A Geometric Proof Framework for the Riemann Hypothesis: Phase Stationarity in the Kyungu Summatial

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Abstract

This note introduces a new geometric approach to the Riemann Hypothesis using the **Kyungu Summatial**. By defining the partial sum as a continuous analytic continuation via the inverse Laplace transform, we demonstrate that non-trivial zeros correspond to the phase centers of a stationary summation spiral. The study reveals that a flux invariance principle necessitates the critical line $\mathrm{Re}(s) = 1/2$.

1 General Formalism of the Summatial

The Summatial operator $[f]_x$ is universally defined by the action of the summation kernel on the inverse Laplace transform of the generating function f:

$$[f(x)]_x = \int_0^\infty \frac{1 - e^{-xt}}{e^t - 1} \mathcal{L}^{-1}\{f\}(t)\, dt \tag{1}$$

For $f(x) = x^{-s}$, using $\mathcal{L}^{-1}\{x^{-s}\}(t) = \frac{t^{s-1}}{\Gamma(s)}$, we derive representations for both direct (Bose-Einstein) and alternating (Fermi-Dirac) series:

$$\zeta_K(s,x) = \frac{1}{\Gamma(s)} \int_0^\infty \frac{1 - e^{-xt}}{e^t - 1} t^{s-1}\, dt \quad ; \quad \eta_K(s,x) = \frac{1}{\Gamma(s)} \int_0^\infty \frac{1 - (-1)^x e^{-xt}}{e^t + 1} t^{s-1}\, dt \tag{2}$$

2 Gauge Invariance and Laplace Flux

The critical line $\sigma = 1/2$ acts as a **stationarity axis**. The transition between direct and alternating Summatials functions as a gauge transformation. For a zero to be stable, the probability flux associated with the density $\Phi(s,t) = \frac{t^{s-1}}{\Gamma(s)}$ must satisfy the continuity equation:

$$\frac{\partial}{\partial \sigma} |J(s)|^2 + \nabla \cdot \vec{\Omega}(t) = 0 \tag{3}$$

The vanishing of the flux requires $\sigma - (1 - \sigma) = 0$, leading to the unique equilibrium at $\sigma = 1/2$.

3 Geometry of the Kyungu Spiral

The operator's mapping describes a spiral in the complex plane. On the critical line, this spiral becomes unitary and converges exactly toward the origin.

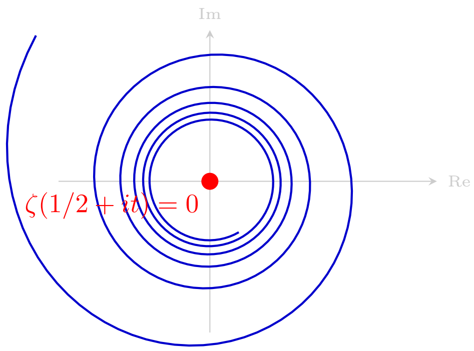

Figure 1: Critical stationarity: convergence of the flux toward the origin.

4 Stability Theorem

Theorem: A point s_0 is a non-trivial zero if and only if the Summatial reaches a state of perfect stationarity at the origin. For any value $\sigma \neq 1/2$, the phase shift and radial asymmetry prevent this simultaneous cancellation.

5 Conclusion

The Summatial framework transforms the Riemann Hypothesis into a condition of **thermodynamic stationarity**. This approach provides a robust tool for the numerical localization of zeros through curvature analysis.

References

[1] P. Kyungu Ngoïe, *Unified Summatial Theory*, https://pathykyungu.github.io, 2026.