Assignment No.2 - Chapter 3, Computer Solution and Sensitivity Analysis

Problem 1

1. Consider the following linear program:

$$\begin{aligned} \text{Max } & 3A + 2B \\ \text{s.t.} & \\ & 1A + 1B \leq 10 \\ & 3A + 1B \leq 24 \\ & 1A + 2B \leq 16 \\ & A, B \geq 0 \end{aligned}$$

a. Use the graphical solution procedure to find the optimal solution.

b. Assume that the objective function coefficient for A changes from 3 to 5. Does the optimal solution change? Use the graphical solution procedure to find the new optimal solution.

c. Assume that the objective function coefficient for A remains 3, but the objective function coefficient for B changes from 2 to 4. Does the optimal solution change? Use the graphical solution procedure to find the new optimal solution.

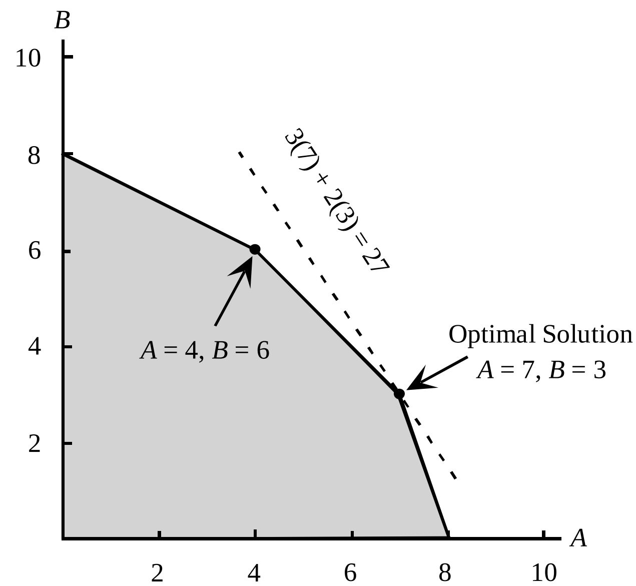
d. The computer solution for the linear program in part (a) provides the following objective coefficient range information:

Variable	Objective Coefficient	Allowable Increase	Allowable Decrease
A	3.00000	3.00000	1.00000
B	2.00000	1.00000	1.00000

Use this objective coefficient range information to answer parts (b) and (c).

Solution:

1. a.



b. The same extreme point, $A = 7$ and $B = 3$, remains optimal.

The value of the objective function becomes $5(7) + 2(3) = 41$

c. A new extreme point, $A = 4$ and $B = 6$, becomes optimal. The value of the objective function becomes $3(4) + 4(6) = 36$.

d. The objective coefficient range for variable A is 2 to 6. Since the change in part (b) is within this range, we know the optimal solution, $A = 7$ and $B = 3$, will not change. The objective coefficient range for variable B is 1 to 3. Since the change in part (c) is outside this range, we have to re-solve the problem to find the new optimal solution.

Problem 3

3. Consider the following linear program:

$$
\begin{aligned}
\text{Min} \quad & 8X + 12Y \\
\text{s.t.} \quad & \\
& 1X + 3Y \geq 9 \\
& 2X + 2Y \geq 10 \\
& 6X + 2Y \geq 18 \\
& A, B \geq 0
\end{aligned}
$$

a. Use the graphical solution procedure to find the optimal solution.

b. Assume that the objective function coefficient for X changes from 8 to 6. Does the optimal solution change? Use the graphical solution procedure to find the new optimal solution.

c. Assume that the objective function coefficient for X remains 8, but the objective function coefficient for Y changes from 12 to 6. Does the optimal solution change? Use the graphical solution procedure to find the new optimal solution.

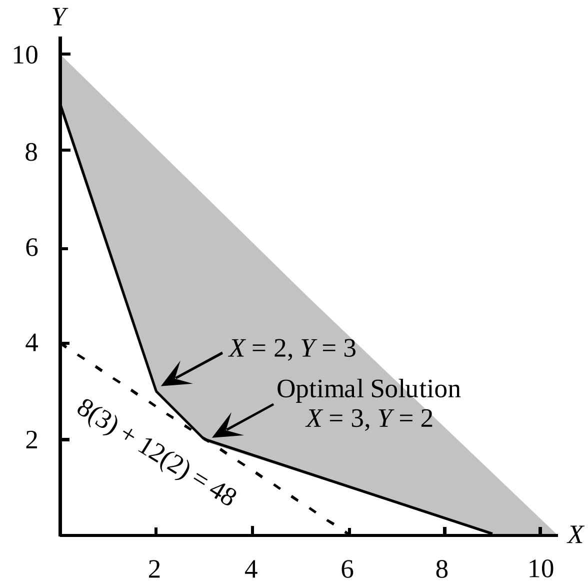
d. The computer solution for the linear program in part (a) provides the following objective coefficient range information:

Variable	Objective Coefficient	Allowable Increase	Allowable Decrease
X	8.00000	4.00000	4.00000
Y	12.00000	12.00000	4.00000

How would this objective coefficient range information help you answer parts (b) and (c) prior to re-solving the problem?

Solution:

3. a.



b. The same extreme point, $X = 3$ and $Y = 2$, remains optimal.

The value of the objective function becomes 6(3) + 12(2) = 42.

c. A new extreme point, $X = 2$ and $Y = 3$, becomes optimal. The value of the objective function becomes 8(2) + 6(3) = 34.

d. The objective coefficient range for variable X is 4 to 12. Since the change in part (b) is within this range, we know that the optimal solution, $X = 3$ and $Y = 2$, will not change. The objective coefficient range for variable Y is 8 to 24. Since the change in part (c) is outside this range, we have to re-solve the problem to find the new optimal solution.

Problem 7

7. Investment Advisors, Inc., is a brokerage firm that manages stock portfolios for a number of clients. A particular portfolio consists of U shares of U.S. Oil and H shares of Huber Steel. The annual return for U.S. Oil is \$3 per share and the annual return for Huber Steel is \$5 per share. U.S. Oil sells for \$25 per share and Huber Steel sells for \$50 per share. The portfolio has \$80,000 to be invested. The portfolio risk index (0.50 per share of U.S. Oil and 0.25 per share for Huber Steel) has a maximum of 700. In addition, the portfolio is limited to a maximum of 1000 shares of U.S. Oil. The linear programming formulation that will maximize the total annual return of the portfolio is as follows:

Max	$3U + 5H$	Maximize total annual return
s.t.		
	$25U + 50H \leq 80{,}000$	Funds available
	$0.50U + 0.25D \leq 700$	Risk maximum
	$1U \leq 1000$	U.S. Oil maximum
	$U, H \geq 0$	

The computer solution of this problem is shown in Figure 3.14.

a. What is the optimal solution, and what is the value of the total annual return?

b. Which constraints are binding? What is your interpretation of these constraints in terms of the problem?

c. What are the dual values for the constraints? Interpret each.

d. Would it be beneficial to increase the maximum amount invested in U.S. Oil? Why or why not?

FIGURE 3.14 THE SOLUTION FOR THE INVESTMENT ADVISORS PROBLEM

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Optimal Objective Value =         8400.00000

      Variable                 Value               Reduced Cost
   --------------         ---------------      -----------------
         U                     800.00000                 0.00000
         H                    1200.00000                 0.00000

     Constraint            Slack/Surplus            Dual Value
   --------------         ---------------      -----------------
          1                      0.00000                 0.09333
          2                      0.00000                 1.33333
          3                    200.00000                 0.00000

                    Objective       Allowable          Allowable
    Variable       Coefficient      Increase           Decrease
   ----------     ------------     ----------         ----------
       U             3.00000        7.00000            0.50000
       H             5.00000        1.00000            3.50000

                       RHS          Allowable          Allowable
   Constraint         Value         Increase           Decrease
   ----------     ------------     ----------         ----------
       1          80000.00000     60000.00000        15000.00000
       2            700.00000        75.00000          300.00000
       3           1000.00000        Infinite          200.00000
```

Solution:

7. a. $U = 800$

   $H = 1200$

   Estimated Annual Return = $8400

   b. Constraints 1 and 2. All funds available are being utilized and the maximum permissible risk is being incurred.

   c.

Constraint	Shadow Prices
Funds Avail.	0.09
Risk Max	1.33
U.S. Oil Max	0

   d. No, the optimal solution does not call for investing the maximum amount in U.S. Oil.

Problem 18

18. Davison Electronics manufactures two models of LCD televisions, identified as model A and model B. Each model has its lowest possible production cost when produced on Davison's new production line. However, the new production line does not have the capacity to handle the total production of both models. As a result, at least some of the production must be routed to a higher-cost, old production line. The following table shows the minimum production requirements for next month, the production line capacities in units per month, and the production cost per unit for each production line:

	Production Cost per Unit		Minimum Production Requirements
Model	**New Line**	**Old Line**	
A	\$30	\$50	50,000
B	\$25	\$40	70,000
Production Line Capacity	80,000	60,000	

Let

AN = Units of model A produced on the new production line
AO = Units of model A produced on the old production line
BN = Units of model B produced on the new production line
BO = Units of model B produced on the old production line

Davison's objective is to determine the minimum cost production plan. The computer solution is shown in Figure 3.21.

FIGURE 3.21 THE SOLUTION FOR THE DAVISON INDUSTRIES PROBLEM

Optimal Objective Value = 3850000.00000

Variable	Value	Reduced Cost
AN	50000.00000	0.00000
AO	0.00000	5.00000
BN	30000.00000	0.00000
BO	40000.00000	0.00000

Constraint	Slack/Surplus	Dual Value
1	0.00000	45.00000
2	0.00000	40.00000
3	0.00000	-15.00000
4	20000.00000	0.00000

Variable	Objective Coefficient	Allowable Increase	Allowable Decrease
AN	30.00000	5.00000	Infinite
AO	50.00000	Infinite	5.00000
BN	25.00000	15.00000	5.00000
BO	40.00000	5.00000	15.00000

Constraint	RHS Value	Allowable Increase	Allowable Decrease
1	50000.00000	20000.00000	40000.00000
2	70000.00000	20000.00000	40000.00000
3	80000.00000	40000.00000	20000.00000
4	60000.00000	Infinite	20000.00000

a. Formulate the linear programming model for this problem using the following four constraints:

Constraint 1: Minimum production for model A
Constraint 2: Minimum production for model B
Constraint 3: Capacity of the new production line
Constraint 4: Capacity of the old production line

b. Using computer solution in Figure 3.21, what is the optimal solution, and what is the total production cost associated with this solution?

c. Which constraints are binding? Explain.

d. The production manager noted that the only constraint with a positive dual value is the constraint on the capacity of the new production line. The manager's interpretation of the dual value was that a one-unit increase in the right-hand side of this constraint would actually increase the total production cost by $15 per unit. Do you agree with this interpretation? Would an increase in capacity for the new production line be desirable? Explain.

e. Would you recommend increasing the capacity of the old production line? Explain.

f. The production cost for model A on the old production line is $50 per unit. How much would this cost have to change to make it worthwhile to produce model A on the old production line? Explain.

g. Suppose that the minimum production requirement for model B is reduced from 70,000 units to 60,000 units. What effect would this change have on the total production cost? Explain.

Solution:

18. a. The linear programming model is as follows:

$$\text{Min} \quad 30AN + 50AO + 25BN + 40BO$$

s.t.

$$\begin{aligned}
AN + AO \quad & \geq 50{,}000 \\
BN + BO & \geq 70{,}000 \\
AN + BN \quad & \leq 80{,}000 \\
AO + BO & \leq 60{,}000
\end{aligned}$$

b. Optimal solution:

	New Line	Old Line
Model A	50,000	0
Model B	30,000	40,000

Total Cost $3,850,000

c. The first three constraints are binding because the values in the Slack/Surplus column for these constraints are zero. The fourth constraint, with a slack of 0 is nonbinding.

d. The dual value for the new production line capacity constraint is -15. Because the dual value is negative, increasing the right-hand side of constraint 3 will cause the objective function value to decrease. Thus, every one unit increase in the right hand side of this constraint will reduce the total production cost by $15. In other words, an increase in capacity for the new production line is desirable.

e. Because constraint 4 is not a binding constraint, any increase in the production line capacity of the old production line will have no effect on the optimal solution. Thus, there is no benefit in increasing the capacity of the old production line.

f. The reduced cost for Model A made on the old production line is 5. Thus, the cost would have to decrease by at least $5 before any units of model A would be produced on the old production line.

g. The right hand side range for constraint 2 shows an allowable decrease of 20,000. Thus, if the minimum production requirement is reduced 10,000 units to 60,000, the dual value of 40 is applicable. Thus, total cost would decrease by 10,000(40) = $400,000.

Problem 26

26. Benson Electronics manufactures three components used to produce cell telephones and other communication devices. In a given production period, demand for the three components may exceed Benson's manufacturing capacity. In this case, the company meets demand by purchasing the components from another manufacturer at an increased cost per unit. Benson's manufacturing cost per unit and purchasing cost per unit for the three components are as follows:

Source	Component 1	Component 2	Component 3
Manufacture	$4.50	$5.00	$2.75
Purchase	$6.50	$8.80	$7.00

Manufacturing times in minutes per unit for Benson's three departments are as follows:

Department	Component 1	Component 2	Component 3
Production	2	3	4
Assembly	1	1.5	3
Testing & Packaging	1.5	2	5

For instance, each unit of component 1 that Benson manufactures requires 2 minutes of production time, 1 minute of assembly time, and 1.5 minutes of testing and packaging time. For the next production period, Benson has capacities of 360 hours in the production department, 250 hours in the assembly department, and 300 hours in the testing and packaging department.

a. Formulate a linear programming model that can be used to determine how many units of each component to manufacture and how many units of each component to purchase. Assume that component demands that must be satisfied are 6000 units for component 1, 4000 units for component 2, and 3500 units for component 3. The objective is to minimize the total manufacturing and purchasing costs.

b. What is the optimal solution? How many units of each component should be manufactured and how many units of each component should be purchased?

c. Which departments are limiting Benson's manufacturing quantities? Use the dual value to determine the value of an *extra hour* in each of these departments.

d. Suppose that Benson had to obtain one additional unit of component 2. Discuss what the dual value for the component 2 constraint tells us about the cost to obtain the additional unit.

Solution:

a. Let M_1 = units of component 1 manufactured

M_2 = units of component 2 manufactured

M_3 = units of component 3 manufactured

P_1 = units of component 1 purchased

P_2 = units of component 2 purchased

P_3 = units of component 3 purchased

$$\text{Min} \quad 4.50M_1 + 5.00M_2 + 2.75M_3 + 6.50P_1 + 8.80P_2 + 7.00P_3$$

$$
\begin{array}{llllllll}
\text{s.t.} & 2M_1 + 3M_2 + 4M_3 & & & & \leq & 21{,}600 & \text{Production} \\
& 1M_1 + 1.5M_2 + 3M_3 & & & & \leq & 15{,}000 & \text{Assembly} \\
& 1.5M_1 + 2M_2 + 5M_3 & & & & \leq & 18{,}000 & \text{Testing/Packaging} \\
& M_1 & + 1P_1 & & & = & 6{,}000 & \text{Component 1} \\
& 1M_2 & & + 1P_2 & & = & 4{,}000 & \text{Component 2} \\
& 1M_3 & & & + 1P_3 & = & 3{,}500 & \text{Component 3}
\end{array}
$$

$$M_1,\ M_2,\ M_3,\ P_1,\ P_2, P_3 \geq 0$$

b.

Source	Component 1	Component 2	Component 3
Manufacture	2000	4000	1400
Purchase	4000	0	2100

Total Cost: $73,550

Constraint	Right-Hand-Side Range
1	4400 to 7440
2	6300 to No Upper Limit
3	100 to 900
4	600 to No Upper Limit
5	700 to No Upper Limit
6	514.29 to 1000

c. Since the slack is 0 in the production and the testing & packaging departments, these department are limiting Benson's manufacturing quantities.

Dual value information:

Production	\$-0.906/minute x 60 minutes = \$-54.36 per hour
Testing/Packaging	\$-0.125/minute x 60 minutes = \$ -7.50 per hour

d. The dual value is \$7.969. This tells us that the value of the optimal solution will worsen (the cost will increase) by \$7.969 for an additional unit of component 2. Note that although component 2 has a purchase cost per unit of \$8.80, it would only cost Benson \$7.969 to obtain an additional unit of component 2.