Covering the sphere with noncontextuality inequalities

Bachelor's thesis in mathematics

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Table of Contents

Introduction

In this Bachelor's thesis the following question was answered:
Does the inequality posed in the article Klyachko et al [2008] cover the real part of the Bloch surface of a 3D quantum system when used as in Kochen and Specker [1967]?

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Remark

However to understand the questions, some introduction is needed.

Table of Contents

A brief introduction

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The basis equation in quantum mechanics is known as Schrödingers equation:

$$i\hbar\frac{\partial}{\partial t}\psi(\vec{x},t) = \hat{H}\psi(\vec{x},t)$$

which is a vector equation and requires the Hamiltonian for a system.

Notation

From here on the vector, also known as wave-function will be denoted as a state: $|\psi\rangle$. Here also the dependent variables are omitted for simplicity such that:

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Example

Using the notation above, systems with a single particle and spin 1/2 can be written

$$\psi\rangle = a_0|+\frac{1}{2}\rangle + a_1|-\frac{1}{2}\rangle$$

Example

However, in the thesis mostly spin 1 is considered, which can be written in the following way:

$$|\psi\rangle = a_0|-1\rangle + a_1|0\rangle + a_2|1\rangle$$

Bloch sphere

The Bloch sphere is a simple way to represent all possible quantum states. Originally used only for spin 1/2, it can be used for spin 1 systems by the restriction in our question, to only include the real part of spin 1 systems.

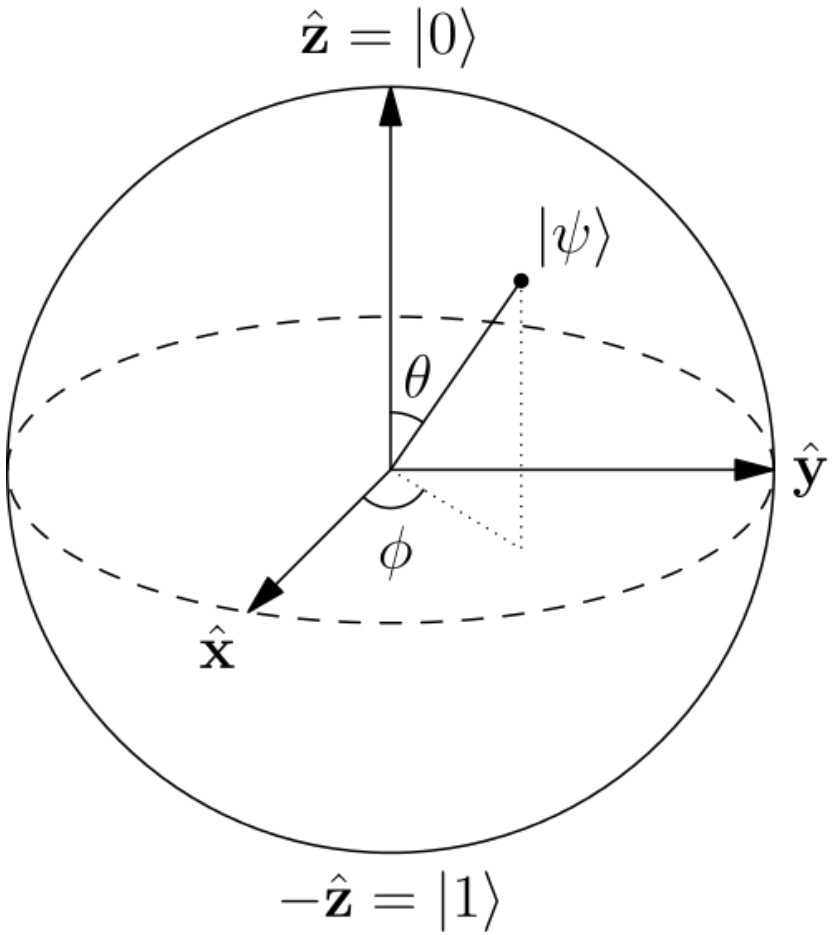

Figure : A figure of the Bloch sphere for a spin 1/2 system.

Table of Contents

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- Contextuality refers to when the result of an experiment depends on the context of the experiment.
- The opposite of this is of course non-contextuality where the commuting property is not dependent on the experiment.
- The noncontextuality inequalities are found through the hidden variable schema which means that they might not hold for a quantum mechanical schema.
- The interesting case is where the inequality is violated in a quantum mechanical description.

The inequality

Klyachko inequality.

$$\langle A_1 A_2 \rangle + \langle A_2 A_3 \rangle + \langle A_3 A_4 \rangle + \langle A_4 A_5 \rangle + \langle A_5 A_1 \rangle \geq -3$$

This inequality, known as the pentagram inequality, is if violated for a state proves that the state has a nonclassical behaviour. The pentagram inequality is valid for only non-contextual hidden variables.

Kochen and Specker

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- Get 117 vectors.

Table of Contents

The vectors

Using the vectors given from Kochen and Specker [1967]. These are used to evaluate the inequality, producing quadratic forms that are then drawn upon the Bloch sphere. Using the following vectors:

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$$|\psi_1>= \begin{pmatrix} 0.0000 \\ 0.0000 \\ 1.0000 \end{pmatrix}, |\psi_2>= \begin{pmatrix} 0.5904 \\ 0.8071 \\ 0.0000 \end{pmatrix}, |\psi_3>= \begin{pmatrix} -0.7071 \\ 0.5172 \\ 0.4822 \end{pmatrix},$$

$$|\psi_4>= \begin{pmatrix} 0.7071 \\ 0.5172 \\ 0.4822 \end{pmatrix}, |\psi_5>= \begin{pmatrix} -0.5904 \\ 0.8071 \\ 0.0000 \end{pmatrix}$$

will produce the following image,

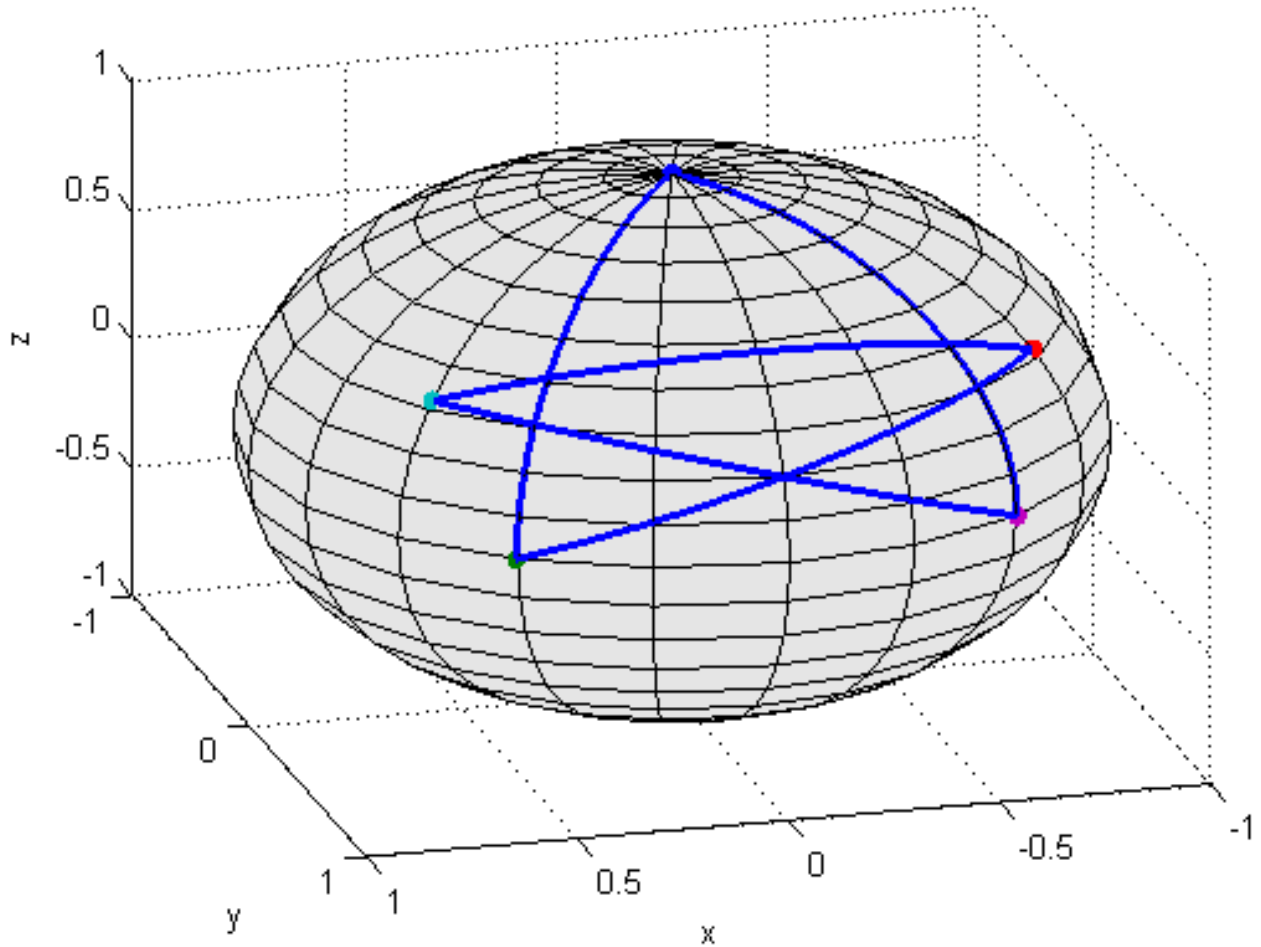

Figure : The vectors plotted on the sphere.

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Continuing covering the sphere

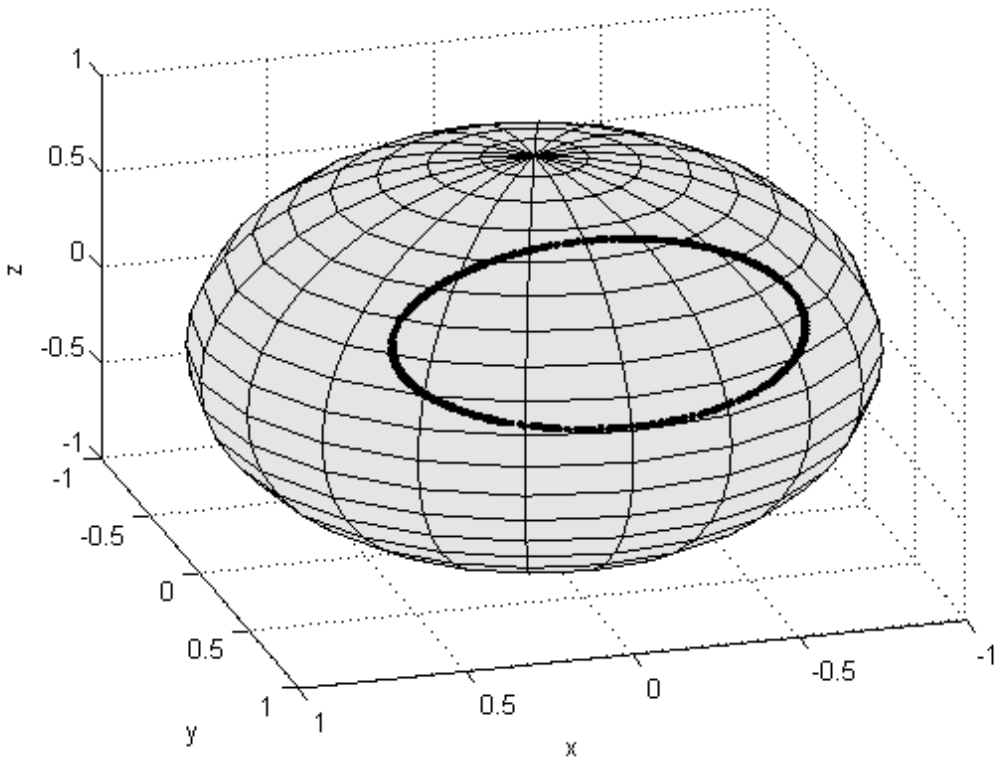


Figure : The Klyachko inequality plotted on the sphere.

More vectors and inequalities can be created by rotating the first vectors around the x-axis and the (1,1,1)-axis.

Table of Contents

Producing bands

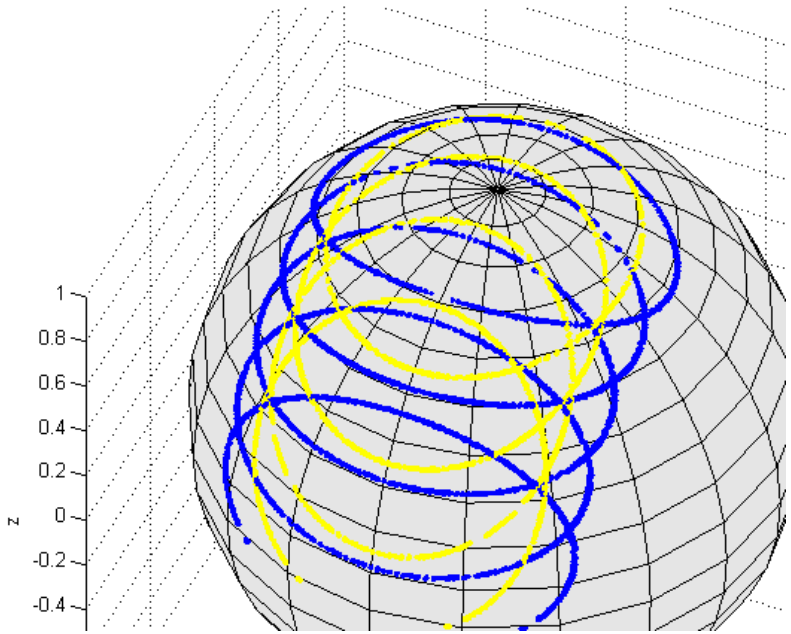
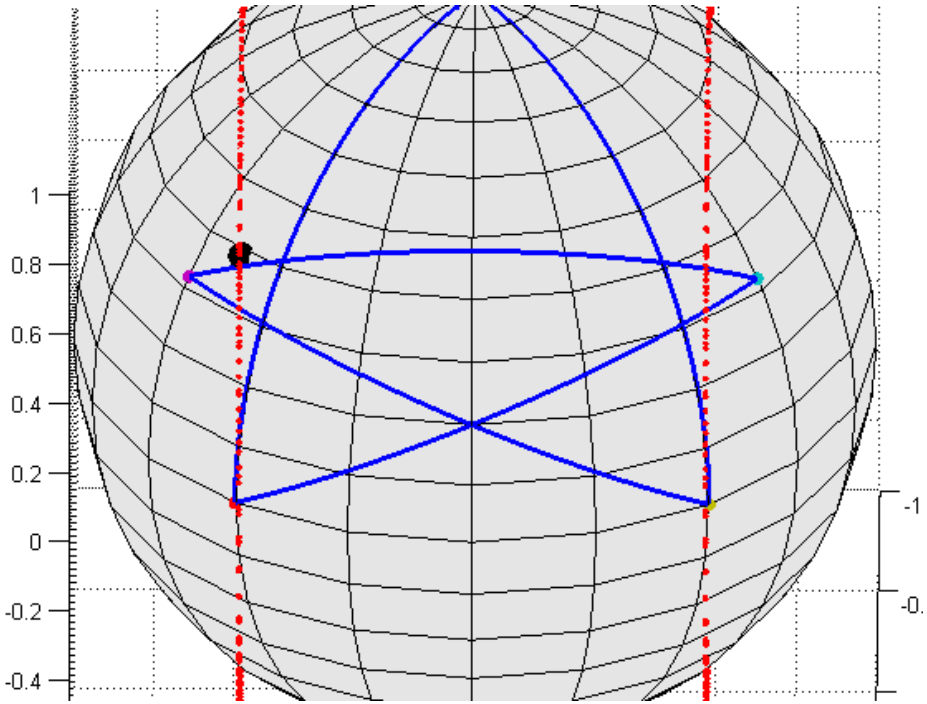


Figure : By rotating the vectors around the x-axis and getting new inequalities a band will be created.



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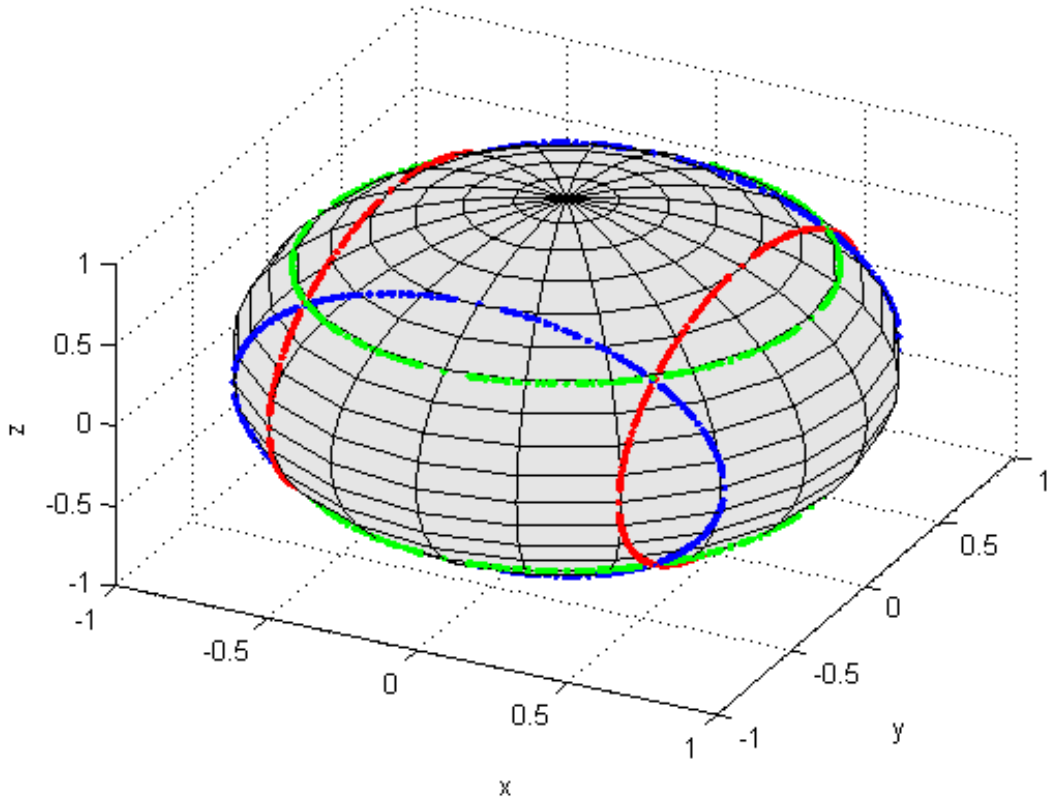

Figure : Rotating the band around the (1,1,1)-axis will produce the following final result.

Table of Contents

Was the question answered?

- It seems that the whole sphere is covered by these inequalities, what does this mean?

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- If the whole Bloch sphere is either inside or on the border of the inequalities as seen in the, this means that all possible states of a spin 1 system can be explained by quantum mechanics.

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- If the whole Bloch sphere is either inside or on the border of the inequalities as seen in the, this means that all possible states of a spin 1 system can be explained by quantum mechanics.
- Remember though that there were a few restrictions given.
- It is quite undeniable that a proof that the inequality covers all of the real states has been found, however nothing has been said about those which require a complex representation.

Further research to be done

- Complex coefficients.
  Would fully answer the question. Not that hard, though would loose the visualization which was prioritized.

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- Complex coefficients.
  Would fully answer the question. Not that hard, though would loose the visualization which was prioritized.
- Other spin systems
  Mostly spin $\frac{1}{2}$ systems are of interest and in this thesis spin 1 systems were discussed. There are more spin systems.