

Why Proofs?

Alexander Shen

LIRMM

Outline

Proofs? Really?

Proof by example

Impossibility proof

Impossibility Proof, II

Conclusion

Do We Need Proofs?

- mathematicians: main tool

Do We Need Proofs?

- mathematicians: main tool
- programmers: more skeptical

Do We Need Proofs?

- mathematicians: main tool
- programmers: more skeptical
 - correctness proof of a video game? or search engine?

Do We Need Proofs?

- mathematicians: main tool
- programmers: more skeptical
 - correctness proof of a video game? or search engine?
 - but real-time operating system or cryptographic protocol is a different thing

What Is a Proof?

What Is a Proof?

- proof: a long formal meaningless manipulation

What Is a Proof?

- proof: a long formal meaningless manipulation
- not at all: an argument that is so convincing that you are ready to use it to convince other people

What Is a Proof?

- proof: a long formal meaningless manipulation
- not at all: an argument that is so convincing that you are ready to use it to convince other people
- a by-product and a sign of understanding

What Is a Proof?

- proof: a long formal meaningless manipulation
- not at all: an argument that is so convincing that you are ready to use it to convince other people
- a by-product and a sign of understanding
- believe or not: proofs are fun

Course Objectives

Course Objectives

- learn by examples how to understand proofs

Course Objectives

- learn by examples how to understand proofs
- how to invent proofs

Course Objectives

- learn by examples how to understand proofs
- how to invent proofs
- how to explain proofs

Course Objectives

- learn by examples how to **understand** proofs
- how to **invent** proofs
- how to **explain** proofs
- how to **enjoy** proofs (most important and ambitious goal)

Course Objectives

- learn by examples how to **understand** proofs
- how to **invent** proofs
- how to **explain** proofs
- how to **enjoy** proofs (most important and ambitious goal)

Intended audience: beginners

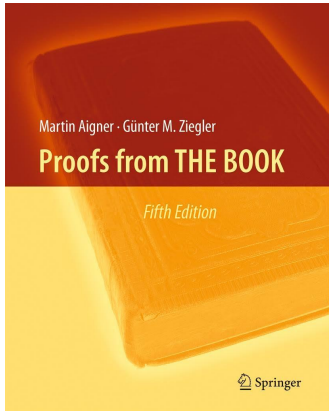
Course Objectives

- learn by examples how to **understand** proofs
- how to **invent** proofs
- how to **explain** proofs
- how to **enjoy** proofs (most important and ambitious goal)

Intended audience: beginners

Prerequisites: curiosity

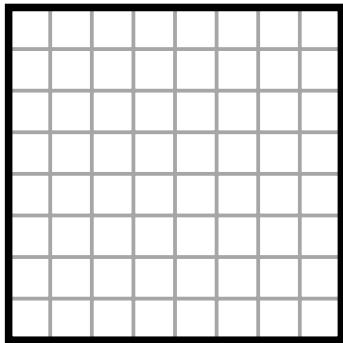
Paul Erdős and The Book



[Google Books and Wikipedia images]

To Begin With...

Can a chessboard 8×8 be tiled (no overlaps or empty space) by domino 1×2 -tiles?



Outline

Proofs? Really?

Proof by example

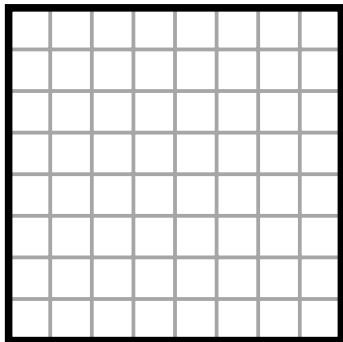
Impossibility proof

Impossibility Proof, II

Conclusion

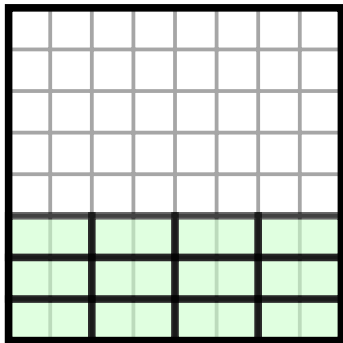
Tiling a Chessboard

Prove that a chessboard 8×8 can be tiled (no overlaps or empty space) by domino 1×2 -tiles.



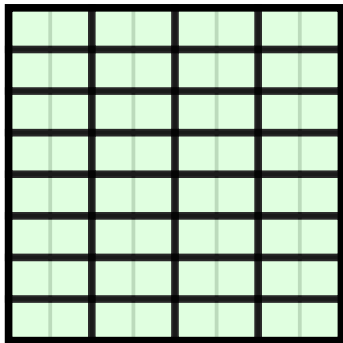
Tiling a Chessboard

Prove that a chessboard 8×8 can be tiled (no overlaps or empty space) by domino 1×2 -tiles.



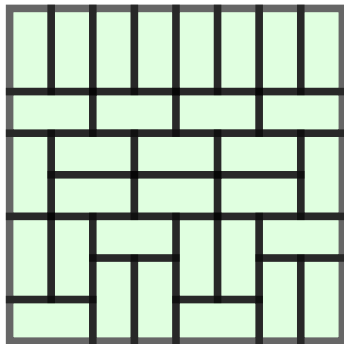
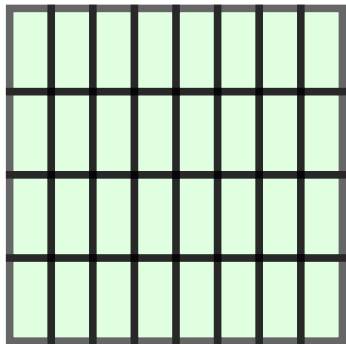
Tiling a Chessboard

Prove that a chessboard 8×8 can be tiled (no overlaps or empty space) by domino 1×2 -tiles.

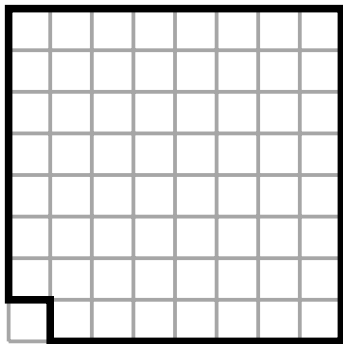


Other Tilings

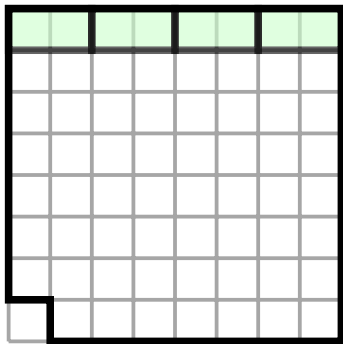
One tiling is enough for the proof. But there are others:



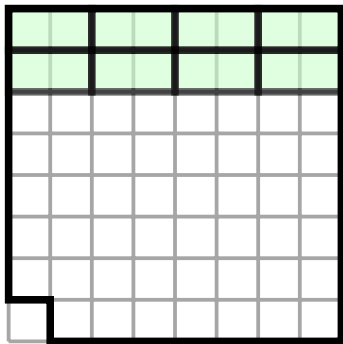
Can We Tile This Board?



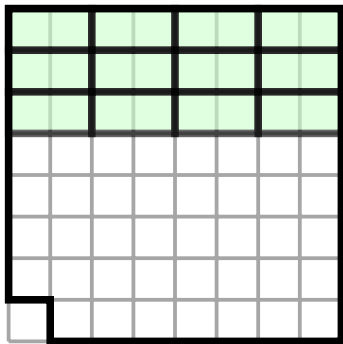
Can We Tile This Board?



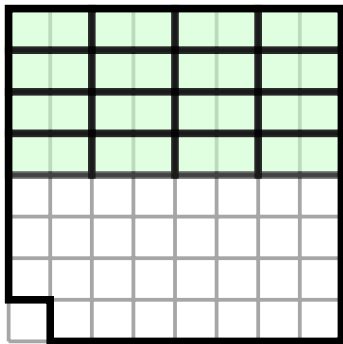
Can We Tile This Board?



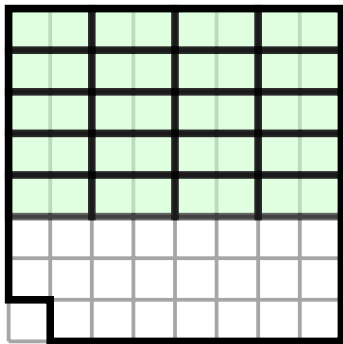
Can We Tile This Board?



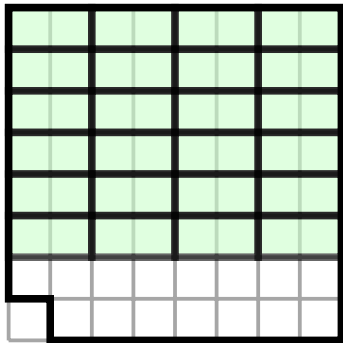
Can We Tile This Board?



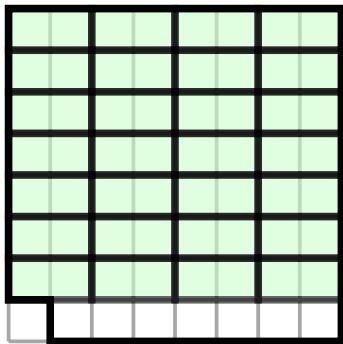
Can We Tile This Board?



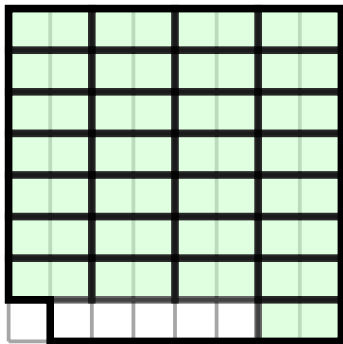
Can We Tile This Board?



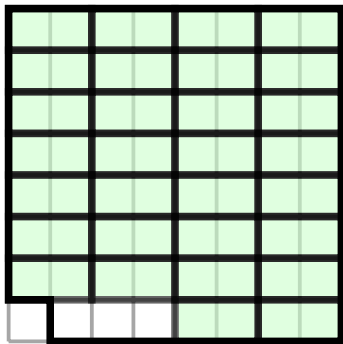
Can We Tile This Board?



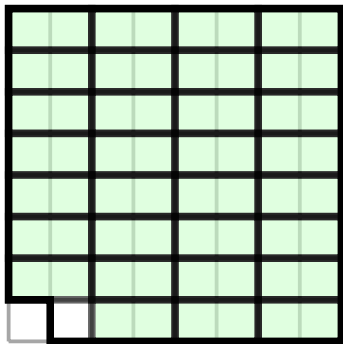
Can We Tile This Board?



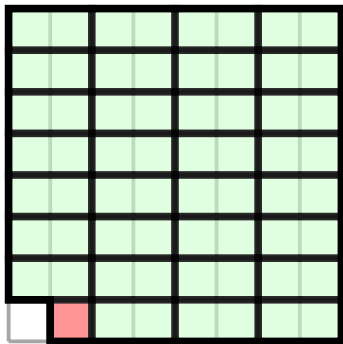
Can We Tile This Board?



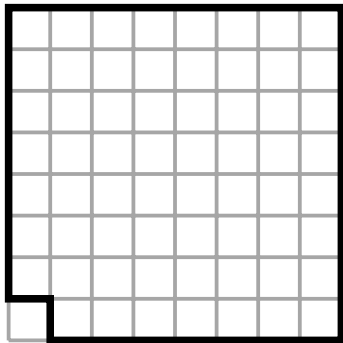
Can We Tile This Board?



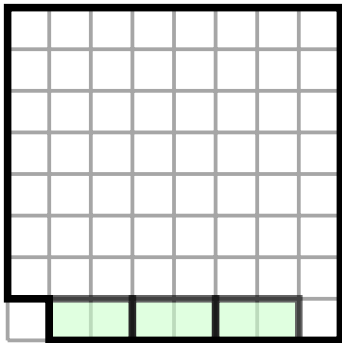
Can We Tile This Board?



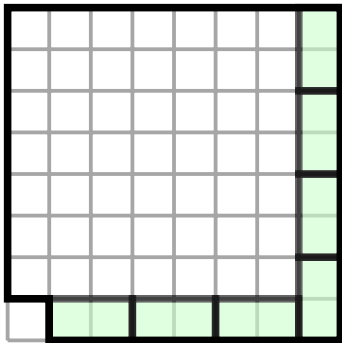
Trying Again



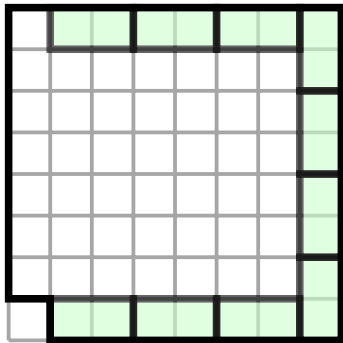
Trying Again



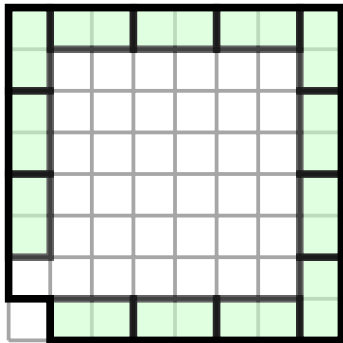
Trying Again



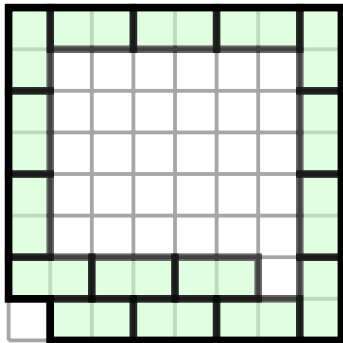
Trying Again



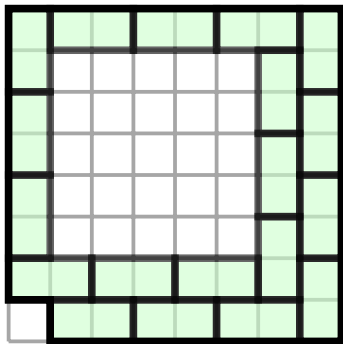
Trying Again



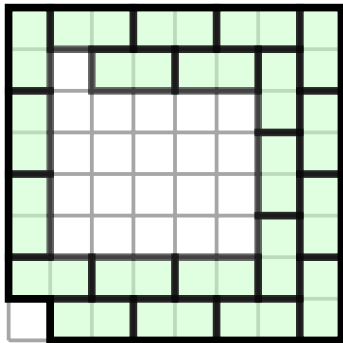
Trying Again



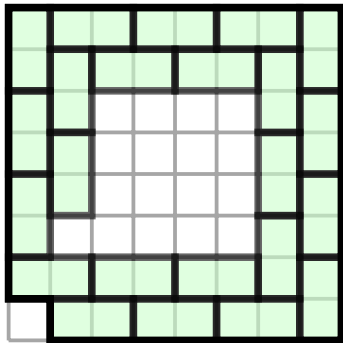
Trying Again



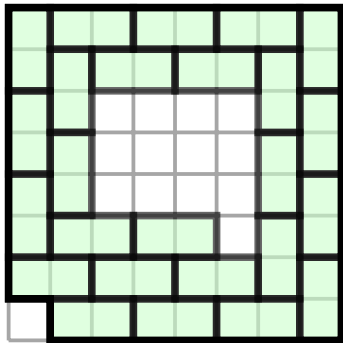
Trying Again



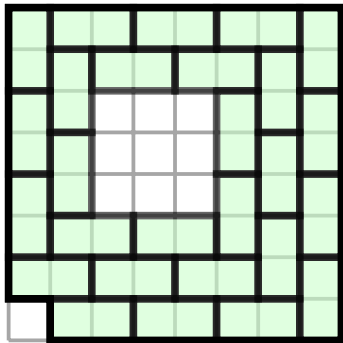
Trying Again



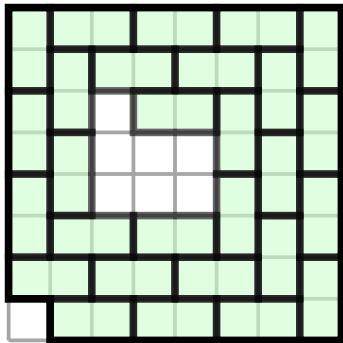
Trying Again



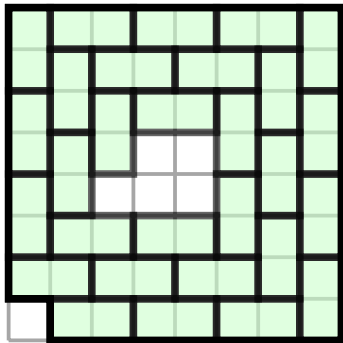
Trying Again



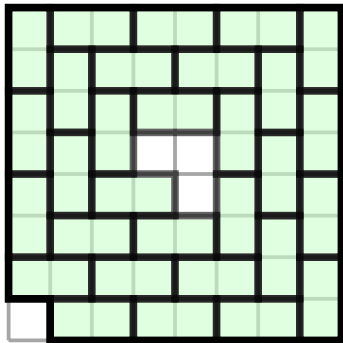
Trying Again



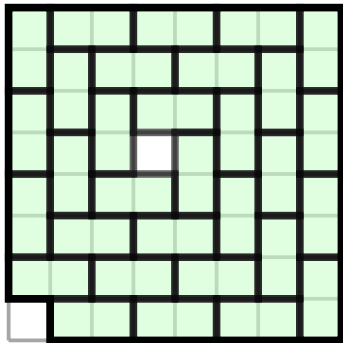
Trying Again



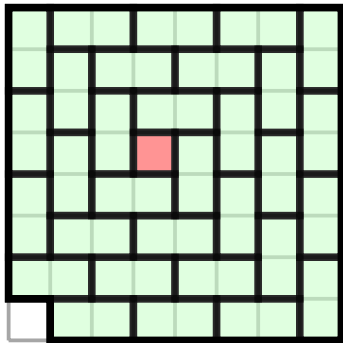
Trying Again



Trying Again



Trying Again



So What?

So What?

- did we find a tiling (and prove that it exists)?

So What?

- did we find a tiling (and prove that it exists)? No.

So What?

- did we find a tiling (and prove that it exists)? No.
- did we prove that tiling does not exist?

So What?

- did we find a tiling (and prove that it exists)? No.
- did we prove that tiling does not exist? No.

So What?

- did we find a tiling (and prove that it exists)? No.
- did we prove that tiling does not exist? No.
- can we do one of these two proofs?

Outline

Proofs? Really?

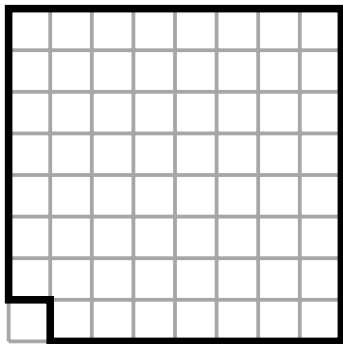
Proof by example

Impossibility proof

Impossibility Proof, II

Conclusion

Can We Tile This Board?



Proof of Impossibility

- one cell will always remain, because...

Proof of Impossibility

- one cell will always remain, because...
- there are $63 = 8 \times 8 - 1$ cells, an odd number

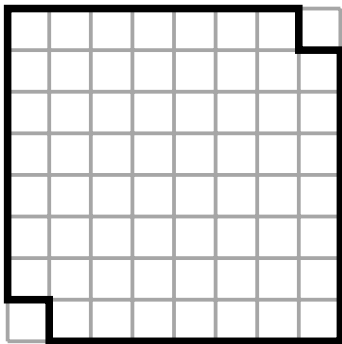
Proof of Impossibility

- one cell will always remain, because...
- there are $63 = 8 \times 8 - 1$ cells, an odd number
- 31 tiles cover 62 cells, one remains

Proof of Impossibility

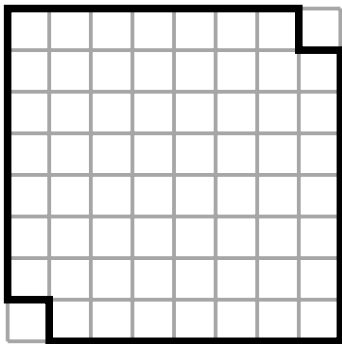
- one cell will always remain, because...
- there are $63 = 8 \times 8 - 1$ cells, an odd number
- 31 tiles cover 62 cells, one remains
- mission *provably* impossible

Two Corners Cut



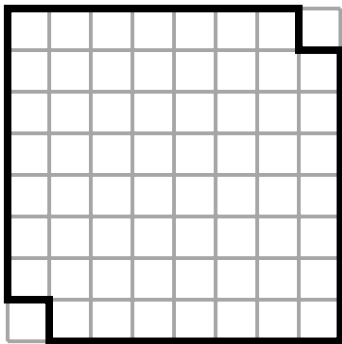
Two Corners Cut

$64 - 2 = 62$ cells, $62/2 = 31$ tiles. So the tiling is possible...



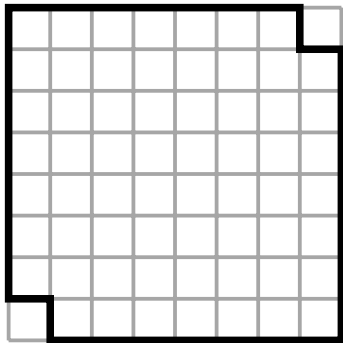
Two Corners Cut

$64 - 2 = 62$ cells, $62/2 = 31$ tiles. So the tiling is possible... is this argument OK?

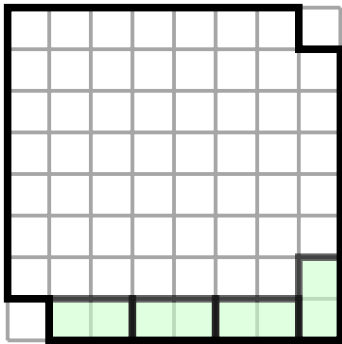


Trying...

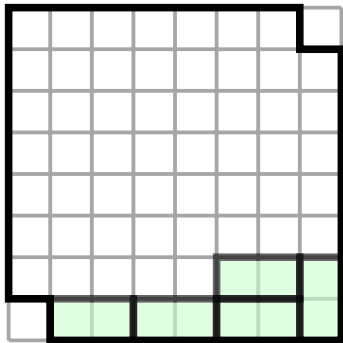
Trying...



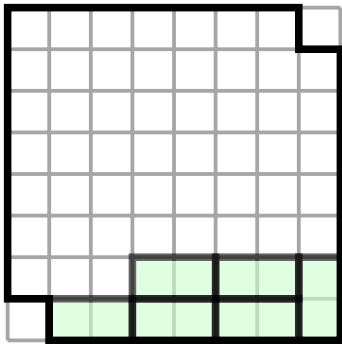
Trying...



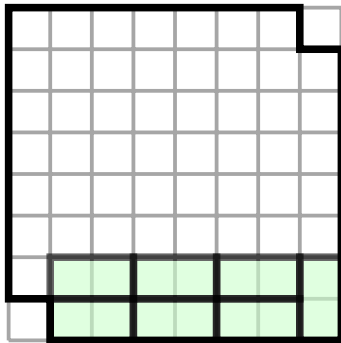
Trying...



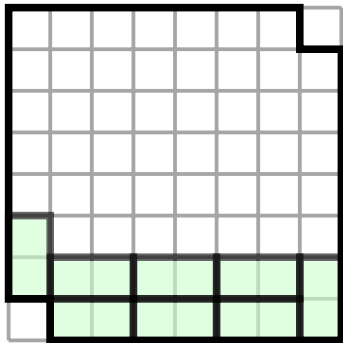
Trying...



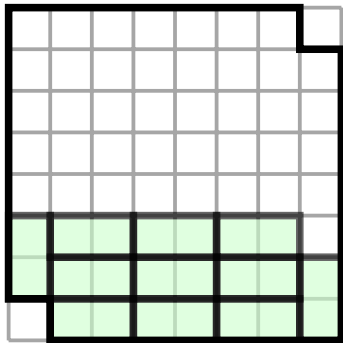
Trying...



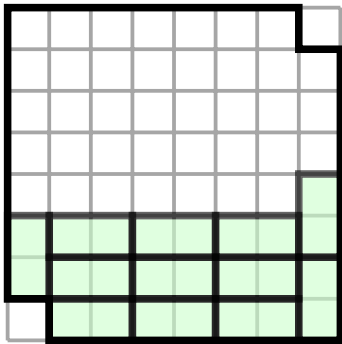
Trying...



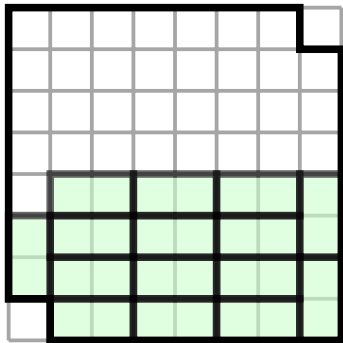
Trying...



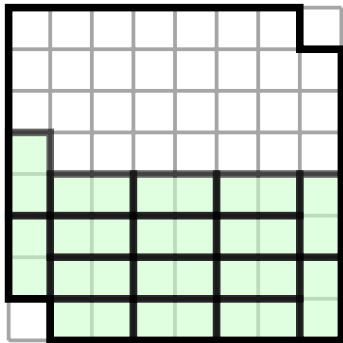
Trying...



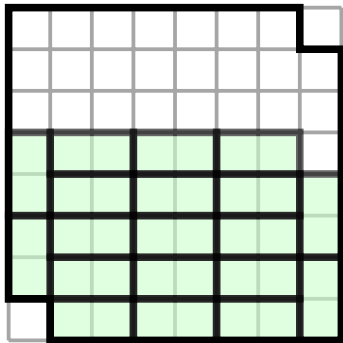
Trying...



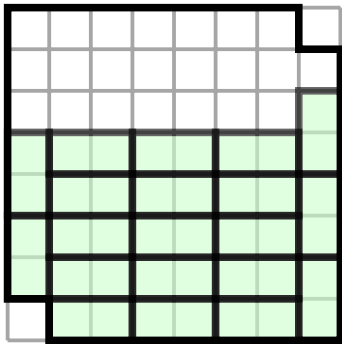
Trying...



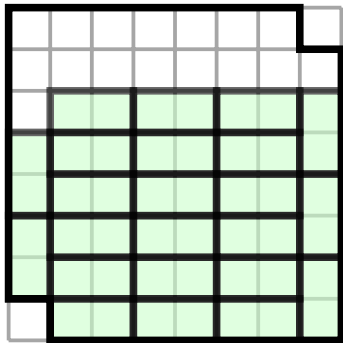
Trying...



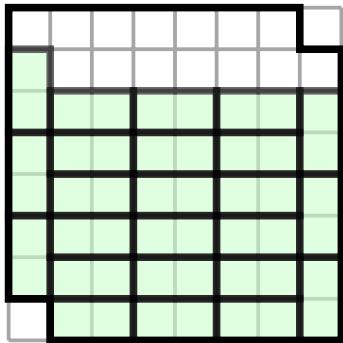
Trying...



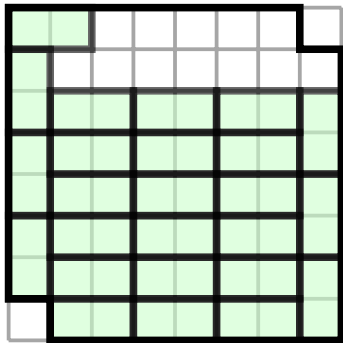
Trying...



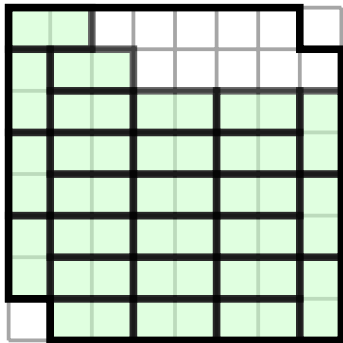
Trying...



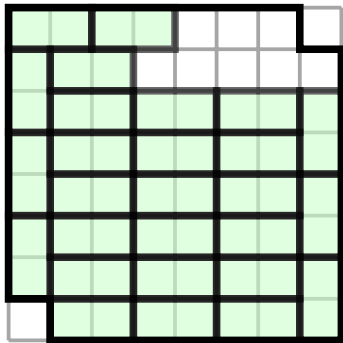
Trying...



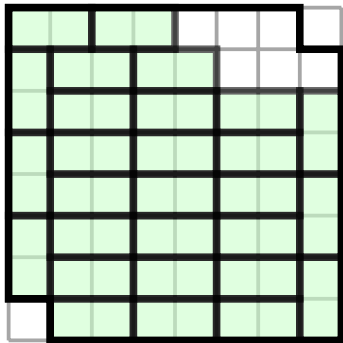
Trying...



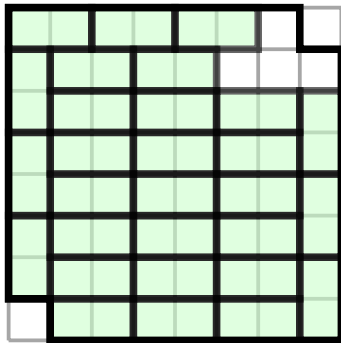
Trying...



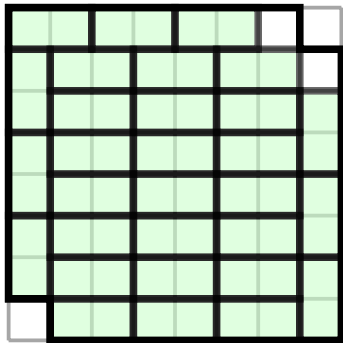
Trying...



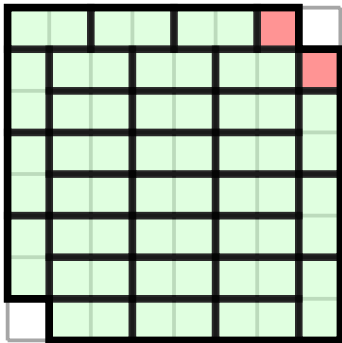
Trying...



Trying...

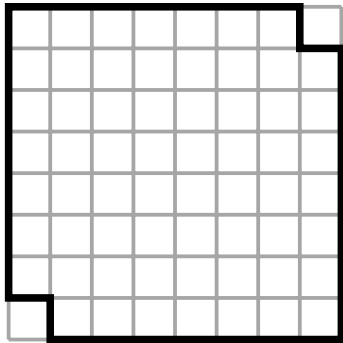


Trying...

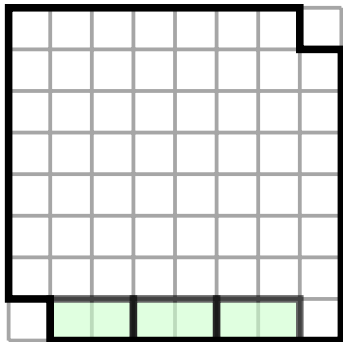


One More Attempt

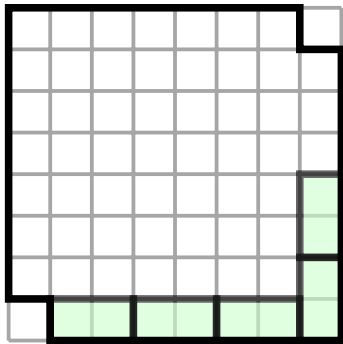
One More Attempt



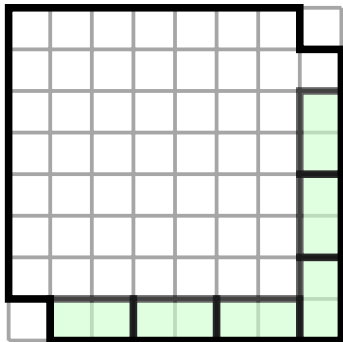
One More Attempt



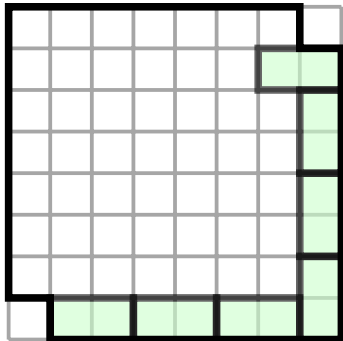
One More Attempt



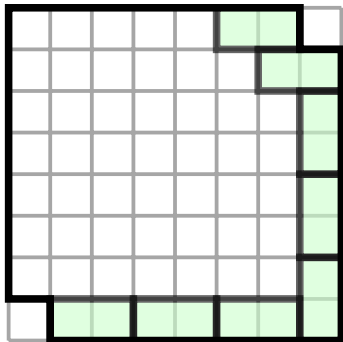
One More Attempt



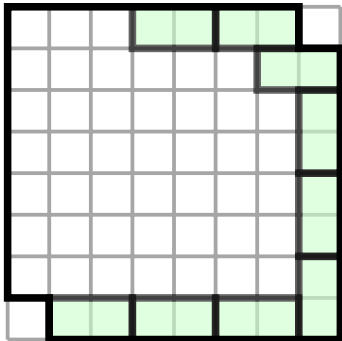
One More Attempt



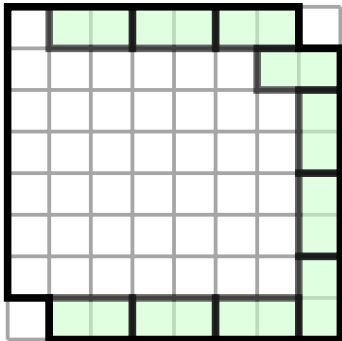
One More Attempt



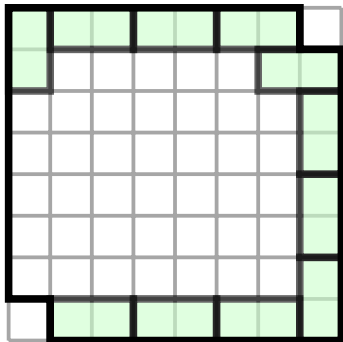
One More Attempt



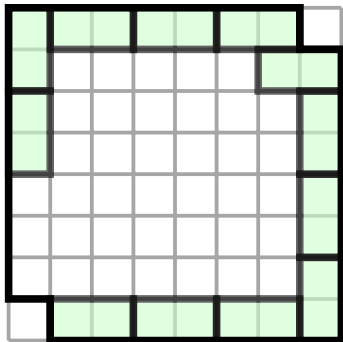
One More Attempt



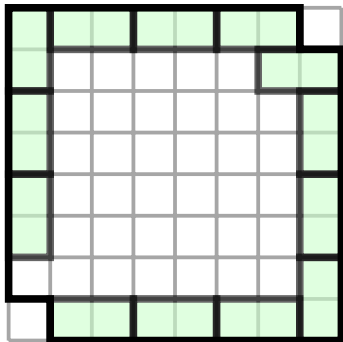
One More Attempt



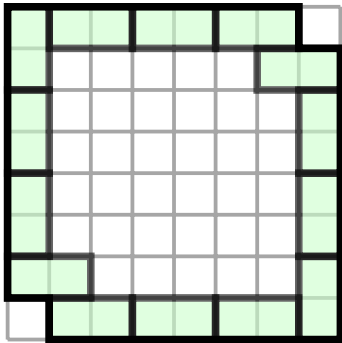
One More Attempt



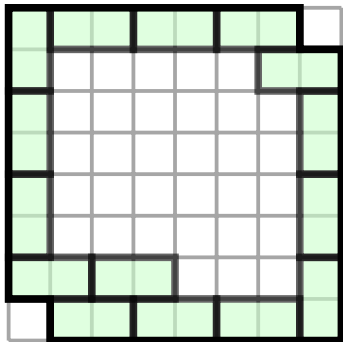
One More Attempt



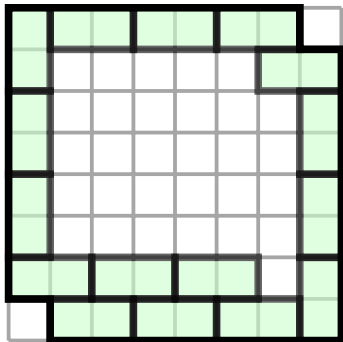
One More Attempt



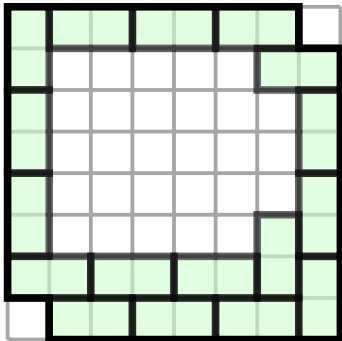
One More Attempt



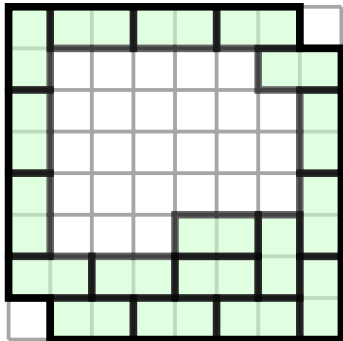
One More Attempt



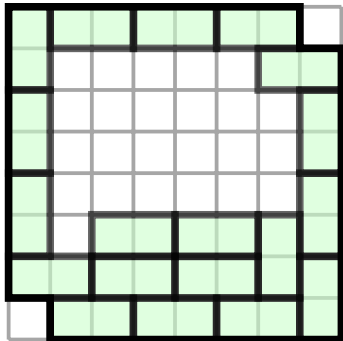
One More Attempt



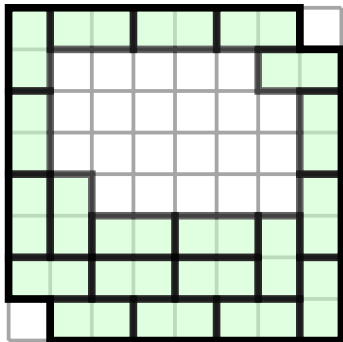
One More Attempt



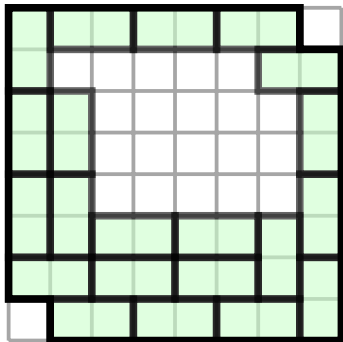
One More Attempt



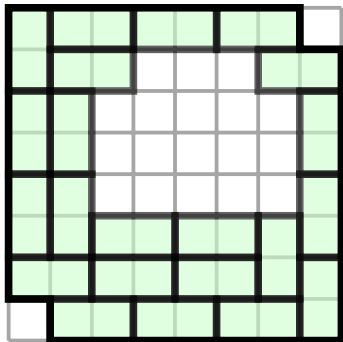
One More Attempt



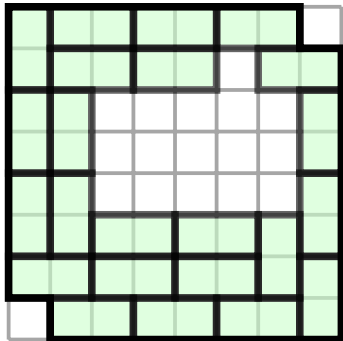
One More Attempt



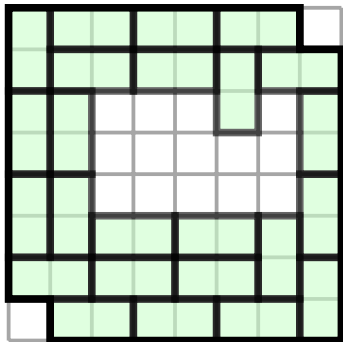
One More Attempt



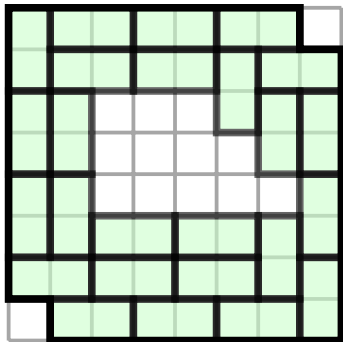
One More Attempt



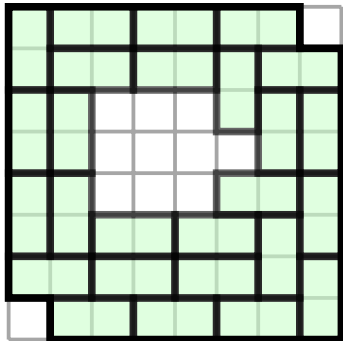
One More Attempt



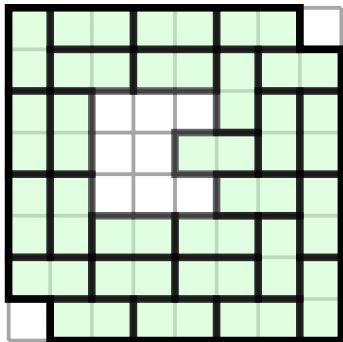
One More Attempt



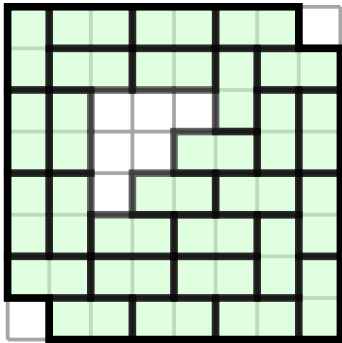
One More Attempt



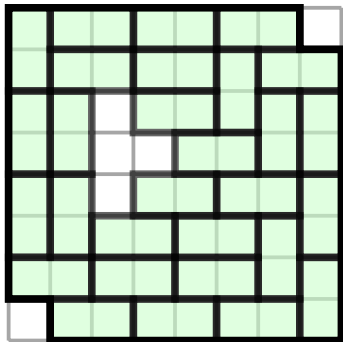
One More Attempt



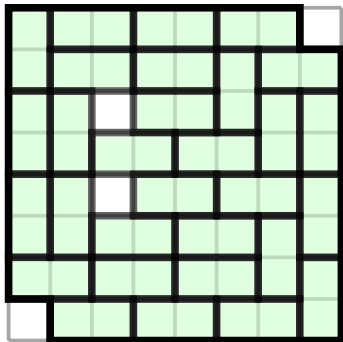
One More Attempt



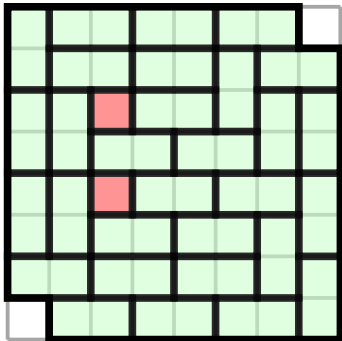
One More Attempt



One More Attempt

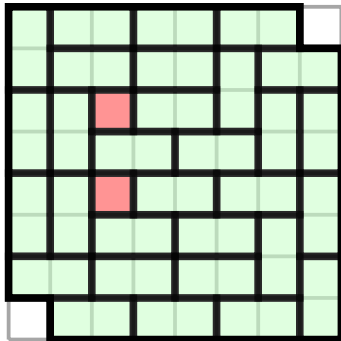


One More Attempt



One More Attempt

should we try harder?



Outline

Proofs? Really?

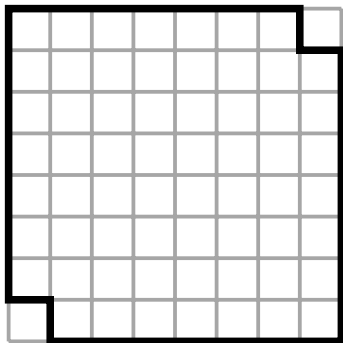
Proof by example

Impossibility proof

Impossibility Proof, II

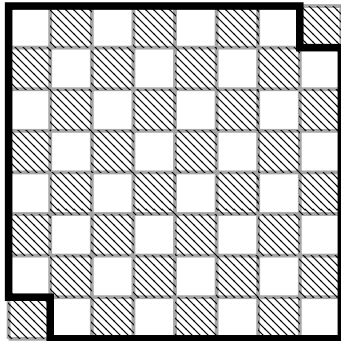
Conclusion

Can We Tile This Board?

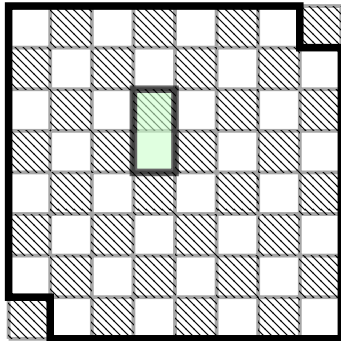


Spoiler

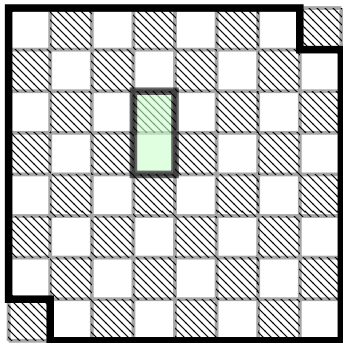
Spoiler



Spoiler

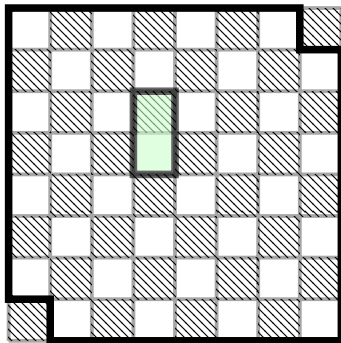


Spoiler



32 white, 30 black

Spoiler



32 white, 30 black \Rightarrow two whites remain

Recap

Theorem. A chess board 8×8 without two opposite corners cannot be tiled by 1×2 dominos.

Recap

Theorem. A chess board 8×8 without two opposite corners cannot be tiled by 1×2 dominos.

Proof.

Recap

Theorem. A chess board 8×8 without two opposite corners cannot be tiled by 1×2 dominos.

Proof.

- black and white cells, 4 black / 4 white

Recap

Theorem. A chess board 8×8 without two opposite corners cannot be tiled by 1×2 dominos.

Proof.

- black and white cells, 4 black / 4 white
- opposite corners are (say) black

Recap

Theorem. A chess board 8×8 without two opposite corners cannot be tiled by 1×2 dominos.

Proof.

- black and white cells, 4 black / 4 white
- opposite corners are (say) black
- 30 black and 32 white

Recap

Theorem. A chess board 8×8 without two opposite corners cannot be tiled by 1×2 dominos.

Proof.

- black and white cells, 4 black / 4 white
- opposite corners are (say) black
- 30 black and 32 white
- a domino: two different colors

Recap

Theorem. A chess board 8×8 without two opposite corners cannot be tiled by 1×2 dominos.

Proof.

- black and white cells, 4 black / 4 white
- opposite corners are (say) black
- 30 black and 32 white
- a domino: two different colors
- (at least) two white remain

Recap

Theorem. A chess board 8×8 without two opposite corners cannot be tiled by 1×2 dominos.

Proof.

- black and white cells, 4 black / 4 white
- opposite corners are (say) black
- 30 black and 32 white
- a domino: two different colors
- (at least) two white remain
- quod erat demonstrandum

Outline

Proofs? Really?

Proof by example

Impossibility proof

Impossibility Proof, II

Conclusion

Take-home Message and Questions

Take-home Message and Questions

- Proofs can be really convincing

Take-home Message and Questions

- Proofs can be really convincing
- What if we cut two non-opposite corners?
Can we tile the rest?

Take-home Message and Questions

- Proofs can be really convincing
- What if we cut two non-opposite corners?
Can we tile the rest?
- Bonus problem: what if we cut any two cells of different colors?

Spoiler

Spoiler

