Linear Algebra

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1 Introduction

This document is intended to summarize the basic concepts of linear algebra used throughout theoretical computer science.

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2 Notation

Notation	Meaning
$\mathbb{R}$	the set of real numbers
$\in$	in
$\forall$	for all

2.1 Other Notes

- We will not present proofs for most of the theorems mentioned in this document. Instead, it is your responsibility to either derive or look up the proof to understand why a specific theorem holds.

3 Vectors

3.1 Vector Basics

A vector x can be defined as a collection of elements $x_1, x_2, \ldots, x_n$, arranged in a column or in a row. Unless otherwise specified, a vector is thought to be in column format:

$$\begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix}.$$

Element x_i is considered to be the i-th element of vector x and the **dimension** of x, the number of elements in x, is most commonly designated by the number n. If all the elements of x are real numbers, i.e. , $x_i \in \mathbb{R}\ \forall i$, then x is considered to be a **real vector of** dimension n, which we indicate with the notation $x \in \mathbb{R}^n$. On the other hand, if x contains complex numbers, then it is a **complex vector**, i.e., $x \in \mathbb{C}^n$. In these notes, we shall only deal with real vectors.

To switch a vector from column to row format and vice versa, we take the **transpose** of the vector, denoted by the superscript $^{\mathrm{T}}$:

$$\begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix}^{\mathrm{T}} = \begin{bmatrix} x_1 & x_2 & \ldots & x_n \end{bmatrix}; \qquad x^{\mathrm{TT}} = x.$$

A vector in $\mathbb{R}^n$ can be viewed as a point in that space, where the Cartesian coordinates are the components x_i.