

A NEW MOTION MODEL FOR TRACKING OF VEHICLES¹

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Abstract: Accurate vehicle motion models are of essential importance for modern car safety systems, as they enable more precise tracking and prediction of the traffic. During normal driving, the driver controls the vehicle almost completely, yet, standard models, such as the constant acceleration kinematic model, fail to acknowledge the impact of the driver. In this paper, we propose a modified version of the above mentioned model, in which the effect of the driver is introduced as an additional acceleration. To calculate this acceleration, we approximate the driver with an optimal regulator, and derive an optimal trajectory through which the acceleration can be found. Our definition of an optimal trajectory, is such that resulting path should be both comfortable, safe, fast and legal. Simulations indicate that the new model can lead to significant gains in both tracking and prediction performance.

Keywords: motion models, active safety, tracking, prediction, driver behavior, Kalman filtering.

1. INTRODUCTION

Today's vehicles are equipped with more and more safety technology, ranging from passive systems such as airbags, to active systems such as lane departure warning systems (Lorenz *et al.*, 1998). In active safety systems, many applications need to keep track of the ego, as well as surrounding, vehicles. Consequently, the performance of the tracking algorithm, will have a substantial influence on the reliability of the safety system.

The objective of the tracking algorithm is to adaptively estimate the target state based on a sequence of measurements. Depending on the sensor setup, different measurement models (or sensor

models) are required to relate the measured data to the quantities of interest. Typically, a second type of model, describing the motion of the target, is used to improve the accuracy of the estimates. The motion model describes, statistically, how the quantities of interest varies in time, and thereby allows us to utilize information in old data to draw conclusions regarding the present state. An additional advantage, is that a well formulated motion model enables enhanced possibilities to produce precise predictions of future positions, velocities, etc.

The most commonly used motion models in current car safety systems are the constant velocity (CV) and the constant acceleration (CA) models. The derivation of these models are based on the assumption that velocity (for the CV model) and acceleration (for the CA model) are approxi-

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mately constant between the sampling instances. In many situation such models enable reasonable predictions of the next state. However, when accurate road geometry information is available, one can often do much better since the car will not move independently of the road; if the road is straight the car will probably not turn, and vice versa.

In this paper, we incorporate the orientation of the road to describe an optimal route. The underlying idea is that, during normal driving, the driver controls the car completely. To decide how to advance, an experienced driver will make a trade off between different objectives such as the desire to travel fast and the wish to travel comfortably. By designing a cost function for the tracks, we are capable of making the same comparison between different trajectories and thereby calculating the optimal path that we believe the driver will choose. As we shall see, by incorporating this information it is possible to greatly improve the tracking performance in scenarios when the quality of the measurements is poor.

2. PROBLEM FORMULATION

In this section, we define the notation and present the tracking problem. Compared to most real world scenarios, the studied one is somewhat simplified. The motivation for this, is to focus the presentation on the proposed modified motion model. Before describing the problem in more detail, the simplifications are discussed. First of all, we restrict the tracking to a single target driving on a known road. In a real application, the road needs to be modelled, for example using a clothoid model (Dickmanns and Zapp, 1986), and the model parameters estimated. Additionally, in a real traffic situation, the actions of the driver will be influenced by the positions and actions of surrounding vehicles. In our modified model, no such consideration is needed since the driver is alone on the road. However, it is not difficult to extend our model to contain more vehicles. Finally, we are using an unrealistically simple data model, which however, does not constrain the usefulness of the motion model.

The quantities of interest are stored in the time continuous state vector $\tilde{\mathbf{x}}(t)$, and the objective is to track a sampled sequence

$$\mathbf{x}(k) = \tilde{\mathbf{x}}(kT) \quad (1)$$

of the state vector. Depending on the motion model, the state vector may contain different parameters, but the position, velocity and acceleration are generally represented. To denote the vectors, it is useful in this work to have two separate

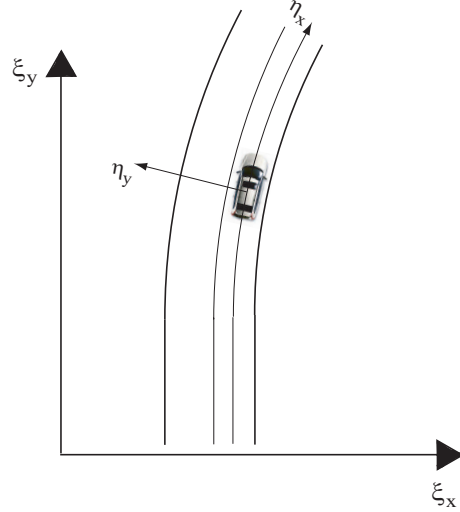


Fig. 1. This figure illustrates the two coordinate systems employed in this paper.

coordinate systems. One is a fixed cartesian coordinate system (ξ_x, ξ_y) and the other is a curved coordinate system which follows the road (η_x, η_y) . A similar idea is presented in (Eidehall, 2004). As illustrated in Fig. 1, η_y is zero when the car is at the center of its lane. Owing to the fact that the road is known, we assume that it is possible to calculate the position (ξ_x, ξ_y) given (η_x, η_y) and vice versa. (Details regarding how this is done are omitted.)

To aid our tracking algorithms, we are given a sequence of measurements, $\mathbf{z}(0), \mathbf{z}(1), \mathbf{z}(2), \dots$, which are sampled at the time instances $t = 0, T, 2T, \dots$, respectively. These samples are related to the parameters of interest through the measurement model

$$\mathbf{z}(k) = \mathbf{x}(k) + \mathbf{w}(k), \quad (2)$$

where $\mathbf{w}(k) \sim \mathcal{N}(\mathbf{0}, \mathbf{C}_w)$. Here, the sample covariance matrix of the noise, \mathbf{C}_w , is assumed known. In safety applications, the noise is often quite large, thus, indirectly, promoting the need for an improved motion model.

3. MOTION MODELS

To describe the motion of the vehicle, a common choice of model structure is to let the state vector evolve according to a linear state space equation

$$\mathbf{x}(k+1) = \mathbf{A}\mathbf{x}(k) + \mathbf{\Gamma}\mathbf{v}(k), \quad (3)$$

where $\{\mathbf{A}, \mathbf{\Gamma}\} \in \mathbb{R}^{n \times n}$ and $\mathbf{v} \sim \mathcal{N}(\mathbf{0}, \sigma_v^2 \mathbf{I})$. By selecting the state vector, $\mathbf{x}(k)$, and the matrices \mathbf{A} and $\mathbf{\Gamma}$ differently, one can obtain models with various properties; important examples include constant velocity models, constant acceleration models and coordinated turn models (Y.Bar-Shalom *et al.*, 2001). In this paper, we study the

CA model in more detail, partly because it can serve as a benchmark, but also since our idea of incorporating the driver actions fits well into its structure.

3.1 Constant acceleration model

The CA model has a state vector containing the following elements

$$\mathbf{x}(k) = [\xi_x(k) \ \xi_y(k) \ \dot{\xi}_x(k) \ \dot{\xi}_y(k) \ \ddot{\xi}_x(k) \ \ddot{\xi}_y(k)]^T. \quad (4)$$

By interpreting the white noise process, $\mathbf{v}(k)$ in (3), as the acceleration increments, the resulting velocity and position increments are given by $\mathbf{v}(k)T$ and $\mathbf{v}(k)T^2/2$ respectively. Consequently, using the notation in (3), the CA model is defined by the matrices

$$\mathbf{A} = \begin{bmatrix} 1 & 0 & T & 0 & T^2/2 & 0 \\ 0 & 1 & 0 & T & 0 & T^2/2 \\ 0 & 0 & 1 & 0 & T & 0 \\ 0 & 0 & 0 & 1 & 0 & T \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}, \quad \mathbf{\Gamma} = \begin{bmatrix} T^2/2 & 0 \\ 0 & T^2/2 \\ T & 0 \\ 0 & T \\ 1 & 0 \\ 0 & 1 \end{bmatrix}. \quad (5)$$

3.2 Modified constant acceleration model

In many applications, the constant acceleration model may be a perfectly reasonable one; if we have no additional information, why believe that the acceleration would change? In traffic scenarios, however, we know that objects are controlled almost completely by the driver. Therefore, by incorporating knowledge regarding driver behavior it may be possible to make intelligent adjustments to the constant acceleration model. Given the assumptions in Section 2, this is particularly obvious since it is very likely that the car will follow the given road, and such information should be used to improve the estimate².

The resulting modified CA model can be expressed as

$$\mathbf{x}(k+1) = \mathbf{A}\mathbf{x}(k) + \mathbf{B}\mathbf{u}(k) + \mathbf{B}\mathbf{v}(k). \quad (6)$$

Here, \mathbf{A} is the same as in (5), whereas

$$\mathbf{B} = \begin{bmatrix} T^2/6 & 0 \\ 0 & T^2/6 \\ T/2 & 0 \\ 0 & T/2 \\ 1 & 0 \\ 0 & 1 \end{bmatrix}, \quad \mathbf{u}(k) = \begin{bmatrix} a_{\xi_x}(k) \\ a_{\xi_y}(k) \end{bmatrix}. \quad (7)$$

The scalar functions $a_{\xi_x}(k)$ and $a_{\xi_y}(k)$ correspond to the accelerations imposed by the driver. (The

reason $\mathbf{B} \neq \mathbf{\Gamma}$ is that we now assume the accelerations change linearly between the sampling instances.) The vector $\mathbf{v}(k)$ is included to account for modelling imperfections in $\mathbf{u}(k)$. Like before, $\mathbf{v}(k)$ is Gaussian white noise so that $\mathbf{v} \sim \mathcal{N}(\mathbf{0}, \sigma_v^2 \mathbf{I})$. Nevertheless, it is important to note that σ_v^2 can be much smaller here, compared to the traditional CA model (if $\mathbf{u}(k)$ takes reasonable values). From a technical perspective, this motivates why the modified CA model can help to improve the prediction performance.

4. DRIVER INTENTIONS

It is understood, from the discussion in Section 3, that further understanding of driver behavior is important for the design of well performing tracking algorithms. An ability to predict future actions of the driver would be particularly useful. Here, we present a new approach for deriving such predictions.

The underlying assumption in this section, is that experienced drivers have a set of driving rules, which they unconsciously seek to obey. We believe the four most important rules are that the journey is *fast*, *safe*, *comfortable* and *legal*. Naturally, the importance and individual priority of these rules varies with different drivers. Our aim is, initially, to describe the rules, or objectives, of an average driver. Thereby, we hope to add knowledge regarding the typical behavior of a normal driver, and hence, regarding a likely motion of the vehicle.

To formulate the driver objectives mathematically, we use a cost function that enables us to order all trajectories according to their costs. The cost is intended to reflect the objectives of the driver so that trajectories corresponding to hazardous, uncomfortable, illegal or slow trips have large costs. Finally, by selecting the trajectory with the lowest cost, we can produce a prediction of the vehicle motion, that incorporates knowledge regarding the road, other objects, speed limits, etc³. In the remaining part of this section, we explain and exemplify our first attempt along these lines.

4.1 Optimal Trajectories

When comparing the costs for different trajectories, we do not employ the motion model from Section 3. First of all, the different trajectories are well known and, hence, there is no uncertainty to consider, and thus, no noise. Also, since there are

² Depending on the application it may be important not to trust this prior information too much.

³ In a sense, we are hereby approximating the driver with an optimal regulator, where the optimality criterion is defined by the cost functions.

no measurement updates in this part, we work with continuous time and denote the state vector, $\tilde{\mathbf{x}}(t) = [\xi_x(t) \ \xi_y(t) \ \dot{\xi}_x(t) \ \dot{\xi}_y(t) \ \ddot{\xi}_x(t) \ \ddot{\xi}_y(t)]^T$. Furthermore, as we shall see, the comfort of a trip depends also on the third derivatives of the positions, $\ddot{\xi}_x(t)$ and $\ddot{\xi}_y(t)$, and we will therefore include these in our algorithms. Clearly, given the state vector at a time T_0 , and the third derivatives for all $t \in (T_0, T_1)$, then $\mathbf{x}(T_1)$ can be calculated through integration. To design the optimal trajectory for $\mathbf{x}(t)$, $T_0 < t \leq T_1$, given $\mathbf{x}(T_0)$, it is therefore sufficient to specify $\xi_x(t)$ and $\xi_y(t)$ in this interval.

The cost function is denoted $c(\mathbf{x}(t), \ddot{\xi}_x(t), \ddot{\xi}_y(t))$, and the total cost in an interval (T_0, T_1) is written

$$c(T_0, T_1) = \int_{T_0}^{T_1} c(\mathbf{x}(t), \ddot{\xi}_x(t), \ddot{\xi}_y(t)) dt. \quad (8)$$

As indicated above, to derive the trajectory we solve

$$\min_{\{\ddot{\xi}_x(t), \ddot{\xi}_y(t)\}} c(T_0, T_1) \quad (9)$$

given the initial state vector $\mathbf{x}(T_0)$. The solution will render predictions of $\mathbf{x}(t)$ for $T_0 \leq t \leq T_1$. Note that these derivations generate predictions of the acceleration $[\ddot{\xi}_x(t), \ddot{\xi}_y(t)]$, that can be used to approximate $[a_{\xi_x}(k), a_{\xi_y}(k)]$ in (7).

4.2 Cost Functions

The objective here, is to design a cost function such that the optimal trajectories approximately captures the behavior of an average driver. To accomplish this, $c(\mathbf{x}(t), \ddot{\xi}_x(t), \ddot{\xi}_y(t))$ is divided into four components related to safety issues $c_s(\mathbf{x}(t))$, traffic regulations $c_{tr}(\mathbf{x}(t))$, time of travel $c_{ti}(\mathbf{x}(t))$ and comfort $c_c(\mathbf{x}(t), \ddot{\xi}_x(t), \ddot{\xi}_y(t))$. The used cost function is the sum of these terms

$$c(\mathbf{x}(t), \ddot{\xi}_x(t), \ddot{\xi}_y(t)) = c_{ti}(\mathbf{x}(t)) + c_{tr}(\mathbf{x}(t)) + c_c(\mathbf{x}(t), \ddot{\xi}_x(t), \ddot{\xi}_y(t)) + c_s(\mathbf{x}(t)). \quad (10)$$

In the following, we present the basic structure of these four underlying functions. For ease of exposition, we first introduce the notation $\ddot{\xi}(t) = \sqrt{\ddot{\xi}_x(t)^2 + \ddot{\xi}_y(t)^2}$ and $\ddot{\xi}(t) = \sqrt{\ddot{\xi}_x(t)^2 + \ddot{\xi}_y(t)^2}$. Also, please recall that the vector $[\eta_x(t) \ \eta_y(t) \ \dot{\eta}_x(t) \ \dot{\eta}_y(t) \ \ddot{\eta}_x(t) \ \ddot{\eta}_y(t)]$ can be calculated from $[\xi_x(t) \ \xi_y(t) \ \dot{\xi}_x(t) \ \dot{\xi}_y(t) \ \ddot{\xi}_x(t) \ \ddot{\xi}_y(t)]$.

Time of Travel, $c_{ti}(\mathbf{x}(t))$. This factor tries to capture the desire to reach the goal of the journey quickly. We use

$$c_{ti}(\mathbf{x}(t)) = -\alpha_{ti} \dot{\eta}_x(t), \quad (11)$$

which renders low costs when the car is travelling fast. The constant α_{ti} is a design parameter that specifies the importance of a fast journey.

Traffic Regulations, $c_{tr}(\mathbf{x}(t))$. Let $s_{\text{limit}}(\eta_x(t))$ denote the speed limit for the part of the road where the vehicle is positioned at time t . Our cost function for exceeding the speed limit is

$$c_{tr}(\mathbf{x}(t)) = \alpha_{tr} (\dot{\eta}_x(t) - s_{\text{limit}}(\eta_x(t)))^2, \quad (12)$$

if $\dot{\eta}_x(t) > s_{\text{limit}}(\eta_x(t))$, and zero otherwise. Similar to above, α_{tr} defines the importance of a legal journey.

Comfort, $c_c(\mathbf{x}(t), \ddot{\xi}_x(t), \ddot{\xi}_y(t))$. We write this function as

$$c_c(\mathbf{x}(t), \ddot{\xi}_x(t), \ddot{\xi}_y(t)) = \alpha_c (c_a(\ddot{\xi}(t)) + c_j(\ddot{\xi}(t))). \quad (13)$$

The scaling parameter α_c represents the drivers eagerness to have a comfortable trip. Based on the information in (VGU, VV Publikation 2004:80, Grundvärden, 2004), we deduce that, the approximate limits, $\ddot{\xi}(t) < 1.5 \text{ m/s}^2$ and $\ddot{\xi}(t) < 0.45 \text{ m/s}^3$ correspond to normal driving, whereas $1.5 < \ddot{\xi}(t) < 3 \text{ m/s}^2$ and $0.45 < \ddot{\xi}(t) < 0.8 \text{ m/s}^3$ are considered rough driving. Accelerations or jerks even larger than 3 m/s^2 and 0.8 m/s^3 , respectively, are perceived as very uncomfortable. To reflect these properties we use

$$c_a(\ddot{\xi}(t)) = \begin{cases} \ddot{\xi}(t) & \text{if } \ddot{\xi}(t) < 0.5 \\ \frac{5}{6} + \frac{2\ddot{\xi}(t)^6}{0.5^5 6} & \text{if } \ddot{\xi}(t) \geq 0.5 \end{cases} \quad (14)$$

$$c_j(\ddot{\xi}(t)) = \begin{cases} \ddot{\xi}(t) & \text{if } \ddot{\xi}(t) < 1.5 \\ -\frac{1}{2} + \frac{2\ddot{\xi}(t)^2}{3} & \text{if } \ddot{\xi}(t) \geq 1.5. \end{cases} \quad (15)$$

Note that the derivatives of c_a and c_j increase with the intensity of the journey.

Safety, $c_s(\mathbf{x}(t))$. The design of this function is greatly simplified by the assumption that there are no other objects in the area. In fact, the only aspect we consider here is that the car should remain in its lane. Let W denote the width of the lane minus the width of the car. The cost function used to reflect the safety of the car is

$$c_s(\mathbf{x}(t)) = \alpha_s (|\eta_y(t)| - W/2)^2, \quad (16)$$

if $|\eta_y(t)| > W/2$ and zero otherwise. Again, the scaling factor, α_s , defines the importance of a safe trip.

4.3 Examples and notes on the implementation

In order to illustrate the optimal trajectories in a few simple scenarios, we need to solve for $\ddot{\xi}_x(t)$ and $\ddot{\xi}_y(t)$ in (9). Unfortunately, it appears to be very difficult to perform this optimization analytically, and we therefore resort to numerical

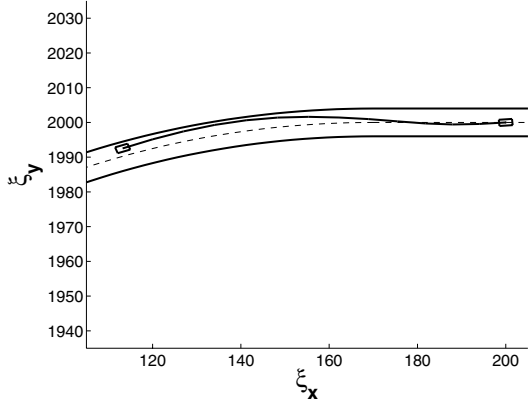


Fig. 2. The calculated optimal trajectory of a car. The trajectory starts in the right hand side of the image, where the car is almost leaving its lane.

solutions. To limit the dimension (the number of free parameters) we approximate $\ddot{\xi}_x(t)$ and $\ddot{\xi}_y(t)$ with constants in the intervals $(T_0, T_0 + \Delta t)$, $(T_0 + \Delta t, T_0 + 2\Delta t)$, etc. In our simulations, we use $\Delta t = 0.5$ s and $T_1 - T_0 = 7$ s rendering 28 free parameters in the expressions. The optimal trajectories can now be calculated using standard numerical software. We use the command 'fminunc' MATLAB's optimization toolbox.

In our two examples, the width of the lane is 4 meters and the car is 1.8 meters wide. The car is also 3 m long and the speed limit is 13.9 m/s (approximately 50 km/h) on the relevant section of the road. Note that the vehicle is travelling from right to left in the images, and that it is supposed to stay on right hand side of the road (the upper half). In both the examples, as well as the simulations in Section 5, the design parameters in the cost function are set to $\alpha_{ti} = 1.4$, $\alpha_{tr} = 0.5$, $\alpha_c = 0.5$ and $\alpha_s = 1000$, respectively.

Example 1 The purpose of this example is to display the optimal trajectory when the car is initially almost outside the lane. It can be seen from Fig. 2 that it makes a smooth yaw and quickly reaches the center of the lane.

Example 2 To further demonstrate the behavior of the trajectories we also study a setting where the car is about to enter a sharp curve. Fig. 3 shows the optimal trajectory in one such scenario, where the radius of the curve is only 40 m. Clearly, the car remains in its lane, but it also attempts to attenuate the centripetal acceleration by approaching the right side of the lane before the curve, and then stay close to the left side of the lane in the curve. Also, if we study the speed of the car one can see that it starts at 13.9 m/s (which is the initial value), slows down to 11 m/s

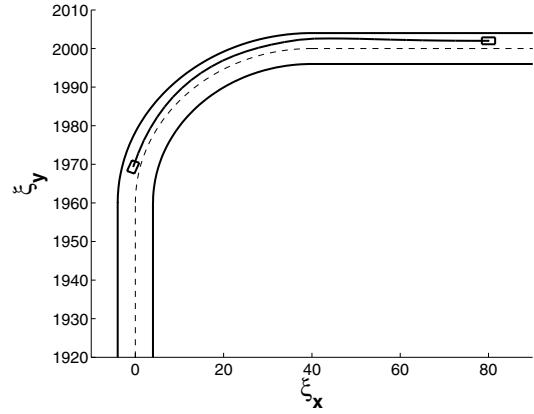


Fig. 3. The calculated optimal trajectory of a car. The trajectory goes from right to left, and illustrates how it handles a sharp curve.

and then increases again to 13 m/s by the end of the curve. In our opinion, this essentially captures the typical behavior of an experienced driver in a similar situation.

5. TRACKING PERFORMANCE

In the previous section, we argued that the derived optimal trajectories have reasonable characteristics. Nevertheless, so far we have not been able to show if the new approach can improve the tracking performance, which is the final objective. The aim in this section is therefore to compare the performance of the proposed algorithm with an algorithm based on the traditional CA model.

The tracking algorithm for the CA model is easily derived, and due to the Gaussian and linear assumption the solution is given by the Kalman filter algorithm, (Kalman, 1960). For the modified CA model, we first need to calculate our estimate of the difference in acceleration, $\mathbf{u}(k)$. By assuming that $\mathbf{x}(T_0) = \hat{\mathbf{x}}(k)$, i.e. that our estimate of the state vector is correct, the third derivatives can be found according to the methodology in Section 4. Finally, the vector $\mathbf{u}(k)$ is calculated by integrating the third derivatives. Once this vector has been calculated, the Kalman filter algorithm applies also to this sequence; the difference compared to the standard implementation is that the predictions are adjusted due to the additional term $\mathbf{B}\mathbf{u}(k)$ in (6).

To obtain a data sequence from the data model in Section 2, we start with a sequence of state vectors $\mathbf{x}(k)$ and add noise $\mathbf{w}(k)$, with a known covariance matrix $\mathbf{C}_w = \sigma_w^2 \mathbf{I}$, where $\sigma_w^2 = 20$. In our simulations, the sequence $\mathbf{x}(k)$ is acquired through a series of reference measurements through which the required road information also is gained. In the studied scenario, the road contains two curves

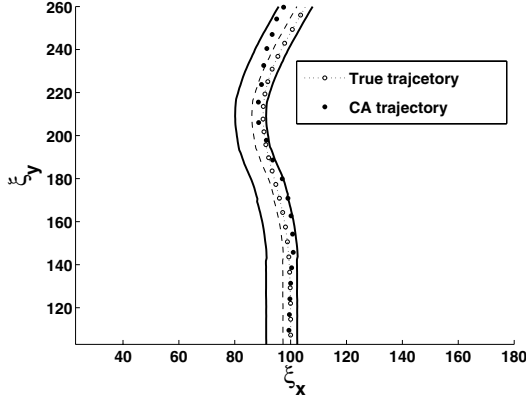


Fig. 4. The estimated trajectory using the CA model is presented together with the true trajectory.

which in total takes the driver almost 13 seconds to complete. During the journey, measurements are collected every 0.1 second.

First the performance of the CA model is presented. The vehicle trajectory is estimated using a Kalman filter, which was initialized with the vector

$$\begin{bmatrix} \eta_x(0) \\ \eta_y(0) \\ \dot{\eta}_x(0) \\ \dot{\eta}_y(0) \\ \ddot{\eta}_x(0) \\ \ddot{\eta}_y(0) \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 13 \\ -0.3 \\ 1 \\ 0 \end{bmatrix}. \quad (17)$$

As can be seen from (17), the vehicle is positioned at the center of its lane, having a lateral velocity of -1 m/s . The uncertainties in the initial state vector are set to 10^{-3} for all elements, and the uncertainty in the CA model is modelled using a noise power $\sigma_v^2 = 10^{-3}$. In Fig. 4, the estimated trajectory using the CA model is presented together with the true trajectory. Evidently, the information provided by the CA model is not enough to aid the noisy measurements in providing an accurate estimate.

To compare the performance of the modified CA model, with the original CA model, we apply it to the same sequence of measurements. The same initialization is used for the Kalman filter, but the model uncertainty is now decreased to $\sigma_v^2 = 10^{-5}$. For each filter iteration, we determine $\mathbf{u}(k)$ by solving (9) for $\Delta t = 0.25 \text{ s}$ and $T_1 - T_0 = 5 \text{ s}$ (thus yielding 40 free parameters). As can be seen in Fig. 5, the resulting tracking algorithm significantly improves the performance. In spite of the poor quality of the measurements, the filter essentially manages to capture the true trajectory. By comparing the average squared errors of the algorithms, we conclude that, in this scenario, our algorithm outperforms the standard algorithm approximately nine times.

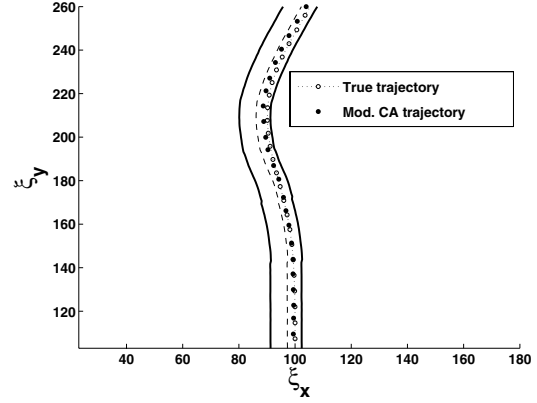


Fig. 5. The estimated trajectory using the modified CA model is presented together with the true trajectory.

6. CONCLUSIONS

A new approach to modelling the motion of vehicles in traffic is proposed. In difference to previous models, the new model incorporates prior knowledge regarding road geometry and typical driver behavior. Through examples, the ability of the new model to produce reasonable predictions of future states is illustrated. Finally, we show that the proposed model can render a much more accurate tracking algorithm, compared to the traditional constant acceleration model, in scenarios where the measurements are poor.

REFERENCES

- Dickmanns, E.D. and A. Zapp (1986). A curvature-based scheme for improving road vehicle guidance by computer vision. In: *Proceedings of the SPIE Conference on Mobile Robots*.
- Eidehall, Andreas (2004). An automotive lane guidance system. Technical Report Licentiate Thesis no. 1122. Department of Electrical Engineering, Linköping University. SE-581 83 Linköping, Sweden.
- Kalman, R.E. (1960). A new approach to linear filtering and prediction problems. *Trans. ASME, Ser. D. Journal Basic Eng.* **82**, 24–45.
- Lorenz, R.W., J. Coleman and S. Mehring (1998). Lane departure warning for CVO in the USA. In: *SAE technical paper 982779*. Indianapolis, IN, USA.
- VGU, VV Publikation 2004:80, Grundvärden (2004). ISSN: 1401-9612.
- Y.Bar-Shalom, X. Rong Li and T. Kirubarajan (2001). *Estimation with applications to tracking and navigation*. Wiley. New York, NY.