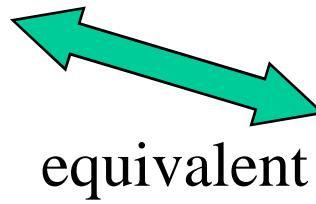


4.4 Simplex Algorithm to Solve Minimization Problems

$$\begin{array}{ll}\min & z = 2x_1 - 3x_2 \\ \text{s.t.} & x_1 + x_2 \leq 4 \\ & x_1 - x_2 \leq 6 \\ & x_1, x_2 \geq 0\end{array}$$



Method 1

$$\begin{array}{ll}\max & -z = -2x_1 + 3x_2 \\ \text{s.t.} & x_1 + x_2 \leq 4 \\ & x_1 - x_2 \leq 6 \\ & x_1, x_2 \geq 0\end{array}$$

Initial Tableau

$-z$	x_1	x_2	s_1	s_2	rhs	BV
1	2	-3	0	0	0	$-z = 0$
0	1	1	1	0	4	$s_1 = 4$
0	1	-1	0	1	6	$s_2 = 6$

Optimal Tableau

$-z$	x_1	x_2	s_1	s_2	rhs	BV
1	5	0	3	0	12	$-z = 12$
0	1	1	1	0	4	$x_2 = 4$
0	2	0	1	1	10	$s_2 = 10$

Nonnegative
(Max problem)

$z = -12$

Method 2

$$\min z = 2x_1 - 3x_2$$

$$\text{s.t. } x_1 + x_2 \leq 4$$

$$x_1 - x_2 \leq 6$$

$$x_1, x_2 \geq 0$$

Initial Tableau

z	x_1	x_2	s_1	s_2	rhs	BV
1	-2	3	0	0	0	$z = 0$
0	1	1	1	0	4	$s_1 = 4$
0	1	-1	0	1	6	$s_2 = 6$

Optimal Tableau

z	x_1	x_2	s_1	s_2	rhs	BV
1	-5	0	-3	0	-12	$z = -12$
0	1	1	1	0	4	$x_2 = 4$
0	2	0	1	1	10	$s_2 = 10$

Nonpositive

Optimal (Min Problem)

Nonpositive coefficient
of NBV in Row 0

$$z - 5x_1 - 3s_1 = -12$$

$$z = -12 + 5x_1 + 3s_1$$

4.5 Alternative Optimal Solution

$$\max z = 60x_1 + 35x_2 + 20x_3$$

Optimal	z	x_1	x_2	x_3	s_1	s_2	s_3	s_4	rhs	BV
Tableau	1	0	0	0	0	10	10	0	280	$z = 280$
	0	0	-2	0	1	2	-8	0	24	$s_1 = 24$
	0	0	-2	1	0	2	-4	0	8	$x_3 = 8$
	0	1	1.25	0	0	-0.5	1.5	0	2	$x_1 = 2^*$
	0	0	1	0	0	0	0	1	5	$s_4 = 5$

Another	z	x_1	x_2	x_3	s_1	s_2	s_3	s_4	rhs	BV
Optimal	1	0	0	0	0	10	10	0	280	$z = 280$
Tableau	0	1.6	0	0	1	1.2	-5.6	0	27.2	$s_1 = 27.2$
	0	1.6	0	1	0	1.2	-1.6	0	11.2	$x_3 = 11.2$
	0	0.8	1	0	0	-0.4	1.2	0	1.6	$x_2 = 1.6$
	0	-0.8	0	0	0	0	-1.2	1	3.4	$s_4 = 3.4$

4.6 Unbounded LPs

$$\max z = 36x_1 + 30x_2 - 3x_3 - 4x_4$$

$$\text{s.t. } x_1 + x_2 - x_3 \leq 5$$

$$6x_1 + 5x_2 - x_4 \leq 10$$

$$x_1, x_2, x_3, x_4 \geq 0$$

Initial Tableau

z	x_1	x_2	x_3	x_4	s_1	s_2	rhs	BV
1	-36	-30	3	4	0	0	0	$z=0$
0	1	1	-1	0	1	0	5	$s_1=5$
0	6	5	0	-1	0	1	10	$s_2=10$

First Tableau

z	x_1	x_2	x_3	x_4	s_1	s_2	rhs	BV
1	0	0	3	-2	0	6	60	$z=60$
0	0	1/6	-1	1/6	1	-1/6	10/3	$s_1=10/3$
0	1	5/6	0	-1/6	0	1/6	5/3	$x_1=5/3$

Second Tableau

z	x_1	x_2	x_3	x_4	s_1	s_2	rhs	BV
1	0	2	-9	0	12	4	100	$z=100$
0	0	1	-6	1	6	-1	20	$x_4=20$
0	1	1	-1	0	1	0	5	$x_1=5$

Impossible to do ratio test

→ Arbitrarily large z values

4.10 How to make Standard Form (Big M Method)

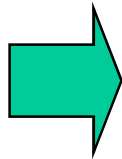
$$\min \quad z = 2x_1 + 3x_2$$

$$\text{s.t.} \quad 1/2 x_1 + 1/4 x_2 \leq 4$$

$$x_1 + 3x_2 \geq 20$$

$$x_1 + x_2 = 10$$

$$x_1, x_2 \geq 0$$



$$z - 2x_1 - 3x_2 = 0$$

$$1/2 x_1 + 1/4 x_2 + s_1 = 4$$

$$x_1 + 3x_2 - e_2 = 20 \quad \text{Excess variable} \quad \text{if } x_1, x_2 = 0$$

$$x_1 + x_2 = 10 \quad \text{Equality}$$

$$x_1, x_2, s_1, e_2 \geq 0$$

How solve?



$$z - 2x_1 - 3x_2 = 0$$

$$1/2 x_1 + 1/4 x_2 + s_1 = 4$$

$$x_1 + 3x_2 - e_2 + a_2 = 20$$

$$x_1 + x_2 + a_3 = 10$$

$$x_1, x_2, s_1, e_2 \geq 0$$

**Artificial
variables**

a_2, a_3

But, artificial variables should be zero in the optimal solution.

4.11 Two-Phase Simplex Method

$$z - 2x_1 - 3x_2 = 0$$

$$1/2 x_1 + 1/4 x_2 + s_1 = 4$$

$$x_1 + 3x_2 - e_2 + a_2 = 20$$

$$x_1 + x_2 + a_3 = 10$$

$$x_1, x_2, s_1, e_2 \geq 0$$



Phase I LP

$$\min w' = a_2 + a_3$$

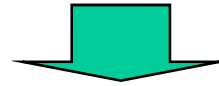
$$\text{s.t. } 1/2 x_1 + 1/4 x_2 + s_1 = 4$$

$$x_1 + 3x_2 - e_2 + a_2 = 20$$

$$x_1 + x_2 + a_3 = 10$$

$$\text{New Row 0 } w' + 2x_1 + 4x_2 - e_2 = 30$$

*eliminate artificial variables from Row 0



Phase II LP

Eliminate column of artificial variables from optimal tableau of phase I and continue simplex method

Initial Tableau of Phase I

	z	w'	x_1	x_2	s_1	e_2	a_2	a_3	rhs
Row z	1	0	-2	-3	0	0	0	0	$z=0$
Row w'	0	1	2	4	0	-1	0	0	$w'=30$
	0	0	$1/2$	$1/4$	1	0	0	0	$s_1=4$
	0	0	1	3	0	-1	1	0	$a_2=20$
	0	0	1	1	0	0	0	1	$a_3=10$

Next Tableau of Phase I

	z	w'	x_1	x_2	s_1	e_2	a_2	a_3	rhs
Row z	1	0	-1	0	0	-1	1	0	$z=20$
Row w'	0	1	$2/3$	0	0	$1/3$	$-4/3$	0	$w'=10/3$
	0	0	$5/12$	0	1	$1/12$	$-1/12$	0	$s_1=7/3$
	0	0	$1/3$	1	0	$-1/3$	$1/3$	0	$x_2=20/3$
	0	0	$2/3$	0	0	$1/3$	$-1/3$	1	$a_3=10/3$

Optimal Tableau of Phase I

	z	w'	x_1	x_2	s_1	e_2	a_2	a_3	rhs
Row z	1	0	0	0	0	$-1/2$	$1/2$	$3/2$	$z=25$
Row w'	0	1	0	0	0	0	-1	-1	$w'=0$
	0	0	0	0	1	$-1/8$	$1/8$	$-5/8$	$s_1=1/4$
	0	0	0	1	0	$-1/2$	$1/2$	$-1/2$	$x_2=5$
	0	0	1	0	0	$1/2$	$-1/2$	$3/2$	$x_1=5$

Initial Tableau of **Phase II**

	z	w'	x_1	x_2	s_1	e_2	rhs
Row z	1	0	0	0	0	$-1/2$	$z=25$
	0	0	0	0	1	$-1/8$	$s_1=1/4$
	0	0	0	1	0	$-1/2$	$x_2=5$
	0	0	1	0	0	$1/2$	$x_1=5$