

## Question 1:

The quality assurance checks on the previous batches of drugs found that — it is 4 times more likely that a drug is able to produce a satisfactory result than not.

Given a small sample of 10 drugs, you are required to find the theoretical probability that at most, 3 drugs are not able to do a satisfactory job.

a.) Propose the type of probability distribution that would accurately portray the above scenario, and list out the three conditions that this distribution follows.

b.) Calculate the required probability.

## Answer:

The batch size is 80,000

As it is 4 times more likely that a drug is able to produce a satisfactory result than not.

$$P(\text{satisfactory}) : 4/5 = 0.8$$

$$P(\text{non-satisfactory}) : 1/5 = 0.2$$

Given a small sample of 10 drugs, we are required to find the theoretical probability that at most, 3 drugs are not able to do a satisfactory job.

So in this case probability of acceptance  $P(a)$  is to find drugs which did not do a satisfactory job and that no is not more than 3

As this example satisfies the rules of Binomial distribution which are as below:

- 1. experiment has n repeated trials ( $n = 10$ )**
- 2. each trial is independent**
- 3. each trial has 2 possibilities (e.g. success or failure)**
- 4. probability of the success is same for each trial**

### **Checkpoint 1:**

So we will apply **binomial distribution** in this case.

Sample size  $n = 10$

Here, we want probability of non-satisfactory for  $P( X \leq 3 )$   
Which is :

$$P(a) = P( X \leq 3 ) = P(X = 0) + P(X = 1) + P(X = 2) + P(X = 3)$$

which will be calculated with formula =  $nCx * P^X * (1-P)^{n-X}$

So as per the below calculation the required probability for acceptance is 0.8791

**Checkpoint 2:**

probability for acceptance  $P(a) = 0.8791$

$$P(\text{satisfactory}) = 4/5 = 0.8$$

$$P(\text{non-satisfactory}) = 1/5 = 0.2$$

$$n = 10 \quad P(x \leq 3) = \sum_{x=0}^{3} {}^{10}C_x 0.2^x 0.8^{(10-x)}$$

$$= \frac{10!}{10! 0!} 0.2^0 0.8^{10} + \frac{10!}{9! 1!} 0.2^1 0.8^9 +$$

$$\frac{10!}{8! 2!} 0.2^2 0.8^8 + \frac{10!}{7! 3!} 0.2^3 0.8^7$$

$$= 0.1074 + 0.2684 + 0.3020 + 0.2013$$

$$P(x \leq 3) = 0.8791$$

## **Question 2:**

For the effectiveness test, a sample of 100 drugs was taken. The mean time of effect was 207 seconds, with the standard deviation coming to 65 seconds. Using this information, you are required to estimate the range in which the population mean might lie — with a 95% confidence level.

- a.) Discuss the main methodology using which you will approach this problem. State all the properties of the required method. Limit your answer to 150 words.
- b.) Find the required range.

## **Answer:**

Sample size **n = 100**

sample mean of  **$\bar{x} = 207 \text{ seconds}$**

sample standard deviation **s = 65 seconds**

we need to estimate range for the population mean with

**confidence Level = 95%**

We will use **central limit theorem** for this analysis as we have a **large sample size** which in turn states that the sample will follow the **normal distribution**.

As the sample size  $> 30$  and population standard deviation ( $\sigma$ ) is unknown we can use sample std. deviation to calculate the range.

The formula would be:  $\bar{x} \pm Z * s/\sqrt{n}$

We need to find the  $Z^*$  score from the % confidence level  
for 95% confidence level the  $Z^*$  score is  $\pm 1.96$

So the confidence interval will be

$$(207 - 1.96 * (65/\sqrt{100}), 207 + 1.96 * (65/\sqrt{100}) )$$

**Outcome:**

**At 95% confidence level, the Confidence interval range = (194.25, 219.74)**

**Question 3:**

- a) The painkiller drug needs to have a time of effect of at most 200 seconds to be considered as having done a satisfactory job. Given the same sample data (size, mean, and standard deviation) of the previous question, test the claim that the newer batch produces a satisfactory result and passes the quality assurance test. Utilize 2 hypothesis testing methods to make your decision. Take the significance level at 5 %. Clearly specify the hypotheses, the calculated test statistics, and the final decision that should be made for each method.
- b) You know that two types of errors can occur during hypothesis testing — namely Type-I and Type-II errors — whose probabilities are denoted by  $\alpha$  and  $\beta$  respectively. For the current sample conditions (sample size, mean, and standard deviation), the value of  $\alpha$  and  $\beta$  come out to be 0.05 and 0.45 respectively.

Now, a different sampling procedure (with different sample size, mean, and standard deviation) is proposed so that when the same hypothesis test is conducted, the values of  $\alpha$  and  $\beta$  are controlled at 0.15 each. Explain under what conditions would either method be more preferred than the other, i.e. give an example of a situation where conducting a hypothesis test having  $\alpha$  and  $\beta$  as 0.05 and 0.45 respectively would be preferred over having them both at 0.15. Similarly, give an example for the reverse scenario - a situation where conducting the hypothesis test with both  $\alpha$  and  $\beta$  values fixed at 0.15 would be preferred over having them at 0.05 and 0.45 respectively. Also, provide suitable reasons for your choice (Assume that only the values of  $\alpha$  and  $\beta$  as mentioned above are provided to you and no other information is available).

**Answer: (a)**

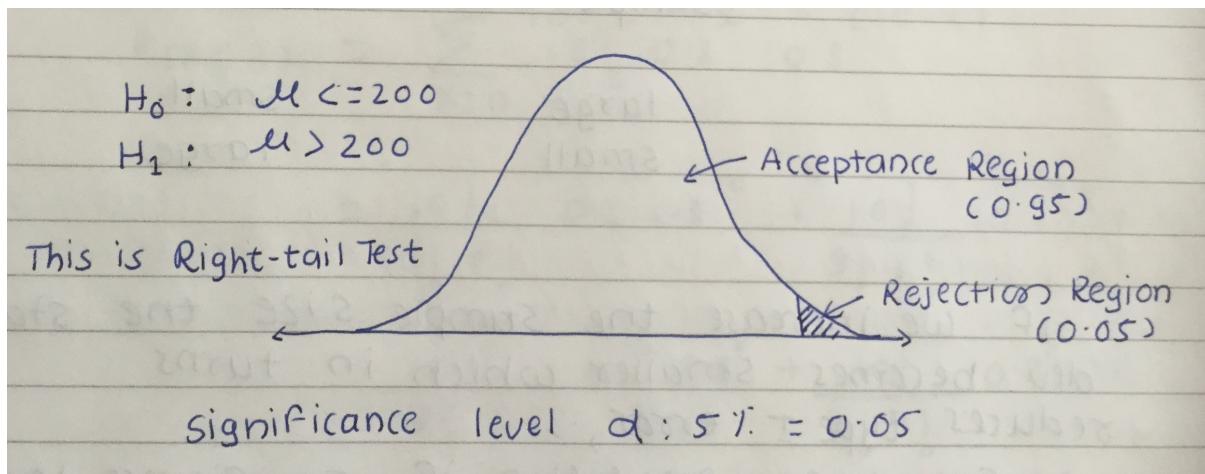
Sample size ,  $n = 100$

sample mean of  $\bar{x} = 207$  seconds

sample standard deviation  $s = 65$  seconds

Null Hypothesis  $H_0: \mu \leq 200$  seconds

Alternate Hypothesis  $H_1: \mu > 200$  seconds



### Method 1: Critical value Method

In this case we need to find the Upper Critical Value (UCV) with significance level of :  $\alpha : 5\%$

$$\text{Formula for UCV} = \mu + (Z_c * (s/\sqrt{n}))$$

As this is right tail test

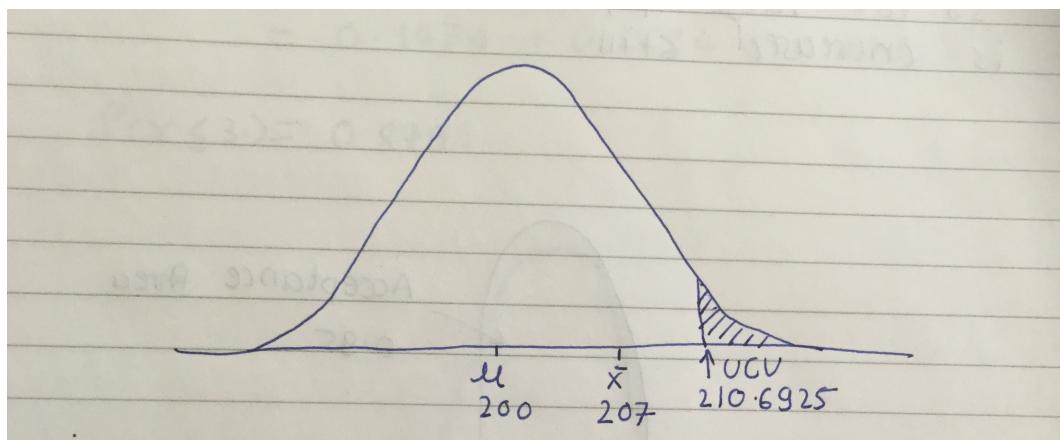
From the Z-table we found the critical Z value for  $1 - \alpha = 0.95$

$$Z_c = 1.645$$

Calculation of UCV

$$\begin{aligned}
 &= 200 + 1.645 * 65/\sqrt{100} \\
 &= 200 + 1.654 * 6.5 \\
 &= 200 + 10.6925
 \end{aligned}$$

$$\text{UCV} = 210.6925$$



As sample mean is less than the UCV, we fail to Reject the null hypothesis

**As the sample mean (207 seconds) is less than the UCV 210.6925 (seconds) we fail to reject the null hypothesis**

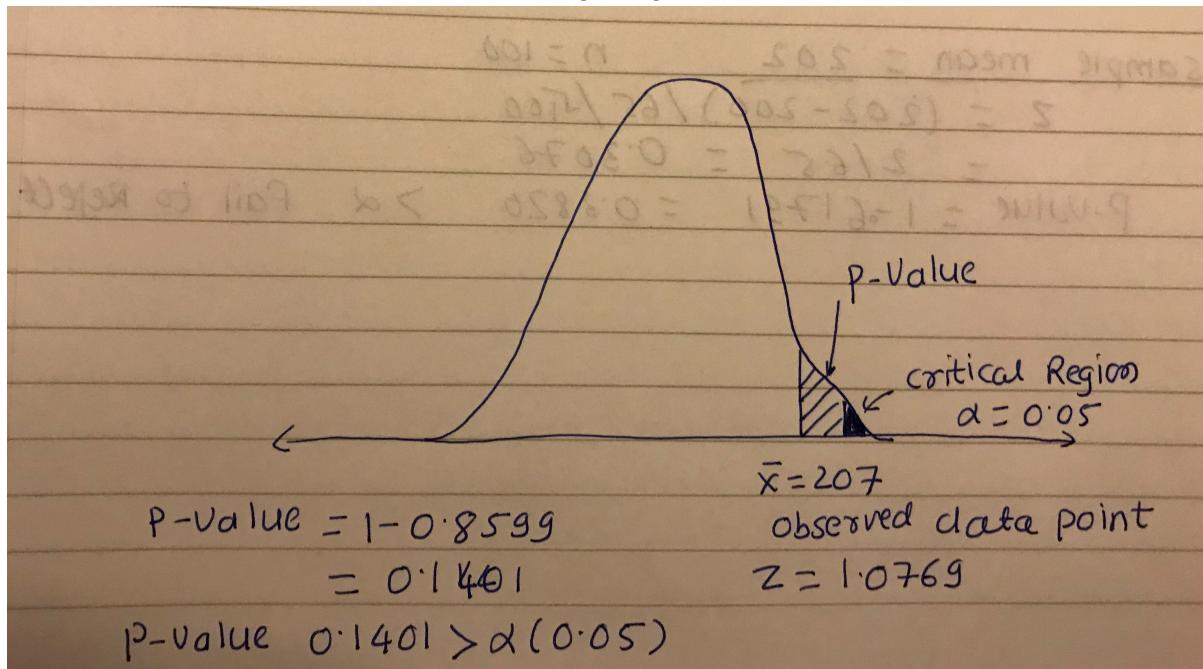
### **Method 2: P - value Method**

significance level :  $\alpha : 5\% = 0.05$

We need to find the Z-score

$$\begin{aligned} \text{Formula: } Z &= (\bar{x} - \mu) / (s / \sqrt{N}) \\ &= (207 - 200) / (65 / \sqrt{100}) \\ &= 7/6.5 \\ Z &= 1.0769 \end{aligned}$$

For above Z-Score, we find the corresponding area from the Z table  
As this is right tail test: P-value =  $1 - 0.8599$   
 $= 0.1401$



**As the P-value (0.1401) >  $\alpha$  (0.05), we fail to reject the Null Hypothesis.**

**Conclusion: As both critical value method and P-Value method confirm failure to reject the null hypothesis, we need to confirm that the Quality Assurance test will pass.**

**(b)** We have 2 experiments with  $\alpha$  and the  $\beta$  values as below

**alpha ( $\alpha$ )**: Probability marking type I error

**beta ( $\beta$ )**: Probability of making a type II error

	$H_0 = \text{true}$	$H_1 = \text{true}$
$H_0$ true	Correct decision	Type II error
$H_1$ true	Type I error	Correct decision

	$\alpha$	$\beta$
Method 1	0.05	0.45
Method 2	0.15	0.15

As our hypothesis remains the same, only the sample data changes such as (different sample size, mean, and standard deviation) , we will need to check conditions when method 1 is suitable and cases where method 2 will be preferred.

**As we don't have any additional information such as sample size, sample mean and sample std. deviation, we cannot user these parameters for our analysis.**

So we will need to user the generic features of the drug which can help us to make a wise decision about selection of the values of  $\alpha$  and  $\beta$  .

Below selection methods tell which one of the two methods will be a suitable and preferred method if our sampling procedure changes.

Null Hypothesis  **$H_0 : \mu \leq 200 \text{ seconds}$**

Alternate Hypothesis  **$H_1 : \mu > 200 \text{ seconds}$**

So the type I & II errors are as below:

type 1 error – confirming  **$\mu > 200 \text{ seconds}$**  even though it is less than 200

type 2 error – confirming  **$\mu \leq 200 \text{ seconds}$**  even though it is greater than 200

**Preferred Method 1:  $\alpha = 0.05$  and  $\beta = 0.45$**

If the painkiller is over the **counter painkiller** with **negligible side-effects**,

We can select these values of  $\alpha$  and  $\beta$  even though type II error is high. As due to type II error, the overall impact won't have serious effects on consumer health.

Also rejection of lot due to type I error will be low.

The second method  $\alpha = 0.15$  and  $\beta = 0.15$ , though it reduces type II error will increase type I error, which might result in false positive conclusion resulting in higher rejection of drug batches.  
as this type of medicine might not need highly effective mean time. We can select Method 1 parameters.

### **Preferred Method 2: : $\alpha = 0.15$ and $\beta = 0.15$**

If the painkiller is being used in the **surgery which needs very accurate and effective pain relief** Or if the drug has **serious side effects**.

Then using  $\alpha = 0.05$  and  $\beta = 0.45$  can have **serious effects**, It might result extreme pain for the consumer without any relief for long time. Also this can result in regulatory actions/lawsuits against company. So company's reputation is at stake in this case.

So in this case true  **$H_0$**  rejection is more safe than not rejecting false  **$H_0$** .

So it is preferred to use  $\alpha = 0.15$  and  $\beta = 0.15$  as this will reduce probability of the Type error which can have serious implications.

## **Question 4:**

Now, once the batch has passed all the quality tests and is ready to be launched in the market, the marketing team needs to plan an effective online ad campaign to attract new customers. Two taglines were proposed

for the campaign, and the team is currently divided on which option to use.

Explain why and how A/B testing can be used to decide which option is more effective. Give a stepwise procedure for the test that needs to be conducted.

## **Answer:**

A/B testing can be used to measure the effectiveness of both the taglines and check which tagline performs better. A/B testing compares the performance of the two taglines and show which tagline is more effective to attract new customers.

A/B testing will split the targeted audience and each group will be shown separate tag line. Based on the number of the customers click on the advert, we will be measuring the effectiveness of each tagline..

Below is the procedure for conducting A/B Testing which we will be following to conduct the A/B test to find the effectiveness of the taglines:

- a. **Objective of the Test:** We are conducting this test to check effectiveness of the two ad complain taglines to attract new customers on an online ad campaign

- b. **Create a Hypothesis:**

Null Hypothesis  $H_0$ : tagline 1 will be more effective than tagline 2

Alternate Hypothesis  $H_1$ : tagline 2 will be more effective than tagline 1

For this test we can use significance level of 5% in absence of any specified significance level

- c. **Identify the targeted customers:**

In this step we will identify the sample customers for our A/B test. We can use random sampling to select random customers or stratified sampling technique to target specific and proportional customers.

- d. **Create control and variable group.**

In this case we can take tagline 1 as control group and tagline 2 as variable group

- e. **Perform the A/B test:**

Once we have identified the targeted customers , we can start the experiment to collect the data.

Running the test parallelly might create some issues about collection data of the conversion rate.

So if we can place a technique in place to track proportional reach to targeted customers for both taglines we can do run the test in parallel or we can run the test once at a time.

**f. Collect the data:**

Collect the data of success, e.g. if the customer click on the advert, the effectiveness of the tagline = 1 else 0

**g. Analyse the data**

Based on the collected data we can measure frequency and perform the 2 sample test to test our hypothesis.

If we have higher p value than the significance level 0.05, we will fail to reject the null hypothesis and can decide accordingly that tagline 1 is more effective than tagline 2.

**h. Repeat the test**

We can continue this experience for additional time to increase the effectiveness of the A/B test.

With the A/B testing we can check the result and the results can be more effective if we can run this test for longer duration so that we have large no of participating customer which can increase our confidence in this test.