5 Rotational Motion

Centre of mass:

"A point at which all the mass of the system can be considered as concentrated."

- Centre of mass is defined for study of extended objects as a particle.
- The centre of mass should be either inside or outside the body.
- It shows the average position of the mass of the component of the object.
- In symmetrical bodies the centre of mass coincides with the geometrical centre, while the centre of mass of irregular shape bodies is towards the heavy mass distribution.
- For two particles system

  $M\vec{r}_{cm} = m_1\vec{r}_1 + m_2\vec{r}_2$

  where $M = m_1 + m_2$
- In component form,

  $M x_{cm} = m_1x_1 + m_2x_2$; $M y_{cm} = m_1y_1 + m_2y_2$
- If centre of mass is at origin then

  $m_1\vec{r}_1 + m_2\vec{r}_2 = 0 \qquad \therefore \vec{r}_2 = -\frac{m_1}{m_2}\vec{r}_1$
- Here the sign of $\vec{r}_1$ and $\vec{r}_2$ are opposite. It shows that the mass m_1 and m_2 are on both the sides of the centre of mass.
- $m_1 > m_2 \Rightarrow r_1 < r_2$. It means that the centre of mass towards the heavy mass.
- If two particles having same mass then,

  $$\vec{r}_{cm} = \frac{\vec{r}_1 + \vec{r}_2}{2}$$

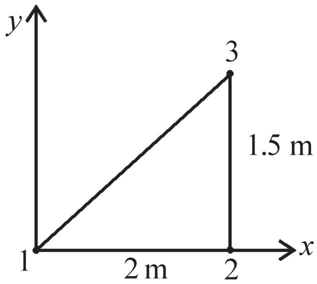
  $$\therefore x_{cm} = \frac{x_1 + x_2}{2} ; \qquad y_{cm} = \frac{y_1 + y_2}{2}$$
- For n- particles system,

  $$M\vec{r}_{cm} = \sum_{i=1}^{n} m_i\vec{r}_i$$

  $$Mx_{cm} = \sum_{i=1}^{n} m_i x_i ;\ My_{cm} = \sum_{i=1}^{n} m_i y_i ;\ Mz_{cm} = \sum_{i=1}^{n} m_i z_i$$
- For a rigid body,

  $$M\vec{r}_{cm} = \int \vec{r}\, dm ; Mx_{cm} = \int x\, dm ;\ My_{cm} = \int y\, dm ;\ Mz_{cm} = \int z\, dm$$

(1) Three particles each of mass 3 kg are placed at three corners of an equilateral triangle as shown in figure. The centre of mass with respect to particle 1 is ...... m.



(A) (0.5, 1.33) m

(B) (1.33, 0.5) m

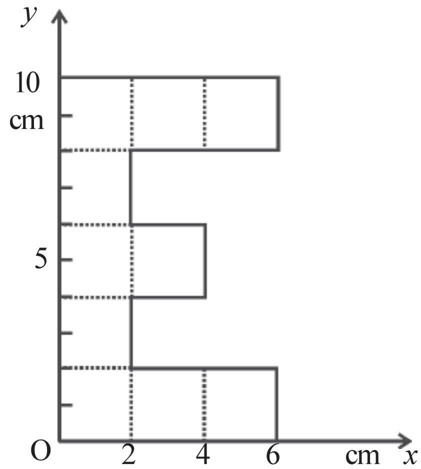
(C) (12, 4.5) m

(D) (4.5, 12) m

(2) Two particles of mass 50 g and 100 g have positions $3\hat{i} + 4\hat{j} + 5\hat{k}$ cm and $-6\hat{i} - 2\hat{k} + 4\hat{j}$ cm with respect to origin. The distance of the centre of mass from the origin is ....

(A) 15 cm (B) $\sqrt{10}$ cm (C) 5 cm (D) $\sqrt{15}$ cm

(3) Find the centre of mass with respect to origin of a E shape having 2 cm thickness and uniform density distribution.



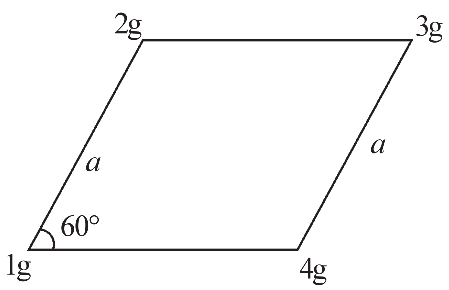
(A) (2.6, 2.4) cm

(B) (2.4, 1.6) cm

(C) (2.4, 5) cm

(D) (1.4, 2.6) cm

(4) Find the centre of mass with respect to a particle of 1g mass of four particles parallelogram shaped system. (The length of each side is a).



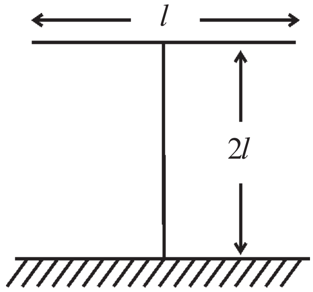
(A) $\left(\frac{\sqrt{3}a}{4}, 0.95a\right)$

(B) $\left(\frac{a}{2}, \frac{3a}{4}\right)$

(C) $\left(\frac{3a}{4}, \frac{a}{2}\right)$

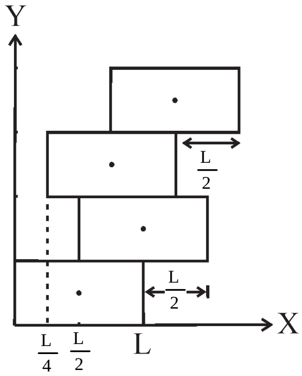
(D) $\left(0.95a, \frac{\sqrt{3}a}{4}\right)$

(5) A 'T'shaped object with dimensions shown in the figure,is lying on a smooth floor. At which point from the surface a force is applied so that the object has only the translation motion ? (Thickness of rod is negligible).



(A) l (B) $\frac{2}{3}l$

(C) $\frac{4}{3}l$ (D) any where.

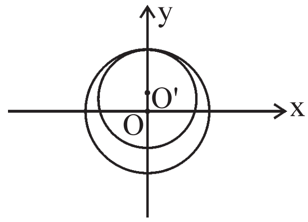
(6) 

Four bricks each of length L and mass m are arranged as shown, from the wall. The distance of the centre of mass of the system from the wall is...... .

(A) $\frac{8}{7}$ L (B) $\frac{7}{8}$ L

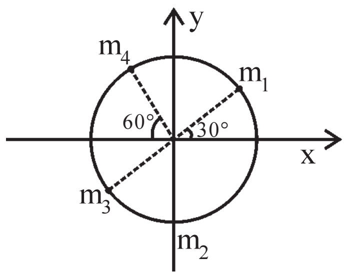
(C) $\frac{11}{12}$ L (D) $\frac{15}{16}$ L

(7) A circular plate of uniform thickness has a diameter of 60 cm. A circular portion of radius 20 cm is removed from one edge of the plate. Then the centre of mass of the remaining portion with respect to origin is ...... cm



(A) (0, –4) (B) (8, 0) (C) (0, –8) (D) (–8, 0)

(8) The particles of 10 g, 20 g, 30 g and 40 g are placed at 2, 6, 8 and 11 hour sysmbols respectively of a weightless dial of clock having radius 8 cm. Find the co - ordinates of centre of mass of this system.



(A) (1.84, –0.136) cm

(B) (1.36, –0.184) cm

(C) (–1.36, –0.184) cm

(D) (–1.49, –0.184) cm

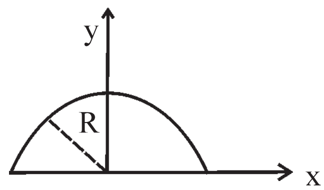
(9) Mass density of a rod of length L changes with respect to distance x from one end is according to equation $\lambda = \beta x$, where β = constant. Then the distance of centre of mass from $x = 0$ is ....

(A) $\frac{L}{2}$ (B) $\frac{L}{3}$ (C) $\frac{2L}{3}$ (D) $\frac{3L}{2}$

(10) The centre of mass of a half portion of a thin ring of mass 2M and radius R having uniform mass density with respect to its centre is ....

(A) $\frac{2R}{\pi}$ (B) $\frac{2\pi}{R}$

(C) $\frac{\pi}{R}$ (D) $\frac{R}{\pi}$



(11) The centre of mass of cone with a uniform mass density with radius R and height h with respect to its top is ....

(A) $\frac{4}{3}$h (B) $\frac{2}{3}$ h (C) $\frac{3}{4}$ h (D) $\frac{1}{3}$ h

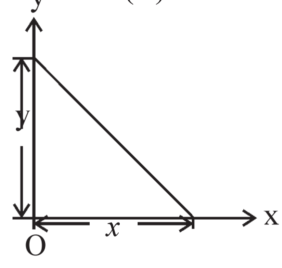
(12) Two spheres of mass M and 4M having radius R and 3R respectively. The distance between their centres is 10 R. If they are move due to gravitational force then before collision the distance travelled by the bigger sphere is ....

(A) 2 R (B) 8 R (C) 4.8 R (D) 1.2 R

(13) Locate the centre of mass of a right angle triangle of uniform mass density.

(A) $\left(\frac{x}{2}, \frac{y}{2}\right)$ (B) $\left(\frac{x}{3}, \frac{y}{3}\right)$

(C) $\left(\frac{x}{2}, \frac{y}{3}\right)$ (D) $\left(\frac{x}{3}, \frac{y}{2}\right)$



(14) The length of a rod is 2 m. Its linear mass density change by equation $\lambda = 3 + x$, then the centre of mass of the rod with respect to $x = 0$ is ....

(A) $\frac{13}{12}$ m (B) $\frac{12}{13}$ m (C) $\frac{15}{12}$ m (D) 1 m

(15) The centre of mass of a half portion of a disc of mass m and radius R having uniform mass density with respect to its centre is ....

(A) $\frac{3R}{4}$ (B) $\frac{4R}{\pi}$ (C) $\frac{4R}{5\pi}$ (D) $\frac{4R}{3\pi}$

Ans. : 1 (B), 2 (A), 3 (C), 4 (D), 5 (C), 6 (B), 7 (C), 8 (D), 9 (C), 10 (A), 11 (c), 12 (D), 13 (B), 14 (A), 15 (D)

Motion of centre of mass :

$$M\vec{v}_{cm} = \sum_{i=1}^{n} m_i \vec{v}_i$$

$$M\vec{a}_{cm} = \sum_{i=1}^{n} m_i \vec{a}_i = \vec{F}_{net} = \text{ Resultant external force.}$$

Linear momentum of the system :

$$\vec{P} = M\vec{v}_{cm}$$

$$\vec{F}_{ext} = \frac{d\vec{P}}{dt} = M\frac{d}{dt}\left(\vec{v}_{cm}\right) = M\vec{a}_{cm} = \sum_{i=1}^{n} m_i \vec{a}_i$$

$$\vec{a}_{cm} = \frac{\vec{F}_{ext}}{M} = \frac{\sum_{i=1}^{n}\vec{F}_i}{\sum_{i=1}^{n} m_i}$$

- Centre of mass behave like a partical of mass M and move due to external force applied on it.

- Conservation of linear momentum : If $\vec{F}_{ext} = 0$ then $\frac{d\vec{p}}{dt} = 0$

$\therefore \vec{P} = \text{constant}$ $\quad \therefore M\vec{a}_{cm} = 0$ $\quad \therefore \vec{a}_{cm} = 0$

$M\frac{d}{dt}(\vec{v}_{cm}) = 0$ $\quad \therefore \vec{v}_{cm} = \text{constant.}$

- The particles of system having individual change in momentum but the total momentum of system remains constant.

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(16) Two particle of mass 2 kg and 4 kg move on a linear path in opposite direction with velocity 2 ms^{-1} and 3 ms^{-1}, then the velocity of centre of mass of system is ....

(A) $\frac{8}{3}$ ms^{-1} in the direction of first object (B) $\frac{4}{3}$ ms^{-1} in the direction of first object

(C) $\frac{4}{3}$ ms^{-1} in the direction of second object (D) $\frac{8}{3}$ ms^{-1} in the direction of second object

(17) The distances of two object of masses m_1 and m_2 from the centre of mass of the system are r_1 and r_2. Due to their gravitational forces the acceleration produced in the object m_2 is ....

(A) $\frac{m_1 G}{(r_1+r_2)^2}$ (B) $\frac{m_1 G}{r_2^2}$ (C) $\frac{m_1 G}{(r_1-r_2)^2}$ (D) $\frac{m_1 G}{(r_1+r_2)^2}$

(18) A gun of mass M put on a friction less horizontal surface. A bullet of mass *m* is fired from it, the distance travel by the gun when the distance travelled by the bullet is *x* ....

(A) $\left(\frac{m+M}{m}\right)x$ (B) $\left(\frac{m}{m+M}\right)x$ (C) $\left(\frac{m}{M}\right)x$ (D) $\left(\frac{M-m}{m}\right)x$

(19) A stone of mass 90 kg tied with a mass less thread of length 10 m and on the other hand a person of mass 60 kg pull this stone. If the surface is frictionless, at which distance the stone and person can meet to eachother ?

(A) At 4m distance from the person (B) At 4m distance from the stone

(C) At 5m distance from the person (D) Can not meet

(20) A freely falling object A suddenly breaks into two fragments. The mass of first fragment is $\frac{3}{4}$ M and the other is $\frac{M}{4}$ and they are freely fall. After breaking the centre of mass of object A is ....

(A) shifted towards heavy fragment.

(B) shifted towards lighter fragment.

(C) move on original path.

(D) shifted in which direction depends on at what height the fragment are made .

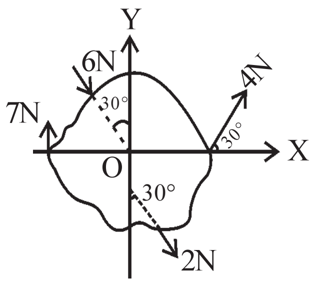
(21) When a object thrown upward it explodes in to two part at a point P of certain height . The mass of bigger piece is four times more than the smaller piece .When the position of small piece is on the right side of point P is 12 cm, the position of big piece from point P is ....

(A) 3 cm on right side (B) 3 cm on left side (C) 4 cm right side (D) 4 cm on left side

(22) The four forces are applied on a object of 2.6 kg as shown in figure. Find out the magnitude of accelaration of the centre of mass of object .

(A) 2 ms^{-2} (B) 3 ms^{-2}

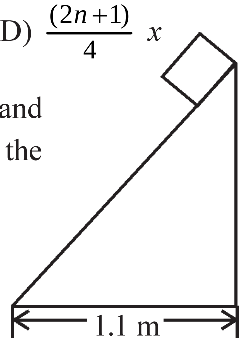
(C) 4 ms^{-2} (D) 3.5 ms^{-2}



(23) The particles of mass m, 2m, 3m.... nm are placed on a line and their position from x = 0 is x, $2x$, $3x$, ... nx, then find out the position of centre of mass from x = 0.

(A) $(2n + 1)x$ (B) $\frac{(2n+1)}{2}x$ (C) $\frac{(2n+1)}{3}x$ (D) $\frac{(2n+1)}{4}x$

(24) What is the displacement of big block (incline) when small block move and reach at the bottom of incline ? The mass of big block is 10 times more than the small block. All the surfaces are friction less.



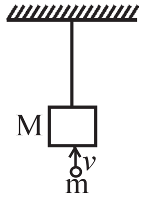
(A) 0.1 m (B) 0.2 m

(C) 0.01 m (D) 0.02 m

(25) A object of mass m moves on X-axis with velocity 5 ms^{-1} collides with a stationary object of mass double then it. Due to this the big object explodes in to two pieces of same mass and the small object is comes in to rest . If one piece move on Y-axis with velocity 3 ms^{-1} than the velocity of second piece is ....

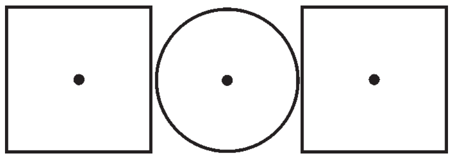
(A) $\sqrt{24}$ ms^{-1} (B) 4 ms^{-1} (C) $\sqrt{34}$ ms^{-1} (D) 2 ms^{-1}

(26) A simple pendulum made of a wooden block of mass 500 g .A bullet of mass 50 g fired from downward side of the block it comes into rest after entering in to the block. If the displacement of block is 1.1 m vertically then the speed of bullet .... ($g = 10\ ms^{-2}$)



(A) $50\sqrt{2}$ ms^{-1} (B) $51\sqrt{2}$ ms^{-1} (C) 51.6 ms^{-1} (D) 50 ms^{-1}

(27) Two square plates of length d and a circular plate of diameter d place as shown in figure . If the mass distribution of all three plates are same than the position of the centre of mass of the system with respect to centre of first plate is ....



(A) < d (B) > d (C) = d (D) = 1.5 d

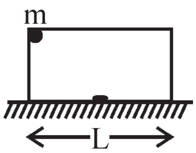
(28) Two spheres of mass 5 kg and 10 kg having same radius placed on a horizontal surface. A sphere of 5 kg mass moves away up to 8 cm from the second sphere. Then how much displacement of heavy sphere is required so that the centre of mass of the system is shifted 1 cm towards the centre of lighter sphere.

(A) 2.5 cm towards lighter sphere (B) 5.5 cm towards lighter sphere
(C) 5.5 cm away from lighter sphere (D) 2.5 cm away from lighter sphere

(29) At the two ends A and B of a stationary boat of mass 50 kg and length 5 m , two person of mass 40 kg and 60 kg are sit. If they meet at the centre of boat for discussion then the displacement of boad .... (neglect the firiction of water)

(A) 0.5 m, towards A (B) 0.3 m, towards A (C) 0.3 m, towards B (D) 0

(30) A symmetric rectangular box of mass M and length L is placed stationary on a frictionless surface. on the bottom of this rectangular a slot is given at the centre. As shown is figure a sphere of mass m is fix at a rolling comes in to the rest in the slot then what is the displacement of box ?



(A) $\frac{mL}{M+m}$ (B) $\frac{M+m}{m}L$ (C) $\frac{2(M+m)}{m}L$ (D) $\frac{mL}{2(M+m)}$

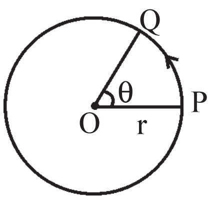
(31) A person with a bag is fall down from a tower of height 100 m. The mass of person and bag is 50 kg and 20 kg respectively. If the person wants to landing in to a lake situated at 0.5 m distance from the bottom of tower, for which velocity the bag can thrown in opposite to the lake when he come at 20 m downward from the top of tower. ($g = 10\ ms^{-2}$)

(A) 0.5 ms^{-1} (B) 0.25 ms^{-1} (C) 1 ms^{-1} (D) 0.1 ms^{-1}

Ans. : 16 (C), 17 (D), 18 (C), 19 (B), 20 (C), 21 (B), 22 (B), 23 (C), 24 (A), 25 (C), 26 (C), 27 (C), 28 (D), 29 (B), 30 (D), 31 (A)

Rotational Motion

Angular displacement :



$$\text{Angular displacement} = \frac{\text{linear displacement}}{\text{radius}}$$

$$\theta = \frac{d}{r}$$

$$\therefore \text{ linear displacement } \vec{d} = \vec{\theta} \times \vec{r}$$

- Angular displacement is a vector quantity and its direction can be find out by using right hand screw rule.
- It is axial vector, so its direction is along the rotational axis.
- Its unit is radian and revolution.
- 1 revolution = 2π rad = $360°$
- In case of the fixed rotational axis the angular displacement is different.

Angular Velocity :

- If the angular displacement is $\Delta\vec{\theta}$ in time interval Δt then,

  $$\text{average angular velocity} = \frac{\text{angular displacement}}{\text{time}}$$

  $$\therefore \ \langle \vec{\omega} \rangle = \frac{\Delta\vec{\theta}}{\Delta t} = \frac{\theta_2 - \theta_1}{\Delta t}$$

- Instantaneous angular velocity $\vec{\omega} = \frac{d\vec{\theta}}{dt}$
- Unit : rad s^{-1}
- direction can be find out according to right hand rule.
- $\vec{\omega}$ is also axial vector so its direction is along the rotational axis.
- Relation with linear velocity : $\vec{v} = \vec{\omega} \times \vec{r}$
- The magnitude of angular velocity is called angular speed.

  $$\omega = \frac{2\pi}{T} = 2\pi f$$

Angular acceleration :

- The change in angular velocity is $\Delta\vec{\omega}$ in time interval Δt then

  $$\text{Average angular acceleration} \quad \langle \vec{\alpha} \rangle = \frac{\Delta\vec{\omega}}{\Delta t}$$

- Instantaneous angular acceleration $\vec{\alpha} = \frac{d\vec{\omega}}{dt} = \frac{d^2\vec{\theta}}{dt^2}$
- Unit : rads^{-2}
- Direction : In the direction of change in angular velocity.

Relation with linear acceleration :

$$\vec{a} = \vec{\alpha} \times \vec{r} + \vec{\omega} \times \vec{v}$$

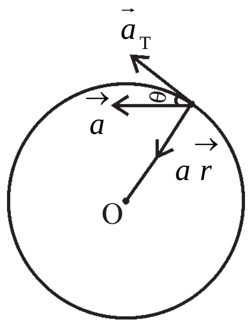
- The direction of $\vec{\alpha} \times \vec{r}$ is found to be along the tangent to the circular path. Hence it is called the tangential component a_T of the linear acceleration. It is responsible for to change only the direction of motion.
- The direction of $\vec{\omega} \times \vec{v}$ is found to be the radial direction towards the centre. Hence $\vec{\omega} \times \vec{v}$ is called the radial component a_r. It is responsible for to change only the magnitude of motion.

$\vec{a_r} = \vec{\omega} \times \vec{v}$

$a_r = \omega v \qquad (\theta = 90°)$

$a_r = \dfrac{v^2}{r}$

$= \omega^2 r$



- $\vec{a} = \vec{a_T} + \vec{a_r}$

$|\vec{a}| = \sqrt{a_T^2 + a_r^2}$

$= \sqrt{\alpha^2 r^2 + \omega^2 v^2}$

$= r\sqrt{\alpha^2 + \omega^4}$

- If $\vec{a}$ makes an angle θ with $\vec{a_t}$ then,

$\tan\theta = \dfrac{a_t}{a_r} = \dfrac{\omega^2}{\alpha}$

- If α = constant then

(1) $\theta = \left(\dfrac{\omega + \omega_0}{2}\right)t$ (2) $\alpha = \dfrac{\omega - \omega_0}{t}$ (3) $\omega = \omega_0 + \alpha t$

(4) $\theta = \omega_0 t + \frac{1}{2}\alpha t^2$ (5) $2\alpha\theta = \omega^2 - \omega_0^2$ (6) $\theta_n th = \omega_0 + \frac{1}{2}\alpha(2n - 1)$

(32) The radius of front and back wheels of a bicycle is r_1 and r_2 respectively, where $r_1 = 2r_2$. If the velocity of the points of both the wheels in contact with ground is v_1 and v_2 respectively than ......

(A) $v_1 = 2v_2$ (B) $v_2 = 2v_1$ (C) $v_1 > v_2$ (D) $v_2 = v_1$

(33) A particle moves with constant tangential component of acceleration on a circular path of radius $\frac{10}{\pi}$ m. After 2.5 rotation the velocity of particle is 50 ms^{-1} tengential acceleration is ....

(A) 25 ms^{-2} (B) 25 rad s^{-2} (C) 2500 π^2 ms^{-2} (D) 2500 π^2 rad s^{-2}

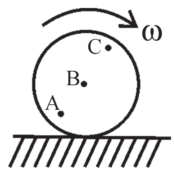
(34) The radius of a wheel is 2 m and its one point is in contact with ground. When the wheel complete half rotation the linear displacement of the particle is ....

(A) 1 m (B) $2\sqrt{4+\pi^2}$ (C) $2\sqrt{2+\pi^2}$ (D) $4\sqrt{4+\pi^2}$

(35) Initial angular speed of a particle is 2 rad s^{-1} and constant angular acceleration is 3 rad s^{-2}, then after 4 s its angular displacement is ...... rad.

(A) 10 (B) 32 (C) 14 (D) 18

(36) As shown in figure a disc is rolling without slipping on a horizontal surface. B is the centre of it and AB = BC. If the velocity of particles A, B and C are v_A, v_B and v_C respectively then ....



(A) $v_C < v_B < v_A$ (B) $v_C = v_A = 2v_B$
(C) $v_C > v_B > v_A$ (D) $v_C < v_B > v_A$

(37) A fan initially at rest get the angular speed of 500 rpm in 4 s with constant angular acceleration.Then the time required to get 250 rpm speed from rest position is ....
(A) 3 s (B) 2.5 s (C) 2 s (D) 1.8 s

(38) A wheel intially at rest get 3 rad angular displacement in 1^{st} s with constant angular acceleration, then the angular displacement in 2^{st} s is ....
(A) 12 rad (B) 15 rad (C) 9 rad (D) 6 rad

(39) A wheel acquires angular displacement of 50 rotation in 4 sec, then after 5sec its angular speed is ...... rad s^{-1}.
(A) 40 π (B) 50 π (C) 30 π (D) 50

(40) A wheel intially at rest moves with constant angular acceleration 3 rad, s^{-2} after this it moves with constant angular speed for 8 s and then in 8 s it come into the rest with constant angular retardation. Then total angular displacement of wheel is ...... rad
(A) 384 (B) 284 (C) 256 (D) 356

(41) A wheel rotating with 5 rad s^{-2} angular acceleration along its axis. After 2 s of motion the magnitude of radial and tangential componant of a particle at a distance 2 cm from the axis is ...... and ...... cms^{-2}.
(A) 50, 5 (B) 25, 10 (C) 25, 5 (D) 50, 10

Ans. : 32 (D), 33 (A), 34 (B), 35 (B), 36 (C), 37 (C), 38 (C), 39 (B), 40 (A), 41 (D)

Moment of Inertia

'A characteristic of a object which oppose the change of motion of object either at rest or move with constant circular motion'

- For a particle I = mr^2
  where r = perpendicular distance of particle from the axis
- For a system of particle (ununiform distribution) I = $\sum_{i=1}^{n} m_i r_i^2$
- For a system of particle (uniform distribution) I = $\int r^2 \, dm$
- SI unit is kg m^2 and dimensional formula is $M^1L^2T^0$
- It depends on the selection of axis and the distribution of mass about it.
- Moment of intertia is a tensor physical quantity.
- Inertia of object is constant while moment of inertia is different.

Radius of Gyration :

'The perpendicular distance from the axis of a particle at which the total mass of the object is concentric'

'The rms perpendicular distances of particles from the axis.'

- For the system of particles $k = \sqrt{\dfrac{r_1^2 + r_2^2 + ... r_n^2}{n}}$
- Moment of inertia $I = Mk^2$
- Radius of gyration does not depends upon the mass of object but depend upon the volume of it.

Theorem of parallel axis :

$I = I_c + Md^2$

I = Moment of inertia of the body about any axis

I_c = Moment of inertia of the body through its center about any axis.

M = Total mass of the substance.

d = Perpendicular distance between the two axis.

Theorem of perpendicular axis :

If we take X and Y axis in the plane of a planar body then,

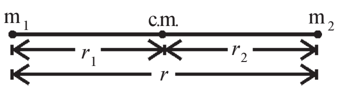
$I_z = I_x + I_y$

Special note :

- (1) For a symmetric two-dimensional object moment of inertia of any axis passing from its centre of mass and parallel to its plane is same. So, in this case it is not necessary that two axis are perpendicular to each other.

  (2) The intersection point of two perpendicular axis is not always the center of mass of the object, it may be the point outside the object.
- The moment of inertia with respect to the axis perpendicular to the joining line of two particles is $I = m_1 r_1^2 + m_2 r_2^2$
- According to the definition of centre of mass, $m_1 r_1 = m_2 r_2$ and $r_1 + r_2 = r$ then,

$r_1 = \dfrac{m_2 r}{m_1 + m_2}$ and $r_2 = \dfrac{m_1 r}{m_1 + m_2}$



$\therefore\ I = \left(\dfrac{m_1 m_2}{m_1 + m_2}\right) r^2$

$\therefore\ I = \mu r^2$ and $\mu = \dfrac{m_1 m_2}{m_1 + m_2}$

= Reduced mass

$\mu < m_1$ and $\mu < m_2$

- **Torque :**

'Torque is the moment of force with respect to a given reference point'

- Torque acting on the particle is

$\vec{\tau} = \vec{r} \times \vec{F}$

$\tau = r F \sin\theta$

where θ = angle between $\vec{r}$ and $\vec{F}$

$\vec{r}$ = position vector

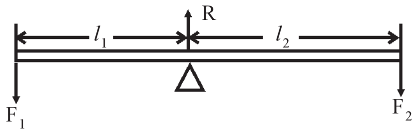
$r \sin\theta$ = perpendicular distance of line of action of force = inertia = lever arm

- according to the right hand screw rule the direction of torques $(\vec{\tau})$ is perpendicular to the plane formed by $\vec{r}$ and $\vec{F}$
- SI unit : Nm or J
- CGS unit : dyne cm

For a system of particles,

$\vec{\tau} = \vec{\tau}_1 + \vec{\tau}_2 + ... + \vec{\tau}_n$

- For equilibrium of rod



$\vec{\tau}_1 + \vec{\tau}_2 = 0$

$\therefore\ F_1 l_1 - F_2 l_2 = 0$

$F_1 l_1 = F_2 l_2$

- $\vec{F} = m\vec{a}$ resembles with the equation $\vec{\tau} = I\vec{\alpha}$

Couple **:**

Two forces of equal magnitude and opposite directions which are not colinear form a couple.

Moment of couple = magnitude of any one of the two force × perpendicular distance between the two forces.

- In a torque applied on a object one force is external while the other force are external. In couple both the force are external.

Angular Momentum **:**

- Angular momentum of a particle is $\vec{l} = \vec{r} \times \vec{P}$

$l = r\ p\sin\theta$

where θ = angle between $\vec{r}$ and $\vec{p}$

p = mv = linear momentum of particle

- Angular momentum is axis vector. Its direction can be find out by using right hand screw rule.
- Unit : J s or erg s
- Angular momentum of a particle = Linear momentum × Perpendicular distance of the vector of linear momentum = moment of linear momentum
- For circular motion $\vec{r} \perp \vec{p}$ so $L = rp = mvr = m\omega r^2. = I\omega$

Also, $\frac{dL}{dt} = I\frac{d\omega}{dt} = I\alpha = \tau$

- If large amount of torque applied for small time interval then 'Angular impulse of torque is

$\vec{J} = \int \vec{\tau}_{a_v}\, dt = \vec{\tau}_{a_v} \int_{t_1}^{t_2} dt = \vec{\tau}_{a_v}\ \Delta t = \Delta\vec{L}$ = change in angular momentum.

Law of conservation of angular momentum :

- $\vec{\tau} = \frac{d\vec{L}}{dt}$

If resultant torque is zero

$$\frac{d\vec{L}}{dt} = 0 \Rightarrow \vec{L} = \text{constant}$$

$$\therefore \vec{L}_1 + \vec{L}_2 + ... + \vec{L}_n = \text{constant}$$

$L = I\omega = \text{constant}$

$\therefore \omega \propto \frac{1}{I}$ from this it is clear that

when a planet come near to the sun its moment of inertia is decrease. So angular speed is increase ($I \propto r^2$)

- If the radius of planet suddenly increase by x time then its periodic time

  $T \propto I \Rightarrow T \propto R^2$

  So, $T_2 = x^2 T_1$

- The work due to torque $W = \tau\,\theta$

  for variable torque $W = \int_{\theta_1}^{\theta_2} \tau\, d\theta$

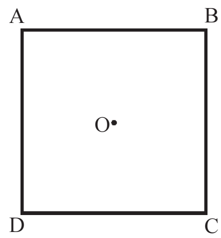
- Rotational kinetic energy $K_R = \frac{1}{2} I\omega^2 = \frac{1}{2} L\omega = \frac{L^2}{2I}$

- Power $P = \tau\omega = I \propto \omega = I\omega \frac{d\omega}{dt}$

  $P = \vec{\tau} \cdot \vec{\omega}$ (In vector form)

---

(42) The object of mass 3kg, 5kg, 6kg, and 2 kg are placed at the vertices A, B, C and D of a square length 1 m. Then moment of inertia with respect to axis perpendicular to its plane and passing through it centre is ....



(A) 6 kg m^2 (B) 8 kg m^2

(C) 4 kg m^2 (D) 16 kg m^2

(43) In above example, find the moment of inertia with respect to AD axis of square.

(A) 11 kg m^2 (B) 13 kg m^2 (C) 16 kg m^2 (D) 8 kg m^2

(44) In above example, the moment of inertia with respect to axis BD is ....

(A) 16 kg m^2 (B) 8 kg m^2 (C) 4.5 kg m^2 (D) 9 kg m^2

(45) The mass of a disc is 10 kg and radius is 0.2 m. It moves with 200 rpm angular speed along the axis passing through its centre and perpendicular to its plane. To let it in stationary position in 15 sec the tangential force required on the circumfrance is ....

(A) 0.5 π N (B) 0.4 π N (C) 0.2 π N (D) 0.44 π N

(46) A disc of mass 5 kg and radius 0.4 m rotates along its axis with 30 rpm. To incerase its angular momentum 20 % in 5 s required torque is ...... Nm.

(A) 2.6 π (B) 0.16 π (C) 1.6 π (D) 0.016 π

(47) When 1500 J energy given to a object its angular speed increase by 1000 rpm to 2500 rpm then the moment of inertia of a object is ...... kg m^2

(A) 0.052 (B) 0.52 (C) 52 (D) 0.026

(48) Two disc rotating about an axis passing through its centre and perpendicular to its plane. The mass of bigger disc is 2 kg and radius is 0.2 m and angular speed 50 rad s^{-1} while for smaller disc the mass, radius and angular speed is 4 kg, 0.15 m and 250 $rads^{-1}$ respectively. The axis of both the disc is coinside when the smaller disc get in connect to bigger disc, then the angular speed of a system of two disc is ...... $rads^{-1}$.

(A) 200 (B) 140 (C) 153 (D) 105

(49) A particle start to move with constant velocity from the point (10, 10) cm and move parallel to y-axis in negative direction, then its angular momentum with respect to orgin is ....

(A) zero (B) constant

(C) increase (D) intially increase and then decrease

(50) The inner and outer radius of a hollow cylinder is a and b respectively. The moment of inertia of it with respect to its axis is ...... (density of material is ρ, length of cylinder is L)

(A) $2\pi L\rho\,(a^2 + b^2)$ (B) $2\pi L\rho\left(\frac{a^4+b^4}{4}\right)$ (C) $2\pi L\rho\left(\frac{b^2-a^2}{2}\right)$ (D) $2\pi L\rho\left(\frac{b^4-a^4}{4}\right)$

(51) If the sixth portion of a disc of mass M and radius R of uniform density distribution cut then its moment of inertia with respect to the axis passing through the centre of original disc and perpendiculr to the plane is ....

(A) $\frac{1}{12}$ MR^2 (B) $\frac{1}{4}$ MR^2 (C) 3 MR^2 (D) $\frac{1}{6}$ MR^2

(52) A circle made of a wire of length l and uniform linear density λ, then its moment of inertia with respect to axis parallel to its plane and passing tangentially is ....

(A) $\frac{8}{3}\,\frac{\pi^2}{\lambda l^3}$ (B) $\frac{3\lambda l^2}{8\pi^2}$ (C) $\frac{3\lambda l^3}{8\pi^2}$ (D) $\frac{\lambda l^3}{8\pi^2}$

(53) The mass of a semicircular ring of radius R is M. The moment of inertia with respect to axis passing through the centre of original of ring and perpendicular to its plane is ....

(A) $\frac{MR^2}{2}$ (B) MR^2 (C) $\frac{MR^2}{4}$ (D) non of the above

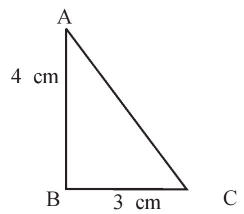
(54) A square made of four rods of length l and mass m. The moment of inertia with respect to the axis passing through its centre and perpendicular to the plane is ....

(A) $\frac{4}{3}$ ml^2 (B) $\frac{1}{3}$ ml^2 (C) $\frac{1}{6}$ ml^2 (D) $\frac{2}{3}$ ml^2

(55) The moment of inertia of a square plate of length l and mass m with respect to axis passing through any one corner and perpendicular to its plane is ....

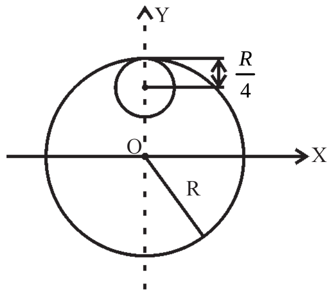
(A) $\frac{4}{3}$ ml^2 (B) $\frac{1}{3}$ ml^2 (C) $\frac{1}{6}$ ml^2 (D) $\frac{2}{3}$ ml^2

(56) If the moment of inertia of a triangle plate of uniform density with respect to sides AB, BC and CA is I_1, I_2 and I_3 respectively then,



(A) $I_2 < I_1$ (B) $I_1 + I_2 = I_3$

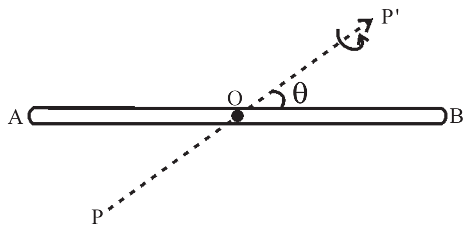
(C) $I_2 > I_1$ (D) I_3 is maximum

(57) 

From a circular disc of radius R and mass 4M, a small disc of radius $\frac{R}{4}$ is removed from the disc. The moment of inertia of the remaining disc about an axis perpendicular to the plane of the disc and passing through O is ....

(A) 2.43 MR^2 (B) 1.23 MR^2 (C) 1.4 MR^2 (D) 1.43 MR^2

(58) The moment of inertia of thin uniform rod of length l and mass m about an axis pp' shown in figure is ....



(A) $\frac{ml^2 \sin^2 \theta}{8}$ (B) $\frac{ml^2 \sin^2 \theta}{12}$

(C) $\frac{ml^2 \cos^2 \theta}{8}$ (D) $\frac{ml^2 \cos^2 \theta}{12}$

(59) A thin wire of uniform density is having length $4l$ and mass m is bent to form a rectangle ABCD. Here side AB is four times the side BC. Then moment of inertia about an axis passing through AD is ....

(A) 0.3 ml^2 (B) 0.5 ml^2 (C) 0.4 ml^2 (D) 0.2 ml^2

(60) A solid metallic sphere is rotating about an axis passing through its diameter. Suddenly its volume is increased by 6 %, then the change in its angular speed will be ....

(A) –2 % (B) +2 % (C) – 4 % (D) +4 %

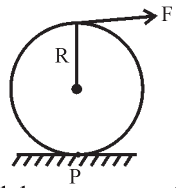
(61) The moment of inertia of a disc of radius 1.5m is 150 kg m^2. The disc can rotate frictionless about vertical axis. A man of mass 60 kg is standing on its edge. Now man walks with speed of 2 ms^{-1} on its edge, then angular speed of a disc will be...... .

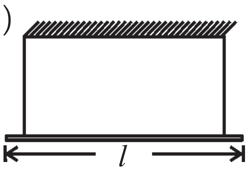
(A) 1 rad s^{-1} (B) $\frac{1.2}{\pi}$ rad s^{-1} (C) $\frac{2}{\pi}$ rad s^{-1} (D) 1.2 rad s^{-1}

(62) A hollow sphere is placed on a rough horizontal surface. On appling force F on it in horizontal direction it rolls down without slipping on the surface. Then its angular acceleration is ....

(A) $\frac{6}{5}$ MR (B) $\frac{6}{5} \frac{F}{MR}$

(C) $\frac{3F}{MR}$ (D) $\frac{3}{5} \frac{F}{MR}$

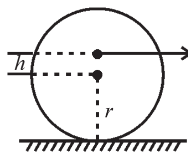


(63) 

A uniform rod of mass m and length l is suspended by means of two right inextensible strings as shown in figure. If one string is cut out then the tension in the other string is ......

(A) $\frac{mg}{4}$ (B) 2 mg (C) mg (D) $\frac{mg}{2}$

(64) If a ball of mass m and radius r is hitted at the height h from its centre, velocity gained by it is v_0. Then angular speed obtained by it is ...... $\left(\frac{hv_0}{r^2}\right)$

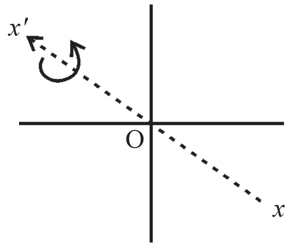


(A) $\frac{2}{5}$ (B) $\frac{5}{2}$ (C) $\frac{5}{4}$ (D) $\frac{4}{5}$

(65) A weightless rod AB of length $2l$ is placed vertically on a frictionless horizontal surface. Two sphere of mass m are attached to two ends of rod AB. impulse of force J is applied parallel to the surface at the point A and also perpendicular to rod AB, then velocity of end A is ....

(A) $\frac{J}{m}$ (B) $\frac{J}{2m}$ (C) 0 (D) $\frac{2J}{m}$

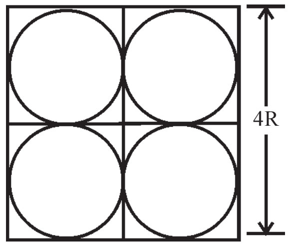
(66) As shown in the figure find the moment of inertia about $x\,x'$ axis of a system of two rods of mass m each and length l each placed perpendicular to each other.



(A) $\frac{ml^2}{6}$ (B) $\frac{ml^2}{\sqrt{2}}$

(C) $\frac{ml^2}{12}$ (D) $\frac{ml^2}{6\sqrt{2}}$

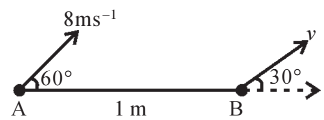
(67) Four dises of radius R are cut from a thin square plate of side 4R and mass M. The moment of inertia of the remaing portion about an axis passing through its centre and perpendicular to its plane is ....



(A) $\frac{5\pi}{8}$ MR^2 (B) $\frac{8}{3}$ MR^2

(C) $\left(\frac{8}{3}+\frac{5\pi}{8}\right)$ MR^2 (D) $\left(\frac{8}{3}-\frac{5\pi}{8}\right)$ MR^2

(68) Two sphere A and B connected to two ends of a 1 m long thin rod and applied difference inpulse of force at the same time. The instantaneous velocities gained by them are shown in the figure. Then the angular speed of sphere B with respect to A will be ....



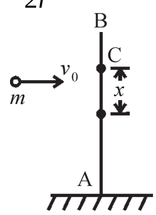
(A) $8\sqrt{3}$ rad s^{-1} (B) $\frac{8}{\sqrt{3}}$ rad s^{-1}

(C) $4\sqrt{3}$ rad s^{-1} (D) $\frac{4}{\sqrt{3}}$ rad s^{-1}

(69) A turn table of radius r is rotating about vertical axis passing through its centre. A bullet of mass m is fired in an opposite direction of turn table's motion from the gun attatched at its edge in the direction tangent to the edge of table. If the speed of bullet is v, then increase in angular speed of turn table is .... (I_0 = moment of inertia of a system of turn table + gun)

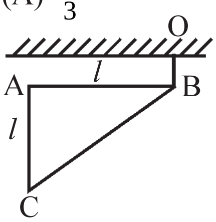
(A) $\frac{mvr}{I_0+mr^2}$ (B) $\frac{2mvr}{I_0+mr^2}$ (C) $\frac{mvr}{I_0}$ (D) $\frac{v}{2r}$

(70) A rod of uniform density is having mass M and length L. It is placed vertical on a smooth horizontal surface. A particle of mass m moving with velocity v_0 collides at point C of the rod and becomes stationary. What should be the height of this point from the centre of the rod so that the end A of the rod remain stationary ?



(A) $\frac{L}{3}$ (B) $\frac{2L}{5}$ (C) $\frac{L}{6}$ (D) $\frac{L}{4}$

(71) A right triangle plate ABC of mass M is free to rotate in the vertical plane about a fixed horizontal axis through A. It is support by a string such that the side AB is horizontal. The reaction at the support A is ....



(A) $\frac{Mg}{3}$ (B) $\frac{2Mg}{3}$ (C) Mg (D) $\frac{3}{2}$Mg

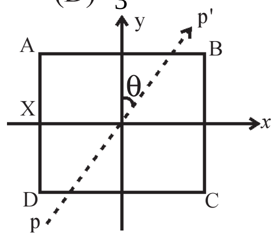
(72) A disc of radius r is formed from a solid sphere of mass M and radius R. If the moment of inertia of the disc about an axis passing through its edge and perpendicular to its plane is equal to the moment of inertia of the sphere about its diameter, then r = ...

(A) $R\sqrt{\frac{2}{15}}$ (B) $\frac{4}{\sqrt{15}}$ R (C) $\frac{R}{4}$ (D) $\frac{7}{\sqrt{15}}$ R

(73) The moment of inertia of a wheel about an axis passing through its centre and perpendicular to its plane is 2.5 kgm^2. It is rotating about this axis with angular speed of 90 rpm. Then the torque required to stop rotating it in 0.5 min is ...... Nm

(A) $\frac{\pi}{2}$ (B) $\frac{\pi}{4}$ (C) π (D) $\frac{\pi}{3}$

(74) The moment of inertia about an axis passing through X axis of a square plate of uniform density is I. Then moment of inertia about an axis PP' making an angle θ with Y-axis will be ....



(A) 2I (B) I $\cos^2\theta$

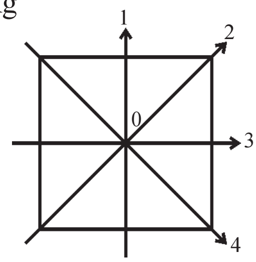
(C) I (D) I $\sin^2\theta$

(75) Two spheres of mass 2 kg and 3 kg are attatched to two ends of a rod of negligible mass. Length of rod is $2m$.The rod is rotating with uniform angular speed about an axis perpendicular to its length. What should be the position of axis of rotation so that minimum work is required to be done to rotate it?

(A) 1.2 m away from 2 kg (B) 1.2 m away from 3 kg

(C) 0.8 m away from 2 kg (D) 0.6 m away from 3 kg

(76) A square plate is shown in the figure. The moment of inertia about an axis passing through plane of plate from 1, 2, 3 and 4 are I_1, I_2, I_3 and I_4 respectively. which of the following is not the moment of inertia about an axis passing through its centre and perpendicular to its plane ?

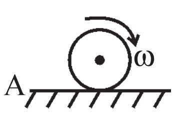


(A) $I_1 + I_2$ (B) $I_1 + I_3$

(C) $I_2 + I_4$ (D) All of the above.

(77) A ring of mass M has radius R. It is rotating with uniform angular speed ω about its axis. If two particles each of mass $\frac{m}{2}$ are placed at diametrically opposite points, then its angular speed will be ....

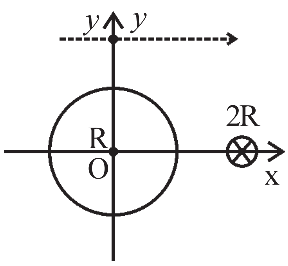
(A) $\frac{M-m}{M}\ \omega$ (B) $\frac{M}{M+m}\ \omega$ (C) $\frac{2M}{2M+m}\ \omega$ (D) $\frac{M+m}{M}\ \omega$

(78) 

A ring of mass M and radius R rolls down on a horizontal surface with angular speed ω. The angular momentum of ring with respect to point A is ....

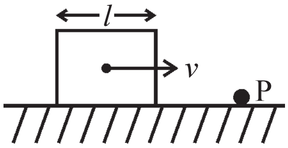
(A) $MR^2\omega$ (B) $\frac{1}{2}\ MR^2\omega$ (C) $2MR^2\omega$ (D) $\frac{3}{2}MR^2\omega$

(79) A disc of uniform density has mass M and radius R. It is placed in a XY plane in such a way that its centre coincides with origin. If its moment of inertia about an axis passing through point (2R, O) is equal to that about an axis passing through the point (y, O) then, y = ...



(A) $\frac{\sqrt{17}}{4}$R (B) $\frac{\sqrt{15}}{2}$R

(C) $\sqrt{\frac{17}{2}}$ R (D) $\frac{\sqrt{17}}{2}$R

(80) 

A cube shaped block of mass m and side l is moving with velocity v on a frictionless horizontal surface. It collides with buldged part at point P and then after it rolls. Then its angular speed after the collision will be ....

(A) $\frac{3}{4}\ \frac{v}{l}$ (B) $\frac{3}{2}\ \frac{v}{l}$ (C) $\frac{3}{4}\ \frac{l}{v}$ (D) $\frac{5}{4}\ \frac{v}{l}$

Ans. : 42 (B), 43 (A), 44 (C), 45 (D), 46 (B), 47 (A), 48 (C), 49 (B), 50 (D), 51 (A), 52 (C), 53 (B), 54 (A), 55 (D), 56 (C), 57 (D), 58 (B), 59 (A), 60 (C), 61 (D), 62 (B), 63 (A), 64 (B), 65 (A), 66 (C), 67 (D), 68 (B), 69 (A), 70 (C), 71 (B), 72 (D), 73 (B), 74 (C), 75 (A), 76 (D), 77 (B), 78 (C), 79 (D), 80 (A)

- **If the body is slipping**

  The friction between the body and the surface is zero

  Translatory kinetic energy, $K_T = \frac{1}{2}mv^2$

  - If the body rotates about fixed axis of rotation, then rotatory kinetic energy $K_R = \frac{1}{2}I\omega^2$

    $$= \frac{1}{2}(mk^2)\frac{v^2}{R^2} = \frac{1}{2}mv^2\left(\frac{k^2}{R^2}\right)$$

    $$\therefore K_R = \left(\frac{k^2}{R^2}\right)(K_T)$$

  - If the body performes rotational motion and its axis of rotation performes linear motion, the body is said to be rolling body. Friction between the body and surface, $F \neq 0$

    Net kinetic energy $K_{Net} = K_T + K_R = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2 = \frac{1}{2}mv^2\left[1+\frac{k^2}{R^2}\right]$

  - If the object rolls across a surface in such a way that there is no relative motion of object and surface at the point of contact, the motion is called rolling without slipping, even in the presence of friction.

  - The works done against friction = energy lost = 0

    Net kinetic energy, $K_{Net} = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$

    $$= \frac{1}{2}mR^2\omega^2 + \frac{1}{2}I\omega^2$$

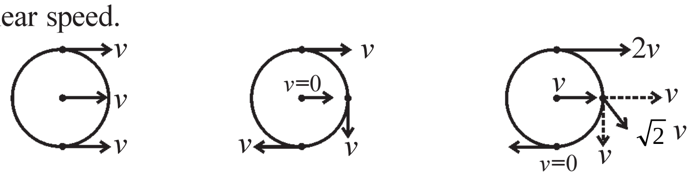
    $$= \frac{1}{2}[I + mR^2]\omega^2$$

    $$K_{Net} = \frac{1}{2}I_p\omega^2$$

    where I = moment of inertia of rolling body about its centre.

    I_P = moment of inertia of rolling body about point of contact P.

  - In case of rolling without slipping, all points of a rigid body have same angular speed but different linear speed.

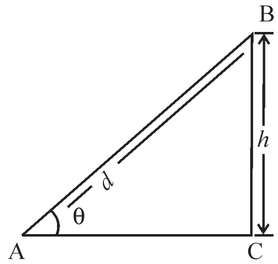


pure translation pure rotation pure rolling

- Rolling without slipping on an inclined plane.

  Potential energy at height h = Total kinetic energy at the bottom at an inclined plane.

  $$mgh = \frac{1}{2}mv^2\left[1+\frac{k^2}{R^2}\right]$$



- Linear velocity at the bottom of an inclined plane. $v = \left[\dfrac{2gh}{1+\dfrac{k^2}{R^2}}\right]^{\frac{1}{2}}$

- constant acceleration in motion, $a = \dfrac{g\sin\theta}{1+\dfrac{k^2}{R^2}}$

- Decrease in acceleration due to friction,

  $a' = g\sin\theta - a$

  $$= \frac{g\sin\theta}{1+\dfrac{R^2}{k^2}}$$

- Frictional force = F = ma'
- Time taken to reach the bottom of an inclined plane,

  $$t = \frac{1}{\sin\theta}\left[\frac{2h}{g}\left(1+\frac{k^2}{R^2}\right)\right]^{\frac{1}{2}}$$

- Here the factor $\dfrac{k^2}{R^2}$ is a measure of moment of inertia of a body and its value is constant for given shape of the body and it does not depend on the mass and radius of a body.
- The values of velocity, linear acceleration and time of descent (for a given inclined plane) is same for bodies of small or large dimensions, for lighter or havier bodies of equal shape.
- The substance, which has more moment of inertia the ratio $\dfrac{k^2}{R^2}$, will be more, so magnitude of velocity and acceleration will be decreased while value of time will be increased.
- For a solid and hollow body of same shape, $I_{solid} < I_{hollow}$. Hence the velocity and acceleration of a solid body are larger and time of descent is lesser. It means solid body will reach the bottom first with greater velocity.
- If given body rolls down on an inclined plane of different angle of inclination.

  $v \propto \theta^{\circ}$, $\quad a \propto \theta \quad t \propto \theta^{-1}$

- The necessary condition for a body rolling without slipping is

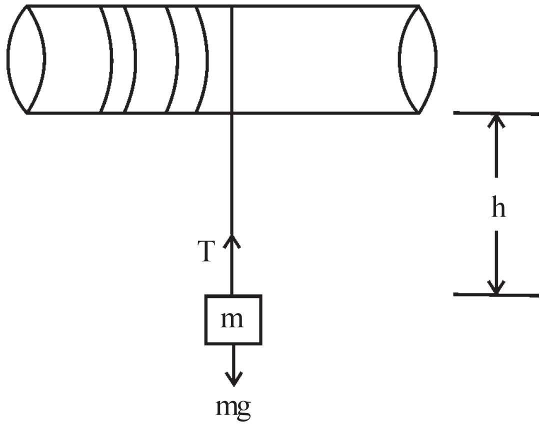
  $$\mu \geq \left(\frac{k^2}{k^2+R^2}\right)\tan\theta$$

- When a body of mass m is tied to one end of a string which is wound around the solid body[cylinder, pulley, disc], when the mass is released,it falls vertically downwards, then

- Downward acceleration

$$a = \frac{g}{1+\frac{M}{m}\left(\frac{k^2}{R^2}\right)}$$

M = mass of a rigid body (rotating body)
k = Radius of gyration of a rotating body



- Tension in string

$$T = mg\left[\frac{I}{I+mR^2}\right]$$

I = moment of inertia of a rotating body.

- Velocity of a body after travelling a distance h in downward direction,

$$v = \sqrt{\frac{2gh}{1+\frac{M}{m}\left(\frac{k^2}{R^2}\right)}}$$

- Time taken to travel a distance h,

$$t = \frac{2h}{g}\left[1+\frac{M}{m}\left(\frac{k^2}{R^2}\right)\right]$$

- Angular speed of a rotating body, $\omega = \sqrt{\frac{2mgh}{I+mR^2}} = \sqrt{\frac{2gh}{R^2+\frac{M}{m}(k^2)}}$

- when a body allows to oscillate about an axis passing from any point except centre of the body of uniform shape, it is called the compound pendulum.

  Its time period is given by, $T = 2\pi\sqrt{\frac{L}{g}}$ where $L = \frac{l^2+k^2}{l}$

  l = distance between the point of suspension to centre of mass of a body,
  k = radius of gyration of a body.

---

(81) A solid sphere, a hollow sphere and a ring are released from an inclined plane (frictionless) of height h, which object will reach the bottom first ?

(A) ring (B) solid sphere

(C) hollow sphere. (D) All of them reach the bottom at the same time.

(82) A hollow sphere of mass M and radius R rolls on a horizontal plane without slipping with the velocity v. Now, it the surface rolls up vertically, the maximum height the body would attain will be ....

(A) $\frac{6}{5}\frac{v^2}{g}$ (B) $\frac{5}{6}\frac{v^2}{g}$ (C) $\frac{7}{10}\frac{v^2}{g}$ (D) $\frac{10}{7}\frac{v^2}{g}$

(83) A small object of uniform density rolls up a curved surface with an initial velocity v. It reaches up to a maximum height of $\frac{3v^2}{4g}$ with respect to the initial position. The object is ....

(A) Ring (B) hollow sphere (C) Disc (D) hollow cylinder

(84) A solid cylinder rolls down an frictionless inclined plane. If angle of inclination with horizontal is θ and the co-efficient of friction between the body and surface is μ, then

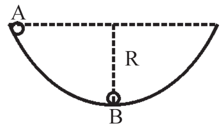
(A) Frictional force equal to $\mu mg \cos \theta$

(B) Frictional force will become resistive force.

(C) Frictional force opposes the linear motion and helps to relational motion.

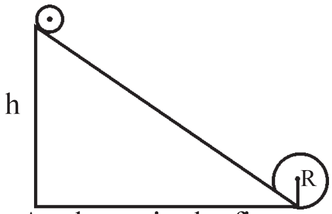
(D) Frictional force increases as θ decreases.

(85) A small sphere of radius r is kept in a hemispherical bowl of radius R. It is released from point A. The angular speed of the sphere when it reaches at the bottom will be ......



(A) $\left(\frac{5}{7}\frac{g}{R-r}\right)^{\frac{1}{2}}$ (B) $\left(\frac{7}{10}\frac{g}{R-r}\right)^{\frac{1}{2}}$ (C) $\left(\frac{2}{5}\frac{g}{R-r}\right)^{\frac{1}{2}}$ (D) $\left(\frac{10}{7}\frac{g}{R-r}\right)^{\frac{1}{2}}$

(86) A hollow sphere of radius r rolls down without slipping on inclined plane of height h. A loop of radius R is formed at the end of an inclined plane. What should be the minimum height of inclined plane to be kept so the hollow sphere can complete the rotation in a loop ?



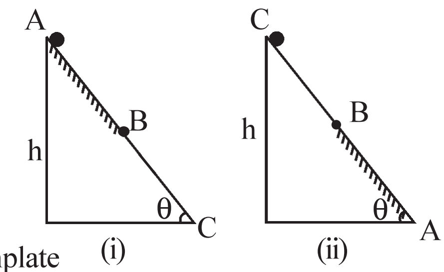
(A) 2.7 R

(B) 1.7 R

(C) $3\frac{R}{2}$

(D) 2.3 R

(87) As shown in the figure two inclined planes are same in all aspects. In which the AB part is rough and the BC part is smooth. The kinetic energy of the body at the bottom of inclined plane is



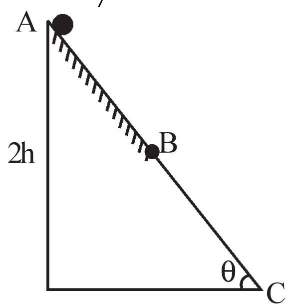
(A) more in (i) (B) more in (ii)

(C) equal in both (i) (ii) (D) information is incomplate

(88) A solid sphere and a solid cylinder have equal mass. They rolled down on inclined planes of height h_1 and h_2 respectively. To have equal velocities for the two objects at the bottom of the inclined planes the ratio of heights $h_1 : h_2$ should be = ...

(A) $\frac{15}{14}$ (B) $\frac{7}{5}$ (C) $\frac{14}{15}$ (D) $\frac{5}{7}$

(89) A solid sphere rolls down without slipping on an inclined plane of height 2h. The surface AB is rough and the suface BC is smooth. The ratio of linear kinetic energy to rotational kinetic energy of sphere at a point C is ....



(A) 4 (B) 6

(C) $\frac{1}{6}$ (D) $\frac{1}{4}$

(90) A horizontal turn table is rotating with constant angular speed ω. A light coin is placed at distance r from the centre of the turn table. Then for which of the following condition, the coin will keep rotating with the turn table ? (co-efficient of static friction is μ)

(A) $r = \mu g \omega^2$ (B) $r < \frac{\omega^2}{\mu g}$ (C) $r \geq \frac{\mu g}{\omega^2}$ (D) $r \leq \frac{\mu g}{\omega^2}$

Ans. : 81 (D), 82 (B), 83 (C), 84 (C), 85 (D), 86 (A), 87 (A), 88 (C), 89 (B), 90 (D)

Assertion - Reason type Question :

Instruction : Read assertion and reason carefully, select proper option from given below.

(a) Both assertion and reason are true and reason explains the assertion.

(b) Both assertion and reason are true but reason does not explain the assertion.

(c) Assertion is true but reason is false.

(d) Assertion is false and reason is true.

(91) **Assertion :** A particle is moving on a straight line with a uniform velocity, its angular momentum is always zero.

Reason : The angular momentum of a particle is given by the formula $\vec{l} = \vec{r} \times \vec{p}$

(A) a (B) b (C) c (D) d

(92) **Assertion :** A hollow cylinder of metal and a solid cylinder of wood having equal mass and dimensions rolled down simultaneously without slipping on an inclined plane. Then the hollow cylinder will reach the bottom first.

Reason : According to law of conservation of kinetic energy, the kinetic energy of both the cylinders is unequal at the bottom of the inclined plane.

(A) a (B) b (C) c (D) d

(93) **Assertion :** Inertia and moment of inertia are same quantities.

Reason : Inertia represents the capacity of a body to oppose its state of motion or rest.

(A) a (B) b (C) c (D) d

(94) **Assertion :** A ladder is more apt to slip, when you are high up on it, then when you just begin to climb.

Reason : At the high up on a ladder, the torque is large and on climbing up the torque is small.

(A) a (B) b (C) c (D) d

(95) **Assertion :** When there is no torque is acting on a body with respect to its centre of mass, its speed remains constant.

Reason : The total linear momentum of an isolated system remains constant.

(A) a (B) b (C) c (D) d

(96) **Assretion :** If a wheel moves down on frictionless inclined, then it can slips only, cannot rolling.

Reason : For rolling the work against the frictional force is zero.

(A) a (B) b (C) c (D) d

(97) **Assertion :** A hollow cylinder is more stronger than a solid cylinder used as shaft in motor.

Reason : The value of torque is more for hollow cylinder as compared to solid cylinder for a given value of angular displacement.

(A) a (B) b (C) c (D) d

(98) **Assertion :** The linear speed for all the particles of a rolling body is same.

Reason : The rotational motion does not affect the linear momentum of a rigid body.

(A) a (B) b (C) c (D) d

(99) **Assertion :** If the earth contracts to half than its initial value (keeping mass constant) the length of the day decrease.

Reason : As the radius of the earth changes, the moment of inertia also changes.

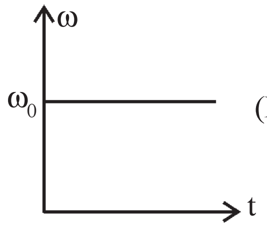
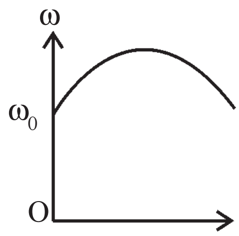
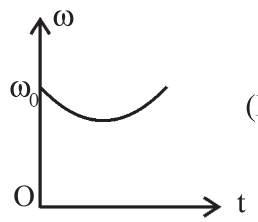
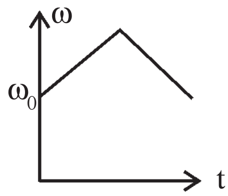
(A) a (B) b (C) c (D) d

(100) **Assertion :** The value of radius of gyration is constant for a given body.

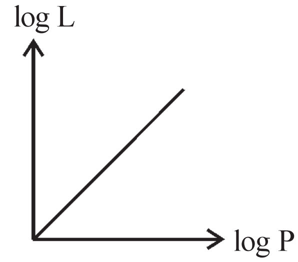
Reason : Radius of gyration means the rms distance of particles from axis of rotation.

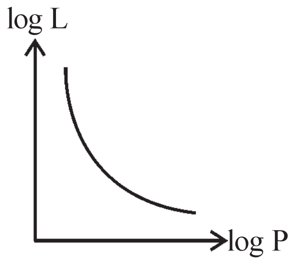
(A) a (B) b (C) c (D) d

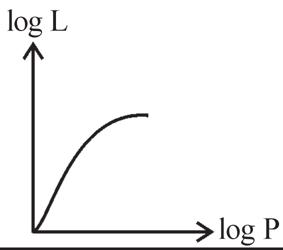
(101) A circular platform is free to rotate in horizontal plane is rotating with angular velocity ω_0. When a child standing on a edge of turn table moves along a chord of the platform and reaches the other end of a turn table.The angular velocity of the platform ω will vary with time t as ......

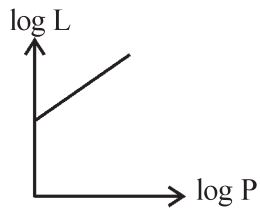
(A)  (B)  (C)  (D) 

(102) The graph between log L and log P is. (L is angular momentum, P = linear momentum)

(A) 

(B) 

(C) 

(D) 

Ans. : 91 (D), 92 (D), 93 (D), 94 (A), 95 (D), 96 (B), 97 (A), 98 (D), 99 (A), 100 (D), 101 (B), 102 (D)

Match the columns :

(103) A object of mass 0.5 kg is projected in the direction making an angle of 45° with horizontal with initial velocity of $10\sqrt{2}$ ms^{-1}. When it moves in the horizontal direction only, then ....

column-1	column-2
with respect to point of projection	(P) 25 SI
(i) Torque acting on the body	(Q) 50 SI
(ii) Angular momentum of the body	(R) 0.4 SI
(iii) Angular velocity of the body	(S) None of these

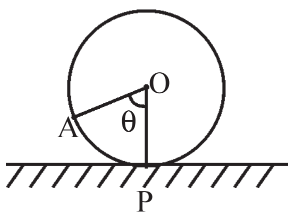
(A) i → R ii → P iii → Q

(B) i → S ii → P iii → R

(C) i → Q ii → P iii → R

(D) i → P ii → Q iii → S

(104) A disc rolls down without slipping on a horizontal surface. The velocity of its centre of mass is v. If the velocity of point A on its edge is v_A and angle between OP and OA is θ as shown in the figure, then



column-1	column-2
(i) $\theta = 60°$	(P) $v_A = \sqrt{2}\,v$
(ii) $\theta = 90°$	(Q) $v_A = v$
(iii) $\theta = 120°$	(R) $v_A = 2v$
(iv) $\theta = 180°$	(S) $v_A = \sqrt{3}\,v$

(A) i → P ii → Q iii → S iv → R
(B) i → S ii → P iii → Q iv → R
(C) i → R ii → Q iii → P iv → S
(D) i → Q ii → P iii → S iv → R

(105) If the resultant force acting on a rigid body is zero, then

column-1	column-2
(i) Linear velocity of centre of mass	(P) P = 0
(ii) Angular velocity of the body	(Q) Q = constant
(iii) The angular velocity about an axis passing from its centre of mass.	(R) R = variable
(iv) The angular velocity about an axis passing from any point other than centre of mass.	

(A) i → Q ii → R iii → R iv → R
(B) i → R ii → Q iii → P iv → R
(C) i → Q ii → R iii → P iv → R
(D) i → R ii → R iii → R iv → R

Ans. : 103 (C), 104 (D), 105 (A)

●