TFYA17 Project

Transmission properties in a short biased quantum wire

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1 Introduction

2 Theory

3 Results

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Introduction

- This project was dedicated to study an electron transport in a biased quantum wire.
- The goals were to construct a solver for the potential and to calculate the transmission and reflection for an electron sent towards the potential.

$$V(x) = \beta * V_{sd} + V_g * tanh(s * (x - dx_1))$$
$$-(V_{sd} + V_g) * tanh(s * (x - dx_2))$$

Goals

- To set and to solve a problem with boundary conditions when an electron is injected into a wire.
- To calculate the transmission and reflection coefficients as functions of energy of injected electron and bias.
- To represent solutions in graphical form.
- Optional: to calculate a conductance.

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Remark

Matlab was used to achieve the goals.

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Finite derivatives

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- Central: $\frac{f(x-h/2)-f(x+h/2)}{h}$

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- Forward: $\frac{f(x+h)-f(x)}{h}$
- Backward: $\frac{f(x)-f(x-h)}{h}$
- Central: $\frac{f(x-h/2)-f(x+h/2)}{h}$
- Second order: $\frac{f(x-h)-2f(x)+f(x+h)}{h^2}$

Finite steps in differential equations

- Second order differential equation without any first order terms $y''(x) = g(y(x), x)$
- Boundary conditions: $y(x_0) = y_0$ and $y(x_N) = y_N$, on the interval $[x_0, x_N]$.
- $h = d$ and $x = n * d$ yields $d * g(y(n * d), n * d) + 2y(n * d) - (y((n+1) * d) + y((n-1) * d)) = 0$.
- $N + 1$ equations with $N + 1$ unknowns.

Quantum barrier problem

(1).
We need to solve the stationary Schrödinger equation with the boundary conditions $y_0 = (I + R)\exp(-ikx_0)$ and $y_N = T\exp(-ikx_N)$ We have put $I = 1$, which gives $y_0 = (1 + R)\exp(-ikx_0)$ to make the equations easier to solve.

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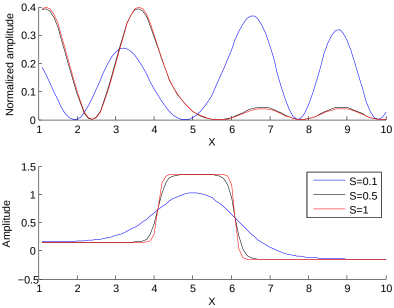
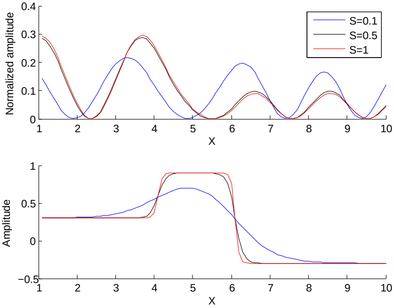
Numerical values

The program produces the following data, as well as images of the wave-functions and the potential barriers.

$$R = |R|^2/(|R|^2 + |T|^2)$$
$$T = |T|^2/(|R|^2 + |T|^2)$$
$$C = T/(R + T)$$


Normalized amplitude
0.4
0.3
0.2
0.1
0
1
2
3
4
5
6
7
8
9
10
X
Amplitude
1.5
1
0.5
0
−0.5
S=0.1
S=0.5
S=1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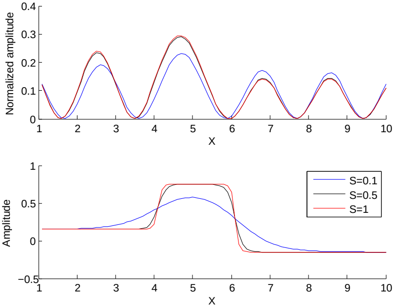
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