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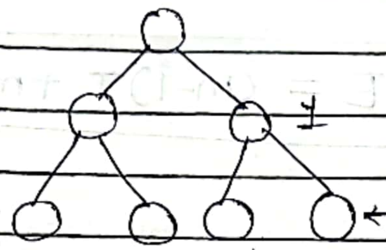
JHB

- **Path length :**

Path length in a Rooted tree :
Path length of a vertex is no. of edges in the path from root to vertex.

eg



- The Height of the tree is maximum of path lengths in a tree.

- 'i' is no. of internal nodes in the tree.
't' is no. of terminal nodes in the tree, then for m-ary tree

$$\boxed{(m-1)\,i = t-1}$$

Que 19 lamps & single electricity outlets. each chord has 4 outlets. How many chords will be required ?

→ $m = 4$
$t = 19.$

$\because (m-1)\,i = t-1$
$\therefore 3i = 18$
$\therefore \boxed{i = 6}$

$\therefore$ 6 chords will be required.

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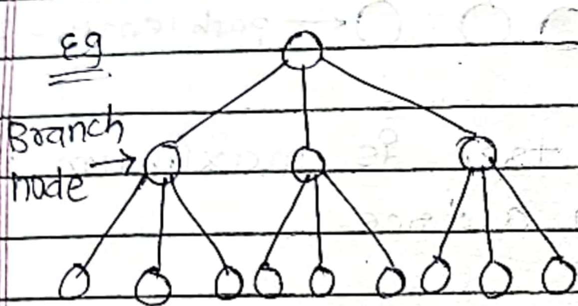
Result: i = no. of internal nodes
I = sum of path lengths of all branch nodes
E = sum of path lengths of leaves

~~E = I +~~

$$\boxed{E = (m-1)I + mi}$$

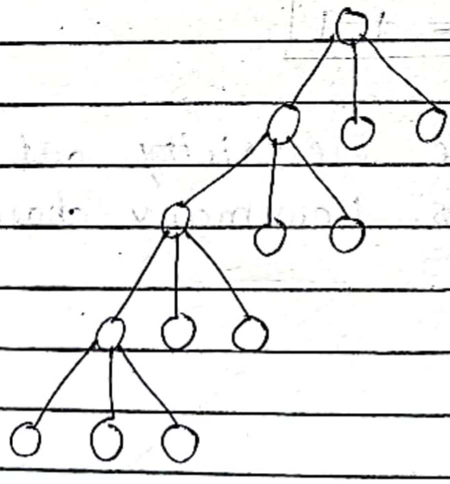
Eg



$I = 3$
$i = 4$

So, $(3-1)3 + 3(4) = \boxed{18}$

Eg.

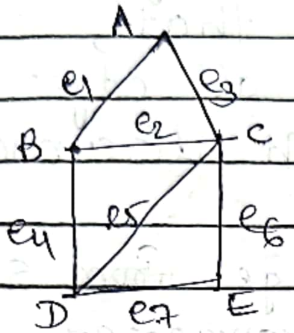


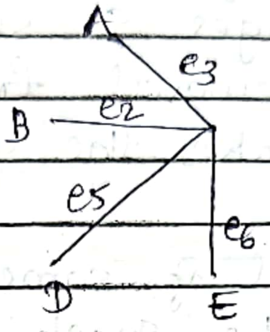
$I = 6$
$i = 4$

So, $(3-1)6 + 3(4)$
$= \boxed{24}$
$= E$
= sum of path lengths of leaves

Spanning Tree

A spanning tree of a connected graph is a spanning subgraph which is a tree.
↳ it should contain all vertices.





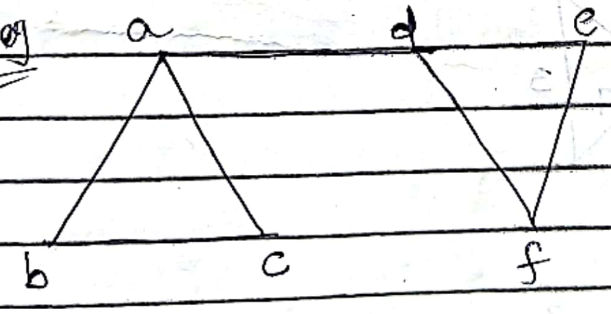
e3, e2, e5, e6
— Branch

- An edge of the tree which is included in the spanning tree is called branch

- An edge of the graph which is not included in the spanning tree is called Chord
  eg e1, e4, e7

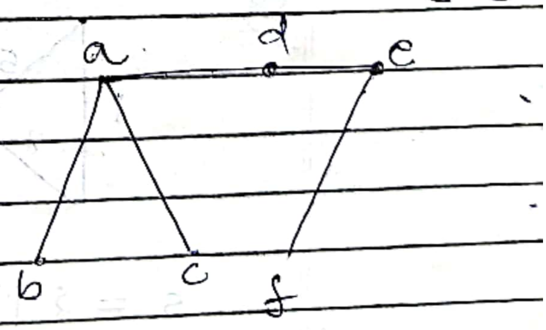
- A connected graph with e edges and v vertices is converted into a spanning tree if ~~there is a graph~~, then no. of branches will be (V−1)

eg



graph

e=7
V=6



spanning tree

e=5
V=6

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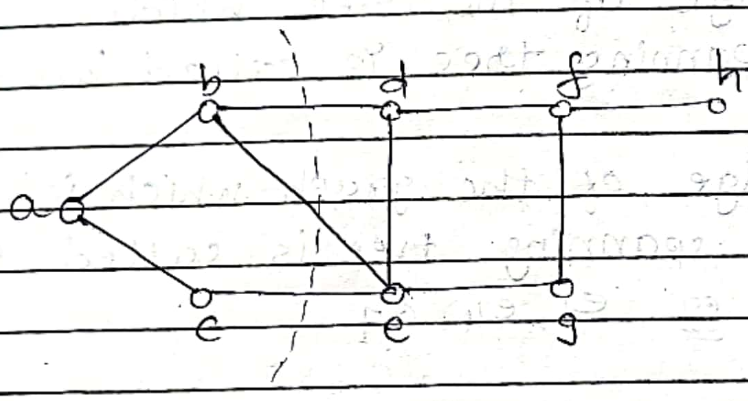
No of chord will be $\boxed{e-v+1}$

- ## Cut Set

A cut set of the connected graph G is set of edges S such that,
i) the removal from all edges from S will make the graph disconnected.

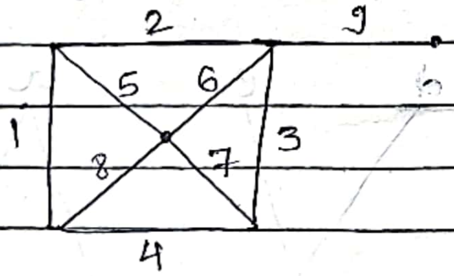
ii) Removal of some edges from S will not make the graph disconnected.

eg



S = {bd, be, ce}
S = {df, eg}

eg



S = {9}
S = {1, 4, 8}
S = {3, 4, 7}
S = {1, 2, 5}
S = {5, 6, 7, 8}

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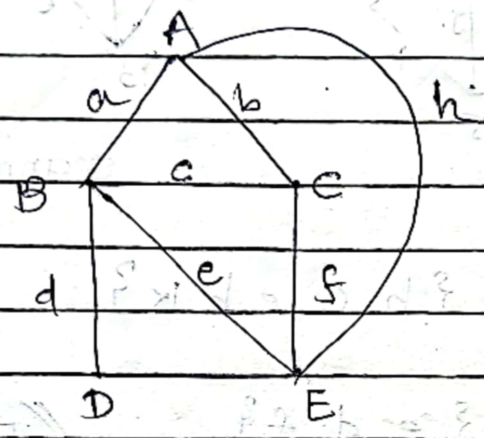
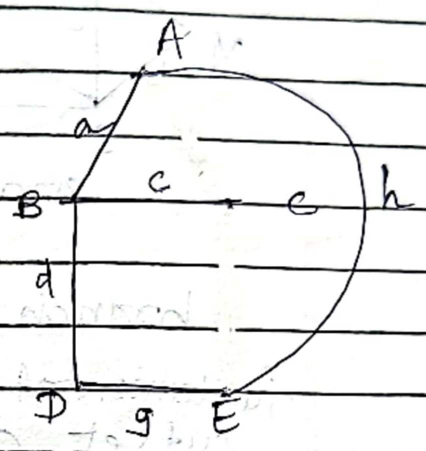
Fundamental Circuit

let T be a spanning tree of connected graph G.
When a chord is added to the spanning tree it will form exactly one circuit. This circuit is called Fundamental circuit.

eg.





chords = {b, e, f, h}

let h chord → add it to the spanning tree. it will form cycle

(1) A - E - D - B - A ← Fundamental circuit

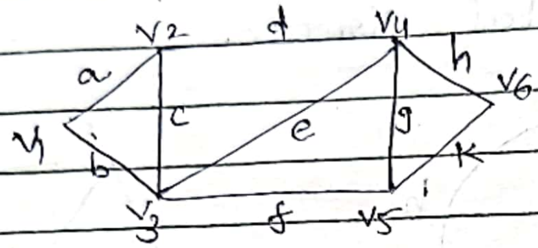
Similarly, add all chords one by one.

→ No. of fundamental circuit = No. of chords.

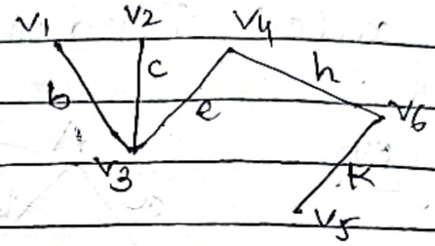
(2) A - B - C - A (b)
(3) B - D - E - B (e)
(4) C - E - D - B - C (f)

Fundamental Cutset

Let T be a spanning tree then fundamental cutset is a cutset formed by exactly one branch



graph.



spanning tree

branch b = {b, c, e, h, K}

~~fundamental~~
cutset C_1 = {e, d, f} // e
C_2 = {d, a, b} // b
C_3 = {a, c, d} // c
C_4 = {h, g, f} // h
C_5 = {K, g, f} // K

- **Result:**
A circuit and complement of any spanning tree must have at least 1 edge in comman

- **Result:**
A cutset and any spanning tree must have at least 1 edge in comman.

- **Result:**
Every circuit has even no. of edges in comman with every cutset.

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Ex. A tree has 2n vertices of degree 1, 3n vertices of degree 2, n vertices of degree 3. Find the no. of vertices & edges in the tree.

→ W.K.T. $\sum_{i=1}^{n} \deg(v_i) = 2e$.

$\Rightarrow (2n)(1) + (3n)(2) + (3)(n) = 2(v-1)$

$\Rightarrow 11n = 2v - 2$

$\Rightarrow 11n = 2(2n+3n+n) - 2$

$\Rightarrow 11n = 12n - 2$

$\Rightarrow \boxed{n=2}$

∴ No of vertices $= 6n = 6(2) = \boxed{12}$

No of edges $= v-1 = \boxed{11}$

Ex. A tree has 50 edges. The removal of certain edge from T creates two disjoint trees T1 and T2. No. of vertices in T1 is equal to no. of edges in T2. Find no. of vertices ~~and edges~~ in T1 and T2.

→ $E = 50$

no. of vertices in T1 + no. of vertices in T2 = 51

$\boxed{n_1 + n_2 = 51}$ — ①

Ans: 25
26

Now, $n_1 = e_2$

$\boxed{n_1 = n_2 - 1}$ — ②

from (1) & (2) $\boxed{n_1 = 25}$ and $\boxed{n_2 = 26}$

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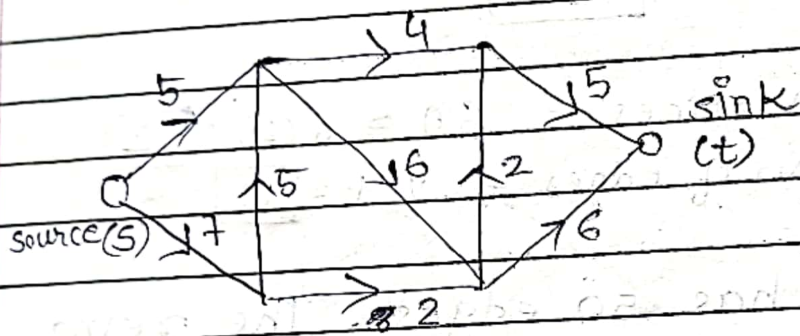
Transport Network

- Transport Network is a weighted directed graph if
  i) It is connected and having no loops.
  ii) There is only 1 vertex which has no incoming edge.
  iii) There is only 1 vertex that has no outgoing edge.
  iv) weight of each edge is a non-negative real number.

ex



- The weight of each edge in the transport network is called capacity of the edge.

- Flow (ϕ) :
  Flow in a transport network is assignment of non-negative number to each edge such that
  i) For each edge, flow $\leq$ capacity.
  ii) For each vertex except source and sink the incoming flow should be equal to outgoing flow.
  iii) total outgoing flow at source should be equal to total incoming flow at sink.

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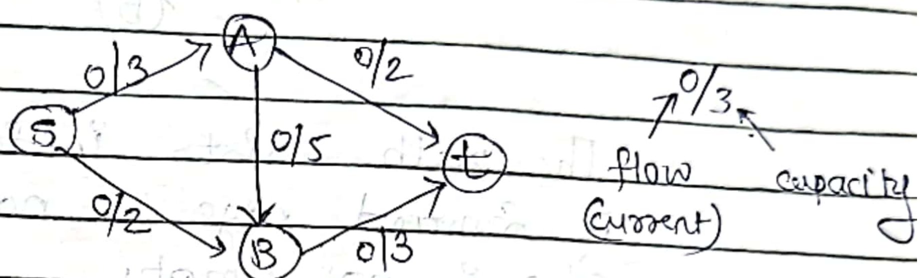
→ Maximum flow in Transport Network

A max. flow is the flow that achieves largest possible value.

⇒ Ford - Fulkerson's Algorithm

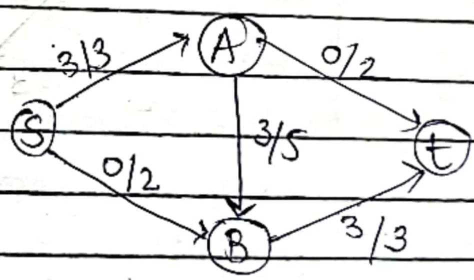
- Used to find max. flow in given transport Network.



i) S-A-T path ⇒ only we can sent flow of 2.

Bottle-neck - max. capacity we can pass through any path.

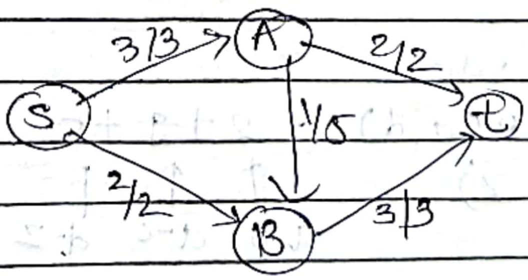
ii) S-A-B-T ⇒ 3



$\boxed{\text{max. flow} = 3}$

But this is not correct

$\boxed{\text{max. flow} = 5}$



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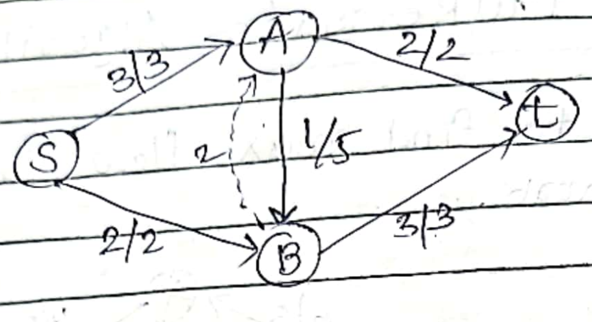
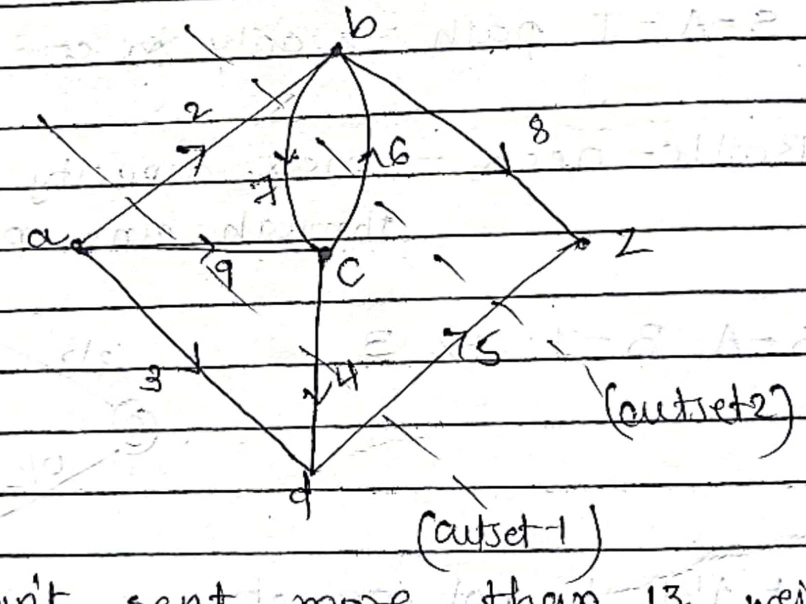
In the ford-fulkerson's Algo. the reverse edges may be considered if they are not empty and if they lead to the max-flow



The path exists in ford-fulkerson Algo if forward edge is not full and backward edge is not empty

eg.



we can't sent more than 13 weight
Here 14 is sent

$W(P, \bar{P})$ where

i) cutset $P(a,d)$ = 2+9+5 = 16 // not possible
$\bar{P}(b,c,z)$

a-b d-c d-z

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ii) $P(a,c,d)$ $\overline{P}(b,z)$ $= 2+6+5 = 13$.

$\uparrow$ a-b $\uparrow$ c-b $\uparrow$ d-z

13 amount of good canbe transfered constantly

- <u>Group Theory</u>

i) Algebraic structure :

A non empty set S is called an algebraic structure with respect to any binary operation (*) if it follows following axioms.

- closure : $(a*b) \in S$, $\forall$ $a,b \in S$.

eg $S = \{-1, 1\}$

$<S,*>$

* is multiplication operation.

$-1 \times 1 = -1 \in S$

$-1 * -1 = 1 \in S$

$1 * -1 = -1 \in S$

$1 * 1 = 1 \in S$

so, $<S,*>$ is an algebraic structure.

Eg.

N+ : ✓		R− : ✓	
N− : X		R* : ✓	
N* : ✓		R/ : X	
N/ : X	Even	E+ : ✓	
Z+ : ✓		E* : ✓	
Z− : ✓	Odd	O+ : X	
Z* : ✓		O* : ✓	
Z/ : X	Matrix	M+ : ✓	
R+ : ✓		M* : ✓	

Q, R

$\frac{1}{2}$, 0.5

in numerator and denominator only int is allowed.

$1.2 \in R$, $0 \in R$

$\frac{1.2}{0} \notin R$

reason:

$2 \in Z$

$5 \in Z$

$\frac{2}{5} \notin Z$

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2) Semi Group:

A non empty set S is called a semigroup w.r.t. binary operation (*) if it follows following axioms:

- closure : $(a*b) \in S$, $\forall a,b \in S$
- Associativity : $a*(b*c) = (a*b)*c$ $\forall a,b,c \in S$

Eg

	A	S		A	S
N+ :	✓	✓	E+ :	✓	✓
N* :	✓	✓	E* :	✓	✓
Z+ :	✓	✓	O* :	✓	✓
Z− :	✓	✗	M+ :	✓	✓
Z* :	✓	✓	M* :	✓	~~✗~~ ✓
R+ :	✓	✓			
R− :	✓	✗			
R* :	✓	✓			

$M_1 * M_2 \neq M_2 * M_1$

but $M_1 * (M_2 * M_3) = (M_1 * M_2) * M_3$

3) Monoid:

A non empty set S is called a monoid with respect to binary operation (*) if it follows foll^n axioms:

- closure : $(a*b) \in S$, $\forall a,b \in S$
- Associativity : $a*(b*c) = (a*b)*c$ $\forall a,b,c \in S$
- Identity element : $\exists e \in S$, such that $a*e = e*a = a$

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eg

	A	S	M
N+ :	✓	✓	✗
N* :	✓	✓	✓
Z+ :	✓	✓	✓
Z* :	✓	✓	✓
R+ :	✓	✓	✓
R* :	✓	✓	✓

	A	S	M	
E+ :	✓	✓	✓	
E* :	✓	✓	✗	$1 \notin E$
O* :	✓	✓	✓	
M+ :	✓	✓	✓	
M* :	✓	✓	✓	

for N+ : 0 is the identity ele. but $0 \notin N$.

for M+ : $M + M_z = M$; M_z is zero-matrix
M_z is an identity element
M* : $M * I = M$; I is the identity matrix

4) Group :

A non empty set S is called a Group with respect to binary operation (*) if it follows foll^n axioms.

- closure : $(a*b) \in S, \ \forall a, b \in S$
- Associativity : $a*(b*c) = (a*b)*c \quad \forall a, b, c \in S$
- Identity element : $\exists e \in S$, such that $a*e = e*a = a$
- Inverse : $\forall a \in G$ (Group), $\exists a^{-1} \in G$ such that $a * a^{-1} = e$

eg

	A	S	M	G
N* :	✓	✓	✓	✗
Z+ :	✓	✓	✓	✓
Z* :	✓	✓	✓	✗
R+ :	✓	✓	✓	✓
R* :	✓	✓	✓	✗

	A	S	M	G
E+ :	✓	✓	✓	✓
O* :	✓	✓	✓	✗
M+ :	✓	✓	✓	✓
M* :	✓	✓	✓	✗

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$N+$: $3 * \frac{1}{3} = 1$ but $\frac{1}{3} \notin N$

$Z+$: $3 + (-3) = 0$
$\forall x \in Z$, $\exists (-x) \in Z$
$-x$ is the inverse of x.

$Z*$: $\forall x \in Z$, $\exists \frac{1}{x} \notin Z$ such that
$x * \frac{1}{x} = e$.

$R*$: $\forall x \in R$, $\exists \frac{1}{x} \notin R$ such that
$x * \frac{1}{x} = e$ [$\because x$ being 0]

$O*$: $x = 3$, $\frac{1}{3} \notin O$ ~~odd~~ fractional no. is not consider as odd

$M*$: For Non-singular matrix inverse is not exist