

Td Primitives

Exercice 1

- 1 Fonctions polynomiales et puissances simples (x^n)

Formule : $\int x^n dx = \frac{x^{n+1}}{n+1}$

a $f(x) = x^3 - 2x + 1$

b $f(x) = (x-1)(x-2)$

c $f(x) = 4x^4 - 2x^2 + 5x$

d $f(x) = x + \frac{1}{x^2}$

e $f(x) = (x+1)^3$

f $f(x) = x + \frac{1}{\sqrt{x}}$

g $f(x) = \frac{2}{x^2} - \frac{3}{x^3}$

h $f(x) = \frac{x^4 + x^2 + 1}{x^2}$

- 2 Forme $u'u^n$ Formule : $\int u'u^n = \frac{u^{n+1}}{n+1}$

a $f(x) = (2x-1)(x^2-x)^2$

b $f(x) = 2x(x^2-1)^5$

c $f(x) = (6x-3)(4x^2-4x+2)^3$

d $f(x) = \sin^2 x \cos x$

e $f(x) = \sin x \cos^3 x$

f $f(x) = (3x^2+2)(x^3+2x-5)^4$

g $f(x) = \frac{1}{x^2} \left(1 - \frac{1}{x}\right)^3$

h $f(x) = e^x(e^x+1)^2$

i $f(x) = \frac{(\ln x)^4}{x}$

j $f(x) = \frac{1}{\cos^2 x} (\tan x + 1)^2$

k $f(x) = (x+1)(x^2+2x-3)^5$

l $f(x) = x^2(x^3+1)^2$

m $f(x) = \sin x \cos^4 x$

n $f(x) = 2(e^{2x}+1)(e^{2x}+2x)^3$

o $f(x) = \frac{1}{\sqrt{x}}(\sqrt{x}+3)^5$

- 3 Forme $\frac{u'}{u^n}$ ($n \geq 2$)

a $f(x) = \frac{1}{(x+1)^3}$

b $f(x) = \frac{2x+1}{(x^2+x+1)^2}$

c $f(x) = \frac{2x}{(x^2+1)^2}$

d $f(x) = \frac{4x+3}{(2x^2+3x+1)^3}$

e $f(x) = \frac{1-x}{(x^2-2x+3)^2}$

f $f(x) = \frac{2 \cos x}{\sin^2 x}$

g $f(x) = \frac{x^2}{(x^3+2)^4}$

h $f(x) = \frac{7e^x}{(e^x+2)^2}$

i $f(x) = \frac{1}{x(\ln x)^3}$

j $f(x) = \frac{\sin x}{\cos^4 x}$

k $f(x) = \frac{1}{(2x+5)^2}$

l $f(x) = \frac{x}{(x^2+4)^3}$

m $f(x) = \frac{x+1}{(x^2+2x-5)^5}$

n $f(x) = \frac{e^{2x}}{(e^{2x}+1)^2}$

o $f(x) = \frac{1}{\cos^2 x \tan^3 x}$

- 4 Forme $\frac{u'}{\sqrt{u}}$

a $f(x) = \frac{1}{\sqrt{x+1}}$

b $f(x) = \frac{3x}{\sqrt{x^2+1}}$

c $f(x) = \frac{x}{\sqrt{x^2 - 1}}$

d $f(x) = \frac{2x + 3}{\sqrt{x^2 + 3x + 2}}$

e $f(x) = \frac{e^x}{\sqrt{e^x + 1}}$

f $f(x) = \frac{\cos x}{\sqrt{\sin x}}$

g $f(x) = \frac{1}{x\sqrt{\ln x}}$

h $f(x) = \frac{e^x - e^{-x}}{\sqrt{e^x + e^{-x}}}$

i $f(x) = \frac{1}{\sqrt{2x + 3}}$

j $f(x) = \frac{\sin x}{\sqrt{\cos x}}$

k $f(x) = \frac{x^2}{\sqrt{x^3 + 1}}$

l $f(x) = \frac{1}{\cos^2 x \sqrt{\tan x}}$

Formes : $\sin(ax + b)$, $u' \cos(u)$, linéarisation ou dérivées de $\tan(x)$

a $f(x) = 3 \sin \frac{\pi x}{2}$

b $f(x) = \sin 3x + \cos(2x + 3)$

c $f(x) = \sin 2x - 2 \cos 2x$

d $f(x) = x \cos x^2$

e $f(x) = \frac{\cos \sqrt{x}}{\sqrt{x}}$

f $f(x) = \frac{1}{x^2} \sin \frac{1}{x}$

g $f(x) = \frac{\tan x}{\cos^2 x}$

h $f(x) = \tan^2 x$

i $f(x) = \tan x + \tan^3 x$

j $f(x) = 1 + \frac{1}{\tan^2 x}$

k $f(x) = \sin^2 x$

l $f(x) = \cos^3 x$

5 Fonctions Trigonométriques

Exercice 2

- 1 Dans chacun des cas suivants, déterminer une primitive F de f sur I après avoir effectuée la transformation d'écriture indiquée.

a $f(x) = \frac{x^2 - 2x}{(x - 1)^2}$ $I =]1; +\infty[$ Indication : Mettre $f(x)$ sous la forme $a + \frac{b}{(x - 1)^2}$

b $f(x) = \frac{3x^2 + 12x - 1}{(x + 2)^2}$, $I =]-2; +\infty[$ Indication : Mettre $f(x)$ sous la forme $a + \frac{b}{(x + 2)^2}$

c $f(x) = \frac{2x^3 + 13x^2 + 24x + 2}{(x + 3)^2}$, $I =]-3; +\infty[$

Indication : Mettre $f(x)$ sous la forme $ax + b + \frac{c}{(x + 3)^2}$

d $f(x) = \frac{x(x^2 + 3)}{(x^2 - 1)^3}$, $I =]-1; 1[$ Indication : Mettre $f(x)$ sous la forme $\frac{a}{(x - 1)^3} + \frac{b}{(x + 1)^3}$