◇◇◇ Lycée de Dindéfélo ◇◇◇			A.S. : 2025/2026
Matière: Mathématiques	Niveau : 2ndS	Date: 00/11/2025	Durée : 4 heures

Devoir n° 1 Du 1$^{\text{ère}}$ Semestre

Correction Exercice 1 : 4 pts (Factoriser les expressions suivantes :)

1. $a^2xy + aby^2 + b^2xy + abx^2$

$$\begin{aligned}(a^2xy + abx^2) + (b^2xy + aby^2) &= ax(ax + by) + by(ax + by)\\ &= (ax + by)(ax + by)\end{aligned}$$

2. $3a^2 + 3b^2 - 4c^2 - 6ab$

$$\begin{aligned}(3a^2 - 6ab + 3b^2) - 4c^2 &= 3(a^2 - 2ab + b^2) - 4c^2\\ &= 3(a-b)^2 - (2\cdot c)^2\\ &= \left[\sqrt{3}(a-b)\right]^2 - (2\cdot c)^2\\ &= (\sqrt{3}(a-b) - 2c)(\sqrt{3}(a-b) + 2c)\end{aligned}$$

3. $y^2 - x^2 + 2x - 1$

$$\begin{aligned}y^2 - (x^2 - 2x + 1) &= y^2 - (x-1)^2\\ &= (y - (x-1))(y + (x-1))\\ &= (y - x + 1)(y + x - 1)\end{aligned}$$

4. $a^2b^2 - 1 + a^2 - b^2$

$$\begin{aligned}a^2b^2 - 1 + a^2 - b^2 &= a^2b^2 - b^2 + a^2 - 1\\ &= b^2(a^2-1) + (a^2-1)\\ &= (a^2-1)(b^2+1)\\ &= (a-1)(a+1)(b^2+1)\end{aligned}$$

5. $(ab-1)^2 - (a-b)^2$

$$\begin{aligned}(ab-1)^2 - (a-b)^2 &= [ab - 1 - (a-b)][ab - 1 + (a-b)]\\ &= (ab - 1 - a + b)(ab - 1 + a - b)\\ &= [a(b-1) + (b-1)][a(b+1) - (1+b)]\\ &= [(a+1)(b-1)][(a+1)(b-1)]\\ &= (a-1)(a+1)(b-1)(b+1)\end{aligned}$$

Exercice 2 : 3 pts

1. Développons $(a+b+c)^2$.
$$\begin{aligned}[(a+b)+c]^2 &= (a+b)^2 + 2c(a+b) + c^2\\ &= \left(a^2+2ab+b^2\right) + 2ac + 2bc + c^2\\ &= a^2+b^2+c^2+2ab+2ac+2bc.\end{aligned}$$

2. Montrons que si $a+b+c=0$ alors $a^2+b^2+c^2=-2(ab+bc+ca)$.

On suppose que $a+b+c=0$.

On a: $(a+b+c)^2 = a^2+b^2+c^2+2ab+2ac+2bc$ donc $(0)^2 = a^2+b^2+c^2+2ab+2ac+2bc$

$$\begin{aligned}a^2+b^2+c^2+2ab+2ac+2bc=0 &\implies a^2+b^2+c^2=-2ab-2ac-2bc\\ &\implies a^2+b^2+c^2=-2(ab+bc+ca)\end{aligned}$$

3. On suppose a, b et c sont non nuls.
Montrons que $\dfrac{1}{a}+\dfrac{1}{b}+\dfrac{1}{c}=0 \implies (a+b+c)^2=a^2+b^2+c^2$.

$$\begin{aligned}\frac{1}{a}+\frac{1}{b}+\frac{1}{c}=0 &\implies \frac{bc+ac+ab}{abc}=0\\ &\implies bc+ac+ab=0\end{aligned}$$

D'après la 1er question, $(a+b+c)^2=a^2+b^2+c^2+2(ab+ac+bc)$
Donc si $bc+ac+ab=0$ alors $(a+b+c)^2=a^2+b^2+c^2$

Exercice 3 : 4 pts

Soit a, b, c trois réels :

1. Développons $(a+b+c)(ab+bc+ca)$ puis $(a+b+c)^3$

$$\begin{aligned}(a+b+c)(ab+bc+ca) &= a(ab+bc+ca)+b(ab+bc+ca)+c(ab+bc+ca)\\ &= a^2b+abc+a^2c+ab^2+b^2c+abc+abc+bc^2+ac^2\\ &= a^2b+a^2c+ab^2+ac^2+b^2c+bc^2+3abc\end{aligned}$$

$$\boxed{(a+b+c)(ab+bc+ca)=a^2b+a^2c+ab^2+ac^2+b^2c+bc^2+3abc}$$

$$\begin{aligned}(a+b+c)^3 &= (a+b)^3+3c(a+b)^2+3(a+b)c^2+c^3\\ &= (a^3+3a^2b+3ab^2+b^3)+3c(a^2+2ab+b^2)+3(ac^2+bc^2)+c^3\\ &= a^3+b^3+c^3+3a^2b+3ab^2+3a^2c+6abc+3b^2c+3ac^2+3bc^2.\end{aligned}$$

$$\boxed{(a+b+c)^3=a^3+b^3+c^3+3a^2b+3ab^2+3a^2c+6abc+3b^2c+3ac^2+3bc^2}$$

2 Démontrons que si $a+b+c=0$ alors $a^3+b^3+c^3=3abc$

$$\begin{cases} (a+b+c)(ab+bc+ca) & = a^2b+a^2c+ab^2+ac^2+b^2c+bc^2+3abc \\ (a+b+c)^3 & = a^3+b^3+c^3+3a^2b+3ab^2+3a^2c+6abc+3b^2c+3ac^2+3bc^2 \end{cases}$$

$$\begin{cases} (a+b+c)(ab+bc+ca) & = a^2b+a^2c+ab^2+ac^2+b^2c+bc^2+3abc \\ (a+b+c)^3 & = 3a^2b+3a^2c+3ab^2+3ac^2+3b^2c+6abc+3bc^2+a^3+b^3+c^3 \end{cases}$$

$$\begin{cases} 3(a+b+c)(ab+bc+ca) = 3a^2b+3a^2c+3ab^2+3ac^2+3b^2c+3bc^2+9abc \\ (a+b+c)^3 = 3a^2b+3a^2c+3ab^2+3ac^2+3b^2c+3bc^2+9abc-3abc+a^3+b^3+c^3 \end{cases}$$

$3(a+b+c)(ab+bc+ca)-(a+b+c)^3 = 3abc-(a^3+b^3+c^3)$

si $a+b+c=0$ alors $3(0)(ab+bc+ca)-(0)^3 = 3abc-(a^3+b^3+c^3)$

donc $3abc-(a^3+b^3+c^3)=0$ d'où $a^3+b^3+c^3=3abc$

3 Déduisons-en que , pour tous réel x, y, z on a :

$$(x+y)^3+(y+z)^3+(z+x)^3 = 3(x+y)(y+z)(z+x)$$

En Posons $\begin{cases} x+y=a \\ y+z=b \\ z+x=c \end{cases}$

donc $a^3+b^3+c^3=3abc$ devient $(x+y)^3+(y+z)^3+(z+x)^3=3(x+y)(y+z)(z+x)$

Exercice 2 : 8 pts

1 Simplifions les expressions suivantes (on suppose que tous les dénominateurs sont non nuls).

$$\begin{aligned} A &= \frac{\dfrac{x+y}{1-xy}-\dfrac{x-y}{1+xy}}{1-\dfrac{x^2-y^2}{1-x^2y^2}} \\ &= \frac{\dfrac{(x+y)(1+xy)-(x-y)(1-xy)}{(1-xy)(1+xy)}}{\dfrac{(1-x^2y^2)-(x^2-y^2)}{1-x^2y^2}} \\ &= \frac{(x+y)(1+xy)-(x-y)(1-xy)}{(1-xy)(1+xy)} \cdot \frac{1-x^2y^2}{(1-x^2y^2)-(x^2-y^2)} \\ &= \frac{2y(1+x^2)}{1-x^2y^2} \cdot \frac{1-x^2y^2}{1-x^2-x^2y^2+y^2} \\ &= \frac{2y(1+x^2)}{1-x^2-x^2y^2+y^2} \\ &= \frac{2y(1+x^2)}{(1+y^2)(1-x^2)}. \end{aligned}$$

$$\boxed{A = \frac{2y(1+x^2)}{(1+y^2)(1-x^2)}}$$

$$\begin{aligned}
B &= \frac{\frac{1}{a}-\frac{1}{b}}{\frac{1}{a}+\frac{1}{b}} \div \frac{a^2-b^2}{(a+b)^2} \\
&= \frac{\frac{b-a}{ab}}{\frac{a+b}{ab}} \cdot \frac{(a+b)^2}{a^2-b^2} \\
&= \frac{b-a}{a+b} \cdot \frac{(a+b)^2}{(a-b)(a+b)} \\
&= \frac{b-a}{a-b} \cdot (a+b) \\
&= -1 \cdot (a+b) \\
&= -(a+b).
\end{aligned}$$

$$\boxed{B = -(a+b)}$$

$$\begin{aligned}
C &= \frac{\frac{1}{a}-\frac{1}{b+c}}{\frac{1}{a}+\frac{1}{b+c}} \times \frac{\frac{1}{b}+\frac{1}{a+c}}{\frac{1}{b}-\frac{1}{a+c}} \\
&= \frac{\frac{b+c-a}{a(b+c)}}{\frac{b+c+a}{a(b+c)}} \times \frac{\frac{a+c+b}{b(a+c)}}{\frac{a+c-b}{b(a+c)}} \\
&= \frac{b+c-a}{a+b+c} \times \frac{a+b+c}{a+c-b} \\
&= \frac{b+c-a}{a+c-b}.
\end{aligned}$$

$$\boxed{C = \frac{b+c-a}{a+c-b}}$$

$$D = \dfrac{\dfrac{1}{a}+\dfrac{1}{b+c}}{\dfrac{1}{a}-\dfrac{1}{b+c}} \div \dfrac{a+b+c}{a-b-c}$$

$$= \dfrac{\dfrac{b+c+a}{a(b+c)}}{\dfrac{b+c-a}{a(b+c)}} \cdot \dfrac{a-b-c}{a+b+c}$$

$$= \dfrac{a+b+c}{b+c-a} \cdot \dfrac{a-b-c}{a+b+c}$$

$$= \dfrac{a-b-c}{b+c-a}.$$

$$\boxed{D = \dfrac{a-b-c}{b+c-a}}$$

2 Écrivons sous la forme $2^m \times 3^n \times 5^p$ (avec m, n, p des entiers relatifs) les réels suivants :

$$A = \dfrac{(0{,}009)^{-3}(0{,}016)^2 \cdot 250}{(0{,}00075)^{-1} \cdot 810^3 \cdot 30}$$

$$= \dfrac{(9 \cdot 10^{-3})^{-3}(16 \cdot 10^{-3})^2 \cdot (25 \cdot 10^1)}{(75 \cdot 10^{-5})^{-1} \cdot (2 \cdot 3^4 \cdot 5)^3 \cdot (3 \cdot 10)}$$

$$= \dfrac{9^{-3}10^9 \cdot 16^2 10^{-6} \cdot 25 \cdot 10}{75^{-1}10^5 \cdot 2^3 3^{12} 5^3 \cdot 3 \cdot 10}$$

$$= \dfrac{3^{-6} \cdot 2^8 \cdot 5^2 \cdot 10^4}{3^{-1}5^{-2}10^6 \cdot 2^3 3^{13} 5^3}$$

$$= 2^{8-3} \cdot 3^{-6+1-13} \cdot 5^{2+2-3} \cdot 10^{4-6}$$

$$= 2^5 \cdot 3^{-18} \cdot 5^1.$$

$$\boxed{C = 2^5 \cdot 3^{-18} \cdot 5^1}$$

$$B = \dfrac{(-6)^4 \cdot 30^{-2} \cdot (-10)^{-3} \cdot 15^4}{(-25)^2 \cdot 36^{-5} \cdot (-12)^3}$$

$$= \dfrac{(2 \cdot 3)^4 \cdot (2 \cdot 3 \cdot 5)^{-2} \cdot (2 \cdot 5)^{-3} \cdot (3 \cdot 5)^4}{(5^2)^2 \cdot (2^2 \cdot 3^2)^{-5} \cdot (2^2 \cdot 3)^3}$$

$$= \dfrac{2^4 3^4 \cdot 2^{-2} 3^{-2} 5^{-2} \cdot 2^{-3} 5^{-3} \cdot 3^4 5^4}{5^4 \cdot 2^{-10} 3^{-10} \cdot 2^6 3^3}$$

$$= 2^{4-2-3} \cdot 3^{4-2+4} \cdot 5^{-2-3+4} \quad \cdot \quad 2^{10-6} \cdot 3^{10-3} \cdot 5^{-4}$$

$$= 2^{-1+4} \cdot 3^{6+7} \cdot 5^{-1-4}$$

$$= 2^3 \cdot 3^{13} \cdot 5^{-5}.$$

$$\boxed{B = 2^3 \cdot 3^{13} \cdot 5^{-5}}$$