CHAPTER 2

MEASUREMENT AND SYSTEM OF UNITS

2.1 Introduction

We observe many phenomena occurring in our surrounding; some are natural and some are man-made. To describe any phenomenon, measurement of different physical quantities associated with it are essential. Let us consider a fruit falling from a tree. To understand this natural phenomenon, we should know, from which height does the fruit fall ? How much time does it take to reach the ground ? What is the speed of fall of the fruit ? To answer all these questions, we need to measure physical quantities like distance, time, mass etc. accurately. For measurement of any physical quantity, we require to decide their appropriate units. In this chapter, we shall study how the physical quantities are measured and how different units are defined. We shall also learn the different types of errors associated in the measurement of physical quantities.

2.2 What should be the Unit of a Physical Quantity ?

The standard measure of any quantity is called a unit of that physical quantity.

(1) The measure of a unit should be definite and unambiguous.

(2) The unit should be such that its measure should not change and if a unit is defined with the help of some phenomenon, that phenomenon must be permanent.

(3) The prototype (replica) of a unit should be easily reproducible and easily available.

2.3 Units of Physical Quantities and Systems of Units

Although the number of physical quantities is very large, we need **only a minimum limited** number of physical quantities for which units should be the units of all other quantities can be expressed defined and with their help of them. These minimum physical quantities are known as **fundamental quantities** and their units are called **fundamental or base units.** The other physical quantities can be expressed as a combination of fundamental quantities. Such physical quantities are called **derived physical quantities** and their units are called **derived units.** Different systems of units have come into existence at different places and times. These systems are as under :

(1) (FPS) system (British System) (foot, pound, second system)

(2) CGS system (centimetre, gram, second system)

(3) MKS system (metre, kilogram, second system)

(4) MKSA system (metre, kilogram, second, ampere system)

(5) SI system. (seven base units)

2.4 SI System of Units (Systeme Internationale)

The International System of units was accepted in 1971 by the 14th General conference on Weights and Measures under the leadership of International Bureau of Weights and Measures, located at Paris in France. Seven quantities are accepted as fundamental quantities. These fundamental quantities, their units, symbols and definition are shown in Table 2.1.

Table 2.1 : SI Units

Base quantity	Name	Symbol	Definition
Length	metre	m	The metre is the length of the path travelled by light in vacuum during a time interval of 1/299,792,458 of a second. (1983)
Mass	kilogram	kg	The kilogram is equal to the mass of the international prototype of the kilogram (a platinum-iridium alloy cylinder) kept at international Bureau of Weights and Measures, at Sevres, near Paris, France. (1889)
Time	second	s	The second is the duration of 9,192,631,770 periods of the radiation corresponding to the transition between the two hyperfine levels of the ground state of the cesium-133 atom. (1967)
Electric current	ampere	A	The ampere is that constant current which, if maintained in two straight parallel conductors of infinite length, of negligible circular cross-section, and placed 1 metre apart in vacuum, would produce between these conductors a force equal to 2×10^{-7} newton per metre of length (1948)
Thermo dynamic Temperature	kelvin	K	The kelvin, is the fraction 1/273.16 of the thermodynamic temperature of the triple point of water. (1967)
Amount of substance	mole	mol	The mole is the amount of substance of a system, which contains as many elementary entities as there are atoms in 0.012 kilogram of carbon - 12. (1971)
Luminous intensity	candela	cd	The candela is the luminous intensity, in a given direction, of a source that emits monochromatic radiation of frequency 540×10^{12} hertz and that has a radiant intensity in that direction of 1/683 watt per steradian. (1979)

Note : (1) The definitions given in above table are only for information.

(2) When mole is used, the elementary entities must be specified. For example, mol of atoms, mol of molecules, mol of ions or mol of electrons.

2.4.1 Derived Units

All the other units of different physical quantities can be expressed as a combination of these seven base (fundamental) units of SI system. Such units are called derived units.

For example, SI unit of acceleration,

$$= \frac{\text{Unit of displacement}}{(\text{Unit of time})^2} = \frac{\text{m}}{\text{s}^2} = \text{m s}^{-2}$$

unit of work = (unit of force) × (unit of displacement)

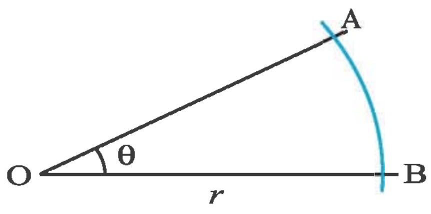
$$= \frac{\text{kg m}}{\text{s}^2} \times (\text{m}) = \text{kg m}^2 \text{ s}^{-2}$$

2.4.2 Supplementary Units

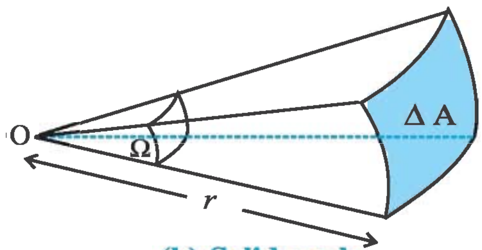
Supplementary physical quantities, their units and symbols in the SI system are shown in Table 2.2.

Table 2.2 : Supplementary Units

No.	Supplementary Physical quantity	SI unit	Symbol	Explanation
1.	Plane angle (θ)	radian	rad	The ratio of length of arc to the radius r of a circle is called plane angle (θ). $\theta = \frac{\text{arc}}{\text{radius}} = \frac{AB}{r}$ (see figure 2.1) one radian is defined as the angle subtended by an arc whose length is equal to the radius. ($1^\circ = \pi/180$ rad)
2.	Solid angle (Ω)	Steradian	Sr	The ratio of the intercepted area of the spherical surface described about the apex O as the centre to the square of radius r is called solid angle. (see fig. 2.1 (b)). $\Omega = \frac{\text{Area }(\Delta A)}{(\text{radius})^2} = \frac{\Delta A}{r^2}$ when $\Delta A = 1\text{m}^2$, $r = 1\text{m}$ then $\Omega = 1$steradian



(a) Plane angle



(b) Solid angle

Figure 2.1

2.4.3 Practical norms for the use of SI system

(1) Unit of every physical quantity should be represented according to its symbol.

(2) No full stop should be used within or at the end of the symbol of a unit. For example, for kilogram, kg should be written instead of kg. or k.g.

(3) Symbols for units do not take plural form. For example m is used to denote many meters also.

(4) The units of physical quantities in numerator and denominator should be written as one ratio only. For example the SI unit of acceleration should be written either as m/s^2 or $m\ s^{-2}$; but not as m/s/s.

(5) Full name of a unit, when it is named after a scientist, is not written with a capital letter; but the symbol for that unit has a capital letter. For example, the unit of force should be written as newton but in symbol it is written as N. The unit of pressure is pascal in symbol it is written as Pa.

2.5 Measurement of Length

You are already familiar with some direct methods for the measurement of length. For example, a metre scale is used for measuring lengths from 10^{-3} m to 10^{2} m. A vernier callipers is used for measuring lengths to an accuracy of 10^{-4} m. A screw gauge and a spherometer can be used to measure lengths down to 10^{-5} m.

To measure the lengths beyond these ranges, and for astronomical distances, we make use of some indirect methods. Now, we shall study a few such methods.

2.5.1 Measurement of large distances : Parallax Method

Large distances such as the distance of a planet or a star from the Earth can be measured with the help of parallax method.

To measure the distance D of a far away planet, we observe it from two different positions (observatories) A and B on the Earth and fix the directions of observations with respect to distant stars.

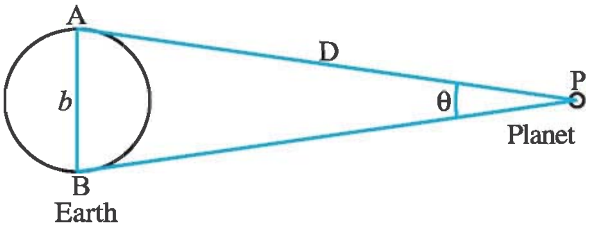


Figure 2.2

For example, observing a planet P simultaneously from two points A and B, situated diametrically opposite on Earth, we get two directions of observations AP and BP.

Since the distance of a planet from the Earth is very large compared to the diameter of the Earth angle θ will be very small (angle θ is called parallax angle). According to the definition of angle in radian,

$$\theta = \frac{\text{arc}}{\text{radius}} = \frac{AB}{AP}$$

$$= \frac{\text{distance between two places of observations, b}}{\text{distance of the planet from the Earth, D}}$$

$$\therefore\ D = \frac{b}{\theta} \qquad (2.5.1)$$

Illustration 1 : The Moon is observed from two diametrically opposite points A and B on the Earth. The angle subtended at the Moon by the two directions of observations is $1°54'$. Given the diameter of Earth to be about 1.276×10^7m, compute the distance of the Moon from the Earth.

Solution : $D = \dfrac{b}{\theta}$

$$\theta = 1°54' = 60' + 54' = 114'$$

$$= \frac{114'}{60}\ \text{degree}$$

$$= \frac{114}{60} \times \frac{\pi}{180}\ \text{rad}$$

$$\therefore\ \theta = 3.32 \times 10^{-2}\ \text{rad}$$

$$b = 1.276 \times 10^7\ \text{m}$$

$$\therefore\ D = \frac{1.276 \times 10^7}{3.32 \times 10^{-2}}$$

$$= 3.84 \times 10^8\text{m}$$

2.5.2 Measurement of the size of a planet or a star

If d is the diameter of a planet, the angle subtended by the diameter of the planet at any point on Earth is called the angular diameter of the planet. The angle α can be determined from any given location on Earth by viewing the diametrically opposite points of the planet through a telescope.

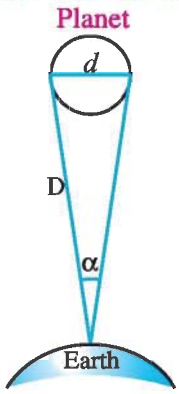


Figure 2.3

$$\alpha = \frac{d}{D}\ \text{(in rad)} \qquad (2.5.2)$$

* **Footnote :** $1°$ (degree) = $60'$ (minute) = $3600''$ (second)

If the distance D of that planet from the Earth is known, the diameter d of the planet can be determined using equation. (2.5.2)

In practice the angle α is very small.

Illustration 2 : The Sun's angular diameter is measured to be 1920″ as the distance D of the Sun from the Earth is 1.496 $\times 10^{11}$ m. What is the diameter of the Sun ?

(1″ = 4.85×10^{-6} rad)

Solution : α = 1920″, D = 1.496×10^{11} m

Using the formula $\alpha = \frac{d}{D}$,

$$d = \alpha D$$

$$= (1920)\ (4.85)\ (10^{-6})\ (1.496 \times 10^{11})$$

$$= 1.393 \times 10^{9} \text{ m}$$

2.5.3 Measurement of very small distances, size of molecule

To measure very small distances like the size of a molecule (10^{-8} m to 10^{-10} m), we cannot use a vernier callipers or micrometre screw gauge or similar instruments. We have to adopt special methods. An optical microscope uses visible light. For visible light the range of wavelength is from 4000 Å to 7000 Å (1 Å = 10^{-10} m). Hence an optical microscope cannot resolve particles with sizes smaller than 4000 Å. The electron microscope, uses electron beam instead of visible light. An electron microscope has resolution of 0.6 Å. It can almost resolve atoms and molecules in a material. (Here, you will be surprised to know that, in electron microscope, electron behaves like a wave, instead of a particle.) In recent times, tunneling microscope has been developed during study of nanotechnology. This has very high resolution needed to estimate the sizes of molecules.

One of the methods of finding the size of a molecule is the method of monomolecular layer. In this method, the thickness of a molecular layer is measured to determine the size of a molecule. For example, the thickness of a layer of steric acid cannot be less than a certain definite value. If we assume that the film has one molecular thickness, this becomes the size or diameter of a molecule.

In physics we deal with very small distances as well as with very large distances. e. g. the size of nucleus of the order of 10^{-14}m and size of galaxy is of the order of 10^{21}m. Therefore, we defined special units of length for short and large distances. These are,

1 fermi = 1 fm = 10^{-15} m

1 angstrom = 1 Å = 10^{-10} m

1 astronomical unit = 1 AU = 1.496×10^{11} m

(Average distance of the Sun from the Earth is called 1 AU.)

1 light year = 1 ly = 9.46×10^{15} m

1 parsec = 3.08×10^{16}

A parsec (pc) is the distance at which 1 AU would subtend on angle of exactly 1 second of arc.

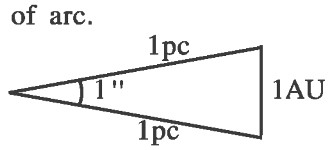


Figure 2.4

$$r = \frac{l}{\theta} = \frac{1 \text{ AU}}{1''}$$

$$= \frac{1.496 \times 10^{11}}{60 \times 60 \times \frac{\pi}{180}} = 3.08 \times 10^{16}\text{m}$$

$$\therefore 1 \text{ PC} = 3.08 \times 10^{16}\text{m}$$

2.6 Measurement of Mass

The amount of matter in the substance is called mass. Since mass is a internal property of matter, it does not depend on external circumstances like temprature and pressure.

The measurement of mass can be done with the help of a simple balance. In this method, the gravitational force on a given object is compared with the gravitational force on some standard object. Remember that the mass playing role in gravitational force (mg) is called gravitational mass. Hence, the mass determined with the help of a simple balance is gravitational mass. The gravitational mass of an object is the same at all places on Earth.

The gravitational force (mg) on an object, of mass m, is called the weight of the object. Hence we can say that the weight of a body at any place depends on the value of gravitational acceleration of that place. For example, the weight of a body on Moon would be different than that on the Earth.

While dealing with atoms and molecules, the kilogram is an inconvenient unit. Therefore, their mass is measured in 'atomic mass unit' called amu. The $\frac{1}{12}^{\text{th}}$ mass of an unexcited atom of C^{12} is called 1 amu. 1 amu = 1.66×10^{-27} kg. It can also be represented as 1u. In physics we come across very wide range of masses from 10^{-30} kg to 10^{55} kg.

Large masses like planets, stars etc. can be determined from Newton's law of gravitation. For measurement of small masses (like atomic particles/atom) we make use of mass spectrograph. (In this method, radius of the trajectory of a charge particle in electric field or magnetic field is proportional to the mass of the charge particle.)

2.7 Measurement of Time

In the early days, time was measured from the length of shadows of objects cast by sunlight. After the invention of the pendulum there is much developement has taken place in the measurement of time. To measure any time interval we need a clock. In order to meet the need for a better standard for time, atomic clocks have been developed. For the measurement of small time intervals, camera, multiflash photography etc. are being used.

2.8 Accuracy and Precision in Measurement

First of all, we shall distinguish between two terms : **accuracy** and **precision. The accuracy of a measurement is a measure of how close the measured value is to the true value of the quantity. Precision tells us to what resolution or limit the quantity is measured.**

For example, your digital watch that shows the time as 10 : 11 : 12 AM is very precise because it has least count of 1 second. On other hand, a grand father clock has no second hand and it gives the time as 10 : 13 AM. The least count of watch is 1 minute which means that it is less precise. If the digital watch runs several minutes slow, then time measured with this watch has no accuracy but it is precise. If the grand father watch shows the correct time then time measured with this watch has high accuracy but less precision. Physical quantity should be measured with high accuracy and high precision. Precision depend upon the least count of instrument. Radius of a sphere measured with micrometer screw gauge will be more precise than measured with verniar callipers.

In the measurement of a physical quantity with accuracy.

(1) Skill of the person doing the experiment

(2) Quality of the instrument used

(3) The Method used for measurement

(4) External and internal factors affecting the result of the experiment.

2.9 Errors in Measurement

When different physical quantities are measured in a laboratory with the help of different apparatus, there would be some inaccuracies in the measurement which must be mentioned along with the result. **The inaccuracy in measurement is called error.**

In physics, the errors in measurement can be broadly classified as :

(1) Systematic error

(2) Random error.

(1) Systematic Error : Systematic errors are those errors that tend to be in one direction, either positive or negative. Such errors cannot be both, positive and negative simultaneously. Some of the sources of systematic errors are as follow :

(a) Instrumental Error : This type of errors arise due to imperfect design or improper calibration of the measuring instrument. For example, when no object is suspended from a spring balance, its pointer shows 0.1g instead of zero. Then all the measurements of more than true wieght will systematically contain this error during experiment.

(b) Error due to Imperfection in Experimental Technique or Procedure : For example, while measuring the temperature of a human body, any improper contact of thermometer with the body would produce an error in the measurement. External factors like temperature, pressure humidity can produce systematic error in measurement.

(c) Personal Error : Such an error arises

arises due to an individual's bias, carelessness in taking observations or improper setting of the apparatus of the experiment.

Systematic errors can be minimised by improving experimental techniques, selecting high quality instruments and removing personal bias.

(2) Random Errors : The random errors are those errors, which arise due to irregular and unpredictable fluctuations in the factors affecting the measurement during experiment.

These types of errors can be both positive and negative. Such errors can be estimated by taking many observations and then taking their mean (average).

2.9.1 Estimation of Errors :

(1) Absolute Error and Average Absolute Error : The magnitude of the difference between an individual measurement and the true value of the quantity is called the **absolute error** of the measurement.

If we do not know the true value, then the average value of measurement is considered as true value.

Suppose the values obtained in several measurement of physical quantity a are a_1, a_2, a_3,...., a_n. If their arithmetic mean is $\overline{a}$, then

$$\overline{a} = \frac{a_1 + a_2 + a_3 + ... + a_n}{n}$$

$$\overline{a} = \frac{1}{n} \sum_{i=1}^{n} a_i$$

The absolute error in individual measurement will be,

$$\Delta a_1 = \overline{a} - a_1$$

$$\Delta a_2 = \overline{a} - a_2$$

...

...

$$\Delta a_n = \overline{a} - a_n$$

Here, Δa_1, Δa_2.... Δa_n are called absolute errors in individual measurement which can be either positive or negative. The arithmetics mean average of absolute error is called **average absolute error.**

$$\Delta \overline{a} = \frac{|\Delta a_1| + |\Delta a_2| + ... + |\Delta a_n|}{n} \quad \text{or}$$

$$\Delta \overline{a} = \frac{1}{n} \sum_{i=1}^{n} |\Delta a_i|$$

Thus, the measure of any physical quantity can represented as :

$$a = \overline{a} \pm \Delta \overline{a}$$

This implies that any measurement of physical quantity a is likely to lie between, $(\overline{a} + \Delta \overline{a})$ and $(\overline{a} - \Delta \overline{a})$

(2) Relative or Fractional Error : The relative error $(\delta \overline{a})$ is the ratio of the mean absolute error $\Delta \overline{a}$ to the mean value $\overline{a}$ of the quantity measured.

$$\therefore \ \delta a = \frac{\Delta \overline{a}}{\overline{a}}$$

(3) Percentage Error : When the relative error is expressed as a percent age it is called percentage error.

$$\text{Percentage error} = \delta a \times 100 \ \%$$

$$= \frac{\Delta \overline{a}}{\overline{a}} \times 100 \ \%$$

Illustration 3 : In an experiment, refractive index of glass was observed to be 1·54, 1·53, 1.44, 1.54, 1.56 and 1.45. Calculate (1) Average absolute error (2) relative error and (3) percentage error. Express the result in terms of absolute error and percentage error.

Solution :

(1) Mean refractive index,

$$\overline{n} = \frac{1{\cdot}54 + 1{\cdot}53 + 1{\cdot}44 + 1{\cdot}54 + 1{\cdot}56 + 1{\cdot}45}{6}$$

$$= 1{\cdot}51$$

Here, an accuracy of two digit after decimal point has been considered. Now the absolute error for each observation will be as follow.

$\Delta n_1 = 1{\cdot}51 - 1{\cdot}54 = -0{\cdot}03$	$\Delta n_4 = 1{\cdot}51 - 1{\cdot}54 = -0{\cdot}03$
$\Delta n_2 = 1{\cdot}51 - 1{\cdot}53 = -0{\cdot}02$	$\Delta n_5 = 1{\cdot}51 - 1{\cdot}56 = -0{\cdot}05$
$\Delta n_3 = 1{\cdot}51 - 1{\cdot}44 = +0{\cdot}07$	$\Delta n_6 = 1{\cdot}51 - 1{\cdot}45 = +0{\cdot}06$

To calculate mean absolute error we take only magnitudes.

$$\Delta \overline{n} = \frac{|\Delta n_1| + |\Delta n_2| + ... + |\Delta n_6|}{6}$$

$$= \frac{|-0{\cdot}03| + |-0{\cdot}02| + |+0{\cdot}07| + |-0{\cdot}03| + |-0{\cdot}05| + |+0{\cdot}06|}{6}$$

$$\Delta \overline{n} = \frac{0{\cdot}26}{6} = 0{\cdot}043 \approx 0{\cdot}04$$

Refractive index of glass with absolute error, $n = 1{\cdot}51 \pm 0{\cdot}04$ i.e. the value of refractive index is between 1.51 and 1.47

(2) Relative error $= \dfrac{\Delta \bar{n}}{\bar{n}} = \dfrac{0{\cdot}04}{1{\cdot}51}$

$= 0{\cdot}02649 = 0{\cdot}03$

(3) Percentage error $= 0{\cdot}03 \times 100 = 3\%$

Refractive index of glass with percentage error $n = 1{\cdot}55 \pm 3\%$

2.9.2 Combination of Errors

When we do an experiment involving several measurements, we must know how the errors in all the measurements combine. For example, in an experiment to determine the density of any substance, we measure the mass and volume of the substance and there would be errors in each of these two measurements. Then we must know what the error will be in the density of the substance.

(1) Errors in Sum and in Difference : Suppose two physical quantities A and B have measured values. $A \pm \Delta A$ and $B \pm \Delta B$ respectively, where ΔA and ΔB are their absolute errors. We wish to find the absolute error ΔZ in the sum

We have by addition, $Z = A + B$

$\therefore Z \pm \Delta Z = (A \pm \Delta A) + (B \pm \Delta B)$

$= (A + B) \pm (\Delta A + \Delta B)$

$\therefore$ The maximum possible absolute error in Z, $\Delta Z = \Delta A + \Delta B$

For the difference,

$Z = A - B$, we have

$\therefore Z \pm \Delta Z = (A \pm \Delta A) - (B \pm \Delta B)$

$= (A - B) \pm \Delta A \mp \Delta B$

$\therefore \quad \pm \Delta Z = \pm \Delta A \mp \Delta B$

Here, there are four possible values $(+\Delta A - \Delta B)$, $(+\Delta A + \Delta B)$, $(-\Delta A - \Delta B)$, $(-\Delta A + \Delta B)$ in which $(+\Delta A + \Delta B)$ is the maximum value. thus, the maximum value of absolute error in Z is again $\Delta A + \Delta B$.

Hence the rule : When two quantities are added or subtracted, the absolute error in the final result is the sum of the absolute errors in the individual quantities.

Illustration 4 : Two resistors of $R_1 = 100 \pm 3\Omega$ and $R_2 = 200 \pm 4\Omega$ are connected in series. Find the maximum absolute error in the equivalent resistance of the combination. Express equivalent resistance with percentage error.

Solution :

$R \pm \Delta R = R_1 + R_2$

$= (100 \pm 3) + (200 \pm 4)$

$= 300 \pm 7\Omega$

$\therefore$ Maximum absolute error $= 7\Omega$

Now, percentage error $= \dfrac{\Delta R}{R} \times 100$

$= \dfrac{7}{300} \times 100$

$= 2.3\ \%$

$\therefore$ Equivalent resistance with percentage error $R = 300 \pm 2.3\ \%$

(2) Errors in product and in division :

Suppose $Z = AB$ and the measured values of A and B are $A \pm \Delta A$ and $B \pm \Delta B$. Then

$Z \pm \Delta Z = (A \pm \Delta A)(B \pm \Delta B)$

$= AB \pm A\Delta B \pm B\Delta A \pm \Delta A\Delta B$

Dividing LHS by Z and RHS by AB we have,

$$1 \pm \frac{\Delta Z}{Z} = 1 \pm \frac{\Delta A}{A} \pm \frac{\Delta B}{B} \pm \frac{\Delta A}{A} \cdot \frac{\Delta B}{B}$$

Since $\dfrac{\Delta A}{A}$ and $\dfrac{\Delta B}{B}$ are very small, we shall ignore their product. Hence, the maximum fractional error in Z is

$$\frac{\Delta Z}{Z} = \frac{\Delta A}{A} + \frac{\Delta B}{B}$$

You can easily verify that this is true for division also.

Hence the rule : When two quantities are multiplied or divided, the maximum relative or fractional error in the result is the sum of the fractional errors in each quantity.

Illustration 5 : In an experiment to determine density of an object mass and volume are recorded as, m = (3 ± 0·12) kg and V = (10 ± 1) m^3 respectively. Calculate fractional error and percentage error in mesurement of density. ($\rho = m/V$)

Solution : $\rho = \dfrac{m}{V}$

Fractional error in density $\dfrac{\Delta\rho}{\rho} = \dfrac{\Delta m}{m} + \dfrac{\Delta V}{V}$

$$= \frac{0.12}{3} + \frac{1}{10}$$

$$= 0.14$$

Percentage error = 0.14 × 100 = 14 %

(3) Error due to the power (index) of a measure quantity :

Suppose $Z = A^2 = A \cdot A$

Then, $\dfrac{\Delta Z}{Z} = \dfrac{\Delta A}{A} + \dfrac{\Delta A}{A}$

$$= 2\frac{\Delta A}{A}$$

Hence, the fractional error in $Z = A^2$, is two times the fractional error in A.

Same way, if $Z = A^n$ than

$$\frac{\Delta Z}{Z} = n\frac{\Delta A}{A}$$

In general, if $Z = \dfrac{A^p B^q}{C^r}$ than

$$\frac{\Delta Z}{Z} = p\frac{\Delta A}{A} + q\frac{\Delta B}{B} + r\frac{\Delta C}{C}$$

Note : The quantity in the formula which has large power is responsible for maximum error. Therefore it should be measured with greater accuracy.

Illustration 6 : In an experiment to determine the density of a sphere, the percentage error in measurement of mass is 0.26 % and percentage error in measurement of radius is 0.38 %. What will be percentage error in the determination of its density ?

Solution : $\dfrac{\Delta m}{m} \times 100 = 0.26\ \%$;

$\dfrac{\Delta r}{r} \times 100 = 0.38\ \%$

Density of sphere $\rho = \dfrac{m}{V} = \dfrac{m}{\frac{4}{3}\pi r^3}$

$\therefore$ Error in density $\dfrac{\Delta\rho}{\rho} = \dfrac{\Delta m}{m} + 3\dfrac{\Delta r}{r}$

Percentage error in density = 0.26 % + 3 (0.38 %)

= 1.40 %

Illustration 7 : If the formula for a physical quantity is $W = \dfrac{a^4 b^3}{c^{\frac{1}{3}}\sqrt{d}}$ and if percentage errors in the measurement of a, b, c and d are 1%, 3%, 3% and 4% respectively. Calculate percentage error in W.

Solution : $W = \dfrac{a^4 b^3}{c^{\frac{1}{3}}\sqrt{d}}$

Percentage error in W,

$$\frac{\Delta W}{W} = 4\frac{\Delta a}{a} + 3\frac{\Delta b}{b} + \frac{1}{3}\frac{\Delta c}{c} + \frac{1}{2}\frac{\Delta d}{d}$$

$$= 4\ (1\ \%) + 3(3\ \%)$$

$$+ \frac{1}{3}(3\ \%) + \frac{1}{2}\ (4\ \%)$$

$$= 16\ \%$$

Illustration 8 : The period of oscillation of a simple pendulum is given by, $T = 2\pi\sqrt{\dfrac{l}{g}}$

The length l of the pendulum is about 10cm and is known to 1mm accuracy. The period of oscillation is about 0.5s. The time of 100 oscillations is measured with a watch of 1s resolution. Calculate percentage error in measurement of g.

Solution : $T = 2\pi\sqrt{\dfrac{l}{g}}$ $\quad\therefore T^2 = \dfrac{4\pi^2 l}{g}$

or, $g = \dfrac{4\pi^2 l}{T^2}$

$$\therefore \frac{\Delta g}{g} = \frac{\Delta l}{l} + 2\frac{\Delta T}{T}$$

Now, $\Delta l = 1mm = 0.1cm$, $l = 10cm$

Total time $t = nT = 0.5 \times 100 = 50$ s and $\Delta t = 1$ s

Now, $T = \frac{t}{n}$ and $\Delta T = \frac{\Delta t}{n}$ hence,

$$\frac{\Delta T}{T} = \frac{\Delta t}{t}$$

$$\therefore \frac{\Delta g}{g} = \frac{\Delta l}{l} + 2\frac{\Delta t}{t}$$

$$\therefore \frac{\Delta g}{g} = \frac{0.1}{10} + 2 \times \frac{1}{50} = 0.05$$

$\therefore$ Percentage error in measurement of $g = 0.05 \times 100 = 5$ %.

2.10 Significant Figures

The accuracy of every measurement has some limitation depending on the least count of the instrument used. For example, time measured with the help of a watch having a second hand can measure the time up to accuracy of 1 second.

Suppose you are measuring the length of a pencil. Keep one end of the pencil on the zero scale of metre and suppose its other end lies between 12.3cm and 12.4cm. The least count of metre scale is 0.1cm. Therefore it does not have any marking between 12.3cm and 12.4cm. Hence we estimate the length as 12.38cm. Here we are centain about digit 1, 2 and 3 but are uncertain for the last digit 8.

The number of digit in a measurement about which we are certain plus one additional digit which is uncertain are known as **significant figures**.

In the above example, 12.38cm has four significant digits 1, 2, 3 and 8.

The larger the number of significant figures obtained in a measurement, the greater is the accuracy of measurement. The number of significant digits depends upon the least count of the instrument being used for a measurement. For exmaple, the radius of the rod measured with vernier callipers is $r = 0.25$cm. For the same rod the radius measured with micrometre screw gauge should be 0.254cm. In the first case, there are two significant digit (2 and 5). While in second case the number of significant digits are three (2, 5 and 4), which shows that the second measurement is more precise.

In mathematics, all the numbers are definite figures. The question of significance arises only when number represents the measured value of a physical quantity.

2.10.1 Rules for Determining Number of Significant Figures :

(1) All the non–zero digits are significant. For example, in measurement of mass 125.63g, there are five significant digits which are 1, 2, 5, 6 and 3.

(2) All the zeros between two non–zero digits are significant, no matter where the decimal point is, if at all.

e.g. in 125.004cm there are six significant figures.

(3) If the number is less than 1, the zero (s) on the right of decimal point but to the left of the first non–zero digit are not significant. e.g., In 0.0 01507, the underline zeros are not significant. There are four significant figures in this number.

(4) In a number without decimal point, the zeros on the right side of the last non–zero digit are not significant.

e.g. 132m = 13200cm = 132000mm has three (1, 3 and 2) significant digits. Here there is no change in the number of significant digits because the zeros after decimal point indicate the position value only. Thus by changing the units even if number of zero increases but the number of significant digits does not change.

(5) The trailing zero (s) in a number with a decimal point are significant.

e.g. 7.900 and 0.0 7 9 0 0 have four significant figures each.

Illustration 9 : Write down the number of significant figures/digits in the following :

(1) 0.003 m^2

(2) 0.1570 g cm^{-2}

(3) 2.64×10^{24} kg

(4) 7.590 J

(5) 6.032 N m^{-2}

(6) 3.012×10^{-4} m^2

Solution :

(1) 0.003 m^2 has only one significant figure (3).

(2) 0.1570 g cm^{-2} has four significant figures. (1, 5 ,7 and 0)

(3) 2.64×10^{24} kg has three significant figures. (2, 6 and 4)

(4) 7.590 J has four significant figures. (7, 5, 9 and 0)

(5) 6.032 N m^{-2} has four significant figures. (6, 0, 3 and 2)

(6) 3.012×10^{-4} m^2 has four significant figures. (3, 0, 1 and 2)

2.10.2 Significant Figures in Addition, Subtraction, Multiplication and Division

Any physics experiment involves a series of measurement and each of these measurement is made upto a certain degree of accuracy. The number of the significant figures depend upon the least count of the measuring instrument. Suppose the resistance of different resistors are measured using meter (ohm meter) of different least count are :

$R_1 = 5.67\,\Omega$, $R_2 = 12.345\,\Omega$ and $R_3 = 0.7\,\Omega$

Here, total resistance would be,

$$R = 5.67\,\Omega + 12.345\,\Omega + 0.7\,\Omega = 18.715\,\Omega$$

Now, the questions is, are we justified in carrying out sum like this ? In the measurement of R_1 (=5.67 Ω) we do not have the information about the third digits after decimal point. In the measurement of R_3 we do not have information about second and third digits after decimal point, it has only one digit after the decimal point. This shows that precision in the measurement of R_3 less than other two resistors. Therefore, in the sum (i.e. 18.715 Ω) second and third digits become insignificant and answer should be expressed to one decimal place, as 18.7 Ω only.

Thus, any result with more than one insignificant or uncertain digit should be **rounded off** up to the correct number of significant digits. For this purpose the following rules should be observed :

(1) If the insignificant digit to be dropped is less than 5, then the preceding digit is left unchanged. For example, l = 10.7<u>43</u> cm is rounded off up to three significant digit as 10.7 cm.

(2) If the digit to be dropped is more than 5, then the preceding digit is increased by 1.

For example, l = 10.6<u>8</u> cm = 10.7cm (Rounded off up to three significant digit)

(3) If the digit to be dropped is 5 then the preceding digit is increased by one if it is odd and is left unchanged if it is even.

e.g. l = 10.4<u>5</u> cm = 10.4 cm and

l = 10.5<u>5</u> cm = 10.6 cm

Addition and Subtraction : The following points should be observed while carrying out the addition / subtraction of two significant numbers :

(1) If all the numbers are integers, the summation or subtraction should be carried out in a normal way.

(2) In addition or subtraction, the final result should retain as many decimal places as are there in number with the least decimal places. In above example, the resistance $R_3 = 0.7\,\Omega$ has only one significant digit after decimal place. The total resistance R = 18.715 Ω should therefore be rounded off to R = 18.7 Ω

Multiplication and Divisions : In any measurement the last digit is uncertain. Multiplication of any number with least significant digit is also insignificant. But the final result should contain only one insignificant digit. Therefore, the following points should be observed while carrying out the multiplication / division.

(1) In multiplication or in division, the final result should retain as many significant as are there in the original number with the least significant.

For example : (i) Suppose the length and breadth of a rectangular plate is 2.613 cm and 1.2 cm respectively.

Hence, area of the plate = 2.613 cm × 1.2 cm = 3.1356 cm^2

But 1.2 cm is the least significant number which has only two significant figures. Therefore the area (= 3.1356 cm^2) should be represented by a number with two significant digit.

Hence, 2.613 cm × 1.2 cm = 3.1 cm^2

(ii) Suppose, the mass of an object is m = 3.523 g and volume is V = 1.47 cm^3.

The density of an object $\rho = \dfrac{3.523g}{1.47\text{cm}^3}$

= 2.4296552 = 2.43 g cm^{-3}

Here, the density is respresented with three significant figures because the measurement of volume has three significant figures.

(2) When two numbers are to be multiplied / divided and out of them which is not a measurement is a definite number, the integers and fraction that occurs in general in physics equations are definite number.

e.g. In equation $v^2 - v_0^2 = 2ad$ the coefficient 2 is exactly 2 and it has infinite number of significant figures (2.000........). In such cases we should not consider the significant figures of definite numbers.

Illustration 10 : The diameter of a sphere is 4.24 cm. Calculate the surface area of the sphere to the correct number of significant figures.

Solution : Diameter D = 4.24 cm

Surface area of sphere

$$= 4\pi R^2 = 4\pi\left(\frac{D}{2}\right)^2$$

$$= 4 \times 3.14 \times \left(\frac{4.24}{2}\right)^2$$

$$= 56.478 \text{ cm}^2$$

$$= 56.5 \text{ cm}^2$$

(In the above equation 4 and 2 are definite number. We do not consider them and measure of D has three significant figures. So the answer is rounded to three significant figures.)

2.11 Dimensions and Dimensional Formulae

Any physical quantity (derived quantity) can be expressed in terms of some combination of seven fundamental or base quantities. For convenience the base quantities are represented by a letter symbol. Generally, mass is denoted by 'M', length by 'L', time by 'T' and electric current by 'A'. The thermodynamic temperature the amount of substance and the luminous intensity are denoted by the symbols 'K', 'mol' and 'cd' respectively. When a physical quantity is expressed with appropriate powers (or exponents) of M, L, T, K, A ... then such an expression for physical quantity is called the **dimensional formula.** The power or exponents of M, L, T, ... are called **dimensions** of that quantity. The dimensional formula of physical quantity is expressed by square brackets [ ] along with the symbol of a physical quantity.

For example, (i) the dimensional formula of velocity can be obtained as follows :

$$\text{Velocity} = \frac{\text{displacement}}{\text{time}}$$

$$\therefore [v] = \frac{\text{Dimension of length}}{\text{Dimension of Time}}$$

$$= \frac{L^1}{T^1}$$

$$= L^1 T^{-1}$$

$$= M^0 L^1 T^{-1}$$

Here, $M^0 L^1 T^{-1}$ is dimensional formula of velocity. The dimensions of velocity are 0 in mass, 1 in length and −1 in time.

(ii) Dimensions of kinetic energy can be obtained as follows :

$$K = \frac{1}{2}mv^2$$

$$[K] = [m]\ [v]^2$$

(Here, $\frac{1}{2}$ is a number and it is dimensionless)

$$= (M^1)\ (M^0 L^1 T^{-1})^2$$

$$[K] = M^1 L^2 T^{-2}$$

Dimensional formule of some physical quantities are given in Table 2.3.

2.11.1 Dimensional Analysis

The method of obtaining the solutions to some several problems in physics by using the formula is called **dimensional analysis.**

Uses of Dimensional Analysis :

(a) To obtain the relation between the units of some physical quantity in two different systems of units.

(b) To check the dimensional consistency of an equation connecting different physical quantities.

(c) To derive an equation for a physical quantity in terms of other (related) physical quantities.

(a) To obtain the relation between the units of a physical quantity in two different systems of units :

The unit of work in MKS system is joule (J) and that in CGS system is erg. The relation between joule and erg can be obtained as follows :

Dimensional formula for work :

$[W] = M^1 \; L^2 \; T^{-2}$

In MKS system		In CGS system
M(kg)	=	10^3 M (g)
L (m)	=	10^2 L (cm)
T (s)	=	10^0 T (s)

$$M^1L^2T^{-2} = (10^3M)^1 \; (10^2L)^2 \; (10^0T)^{-2}$$

$$= 10^3(M^1) \; 10^4 \; (L^2) \; (T^{-2})$$

$$= 10^7 \; M^1 \; L^2 \; T^{-2}$$

So, MKS Unit of work = $10^7 \times$ CGS unit of work.

$\therefore$ 1 joule = 10^7 erg

(b) To verify the dimensional consistency of an equation connecting different physical quantities :

In any equation relating different physical quantities, if the dimensions of terms on both sides are same then that equation is said to be consistent dimensionally.

For exmaple, the centripetal force acting on an object in uniform circular motion is given by,

$F = \dfrac{mv^2}{r}$ where, m = mass of the object,

v = velocity of the object and

r = radius of the circular path.

Now, we will check the dimensional consistency of this equation,

For left side of the equation

$[F] = M^1 \; L^1 \; T^{-2}$

For the term on right side of equation,

$$\left[\frac{mv^2}{r}\right] = \frac{[m][v]^2}{r}$$

$$= \frac{(M^1)(L^1T^{-1})^2}{(L^1)}$$

$$= \frac{(M^1)(L^2T^{-2})}{(L^1)}$$

$$= M^1L^1T^{-2}$$

Thus, $[F] = \left[\dfrac{mv^2}{r}\right]$

Since the dimensions of LHS and RHS are same, the given equation is dimensionally correct.

Note : If an equation has constants which are dimensionless, it can not be verified with dimension analysis.

(c) To obtain the equation for a physical quantity in terms of an other physical quantities :

Suppose we want to obtain the expression for the periodic time of a simple pendulum. The periodic time (T) of simple pendulum depends on the length (l) of the pendulum, the mass (m) of the bob and gravitational acceleration (g).

Suppose, periodic time $T \propto m^a$

$\propto l^b$

$\propto g^c$

$T \propto m^a \; l^b \; g^c$

$$\therefore \; T = km^a \; l^b \; g^c \qquad (2.11.1)$$

where, k is constant of proportionality and it is dimensionless, $a, b, c \in R$

Writing the dimensional formula for both the sides of equation (2.11.1)

$$(M^0L^0T^1) = (M^1)^a \; (L^1)^b \; (M^0L^1T^{-2})^c$$

$$= (M^a) \; (L^b) \; (M^0L^cT^{-2c})$$

$$M^0L^0T^1 = M^a \; L^{b+c} \; T^{-2c}$$

Comparing the dimensions of corresponding quantities on both the sides, we get

$a = 0$ $\quad\therefore c = -\frac{1}{2}$

$b + c = 0$ $\quad\therefore b = \frac{1}{2}$

$-2c = 1$

Substituting these values of a, b and c in equation (2.11.1)

$$T = km^0 l^{\frac{1}{2}} g^{-\frac{1}{2}}$$

or $T = k\sqrt{\frac{l}{g}}$

The value of k is obtained experimentally and it is 2π. Hence,

$$T = 2\pi\sqrt{\frac{l}{g}} \qquad (2.11.2)$$

which is the formula for periodic time of a simple pendulum.

2.11.2 Limitations of dimensional analysis

(1) In any dimensional formula containing M, L and T, we get at the most three equations by equating power of M, L and T. Hence dimensional analysis cannot be used to derive the exact form of a physical relation if a physical quantity depends upon more than three physical quantities.

(2) Information about dimensionless constant cannot be obtained. e.g. In $T = k\sqrt{\frac{l}{g}}$, the value of $k = 2\pi$ can be obtained with the help of experiment.

(3) Dimensional analysis can not be used to derive relations involving trigonometrical, exponential and logarithmic functions. Such functions are dimensionless. For example, in $\sin\omega t$ and e^{-kx} functions ωt and kx are dimensionless respectively.

(4) This method is not useful if a constant of proportionality is not dimensionless. For example, in the equation $F = G\frac{m_1 m_2}{r^2}$, G has a unit of N m^2 kg^{-2}. Such equations can not be obtained by dimensional analysis.

Illustration 11 : If the velocity of light is taken as the unit of velocity and year as the unit of time, then find the unit of distance. (velocity of light = 3×10^8 m s^{-1})

Solution :

Distance = velocity $\times$ time

Unit of distance = unit of velocity $\times$ unit of time

$= (3 \times 10^8 \text{ m s}^{-1}) \times (1 \text{ year})$

$= (3 \times 10^8 \text{ m s}^{-1}) \times (365.25 \times 24 \times 3600\text{s})$

$= 9.468 \times 10^{15}$ m

This unit of distance is called **light year.**

Illustration 12 : In a new system, the unit of length, mass and time are chosen to be 10cm, 10g and 0.1s respectively. What will be the new unit of force in newton in this system ?

Solution :

Dimensional formula of force [F]

$= M^1L^1T^{-2}$

Unit of force in new system

$= [(10\text{g})^1 (10\text{cm})^1 (0.1\text{s})^{-2}]$

$= (10^{-2}\text{kg})^1 (10^{-1}\text{m})^1 (10^2\text{s}^{-2})$

$= 10^{-1}$kg m s^{-2}

$= 0.1$ newton

Illustration 13 : When a metallic rod through which heat is being conductivity conducting heat through it is in thermal steady state, the amount of heat passing through it in time t is given by $Q = \frac{kA(T_1 - T_2)t}{L}$ where k = thermal conductivity of the material of the rod, A = cross sectional area of the rod, T_1 and T_2 are the temperatures of hot and cold ends, respectively, of the rod, t = time and L = lengh of the rod. Obtain the dimensional formula for k.

Solution :

$$Q = \frac{kA(T_1 - T_2)t}{L}$$

$$\therefore k = \frac{QL}{A(T_1 - T_2)t} \qquad (1)$$

where, heat energy, $[Q] = M^1L^2T^{-2}$

length, $[L] = L^1$

area, $[A] = L^2$

difference of temperature,

$(T_1 - T_2) = [\Delta T] = K^1$

time, $[t] = T^1$

Note that here we have included K (for temperature) along with M, L and T. Substituting these dimensional formula in equation (1), we get,

$$[k] = \frac{M^1L^2T^{-2}L^1}{L^2K^1T^1} = M^1L^1T^{-3}K^{-1}$$

Note : In some books θ is used in place of K.

Illustration 14 : Obtain the dimensional formula of the following physical quantities :

(i) electric charge (Q), (ii) potential difference (V), (iii) capacitance (C) , (iv) resistance (R).

The formulas connecting these physical quantities are as follows :

$Q = It$, $W = VIt$, $Q = CV$, $V = IR$, where I = electric current, t = time, W = energy.

Solution :

(i) $Q = It$

$\therefore [Q] = M^0L^0A^1T^1$

Where A is the symbol for ampere.

(ii) $W = VIt$

$$\therefore [V] = \frac{M^1L^2T^{-2}}{AT^1} = M^1L^2T^{-3}A^{-1}$$

(iii) $Q = CV$

$$C = \frac{Q}{V} = \frac{It}{W/It}$$

$$\therefore C = \frac{I^2t^2}{W} \Rightarrow [C] = \frac{A^2T^2}{M^1L^2T^{-2}}$$

$\therefore [C] = M^{-1}L^{-2}T^4A^2$

(iv) $V = IR$

$$\therefore R = \frac{V}{I} = \frac{W/It}{I} = \frac{W}{I^2t}$$

$$\Rightarrow [R] = \frac{M^1L^2T^{-2}}{A^2T^1}$$

$\therefore [R] = M^1L^2T^{-3}A^{-2}$

Illustration 15 : By taking velocity, time and force as base quantities, obtain the dimensional formula of mass.

Solution :

Use the symbols F for force, T for time and v for velocity

Force = mass $\times$ acceleration

$$= \text{mass} \times \frac{\text{velocity}}{\text{time}}$$

$$\therefore \text{mass} = \frac{\text{force} \times \text{time}}{\text{velocity}}$$

$$\therefore [m] = \frac{F^1T^1}{v^1}$$

$\therefore [m] = F^1T^1v^{-1}$

Illustration 16 : Heat produced in a current carrying conducting wire depends on current I, resistance R of the wire and time t for which current is passed. Using these facts, obtain the formula for heat energy.

Solution :

Suppose heat energy $H \propto I^a R^b t^c$

$\therefore H = k\, I^a R^b t^c$ (1)

(where $a, b, c \in$ R and k is a dimensionless constant)

Writing the dimensional formula for all the physical quantities in eqn. (1),

$M^1L^2T^{-2} = (A)^a (M^1L^2T^{-3}A^{-2})^b (T)^c$

$= A^{a-2b} M^bL^{2b}T^{c-3b}$

Equating the indices on both the sides, we get,

$a - 2b = 0$, $b = 1$, $-3b + c = -2$

Thus, $a = 2$ and $c = 1$

Substituting these values of a, b and c in eqn. (1),

$\therefore H = kI^2Rt$

Experimentally $k = 1$ is obtained

$\therefore H = I^2Rt$

Table 2.3 : Some physical quantities, their SI units and dimensional formula :

Sr. No.	Physical Quantity	Relation with other Physical Quantities	Dimensional Formula	SI Unit
1.	Distance (*d*)	–	$M^0L^1T^0$	m
2.	Mass (*m*)	–	$M^1L^0T^0$	kg
3.	Time (T)	–	$M^0L^0T^1$	s
4.	Plane angle (θ)	arc / radius	$M^0L^0T^0$	rad
5.	Solid angle (Ω)	Area / $(\text{radius})^2$	$M^0L^0T^0$	sr
6.	Area (A)	length × breadth	$M^0L^2T^0$	m^2
7.	Volume (V)	length × breadth × height	$M^0L^3T^0$	m^3
8.	Density (ρ)	mass / volume	$M^1L^{-3}T^0$	kg m^{-3}
9.	Speed/Velocity(*v*)	Distance / time	$M^0L^1T^{-1}$	m s^{-1}
10.	Acceleration (*a*)	Change in velocity / time	$M^0L^1T^{-2}$	m s^{-2}
11.	Force (F)	mass × acceleration	$M^1L^1T^{-2}$	kg m s^{-2} (newton)
12.	Work (W)	Force × distance	$M^1L^2T^{-2}$	joule (J)
13.	Power (P)	work / time	$M^1L^2T^{-3}$	J/s, watt
14.	Energy, (Kinetic energy, Potential energy, Heat energy etc)	work	$M^1L^2T^{-2}$	joule (J)
15.	Momentum (p)	mass × velocity	$M^1L^1T^{-1}$	kg m s^{-1}
16.	Pressure (P)	Force / Area	$M^1L^{-1}T^{-2}$	N m^{-2}, Pa
17.	Periodic time (T)	time	$M^0L^0T^1$	s
18.	Frequency (*f*)	1 / periodic time	$M^0L^0T^{-1}$	s^{-1}, Hz
19.	Angular displacement (θ)	arc / radius	$M^0L^0T^0$	rad
20.	Angular velocity(ω)	angular displacement / time	$M^0L^0T^{-1}$	rad s^{-1}
21.	Angular acceleration (α)	angular velocity / time	$M^0L^0T^{-2}$	rad s^{-2}
22.	Moment of Inertia (I)	mass × $(\text{distance})^2$	$M^1L^2T^0$	kg m^2
23.	Torque (τ)	Force × Perpendicular distance	$M^1L^2T^{-2}$	N m
24.	Impulse of force	Force × time	$M^1L^1T^{-1}$	N s^{-1}
25.	Surface tension (T)	Force / distance	$M^1L^0T^{-2}$	N m^{-1}

26.	Specific Heat (C)	$\frac{\text{Heat energy}}{\text{mass} \times \text{temperature}}$	$M^0L^2T^{-2}K^{-1}$	J kg^{-1} K^{-1}
27.	Thermal Conductivity	$\frac{\text{Heat energy} \times \text{thickness}}{\text{Area} \times \text{temperature} \times \text{time}}$	$M^1L^1T^{-3}K^{-1}$	J m^{-1} s^{-1} K^{-1}
28.	Electric Current (I)	–	$M^0L^0T^0A^1$	A
29.	Electric Charge (Q)	Electric Current × time	$M^0L^0T^1A^1$	C (Coloumb)
30.	Potential Differnce (V)	work / charge	$M^1L^2T^{-3}A^{-1}$	V (Volt)
31.	Resistance (R)	$\frac{\text{Potential difference}}{\text{current}}$	$M^1L^2T^{-3}A^{-2}$	Ω (ohm)
32.	Capacitance (C)	Charge / Potential Difference	$M^{-1}L^{-2}T^4A^2$	F (faraday)

Table 2.4 : Multiples and Submultiples of SI units

Multiples

Value	Prefix	Symbol
10^{18}	exa	E
10^{15}	peta	P
10^{12}	tera	T
10^9	giga	G
10^6	mega	M
10^3	kilo	k
10^2	hecto	h
10	deca	da

Submiltiples

Value	Prefix	Symbol
10^{-1}	deci	*d*
10^{-2}	centi	*c*
10^{-3}	mili	*m*
10^{-6}	micro	μ
10^{-9}	nano	*n*
10^{-12}	pico	*p*
10^{-15}	femto	*f*
10^{-18}	atto	*a*

SUMMARY

1. The standard measure of any quantity is called the unit of that physical quantity.
2. The number of physical quantities is large. We select a limited number of physical quantities to express other quantities. These physical quantities are called fundamental or base quantities. The other physical quantities are called derived physical quantities.
3. There are seven fundamental quantities in SI system of units. They are length, mass, time, electric current, thermodynamic, temperature, luminious intensity and amount of substance.
4. There are two supplementary quantities in SI system. They are plane angle (θ) and solid angle (Ω) and their units are radian (rad) and steradian (sr) respectively.

5. Small distances can be measured using either metrescale, vernier callipers or micrometer screw guage. A screw gauge can be used for measuring length in order of 10^{-5}m for the measurement of large distances or astronomical distances indirect methods are used. e.g. Parallax method.

6. **Mass and weight** : The amount of matter in any substance is called mass (*m*). It is an internal property of substance. The gravitational force acting on the substance is called weight (W).

7. The accuracy of measurement is a measure of how close the measured value is to the true value of the quantity. Precision tells us about the resolution or the limit to which the quantity is measured.

8. **Error** : Inaccuracy in measurement of physical quantity is called error. There are two types of error. (i) Systematic error (ii) Random error.

9. The magnitude of the difference between the individual measured value of any physical quantity and its mean value is called absolute error.

10. The ratio of the mean absolute error to the mean value of quantity measured is called relative error or fractional error. When the relative error is expressed in percentage, it is called the percentage error.

11. **Combination of errors** : When more than one physical quantities are measured, then the maximum error that occurs in the final result will be as follows :

Sr. No.	Mathematical operation	Error
1.	Addition : $Z = A + B$	$\Delta Z = \Delta A + \Delta B$
2.	Subtraction : $Z = A - B$	$\Delta Z = \Delta A + \Delta B$
3.	Division : $Z = \frac{A}{B}$	$\frac{\Delta Z}{Z} = \frac{\Delta A}{A} + \frac{\Delta B}{B}$
4.	Multiplication : $Z = A \cdot B$	$\frac{\Delta Z}{Z} = \frac{\Delta A}{A} + \frac{\Delta B}{B}$
5.	Exponential power : $Z = A^n$	$\frac{\Delta Z}{Z} = n\frac{\Delta A}{A}$

12. The number of digits in a measurement about which we are certain plus one additional digit which is uncertain are known as significant digits. The greater the numbers of significant digits obtained in a measurement, the more accurate is the measurement.

13. When any physical quantity is expressed with appropriate powers of M, L, T, ... then such an expression for physical quantity is called its dimensional formula.

14. With the help of dimensional analysis, we can obtain relation between the units of different system of units, can verify dimensional consisteney of physical equation as well as relation between different physical quantities can be obtained.

EXERCISES

Choose the correct option from the given options :

1. Which of the following physical quantities is a derived one ?

(A) mass (B) force (C) plane angle (D) time

2. Which of the following physical quantities is not a fundamental physical quantity in the SI system ?

(A) Luminous intensity (B) Electric current

(C) Solid angle (D) Quantity of matter

3. $\frac{1\mu m}{1 fm}$ = ...

(A) 10^9 (B) 10^{-9} (C) 10^{15} (D) 10^6

4. Unit of plane angle in SI system is ....

(A) degree (B) radian (C) steradian (D) candela

5. The percentage error in the distance 125.0 $\pm$ 0.5 cm is ....

(A) 4 % (B) 0.04 % (C) 0.4 % (D) 40 %

6. In an experiment to determine the density of a cube, the percentage error in the measurement of mass is 0.26 % and percentage error in the measurement of length is 0.38%. What will be percentage error in the determination of its density ?

(A) 14 % (B) 1.40 % (C) 1.04 % (D) 1.44 %

7. If Z = A^3, then relative error in Z is ....

(A) $(\Delta A)^3$ (B) $\frac{(\Delta A)^3}{A}$ (C) $3\frac{\Delta A}{A}$ (D) $\frac{\Delta A}{A}$

8. If $x = ab^{-1}$ and Δa and Δb are the errors in the measurement of a and b respectively, then the maximum percentage error in the value of x will be ....

(A) $\left(\frac{\Delta a}{a} + \frac{\Delta b}{b}\right) \times 100$ (B) $\left(\frac{\Delta a}{a} - \frac{\Delta b}{b}\right) \times 100$

(C) $\left(\frac{\Delta a}{a-b} + \frac{\Delta b}{a-b}\right) \times 100$ (D) $\left(\frac{\Delta a}{a-b} - \frac{\Delta b}{a-b}\right) \times 100$

9. The dimensional formula of physical quantity Z is $M^aL^bT^{-c}$. The percentage error in measurement of mass, length and time are α %, β % and γ % respectively. The percentage error in Z would be ....

(A) $(\alpha + \beta + \gamma)$ % (B) $(\alpha + \beta - \gamma)$ %

(C) $(a\alpha + b\beta + c\gamma)$ % (D) $(a\alpha + b\beta - c\gamma)$ %

10. A student performs an experiment for determination of $g\left(= \frac{4\pi^2 l}{T^2}\right)$. The error in length l is Δl and in time T is ΔT and n is the number of times readings were taken. The measurement of g is most accurate for ....

	Δl	ΔT	n
(A)	5mm	0.2s	10
(B)	5mm	0.2s	20
(C)	5mm	0.1s	10
(D)	1mm	0.1s	50

11. When a current of (2.5 $\pm$ 0.5) A flows through a wire, it develops a potential difference of (20 $\pm$ 1)V. The resistance of the wire is ....

(A) $(8 \pm 2)\Omega$ (B) $(8 \pm 1.5)\Omega$

(C) $(8 \pm 0.5)\Omega$ (D) $(8 \pm 3)\Omega$

12. Numbers of significant figures in 5.055 and 0.005055 are .... respectively.

(A) 4 and 3 (B) 3 and 3 (C) 4 and 4 (D) 4 and 6.

13. The number of significant figures in 0.0060 is ....

(A) 4 (B) 3 (C) 2 (D) 1.

14. The gravitational force F between two masses m_1 and m_2 separated by a distance r is given by $F = G\frac{m_1 m_2}{r^2}$, where G is the universal gravitational constant. What are the dimensions of G ?

(A) $M^{-1}L^3T^{-2}$ (B) $M^1L^3T^{-2}$ (C) $M^1L^3T^{-3}$ (D) $M^{-1}L^2T^{-3}$

15. According to quantum theory, the energy E of a photon of frequency f is given by $E = hf$, where h is planck's constant. What is dimensional formula for h ?

(A) $M^1L^2T^{-2}$ (B) $M^1L^2T^{-1}$ (C) $M^1L^2T^1$ (D) $M^1L^2T^2$

16. The dimensional formula of 'light year' is...

(A) L^{-1} (B) T^{-1} (C) L^1 (D) T^1

17. What is the dimensional formula of a solid angle ?

(A) $M^1L^1T^1$ (B) $M^0L^0T^1$ (C) $M^1L^0T^{-2}$ (D) $M^0L^0T^0$

18. The time dependance of a physical quantity P is given by $P = P_0 \exp(-\alpha t^2)$. where α is a constant and t is the time. P is the pressure. The dimensional formula of α is...

(A) $M^0L^0T^{-2}$ (B) $M^0L^0T^2$

(C) $M^0L^0T^0$ (D) $M^1L^{-1}T^{-2}$

19. If energy (E), momentum (p) and force (F) were to be chosen as fundamental units, what would be the dimensions of mass in the new system ?

(A) $E^{-1}P^2F^0$ (B) $E^1P^{-2}F^0$

(C) $E^{-1}P^2F^{-2}$ (D) $E^{-2}P^1F^2$

20. The number of particles crossing a unit area perpendicular to the X–axis in unit time is given by,

$n = -D\left(\frac{n_2 - n_1}{x_2 - x_1}\right)$ Where n_1 and n_2 are the number of particles per unit volume at $x = x_1$ and $x = x_2$ respectively and D is diffusion constant. The dimensions of D are...

(A) $M^0L^1T^{-2}$ (B) $M^0L^2T^{-4}$ (C) $M^0L^1T^{-3}$ (D) $M^0L^2T^{-1}$

21. The speed of gravity waves in water is proportional to $\lambda^{\alpha}\ \rho^{\beta}\ g^{\gamma}$ where λ is the wavelength, ρ is the density of water and g is acceleration due to gravity. Which of the following relations is correct ?

(A) $\alpha = \beta = \gamma$ (B) $\alpha \neq \beta \neq \gamma$

(C) $\alpha \neq \gamma = \beta$ (D) $\alpha = \gamma \neq \beta$

22. If the distance between two charges is $2a$, then the dipole moment of this system is given by $p = (2a)q$, where q is electric charge. The dimensional formula of p is ....

(A) $M^0L^{-1}T^1A^1$ (B) $M^0L^1T^{-1}A^{-1}$ (C) $M^0L^1T^{-1}A^1$ (D) $M^0L^1T^1A^1$

23. If 1 $gcms^{-1} = x$ N s then x = ...

(A) 1×10^{-1} (B) 3.6×10^{-3} (C) 1×10^{-5} (D) 6×10^{-4}

24. The equation of stationary wave is $y = 2A\sin kx \cos wt$ (in metre). Where A and x are in metre. w is angular frequency. Dimensions of A/k are ....

(A) $M^0L^0T^0$ (B) $M^0L^{-2}T^0$ (C) $M^0L^{-1}T^1$ (D) $M^0L^2T^0$

25. In $\left(P + \frac{a}{V^2}\right)(V - b) = RT$ equation, the dimensional formula of $\frac{a}{b}$ will be .... where, P = Pressure, V = Volume and T is temperature.

(A) $M^1L^2T^{-2}$ (B) $M^1L^2T^{-2}K^1$ (C) $M^1L^{-2}T^2$ (D) $M^1L^2T^{-2}K^{-1}$

ANSWERS

1. (B)	**2.** (C)	**3.** (A)	**4.** (B)	**5.** (C)	**6.** (B)	
7. (C)	**8.** (A)	**9.** (C)	**10.** (D)	**11.** (A)	**12.** (C)	
13. (C)	**14.** (A)	**15.** (B)	**16.** (C)	**17.** (D)	**18.** (A)	
19. (A)	**20.** (D)	**21.** (D)	**22.** (D)	**23.** (C)	**24.** (D)	**25.** (A)

Answer the following in short :

1. What is a unit ? What are derived units ?

2. Which are the supplementary units of SI system ?

3. What is dimensional formula ?

4. Which physical quantity has the unit amu ?

5. $1g/cm^3$ = ... kg/m^3

6. In any experiment, physical quantity in the formula which has the maximum power should be measured more accurately. Justify the statement ?

7. The mass of an object is $225 \pm 0.05g$. Calculate percentage error in measurement.

8. Write the dimensional formula of capacitance.

9. Give the difference between accuracy and precision.

10. If $\theta_1 = 25.5 \pm 0.1$ °C and $\theta_2 = 35.3 \pm 0.1$ °C calculate $\theta_1 - \theta_2$

11. Subtract with due regard to significant figures : $3.9 \times 10^5 - 2.5 \times 10^4$

Answer the following questions :

1. Which are the fundamental and supplementary units of the SI system ? Give their units with symbols.
2. Explain the parallax method to determine the distance between the earth and a planet.
3. Explain the different types of errors that occur during measurement of a physical quantity.
4. Explain absolute error, average absolute error, relative error and percentage error.
5. Explain the dimensional consistency of an equation can be checked using dimensional analysis ?
6. Give the limitations of dimensional analysis.

Solve the following Problems :

1. In Ohm's experiment, the values of an unknown resistance were found to be $4.12\,\Omega$, $4.08\,\Omega$, $4.22\,\Omega$ and $4.14\,\Omega$. Calculate absolute error, relative error and percentage error in these measurement.

   [**Ans. :** 0.04, 0.0096, 0.96 %]
2. If the length of a cylinder is $l = (4.00 \pm 0.01) cm$, radius $r = (0.250 \pm 0.001)\ cm$ and mass $m = 6.25 \pm 0.01 g$. Calculate the percentage error in determination of density.

   [**Ans. :** 1.21 %]
3. The acceleration due to gravity (g) is determined by using simple pendulum of length $l = (100 \pm 0.1)$cm. If the time period is $T = (2 \pm 0.01)$s, find the maximum percentage error in the measurement of g.

   [**Ans. :** 1.1 %]
4. The length, breadth and thickness of a metal sheet are 4.234m, 1.005m and 2.01cm respectively. Calculate the total area and volume of the sheet to the correct number of significant figures.

   [**Ans. :** 8.72 m^2, 0.086 m^3]
5. The electric force between two electric charges is given by

   $F = \dfrac{1}{4\pi\varepsilon_0}\dfrac{q_1\, q_2}{r^2}$, Where r is the distance between q_1 and q_2. Give the unit and dimensional formula of ε_0.

   [**Ans. :** $N^{-1}C^2\, m^{-2}$; $M^{-1}L^{-3}T^4A^2$]
6. Check the dimensional validity of the following equations :

   (i) Pressure $P = \rho gh$

   ρ = density of matter, g = acceleration due to gravity, h = height

   (ii) $F.s = \frac{1}{2}mv^2 - \frac{1}{2}mv_0^{\,2}$

   Where F = force, s = displacement, m = mass, v = final velocity and v_0 = initial velocity

(iii) $s = v_0 t + \frac{1}{2}(at)^2$

s = displacement, v_0= initial velocity, a = acceleration and t = time.

(iv) $F = \frac{m \times a \times s}{t}$

Where m = mass, a = acceleration, s = distance and t = time.

[**Ans. :** (i) and (ii) is dimensionally valid and (iii) and (iv) are not valid.]

7. If the velocity of light, acceleration due to gravity and normal pressure are chosen as the fundamental units, find the unit of mass, length and time. Given that velocity of light, $c = 3 \times 10^8 \text{m s}^{-1}$, $g = 10\text{ms}^{-1}$ and normal atmospheric pressure, $P = 10^5 \text{N/m}^2$

[**Ans. :** unit of mass = 8.1×10^{35}kg, unit of length = $L = 9 \times 10^{15}$m and unit of time $T = 3 \times 10^7$s]

8. $v = at + \frac{b}{t + c} + v_0$ is a dimensionally valid equation. Obtain the dimensional formula for a, b and c. where, v is velocity, t is time and v_0 is initial velocity.

[**Ans. :** $[a] = M^0L^1T^{-2}$, $[b] = M^0L^1T^0$, $[c] = M^0L^0T^1$]

9. An object is falling freely under the gravitational force. Its velocity after traversing a distance h is v. If v depends upon gravitational acceleration g and distance has prove with the help of dimensional analysis that $v = k\sqrt{gh}$, where k is a constant.

10. A gas bubble from an explosion under water oscillates with a period T proportional to $P^a \rho^b E^c$ where P is the static pressure, ρ is the density of water and E is the total energy of the explosion. Find the values of a, b and c.

[**Ans. :** $a = -\frac{5}{6}$, $b = \frac{1}{2}$, $c = \frac{1}{3}$]