◇◇◇ Lycée de Dindéfélo ◇◇◇			A.S. : 2025/2026
Matière: Mathématiques	Niveau : 2ndS	Date: 00/11/2025	Durée : 4 heures

Devoir n° 1 Du 1ère Semestre

Correction Exercice 1 : 4 pts (Factoriser les expressions suivantes :)

1. $a^2xy + aby^2 + b^2xy + abx^2$

$$\begin{aligned}(a^2xy + abx^2) + (b^2xy + aby^2) &= ax(ax+by) + by(ax+by)\\ &= (ax+by)(ax+by)\end{aligned}$$

2. $3a^2 + 3b^2 - 4c^2 - 6ab$

$$\begin{aligned}(3a^2 - 6ab + 3b^2) - 4c^2 &= 3(a^2 - 2ab + b^2) - 4c^2\\ &= 3(a-b)^2 - (2\cdot c)^2\\ &= \left[\sqrt{3}(a-b)\right]^2 - (2\cdot c)^2\\ &= (\sqrt{3}(a-b) - 2c)(\sqrt{3}(a-b) + 2c)\end{aligned}$$

3. $y^2 - x^2 + 2x - 1$

$$\begin{aligned}y^2 - (x^2 - 2x + 1) &= y^2 - (x-1)^2\\ &= (y - (x-1))(y + (x-1))\\ &= (y - x + 1)(y + x - 1)\end{aligned}$$

4. $a^2b^2 - 1 + a^2 - b^2$

$$\begin{aligned}a^2b^2 - 1 + a^2 - b^2 &= a^2b^2 - b^2 + a^2 - 1\\ &= b^2(a^2-1) + (a^2-1)\\ &= (a^2-1)(b^2+1)\\ &= (a-1)(a+1)(b^2+1)\end{aligned}$$

5. $(ab-1)^2 - (a-b)^2$

$$\begin{aligned}(ab-1)^2 - (a-b)^2 &= [ab - 1 - (a-b)][ab - 1 + (a-b)]\\ &= (ab - 1 - a + b)(ab - 1 + a - b)\\ &= [a(b-1) + (b-1)][a(b+1) - (1+b)]\\ &= [(a+1)(b-1)][(a+1)(b-1)]\\ &= (a-1)(a+1)(b-1)(b+1)\end{aligned}$$

Exercice 2 : 3 pts

1. Développons $(a+b+c)^2$.

$$\begin{aligned}[(a+b)+c]^2 &= (a+b)^2 + 2c(a+b) + c^2 \\ &= \left(a^2+2ab+b^2\right) + 2ac + 2bc + c^2 \\ &= a^2+b^2+c^2+2ab+2ac+2bc.\end{aligned}$$

2. Montrons que si $a+b+c=0$ alors $a^2+b^2+c^2=-2(ab+bc+ca)$.

On suppose que $a+b+c=0$.

On a: $(a+b+c)^2 = a^2+b^2+c^2+2ab+2ac+2bc$ donc $(0)^2 = a^2+b^2+c^2+2ab+2ac+2bc$

$$\begin{aligned}a^2+b^2+c^2+2ab+2ac+2bc=0 &\implies a^2+b^2+c^2=-2ab-2ac-2bc \\ &\implies a^2+b^2+c^2=-2(ab+bc+ca)\end{aligned}$$

3. On suppose a, b et c sont non nuls.
Montrons que $\dfrac{1}{a}+\dfrac{1}{b}+\dfrac{1}{c}=0 \implies (a+b+c)^2=a^2+b^2+c^2$.

$$\begin{aligned}\frac{1}{a}+\frac{1}{b}+\frac{1}{c}=0 &\implies \frac{bc+ac+ab}{abc}=0 \\ &\implies bc+ac+ab=0\end{aligned}$$

D'après la 1er question, $(a+b+c)^2=a^2+b^2+c^2+2(ab+ac+bc)$
Donc si $bc+ac+ab=0$ alors $(a+b+c)^2=a^2+b^2+c^2$

Exercice 3 : 4 pts

Soit a, b, c trois réels :

1. Développons $(a+b+c)(ab+bc+ca)$ puis $(a+b+c)^3$

$$\begin{aligned}(a+b+c)(ab+bc+ca) &= a(ab+bc+ca)+b(ab+bc+ca)+c(ab+bc+ca) \\ &= a^2b+abc+a^2c+ab^2+b^2c+abc+abc+bc^2+ac^2 \\ &= a^2b+a^2c+ab^2+ac^2+b^2c+bc^2+3abc\end{aligned}$$

$$\boxed{(a+b+c)(ab+bc+ca)=a^2b+a^2c+ab^2+ac^2+b^2c+bc^2+3abc}$$

$$\begin{aligned}(a+b+c)^3 &= (a+b)^3+3c(a+b)^2+3(a+b)c^2+c^3 \\ &= (a^3+3a^2b+3ab^2+b^3)+3c(a^2+2ab+b^2)+3(ac^2+bc^2)+c^3 \\ &= a^3+b^3+c^3+3a^2b+3ab^2+3a^2c+6abc+3b^2c+3ac^2+3bc^2.\end{aligned}$$

$$\boxed{(a+b+c)^3=a^3+b^3+c^3+3a^2b+3ab^2+3a^2c+6abc+3b^2c+3ac^2+3bc^2}$$

2 Démontrons que si $a+b+c=0$ alors $a^3+b^3+c^3=3abc$

$$\begin{cases}(a+b+c)(ab+bc+ca) & = a^2b+a^2c+ab^2+ac^2+b^2c+bc^2+3abc \\ (a+b+c)^3 & = a^3+b^3+c^3+3a^2b+3ab^2+3a^2c+6abc+3b^2c+3ac^2+3bc^2\end{cases}$$

$$\begin{cases}(a+b+c)(ab+bc+ca) & = a^2b+a^2c+ab^2+ac^2+b^2c+bc^2+3abc \\ (a+b+c)^3 & = 3a^2b+3a^2c+3ab^2+3ac^2+3b^2c+6abc+3bc^2+a^3+b^3+c^3\end{cases}$$

$$\begin{cases}3(a+b+c)(ab+bc+ca) = 3a^2b+3a^2c+3ab^2+3ac^2+3b^2c+3bc^2+9abc \\ (a+b+c)^3 = 3a^2b+3a^2c+3ab^2+3ac^2+3b^2c+3bc^2+9abc-3abc+a^3+b^3+c^3\end{cases}$$

$3(a+b+c)(ab+bc+ca)-(a+b+c)^3 = 3abc-(a^3+b^3+c^3)$

si $a+b+c=0$ alors $3(0)(ab+bc+ca)-(0)^3 = 3abc-(a^3+b^3+c^3)$

donc $3abc-(a^3+b^3+c^3)=0$ d'où $a^3+b^3+c^3=3abc$

3 Déduisons-en que , pour tous réel x, y, z on a :

$$(x+y)^3+(y+z)^3+(z+x)^3 = 3(x+y)(y+z)(z+x)$$

En Posons $\begin{cases}x+y=a \\ y+z=b \\ z+x=c\end{cases}$ donc $a^3+b^3+c^3=3abc$ devient $(x+y)^3+(y+z)^3+(z+x)^3=3(x+y)(y+z)(z+x)$