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CS 225 week 1 part 2

2.1: 22, 42, 45

2.2: 11, 13.b, 15, 20, 38, 41, 43, 45

**2.1:**

Determine whether the statement forms in 16-24 are logically equivalent. In each case, construct a truth table and include a sentence justifying your answer. Your sentence should show that you understand the meaning of logical equivalence.

22. p **∧** (q **∨** r) and (p **∧** q) **∨** (p **∧** r)

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
| p | q | r | (q **∨** r) | (p **∧** q) | (p **∧** r) | p **∧** (q **∨** r) | (p **∧** q) **∨** (p **∧** r) |
| T | T | T | T | T | T | T | T |
| T | T | F | T | T | F | T | T |
| T | F | T | T | F | T | T | T |
| T | F | F | F | F | F | F | F |
| F | T | T | T | F | F | F | F |
| F | T | F | T | F | F | F | F |
| F | F | T | T | F | F | F | F |
| F | F | F | F | F | F | F | F |

Because the true and false values for the p **∧** (q **∨** r) column and the (p **∧** q) **∨** (p **∧** r) column are identical on each row of the table these statements are said to be logically equivalent.

Use truth tables to establish which of the statement forms in 40-43 are tautologies and which are contradictions.

42. ((~p **∧** q) **∧** (q **∧** r)) **∧** ~q

|  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- |
| p | q | r | ~p | ~q | (~p **∧** q) | (q **∧** r) | (~p **∧** q) **∧** (q **∧** r) | ((~p **∧** q) **∧** (q **∧** r)) **∧** ~q |
| T | T | T | F | F | F | T | F | F |
| T | T | F | F | F | F | F | F | F |
| T | F | T | F | T | F | F | F | F |
| T | F | F | F | T | F | F | F | F |
| F | T | T | T | F | T | T | T | F |
| F | T | F | T | F | T | F | F | F |
| F | F | T | T | T | F | F | F | F |
| F | F | F | T | T | F | F | F | F |

Contradiction.

In 44 and 45, determine whether the statement in (a) and (b) are logically equivalent.

45.

a. Bob is a math and computer science major and Ann is a math major, but Ann is not both a math and computer science major.

b. it is not the case that both Bob and Ann are both math and computer science majors, but it is the case that Ann is a math major and Bob is both a math and computer science major.

Let p = Bob is a math and computer science major

q = Ann is a math major

r = Ann is a math and computer science major

a. p **∧** (q **∧** ~r) b. ~(p **∧** r) **∧** (q **∧** p)

|  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- |
| p | q | r | ~r | (q **∧** ~r) | ~(p **∧** r) | (q **∧** p) | p **∧** (q **∧** ~r) | ~(p **∧** r) **∧** (q **∧** p) |
| T | T | T | F | F | F | T | F | F |
| T | T | F | T | T | T | T | T | T |
| T | F | T | F | F | F | F | F | F |
| T | F | F | T | F | T | F | F | F |
| F | T | T | F | F | T | F | F | F |
| F | T | F | T | T | T | F | F | F |
| F | F | T | F | F | T | F | F | F |
| F | F | F | T | F | T | F | F | F |

The table above proves that the statements in a. and b. are logically equivalent

**2.2: ∧ ∨ ≡ → ↔**

Construct truth tables for the statement forms in 5 – 11.

11. (p **→** (q **→** r)) **↔** ((p **∧** q) **→** r)

let x = (p **→** (q **→** r))

y = ((p **∧** q) **→** r)

|  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| p | q | r | q **→** r | p **∧** q | p **→** (q **→** r) | (p **∧** q) **→** r | x **→** y | y **→** x | x **∧** y |
| T | T | T | T | T | T | T | T | T | T |
| T | T | F | F | T | F | F | T | T | T |
| T | F | T | T | F | T | T | T | T | T |
| T | F | F | T | F | T | T | T | T | T |
| F | T | T | T | F | T | T | T | T | T |
| F | T | F | F | F | T | T | T | T | T |
| F | F | T | T | F | T | T | T | T | T |
| F | F | F | T | F | T | T | T | T | T |

13. Use truth tables to verify the following logical equivalences. Include a few words of explanation with your answers.

b. ~(p **→** q) **≡** p **∧** ~q

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| p | q | ~q | (p **→** q) | ~(p **→** q) | p **∧** ~q |
| T | T | F | T | F | F |
| T | F | T | F | T | T |
| F | T | F | T | F | F |
| F | F | T | T | F | F |

Because the ~(p **→** q column and the p **∧** ~q column have the same truth and false values on each row of the table there is a logical equivalence between the two statement forms.

15. Determine whether the following statement forms are logically equivalent:

p **→** (q **→** r) and (p **→** q) **→** r

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| p | q | r | q **→** r | p **→** q | p **→** (q **→** r) | (p **→** q) **→** r |
| T | T | T | T | T | T | T |
| T | T | F | F | T | F | F |
| T | F | T | T | F | T | T |
| T | F | F | T | F | T | T |
| F | T | T | T | T | T | T |
| F | T | F | F | T | T | F |
| F | F | T | T | T | T | T |
| F | F | F | T | T | T | T |

They are not logically equivalent.

20. Write negations for each of the following statements. (Assume that all variables represent fixed quantities or entities, as appropriate.)

1. If P is a square, then P is a rectangle.  
   **Negation:** P is a square and P is not a rectangle.
2. If today is New Year’s Eve, then tomorrow is January.  
   **Negation:** Today is New Year’s Eve and tomorrow is not January.
3. If the decimal expansion of r is terminating, then r is rational.  
   **Negation:** The decimal expansion of r is terminating and r is not rational.
4. If n is prime, then n is odd or n is 2.  
   Negation: n is prime but n is not odd and n is not 2.
5. If x is nonnegative, then x is positive or x is 0.  
   **Negation:** x is nonnegative but x is not positive and x is not 0.
6. If Tom is Ann’s father, then Jim is her uncle and Sue is her aunt.  
   **Negation:** Tom is Ann’s father but Jim is not her uncle or Sue is not her aunt.
7. If n is divisible by 6, then n is divisible by 2 and n is divisible by 3.  
   **Negation:** n is divisible by 6 and n is not divisible by 2 or n is not divisible by 3.

In 37-39, rewrite the statements in if-then form.

38. Ann will go unless it rains.

**If-then:** If it does not rain then Ann will go.

Rewrite the statements in 40 and 41 in if-then form.

41. Having two 45° angles is a sufficient condition for this triangle to be a right triangle.

**If-then:** If this triangle has two 45° angles then it is a right triangle.

Use the contrapositive to rewrite the statements in 42 and 43 in if-then form in two ways.

43. Doing homework regularly is a necessary condition for Jim to pass the course.

**Contrapositive If-then:** If not doing homework regularly then Jim does not pass the course.

**If-then:** If Jim passes the course then he is doing homework regularly.

Note that “a sufficient condition for s is r” means r is a sufficient condition for s and that “a necessary condition for s is r” means r is a necessary condition for s. Rewrite the statements in 44 and 45 in if-then form.

45. A necessary condition for this computer program to be correct is that it not produce error messages during translation.

**If-then:** If the computer program is correct then it does not produce error messages during translation.