Calculation of coefficients

The function $u(x, y)$ is approximated by $u^h(x, y)$ which is a linear combination of legendre polynomials upto degree q.

$$u^h(x(r,s), y(r,s)) = \sum_{k=0}^{q} \sum_{l=0}^{q} \hat{u}^h_{k,l} P_k(r) P_l(s) \tag{1}$$

where x(r,s) and y(r,s) is the mapping from the r - s plane to the element in question in the x - y plane. We aim to find the co-efficients $\hat{u}^h_{k,l}$.

Multiplying both sides by $P_m(r)P_n(s)$ and performing a double integral in the r - s plane over the square $[-1, 1]^2$, we get

$$\begin{aligned}
\int_{-1}^{1}\int_{-1}^{1} u^h(x(r,s), y(r,s)) P_m(r) P_n(s) dr ds &= \int_{-1}^{1}\int_{-1}^{1} \sum_{k=0}^{q} \sum_{l=0}^{q} \hat{u}^h_{k,l} P_k(r) P_l(s) P_m(r) P_n(s) dr ds \\
&= \sum_{k=0}^{q} \sum_{l=0}^{q} \hat{u}^h_{k,l} \Big(\int_{-1}^{1} P_k(r) P_m(r) dr \Big) \Big(\int_{-1}^{1} P_l(s) P_n(s) ds \Big) \\
&= \sum_{k=0}^{q} \sum_{l=0}^{q} \hat{u}^h_{k,l} \delta_{km} \delta_{ln} \frac{2}{2k+1} \frac{2}{2l+1} \\
&= \hat{u}^h_{m,n} \frac{4}{(2m+1)(2n+1)}
\end{aligned}$$

$$\hat{u}^h_{m,n} = \frac{(2m+1)(2n+1)}{4} \int_{-1}^{1}\int_{-1}^{1} u^h(x(r,s), y(r,s)) P_m(r) P_n(s) dr ds \tag{2}$$

Both of these integrals are evaluated using $q + 1$ point Gauss-Legendre quadrature.