2 Kinematics

Vector

	Equal vectors	**Parallel vectors**	**Opposite vectors**	**Antiparallel vectors**	**Aparallel vectors**
Magnitude	Equal	Unequal	Equal	Unequal	Equal or Unequal
Direction	Equal	Equal	Opposite	Opposite	any
Angle between two vectors	$\theta = 0°$	$\theta = 0°$	$\theta = 180°$ or π rad	$\theta = 180°$ or π rad	$\theta \neq 0°, \theta \neq 180°$ $0 < \theta < 360°$

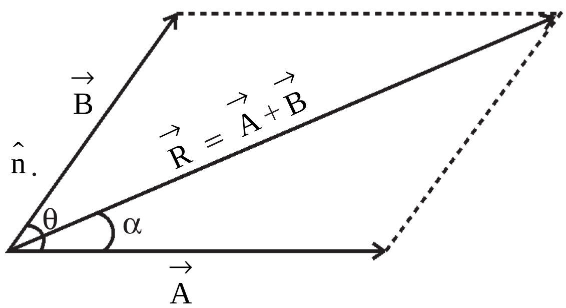
- When a vector is multiply by any value or scaler, then direction remain as it is but magnitude becomes scaler times.

Addition or Subtraction of vectors

- Method of triangle
- Method of parallelogram and
- Algebraic or analytical Method
- It $\vec{R}$ is resultant vector of addition of vector $\vec{A}$ and vector $\vec{B}$, then $|\vec{R}| = \sqrt{A^2+B^2+2AB\cos\theta}$.

  and $\vec{R}$ makes an angle α with vector $\vec{A}$, then

  $$\tan\alpha = \frac{B\sin\theta}{A+B\cos\theta} = \frac{R_y}{R_x}$$

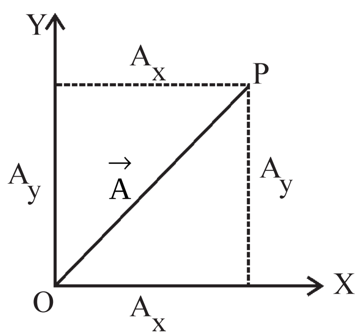


- Magnitude of unit vector is one unit, it is expressed as $\hat{n}$.
- Unit vector in direction of $\vec{A}$,

  $$\hat{n}_A = \frac{\vec{A}}{|\vec{A}|} = \frac{\text{Vector}}{\text{magnitude of vector}}$$

- Unit vectors in three dimensions (i.e. cartesian co-ordinate system) X-axis $\rightarrow \hat{i}$, Y-axis $\rightarrow \hat{j}$, Z-axis $\rightarrow \hat{k}$

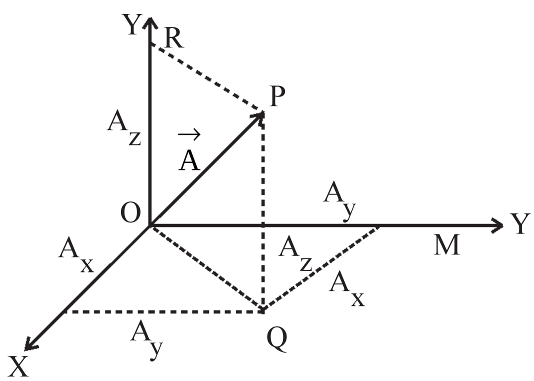
Resolution of a vector



In two dimension

$\vec{A} = A_x\hat{i} + A_y\hat{j}$ and

$|\vec{A}| = \sqrt{A_x^2 + A_y^2}$



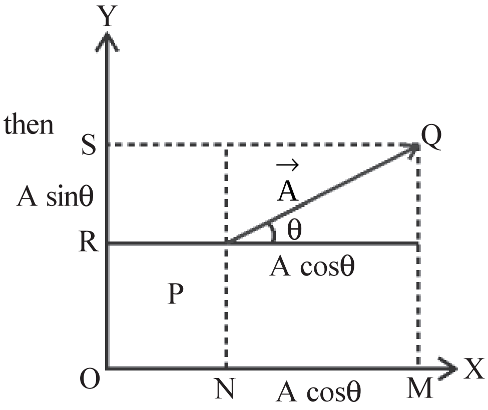
In three dimension

$\vec{A} = A_x\hat{i} + A_y\hat{j} + A_z\hat{k}$

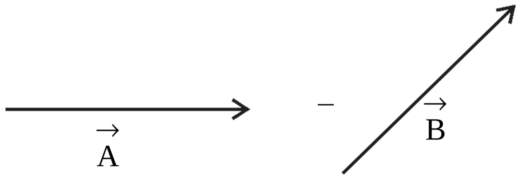
$|\vec{A}| = \sqrt{A_x^2 + A_y^2 + A_z^2}$

- **Angular resolution of a vector (In two dimensions)**
- $\vec{A} = A\cos\theta\,\hat{i} + A\sin\theta\,\hat{j}$
- A vectors makes an angle θ with x-axis in two dimensions, then

$\tan\theta = \dfrac{A_y}{A_x}$ $\quad\therefore\ \theta = \tan^{-1}\left(\dfrac{A_y}{A_x}\right)$



- **Subtraction of vectors :**
To make subtraction, add an opposite vector of which will be subtract, in given vector.



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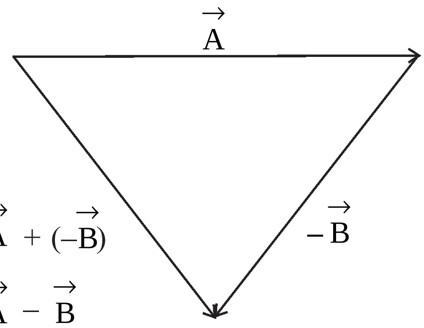


$\vec{R} = \vec{A} + (-\vec{B})$

$\vec{R} = \vec{A} - \vec{B}$

- If $\vec{R} = \vec{A} - \vec{B}$ then

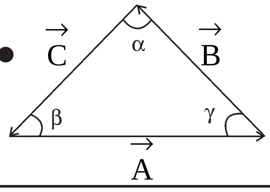
$|\vec{R}| = \sqrt{A^2 + B^2 - 2AB\cos\theta}$ and an angle of $\vec{R}$ with $\vec{A}$ is α then, $\tan\alpha = \dfrac{-B\sin\theta}{A - B\cos\theta}$

Multiplication of vectors

Scaler product (dot product) | Vector Product (cross product)

- $\vec{A}\cdot\vec{B} = |\vec{A}|\,|\vec{B}|\cos\theta$
- $\vec{A}\cdot\vec{B}$ = The product of the magnitude of first vector with the projection of second vector on the first vector
- $\vec{A}\cdot\vec{B} = \vec{B}\cdot\vec{A}$
- $\vec{A}\cdot(\vec{B}+\vec{C}) = \vec{A}\cdot\vec{B} + \vec{A}\cdot\vec{C}$
- If $\vec{A}$ and $\vec{B}$ are parallel then,
$\vec{A}\cdot\vec{B} = |\vec{A}|\,|\vec{B}|$
- If $\vec{A}$ and $\vec{B}$ are antiparallel, then
$\vec{A}\cdot\vec{B} = -|\vec{A}|\,|\vec{B}|$

- $\vec{A}\times\vec{B} = |\vec{A}|\,|\vec{B}|\sin\theta\ \hat{n}$
Where $\hat{n}$ is unit vector in the direction obtained by right handed screw rule.
- $\vec{A}\times\vec{B} = -\vec{B}\times\vec{A}$
- $\vec{A}\times(\vec{B}+\vec{C}) = \left(\vec{A}\times\vec{B}\right) + \left(\vec{A}\times\vec{C}\right)$
- If $\vec{A}$ and $\vec{B}$ are parallel or anti parallel $\left(\vec{A}\,\|\,\vec{B}\right)$, then $\vec{A}\times\vec{B} = 0$
- If $\vec{A}$ and $\vec{B}$ are perpendicular $\left(\vec{A}\perp\vec{B}\right)$
then, $\vec{A}\times\vec{B} = |\vec{A}|\,|\vec{B}|\ \hat{n}$

- If $\vec{A}$ and $\vec{B}$ are perpendicular then $\vec{A}\cdot\vec{B} = 0$
- $\vec{A}\times\vec{B} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix}$
- $\vec{A}\cdot\vec{B} = A_xB_x + A_yB_y + A_zB_z$
- Area of the triangle formed by $\vec{A}$ and $\vec{B}$, $\Delta = \frac{1}{2}|\vec{A}\times\vec{B}|$
- $\theta = \cos^{-1}\left[\dfrac{A_xB_x + A_yB_y + A_zB_z}{(\sqrt{A_x^2 + A_y^2 + A_z^2})(\sqrt{B_x^2 + B_y^2 + B_z^2})}\right]$
- 

$\dfrac{\text{Sin}\,\alpha}{A} = \dfrac{\text{Sin}\,\beta}{B} = \dfrac{\text{Sin}\,\gamma}{C}$

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(1) Magnitude of $\vec{A}$ and $\vec{B}$ are 10 unit and 20 unit respectively. If the angle between this two vectors is 30°, the magnitude of resultant (R) is ...... unit and angle with vector $\vec{A}$ is ...... ($\sqrt{3} = 1.7$).

(A) 30 unit, 30° (B) 10 unit, 30° (C) 29 unit, 20° 19' (D) 39 unit, 30° 30'

(2) A force act on a substance having mass m is $\vec{F} = 5\hat{i} - 4\hat{j} + 2\hat{k}$ N. An angle of this force with X-axis is ......

(A) $\text{Cos}^{-1}\left(\frac{1}{3\sqrt{5}}\right)$ (B) $\text{Cos}^{-1}\left(\frac{\sqrt{5}}{3}\right)$ (C) $\text{Cos}^{-1}\left(\frac{3}{\sqrt{5}}\right)$ (D) $\text{Cos}^{-1}\left(3\sqrt{5}\right)$

(3) $|\vec{A}\times\vec{B}|$ is greater by 73 % than $\vec{A}\cdot\vec{B}$ for given $\vec{A}$ and $\vec{B}$. An angle between $\vec{A}$ and $\vec{B}$ is ......

(A) 0° (B) 30° (C) 60° (D) 90°

(4) If the addition vector $\vec{R}$ of $\vec{A}$ and $\vec{B}$ makes an angle α with $\vec{A}$ and β with $\vec{B}$. Then ......

(A) Always $\alpha < \beta$ (B) If $A < B$, $\alpha < \beta$
(C) If $A = B$, $\alpha < \beta$ (D) If $A > B$, $\alpha < \beta$

(5) An angle between $\vec{A} = 3\hat{i} - 4\hat{j} - 2\hat{k}$ and $\vec{B} = 8\hat{i} + 3\hat{j} + 6\hat{k}$ is ......

(A) 90° (B) 60° (C) 180° (D) 0°

(6) If the vectors $\vec{A} = 5\hat{i} + 7\hat{j} - 3\hat{k}$ and $\vec{B} = -2\hat{i} + m\hat{j} + 6\hat{k}$ are mutually perpendicular, then m = ...

(A) 1 (B) 4 (C) 3 (D) -4

(7) If $\vec{A} = 2\hat{i} - \hat{j} + 3\hat{k}$ and $\vec{B} = \hat{i} + 2\hat{j} - \hat{k}$ then unit vector in the direction of $\vec{A}\times\vec{B}$ is ......

(A) $\frac{1}{\sqrt{14}}(3\hat{i} + \hat{j} + 2\hat{k})$ (B) $\hat{i} - \hat{j} + \hat{k}$

(C) $\frac{1}{\sqrt{3}}(\hat{i} + \hat{j} + \hat{k})$ (D) $\frac{1}{\sqrt{3}}(-\hat{i} + \hat{j} + \hat{k})$

(8) Area of triangle formed by $\vec{A} = 4\hat{i} + 3\hat{j} - \hat{k}$ and $\vec{B} = \hat{i} + 2\hat{j} + \hat{k}$ is ...... unit.

(A) $10\sqrt{3}$ (B) $\frac{5\sqrt{3}}{2}$ (C) $\frac{\sqrt{3}}{2}$ (D) $5\sqrt{3}$

(9) $|\vec{A} \times \vec{B}|^2 + |\vec{A} \cdot \vec{B}|^2 =$ ......

(A) zero (B) $\sqrt{AB}$ (C) AB (D) A^2B^2

(10) If $|\vec{A} + \vec{B}| = |\vec{A}| = |\vec{B}|$, then the angle between $\vec{A}$ and $\vec{B}$ is ....

(A) 0° (B) 30° (C) 90° (D) 120°

(11) The unit vector, which is perpendicular to $\vec{A}$ and $\vec{B}$ both is ....

(A) $\frac{\vec{A} - \vec{B}}{AB}$ (B) $\frac{\vec{A} \times \vec{B}}{AB\sin\theta}$ (C) $\frac{\vec{A} \times \vec{B}}{AB\cos\theta}$ (D) $\frac{\vec{A} \cdot \vec{B}}{AB\sin\theta}$

(12) If $\frac{2}{3}\hat{i} + \frac{1}{3}\hat{j} + m\hat{k}$ is unit vector, then m = ...

(A) $\frac{2}{3}$ (B) $-\frac{1}{3}$ (C) 1 (D) $\frac{1}{3}$

(13) If $|\vec{A}| = 4$, $|\vec{B}| = 2.5$, $\vec{A} \cdot \vec{B} = 8$ and an angle between $\vec{A}$ and $\vec{B}$ is acute angle then $|\vec{A} \times \vec{B}| =$ ....

(A) 6 (B) 10 (C) 3.2 (D) 2

(14) Aproximate area of parallelogram fromed by $\vec{A} = 2\hat{i} - 3\hat{j} - \hat{k}$ and $\vec{B} = 4\hat{i} + 3\hat{k}$ is ...... unit.

(A) 5 (B) 11 (C) 18 (D) 15

(15) The component of $\vec{A} = 3\hat{i} - 2\hat{j} + \hat{k}$ in the direction of $\vec{B} = \hat{i} + \hat{j}$ is ....

(A) 1 (B) $\sqrt{2}$ (C) $\frac{1}{\sqrt{2}}$ (D) $\sqrt{14}$

Ans : 1 (C), 2 (B), 3 (C), 4 (D), 5 (A), 6 (B), 7 (D), 8 (B), 9 (D), 10 (D), 11 (B), 12 (A), 13 (A), 14 (C), 15 (C)

Mechanics

- A branch of physics dealing with properties of a moving body, causes of motion, results due to motion etc. is known as mechanics.
- Mechanics devides in two branches.
  - **Kinematics :** A branch of mechanics dealing with motion without considering its causes is known as kinematics.
  - **Dynamics :** A branch of mechanics describing motion along with its causes and properties of moving body is called dynamics.

Concept of a particle and frame of reference :

- A point like object having mass can be considered as a particle.
- If the distance between two object is very large as compared to their dimensions, these objects can be treated as particle.

- All the particles of a solid body performing linear motion cover the same distance in the same time. Hence, the motion of such a body can be described in terms of the motion of any of its constituent particles.
- Particle is relative concept.
- Motion is also a relative concept.
- Motion is described according to a place and a situation of an observer.
- A place and a situation from where an observer takes his observation is called frame of reference. Frame of reference is arbitrary.
- Frame of reference will be inertial or non-inertial.

Position, Path length and Displacement :

- To describe a motion and position of particle, we can choose three mutually perpeducular axis name them X, Y and Z.
- If all the three co-ordinates of a particle remain unchanged with time, the particle is considered at rest with respect to time in this frame. If one or more co-ordinates of a particle changes with time, we say that particle is in motion with respect to this frame.

Path length	Displacement
● The total distance travelled by a particle in some time interval is called the path length or total distance.	● The change in position of a particle in some time interval is called displacement.
● Path length depends on path.	● Displacement does not depends on path. Its depends only on initial and final position.
● Path length is always positive.	● Displacement may be positive, negative or zero.
● Path length is scaler quantity.	● Displacement is vector quantity.
● In a normal condition, ratio of path length to displacement is one or more than one.	

(16) In a joggers park, a man complete two rotation on a circular path of radius 30 m. Then difference of path length and displacement is ...... m in given motion.

(A) 120 π (B) 60 π (C) zero (D) 30 π

(17) A person travelled a distance of 40 m along a straight line, then he travelled 30 m in right side. So, the ratio of total distance to displacement is ......

(A) 1 (B) 1.33 (C) 0.75 (D) 1.4

(18) An ant experienced 60 cm displacement on a cicular path of 60 cm radius. Then total distance travelled by it is ...... cm.

(A) $\frac{\pi}{3}$ (B) 20 π (C) 60 π (D) 120 π

Ans. : 16 (A), 17 (D), 18 (B)

Average speed and average velocity :

- With help of average speed, we know how fast the position of object is changing.
- The ratio of total distance travelled (i.e. path length) to the time interval during which the motion has been taken place is known as average speed. Thus, distance travelled by object in unit time is known as average speed.

average speed = <v> = $\bar{v}$ = $\dfrac{\text{Total travelled distance}}{\text{time interval}}$ = $\dfrac{\text{Path length}}{\text{time interval}}$

- Average velocity is the ratio of displacement to the time interval, in which the displacement occurs.Thus, the displacement covered by object in unit time is known as average velocity.

  average velocity = $\dfrac{\text{displacement}}{\text{time interval}}$

- Unit of average speed and average velocity is ms^{-1}.
- Average speed is scaler while average velocity is vector physical quantity.

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(19) A vehicle travels different distances with different speeds in the same direction. The average speed of the vehicle is ......

(A) $v_1 + v_2 + v_3 + ...$ (B) $\dfrac{v_1+v_2+v_3+.....}{t_1+t_2+t_3+....}$

(C) $\dfrac{d_1}{t_1} + \dfrac{d_2}{t_2} + \dfrac{d_3}{t_3} + ...$ (D) $\dfrac{d_1+d_2+d_3+....}{\dfrac{d_1}{v_1}+\dfrac{d_2}{v_2}+\dfrac{d_3}{v_3}+....}$

(20) A cyclist travelled third part of total distance with a speed of 12 kmh^{-1} and remaining distance with a speed of 18 kmh^{-1}. so, average speed of his is ...... kmh^{-1}.

(A) 15 (B) 12 (C) 18 (D) 15.43

(21) A person wallking in a straight line, covers $\frac{1}{3}$ part of the distance to be travelled with a speed of v_1 and remaining distance with speed v_2. So, avrage speed of his is ......

(A) $\dfrac{3v_1 v_2}{v_1+2v_2}$ (B) $\dfrac{3v_1 v_2}{2v_1+v_2}$ (C) $\dfrac{2v_1 v_2}{v_1+2v_2}$ (D) $\dfrac{2v_1 v_2}{2v_1+v_2}$

(22) A car covered 120 km straight line disance in 2 hour, then it terned left and travelled 50 km distance in 1 hour. So average speed and average velocity during this is ...... and ...... respectively.

(A) 40 kmh^{-1}, 50 kmh^{-1} (B) 56.67 kmh^{-1}, 43.33 kmh^{-1}

(C) 43.33 kmh^{-1}, 56.67 kmh^{-1} (D) 50 kmh^{-1}, 40 kmh^{-1}

(23) A length of second arm of a clock is 10 cm. This second arm is moves to 9 from 3. Average speed and average velocity of this second arm is ...... cms^{-1} and ...... cms^{-1} respectively.

(A) 3.33, 3.33 (B) 1.05, 0.6666 (C) 1.111, 3.00 (D) 1.666, 3.33

Ans. : 19 (D), 20 (D), 21 (B), 22 (B), 23 (B)

Instantaneous velocity and Instantaneous speed :

- Instantaneous velocity is obtained by time derivative of position.
- Instantaneous velocity is vector physics quantity and its direction is in direction of tangent of path.

- Magnitude of instantaneous velocity is instantaneous speed.
- In a graph of $x \rightarrow t$, instantaneous speed is slope of tangent at given point of path.

(24) A motion of object is expressed by $x(t) = 2t^2 - 3t + 4$ m. Average speed of object for initial 3 s and instantaneous speed at t = 3 s is ...... and ...... respectively.

(A) 3 ms^{-1}, 9 ms^{-1} (B) 9 ms^{-1}, 9 ms^{-1}

(C) 4.5 ms^{-1}, 3 ms^{-1} (D) 4.5 ms^{-1}, 4.5 ms^{-1}

(25) An equation of motion for a particle is given by $v = 2t + 3$ ms^{-1}. So, distance travelled by particle in initial four second is ......

(A) 11 m (B) 3 m (C) 19 m (D) 28 m

(26) An object is moving on a circular path with velocity $\vec{v}$ at a given instant. When it completes half rotation change in its velocity will be ......

(A) zero (B) $-\vec{v}$ (C) $-2\vec{v}$ (D) $2\vec{v}$

(27) Motion of an object in a straight line is given by $x = 5t^2 - 6t + 2.2$ m. The position of the object when its velocity is zero is given by ...... m.

(A) 0.6 (B) 0 (C) 0.4 (D) 1.0

Ans. : 24 (A), 25 (D), 26 (C), 27 (C)

- **Acceleration :**
  - The time rate of change of velocity of moving body is called acceleration.

Average acceleration = $\dfrac{\text{change in velocity}}{\text{time interval}}$

$\bar{a} = \langle a \rangle = \dfrac{\Delta v}{\Delta t}$

Instantaneous acceleration $\vec{a} = \dfrac{d\vec{v}}{dt}$ = time derivative of velocity = $\dot{\vec{v}}$

$= \dfrac{d^2\vec{x}}{dt^2}$ = Second derivative of position with respect to time = $\ddot{\vec{x}}$

(28) A motion of particle is given by $x(t) = 4t^2 + 4t - 3$ m. So, instantaneous acceleration of particle for 4^{th} second will be ...... ms^{-2}.

(A) zero (B) 8 (C) 4 (D) 16

(29) For a moving particle, the relation between time and position is given by $t = 4x^2 + 3x$. Where t is in second and x is in meter. So, acceleration for this particle a = ... (as a function of velocity).

(A) $12v$ (B) $8v^2$ (C) $-8v^3$ (D) $-12v^2$

(30) For moving particle, If $x = At^2 + Bt + C$ then equation for velocity ...... and quation for acceleration ....

(A) 2At + B, 2A (B) 2t + B, 2B (C) At + B, 2A (D) At^2, Bt

(31) The velocity of a particle becomes $9\hat{i} + 2\hat{j}$ ms^{-1} from $(4\hat{i} - 3\hat{j})$ ms^{-1} in 5 second. Then average acceleration is ...... ms^{-2}.

(A) $\hat{i} + \hat{j}$ (B) $13\hat{i} - 5\hat{j}$ (C) $5\hat{i} - 5\hat{j}$ (D) $\hat{i} - \hat{j}$

(32) An acceleration of moving particle is $a = 4t$ ms^{-2}. So, velocity at third second will be ...... ms^{-1}.

(A) 12 (B) 18 (C) 36 (D) zero

Ans. : 28 (B), 29 (C), 30 (A), 31 (A), 32 (B)

Kinetic equations for uniformly acceleration motion :

At $t = 0$ position of particle $x_0 = 0$ then,

at $t = t$ velocity of particle is $v = v_0 + at$.

displacement of particle is $x = v_0 t + \frac{1}{2} at^2$

$$x = \left(\frac{v+v_0}{2}\right)t$$

$$x = \frac{v^2 - v_0^2}{2a}$$

In general, at $t = 0$, position of particle $x = x_0$ then

$v = v_0 + at$

$$x - x_0 = v_0 t + \frac{1}{2} at^2$$

$$x - x_0 = \left(\frac{v+v_0}{2}\right)t$$

$$x - x_0 = \frac{v^2 - v_0^2}{2a}$$

A distance travelled in n^{th} second by a perticle, which moves with constant acceleration and v_0 as initial velocity.

$$d_{n^{th}} = v_0 + \frac{a}{2}(2n - 1)$$

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(33) A car which stands at traffic signal, starts its motion with acceleration 5 ms^{-2}. Then velocity of car after 4 s and distance travelled by car during this 4 s will be ...... and ...... respectively.

(A) 40 ms^{-1}, 20 m (B) 20 ms^{-1}, 20 m (C) 40 ms^{-1}, 40 m (D) 20 ms^{-1}, 40 m

(34) The ratio of the distances travelled in the fifth and the third second by a particle moving start from rest over a straight path with constant acceleration 8 ms^{-2} is ......

(A) $\frac{5}{3}$ (B) $\frac{9}{5}$ (C) $\frac{10}{3}$ (D) $\frac{5}{9}$

(35) An object starts its constant accelerated motion with 1 ms^{-1} initial velocity and it covered same distance in third second as it covered in initial two seconds. Then constant acceleration of object during this motion is ...... ms^{-2}.

(A) 1 (B) 5 (C) 2 (D) 3

(36) The velocity of an object moving with constant acceleration is 45 ms^{-1} at the end of 10 s and becomes 53 ms^{-1} at the end of 12 second. Then the distance travelles by object in 15 second will be ...... m.

(A) 65 (B) 82.5 (C) 110 (D) 525

(37) The object srarts from stationary, moves with constant acceleration 4 ms^{-2} for 10 s then it moves with constant velocity for 50 s and finally it stops with 2 ms^{-2} deceleration. Total distance coverd by object will be ......

(A) 1200 m (B) 1300 m (C) 2000 m (D) 2600 m

(38) In a constant acceleration motion, an object losing $\frac{2}{3}v_0$ velocity in time t. Where v_0 is initial velocity. So time required to velocity becomes zero will be ....

(A) t (B) 2t (C) $\frac{3t}{2}$ (D) $\frac{2t}{3}$

(39) Speeds of on object moving with constant acceleration at point A and B are u and v respectively. Then the speed of object at midpoint of A and B is ....

(A) $\sqrt{\frac{v^2+u^2}{2}}$ (B) $\frac{v+u}{2}$ (C) $\frac{v^2+u^2}{2}$ (D) $\frac{\sqrt{v^2+u^2}}{2}$

(40) An object starts its motion with 2 ms^{-1} velocity and acceleration given by $a = 6t^2 - 2t + 3$ ms^{-2}. So, velocity of object at t = 3 s is ...... ms^{-1}.

(A) 20 (B) 56 (C) 48 (D) 51

Ans. : 33 (D), 34 (B), 35 (C), 36 (D), 37 (D), 38 (C), 39 (A), 40 (B)

Kinetic equations for freely falling body :

- For freely falling body $v_0 = 0$ and $a = -g$.

  upward direction $\rightarrow$ Positive y-axis $\rightarrow$ Positive.

  downward direction $\rightarrow$ negative y-axis $\rightarrow$ negative.

- For freely falling body,

  $v = -gt$ $\qquad -2gy = v^2$

  $y = -\frac{1}{2}gt^2$ $\qquad -y = \frac{1}{2}vt$

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(41) Two different substances are allowed to free falling from towers of height h_1 and h_2. Time taken by them to reach on ground are t_1 and t_2 respetively. So $\frac{t_1}{t_2}$ = ...

(A) $\frac{h_1}{h_2}$ (B) $\sqrt{\frac{h_1}{h_2}}$ (C) $\frac{h_1^2}{h_2^2}$ (D) $\sqrt{\frac{h_2}{h_1}}$

(42) An object is allowed to fall freely from a tower of 80 m height. Distance covered by it in last second before touching ground is ...... m. (g = 10 ms^{-2})

(A) 4 (B) 45 (C) 35 (D) 10

(43) A freely falling body passed a building of height 25 m in last second. Then from ...... m height that body will be allowed to free fall. (g = 10 ms^{-2})

(A) 125 (B) 100 (C) 85 (D) 45

(44) The time taken to reach on ground by a freely falling object from height h_1 and h_2 are t_1 and t_2 respectively then $\frac{t_2}{t_1}$ = ...

(A) $\frac{h_2}{h_1}$ (B) $\sqrt{\frac{h_2}{h_1}}$ (C) $\frac{h_2^2}{h_1^2}$ (D) $\sqrt{\frac{h_1}{h_2}}$

(45) An object is thrown from a tower of height h in vertically upward direction with speed v. So time taken by object to touch a ground is ......

(A) $\sqrt{1+\frac{2gh}{v^2}}$ (B) $\frac{v}{g}\sqrt{1+\frac{hg}{v^2}}$ (C) $\frac{v}{g}\sqrt{1-\frac{2hg}{v^2}}$ (D) $\frac{v}{g}\left[1+\sqrt{1+\frac{2hg}{v^2}}\right]$

(46) A stone is dropped from a balloon, starts its motion with 4 ms^{-2} in vertically upward direction after 5 s. So, minimum height from ground obtain by this stone is ...... m. (g = 10 ms^{-2})

(A) 55 (B) 70 (C) 90 (D) 105

(47) An object is thrown from a tower of height H in vertically upward direction with velocity v_0, when it touch the ground its velocity becomes $2v_0$. then H = ...

(A) $\frac{3v_0^2}{2g}$ (B) $\frac{3v_0^2}{g}$ (C) $\frac{v_0^2}{2g}$ (D) $\frac{v_0}{2g}$

(48) An object is allowed to fall freely from a tower of hight 150 m, exactly at the same time another object is thrown from the bottom of the tower in vertically upward direction with velocity of 30 ms^{-1}. So, two object will be meet after ...... time.

(A) 10 s (B) 45 s (C) 5 s (D) 12 s

(49) A ball is allowed to fall freely from 10 m height from a surface. The ball reaches at 5 m height after colliding with surface. If time of contact for ball and surface is 0.05 s then average acceleration during contact is .... (g = 10 ms^{-2})

(A) 241.4 ms^{-2} (B) 965.2 ms^{-2} (C) 4828 ms^{-2} (D) 482.8 ms^{-2}

Ans. : 41 (B), 42 (C), 43 (D), 44 (B), 45 (D), 46 (B), 47 (A), 48 (C), 49 (D)

Relative velocity :

- velocity of P with respect to A =v_{PA}, velocity of P with respect to B = v_{PB}, velocity of B with respect to A = v_{BA}.
  According to this $v_{PA} = v_{PB} + v_{BA}$ (B will be eliminated.)
- Thus, velocity of P with respect to A is obtained by addition of velocity of P with respect to B and velocity of B with respect to A.
- Above matter can be expressed as below :

$$v_{BA} = v_{PA} - v_{PB}$$
$$= -v_{AP} - (-v_{BP})$$
$$= v_{BP} - v_{AP}$$ (P will be eliminated) $v_{BA} = v_B - v_A$

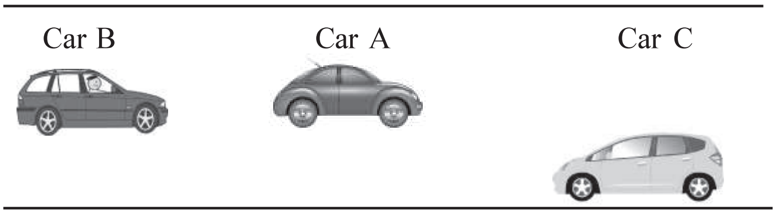
Thus relative velocity v_{BA} of moving body B with respect to moving body A will be obtained by subtraction of v_{BP} (velocity of moving body B with respect to any stationary object or point P) and v_{AP} (velocity of moving body A with respect to any stationary object or point P.)

Stationary object means reference object or point will be taken any object or point.

(50) A truck passes through a traffic signal with a constant speed 15 ms^{-1} while at same instant, a car start its motion with constant acceleration 4 ms^{-2} in the direction of truck's motion. So, relative velocity of car with respect to truck after 6 sec is ....

(A) 9 ms^{-1} (B) –9 ms^{-1} (C) 39 ms^{-1} (D) –39 ms^{-1}

(51) Car A is moving on a road with constant speed 54 kmh^{-1}. Now car B and car C are moving with 72 kmh^{-1} in mutually opposite direction on the same road. At any cretain time (instant), distance of car B and car C from car A is 1.4 km, driver of car B think to overtake car A. So, acceleration of car B will be ....



(A) 1 ms^{-2}

(B) 1.5 ms^{-2}

(C) 2 ms^{-2}

(D) –1 ms^{-2}

(52) The rain is falling verticully downward with speed of 12 kmh^{-1}. A man running with 5 kmh^{-1} on a straight way. velocity of rain experience by man will be ....

(A) 7 kmh^{-1} (B) 13 kmh^{-1} (C) 17 kmh^{-1} (D) $\sqrt{119}$ kmh^{-1}

(53) The speed of boat is 5 kmh^{-1} in still water. It crosses a river of width 1.0 km along the shortest possible path in 20 min. Then, speed of river will be ...... kmh^{-1}.

(A) 1 (B) 3 (C) 4 (D) 5

(54) A stationary man observes that the rain is falling vertically downward. When he starts running with a velocity of 10 kmh^{-1}, he observes that the rain is falling at an angle 30° with the vertical direction. The actual velocity of rain is ....

(A) $10\sqrt{3}$ kmh^{-1} (B) $\frac{10}{\sqrt{3}}$ kmh^{-1} (C) $5\sqrt{3}$ kmh^{-1} (D) 15 kmh^{-1}

Ans : 50 (A), 51 (B), 52 (B), 53 (C), 54 (A)

Relative displacement :

At t = 0, relative positions of particle A and B with respect to reference point (origin) are x_{A0} and x_{B0} and relative velocities of particles A and B with respect to reference point are v_A and v_B. So, relative displacement of particle B with respect to particle A at t = t will be,

$$x_B - x_A = \underbrace{(x_{B0} - x_{A0})}_{x_{BA}} + \underbrace{(v_B - v_A)}_{v_{BA}}\, t$$

Position of particle B with respect to particle A at t = 0

relative velocity of particle B with respect to particle A.

(55) At t = 0, car A and car B are at 50 m and 130 m away from origin respectively. Both car starts their motion in same direction with constant velocities 15 ms^{-1} and 7 ms^{-1} respectively. When and where they overtake each other ?

(A) 10 s, 200 m (B) 15 s, 170 m (C) 8 s, 180 m (D) 12 s, 150 m

(56) When particles A and B move uniformly towards each other, they get 16 m closer in 2 second and when they move uniformly in same direction with original speeds they get 16 m closer in 8 second. So, the speeds of these particles will be ...... and ....

(A) 5 ms^{-1} and 3 ms^{-1} (B) 8 ms^{-1} and 0.5 ms^{-1}

(C) 4 ms^{-1} and 4 ms^{-1} (D) 32 ms^{-1} and 2 ms^{-1}

Ans. : 55 (A), 56 (A)

Motion in one, two and three dimensions :

	Motion in one dimension	**motion in two dimension**	**motion in three dimension**
Position	x	$\vec{r} = x\hat{i} + y\hat{j}$	$\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$
displacement	$\Delta x = x - x_0$	$\Delta\vec{r} = \vec{r} - \vec{r_0}$ $= (x - x_0)\hat{i} + (y - y_0)\hat{j}$	$\Delta\vec{r} = \vec{r} - \vec{r_0}$ $= (x - x_0)\hat{i} + (y - y_0)\hat{j}$ $+(z - z_0)\hat{k}$
average speed	$<v> = \frac{\Delta x}{\Delta t}$	$\lvert\vec{v}\rvert = \frac{\lvert\Delta\vec{r}\rvert}{\Delta t}$	$\lvert\vec{v}\rvert = \frac{\lvert\Delta\vec{r}\rvert}{\Delta t}$
Instantaneous velocity	$v = \frac{dx}{dt}$	$\vec{v} = \frac{d\vec{r}}{dt}$	$\vec{v} = \frac{d\vec{r}}{dt}$
average acceleration	$<a> = \frac{\Delta v}{\Delta t}$	$<\vec{a}> = \frac{\Delta\vec{v}}{\Delta t}$	$<\vec{a}> = \frac{\Delta\vec{v}}{\Delta t}$
Instantaneous acceleration	$a = \frac{dv}{dt} = \frac{d^2x}{dt^2}$	$\vec{a} = \frac{d\vec{v}}{dt} = \frac{d^2\vec{r}}{dt^2}$	$\vec{a} = \frac{d}{dt}(\vec{v}) = \frac{d^2}{dt^2}(\vec{r})$
Kinematic equations of uniformly accel-ereted motion	$v = v_0 + at$ $x = v_0t + \frac{1}{2}at^2$ $x = \left(\frac{v+v_0}{2}\right)t$ $2ax = v^2 - v_0^2$	$\vec{v} = \vec{v_0} + \vec{a}t$ $\vec{d} = \vec{v_0}t + \frac{1}{2}\vec{a}t^2$ $\vec{d} = \left(\frac{\vec{v}+\vec{v_0}}{2}\right)t$ $2\vec{a}\cdot\vec{d} = v^2 - v_0^2$	$\vec{v} = \vec{v_0} + \vec{a}t$ $d = \vec{v_0}t + \frac{1}{2}\vec{a}t^2$ $\vec{d} = \left(\frac{\vec{v}+\vec{v_0}}{2}\right)t$ $2\vec{a}\cdot\vec{d} = v^2 - v_0^2$

- Dircetion of instantaneous velocity is in a direction of tangent drawn at given point of path.
- Instantaneous velocity in two dimensions is $\vec{v} = v_x\hat{i} + v_y\hat{j}$, then an angle made by instantaneous velocity with x-axis is obtained by equation. $tan\,\theta = \frac{v_y}{v_x}$
- Velocity is vector physical quantity, hence it can change in three ways :
  (1) Magnitude of velocity will change. (In a direction of velocity)
  (2) Direction of velocity will change. (In a perpendicular direction to velocity)
  (3) Magnitude and direction both will change. (In direction and perpendicular direction of velocity)
- Acceleration is result of change in velocity, hence it results three different ways.
- If only magnitude of velocity changes then acceleration is parallel to velocity. It known as $a_{\parallel}$.

- If only direction of velocity changes then acceleration is perpendicular to velocity. It known as $a_\perp$.
- If magnitude and direction of velocity both changes then acceleration will be parallel and perpendicular to velocity. so,

  $\vec{a} = \vec{a}_{\parallel} + \vec{a}_{\perp}$ Here $\vec{a}_{\parallel}$ and $\vec{a}_{\perp}$ are mutually perpendicular to each other.

  $\therefore |\vec{a}| = \sqrt{a_{\parallel}^2 + a_{\perp}^2}$
- During uniform circular motion, only direction of velocity changes. The direction of acceleration is perpendicular to the direction of velocity, it means in the direction of centre of circular path. That's why this acceleration is called Radial acceleration or Centripetal acceleration (a_r or a_c.)

  $a_r = a_c = \frac{v^2}{r}$

---

(57) The position vector of a moving particle depends on time according to $\vec{r} = \alpha t^2 \hat{i} + (\beta t - 3)\hat{j}$ m, then magnitude of velocity and magnitude of acceleration at time t will be ...... and ...... respectively.

(A) $\alpha t^2 + \beta t - 3, \alpha t^2$ (B) $2\alpha t + \beta, 2\alpha$ (C) $\sqrt{4\alpha^2 t^2 + \beta^2}$, 2α (D) $2\alpha t + \beta - 3, 2\alpha t$

(58) Ships A and B are standing 20 km far from each other on a line joining north south. Ship A moves with 10 kmh^{-1} in west direction and ship B moves with 10 kmh^{-1} in north direction. Then minimum distance between these two ships will be ...... km obtain after ...... minute.

(A) $10\sqrt{2}$, 60 (B) $20\sqrt{2}$, 60 (C) $\frac{10}{\sqrt{2}}$, 15 (D) $\frac{20}{\sqrt{2}}$, 15

(59) A particle starts its motion from the origin with velocity $3\hat{j}$ ms^{-1} and moves in xy plane with uniform acceleration $2\hat{i} + \hat{j}$ ms^{-2}. When the value of x co-ordinate is 25 m, then the value of y co-ordinate will be ...... m.

(A) 25 (B) 54 (C) 27.5 (D) 55

Ans. : 57 (C), 58 (A), 59 (C)

Projectile motion

When an object is thrown in gravitational field of earth it moves with constant horizontal velocity and only gravitational acceleration. Such two dimensional motion is called a Projectile motion.

For projectile motion,

Horizontal direction	Vertical direction
$v_{0x} = v_0 \cos\theta_0$ = constant $a_x = 0$ Position at time t, $x = (v_0\cos\theta_0)t$ velocity at time t, $v_x = v_0\cos\theta_0$	$v_{0y} = v_0 \sin\theta_0$ $a_y = -g$ Position at time t, $y = (v_0\sin\theta_0)t - \frac{1}{2}gt^2$ velocity at time t, $v_y = v_0\sin\theta_0 - gt$

- Position of projectile at time t, $\vec{r} = (v_0\cos\theta_0)t\,\hat{i} + [(v_0\sin\theta_0)t - \frac{1}{2}gt^2]\,\hat{j}$

  velocity of projectile at time t, $\vec{v} = v_0\cos\theta_0\,\hat{i} + (v_0\sin\theta_0 - gt)\,\hat{j}$

- Equation of trajectory of a Projectile, y = $(\tan\theta_0)x - \dfrac{g}{2(v_0\cos\theta_0)^2}\,x^2$
- The maximum height (H) reached by the projectile, H = $\dfrac{v_0^{\,2}\sin^2\theta_0}{2g}$
- Time taken to achieve maximum height by the Projectile, $t_m = \dfrac{v_0\sin\theta_0}{g}$
- Time of flight for Projectile, $t_F = 2t_m = \dfrac{2v_0\sin\theta_0}{g}$
- Range of a projectile (Distance travelled in horizontal direction) : R = $\dfrac{2v_0^{\,2}\sin\theta_0\cos\theta_0}{g} = \dfrac{v_0^{\,2}\sin 2\theta_0}{g}$
- Maximum Range $R_{max} = \dfrac{v_0^{\,2}}{g}$ (For given v_0, θ Should be 45° for maximum Range.)
- Relation between range and maximum height for projectile $\tan\theta_0 = \dfrac{4H}{R}$
- For θ and 90° – θ, ranges covered by projectiles are equal for given v_0.
- The ranges covered by projectile at 45° + α and 45° – α are equal.

---

(60) Two objects are projected with the same velocity at different angles with the horizontal and if the range is same for both of them. It t_1 and t_2 are their time of flights then $t_1 . t_2$ = ...

(A) 2 Rg (B) 2 Hg (C) $\dfrac{2R}{g}$ (D) $\dfrac{R}{2g}$

(61) An object is projected at angle of 30° with the horizontal, with kinetic energy K. Its kinetic energy at maximum height is ......

(A) 0 (B) $\dfrac{3K}{4}$ (C) $\dfrac{K}{\sqrt{2}}$ (D) $\dfrac{\sqrt{3}K}{2}$

(62) For a projectile motion $y(t) = 12t - 5t^2$ and $x(t) = 5t$. Where x and y are in meter and t in second. So initial velocity ......

(A) 6 ms^{-1} (B) 12 ms^{-1} (C) 5 ms^{-1} (D) 13 ms^{-1}

(63) Bullets are fired with the same initial velocity v in different direction on a plane surface. These bullets would fall on the maximum area of ...... on this surface.

(A) $\dfrac{\pi v^2}{g}$ (B) $\dfrac{\pi^2 v^2}{g^2}$ (C) $\dfrac{\pi v^4}{g^2}$ (D) $\dfrac{\pi v^2}{g^2}$

(64) In a projectile motion, if the maximum height H is one-forth of the range (R), then angle of projections with horizontal is ......

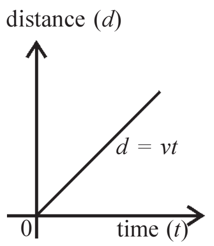
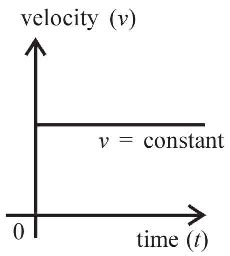
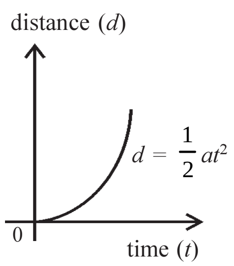
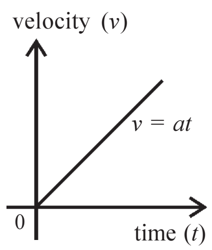
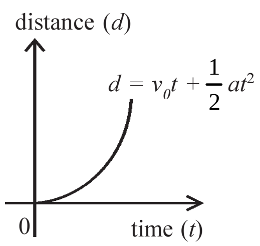
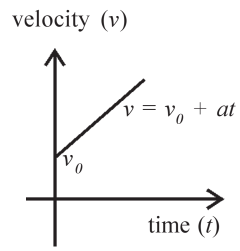
(A) zero (B) 30° (C) 45° (D) 60°

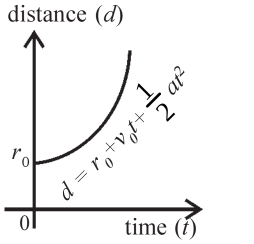
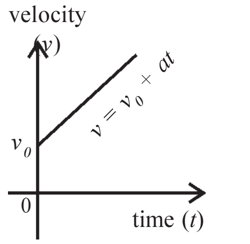
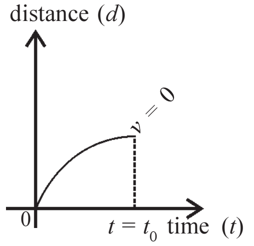
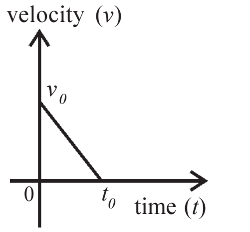
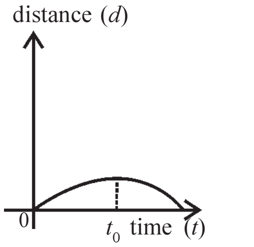
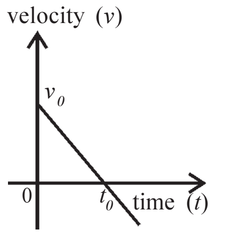
Ans. : 60 (C), 61 (B), 62 (D), 63 (C), 64 (C)

Graphs :

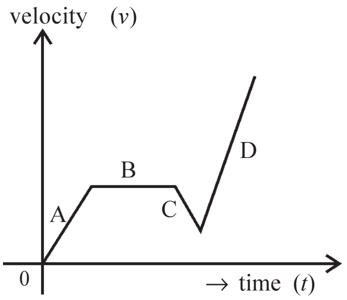
- Graph is always y → x.
- Shape of graph can be decided by power of y and x in a equation express in terms of y.
  - If both have a power 1-1 then graph is straight line, otherwise different shape for different power. (generally they are considered as a parabolic).
- For a straight line,
  - Co-efficient of x (with sign) gives a slope of graph when equation is expressed in terms of Y.
  - To obtain a segment on Y-axis, put x-co-ordinate zero and get the value of y.
  - To obtain a segment on X-axis, put y-co-ordinate zero and get the value of x.
- If graph is strait line, then equation of line will be y = mx + c.

  Where m = slope of line and c = segment on y axis.
- From a graph, a physical quantity which is obtained by product of x and y-axis will be the area under the curve and x-axis.
- From a graph, a ratio of y to x is obtained by a slope of curve at a point.

<table>
<tr><th></th><th>Distance → time graph</th><th>velocity → time graph</th><th>Important-point</th></tr>
<tr><td>Uniform motion (motion with constant velocity)</td><td>
</td><td>
</td><td>(1) d = 0 at t = 0, in graph of displacement → time.
(2) Slope of graph of displacement → time = velocity = constant
(3) Graph of velocity → time is parallel to time x-axis, so slope = 0 it means acceleration = 0</td></tr>
<tr><td>Constant acceleration motion
(i) initial velocity and initial position is zero</td><td>
</td><td>
</td><td>(1) Graph of displacement → time is Parabolic
(2) Slope of velocity → time graph = acceleration = constant
(3) Initial velocity and initial position Q is zero, So graph starts from origin.</td></tr>
<tr><td>(ii) Initial velocity ≠ 0 Initial position = 0</td><td>
</td><td>
</td><td>(1) Here initial position is zero, so graph of displacement → time starts from origin.
(2) Initial velocity is not zero, so graph of velocity → time does not start from origin
(3) Slope of velocity → time graph = acceleration = constant.</td></tr>
</table>

(iii) Initial velocity $\neq 0$ Initial position $\neq 0$			(1) Initial position and Initial velocity are non zero so, graphs do not pass through origin. (2) Slope of velocity $\rightarrow$ time = acceleration = constant
constant decelerated motion (i) Till final velocity becomes zero			(1) Slope of graph velocity $\rightarrow$ time is constant but negative (2) Final velocity becomes zero, So motion will be posible till $t = t_0$.
(i) It final velocity becomes negative			(1) Slope of graph velocity $\rightarrow$ time is constant but negative. (2) Final velocity not become zero, So motion will continue after $t = t_0$

(65) Figure shows a graph of velocity $\rightarrow$ time for any particle. It is clear from graph that ...... on particle in part B.



(A) Force act in direction of motion.

(B) Force act in opposite direction of motion.

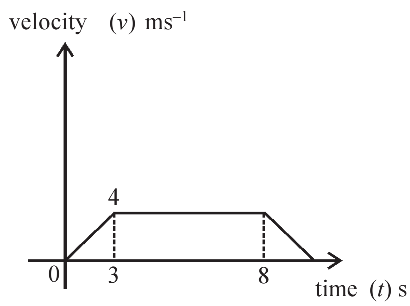
(C) Force will be zero

(D) nothing is clear about force.

(66) A graph of displacement $\rightarrow$ time makes 30° angle with time axis. After 2s it makes 60° angle with time angle. So, average acceleration during this motion is ......

(A) $\sqrt{3}$ (B) 1 (C) $2\sqrt{3}$ (D) $\frac{1}{\sqrt{3}}$

(67) Figure shows a velocity $\rightarrow$ time graph for a upward going lift. So, at ...... m height lift becomes stationary.



(A) 12

(B) 32

(C) 44

(D) 24

Ans. : 65 (C), 66 (D), 67 (B)

Assertion - Reason type Question :

Instruction : Read assertion and reason carefully, select proper option from given below.

(a) Both assertion and reason are true and reason explains the assertion.

(b) Both assertion and reason are true but reason does not explain the assertion.

(c) Assertion is true but reason is false.

(d) Assertion is false and reason is true.

(68) **Assertion :** Speed will be changed without changing velocity of object.

Reason : When velocity of object is zero then acceleration of it may not be zero.

(A) a (B) b (C) c (D) d

(69) **Assertion :** An object becomes stationary for a moment while the direction of motion will changed.

Reason : At given instant, If velocity of object is zero then acceleration of object is also zero.

(A) a (B) b (C) c (D) d

(70) **Assertion :** The $v \rightarrow$ t graph perpendicular to time axis is not possible in practice.

Reason : Infinite acceleration can not be realized in practice.

(A) a (B) b (C) c (D) d

(71) **Assertion :** Magnitude of average velocity is equal to average speed if velocity is constant.

Reason : If velocity is constant, then there is no change in the direction of motion.

(A) a (B) b (C) c (D) d

(72) **Assertion :** Particle A is moving northwards and partide B eastwards with same speed. Then, velocity of A with respect to B is in southeast direction.

Reason : Relative velocity between them is zero as their speeds are same.

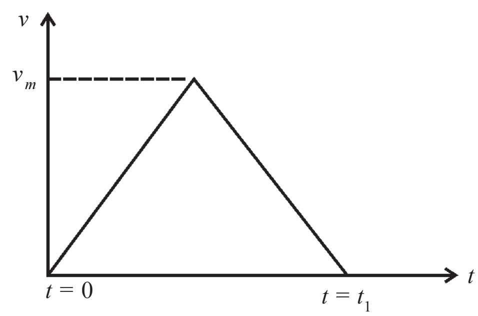
(A) a (B) b (C) c (D) d

(73) **Assertion :** In the $v \rightarrow t$ diagram shown in figure, average velocity between the interval $t = 0$ and $t = t_1$ is independent of t_1.

Reason : Average velocity for given interval is $\frac{v_m}{2}$.

(A) a (B) b

(C) c (D) d



Match the columns :

(74) Find the appropriate option from column 2 for column-1, when motion is described by $d = 3 + 8t - 4t^2$.

column-1		column-2	
(a)	Initial acceleration	(p)	-16 unit
(b)	velocity at end of 3 s	(q)	3 unit
(c)	distance travelled in 2 s	(r)	7 unit
(d)	displacement at 1 s	(s)	-8 unit

(A) a $\rightarrow$ s, b $\rightarrow$ r, c $\rightarrow$ q, d $\rightarrow$ p

(B) a $\rightarrow$ s, b $\rightarrow$ p, c $\rightarrow$ q, d $\rightarrow$ r

(C) a $\rightarrow$ q, b $\rightarrow$ r, c $\rightarrow$ s, d $\rightarrow$ p

(D) a $\rightarrow$ q, b $\rightarrow$ r, c $\rightarrow$ p, d $\rightarrow$ s

Ans. : 68 (A), 69 (C), 70 (A), 71 (A), 72 (C), 73 (A), 74 (B)