

# TFYA17 Project

Transmission properties in a short biased quantum wire

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# Introduction

- This project was dedicated to study an electron transport in a biased quantum wire.
- The goals were to construct a solver for the potential and to calculate the transmission and reflection coefficients for an electron sent towards the potential.

The potential used in this presentation:

$$V(x) = \beta e V_{sd} + V_g \tanh(s(x - \Delta x_1)) \\ - (V_{sd} + V_g) \tanh(s(x - \Delta x_2))$$

## Remark

- $\beta$  = Potential drop, if 0.5 then symmetric as in our case.
- $V_{sd}$  = Bias voltage.
- $V_g$  = Potential height
- $S$  = Potential steepness.

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Matlab was used to achieve the goals.

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- Second order:  $\frac{f(x-h)-2f(x)+f(x+h)}{h^2}$



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- Boundary conditions:  $y(x_0) = y_0$  and  $y(x_N) = y_N$ , on the interval  $[x_0, x_N]$  where  $d = \left(\frac{x_0 - x_N}{N}\right)$ .

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- $h = d$  and  $x = nd$  yields  $dg(y(nd), nd) + 2y(nd) - (y((n+1)d) + y((n-1)d)) = 0$ .
- $N + 1$  equations with  $N + 1$  unknowns.

# Quantum barrier problem

Solve the stationary Schrödinger equation. With

$$k = \sqrt{\frac{2m^*}{\hbar^2}(E - \beta eV_{sd})} \text{ and } k' = \sqrt{\frac{2m^*}{\hbar^2}(E + (1 - \beta)eV_{sd})}$$

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- $R$  and  $T$  give  $N + 3$  unknowns but only  $N + 1$  equations.
- Continuous at boundaries yielding:  $N + 3$  unknowns and  $N + 3$  equations which will have a unique solution.

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# Numerical values

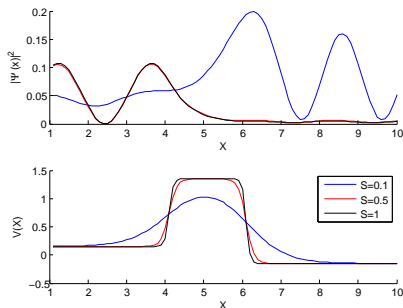
The program produces the following data, as well as images of the wave-functions and the potential barriers.

$$R = |r|^2 / (|r|^2 + |t|^2)$$

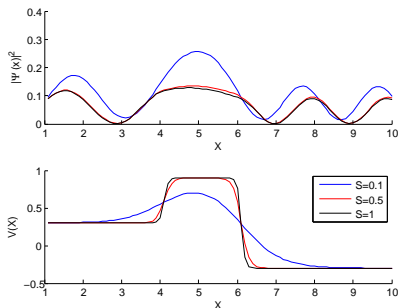
$$T = |t|^2 / (|r|^2 + |t|^2)$$

$$C \propto T / (R + T)$$

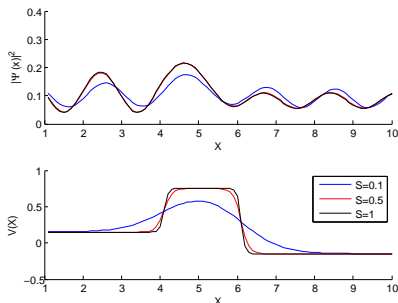
**Figure :** Amplitudes of wave-functions (upper graph) and potential barriers (lower graph) for the case:  $E=1$ ,  $\beta = 0.5$ ,  $x_1 = 4$ ,  $x_2 = 6$ ,  $V_{sd} = 0.3$ ,  $V_g = 0.6$



**Figure :** Amplitudes of wave-functions (upper graph) and potential barriers (lower graph) for the case:  $E=1$ ,  $\beta = 0.5$ ,  $x_1 = 4$ ,  $x_2 = 6$ ,  $V_{sd} = 0.6$ ,  $V_g = 0.3$



**Figure :** Amplitudes of wave-functions (upper graph) and potential barriers (lower graph) for the case:  $E=1.5$ , passing over the barrier,  $\beta = 0.5$ ,  $x_1 = 4$ ,  $x_2 = 6$ ,  $V_{sd} = V_g = 0.3$



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We produced a solver that plotted the wavefunction and calculated transmission and reflection coefficients.