**0**	X'Y
0	0	**1**	X'Y'

Now add all the product term having output Z=1 : **XY + XY' + X'Y' = Z**
So this is purely sum of minterm called **Canonical Sum-of-Product**

Sum-of-Product

SOP of Three variables X, Y and Z output is F :

Output [F] will be 1 if the inputs of 1 will be odd instead of all 1s

X	Y	Z	F	Product
1	1	1	**1**	XYZ
1	1	0	**0**	XYZ’
1	0	1	**0**	XY’Z
1	0	0	**1**	XY’Z’
0	1	1	**0**	X’YZ
0	1	0	**1**	X’YZ’
0	0	1	**1**	X’Y’Z
0	0	0	**0**	X’Y’Z’

Now add all the product term having output Z=1 : **XYZ+ XY’Z’ + X’YZ’ + X’Y’Z = F**
So this is purely sum of minterm called **Canonical Sum-of-Product**

- **Convert ((X’Y) + X’Y’))’ into canonical SOP.**

Sol. : ((X’Y) + X’Y’))’ = (X’Y)’ . (X’Y’)’ [(A+B)’ = A’ . B’]

= (X + Y’) (X + Z) [(AB)’ = A’ + B’]

= X + (Y’Z)

= X(Y+Y’) (Z+Z’) + (X+X’)Y’Z

= (XY+XY’) (Z+Z’) + XY’Z + X’Y’Z

= Z(XY+XY’) + Z’ (XY+XY’) + XY’Z + X’Y’Z

= ZXY+ZXY’ + Z’XY+Z’XY’ + XY’Z + X’Y’Z

= XYZ + XY’Z + XYZ’ + XY’Z’ + XY’Z + X’Y’Z

= **XYZ + XY’Z + XYZ’ + XY’Z’ + X’Y’Z**

De morgn’s

Fill variable with missing variables with or without bar

- **Convert F=∑(0,1,2,5) into Canonical SOP suing Short Hand.**

Sol : $F = m_0 + m_1 + m_2 + m_5$

m_0 = 000 » X’Y’Z’

m_1 = 001 » X’Y’Z

m_2 = 010 » X’YZ’

m_5 = 101 » XY’Z

So Canonical form of expression is **X’Y’Z’ + X’Y’Z + X’YZ’ + XY’Z**

Product-of-Sum

Output [F] will be 1 if the inputs of 1 will be odd.

POS of Three variables X, Y and Z   output is F :

X	Y	Z	F	Product
1	1	1	**1**	X’+Y’+Z’
1	1	0	**0**	X’+Y’+Z
1	0	1	**0**	X’+Y+Z’
1	0	0	**1**	X’+Y+Z
0	1	1	**0**	X+Y’+Z’
0	1	0	**1**	X+Y’+Z
0	0	1	**1**	X+Y+Z’
0	0	0	**0**	X+Y+Z

Now add all the product term having output Z=1   :

(X’+Y’+Z) . (X’+Y+Z’) + (X+Y’+Z’).(X+Y+Z) = F

So this is purely sum of minterm called **Canonical Sum-of-Product**

•Convert F=∏(0,1,2,5) into Canonical POS(Maxterm) using Short Hand.

Sol : $F = m_0 \ . \ m_1 \ . \ m_2 \ . \ m_5$

$m_0 = 000$ » $X+Y+Z$

$m_1 = 001$ » $X+Y+Z'$

$m_2 = 010$ » $X+Y'+Z$

$m_5 = 101$ » $X'+Y+Z'$

So Canonical form of expression is **(X+Y+Z) . (X+Y+Z') . (X+Y'+Z) .(X'+Y+Z')**

Note : Above is Canonical where 1s and 0s is fixed at odd places but it can be asked only SOP or POS where the function may be any no. of 1s.

eg. **POS : F= ∏(1,2,3,4,6)** **or**

SOP : F=∑(1,2,3,4,6) etc.

Qus. Simplify $AB'CD' + AB'CD + ABCD' + ABCD$

$= AB'C(D'+D) + ABC(D'+D)$

$= AB'C.1 + ABC.1$ [D'+D=1]

$= AB'C + ABC$ [X.1=X]

$= AC(B'+B)$

$= AC.1$

$= \mathbf{AC}$

De Morgan's

Qus. Simplify $(XY)' + X' + XY$

$= X' + Y' + X' + XY$ [(XY)' = X' + Y']

$= X' + Y' + XY$ [eliminate common]

$= X' + (Y' + XY'')$ [Y'' = Y]

$= X' + Y' + X$

$= (X' + X) + Y'$

$= 1 + Y'$ [1 + Y' = 1]

$\mathbf{= 1}$

Karnaugh Map [K-Map]

K-Map is the graphical representation of the fundamental products in a truth table. Where each squire represents the Minterm or Maxterm.

K-Map of 02 variables X & Y

X \ Y	Y'	Y
X'	X' Y'	X' Y
X	X Y'	X Y

X \ Y	0	1
0	0	1
1	2	3

K-Map of 03 Variable   X, Y & Z

X \ YZ	00	01	11	10
0	0	1	2	3
1	4	5	6	7

X \ YZ	Y’Z’	Y’Z	YZ	YZ’
X’	X’ Y’ Z’ 0	X’ Y’ Z 1	X’ Y Z 2	X’ Y Z’ 3
X	X Y’ Z’ 4	X Y’ Z 5	X Y Z 6	X Y Z’ 7

K-Map of 04 Variable   W, X, Y & Z

WX \ YZ	00	01	11	10
00	0	1	3	2
01	4	5	7	6
11	12	13	15	14
10	8	9	11	10

K-Map of 04 Variable W, X, Y & Z

WX \ YZ	Y’Z’	Y’Z	YZ	YZ’
W’X’	W’ X’ Y’ Z’ **0**	W’ X’ Y’ Z **1**	W’ X’ Y Z **3**	W’ X’ Y Z’ **2**
W’X	W’ X Y’ Z’ **4**	W’ X Y’ Z **5**	W’ X Y Z **7**	W’ X Y Z’ **6**
WX	W X Y’ Z’ **12**	W X Y’ Z **13**	W X Y Z **15**	W X Y Z’ **14**
WX’	W X’ Y Z’ **8**	W X’ Y Z’ **9**	W X’ Y Z **11**	W X’ Y Z’ **10**

Reducing functions Through K-Map

Taking example of 4 variables

- SOP [SOP $\sum$]
- POS [POS $\prod$]

Example : Reduce F(W,X,Y,Z) = ∑(0,2,7,8,10,15)

WX \ YZ	00	01	11	10
00	1 0	1	3	1 2
01	4	5	1 7	6
11	12	13	1 15	14
10	1 8	9	11	1 10

Pair : $m_7 + m_{15}$ = X Y Z

Quad : $m_0 + m_2 + m_8 + m_{10}$m = X'Z'

so **X Y Z + X' Y'**

Now solve same func. With algebraic method Reduce

$F(W,X,Y,Z) = \sum(0,2,7,8,10,15)$

m0	=	0000	=	$W'X'Y'Z'$
m2	=	0010	=	$W'X'YZ'$
m7	=	0111	=	$W'XYZ$
m8	=	1000	=	$WX'Y'Z'$
m10	=	1010	=	$WX'YZ'$
m15	=	1111	=	$WXYZ$

$F= \mathbf{W'X'Y'Z'} + W'X'YZ' + \mathbf{W'XYZ} + \mathbf{WX'Y'Z'} + WX'YZ' + \mathbf{WXYZ}$

$= \mathbf{(WXYZ+W'XYZ)} + \mathbf{(W'X'Y'Z'+WX'Y'Z')} + (W'X'YZ' + WX'YZ')$

$= XYZ(W+W') + X'Y'Z'(W'+W) + X'YZ'(W' + W)$

$= XYZ + X'Y'Z' + X'YZ'$

$= XYZ + X'Z'(Y' + Y)$

$= \mathbf{XYZ + X'Y'}$ **solved**

Example : Reduce $F(W,X,Y,Z) = \prod(0,2,7,8,10,15)$

W+X \ Y+Z	00	01	11	10
00	0 0	1	3	0 2
01	4	5	0 7	6
11	12	13	0 15	14
10	0 8	9	11	0 10

Pair : $m_7 . m_{15}$ = X +Y+ Z

Quad : $m_0 . m_2 . m_8 . m_{10}$ = X'+Z' so (X+ Y+ Z) . (X' + Y')