# **Frequent Pattern Mining**

## **Agenda**

- Frequent Pattern : market basket analysis
- Association Rules
- Apriori Algorithm
- FP-growth: FP-tree
- FP Mining with Vertical Data Format
- Lift: Correlation measure

## **Basic Concepts**

- Frequent Pattern: a pattern (a set of items, subsequences, substructures, etc.) that occurs frequently in a data set
- Goal: finding inherent regularities in data
  - → What products were often purchased together?— Milk and Bread?!
  - → What are the subsequent purchases after buying a PC?
  - → What kinds of DNA are sensitive to this new drug?
  - → Can we automatically classify Web documents?

#### Applications:

- → A typical example of frequent itemset mining is market basket analysis.
- → cross-marketing, catalog design, sale campaign analysis, Web log (click stream) analysis, and DNA sequence analysis.

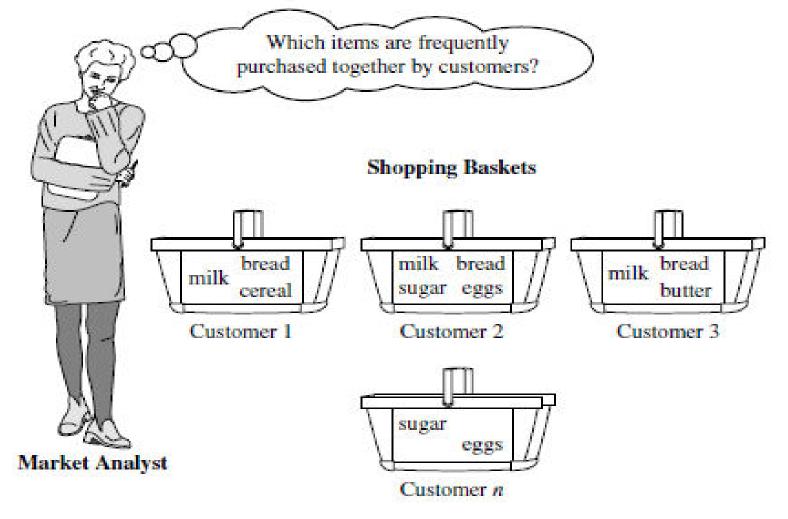


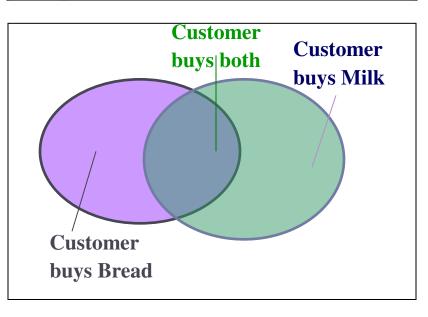
Figure 1 Market basket analysis.

First proposed by Agrawal, Imielinski, and Swami [AIS93] in the context of frequent itemsets and association rule mining (Cited by 25688)



## **Frequent Patterns**

Tid	Items bought
10	Milk, Bread, Cookies, Juice
20	Milk, Juice
30	Milk, Eggs
40	Bread, Cookies, Coffee

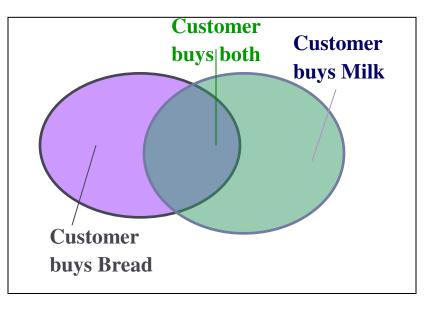


- Itemset  $X = \{x_1, ..., x_k\}$
- Find all the rules  $X \rightarrow Y$  with minimum support and confidence
  - support, s, probability that a transaction contains  $X \cup Y$
  - confidence, c, conditional probability that a transaction having X also contains Y
- An itemset X is **frequent** if X's support is no less than a *minsup* threshold

### **Association Rules**

Tid	Items bought
10	Milk, Bread, Cookies, Juice
20	Milk, Juice
30	Milk, Eggs
40	Bread, Cookies, Coffee

- Find all the rules  $X \rightarrow Y$  with minimum support and confidence threshold
  - → support , s, probability that a transaction contains X ∪ Y
  - → confidence, c, conditional probability that a transaction having X also contains Y



Let Milk → Juice and Bread → Juice

- The support of Milk→ Juice = 50% (2/4)
- ▶ The Conf of Milk  $\rightarrow$  Juice = 66.7% (2/3)
- ▶ The support of Bread  $\rightarrow$  Juice = 25% ?
- The Conf of Bread → Juice = 50% ?
- Rules that satisfy both minsup and minconf are called strong rules

### **Closed Patterns and Max-Patterns**

- A long pattern contains a combinatorial number of sub-patterns, e.g.,  $\{a_1, ..., a_{100}\}$  contains  $\binom{1}{100} + \binom{1}{100} + ... + \binom{1}{1000} = 2^{100} 1 = 1.27*10^{30}$  sub-patterns!
- Solution: Mine closed patterns and max-patterns instead
- An itemset X is closed if X is frequent and there exists no superpattern Y > X, with the same support as X
- An itemset X is a max-pattern if X is frequent and there exists no frequent super-pattern Y > X
- Closed pattern is a lossless compression of freq. patterns
  - → Reducing the number of patterns and rules

## **Closed Patterns and Max-Patterns**

#### **Example**

- $DB = \{ <\alpha_1, ..., \alpha_{100} >, <\alpha_1, ..., \alpha_{50} > \}$
- Min\_sup=1
- What is the set of closed itemset?
  - <a1, ..., a100>: 1
  - → < a1, ..., a50>: 2
- What is the set of max-pattern?
  - <a1, ..., a100>: 1

# **Computational Complexity**

- How many itemsets are potentially to be generated in the worst case?
  - → The number of frequent itemsets to be generated is sensitive to the minsup threshold
  - → When minsup is low, there exist potentially an exponential number of frequent itemsets
  - → The worst case: MN where M: # distinct items, and N: max length of transactions

## **Apriori: Concepts and Principle**

- ▶ The **downward closure** property of frequent patterns
  - → Any subset of a frequent itemset must be frequent
  - → If {Milk, Bread, Juice} is frequent, so is {Milk, Bread}
  - → i.e., every transaction having {Milk, Bread, Juice} also contains {Milk, Bread}
- Apriori pruning principle: If there is any itemset which is infrequent, its superset should not be generated/tested

## **Apriori Principle**

- Initially, scan DB once to get frequent 1-itemset
- ▶ **Generate** length (k+1) **candidate** itemsets from length k frequent itemsets
- ▶ Test the candidates against DB
- ▶ Terminate when no frequent or candidate set can be generated

# **Apriori: Example**

## $Sup_{min} = 2$

### Database

Tid	Items
10	11, 13, 14
20	12, 13, 15
30	11, 12, 13, 15
40	12, 15

 $C_{i}$ 

1st scan

Itemset	sup
{11}	2
{12}	3
{13}	3
{14}	1
{15}	3

 $L_1$ 

	Itemset	sup
	{I1}	2
	{I2}	3
٠	{I3}	3
	{I5}	3

$L_2$	Itemset	sup
_	{11, 13}	2
	{12, 13}	2
	{12, 15}	3
1	{13, 15}	2

- Z

,	Itemset	sup
′	{11, 12}	1
	{11, 13}	2
	{11, 15}	1
	{12, 13}	2
	{12, 15}	3
	{13, 15}	2

2<sup>nd</sup> scan

Ite	emset
{	1, I2}
{	1, I3}
{	1, I5}
{	2, 13}
{	2, 15}
{	3, 15}

$C_3$	Itemset	
3	{12, 13, 15}	

3 <sup>rd</sup> scan	$L_3$

Itemset	sup
{12, 13, 15}	2



## **Apriori Algorithm**

```
C_k: Candidate itemset of size k
L_k: frequent itemset of size k
L_1 = \{\text{frequent items}\};
for (k = 1; L_k !=\varnothing; k++) do begin
  C_{k+1} = candidates generated from L_k;
  for each transaction t in database do
    increment the count of all candidates in C_{k+1} that are
     contained in t
  L_{k+1} = candidates in C_{k+1} with min_support
  end
return \cup_k L_k;
```

### **Candidate Generation**

- How to generate candidates?
  - → Step 1: self-joining L<sub>k</sub>
  - → Step 2: pruning

```
Join L_k p with L_k q, as follows: 

insert into C_{k+1} 

select {p.item<sub>i</sub>}<sub>{1,...,k-1}</sub>, p.item<sub>k</sub>, q.item<sub>k</sub>

from L_k p, L_k q

where {p.item<sub>i</sub>}<sub>{1,...k-1}</sub> = {q.item<sub>i</sub>}<sub>{1,...k-1}</sub> and p.item<sub>k</sub> < q.item<sub>k</sub>
```

## **Example of Candidate Generation**

Suppose we have the following frequent 3-itemsets and we would like to generate the 4-itemsets candidates

- L3={{I1, I2, I3}, {I1, I2, I4}, {I1, I3, I4}, {I1, I3, I5}, {I2,I3,I4}}
- Self-joining: L3\*L3 gives: {11,12,13,14} from {11, 12, 13}, {11, 12, 14}, and {12,13,14} {11,13,14,15} from {11, 13, 14} and {11, 13, 15}
- Pruning: {11,13,14,15} is removed because {11,14,15} is not in L3
- $C4 = \{11,12,13,14\}$

## **Generating Association Rules**

- Once the frequent itemsets have been found, it is straightforward to generate strong association rules that satisfy:
  - → **minimum** support
  - → minimum confidence
- Relation between support and confidence:

$$support\_count(A \cup B)$$

$$Confidence(A \Rightarrow B) = P(B \mid A) = \frac{}{support\_count(A)}$$

- → Support\_count(A $\cup$ B) is the number of transactions containing the itemsets A $\cup$ B
- → Support\_count(A) is the number of transactions containing the itemset A.

## **Generating Association Rules**

- For each frequent itemset L, generate all non empty subsets of L
- For every non empty subset S of L, output the rule:

$$S \Rightarrow (L-S)$$

If (support\_count(L)/support\_count(S)) >= min\_conf

## **Example**

- → Suppose the frequent Itemset L={11,12,15}
- → Subsets of L are: {I1,I2},{I1,I5},{I2,I5},{I1},{I2},{I5}
- → Association rules:

$11 \land 12 \Rightarrow 15$	confidence = 2/4= 50%
I1 ∧ I5 ⇒ I2	confidence=2/2=100%
<b>I2</b> ∧ <b>I5</b> ⇒ <b>I1</b>	confidence=2/2=100%
I1 ⇒ I2 ∧ I5	confidence=2/6=33%
<b>I2</b> ⇒ <b>I</b> 1 ∧ <b>I</b> 5	confidence=2/7=29%
15 - 12 , 12	confidence=2/2=100%

If the minimum confidence =70%

#### **Transactional Database**

TID	List of item IDS	
T100	11,12,15	
T200	12,14	
T300	12,13	
T400	11,12,14	
T500	I1,I3	
T600	12,13	
T700	11,13	
T800	11,12,13,15	
T900	11,12,13	

 $L_2$ 

-		_
Itemset	Sup. count	C
{I1, I2}	4	
{I1, I3}	4	Ţ
{I1, I5}	2	1.
$\{12, 13\}$	4	. 1
{I2, I4}	2	į
{I2, I5}	2	

 $: L_{:}$ 

Itemset	Sup. count
{I1, I2, I3}	2
{11, 12, 15}	2

# Improving the Efficiency of Apriori

#### Major computational challenges

- Huge number of candidates
- Multiple scans of transaction database
- Tedious workload of support counting for candidates

#### Improving Apriori: general ideas

- → Hash-based technique
- → Transaction reduction
- → Partitioning
- → Sampling

## **Exercises**

A database has five transactions. Let min sup D 60% and min conf D 80%.

TID	items_bought		
T100	$\{M, O, N, K, E, Y\}$		
T200	{D, O, N, K, E, Y}		
T300	{M, A, K, E}		
T400	$\{M, U, C, K, Y\}$		
T500	{C, O, O, K, I, E}		

- Find all frequent itemsets using Apriori.
- List all the *strong association rules*.

# FP-growth: Frequent Pattern-Growth

- Adopts a divide and conquer strategy
- Compress the database representing frequent items into a frequent -pattern tree or FP-tree
  - → Retains the itemset association information.
- Divid the compressed database into a set of conditional databases, each associated with one frequent item

Mine each such databases separately

## **Example: FP-growth**

- The first scan of data is the same as Apriori
- Derive the set of frequent 1itemsets
- Let min-sup=2
- Generate a set of ordered items

Item ID	Support count
12	7
11	6
13	6
14	2
15	2

#### **Transactional Database**

TID	List of item IDS	
T100	11,12,15	
T200	12,14	
T300	12,13	
T400	11,12,14	
T500	11,13	
T600	12,13	
T700	11,13	
T800	11,12,13,15	
T900	11,12,13	

#### **Transactional Database**

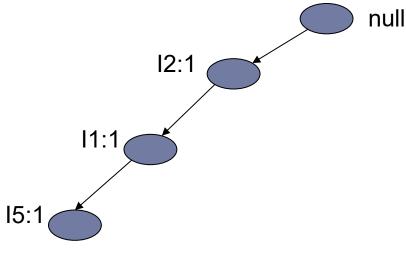
TID	Items	TID	Items	TID	Items
T100	11,12,15	T400	11,12,14	T700	11,13
T200	12,14	T500	11,13	T800	11,12,13,15
T300	12,13	T600	12,13	T900	11,12,13

- Create a branch for each transaction
- Items in each transaction are processed in order

Item ID	Support count
12	7
11	6
13	6
14	2
15	2

- 1- Order the items T100: {I2,I1,I5}
- **2-** Construct the first branch:

<|2:1>, <|1:1>,<|5:1>



#### **Transactional Database**

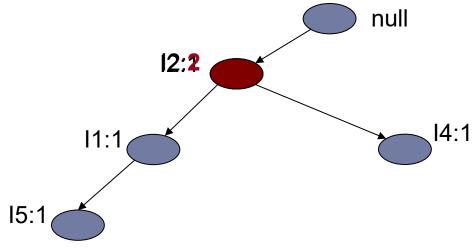
TID	Items	TID	Items	TID	Items
T100	11,12,15	T400	11,12,14	T700	11,13
T200	12,14	T500	11,13	T800	11,12,13,15
T300	12,13	T600	12,13	T900	11,12,13

- Create a branch for each transaction
- Items in each transaction are processed in order

Item ID	Support count
12	7
11	6
13	6
14	2
15	2

- 1- Order the items T200: {I2,I4}
- 2- Construct the second branch:

<12:1>, <14:1>



#### **Transactional Database**

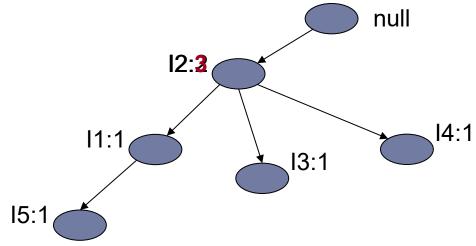
TID	Items	TID	Items	TID	Items
T100	11,12,15	T400	11,12,14	T700	11,13
T200	12,14	T500	11,13	T800	11,12,13,15
T300	12,13	T600	12,13	T900	11,12,13

- Create a branch for each transaction
- Items in each transaction are processed in order

Item ID	Support count
12	7
11	6
13	6
14	2
15	2

- 1- Order the items T300: {I2,I3}
- **2-** Construct the third branch:

<I2:2>, <I3:1>



#### **Transactional Database**

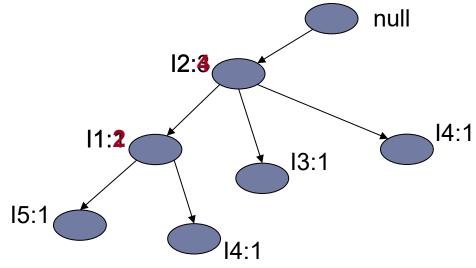
TID	Items	TID	Items	TID	Items
T100	11,12,15	T400	11,12,14	T700	11,13
T200	12,14	T500	11,13	T800	11,12,13,15
T300	12,13	T600	12,13	T900	11,12,13

- Create a branch for each transaction
- Items in each transaction are processed in order

Item ID	Support count
12	7
11	6
13	6
14	2
15	2

- 1- Order the items T400: {I2,I1,I4}
- **2-** Construct the fourth branch:

<|2:3>, <|1:1>,<|4:1>



#### **Transactional Database**

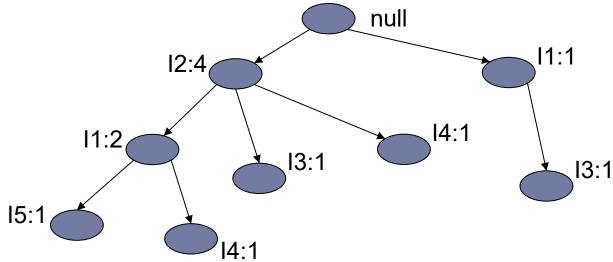
TID	Items	TID	Items	TID	Items
T100	11,12,15	T400	11,12,14	T700	11,13
T200	12,14	T500	11,13	T800	11,12,13,15
T300	12,13	T600	12,13	T900	11,12,13

- Create a branch for each transaction
- Items in each transaction are processed in order

Item ID	Support count
12	7
11	6
13	6
14	2
15	2

- 1- Order the items T400: {I1,I3}
- 2- Construct the fifth branch:

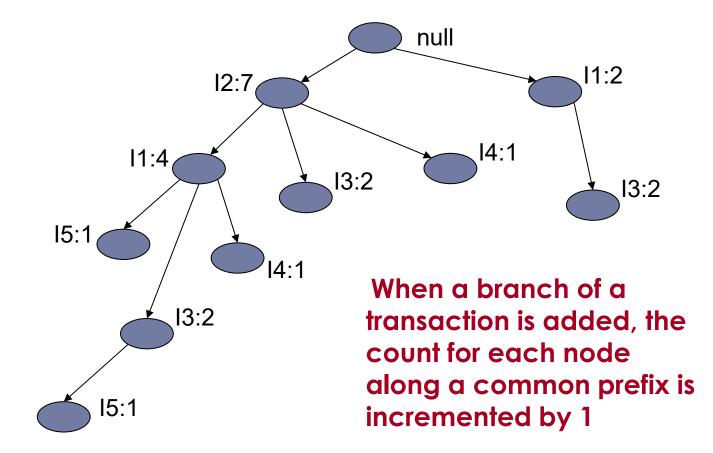
<l1:1>, <l3:1>

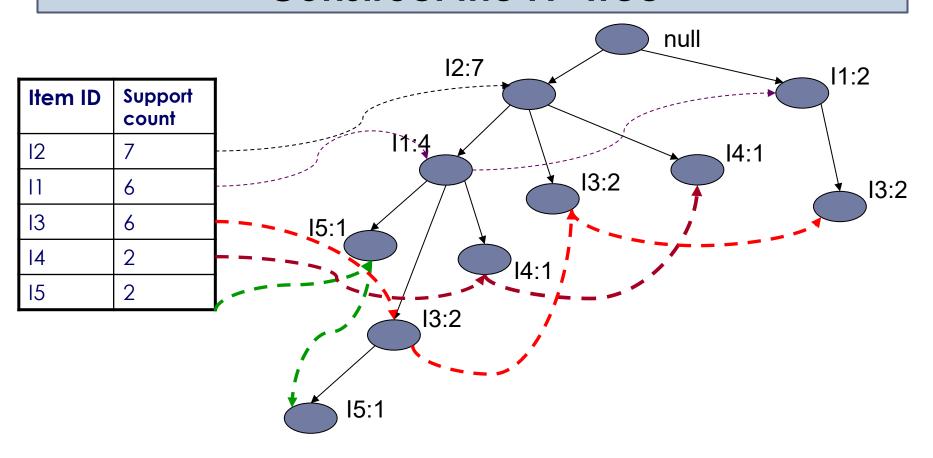


#### **Transactional Database**

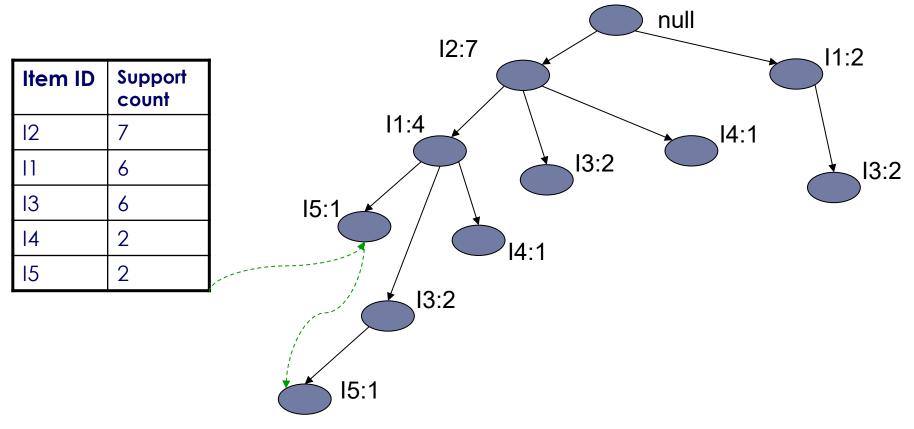
TID	Items	TID	Items	TID	Items
T100	11,12,15	T400	11,12,14	T700	11,13
T200	12,14	T500	11,13	T800	11,12,13,15
T300	12,13	T600	12,13	T900	11,12,13

Item ID	Support count
12	7
11	6
13	6
14	2
15	2



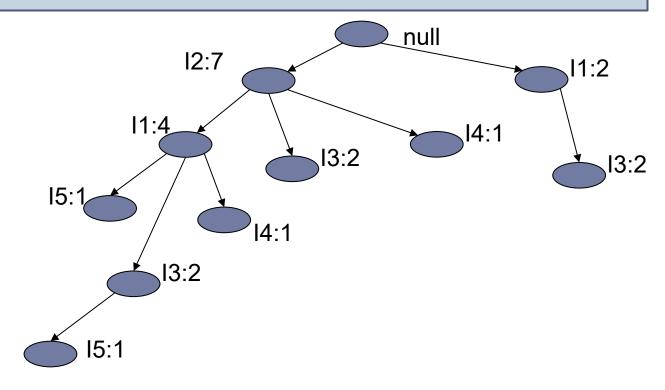


The problem of mining frequent patterns in databases is transformed to that of mining the FP-tree



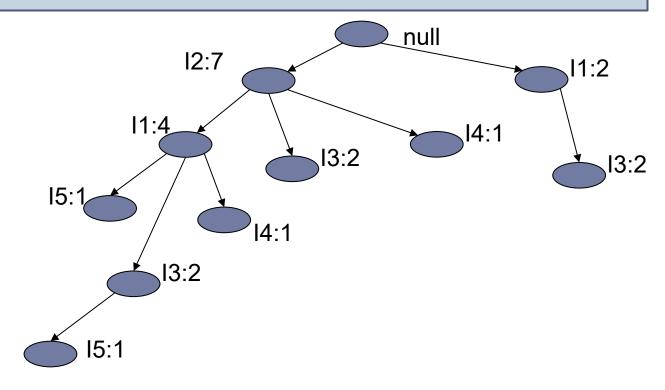
- -Occurrences of I5: <12,11,15> and <12,11,13,15>
- **-Two prefix Paths** <12, 11: 1> and <12,11,13: 1>
- -Conditional FP tree contains only <12: 2, 11: 2>, 13 is not considered because its support count of 1 is less than the minimum support count.
- **-Frequent patterns** {|2,|5:2}, {|1,|5:2},{|2,|1,|5:2}

Item ID	Support count
12	7
11	6
13	6
14	2
15	2



TID	Conditional Pattern Base	Conditional FP-tree
15	{{I2,I1:1},{I2,I1,I3:1}}	< 2:2, 1:2>
14	{{ 2, 1:1},{ 2,1}}	< 2:2>
13	{{ 2, 1: <mark>2</mark> },{ 2: <mark>2</mark> }, { 1: <mark>2</mark> }}	< 2:4, 1:2>, <b>&lt; 1:2&gt;</b>
11	{l2, <b>4</b> }	< 2:4>

Item ID	Support count
12	7
11	6
13	6
14	2
15	2



TID	Conditional FP-tree	Frequent Patterns Generated
15	<12:2,11:2>	{ 2, 5:2}, { 1, 5:2},{ 2, 1, 5:2}
14	<12:2>	{ 2, 4:2}
13	< 2:4, 1:2>, <b>&lt; 1:2&gt;</b>	{ 2, 3:4},{ 1, 3:4}, <b>{ 2, 1, 3:2}</b>
11	<12:4>	{12,11:4}

## **FP-growth properties**

- FP-growth transforms the problem of finding long frequent patterns to searching for shorter once recursively and concatenating the suffix
- It uses the least frequent suffix offering a good selectivity
- It reduces the search cost
- If the tree does not fit into main memory, partition the database
- Efficient and scalable for mining both long and short frequent patterns

## FP Mining with Vertical Data Format

Both Apriori and FP-growth use horizontal data format

TID	List of item IDS
T100	11,12,15
T200	12,14
T300	12,13
T400	11,12,14
T500	11,13
T600	12,13
T700	11,13
T800	11,12,13,15
T900	11,12,13

 Alternatively data can also be represented in vertical format

itemset	TID_set
11	{T100,T400,T500,T700,T800,T900}
12	{T100,T200,T300,T400,T600,T800,T900}
13	{T300,T500,T600,T700,T800,T900}
14	{T200,T400}
15	{T100,T800}

## **Example**

Transform the horizontally formatted data to the vertical format by scanning the database once

TID	List of item IDS			
T100	11,12,15			
T200	12,14		itemse	TID_set
T300	12,13		t	
T400	11,12,14	<b>│</b>	11	{T100,T400,T500,T700,T800,T900}
T500	11,13		12	{T100,T200,T300,T400,T600,T800,T900}
T600	12,13		13	{T300,T500,T600,T700,T800,T900}
T700	11,13		14	{T200,T400}
T800	11,12,13,15		15	{T100,T800}
T900	11,12,13			

The support count of an itemset is simply the length of the TID\_set of the itemset

## Example ...

#### Frequent 1-itemsets in vertical format

min\_sup=2

itemset	TID_set
11	{T100,T400,T500,T700,T800,T900}
12	{T100,T200,T300,T400,T600,T800,T900}
13	{T300,T500,T600,T700,T800,T900}
14	{T200,T400}
15	{T100,T800}

The frequent k-itemsets can be used to construct the candidate (k+1)-itemsets based on the Apriori property

#### Frequent 2-itemsets in vertical format

itemset	TID_set
{11,12}	{T100,T400,T800,T900}
{11,13}	{T500,T700,T800,T900}
{11,14}	{T400}
{11,15}	{T100,T800}
{12,13}	{T300,T600,T800,T900}
{12,14}	{T200,T400}
{12,15}	{T100,T800}
{13,15}	{T800}

## Example ...

#### Frequent 3-itemsets in vertical format

min\_sup=2

itemset	TID_set
{11,12,13}	{T800,T900}
{11,12,15}	{T100,T800}

- This process repeats, with k incremented by 1 each time, until no frequent items or no candidate itemsets can be found
- Properties of mining with vertical data format
  - → Take the advantage of the Apriori property in the generation of candidate (k+1)-itemset from k-itemsets
  - → No need to scan the database to find the support of (k+1) itemsets, for k>=1
  - → The TID\_set of each k-itemset carries the complete information required for counting such support
  - → The TID-sets can be quite long, hence expensive to manipulate

# Strong Rules Are Not Necessarily Interesting

- Whether a rule is interesting or not can be assessed either subjectively or objectively
- Objective interestingness measures can be used as one step toward the goal of finding interesting rules for the user
- Example of a misleading "strong" association rule
  - → Analyze transactions of AllElectronics data about computer games and videos
  - → Of the 10,000 transactions analyzed
    - 6,000 of the transactions include computer games
    - 7,500 of the transactions include videos
    - 4,000 of the transactions include both
  - → Suppose that min\_sup=30% and min\_confidence=60%
  - → The following association rule is discovered:

# Strong Rules Are Not Necessarily Interesting

Buys(X, "computer games")  $\Rightarrow$  buys(X, "videos")[support 40%, confidence=66%]

- This rule is strong but it is misleading
- The probability of purchasing videos is 75% which is even larger than 66%
- In fact computer games and videos are negatively associated because the purchase of one of these items actually decreases the likelihood of purchasing the other
- ▶ The confidence of a rule  $A \Rightarrow B$  can be deceiving
- → It is only an estimate of the conditional probability of itemset B given itemset A.
- → It does not measure the real strength of the correlation implication between A and B
- Need to use Correlation Analysis

## From Association to Correlation Analysis

A misleading "strong" association rule. Suppose we are interested in analyzing transactions at *AllElectronics* with respect to the purchase of computer games and videos. Let *game* refer to the transactions containing computer games, and *video* refer to those containing videos. Of the 10,000 transactions analyzed, the data show that 6000 of the customer transactions included computer games, while 7500 included videos, and 4000 included both computer games and videos. Suppose that a data mining program for discovering association rules is run on the data, using a minimum support of, say, 30% and a minimum confidence of 60%. The following association rule is discovered:

$$buys(X, "computer games") \Rightarrow buys(X, "videos")$$
  
 $[support = 40\%, confidence = 66\%].$  (6.6)

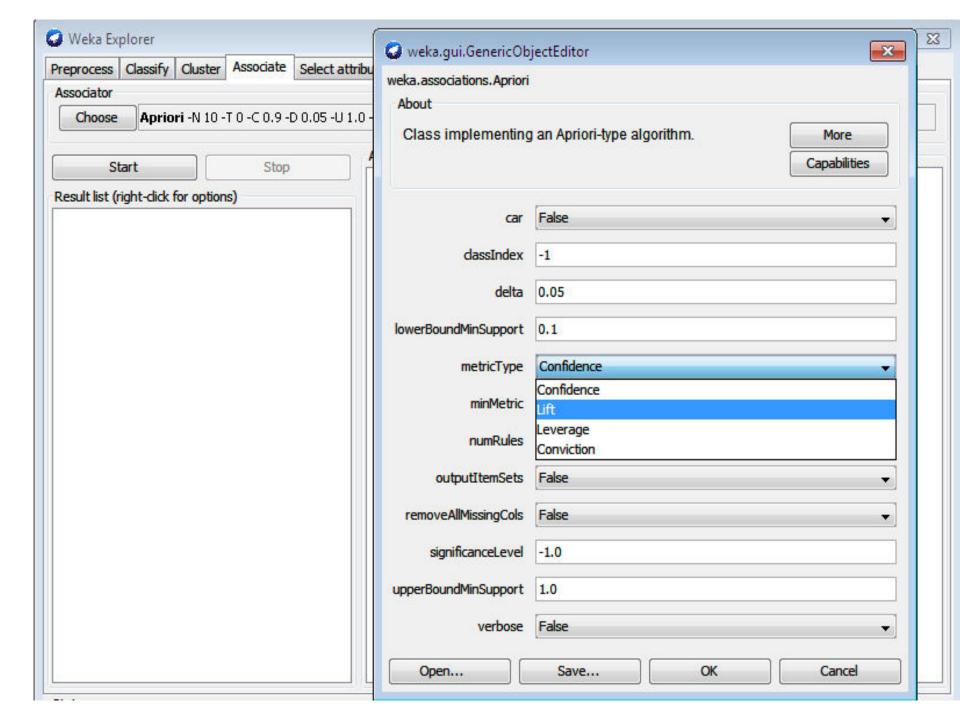
Rule (6.6) is a strong association rule and would therefore be reported, since its support value of  $\frac{4000}{10,000} = 40\%$  and confidence value of  $\frac{4000}{6000} = 66\%$  satisfy the minimum support and minimum confidence thresholds, respectively. However, Rule (6.6) is misleading because the probability of purchasing videos is 75%, which is even larger than 66%. In fact, computer games and videos are negatively associated because the purchase of one of these items actually decreases the likelihood of purchasing the other. Without fully understanding this phenomenon, we could easily make unwise business decisions based on Rule (6.6).

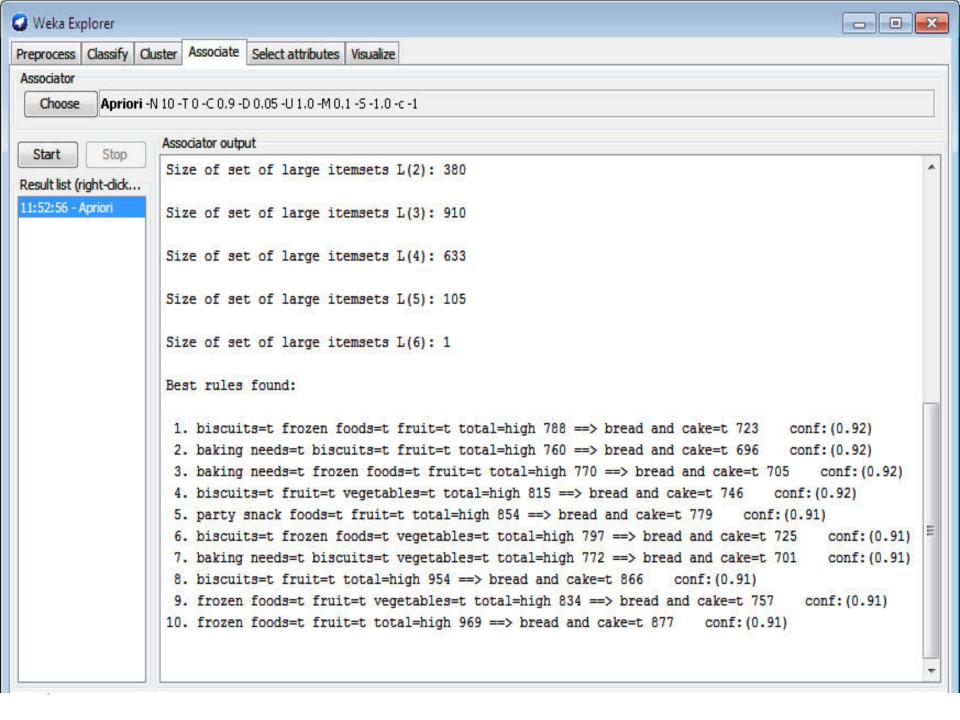
## From Association to Correlation Analysis

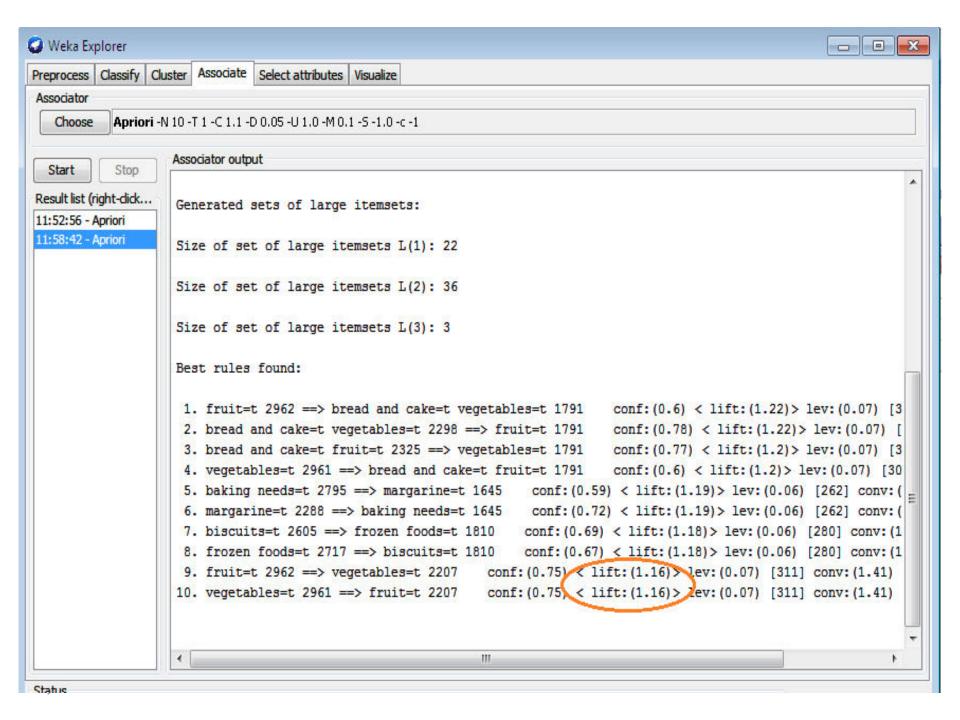
- Use Lift, a simple correlation measure
- The occurrence of itemset A is independent of the occurrence of itemset B if P(A∪B)=P(A)P(B), otherwise itemsets A and B are dependent and correlated as events
- ▶ The lift between the occurences of **A** and **B** is given by

$$Lift(A,B)=P(A\cup B)/P(A)P(B)$$

- → If > 1, then A and B are positively correlated (the occurrence of one implies the occurrence of the other)
- → If <1, then A and B are negatively correlated
- $\rightarrow$  If =1, then A and B are independent
- Example:  $P(\{game, video\})=0.4/(0.60 \times 0.75)=0.89$







## Summary

- Basic Concepts: association rules, support-confident framework, closed and max patterns
- Scalable frequent pattern mining methods
  - → Apriori (Candidate generation & test)
  - → Pattern-Growth Approach (FPgrowth)
  - → Vertical format approach
- Interesting Patterns
  - → Correlation analysis