

# Existence Proofs

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Alexander Shen

LIRMM / CNRS, University of Montpellier, France

# Outline

When One Example is Enough

Splitting an Octagon

Making Fun in Real Life

Know Your Rights

Nobody Can Win All The Times

# Know What Are You Looking For



# Proofs For Existential Statements

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- one example is enough

# Cutting Figures

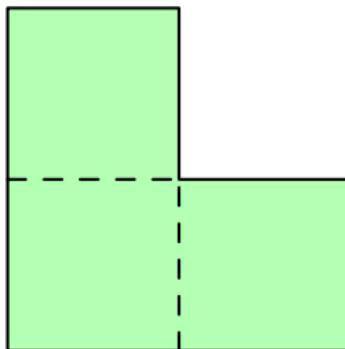
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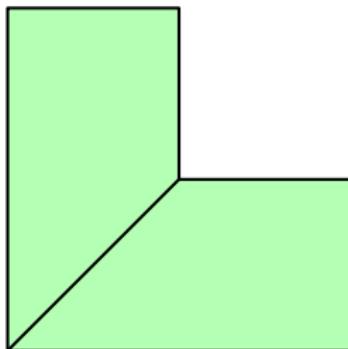
*Prove that this figure can be cut into 2 congruent pieces*



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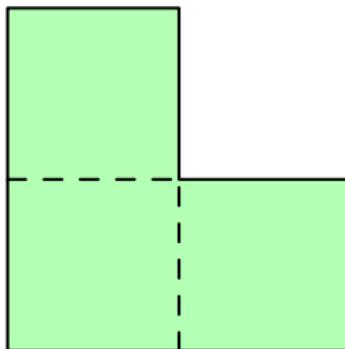
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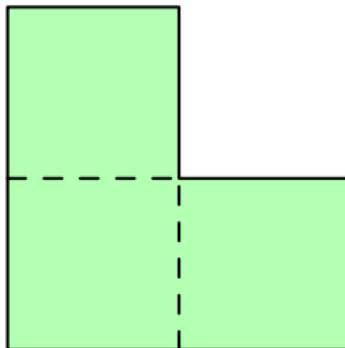
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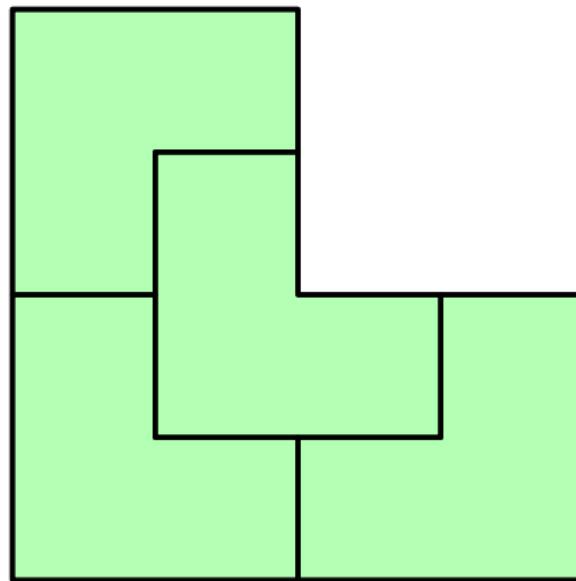
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what about 4 pieces?

# Spoiler

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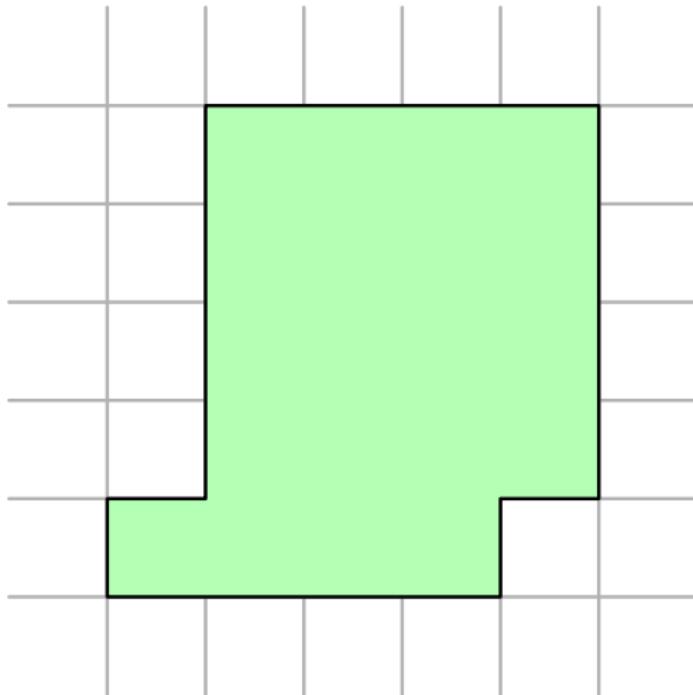
Making Fun in Real Life

Know Your Rights

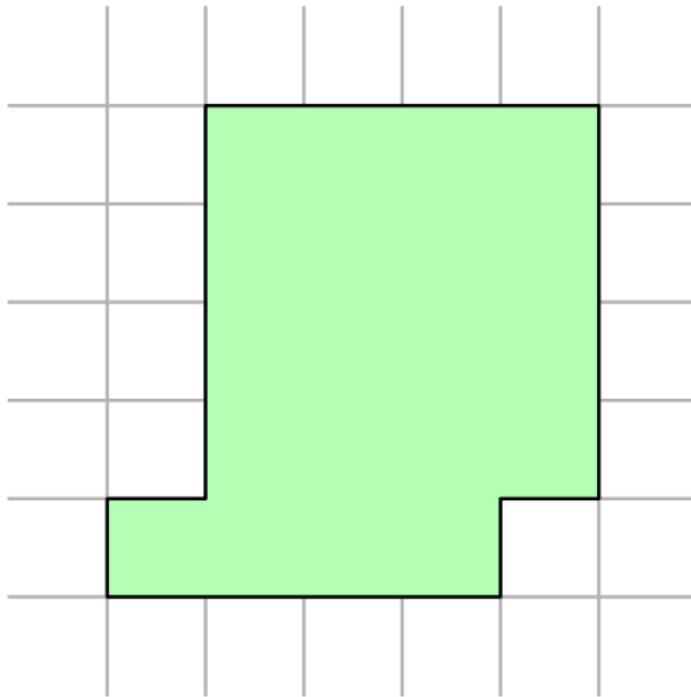
Nobody Can Win All The Times

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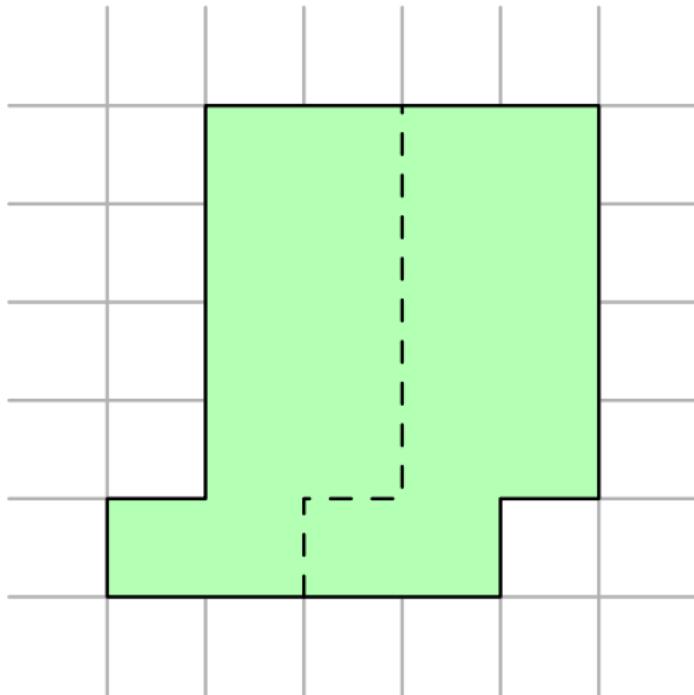
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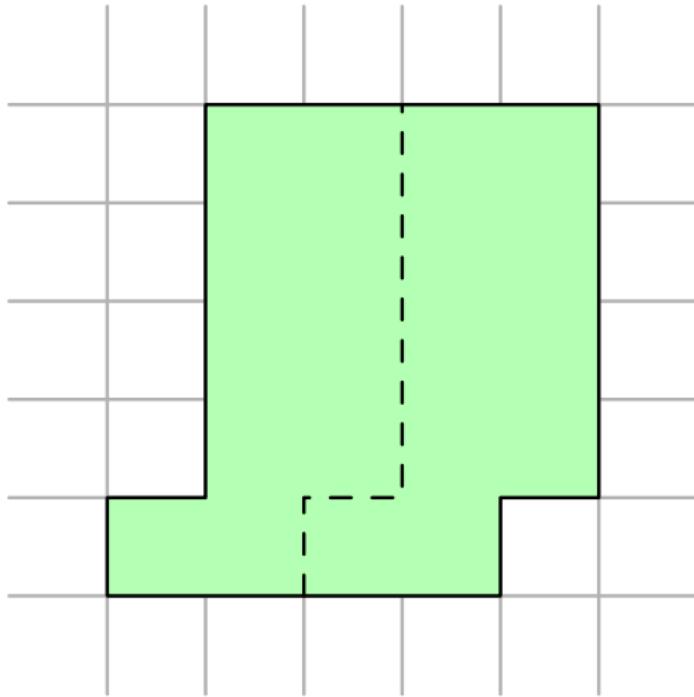
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what about three congruent pieces?

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# Tensegrities

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- drinking straws and thread



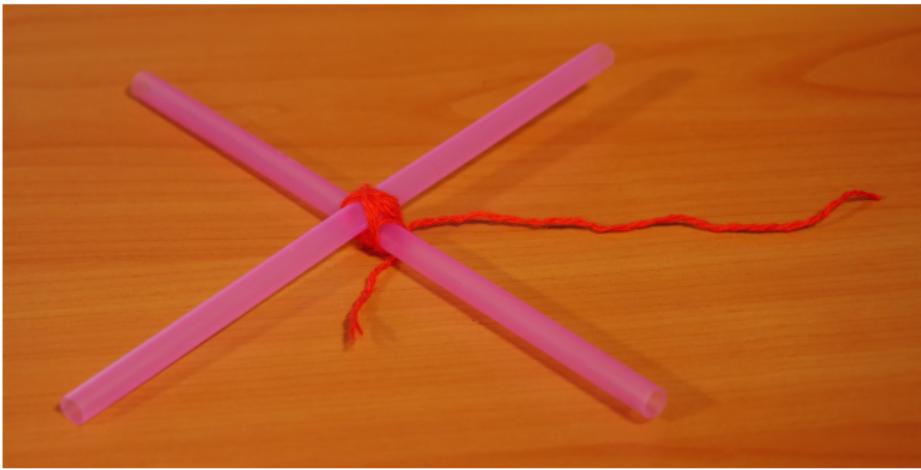
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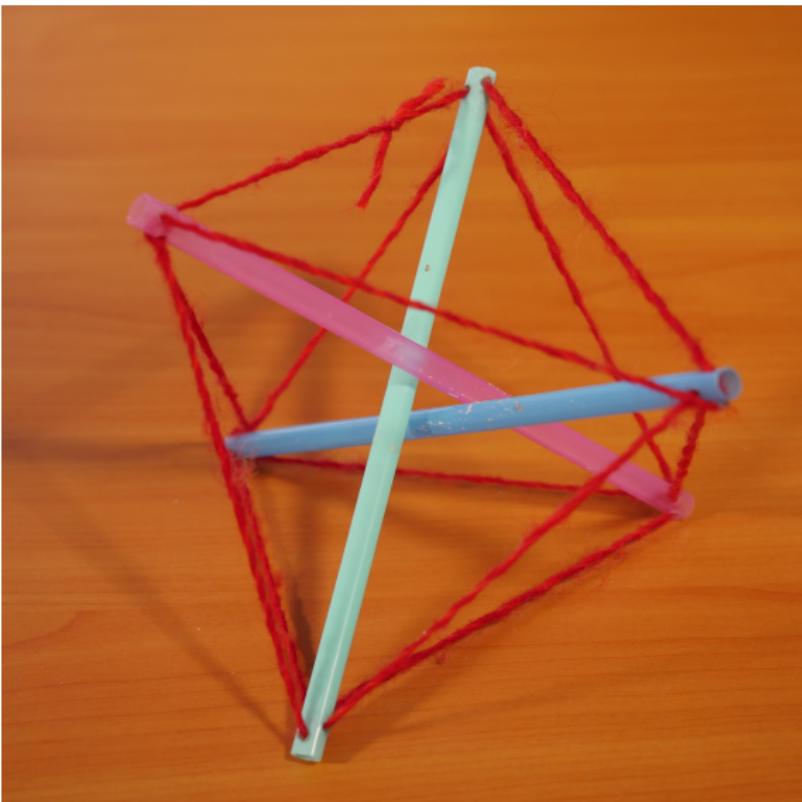


- there exists a “tensegrity”: a solid construction; straws do not touch each other; connected by threads

# Not Allowed



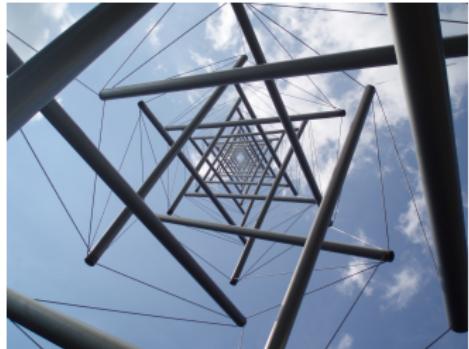
# Tensegrity Finished



# A Tensegrity: Animation

[Source: [https://commons.wikimedia.org/wiki/File:Tensegrity\\_simple\\_3.gif](https://commons.wikimedia.org/wiki/File:Tensegrity_simple_3.gif)]

# Tensegrities in the Real Life



[Source:[https://en.wikipedia.org/wiki/Needle\\_Tower](https://en.wikipedia.org/wiki/Needle_Tower)]

made by Kenneth Snelson, a student of Buckminster Fuller (who invented the word and made many of them)

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- you say:  $7125 = 57 \cdot 125$

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- divisible by 7: 14, 21, 28, 35, 42, 49, 56, 63
- but what if we asked for a number that becomes 57 times smaller?
- you say:  $7125 = 57 \cdot 125$
- no need to explain how you found it

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- 1000 works,  $X = 125$
- also  $71250 = 57 \times 1250$ , etc.

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- $+1 + 2 - 3 = 0$

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- find a group of weight 11
- easy:  $4 + 7$  (also  $1 + 2 + 3 + 5 = 11$ )

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- total weight 21: not a multiple of 2
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- what about weights 2, 4, 6, 8, 10, 12?
- hint: just changing the units

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- NP-complete  $\approx$  infeasible

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- claim: *an object with some property exists*
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- no need to disclose the sources
- beware: claim may be false!