Mathematics for Management Concept Summary

Algebra

Solving Linear Equations in One Variable

Manipulate the equation using Rule 1 so that all the terms involving the variable (call it x) are on one side of the equation and all constants are on the other side. Then use Rule 2 to solve for x.

Rule 1: Adding the same quantity to both sides of an equation does not change the set of solutions to that equation.

Rule 2: Multiplying or dividing both sides of an equation by the same nonzero number does not change the set of solutions to that equation.

Straight Lines: Slope Intercept Form

A straight line with slope m and y-intercept (b, 0) has the equation $y = mx + b$.

Point Slope Form of a Line Equation

Given two points on a line, (x_0, y_0) and (x_1, y_1), find the line's slope m $= \frac{y_1 - y_0}{x_1 - x_0}$.

Then the equation of the line may be written as $y - y_0 = m(x - x_0)$.

Solving Two Linear Equations

Two linear equations in two variables (call them x and y) have no solution, an infinite number of solutions, or a unique solution. You may solve two linear equations by either substitution or elimination.

- **Substitution**: Use one equation to solve for one variable in terms of the other (say, x in terms of y). Then substitute this relationship for each occurrence of x in the remaining equation. Now solve the remaining equation for y. Given that you know x in terms of y, you also know y.

- **Elimination:** Add a multiple of one equation to eliminate a variable (say, x) from the other equation. Solve the resulting equation for the remaining variable (y). Substitute this value of y in either of the original equations to find x.

Linear Inequalities: One Variable

Use the following rules to solve for the set of values satisfying a linear inequality.

- If you add the same number to both sides of an inequality, the resulting inequality has the same direction as the original inequality. For example, if the original inequality were a "less than" inequality, the resulting inequality would also be a "less than" inequality.

- If you multiply or divide both sides of an inequality by the same positive number, the resulting inequality has the same direction as the original inequality. For example, if the original inequality were a "less than" inequality, the resulting inequality would also be a "less than" inequality.
- If you multiply or divide both sides of an inequality by the same negative number, the resulting inequality has the opposite direction as the original inequality. For example, if the original inequality were a "less than" inequality, the resulting inequality would be a "greater than" inequality .

Linear Inequalities: Two Variables

Change the inequality sign to an equals sign and graph the resulting line. The set of points satisfying the linear inequality will be all points on one side of the line (including the line). To find the side of the line that satisfies the inequality, simply choose a point (call it P) not on the line. If the point satisfies the inequality, then shade all points on P's side of the line; otherwise, shade all points not on the same side of the line as P.

Parabolas

The graph of $y = ax^2 + bx + c$ is called a **parabola**. If a < 0 , as x increases to the value –b/2a, the value of y will increase, and then decrease. If a > 0, as x increases to –b/2a, the value of y will decrease, and then increase. Any parabola is symmetric about the line x = –b/2a. That is, if x is k times larger than –b/2a and k times smaller than –b/2a, the function has the same value for both these values of x.

The Quadratic Formula

To solve for the values of x satisfying $ax^2 + bx + c = 0$, substitute a, b, and c into the following equation, called the **quadratic formula**:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}.$$

Exponents

An **exponent** is shorthand for repeated multiplication. For example, $2^4 = 2 \times 2 \times 2 \times 2$. The following rules apply to exponents:

Rule 1: $x^0 = 1$

Rule 2: When multiplying like terms involving exponents, add the exponents. That is, $x^a \times x^b = x^{a+b}$.

Rule 3: $(xy)^a = x^a \times y^a$.

Rule 4: $(x^a)^b = x^{ab}$.

Rule 5: $(x/y)^a = \frac{x^a}{y^a}$.

Rule 6: $x^{-a} = \frac{1}{x^a}$.

Rule 7: $\frac{x^a}{x^b} = x^{a-b}$.

Power Function

The function $y = ax^b$ is called a **power function**. Usually in business, $a > 0$ and $x > 0$. Then for $b > 1$, as x increases, y increases and the graph gets steeper. For $0 < b < 1$, as x increases, y increases but the graph gets flatter. For $b < 0$, as x increases, y decreases and the graph gets flatter.

Cobb Douglas Function

If K = capital, L = labor, and $0 < a < 1$, the output of an organization or economy is often modeled by the Cobb-Douglas production function, which is of the form $f(K,L) = K^a L^{1-a}$.

Order of Operations

In the evaluation of mathematical expressions, the order of operations is as follows:

- P (Parentheses): If the expression contains parentheses, first evaluate all expressions within parentheses, working from the innermost set of parentheses out.
- E (Exponents): Next, perform all operations involving exponents.
- MD (Multiplication and Division): Next, perform all multiplication and division calculations from left to right.
- AS (Addition and Subtraction): Finally, perform all addition and subtraction calculations from left to right.

This hierarchy is easily remembered by using the pneumonic device PEMDAS, or Please Excuse My Dear Aunt Sally.

Entering and Graphing Functions in Excel

- All Excel formulas begin with an = sign.
- Use the ^ symbol to raise a number to a power and * for multiplication.
- Excel follows PEMDAS.

- Use the Scatter option from the Insert tab of the Excel ribbon to graph a function.

Inverse Function

If $y = f(x)$, the **inverse function** of f (call it g) is found by solving for x in terms of y. The expression is often written as $x = g(y)$.

Ratio

The **ratio** of two numbers reflects their relative sizes. For example, if you want to divide an inheritance between two siblings in the ratio 2:3, the first sibling will get 2/3 as much as the second sibling or, equivalently, the second sibling will get 3/2 as much as the first sibling.

Percentage

A **percentage** is simply mathematical shorthand for one hundredth. For example, if a bookstore marks up the price of a book 40% over the wholesale price, you can compute the retail price by adding forty hundredths, or 0.4 times, the wholesale price to the wholesale price.

Elasticity of Demand

The demand **elasticity** (E) for a product is the percentage change in demand that results from a 1% increase in the product's price. If $E < -1$, the demand is elastic; if $-1 < E < 0$, the demand is inelastic.

Logarithm

- Assuming a positive number b, the logarithm of a number x to the base b is the power to which b must be raised to result in the number x.
- If you write $\text{Log}_b\, x = c$, then $b^c = x$.
- When b = e (e is approximately 2.7182), write Ln x instead of $\text{Log}_e\, x$.
- The Excel function LOG(x,b) returns $\text{Log}_b\, x$.
- The following rules apply to logarithms:

  Rule 1: $\text{Log}_b\, x + \text{Log}_b\, y = \text{Log}_b\, x*y$.

  Rule 2: $\text{Log}_b\, x - \text{Log}_b\, y = \text{Log}_b\, x/y$.

  Rule 3: $\text{Log}_b\, x^c = c*\text{Log}_b\, x$.

Index Numbers

An **index number** indicates the percentage change in a quantity, relative to a base level that is assigned a value of 100. For example, suppose that the base year for GNP is 2000 and that the GNP in 2000 is $4 trillion. If the GNP in 2008 is $6 trillion, the 2008 GNP index is 150.

Calculus

Derivatives and Rules for Finding Derivatives

The derivative of a function for a value of *x* (written as *f*'(x) or $\frac{dy}{dx}$) is the slope of the function for that value of *x*. The following rules make it easy to find the slope of a curve for a variety of functions.

Rule 1: If *f*(*x*) = k, where k is a constant, then $\frac{dy}{dx}$=0.

Rule 2: If *f*(*x*) = x^n, where n is any number, then $\frac{dy}{dx}$=nx^{n-1}.

Rule 3: If *f*(*x*) = k*g*(*x*), where k is a constant, then y' = k*g*'(*x*), where *g*'(*x*) is the derivative of *g*(*x*).

Rule 4: If *f*(*x*) can be written as the sum of two functions, *g*(*x*) and *h*(*x*), then *f*'(*x*) = *g*'(*x*) + *h*'(*x*).

Second Derivatives, Convex Functions, and Concave Functions

- The **second derivative** of a function (written as *f*''(*x*) or $\frac{d^2y}{dx^2}$.) is simply the derivative of the function's first derivative.
- If the second derivative is less than or equal to 0 for a value of *x*, then *f*(*x*) is **concave** at *x*.
- If the second derivative is greater than or equal to 0 for a value of *x*, then *f*(*x*) is **convex** at *x*.
- If *f*''(*x*) ≤ 0 for all *x*, then *f*(*x*) is a **concave function**.
- If *f*''(*x*) ≥ 0 for all *x*, then *f*(*x*) is a **convex function**.

Maximizing or Minimizing a Function

- A concave function is maximized for any value of *x* where *f*'(*x*) = 0.
- A convex function is minimized for any value of *x* where *f*''(*x*) = 0.

Inflection Points

An **inflection point** for a function is a point where the function changes from convex to concave or convex to concave. To find an inflection point, find an *x* such that *f*''(x) = 0 and check that the function changes from convex to concave or concave to convex at *x*.

Statistics

Histograms

To summarize data with a **histogram**, begin by dividing the possible data values into 5 to 10 bin ranges and determine how many data points fall into each bin. The histogram option of the Excel Data Analysis Toolpak makes it easy to create a histogram.

Measures of Central Location

The **mean, median,** and **mode** are the primary measures used to summarize the typical value or central location for a data set.

- The **mean** is just the average of the numbers in a data set.
- Roughly speaking, the **median** is the point in a data set where half the observations are less than the median and half the observations are more than the median. If the data set consists of an even number of data points, the median is the average of the two middle observations. If the data set consists of an odd number of data points, the median is the middle observation.
- The **mode** is the most frequently occurring value in a data set.
- Excel's AVERAGE, MEDIAN, and MODE functions can be used to compute measures of central location.

Skewness

If a data set exhibits a lot of skewness, then the median is a better measure of central location than the mean. Otherwise, the mean is a better measure of central location.

Measures of Variability

The sample variance and sample standard deviation are measures of a data set's spread about the mean or variability. Given data points x_1, $x_{2, \ldots}$ x_n, the sample variance of the data set (written as S^2) is defined as

$S^2 = \frac{1}{n-1}\sum_{i=1}^{i=n}(x_i - \bar{x})^2$. The sample standard deviation is the square root of the sample variance. Excel's VAR function finds the sample variance, and Excel's STDEV function finds the sample standard deviation.

Rule of Thumb and Outliers

When a data set has a fairly symmetric histogram, approximately 68% of the data is within one standard deviation of the mean and approximately 95% of the data is within two standard deviations of the mean. Any data point that is more than 2 standard deviations away from the mean is called an **outlier**.

Covariance and Correlation

Covariance and correlation are measures of linear association between two sets of data X and Y consisting of the points (x_1, y_1), (x_2, y_2)… (x_n, y_n).. The sample covariance between data sets X and Y is given by $\frac{\sum_{i=1}^{i=n}(x_i - \bar{x})(y_i - \bar{y})}{(n-1)}$. The sample correlation between two data sets X and Y is given by $\frac{Covariance\ (X,Y)}{S_X S_Y}$, where S_X and S_Y are the standard deviations of the data sets. Correlation is a unit-free measure of linear association, so it is used more often than covariance.

Probability

Probability of Two Events

Given two events E_1 and E_2, P(E_1 or E_2) = P(E_1) + P(E_2) – P(E_1 and E_2).

Probability of Complementary Events

For a given event E, the probability of the **complementary event** ($\overline{\mathrm{E}}$) may be found using the formula P*($\overline{\overline{\mathrm{E}}}$)* P($\overline{\mathrm{E}}$) = 1 – P(E)

Conditional Probability

Given two events A and B, the **conditional probability** that A will occur, given that you know that B has occurred, may be computed from P(A|B)=$\frac{\mathrm{P(A\ and\ B)}}{\mathrm{P(B)}}$.)

Joint Probabilities

Given two events A and B, the **joint probability** that events A and B both occur can be computed from P(A and B) = P(B)P(A|B).

Independent Events

Two events A and B are **independent events** if knowledge that one event has occurred does not change the probability that the other event has occurred. If a set of events is independent, you can find the joint probability that a subset of the independent events occurs simply by multiplying the probabilities of the individual events.

Random Variables

A **random variable** is a function that associates a value with every point in an experiment's sample space. For a **discrete random variable**, assuming values x_i with probability p_i (i = 1, 2, …, n), the expected value of the random variable equals $\sum_{i=1}^{i=n} x_i p_i$. The variance of a discrete random variable may be computed as $\sum_{i=1}^{i=n}(x_i - E(\boldsymbol{X}))^2$.

Continuous Random Variables and Probability Density Functions

Every **continuous random variable** has a **probability density function (PDF).** Probabilities for a continuous random variable correspond to an area under the continuous random variable's PDF.

Normal Random Variable

The **normal random variable** is used to model many uncertain quantities such as height, weight, and monthly demand for a product. A normal random variable is specified by two parameters: the mean and the standard deviation. Excel's NORMDIST function makes it easy to compute probabilities involving the normal random variable.

Finance

Net Present Value (NPV)

If a cash flow of c_t is received at time t, (t = 0, 1, 2 … n), and cash flows are discounted at a rate r per period, the value in today's dollars of the cash flows (called Net Present Value) is expressed as

$\sum_{t=0}^{t=n} \frac{c_t}{(1+r)^t}$. Excel's NPV function can be used to compute NPVs. Remember that the Excel NPV function assumes that the first cash flow is received one period from now, not today.

Internal Rate of Return (IRR)

The **IRR** of a stream of cash flows is the interest rate that makes the NPV of a sequence of cash flows equal to 0. In most cases, a sequence of cash flows will have a unique IRR. In rare cases, however, a sequence of cash flows will have multiple IRRs or no IRR. In those rare cases, it is difficult or impossible to glean any useful insights using IRR. Excel's IRR function can be used to compute IRRs.

Payback Period

An investment's **payback period** is the length of time needed to pay back the investment's initial cash outflow.

Future Value

The value of a cash flow brought forward to a future point in time is called the **future value**. If the rate of return is r per period, then $1 today has a future value t periods from now of $(1 + r)^t$.

Annuity

An **annuity** is a stream of equal cash flows (say, $C) received at times 1, 2… n. If the interest rate is r, the present value of an annuity may be found using the formula

$C[\frac{1}{r} - \frac{1}{r(1+r)^N}]$ The present value of an annuity may also be found using Excel's PV function.

Perpetuity

A **perpetuity** is an infinite stream of equal cash flows (say, $C) received at times 1, 2… n. If the interest rate is r, the present value of the perpetuity is C/r.

Growing Perpetuity

A **growing perpetuity** is an infinite stream of cash flows that begins with a cash flow of \$C received at Time 1 that grows by a rate g per period. The present value of a growing perpetuity (assuming r > g) is given by C/(r – g).

Compound Interest

If the annual interest rate is r and interest is **compounded** m times per year, \$1 will grow in t years to $(1+\frac{r}{m})^{mt}$ dollars.

Continuously Compound Interest

If interest is **compounded continuously** at an annual rate r, then in t years, \$1 will grow to e^{rt} dollars.

Effective Interest Rate

The **effective interest rate** is the simple interest rate that yields the same ending cash position as compounded interest.

Pricing a Zero Coupon Bond

Given a required interest rate r, a zero coupon bond paying \$F N years from now should sell for a price of $F/(1 + r)^N$ dollars.

Pricing a Bond with Annual Coupons

Given a required interest rate r, a bond that pays an annual coupon of \$C for N years and a face value of \$F N years from now should sell for a price of $C[\frac{1}{r}-\frac{1}{r(1+r)^N}]+\frac{F}{(1+r)^n}$.

Bond Yield

The **yield** of a bond is the required rate of return that makes the NPV of the bond's cash flows equal to its price. You can find the yield of the bond in Excel by listing the bond's cash flows (including the bond price as a negative cash flow) and then using the IRR function to determine the bond's yield. Of course, if coupons are received n times per year, the IRR function will return an annual yield/n.

Compound Annual Growth Rate (CAGR)

The **compounded annual growth rate (CAGR)** of a sequence of investment returns gives a measure of a typical return that adjusts for the variability in the investment's returns. Given annual returns R_1, R_2… R_n on an investment, the investment's CAGR may be calculated as

$$\sqrt[n]{(1+R_1)(1+R_2)\ldots(1+R_n)}-1.$$

European Call and Put Options

A **European call option** gives the owner the right to buy a share of stock for an exercise price X at a time T years from now. A **European put option** gives the owner the right to sell a share of stock for an exercise price X at a time T years from now. Given the following inputs . . .

- S = today's stock price

- T = duration of option in years
- X = exercise or strike price for option
- R = risk-free rate (expressed as a compound rate per year)
- V = annual volatility of stock
- D = annual dividend rate as a percentage of stock price

. . . the famous Black-Scholes formula can be used to price European puts and calls. The file Bstemp.xlsx makes it easy to price a European put or call option.