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ME314 Project – Spinning Top

Brief description of project proposal:

a. Developing a mathematical model to simulate behavior of spinning top
b. Deducing the precession[1] and nutation[2] behavior of the top using the model
c. Generating a 3D graphics model
d. Animating the 3D model to illustrate behaviors of the spinning top

In the proposal, it was mentioned that energy method would be used to deduce precession and nutation behavior. However, the use of Euler-Lagrange method and the structure of the code led a to different approach. In fact, changing some of configuration states allowed modeling of precession and nutation behavior.

Drawing of System:

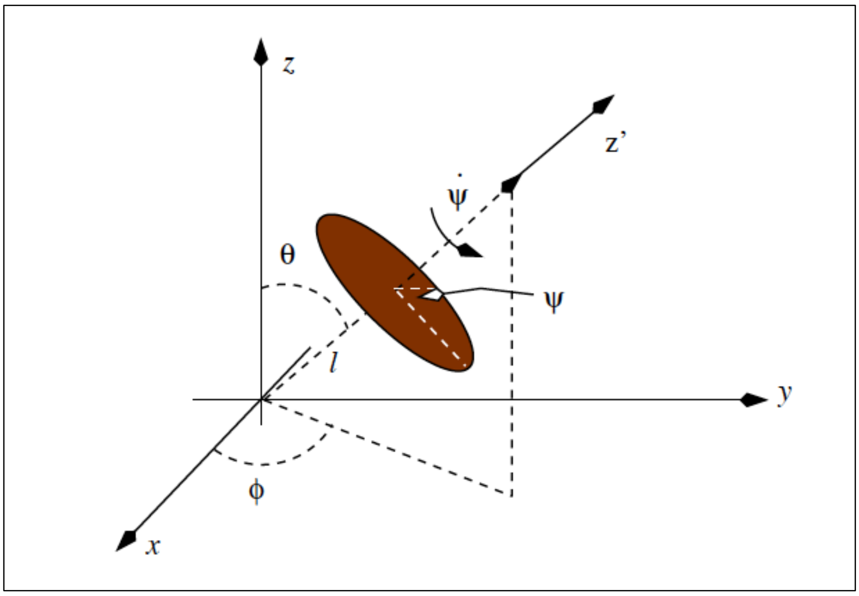

Figure 1: 'Wheel top' with configuration variables

Explanation:

- A polar coordinate system is chosen
- 3 configuration variables are required (these are the 3 Euler angles) as shown in Fig. 1 (Ψ, ϕ and θ)
- Transformation from the base frame to the 'body' frame is obtained using Euler's rotation theorem (adapted from *Wolfram MathWorld*) [1]
- The angular velocities (in all 3 directions) are determined from *Wolfram MathWorld* website [1]
- The required moment of inertia are calculated, one for the thin wheel and the other for the entire system
- Kinetic energies and potential energies are computed to in turn formulate the Lagrange equation

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[1] Precession: refers to revolution of top about vertical axis
[2] Nutation: refers to changes in lean angle of top (bobbing up and down)

- Continuing with the Euler Lagrange theorem, ordinary differential equations are numerically solved to simulate the model (The 'NDSolve' is generated as a function that takes in different precession and nutation values as input parameters)

Results: *(Code take about **10 seconds** to run – all animations are significantly better when run in Mathematica than in .mov files)*

3 types of simulations are produced to study how the top behaves (its precession and nutation behavior) when certain parameters are altered: *(Animations for each case are uploaded on Canvas)*

a) Initial precession rate is zero ($\dot{\phi}_0 = 0$)
As it can be observed from Fig. 3a , the top oscillates between the start lean angle (θ) of $\pi/4$ and 1.3 radians. The pointed ends in the graph disappear when a slight 'push' is given to the top initially ($\dot{\phi}_0 > 0$).

This behavior is expected and can be explained qualitatively – The top, when released from rest with an initial lean angle, initially starts to drop under gravity. That is, lean angle θ, increases. However, as θ starts to increase, $\dot{\phi}_0$ must increase to keep p_ϕ remains constant (Equation is shown in Mathematica code).

b) Initial precession rate is greater than zero ($\dot{\phi}_0 > 0$)
A smooth trajectory and motion of top is obtained when a slight 'push' is given to the top (Fig. 3b). The trajectory is almost sinusoidal. In practice, the wobbles die down due to frictional losses and a 'perfect' spinning top with precessional movement is obtained.

It is also determined that the precession rate is inversely proportional to the top's spin rate as seen in Fig. 2. This explains the fact why precession tends to be more pronounced as the top slows down.

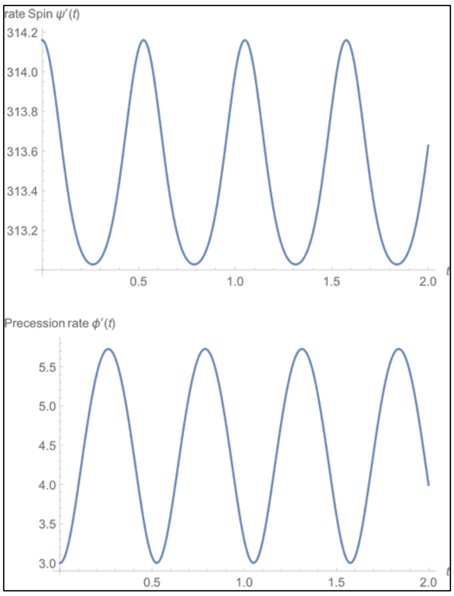

Figure 2: Plots of spin rate against time and precession rate against time

c) <u>Initial precession rate is less than zero ($\dot{\phi}_0 < 0$)</u>
A 'curly' trajectory is obtained when the precession rate is negative (Fig. 3c). This is because the velocity periodically goes to negative as the top spins.

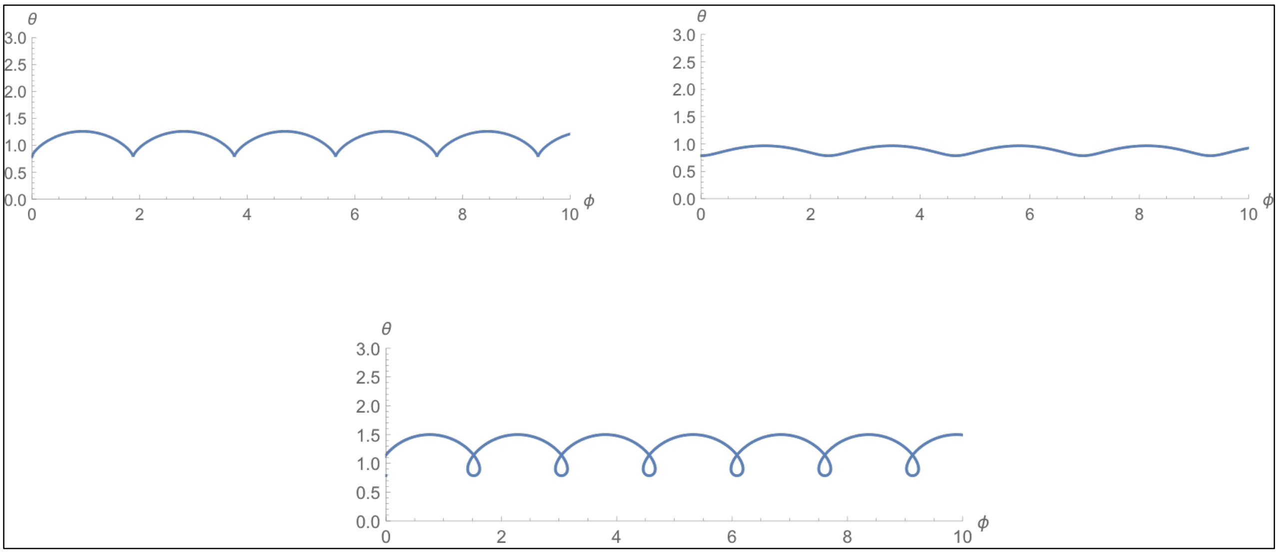

Figure 3: (Clockwise from top left)
(a) Trajectory when $\dot{\phi}_0 = 0$; (b) Trajectory when $\dot{\phi}_0 > 0$; (c) Trajectory when $\dot{\phi}_0 < 0$

References:

1. Euler Angles: http://mathworld.wolfram.com/EulerAngles.html