Matrix: A rectangular array of numbers like $\begin{bmatrix} 1402 & 191 \\ 1371 & 821 \\ 2321 & 7912 \\ 5641 & 9089 \end{bmatrix}$
(4 x 2)

A **Vector** is a matrix with one row and many columns:

$$\begin{bmatrix} 1 & 9 \end{bmatrix}$$

Some important points to know:

- Matrices are denoted by uppercase names while vectors are lowercase.
- $\mathbb{R}$ refers to the set of scalar real numbers.
- $\mathbb{R}^n$ refers to the set of n-dimensional vectors of real numbers.

Addition and Scalar Multiplication

(3 x 2) (3 x 2). (3 x 2)

$$\begin{bmatrix} 1 & 0 \\ 2 & 5 \\ 3 & 1 \end{bmatrix} + \begin{bmatrix} 4 & 0.5 \\ 2 & 5 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 5 & 0.5 \\ 4 & 10 \\ 3 & 2 \end{bmatrix}$$

You can only add matrices of the same dimension.

$3 * \begin{bmatrix} 1 & 0 \\ 2 & 5 \\ 3 & 1 \end{bmatrix} = \begin{bmatrix} 3 & 0 \\ 6 & 15 \\ 9 & 3 \end{bmatrix}$ - This operation is commutative and works if the scalar is a fraction.

Combinations of operands:

$$3 * \begin{bmatrix} 1 \\ 4 \\ 2 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 5 \end{bmatrix} - \begin{bmatrix} 3 \\ 0 \\ 2 \end{bmatrix} * \frac{1}{3} = \begin{bmatrix} 4 \\ 12 \\ 10.33 \end{bmatrix}$$

Matrix-Vector Multiplication

If **A** is a **m x n** matrix and **v** is a **n x 1** vector, then the resulting vector will be **m x 1.**

$$\begin{bmatrix} A_{11} & \cdots & A_{1n} \\ \vdots & \ddots & \vdots \\ A_{m1} & \cdots & A_{mn} \end{bmatrix} * \begin{bmatrix} v_1 \\ \vdots \\ v_n \end{bmatrix} = \begin{bmatrix} \sum_{i=1}^{n} A_{1n} * v_n \\ \sum_{i=1}^{n} A_{2n} * v_n \\ \vdots \\ \sum_{i=1}^{n} A_{mn} * v_n \end{bmatrix}$$

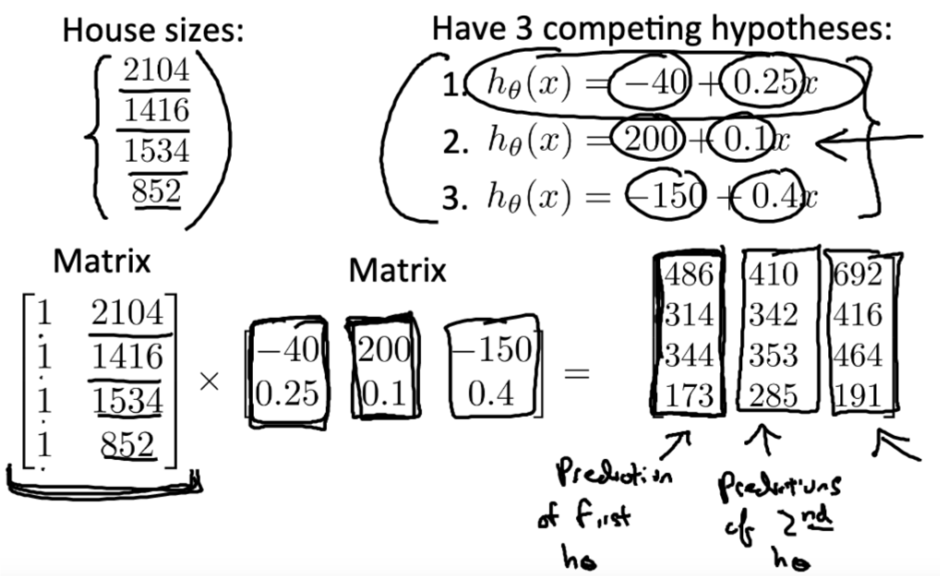
Matrix-Vector multiplication must be conformable. The number of **columns** of the matrix must equal the number of **rows** of the vector.

A great, and efficient, application of Matrix-Vector multiplication is when you want to use a hypothesis function, a linear regression equation, to make predictions. For instance, you can multiply a matrix of house sizes with a vector of the hypothesis function parameters to get a vector of predicted prices.

$$\begin{bmatrix} 1 & 2104 \\ 1 & 1416 \\ 1 & 1534 \\ 1 & 852 \end{bmatrix} * \begin{bmatrix} -40 \\ 0.25 \end{bmatrix} = \begin{bmatrix} 486 \\ 314 \\ 343.5 \\ 173 \end{bmatrix}$$

<u>Matrix-Matrix Multiplication</u>

Using the same example about predicting house prices using house size, we can use matrix multiplication to make predictions resulting from three hypothesis functions as shown below. Using just one matrix-matrix multiplication, we are able to get 12 different predictions.



In general, to multiply two matrices, the number of **columns** of the first matrix must equal the number of **rows** of the second matrix.

$$If\ A = \begin{bmatrix} A_{11} & \cdots & A_{1n} \\ \vdots & \ddots & \vdots \\ A_{m1} & \cdots & A_{mn} \end{bmatrix}, and\ B = \begin{bmatrix} B_{11} & \cdots & B_{1p} \\ \vdots & \ddots & \vdots \\ B_{n1} & \cdots & B_{np} \end{bmatrix},$$

$$then\ C\ is\ defined\ by\ an\ m * p\ matrix. C = \begin{bmatrix} C_{11} & \cdots & C_{1p} \\ \vdots & \ddots & \vdots \\ A_{m1} & \cdots & C_{mp} \end{bmatrix}$$

Properties of Matrix Multiplication

1. Matrix multiplication is not commutative. A x B does not equal B x A even if they are conformable.
2. Matrix multiplication is associative. (A x B) x C = A x (B x C).
3. Commutativity does hold when A x I = I x A = A, where I is a square matrix.

Inverses

If A is a non-singular matrix, then it is a square matrix and its determinant is non-zero. Then and only then, A will have an inverse A^{-1} such that $A * A^{-1} = A^{-1} * A = I$. This implies that not all matrices have an inverse.

Transposes

If A is an $m * n$ matrix and $B = A^T$, than B is an $n * m$ matrix, and $B_{yz} = A_{zy}$