6 Gravitation

Newton's Universal law of Gravitation :

"Every particle in the universe attracts every other particle with a force which is directly proportional to the product of their mass and inversely proportional to the square of the distance between them and the direction of this force is along the line joining them."

- The magnitude of the gravitational force acting between two particles of mass m_1 and m_2 lying at distance r from each other is,

  $F = \frac{G m_1 m_2}{r^2}$ (obeys inverse square law)

  Where G = universal constant of gravitation

  its value is 6.67×10^{-11} Nm^2 kg^{-2} and the dimensional formula is $M^{-1}L^3T^{-2}$.

- **Definition of 'G' :** The gravitational force of attraction between two objects of unit mass each and placed unit distance apart is called the universal gravitational constant.

  Gravitational force in vector form :

  Gravitational force acting on the particle of mass m_1 by the particle of mass m_2 is,

  $\vec{F}_{12} = \frac{G m_1 m_2}{r^2} \hat{r}_{12}$

  Where $\hat{r}_{12}$ is the unit vector in the direction of $\vec{F}_{12}$.

  Similarly, the gravitational force acting on the particle of mass m_2 by the particle of mass m_1 is,

  $\vec{F}_{21} = \frac{G m_1 m_2}{r^2} \hat{r}_{21}$ or $\vec{F}_{21} = \frac{-G m_1 m_2}{r^2} \hat{r}_{12}$

  where $\hat{r}_{21}$ is the unit vector in the direction of $\vec{F}_{21}$.

  Here $\vec{F}_{12} = -\vec{F}_{21}$ and $|\vec{F}_{12}| = |\vec{F}_{21}|$

- **Important features of Gravitational force :**
  - It is acting between any two bodies by virtue of their mass.
  - It is always attractive in nature.
  - The gravitational force between two objects is independent of intervening medium.
  - The gravitational forces are mutually interactive forces.
  - The gravitational force is a central force.
  - The gravitational force is a conservative force. The work done on the object by it does not depend on the path taken but only depends on initial and final position. **or** The work done by it on closed path is zero.
  - The gravitational force between two bodies is independent of the presence of other bodies. (Two body force)
  - The gravitational force by a hollow spherical shell of uniform density on a particle out side the shell is equal to the force which can be obtained by considering the entire mass of the shell as concentrated on its centre.
  - The force on a particle at any point inside a hollow spherical shell of uniform density is zero.

(1) Calculate the value of gravitational force acting between two spheres each of mass 2 kg, when their centres are 20 cm apart. ($G = 6.67 \times 10^{-11}\ Nm^2kg^{-2}$)

(A) 6.67×10^{-9} N (B) 6.67×10^{-11} N (C) 6.67×10^{-7} N (D) 6.67×10^{-5} N

(2) Three uniform spheres, each having mass m and radius r, are kept in such a way that each touches the other two. The magnitude of the gravitational force on any sphere due to the other two is ......

(A) $\frac{Gm^2}{r^2}$ (B) $\frac{Gm^2}{4r^2}$ (C) $\frac{\sqrt{2}\,Gm^2}{4r^2}$ (D) $\frac{\sqrt{3}\,Gm^2}{4r^2}$

(3) Three masses, each equal to m, are placed at the three corners of a square of side l. The magnitude of gravitational force on unit mass at the fourth corner will be ....

(A) $\frac{Gm}{3l^2}$ (B) $\frac{\sqrt{3}\,Gm}{l^2}$ (C) $\frac{Gm}{l^2}\left(\frac{1}{2}+\sqrt{2}\right)$ (D) $\frac{3Gm}{l^2}$

(4) Two identical solid brass spheres of radius R are placed in contact with each other. The gravitational force between them is proportional to ......

(A) R^2 (B) R^{-4} (C) R^3 (D) R^4

(5) Two point masses A and B having mass in the ratio 4 : 5 are separated by a distance of 1 m. When another point mass C of mass M is placed in between A and B, the gravitational force between A and C is $\frac{1}{5}$ times the gravitational force between B and C. Then the distance of C from A is ......

(A) $\frac{2}{3}$ m (B) $\frac{2}{7}$ m (C) $\frac{1}{3}$ m (D) $\frac{2}{5}$ m

(6) The magnitude of gravitational force acting between two particles of mass m_1 and m_2 separated by a distance r is F. What would be the change in the distance between them so that the gravitational force acting between them will become 2F ?

(A) decreased by 29.3 % (B) increased by 29.3 %

(C) decreased by 50 % (D) decreased by 25 %

(7) The gravitational force due to earth on a body of mass m at a height h from the Earth's surface is $\frac{1}{3}$ times the force on it at sea level (at surface of the earth). Then $\frac{h}{R}$ = ...

Where R = radius of the earth

(A) 0.414 (B) 0.732 (C) 0.500 (D) 0.314

(8) The distance of the centres of earth and moon is r. The mass of earth is 81 times the mass of the moon. At what distance on the line joining their centres from the center of the earth, the gravitational force on any object will be zero ?

(A) 0.9 r (B) 0.7 r (C) 0.5 r (D) 0.25 r

(9) Three particles each of mass m are placed at the three vertices of an equilateral triangle of side l. What is the resultant gravitational force due to this system of particles acting on another particle of mass (M) placed at the mid-point of any side ?

(A) $\frac{3GMm}{4l^2}$ (B) $\frac{4GMm}{3l^2}$ (C) $\frac{GMm}{4l^2}$ (D) $\frac{4GMm}{l^2}$

(10) A mass M is split into two parts, m and M – m. If the gravitational force acting between the two parts is maximum for a given distance, then the ratio $\frac{m}{M}$ = ...

(A) $\frac{1}{2}$ (B) $\frac{1}{4}$ (C) $\frac{3}{4}$ (D) $\frac{1}{5}$

(11) The gravitational force acting between two spheres of mass m and M situated at a distance r in air is F. Now these spheres are kept in the liquid of specific gravity 5 at a distance r, then the gravitational force will be ......

(A) 5F (B) $\frac{F}{5}$ (C) F (D) $\frac{F}{25}$

(12) The gravitational force by earth on a body of mass 1 kg at earth's surface is 10 N. Then the gravitational force on a satellite of mass 100 kg revolving around the earth in a orbit at average distence $\frac{3}{2}$R from the centre of the earth will be ......
(R = radius of earth)

(A) 4.44×10^2 N (B) 6.66×10^2 N (C) 500 N (D) 3.33×10^2 N

(13) The centripetal force acting on a satellite orbiting around the earth is F and the gravitational force acting on a satellite due to earth is also F. The resultant force acting on satellite will be ......

(A) F (B) 2F (C) Zero (D) $\sqrt{2}$ F

(14) Two particles of equal mass m go round a circle of radius R under the action of their mutual gravitational attraction. The speed of each particle is ......

(A) $\sqrt{\frac{Gm}{2R}}$ (B) $\sqrt{\frac{4Gm}{R}}$ (C) $\frac{1}{2}\sqrt{\frac{Gm}{R}}$ (D) $\frac{1}{2R}\sqrt{\frac{1}{Gm}}$

(15) The gravitational force is a ...... force.

(A) conservative (B) non conservative (C) electrostatic (D) repulsive

Ans. : 1 (A), 2 (D), 3 (C), 4 (D), 5 (A), 6 (A), 7 (B), 8 (A), 9 (B), 10 (A), 11 (C), 12 (A), 13 (A), 14 (C), 15 (A)

Gravitational Acceleration

'The acceleration produced in the body due to the gravitational force of the earth is called the gravitational acceleration or the acceleration due to gravity (g).

The gravitational acceleration at a distance r ($r > R_e$) from the centre of the earth is

$g = \frac{GM_e}{r^2}$; where M_e = mass of the earth and R_e = Radius of the earth.

At the surface of the earth, $r = R_e$

$\therefore\ g = \frac{GM_e}{R_e^2}$

- The value of the g does not depend on the mass, shape and size of the body but depends on the mass of the earth and height or depth from the surface of the earth.
- The value of 'g' at the surface of the earth is 9.8 ms^{-2}.
- 'g' is a vector quantity and its direction is always towards the centre of the earth.

Variations in Acceleration Due to Gravity

(1) **Due to shape of the earth :**

The earth is not completely spherical but is slightly bulged out at the equator and flattened at the poles. The radius of the earth at equator is nearly 21 km more than the radius at the poles.

$\therefore\ g_{pole} > g_{equator}$ (more by 0.018 ms^{-2} approximately.)

(2) **Variation in 'g' with altitude :**

The gravitional acceleration at a height h from the surface of the earth is,

$$g(h) = \frac{GM_e}{(R_e+h)^2} \quad \textbf{OR}$$

$$g(h) = \frac{g}{\left(1+\frac{h}{R_e}\right)^2} \text{ (for any height)} \quad \textbf{OR}$$

$$g(h) = g\left(1-\frac{2h}{R_e}\right) \text{ (for } h << R_e\text{)}$$

for $h << R_e$,

The absolute decrease, $\Delta g = g - g(h) = \frac{2hg}{R_e}$

The fractional decrease, $\frac{\Delta g}{g} = \frac{g-g(h)}{g} = \frac{2h}{R_e}$

The percentage decrease, $\frac{\Delta g}{g} \times 100\ \% = \frac{2h}{R_e} \times 100\ \%$

$\therefore$ The loss in the weight of a body of mass m at a height h from the surface of the earth $= \frac{2mgh}{R_e}$

(3) **Variation in 'g' with depth :**

The gravitational acceleration at a distance r ($r < R_e$) from the centre of the earth is

$g' = \frac{4}{3}\pi G r \rho$ where ρ = uniform density of the earth.

At the surface of the earth, $r = R_e$

$\therefore g = \frac{4}{3}\pi G R_e \rho$

- The gravitational acceleration at depth d from the surface of the earth (at distance $r < R_e$ from the centre of the earth)

$$g' = g\left(1-\frac{d}{R_e}\right)$$

At the centre of the earth, $d = R_e$

$$\therefore g' = g\left(1-\frac{R_e}{R_e}\right) = 0$$

Thus, the value of the gravitational acceleration at the centre of the earth is zero.

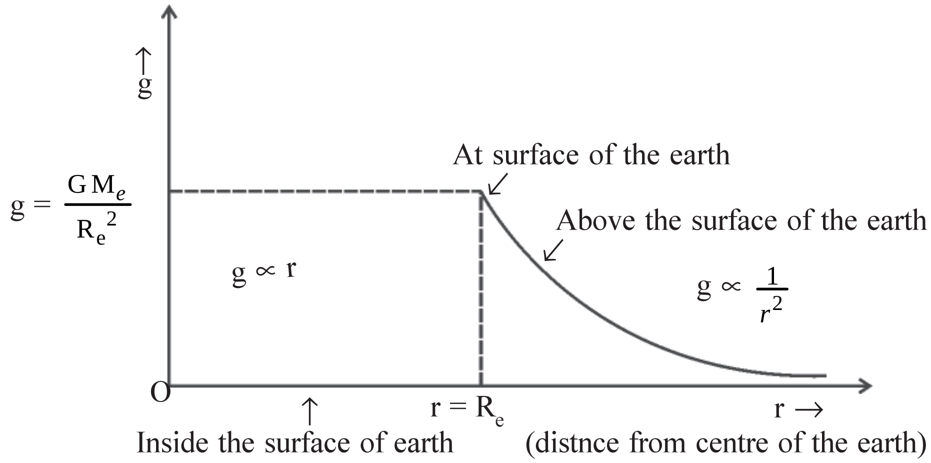
The absolute decrease $= \Delta g = g - g' = d\left(\frac{g}{R_e}\right)$

The fractional decrease $= \frac{\Delta g}{g} = \frac{g-g'}{g} = \frac{d}{R_e}$

The percentage decrease $= \frac{\Delta g}{g} \times 100\ \% = \frac{d}{R_e} \times 100\ \%$

- The rate of decrease of 'g' outside the surface of the earth (for $h << R_e$) is double to that of inside the surface of the earth.

- The graph of $g \to r$:



(4) **Variation in effective Gravitational Acceleration 'g' with latitude due to earth's Rotation :**

Equtorial Plane : The plane passing through the center of the earth and perpendicular to its axis of rotation is called equatorial plane.

Latitude : The angle made by the line joining a given place on the Earth's surface to the centre of the Earth with the equatorial line is called the latitude (λ) of that place.

At the equator $\lambda = 0°$ and at the poles $\lambda = 90°$

The effective gravitational acceleration at the place having latitude λ is.

$g' = g - R_e \omega^2 \cos^2 \lambda$

$= g\left(1 - \frac{R_e \omega^2 \cos^2 \lambda}{g}\right)$, Where ω = rate of rotation about its own axis.

(i) At the equator $\lambda = 0° \Rightarrow \cos \lambda = 1$

$g' = g\left(1 - \frac{R_e \omega^2}{g}\right) \quad = g - R_e\omega^2$

= minimum value of effective gravitational acceleration.

(ii) At the poles $\lambda = 90° \Rightarrow \cos \lambda = 0$

$\therefore\ g' = g$

= maximum value of effective gravitational acceleration.

- When a body of mass m is moved from the equator to the poles, its weight increases by an amount, $m(g_p - g_e) = m\omega^2 R_e$
  where g_p = gravitational acceleration at poles, g_e = gravitational acceleration at equator.
- If earth stops rotating about its own axis then at the equator the value of g increases by $\omega^2 R_e$ and consequently the weight of the body of mass m lying there increases by $m\omega^2 R_e$.
- **Average density of the earth in terms of 'g' and 'G'.**
  Accepting the earth as a solid sphere of uniform density,

$$\rho = \frac{M_e}{\frac{4}{3}\pi R_e^3} = \frac{3M_e}{4\pi R_e^3} = \frac{3\left(\frac{gR_e^2}{G}\right)}{4\pi R_e^3} \qquad \left(\therefore\ M_e = \frac{gR_e^2}{G}\right)$$

$$= \frac{3g}{4\pi R_e G}$$

(16) A body weighs 81 *kgf* on the surface of the earth. How much will it weigh on the surface of Mass whose mass and radius are $\frac{1}{9}$ times and $\frac{1}{2}$ times respectively that of the earth ?

(A) 40 kgf (B) 36 kgf (C) 24 kgf (D) 162 kgf

(17) If the earth were a sphere made completely of lead, then what would be the value of gravitational acceleration on its surface ? (Radius of the Earth = 6.4×10^6 m, G = 6.67×10^{-11} SI, Relative density of lead = 11.3)

(A) 22.21 ms^{-2} (B) 34.49 ms^{-2} (C) 28.72 ms^{-2} (D) 14.67 ms^{-2}

(18) The mass of two planets are in the ratio 1:2. Their diameters are in the ratio 1:3. The acceleration due to gravity on the surface of the planets are in the ratio ......

(A) 2 : 1 (B) 3 : 2 (C) 2 : 3 (D) 9 : 2

(19) If the radius of the earth is made three times that of present value. Then for what should be the approximate change in the value of density of earth, so that the value of gravitatonal acceleration on the surface of the earth remains constant.

(A) decreased by 67 % (B) increased by 67 %

(C) decreased by 33 % (D) increased by 33 %

(20) A man can jump to a height of 2 m on a planet A. What is the height he may be able to jump on another planet whose density and radius are respectively, one - quarter and one third that of planet A.

(A) 18 m (B) 24 m (C) 36 m (D) 15 m

(21) The weight of a body on the surface of the earth is 54 N. What would be its weight at height $\frac{R_e}{2}$ from the surface of the earth ? Where R_e = radius of the earth.

(A) 72 N (B) 36 N (C) 18 N (D) 24 N

(22) At what height from the surface of the earth, the value of gravitational acceleration will be half that on the surface of the earth ? Radius of the earth R = 6400 km

(A) 2650 km (B) 3366 km (C) 1325 km (D) 414 km

(23) A body hanging from a massless spring stretches it by 1 cm at the earth surface. How much will the same body stretch the spring at a place 1600 km above the earth's surface ?
(Radius of earth R = 6400 km)

(A) 0.32 cm (B) 0.64 cm (C) 0.16 cm (D) 0.86 cm

(24) At what distance from the centre of the earth the weight of body becomes $\frac{1}{16}$ times its weight that on the surface of the earth ? Radius of the earth is R.

(A) 3R (B) 4R (C) 5R (D) 8R

(25) At what height above the earth's surface the value of gravitational acceleration be same as that the gravitational acceleration at a depth of 100 km from the surface of the earth ?

(A) 50 km (B) 100 km (C) 200 km (D) 25 km

(26) How much below the surface of the earth does the acceleration due to gravity become 10 % of its value at the earth's surface ? (Radias of the earth R = 6400 km)

(A) 6336 km (B) 5400 km (C) 5760 km (D) 5980 km

(27) The ratio of weights of a body of mass m at a height of 30 km above earth's surface to a depth of 30 km from the surface of the earth is ......

(A) 0.946 (B) 0.962 (C) 0.984 (D) 0.995

(28) Suppose the earth is a uniform sphere of radius R. If the acceleration due to gravity at a place having latitude 45° and at equator are g' and g'' respectively. Then g' – g'' = ... (gravitational acceleration at the poles = g)

(A) $\frac{3R\omega^2}{2}$ (B) $\frac{R\omega^2}{2}$ (C) $\frac{R\omega^2}{3}$ (D) $\frac{2R\omega^2}{3}$

(29) The angular velocity of the earth with which it has to rotate so that acceleration due to gravity on 60° latitude becomes zero is ...... (Radius of earth R = 6400 km, at the poles g = 10 ms^{-2})

(A) 2.5×10^{-3} rad s^{-1} (B) 1.25×10^{-3} rad s^{-1} (C) 2.5×10^{-2} rad s^{-1} (D) 1.25×10^{-2} rad s^{-1}

(30) The angular velocity of the earth with which it has to rolate so that the weight of a body becomes $\frac{1}{3}$ times the weight at body at equator ...... (Redius of earth R = 6.4×10^6 m, at the poles g = 9.8 ms^{-2})

(A) 7.8×10^{-4} rad s^{-1} (B) 6.7×10^{-4} rad s^{-1} (C) 8.7×10^{-4} rad s^{-1} (D) 10×10^{-4} rad s^{-1}

(31) The weight of a body on the surface of earth is W. Then the weight of a body at half way mark from the surface of the earth to centre of the earth is ...... (Consider the density of the earth to be uniform)

(A) $\frac{W}{2}$ (B) W (C) $\frac{W}{4}$ (D) $\frac{W}{8}$

(32) The density of a planet is twice that of the earth and radius is $\frac{3}{2}$ times that of the earth. Then the value of gravitational acceleration on the surface of the planet is how many times the value at surface of the earth ?

(A) $\frac{4}{3}$ (B) 3 (C) 6 (D) $\frac{3}{4}$

(33) The mass of a body on a surface of the earth ia M. Then the mass of the same body at the surface of the moon is ....

(A) $\frac{M}{6}$ (B) M (C) zero (D) infinite

(34) The rate of change of gravitational acceleration (g) at a depth x from the surface of the earth is ......

(A) $-\frac{4}{3}G\pi\rho$ (B) $-\frac{2}{3}G\pi\rho$ (C) $\frac{-8G\pi\rho}{3}$ (D) $-G\pi\rho$

(35) The value of acceleration due to gravity at a height 1600 km above the earth's surface is ...... (Value of g at surface of the earth = 9.8 ms^{-2}, radius of earth R = 6400 km)

(A) 8.73 ms^{-2} (B) 7.59 ms^{-2} (C) 6.27 ms^{-2} (D) 9.12 ms^{-2}

(36) If the earth stops rotating about its own axis, then the change in the value of gravitational acceleration at a place having latitude of 45° is ...... (radius of the earth R = 6.4×10^6 m)

(A) 2.74 cms^{-2} (B) 1.68 cms^{-2} (C) 1.12 cms^{-2} (D) 3.34 cms^{-2}

(37) The radius and mass of the earth are R and M respectively. Then the ratio $\frac{g}{G}$ = ...

(Where g = gravitational acceleration, G = universal constant)

(A) MR^2 (B) $\frac{M}{R^2}$ (C) $\frac{M}{R}$ (D) $\frac{R^2}{M}$

(38) Assume earth to be complete sphere of radius R. If values of gravitational accelerations at a place having latitude of 30° and at equator are g_{30} and g respectively. Then, $g - g_{30}$ = ...

(A) $\omega^2 R$ (B) $\frac{3}{4}\omega^2 R$ (C) $\frac{\omega^2 R}{2}$ (D) $\frac{\omega^2 R}{4}$

(39) Assume that the acceleration due to gravity on the surface of the moon is 0.2 times the acceleration due to gravity on the surface of the earth. If R_e is the maximum range of a projectile on the earth's surface. What is the maximum range on the surface of the moon ...... (Assume initial velocity of projection to be constant)

(A) 0.2 R_e (B) 0.5 R_e (C) 2 R_e (D) 5 R_e

Ans. :	**16 (B), 17 (A), 18 (D), 19 (A), 20 (B), 21 (D), 22 (A), 23 (B), 24 (B), 25 (A), 26 (C), 27 (D), 28 (B), 29 (A), 30 (D), 31 (A), 32 (B), 33 (B), 34 (A), 35 (C), 36 (B), 37 (B), 38 (B), 39 (D)**

Mass and Weight

The quantity of matter in the body is called mass. it is the fundamental intrisic property of the body. Mases are of two types :

(i) Inertial mass : The ratio of the external force applied on a body to the acceleration produced in it due to the external force is called the inertial mass (m_i).

$$m_i = \frac{\text{Applied external force}}{\text{Acceleration produced}} = \frac{F}{a}$$

($\because$ according to Newton's second law of motion, mass $= \frac{\text{Froce}}{\text{acceleration}}$).

- The inertial mass of a body is a measure of its inertia.
- It is the measure of ability of the body to oppose the production of acceleration in its motion by an external force.

(ii) Gravitational mass : The ratio of the gravitational pull of the earth on a body to the acceleration produced in it due to gravitational force is called the gravitational mass (m_g).

$$\therefore m_g = \frac{F}{g}$$

From the experiments $m_i = m_g = m$

Weight of the body : The gravitational force exerted by earth on a body is called weight of a body.

$$W = \frac{GM_e m}{R_e^2}$$

(Where M_e = mass of the earth, R_e = radius of the earth, G = universal gravitational constant)

$\because$ W = mg

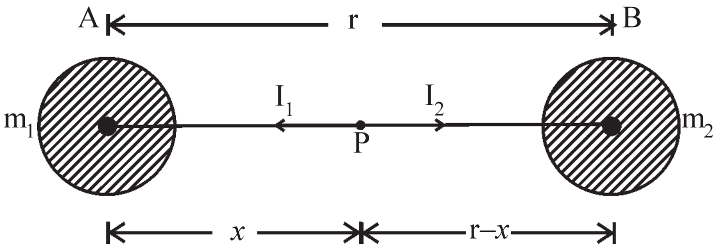
Its unit is N and directed towards the centre of the earth.

Gravitational Intensity (Gravitational field) :

- **Defination :** The gravitational force exerted by the given body on a body of unit mass (test mass) at a given point is called the intensity of gravitational field (I) at that point.

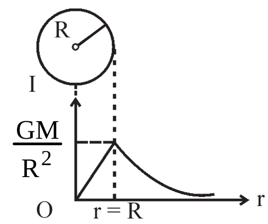
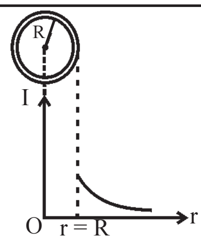
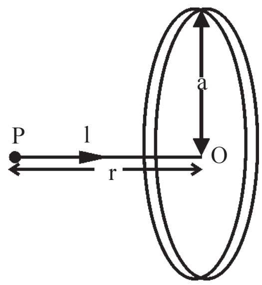
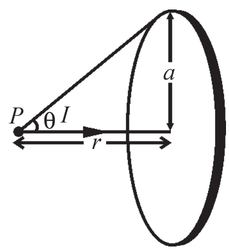
  $\therefore \vec{I} = \frac{\vec{F}}{m}$ where m = test mass
- It is a vector quantity and its direction is towards the centre of gravity of a body whose gravitational field is considered.
- Its unit is N kg^{-1} and dimensions are $M^0L^1T^{-2}$.
- The gravitational intensity due to body of mass on at a distance r is, $I = \frac{GM}{r^2} \Rightarrow I \propto \frac{1}{r^2}$
- If the gravitational intensiry is known at a given point the gravitotional force acting on any body kept at that point in the field can be determined.
- The value of gravitational field intensity at any point is equal to the value of gravitational acceleration at that point.
- gravitational intensity and gravitational acceleration are different quantities. Their units are different but equivalent. ($\therefore$ N kg^{-1} and ms^{-2}).
- I $\rightarrow$ r graph for the earth is gravitational field would be the same as g $\rightarrow$ r graph.

- As shown in the figure suppose at point P the gravitational intensities are equal and opposite due to two bodies A and B, Thus at point P resultant gravitational intensity is zero.



here $x = \left(\frac{\sqrt{m_1}}{\sqrt{m_1}+\sqrt{m_2}}\right) r$ and $r - x = \left(\frac{\sqrt{m_2}}{\sqrt{m_1}+\sqrt{m_2}}\right) r$

Gravitational Field Intensity for Bodies of Different Shape

Body	**Position**	**Gravitational intensity**	**Figure $I \rightarrow r$ (graph)**
Uniform solid sphere	(i) Outside the surface $r > R$ (ii) On the surface $r = R$ (iii) Inside the surface $r < R$	$I = \frac{GM}{r^2}$ $I = \frac{GM}{R^2}$ $I = \frac{GMr}{R^3}$	
Uniform spherical shell	(i) Outside the surface $r > R$ (ii) On the surface $r = R$ (iii) Inside the surface $r < R$	$I = \frac{GM}{r^2}$ $I = \frac{GM}{R^2}$ $I = 0$	
Thin uniform circular ring	(i) At a point on its axis (ii) At the centre of the ring	$I = \frac{GMr}{(a^2+r^2)^{\frac{3}{2}}}$ $I = 0$	
Uniform Disc	(i) At a point on its axis (ii) At the centre of the disc	$I = \frac{2GMr}{a^2}\left[\frac{1}{r}-\frac{1}{\sqrt{r^2+a^2}}\right]$ or $I = \frac{2GM}{a^2}(1-\cos\theta)$ $I = 0$	

Gravitational Potential

'The negative of the work done by the gravitational force in bringing a body of unit mass, from infinite distance to the given point in the gravitational field is called the gravitational potential (ϕ) at that point.'

$$\phi = -\frac{W}{m} = -\int_{\infty}^{r} \frac{\vec{F}.d\vec{r}}{m} = -\int_{\infty}^{r} \vec{I}.d\vec{r} \qquad \left(\because \vec{I} = \frac{\vec{F}}{m}\right)$$

$$= -\int_{\infty}^{r} \frac{Gm}{r^2}\,dr$$

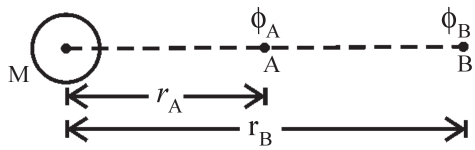
Gravitational Potential, $\phi = \dfrac{-Gm}{r}$

If $r = \infty$ then $\phi = 0 = \phi_{max}$

- It is a scalar quantity. Its SI unit is Jkg^{-1} and dimensional formula is $M^0L^2T^{-2}$.
- $\phi = -\int \vec{I}.d\vec{r} \Leftrightarrow I = \dfrac{-d\phi}{dr}$

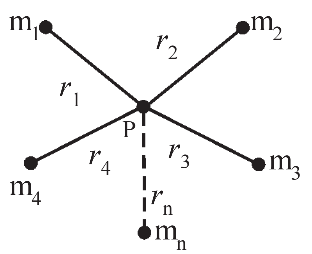
- **Gravitational Potential Difference :**

'It is defined as the work done to move a unit mass from one point to the other in the gravitational field.



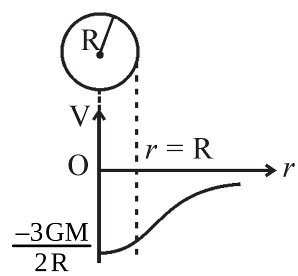
$$\Delta\phi = \phi_B - \phi_A = \frac{W_{A\to B}}{m} = -GM\left(\frac{1}{r_B} - \frac{1}{r_A}\right)$$

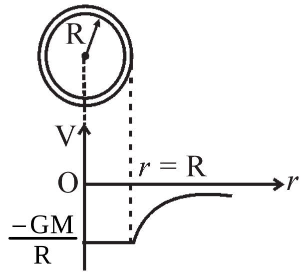
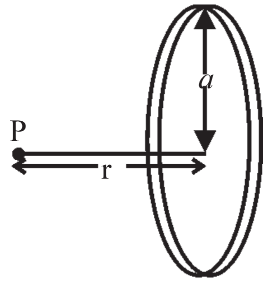
- Potential due to large numbers of particles is given by scalar addition of all the potentials. In the figure at point P, total gravitational potential, is given by,



$$\phi = \frac{-Gm_1}{r_1} - \frac{Gm_2}{r_2} - \frac{Gm_3}{r_3} \ldots\ldots\ldots$$

Gravitational Potential for bodies of Different shape

Body	Position	Gravitational intensity	Figure $I \rightarrow r$ (graph)
Uniform solid sphere	(i) Outside the surface $r > R$	$\phi = \dfrac{-GM}{r}$	
	(ii) On the surface $r = R$	$\phi_{surface} = \dfrac{-GM}{R}$	
	(iii) Inside the surface $r < R$	$\phi = \dfrac{-GM}{2R}\left[3-\left(\dfrac{r}{R}\right)^2\right]$	
	(iv) At the centre of the sphere $(r = 0)$	$\phi_{center} = \dfrac{-3GM}{2R} = \dfrac{3}{2}\phi_{Surface}$	

Uniform spherical shell	(i) Outside the surface r > R (ii) On the surface r = R (iii) Inside the surface r < R	$\phi = \frac{-GM}{r}$ $\phi = \frac{-GM}{R}$ $\phi = \frac{-GM}{R}$	
Thin uniform circular ring	(i) At a point on its axis (ii) At the centre of the ring	$\phi = \frac{-GM}{\sqrt{a^2+r^2}}$ $\phi = \frac{-GM}{a}$	

Gravitational Potential Energy

'The negative of the work done by the gravitational force in bringing a given body (of mass m) in the gravitational field of the Earth from infinite distance at the given point is called the gravitational potential energy (U) of that body at that point.'

The gravitational potential energy of a body of mass m at a distance r from the centre of the earth ($r \geq R_e$) is,

$$U = \frac{-GM_e\, m}{r} = \phi m \quad (\because\ \phi = \frac{-GM_e}{r})$$

- It is a scalar quantity. Its unit is J and dimensional formula is $M^1L^2T^{-2}$.
- Gravitational potential energy is always negative because gravitational forces are attractive in nature.
- As the distance increases, gravitational potential energy increases. (becomes less negative)
- At infinite distance Gravitational potential energy $U = 0 = U_{max}$
- Here the potential energy U is of the system consisting of the Earth and the body.
- From the centre of body of mass M is the body of mass m is moved from a point at a distance r_1 to a point at distance r_2 ($r_1 > r_2$) then change in potential energy,

  $$\Delta U = U_2 - U_1 = GMm\left(\frac{1}{r_1} - \frac{1}{r_2}\right)$$
- As r_1 is greater than r_2, ΔU will be negative. It means that if a body is brought closer to earth it's potential energy decreases.
- Gravitational potential energy at the centre of the earth,

  $$U_{centre} = m\phi_{centre} = m\left(\frac{-3}{2}\frac{GM_e}{R_e}\right) = \frac{-3}{2}\frac{GM_e m}{R_e}$$
- If the body of mass m is taken at a height h from the surface of the earth, then change in potential energy,

  $$\Delta U = U_2 - U_1 = \frac{m\,gh}{1+\frac{h}{R_e}}$$

  (i) If $h = nR_e$; $\Delta U = \left(\frac{m\,g\,R_e}{(n+1)}\right) n$

  (ii) If $h \ll R_e$; $\Delta U = mgh$ ($\because \frac{h}{R_e} \to 0$)

  (iii) If $h = R_e$; $\Delta U = \frac{1}{2}\, mgR_e$

- In case of discrete distribution of mass total potential energy

$$U = \Sigma U_i = -\left[\frac{G m_1 m_2}{r_{12}} + \frac{G m_2 m_3}{r_{23}} + ...\right]$$

- Total $\frac{n(n-1)}{2}$ pairs are formed for a system of n particles.

---

(40) If gravitational force acting on a body of mass 50 g at piont is 2 N, then what would the magnitude of intensity of the gravitational field at that point ?

(A) 40 N kg^{-1} (B) 0.4 N kg^{-1} (C) 2 N kg^{-1} (D) 100 N kg^{-1}

(41) The distance at which the magnitute of gravitational field intensity due to thin uniform ring of mass M and radius R from the centre of the ring on its axis will be zero.

(A) $\frac{R}{\sqrt{2}}$ (B) $\frac{R}{\sqrt{3}}$ (C) $\frac{R}{2}$ (D) $\frac{2R}{3}$

(42) The magnitude of gravitational intensity at a point is 20 N kg^{-1}. What would be the magnitude of the gravitational force on a body of 10 kg mass at this point ?

(A) 100 N (B) 200 N (C) 50 N (D) 400 N

(43) Two objects of equal mass m are placed at a distance d from each other on a horizontal surface. The value of gravitational potential at a mid point on line joining their centres is ...... (G = universal gravitational constant)

(A) $\frac{-Gm}{d}$ (B) $\frac{-2Gm}{d}$ (C) $\frac{-4Gm}{d}$ (D) Zero

(44) Three particles each of mass m are kept at the three vertices of an equilateral triangle of side b. The gravitational potential at the centroid of this equilateral triangle is ......

(A) $\frac{-3\sqrt{3}Gm}{b}$ (B) $\frac{-2\sqrt{3}Gm}{b}$ (C) $\frac{-3\sqrt{2}Gm}{b}$ (D) $\frac{-9Gm}{b}$

(45) Two bodies of mass m and 9m are placed at a distance *r*. The gravitational potential at a point on the line joining them where the gravitational field is zero, will be ....

(A) $\frac{-6Gm}{r}$ (B) $\frac{-9Gm}{r}$ (C) $\frac{-4Gm}{r}$ (D) $\frac{-16Gm}{r}$

(46) Four particles each of mass m are kept at the four vertices of a square with side *l*. The gravitational potential at the centre of the square is ......

(A) $\frac{-2\sqrt{3}Gm}{l}$ (B) $\frac{-4\sqrt{2}Gm}{l}$ (C) $\frac{-3\sqrt{2}Gm}{l}$ (D) $\frac{-5\sqrt{2}Gm}{l}$

(47) A body of mass *m* is taken from earth surface to the height h = $\frac{R}{5}$. The increase in its potential energy will be ......
(accleration due to gravity on the surface of the earth = g, radius of the earth = R).

(A) $\frac{6}{7}$ mgh (B) $\frac{5}{6}$ mgh (C) $\frac{3}{4}$ mgh (D) $\frac{2}{3}$ mgh

(48) Three particles each of mass 2m are kept at the three vertices of an equilateral triangle of side *l*. The gravitational potential energy of this system is ......

(A) $\frac{-3Gm^2}{l}$ (B) $\frac{-6Gm^2}{l}$ (C) $\frac{-12Gm^2}{l}$ (D) $\frac{-Gm^2}{2l}$

(49) A body of mass m starts falling from a distance 3R above the Earth's surface. Its kinetic energy when it has fallen to distance R above the Earth's surface is ......

Radius of Earth = R, mass of Earth = M, Universal Gravitational constant = G

(A) $\frac{GMm}{2R}$ (B) $\frac{GMm}{4R}$ (C) $\frac{GMm}{3R}$ (D) $\frac{GMm}{6R}$

(50) A body of mass 1 kg is placed at a distance of 4 m from the centre and on the axis of a uniform ring of mass 5 kg and radius 3 m. Calculate the work required to be done to increases the distance of the body from 4 m to $3\sqrt{3}$ m. (Gravitational constant = G)

(A) $\frac{G}{6}$ (B) $\frac{2G}{3}$ (C) $\frac{3G}{2}$ (D) $\frac{G}{5}$

Ans. : 40 (A), 41 (A), 42 (B), 43 (C), 44 (A), 45 (D), 46 (B), 47 (B), 48 (C), 49 (B), 50 (A)

Escape velocity

The minimum velocity with which a body must be projected from the surface of earth so that it escapes from the gravitational field of the earth is known as escape velocity (v_e).

Escape velocity $v_e = \sqrt{\frac{2GM_e}{R_e}} = \sqrt{2gR_e}$ $\quad (\because GM_e = gR_e^2)$

$= \sqrt{2(\frac{4}{3}\pi\rho G R_e)R_e} = R_e\sqrt{\frac{8}{3}\pi G\rho}$

- The value of escape velocity does not depend upon the mass of the projected body but it depends on the mass and radius of the planet (Here Earth) from which it is being escaped. (projected).
- On the surface of the earth, $v_e = \sqrt{2gR_e}$

  substituting the values of g and R_e, $v_e = 11.2\ \text{kms}^{-1}$
- If the escape velocity required for the body lying on the surface of moon, to make free from the moon's gravitational field is v_e', then

  $v_e' = \sqrt{\frac{2GM_m}{R_m}}$ Where M_m = mass of the moon and R_m = radius of the moon.

  substituting all these values, in this case, $v_e' = 2.3\ \text{kms}^{-1}$

  Which is nearly $\frac{1}{6}$ times the escape velocity at the earth's surface.
- A planet will have atmosphere if the speed of molecule in its atmosphere at the temperature prevailing there is less than the escape speed.
- speed of molecules of gas = $v_{rms} = \sqrt{\frac{3RT}{M}}$

  $\therefore$ That's why the earth has atmosphere as at earth $v_{rms} < v_e$

  while moon has no atomosphere as at moon $_{rms} > v_e$

Escape Energy

'The minimum energy to be supplied to the body to make it free from Earth's gravitational field (from binding with the earth) is called the escape energy of that body. It is often called the binding energy of the body.'

$\therefore$ The escape energy of the body of mass m lying on the surface of the Earth = $\frac{GM_e m}{R_e}$

(51) The escape velocity for a body projected vertically upwards from the surface of the earth is 11.2 kms^{-1}. If the body is projected an angle of 45° with the vertical, the escape velocity will be ...... kms^{-1}

(A) 11.2 (B) $11.2 \times \left(\frac{1}{\sqrt{2}}\right)$ (C) $11.2 \times \left(\frac{\sqrt{3}}{2}\right)$ (D) $11.2 \times \left(\frac{1}{3}\right)$

(52) The escape velocity of a body on the surface of the earth is v_e, then the escape velocity on a planet whose radius is three times and mass is three times that of the earth, is ......

(A) 3 v_e (B) 9 v_e (C) v_e (D) 27 v_e

(53) The escape velocity of a body on the surface of the earth is v_e. The radius of the earth is 6400 km. The value of radius of the earth (by contracting the earth), for which the escape velocity would become 10 times the escape velocity of its present value ...... (Assume mass of the earth to be constant.)

(A) 6.4 km (B) 64 km (C) 640 km (D) 4800 km

(54) A satellite with kinetic energy K is revolving round the earth in a circular orbit. How much more kinetic energy should be given to it so that it may just escape from the gravitational field of the earth ?

(A) $\sqrt{2}$ K (B) 2K (C) K (D) $\frac{K}{\sqrt{2}}$

(55) A satellite is orbiting close to the surface of the earth. How much additional velocity (appronimately) should be given to it so that it may just escape into outer space (radius of earth = 6400 km, gravitational acceleration g = 9.8 ms^{-2})

(A) 11.2 km s^{-1} (B) 3.2 km s^{-1} (C) 8 km s^{-1} (D) 20.2 km s^{-1}

(56) The escape velocity of a body on the surface of the earth is v_e. If mass of the earth is made twice and radius is made halved, then the escape velocity of a body would become ......

(A) 2 v_e (B) $\frac{3}{2}$ v_e (C) $\frac{4}{3}$ v_e (D) 3 v_e

(57) The escape velocity on the surface of the earth is v_1. The escape velocity on the surface of a planet whose radius and density are 4 times and 9 times respectively than that of earth is v_2. Then $\frac{v_1}{v_2}$ = ...

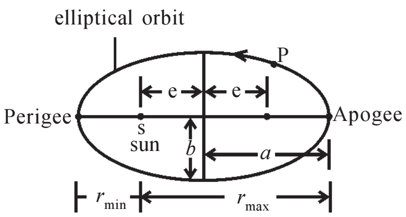
(A) $\frac{1}{6}$ (B) $\frac{1}{12}$ (C) $\frac{4}{3}$ (D) $\frac{3}{4}$

Ans. : 51 (A), 52 (C), 53 (B), 54 (C), 55 (B), 56 (A), 57 (B)

Kepler's Laws

- **First Law (Law of orbits)**

"All the planets move in the elliptical orbits with the sun situated at one of the foci."



a = Semi major axis

b = Semi minor axis

Aphelion distance = Largest distance of planet from the sun.

Perihelion distance = Shortest distance of planet from the sun.

- When planet is far away from the sun it moves slower in the orbit. Thus its kinetic energy is minimum and potential energy is maximum.

- When planet is near to the sun it moves faster in the orbit. Thus its kinetic energy is maximum and potential energy is minimum.

$r_{max} = a + ea = a\ (1 + e)$

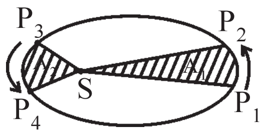
$r_{min} = a - ea = a\ (1 - e)$

Where e is the dimensionless number having value between 0 to 1 called eccentricity of the ellipse. If e = 0, the ellipse is a circle. For earth e = 0.017.

Second Law (Law of Areas) :

"The line joining the sun and the planet sweeps equal areas in equal interval of time it means the areal velocity $\left(\frac{dA}{dt}\right)$ remains constant.."

- The areal velocity being constant is the geometrical representation of the law of conservation of angular momentum.



In equal interval of time,

area of SP_1P_2 = area of SP_3P_4

$\therefore A_1 = A_2$

Third Law (Law of Periods) :

"The square of the time - period (T) of the revolution of a planet is proportional to the cube of the semi major axis (a) of its elliptical orbit."

$\therefore T^2 \propto a^3 \Rightarrow T \propto a^{3/2}$

Satellites

A body revolving around a planet is called its satellite.

- The orbital motion of the salellite depends on the gravitational force by the planet and the initial conditions.
- The path of these satellites are elliptical with the centre of Earth at a focus. However, the difference in semi major axis and semi minor axis is so small that they can be treated as nearly circular.

Orbital velocity

The minimum velocity required to put a satellite into its orbit is known as orbital velocity. The orbital velocity of a satellite at a distance r ($r > R_e$) from the centre of the earth is,

$$v_0 = \sqrt{\frac{GM_e}{r}} = \sqrt{\frac{GM_e}{R_e + h}}$$

Very close to the surface of the earth, $v_0 = \sqrt{\frac{GM_e}{R_e}} = \sqrt{gR_e}$

- The value of orbital velocity does not depend on the mass of the satellite, but depends on the mass of the planet (here earth) about which it revolves and the radius of the orbit.
- The orbital speed of a satellite when it revolves very close to the surface of the earth,

$v_0 = \sqrt{9.8 \times 6.4 \times 10^6} = 7.92$ km s^{-1}

- The work done by the satellite in a complete orbit (i.e. one complete revolution) is zero.
- If the orbital velocity of a satellite orbiting near the surface of the earth is increased by 41.4 % ($\sqrt{2}$ times) then it will escape from the gravitational field of the earth.

Different orbital shapes cossesponding to different velocities of a satellite.

(1) If $v < v_o$ (v_o is the velocity require to maintain satellite in the orbit)

(i) The path would not be circular, rather it will be spiral. The satellite finally falls on the Earth.

(ii) Kinetic energy is less than potential energy $\Rightarrow$ Total energy is negative.

(2) If $v = v_0$

(i) The satellite revolves in a circular orbit.

(ii) e = 0 (e $\rightarrow$ eccentricity).

(iii) Kinetic energy is less than potential energy $\Rightarrow$ Total energy is negative.

(3) If $v_0 < v < v_e$ (Where v_e = Escape Velocity)

(i) The satellite revolves in a elliptical orbit.

(ii) e < 1

(iii) Kinetic energy is less than potential energy $\Rightarrow$ Total energy is negative.

(4) If $v = v_e$

(i) The satellite will move along a parabolic path and escape out of the gravitational field of earth.

(ii) e = 1

(iii) The kinetic energy is equal to the potential energy. $\Rightarrow$ Total energy becomes zero.

(5) If $v > v_e$

(i) The satellite will move along a hyperbolic path and escape out of the gravitational field of earth.

(ii) e > 1

(iii) The kinentic energy is greater than the potential energy $\Rightarrow$ Total energy becomes positive.

Time Period of Satellife (T) :

$$T = \frac{2\pi r}{v_0} = \frac{2\pi r}{\sqrt{\frac{GM_e}{r}}} = 2\pi\sqrt{\frac{r^3}{GM_e}} \Rightarrow T^2 \propto r^3 \qquad (\because \ GM_e \text{ is constant})$$

- The periodic time of satellite is independent of mass of a satellite but depends on the mass of the planet (here earth) about which it revolves and the radius of the orbit.

Height of satellite from the surface of the Earth

$$h = \left(\frac{g R_e^2 T^2}{4\pi^2}\right)^{\frac{1}{3}} - R_e$$

Energy of Satellite :

(i) Kinetic Energy (K) :

$$K = \frac{1}{2}mv_0^2 = \frac{GM_e m}{2r} \quad ... (1)$$

Angular momentum of satellite is $L = mv_0 r$, then kinetic energy is $K = \frac{L^2}{2mr^2}$

(ii) Potential Energy (U) :

The potential energy of a satellite at a distance r from the centre of the earth is,

$$U = \frac{-GM_e\, m}{r} \quad ......\ (2) \quad \text{(Note : From formula } \phi = \frac{-GM_e}{r},\ U = m\phi.)$$

$$= \frac{-L^2}{m r^2} \quad \text{(in terms of angular momentum)}$$

(iii) Total energy (E) :

E = Potential energy + Kinetic energy

$$= \frac{-GM_e\, m}{2r} \quad ......\ (3)$$

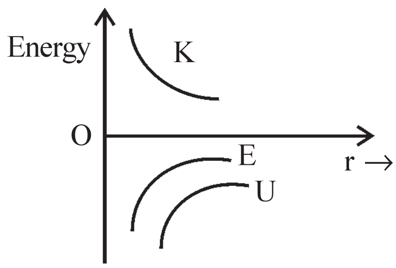
$$= \frac{-L^2}{2 m r^2} \quad \text{(in terms of angular momentum)}$$

- Total energy of a satellite is negative.

  From equations (1), (2) and (3)

  $\frac{E}{K} = -1 \Rightarrow K = -E$ and

  $\frac{U}{E} = 2 \Rightarrow U = 2E$



- For any value of r, values of U and E are negative and value of K is positive and K = – E.
- As $r \to \infty$ all three energy curves approach a value of zero.

Binding Energy of Satellite :

Total energy of satellite is $E = \frac{-GM_e\, m}{2r}$. Negative sign indicates that this satellite is in the bound state by an attractive force of central body. Thus energy must be supplied to remove it from the orbit to infinity. The energy required to remove the satellite from the orbit (from the gravitational field of the planet here the earth) to infinity is called Binding Energy of the system. i.e.

$\therefore$ Binding Energy (B.E.) $= -E = \frac{GM_e\, m}{2r}$

If the orbit of a satellite is elliptical

then, Total Energy $E = -\frac{GM_e\, m}{2a}$ = constant.

Where a = semi major axis

- When the satellite is closest to the central body (about which it revolves) (at perigee), then kinetic energy of satellite is maximum. i.e. the potential energy is minimum. (from formula U = E – K) and when the satellite is farthest from the central body (at apogee), then kinetic energy of satellite is minimum and potential energy is maximum.
- When the satellite is transferred from one circular orbit of radius r_1 to other circular orbit of radius r_2 ($r_2 > r_1$) then the variation in different quantities can be shown by the following table.

	Quantity	Variation	Relation with *r*
(1)	Orbital Velocity	Decreases	$v_0 \propto \frac{1}{\sqrt{r}}$
(2)	Time Period	Increases	$T \propto r^{\frac{3}{2}}$
(3)	Linear momentum	Decreases	$P \propto \frac{1}{\sqrt{r}}$
(4)	Angular momentum	Increases	$L \propto \sqrt{r}$
(5)	Kinetic Energy	Decreases	$K \propto \frac{1}{r}$
(6)	Potential Energy	Increases	$U \propto -\frac{1}{r}$
(7)	Total Energy	Increases	$E \propto -\frac{1}{r}$
(8)	Binding Energy	Decreases	$B.E. \propto \frac{1}{r}$

Geo-Stationary Satellite (Geo-Synchronous satellite) (parking satellite)

- The Earth's satellite having orbital periodic time of 24 hours (equal to the periodic time of rotation of the Earth about it's own axis), is called the geo-stationary satellite.
- Geo-stationary satellite revolve around the Earth in the equitorial plane in east west direction.

Height of Geo-stationary satellite from the surface of the Earth :

$$T^2 = \frac{4\pi^2 r^3}{GM_e} \Rightarrow r = \left(\frac{GM_e T^2}{4\pi^2}\right)^{\frac{1}{3}}$$

Now substituting all values in above equation, we get

$\therefore$ r = 42,260 km

$\therefore$ $h = r - R_e = 42{,}260 - 6400 = 35860$ km

Such an orbit of satellite is known as parking orbit.

The orbital speed of satellite using equation $\sqrt{\frac{GM_e}{r}}$ is 3.08 kms^{-1}

Polar Satellite :

- These satellites revolve around the Earth in north south direction at height nearly of 800 km from the surface of the Earth.
- The time period of these satellites is almost 100 min.

Maximum height attained by a projectile :

Suppose body of mass m is thrown with velocity *v* in vertically upward direction from the surface of the earth and it attains maximum height of H. At maximum height its velocity is zero.

According to law of conservation of mechanical energy,

Total energy at the Earth's surface = Total energy at a height from Earth's surface.

$$\therefore \frac{1}{2} mv^2 + \left(-\frac{GM_e m}{R_e}\right) = 0 + \left(-\frac{GM_e m}{R_e + H}\right)$$

Where R_e = Radius of the Earth, M_e = Mass of the Earth.

$$\therefore\ v^2 = 2GM_e\left[\frac{1}{R_e} - \frac{1}{R_e+H}\right]$$

$$= 2GM_e\left(\frac{H}{(R_e)(R_e+H)}\right)$$

$$= 2gR_e^{\ 2}\left(\frac{H}{(R_e)R_e(1+H/R_e)}\right) \qquad (\because\ GM_e = gR_e^{\ 2})$$

$$v^2 = \frac{2gH}{1+\frac{H}{R_e}} \Rightarrow v = \left(\frac{2gH}{\left(1+\frac{H}{R_e}\right)}\right)^{\frac{1}{2}}$$

$$\text{Now, } v^2 = \frac{2gHR_e}{R_e+H}$$

$\therefore\ (v^2)(R_e + H) = 2gHR_e \Rightarrow v^2R_e + v^2H = 2gHR_e$

$\therefore\ v^2R_e = (2gR_e - v^2)H$

$$\Rightarrow H = \frac{v^2 R_e}{2gR_e - v^2}$$

Relative angular velocity of satellite

If satellite revolve around the Earth in the equitorial plane in same sense of rotations as that of earth about its own axis (from west to east), Then the relative angular velocity of satellite for an observer lying at the Earth surface is,

$\omega_{relative} = \omega_s - \omega_E$

Where ω_s = angular velocity of satellite

ω_E = angular velocity of Earth

Now using equation $\omega = \dfrac{2\pi}{T}$,

$$T = \frac{2\pi}{\omega_{relative}} = \frac{2\pi}{\omega_s - \omega_E} = \frac{2\pi}{\frac{2\pi}{T_s} - \frac{2\pi}{T_E}} = \frac{T_E\ T_s}{T_E - T_s}$$

---

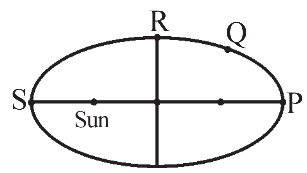
(58) The time period of a satellite orbiting close to the surface of the earth is 50 min. The time period of a satellite orbiting at height three times the radius of the earth from the surface of the earth is ......

(A) 100 min (B) 400 min (C) $50 \times \sqrt{8}$ min (D) 150 min

(59) The rate of rotation of a planet is 8 times the rate of rotation of earth around the sun. Then ratio of their radii of orbits of rotation is ......

(A) $\frac{1}{24}$ (B) $\frac{1}{4}$ (C) $\frac{1}{16}$ (D) $\frac{1}{2}$

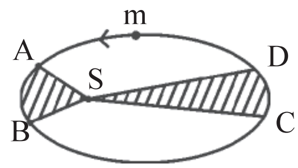
(60) The mercury (A planet) is revolving around the sun as shown in elliptical path. The potential energy of the mercury will be minimum at the point ......



(A) P (B) Q

(C) R (D) S

(61) The figure shows elliptical orbit of a planet of mass m about the sun S. The shaded area SCD is thrice the shaded area SAB. If t_1 is the time for the planet to move from A to B and t_2 is time to move from C to D then $\frac{t_1}{t_2}$ = ...



(A) 1 (B) 2 (C) 3 (D) 4

(62) The period of revolution of planet A is 27 times that of B. The distance of A from the sun is how many times greater than that of B from the sun ......

(A) 9 (B) 8 (C) 4 (D) 6

(63) Two satellites are revolving into a circular orbit of radii r and 1.01 r respectively around the earth. Their orbital time periods are T_1 and T_2 respectively. The period of second satellite is larger than that of the first one by approximately ......

(A) 0.5 % (B) 1.0 % (C) 1.5 % (D) 3.0 %

(64) The time period of a satellite revolving at a height equal to the radius of the earth from the surface of the earth is ...... (radius of the earth R, acceleration due to gravity = g)

(A) $2\pi\sqrt{\frac{2R}{g}}$ (B) $4\sqrt{2}\,\pi\sqrt{\frac{R}{g}}$ (C) $2\pi\sqrt{\frac{R}{g}}$ (D) $8\pi\sqrt{\frac{R}{g}}$

(65) A geo-stationary satellite is orbiting the earth at a height 6R above the surface of earth. R being the radius of the earth. The time period of another satellite at a height of 2.5R from the surface of the earth will be ......

(A) $6\sqrt{2}$ hr (B) 6 hr (C) 10 hr (D) $2\sqrt{6}$ hr

(66) Two satellites A and B are revolving in the circular orbit of equal radii around the earth. The mass of A is 100 times the mass of B. Their time periods are in the ratio of ......

(A) 1 : 100 (B) 100 : 1 (C) 1 : 1 (D) 10 : 1

(67) The rotation period of a satellite of mass m revoling close to the surface of the earth is ......

(A) 72 min (B) 62.2 min (C) 84.6 min (D) 104 min

(68) The time period of revolution of a satellite orbiting close to the surface of the earth in terms of density of the earth (ρ) is ......

(A) $\sqrt{\frac{3\pi}{G\rho}}$ (B) $\sqrt{\frac{4\pi}{G\rho}}$ (C) $\sqrt[3]{\frac{\pi}{G\rho}}$ (D) $\frac{2\pi}{\sqrt{G\rho}}$

(69) Two satellites A and B go round the earth in circular orbits having radii 4R and R. If the orbital velocity of satellite A is $3v$, then the orbital velocity of satellite B is ......

(A) $6\,v$ (B) $12\,v$ (C) $\frac{4}{3}\,v$ (D) $\frac{3}{2}\,v$

(70) The orbital velocily of a satellite revolving around the earth in a circular orbit close to the surface of the earth is v_0. The orbital velocity of another satellite revolving at a height one half of the radius of the earth, from the surface of the earth, is ......

(A) $\sqrt{\frac{3}{2}}\,v_0$ (B) $\sqrt{\frac{2}{3}}\,v_0$ (C) $\frac{2}{3}\,v_0$ (D) $\frac{3}{2}\,v_0$

(71) The orbital time period of satellite revolving around the earth in the orbit of radius r is T. If the same satellite is revolving in the orbit of radius $2r$, the new periodic time is ......

(A) 2T (B) 1.5 T (C) 2.8 T (D) 0.5 T

(72) If orbital period of a satellite is T, then its kinetic energy is proportional to ......

(A) $\frac{1}{T}$ (B) $\frac{1}{T^3}$ (C) $T^{\frac{2}{3}}$ (D) $T^{\frac{-2}{3}}$

(73) A satellite of mass m and having kinetic energy K is orbiting around the earth in circular orbit of radius r. The angular momentum of the satellite is ......

(A) $\sqrt{\frac{K}{mr^2}}$ (B) $\frac{K}{2mr^2}$ (C) $\sqrt{2Kmr^2}$ (D) $\sqrt{2Kmr}$

Ans. : 58 (B), 59 (B), 60 (D), 61 (C), 62 (B), 63 (C), 64 (B), 65 (A), 66 (C), 67 (C), 68 (A), 69 (A), 70 (B), 71 (C), 72 (D), 73 (C)

Assertion - Reason type Question :

Instruction : Read assertion and reason carefully, select proper option from given below.

(a) Both assertion and reason are true and reason explains the assertion.

(b) Both assertion and reason are true but reason does not explain the assertion.

(c) Assertion is true but reason is false.

(d) Assertion is false and reason is true.

(74) **Assertion :** The earth suddenly stops rotating about its axis, then the value of acceleration due to gravity will become same at all the places.

Reason : The value of acceleration due to gravity is independent of rotation of the earth.

(A) a (B) b (C) c (D) d

(75) **Assertion :** The escape velocities for two objects projected in the direction making an angle of 30° and 60° with the surface of the earth, from the surface of the earth are $v_1 = 2v_e$ and $v_2 = \frac{2v_e}{\sqrt{3}}$ respectively.

Reason : The value of escape velocity does not depend on the angle of projection.

(A) a (B) b (C) c (D) d

(76) **Assertion :** For the planets orbiting around the sun, angular speed, linear speed, kinetic energy changes with time but the angular momentum remains constant.

Reason : No torque is acting on the rotating planet. So its angular momentum is constant.

(A) a (B) b (C) c (D) d

(77) **Assertion :** The weight of a body on the surface of the earth is more at mid night time that of noon time.

Reason : The gravitational forces exerted on the body by the earth and by the sun are in opposite direction to each other at noon time.

(A) a (B) b (C) c (D) d

(78) **Assertion :** The orbital time period of a satellite revolving close to the sarface of the earth is smaller than that the satellite revolving far away from the surface of the earth.

Reason : The square of the orbital time period is directly proportinal to the cube of the orbital radius.

(A) a (B) b (C) c (D) d

(79) **Assertion :** The orbital speed of a satellite is greater than its escape speed.

Reason : Orbit of a satellite is within the gravitational field of earth, whereas escaping is beyond the gravitational field of earth.

(A) a (B) b (C) c (D) d

(80) **Assertion :** Different planets have different values of escape velocity.

Reason : The value of escape velocity is not a universal constant.

(A) a (B) b (C) c (D) d

(81) **Assertion :** The gravitational froce exerted on a body by the moon is less than that by the earth.

Reason : The value of gravitational force depends on the factor $\frac{M}{r^2}$ for a given mass m and it is very small for the moon. Where r = distance from the centre.

(A) a (B) b (C) c (D) d

(82) **Assertion :** Gravitational force between two particles is negligibly small compared to the electrical force.

Reason : The electrical force is experienced by the charged particles only.

(A) a (B) b (C) c (D) d

(83) **Assertion :** Body becomes weightless at the Earth's centre.

Reason : The gravitational acceleration increases when distance decreases from surface of Earth.

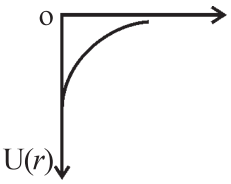
(A) a (B) b (C) c (D) d

And. : 74 (C), 75 (D), 76 (A), 77 (D), 78 (A), 79 (D), 80 (A), 81 (A), 82 (B), 83 (C)

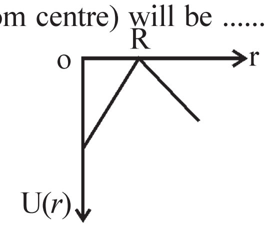
Graph based questions :

(84) A shell of mass M and radius R has a point mass m placed at a distance r from its centre. The gravitational potential energy U (r) versus r (distance from centre) will be ......

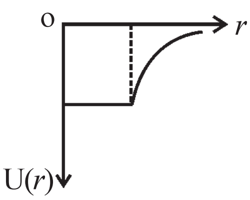
(A)



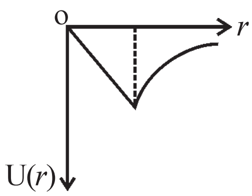
(B)



(C)

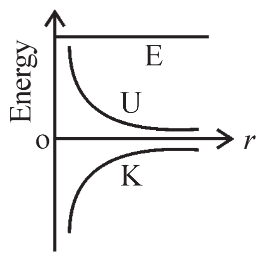


(D)

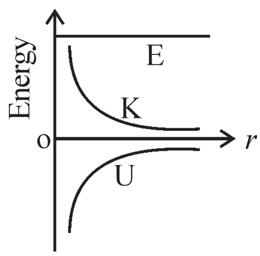


(85) The correct graph representing the variation of total energy (E), Kinetic energy (K) and potential energy (U) of a satellite with its distance from the centre of earth is ......

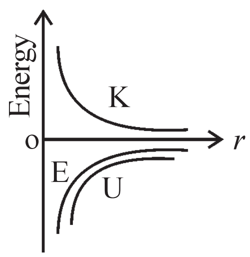
(A)



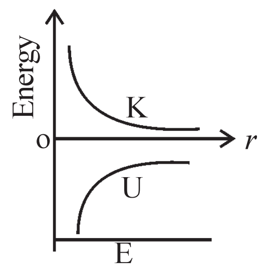
(B)



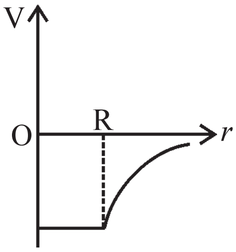
(C)

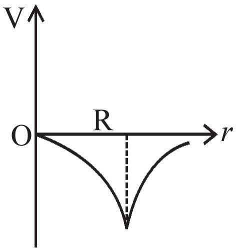


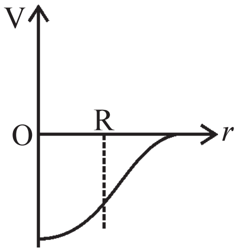
(D)

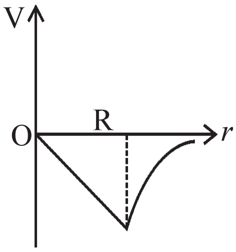


(86) The diagram showing the variation of gravitational potential of earth with distance from the centre of earth is ......

(A) 

(B) 

(C) 

(D) 

Ans. : 84 (C), 85 (C), 86 (C)

Comprehension Type Questions :

Paragraph-1

The gravitational field in a region is given by $\vec{I} = 5\hat{i} + 12\hat{j}$ N kg^{-1}. Answer the following questions.

(87) Find the magnitude of the gravitational force acting on a body of mass 2 kg placed at the origin ......

(A) 26 N (B) 30 N (C) 20 N (D) 35 N

(88) Find the potential at points (12 m, 0) and (0, 5 m), if the potential at the origin is taken to be zero.

(A) –30 J kg^{-1}, –30 J kg^{-1} (B) –40 J kg^{-1}, –30 J kg^{-1}

(C) –60 J kg^{-1}, –60 J kg^{-1} (D) –40 J kg^{-1}, –50 J kg^{-1}

(89) Find the change in gravitational potential energy if a body of mass 2 kg is taken from the origin to the point (12 m, 5 m) ......

(A) –225 J (B) –240 J (C) – 245 J (D) –480 J

(90) Find the change in potential energy if the body is taken from (12 m, 0) to (0, 5m)

(A) –10 J (B) – 50 J (C) 0 (D) – 60 J

Paragraph - 2

Assume that orbits of Earth and the Mars around the sun to be circular. An artificial satellite is launched from the earth which can revolve around the sun in such a way that its apogee is lying on the axis of rotation of Mars and the perigee is lying on the axis of rotation of earth. The orbital time periods for earth and the Mars are T_e and T_m respectively around the sun and symbols for different parameters are as under :

M_e = mass of earth, M_m = mass of Mars, M = mass of artificial satellite, L_e = angular momentum of earth around sun, L_m = angular momeatum of Mars around sun, R_e = semi major axis of arbit of Earth, R_m = Semi major axis of orbit of Mars, E_e = total energy of the earth, E_m = total energy of Mars.

(91) The orbital time period of a satellite around the sun is ...... (Neglect the effect of gravitational field by earth and by Mars.)

(A) $\frac{T_e + T_m}{2}$ (B) $\sqrt{T_e T_m}$ (C) $\frac{2T_e T_m}{T_e + T_m}$ (D) $\left[\frac{T_e^{\frac{2}{3}} + T_m^{\frac{2}{3}}}{2}\right]^{\frac{3}{2}}$

(92) Total energy of the satellite is ......

(A) $\frac{2M}{M_e}\left(\frac{R_e E_e}{R_e + R_m}\right)$ (B) $\frac{2M}{M_m}\left(\frac{R_e E_e}{R_e + R_m}\right)$

(C) $\frac{2E_e M}{M_m}\left(\frac{R_e + R_m}{R_m}\right)$ (D) $\frac{2E_e M}{M_e}\left(\frac{R_e + R_m}{\sqrt{R_e^2 + R_m^2}}\right)$

(93) Areal velocity of a satellite around the sun is ......

(A) Less than that of the areal velocity of earth.

(B) Greater than that of the areal velocity of Mars.

(C) Same as that of the areal velocity of earth.

(D) Greater than that of the areal velocity of earth.

Ans : 87 (A), 88 (C), 89 (B), 90 (B), 91 (D), 92 (A), 93 (D)

Match the columns :

(94) A satellite is projected vertically near the surface of a planet with speed v. The value of acceleration of a freely falling body near this planet is found to be 4.9 ms^{-2}. Radius of the planet is 3200 km. For various values of v, the path of satellite can be predicted. Match the velocity of satellite with its respective path $\left(\sqrt{2} = 1.4\right)$

Column-1		Column-2	
(a)	$v = 4$ km s^{-1}	(p)	Elliptical
(b)	$v = 5$ km s^{-1}	(q)	Circular
(c)	$v = 5.6$ km s^{-1}	(r)	Hyperbolic
(d)	$v = 6.6$ km s^{-1}	(s)	Parabolic

(A) a → q b → p c → s d → r

(B) a → p b → q c → r d → s

(C) a → s b → r c → p d → q

(D) a → r b → s c → q d → p

(95) **Match Column 1 and Column 2**

Column-1		Column-2	
(a)	Elliptical orbit of a planet	(p)	Conservation of kinetic energy
(b)	Circular orbit of a satellite	(q)	Conservation of angular momentum
(c)	Escape velocity	(r)	Independent of mass of a satellite
(d)	Orbital velocity	(s)	$\sqrt{\frac{GM}{R}}$
		(t)	Constant areal velocity

(A) a → q, t b → p, q, t c → r d → r, s

(B) a → p, r b → q, r c → t d → p

(C) a → s b → r c → s d → t

(D) a → p b → q c → r d → s

Ans. : 94 (A), 95 (A)

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