TFYA17 Project

Transmission properties in a short biased quantum wire

Patrik Hallsjö, Felix Faber

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Introduction

- This project was dedicated to study an electron transport in a biased quantum wire.
- The goals were to construct a solver for the potential and to calculate the transmission and reflection coefficients for an electron sent towards the potential.

The potential used in this presentation:

$$V(x) = \beta e V_{sd} + V_g tanh(s(x - \Delta x_1)) \\ -(V_{sd} + V_g) tanh(s(x - \Delta x_2))$$

Remark

- $\beta =$ Potential drop, if 0.5 then symmetric as in our case.
- $V_{sd} =$ Bias voltage.
- $V_g =$ Potential height
- $S =$ Potential steepness.

Goals

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Matlab was used to achieve the goals.

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- Central: $\frac{f(x-h/2)-f(x+h/2)}{h}$
- Second order: $\frac{f(x-h)-2f(x)+f(x+h)}{h^2}$

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- $h = d$ and $x = nd$ yields $dg(y(nd), nd) + 2y(nd) - (y((n+1)d) + y((n-1)d)) = 0.$
- $N + 1$ equations with $N + 1$ unknowns.

Quantum barrier problem

Solve the stationary Schrödinger equation. With $k = \sqrt{\frac{2m^*}{hbar^2}(E - \beta eV_{Sd})}$ and $k' = \sqrt{\frac{2m^*}{hbar^2}(E + (1-\beta)eV_{Sd})}$

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- R and T give $N + 3$ unknowns but only $N + 1$ equations.
- Continuous at boundaries yeilding: $N + 3$ unknowns and $N + 3$ equations which will have a unique solution.

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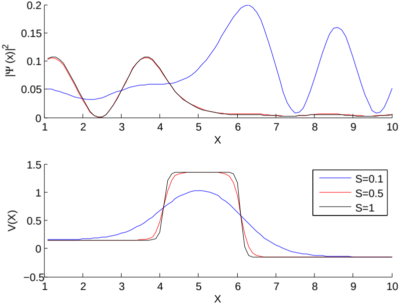
Numerical values

The program produces the following data, as well as images of the wave-functions and the potential barriers.

$$R = |r|^2/(|r|^2 + |t|^2)$$
$$T = |t|^2/(|r|^2 + |t|^2)$$
$$C \propto T/(R + T)$$

Figure: Amplitudes of wave-functions (upper graph) and potential barriers (lower graph) for the case: E=1, $\beta = 0.5$, $x_1 = 4$, $x_2 = 6$, $V_{sd} = 0.3$, $V_g = 0.6$



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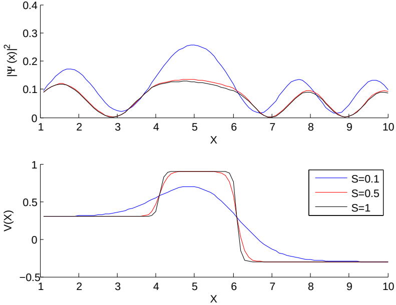
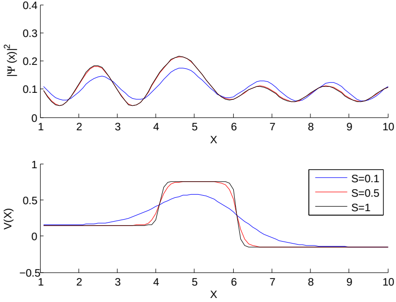


Figure: Amplitudes of wave-functions (upper graph) and potential barriers (lower graph) for the case: E=1.5, passing over the barrier, $\beta = 0.5$, $x_1 = 4$, $x_2 = 6$, $V_{sd} = V_g = 0.3$



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We produced a solver that plotted the wavefunction and calculated transmition and reflection coefficients.