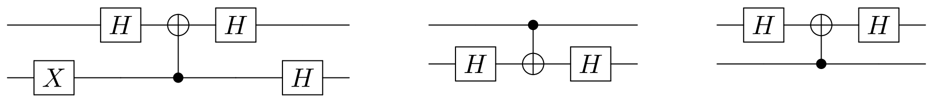
1 Exercise 2.1

Which of the following two states can be written as a product state? Give proofs of your answers.

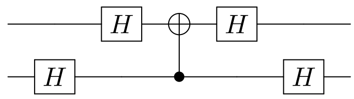
$$|\psi_1\rangle = \frac{36}{65}|00\rangle + \frac{48}{65}|01\rangle - \frac{3}{13}|10\rangle + \frac{4}{13}|11\rangle,$$
$$|\psi_2\rangle = \frac{36}{65}|00\rangle + \frac{48i}{65}|01\rangle - \frac{3}{13}|10\rangle + \frac{4i}{13}|11\rangle$$

2 Exercise 2.2

(a) Compute the unitary matrix corresponding to each of the following quantum circuits:

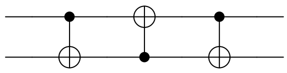


(b) Give an equivalent quantum circuit to the following circuit, using fewer gates, where the gates can be either H or CX.



3 Exercise 2.3

(a) Alicia's quantum computer is "working too well" and instead of computing one CX gate, it computes three CX gates at once as shown in the following circuit:



Describe the action of this circuit on an input $|xy\rangle$, where x, y $\in \{0, 1\}$. For the input $|x0\rangle$ (i.e., y $= 0$), show that the circuit can be reduced to just two gates, where the gates are either CX or elementary single-qubit gates.

(b) Berthold's quantum computer is "stuck in forward" mode: it can apply a CX gate from qubit 1 to qubit 2, but not vice versa. On the other hand, single-qubit operations work fine and he is able to apply any such operation. How can Berthold still apply a CX gate from qubit 2 to qubit 1?

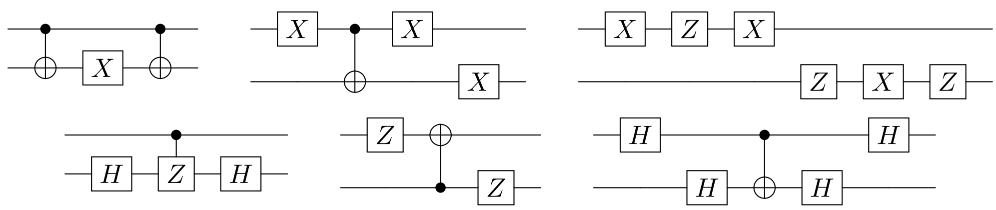
(c) Cecelia claims that she invented a cloning machine that maps

$$|\psi\rangle|0\rangle|g\rangle \mapsto |\psi\rangle|\psi\rangle|g'\rangle$$

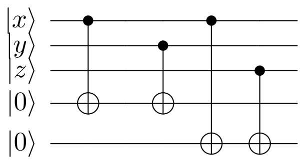
for all states $|\psi\rangle$, where $|g\rangle$ is an input state that is independent of $|\psi\rangle$, and $|g\prime\rangle$ is an arbitrary state. Do you believe her? Give a proof of your answer.

4 Exercise 2.4

(a) Simplify the following 6 quantum circuits by rewriting each one of them using at most 2 elementary quantum gates (i.e., CX or single-qubit gates):



(b) Determine the output state of the following circuit, provided that the input is given by $|xyz00\rangle$, with $x, y, z \in \{0, 1\}$:



5 Exercise 2.5

Let P be the operation which is defined by

$$P|x\rangle = \big|(x+1) \bmod 8\big\rangle, x \in \{0, 1, 2, 3, 4, 5, 6, 7\}.$$

(a) Show that P is a unitary matrix and find its inverse.

(b) Find the matrices P^2, P^3, P^4, P^5, P^6, P^7, P^8.

(c) Draw a quantum circuit that implements P using only the gates *CCX, CX, X.*