FNCE-921
Cross Sectional Asset Pricing Tests

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Spring 2019

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Outline

Asset Pricing Tests

Campbell, Lo, and MacKinlay (1997, Chapter 5)

Let Z_i be the return on the i-th asset in excess of the risk free rate and Z_m denote market excess return. The Sharpe-Lintner CAPM says that:

$$\mathrm{E}[Z_i] = \beta_{im}\mathrm{E}[Z_m] \quad \text{where} \quad \beta_{im} = \frac{\mathrm{Cov}[Z_i, Z_m]}{\mathrm{Var}[Z_m]}$$

Three testable implications of the Sharpe-Lintner CAPM:

- The intercept is zero (mean-variance efficiency of the market portfolio)
- Beta completely captures cross-sectional variations in average returns
- The market risk premium is positive

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Asset Pricing Tests

Statistical framework

Define $\mathbf{Z}_t$ as an $N \times 1$ vector of excess returns for N assets:

$$
\begin{aligned}
\mathbf{Z}_t &= \alpha + \beta Z_{mt} + \epsilon_t \\
\mathrm{E}[\epsilon_t] &= 0; \quad \mathrm{E}[\epsilon_t \epsilon_t'] = \Sigma \\
\mathrm{E}[Z_{mt}] &= \mu_m; \quad \mathrm{E}[(Z_{mt} - \mu_m)^2] = \sigma_m^2 \\
0 &= \mathrm{Cov}(Z_{mt}, \epsilon_t)
\end{aligned}
$$

Asset Pricing Tests

The MLS/OLS Estimators

$$
\begin{aligned}
\hat{\alpha} &= \hat{\mu} - \hat{\beta}\hat{\mu}_m \\
\hat{\beta} &= \frac{\sum_{t=1}^{T}(\mathbf{Z}_t - \hat{\mu})(Z_{mt} - \hat{\mu}_m)}{\sum_{t=1}^{T}(Z_{mt} - \hat{\mu}_m)^2} \\
\hat{\Sigma} &= \frac{1}{T}\sum_{t=1}^{T}(\mathbf{Z}_t - \hat{\alpha} - \hat{\beta}Z_{mt})(\mathbf{Z}_t - \hat{\alpha} - \hat{\beta}Z_{mt})' \\
\hat{\mu} &= \frac{1}{T}\sum_{t=1}^{T}\mathbf{Z}_t; \quad \hat{\mu}_m = \frac{1}{T}\sum_{t=1}^{T}Z_{mt}
\end{aligned}
$$

Asset Pricing Tests

Hypothesis tests in the multivariate normal framework

The conditional distributions of $\hat{\alpha}$ are:

$$\hat{\alpha} \sim \mathcal{N}\left(\alpha, \frac{1}{T}\left[1 + \frac{\hat{\mu}_m^2}{\hat{\sigma}_m^2}\right]\Sigma\right) \quad \text{where} \quad \hat{\sigma}_m^2 = \frac{1}{T}\sum_{t=1}^{T}(Z_{mt} - \hat{\mu}_m)^2$$

Form a Wald statistic for the null: $H_0 : \alpha = 0$ as

$$J_0 = T\left[1 + \frac{\hat{\mu}_m^2}{\hat{\sigma}_m^2}\right]^{-1} \hat{\alpha}'\hat{\Sigma}^{-1}\hat{\alpha} \quad \sim^a \quad \chi_N^2$$

Gibbons, Ross, and Shanken (1989) construct a finite-sample F test:

$$J_1 = \frac{(T-N-1)}{N}\left[1 + \frac{\hat{\mu}_m^2}{\hat{\sigma}_m^2}\right]^{-1} \hat{\alpha}'\hat{\Sigma}^{-1}\hat{\alpha} \quad \sim \quad \mathcal{F}_{N,T-N-1}$$

GRS Statistic

- Consider the following portfolio problem –given the *observed* asset data, we want to find a portfolio with a given mean and the lowest variance.

$$Min_{\omega} \quad \omega' \hat{V} \omega \quad s.t. \quad \omega' \hat{\mu} = \mu$$

- where
  - $\hat{V}$ is the var-cov matrix of N assets and the wealth portfolio (note this is the estimated one –not necessarily the population one)
  - ω is an $N \times 1$ vector of portfolio weights
  - $\hat{\mu}$–$(N+1)$ vector of sample mean excess returns
- Solution to the minimization problem implies the following Sharpe ratio of the *ex-post* tangency portfolio:

$$\hat{S}R_q^2 = \frac{\hat{\mu}^2}{\hat{\sigma}_q^2} = \hat{\mu} \hat{V}^{-1} \hat{\mu}$$

- Partition the inverse of the var-cov matrix as

$$\hat{V}^{-1} = \begin{bmatrix} \hat{\Sigma}^{-1} & -\hat{\Sigma}^{-1}\hat{\beta} \\ -\hat{\Sigma}^{-1}\hat{\beta} & (\hat{\sigma}(R_w^e))^{-2} + \hat{\beta}'\hat{\Sigma}^{-1}\hat{\beta} \end{bmatrix}$$

GRS statistic

- It follows that

$$\hat{S}R_q^2 = \left(\frac{\hat{E}(R_w^e)}{\hat{\sigma}(R_w^e)}\right)^2 + [\hat{E}(R^e) - \hat{\beta}\hat{E}(R_w^e)]'\hat{\Sigma}^{-1}[\hat{E}(R^e) - \hat{\beta}\hat{E}(R_w^e)] = \hat{S}R_w^2 + \hat{\alpha}'\hat{\Sigma}^{-1}\hat{\alpha}$$

- Hence the Wald test statistic can be presented as:

$$\frac{T-N-1}{N}\left[1 + \left(\frac{\hat{E}(R_w^e)}{\hat{\sigma}(R_w^e)}\right)^2\right]^{-1}\hat{\alpha}'\hat{\Sigma}^{-1}\hat{\alpha} = \frac{T-N-1}{N}\left(\frac{\widehat{SR}_q^2 - \widehat{SR}_m^2}{1 + \widehat{SR}_m^2}\right)$$

- Gibbons, Ross and Shanken (1989) show:

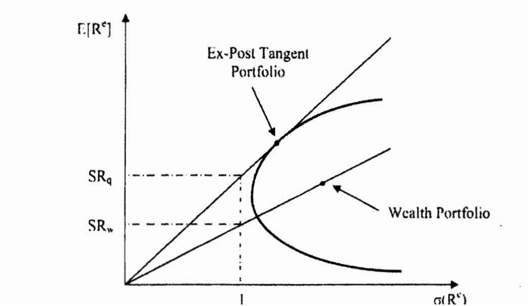
$$J_1 = \frac{T-N-1}{N}\left(\frac{\widehat{SR}_q^2 - \widehat{SR}_m^2}{1+\widehat{SR}_m^2}\right)$$

- where $\widehat{SR}_q^2$ and $\widehat{SR}_m^2$ are the estimated squared Sharpe ratios of the *ex post* tangency portfolio (formed from N included assets and the market portfolio) and the candidate MVE portfolio (market portfolio in the case of CAPM), respectively.

- **Interpretation**: Sharpe ratio measures the expected return per unit risk. Thus, testing the mean-variance efficiency of the market portfolio is equivalent to testing how far is SR of the market from that of the tangency portfolio (which has the maximum SR among all portfolios of risky assets).

CAPM: GRS intuition

- What is the ex-post tangency portfolio –this is the portfolio that delivers the best in sample mean variance trade-off
- According to CAPM, wealth portfolio is efficient ex-ante
- If CAPM is true, after STD-ERR, the wealth portfolio should not be too far inside the sample frontier –this is what GRS tests.



CAPM: Estimation and Testing

- We can test the null of $\alpha = 0$ relying on the likelihood ratio test:
  - Run LogL unrestricted (that is estimate α)   delivers $\hat{\alpha}$, $\hat{\beta}$, $\hat{\Sigma}$
  - Run LogL restricted/constrained (such that $\alpha = 0$)   delivers β^*, $\hat{\Sigma}^*$
- Likelihood Ratio test statistic
  (using constrained (*) and unconstrained MLE):

$$LR = T[\log|\hat{\Sigma}^*| - \log|\hat{\Sigma}|] \quad \overset{a}{\sim} \quad \chi^2_{N-1}$$

- To improve the finite-sample propertied of the test statistic, the following adjustment has been proposed:

$$LR = (T - \frac{N}{2} - 2)[\log|\hat{\Sigma}^*| - \log|\hat{\Sigma}|] \quad \overset{a}{\sim} \quad \chi^2_{N-1}$$

- Keep in mind this is all under assumption of iid normality

CAPM: Estimation and Testing

- **Black version** (a risk-free asset does not exist)
- Let γ be the zero-beta expected return:

$$E[r_t] = \vec{1}\gamma + \beta(E[r_{mt}] - \gamma) = (\vec{1} - \beta)\gamma + \beta E[r_{mt}]$$

- Consider unconstrained real-return market model:

$$r_t = \alpha + \beta r_{mt} + \epsilon_t$$

- Testable implications

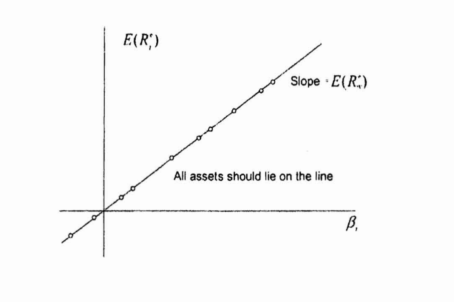
$$H_0 : \quad \alpha = (\vec{1} - \beta)\gamma$$
$$H_A : \quad \alpha \neq (\vec{1} - \beta)\gamma$$

Cross Sectional Restrictions

- If CAPM is true then for any asset i

$$E[R^e_{i,t}] = \beta_i E[R^e_{w,t}] = \beta_i \lambda$$

- This is the cross sectional restriction of the CAPM:



Cross-Sectional Tests

- How to test the CAPM in the cross sectional dimension–run a cross sectional regression of average returns on the betas
- That is run the following regression

$$\bar{R}_i^e = \beta_i \lambda + \alpha_i$$

- Two pass procedure
  - Run for each asset estimate β_i by

$$R_{i,t}^e = a_i + \beta_i R_{w,t}^e + \epsilon_{i,t}$$

  - Run a cross sectional regression of mean excess returns on estimated betas

$$\bar{R}_i^e = \hat{\beta}_i \lambda + \alpha_i$$

Cross Sectional Tests

- The cross sectional estimates of $\hat{\lambda}$ and α and their sampling distribution can be used to test the CAPM
- Can use the χ^2 test to test the pricing errors are jointly zero:

$$H_0 : \alpha_i = 0 \quad \Rightarrow \quad \hat{\alpha}' Cov(\hat{\alpha})^{-1} \hat{\alpha} \sim \chi^2_{N-1}$$

- To thest the price of risk is positive we can use the usual test statistic:

$$H_0 : \lambda > 0 \Rightarrow t - stat = \hat{\lambda}/\sigma(\hat{\lambda})$$

- Can also test that the slope of the cross sectional regression is equal to the market risk premium
- More details to follow

Errors in variables

- **Note**: Since the market betas are not known, this procedure induces an *error-in-variables* problem. To minimize the EIV problem, Fama and MacBeth group stocks in portfolios. Another approach is to adjust the SE for the biases introduced by the EIV problem (Shanken (1992)).
- Roll's Critique: Roll (1977) argues that if market portfolio is efficient then the CAPM implications (considered by Fama and MacBeth) are tautological and, hence, are not independently testable. There is only a single testable hypothesis: *market portfolio is ex-ante efficient*. However, the true market portfolio returns are unobservable. This implies that the theory is not testable.
- In response to this critique, Kandel and Stambaugh (1987) and Shanken (1987) show that if the correlation between the market proxy return and the true market return exceeds about 0.7, then the rejection of the CAPM with a market proxy would also imply the rejection of the CAPM with the true market portfolio.

Cross Sectional Tests

Fama and MacBeth (1973)

To test whether beta completely explains the cross-section of average returns, project the returns on the betas for each cross section and then aggregate the estimates in the time dimension

The regression model for the t-th cross section of N assets is:

$$Z_t = \gamma_{0t}\iota + \gamma_{1t}\beta_m + \eta_t$$

given T periods of data, run the cross-sectional regression for each t and obtain T observations of γ_{0t} and γ_{1t}

Define $\gamma_0 = \mathrm{E}[\gamma_{0t}]$ and $\gamma_1 = \mathrm{E}[\gamma_{1t}]$; test the null $\gamma_0 = 0$ and $\gamma_1 > 0$ using

$$w(\hat{\gamma}_j) = \frac{\hat{\gamma}_j}{\hat{\sigma}_{\gamma_j}} \sim t_{T-1}; \quad \hat{\gamma}_j = \frac{1}{T}\sum_{t=1}^{T}\hat{\gamma}_{jt}; \quad \hat{\sigma}^2_{\gamma_j} = \frac{1}{T(T-1)}\sum_{t=1}^{T}(\hat{\gamma}_{jt} - \hat{\gamma}_j)^2$$

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Cross Sectional Tests

Fama and MacBeth (1973), continued

To test whether beta completely explains the cross-section, define ς_t be the $N \times 1$ vector of firm characteristics (e.g. size) at the beginning of time t

Then test on $\gamma_2 = 0$ from the regression:

$$Z_t = \gamma_{0t}\iota + \gamma_{1t}\beta_m + \gamma_{2t}\varsigma_t + \eta_t$$

Errors-in-variables problem from first-pass beta estimation:

- Estimate betas for portfolios not individual stocks
- Shanken (1992) adjustment: multiplying $\hat{\sigma}^2_{\gamma_j}$ in $w(\hat{\gamma}_j)$ with

$$1 + \frac{(\hat{\mu}_m - \hat{\gamma}_0)^2}{\hat{\sigma}^2_m}$$

Cross Sectional Tests

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Fama-MacBeth (1973) procedure

- STEP 1: Find beta and non-beta risk estimates using *time series* regressions
  - FM use rolling 5 year regressions, but we can also use the full sample or other subsample
  - As a measure of non-beta risk FM use standard deviation of the OLS residuals $\hat{\epsilon}_{i,t}$, from the market model

$$R_{i,t} = a_i + \beta_i R_{w,t} + \epsilon_{i,t}$$

- STEP 2: Each time period t run a *cross sectional* regression:

$$R_t = \gamma_{1,t}\vec{1} + \gamma_{2,t}\hat{\beta}_t + \gamma_{3,t}\hat{\beta}^2 + \gamma_{4,t}\sigma(\epsilon_t) + \eta_t$$

- Get a time series of $\{\gamma_{i,t}\}_{t=1}^{T} \quad i = 1, 2, ..4$

  where
  - R_t – $(N \times 1)$ vector of asset returns at time
  - $\hat{\beta}_t$ – $(N \times 1)$ vector of estimated betas
  - $\hat{\sigma}(\epsilon)$ – $(N \times 1)$ vector of the estimated non beta risks

Fama-MacBeth (1973) procedure

- STEP 3: Use the cross-sectional estimates to test the CAPM restrictions:

$$\hat{E}[\gamma_{jt}] = \frac{1}{T}\sum_{t=1}^{T}\hat{\gamma}_{jt} \quad j = 1, 2, 3, 4$$

- Sampling errors for the estimates are constructed using the standard deviation of the cross-sectional estimates:

$$\sigma^2(\bar{\hat{\gamma}}_j) = \frac{1}{T^2}\sum_{t=1}^{T}(\hat{\gamma}_j - \bar{\hat{\gamma}}_j)^2 \quad j = 1, 2, 3, 4$$

- The cross sectional restrictions can now be tested using the standard t-test:

$$t - test = \frac{\bar{\hat{\gamma}}_j}{\sigma(\bar{\hat{\gamma}})}$$

- Note that $\sigma(\bar{\hat{\gamma}})$ defined above assumes $\hat{\gamma}$ are serially uncorrelated. In case of autocrrelation, standard errors should be appropriately adjusted.

Cross-Sectional Regressions

- **Fama-MacBeth (1973) procedure**

$$E[r_i] = E[r_0] + \beta_i(E[r_m] - E[r_0]) + q\beta_i^2 + d\sigma(\epsilon_i)$$

- This can be implemented via the regression:

$$r_{it} = \gamma_{1t} + \gamma_{2t}\beta_i + \gamma_{3t}\beta_i^2 + \gamma_{4t}\sigma(\epsilon_i) + \eta_{it}, \quad i = 1..N$$

- CAPM implications:
  - Sharpe-Lintner hypothesis: $E[\gamma_{1t}] = r_f$
  - Positive expected return-risk tradeoff: $E[\gamma_{2t}] > 0$
  - Linearity: $E[\gamma_{3t}] = 0$
  - No systematic effects of non-beta risk: $E[\gamma_{4t}] = 0$

Portfolio Approach to the Tests

- Consider the following model:

$$E[r_i] = E[r_0] + (E[r_m] - E[r_0])\beta_i + q\beta_i^2 + d\sigma(\epsilon_i)$$

- where: β_i - market betas; $\sigma(\epsilon_i)$ - a measure of non-beta risk.
- We are after the following tests:
  - $E[R_0] = R_f$
  - $E[R_m - R_f] > 0$
  - $q = 0$
  - $d = 0$
- For the last two restrictions we obviously need estimates of q and d
- Construct portfolios whose returns have the expected value equal to q and d –call them q and d portfolios
- The means of the time-series of the returns on these portfolios give us estimates of q and d –use those to test zero restrictions

Portfolio Approach to the Tests

- Consider a portfolio expected return with N assets and weights x_i:

$$
\begin{aligned}
E[R_p] &= \sum_{i=1}^{n} x_{ip} E[R_i] \\
&= E[R_0] \sum_{i=1}^{n} x_{ip} + (E[R_m] - E[R_0]) \sum_{i=1}^{n} x_{ip}\beta_i + q \sum_{i=1}^{n} x_{ip}\beta_i^2 + d \sum_{i=1}^{n} x_{ip}\sigma(\epsilon_i)
\end{aligned}
$$

- Let's choose such weights x_{ip} so as to get a portfolio that has expected return equal to $E[R_m] - E[R_0]$.
- Consider weights that satisfy:

$$
\begin{cases}
\sum_{i=0}^{n} x_{ip} = 0 \\
\sum_{i=0}^{n} x_{ip}\beta_i = 1 \\
\sum_{i=0}^{n} x_{ip}\beta_i^2 = 0 \\
\sum_{i=0}^{n} x_{ip}\sigma(\epsilon_i) = 0
\end{cases}
$$

- Portfolio with such weights is what we are looking for: it has *unit* exposure to the market and *zero* exposure to all other risks. Expected return on this portfolio is $E[R_m] - E[R_0]$.

Portfolio Intuition of FM Cross sectional Tests

- To construct q and d portfolio need to choose weights

$$
\begin{cases}
\underline{q} & \underline{d} \\
\sum_{i=0}^{n} x_{ip} = 0 & \sum_{i=0}^{n} x_{ip} = 0 \\
\sum_{i=0}^{n} x_{ip}\beta_i = 0 & \sum_{i=0}^{n} x_{ip}\beta_i = 0 \\
\sum_{i=0}^{n} x_{ip}\beta_i^2 = 1 & \sum_{i=0}^{n} x_{ip}\beta_i^2 = 0 \\
\sum_{i=0}^{n} x_{ip}\sigma(\epsilon_i) = 0 & \sum_{i=0}^{n} x_{ip}\sigma(\epsilon_i) = 1
\end{cases}
$$

- These portfolio weights have unit exposure to β_i^2 and $\sigma(\epsilon_i)$ respectively.

LS Coefficients as Portfolio Returns

- Notice that if the number of assets is greater than 4, q and d portfolios are not unique.
- We want to chose q and d portfolios with the smallest variance among all such portfolios
- Consider the FM regression:

$$r_{it} = \gamma_{1t} + \gamma_{2t}\beta_i + \gamma_{3t}\beta_i^2 + \gamma_{4t}\sigma(\epsilon_i) + \eta_{it}, \quad i = 1 \ldots n$$

- Stack variables:

$$r_t = \begin{pmatrix} r_{1t} \\ \vdots \\ r_{nt} \end{pmatrix} \quad C = \begin{bmatrix} 1 & \beta_1 & \beta_1^2 & \sigma(\epsilon_1) \\ \vdots & \vdots & \vdots & \vdots \\ 1 & \beta_n & \beta_n^2 & \sigma(\epsilon_n) \end{bmatrix} \quad \eta_t = \begin{pmatrix} \eta_{1t} \\ \vdots \\ \eta_{nt} \end{pmatrix}$$

$$\gamma_t = \begin{pmatrix} \gamma_{1t} \\ \gamma_{2t} \\ \gamma_{3t} \\ \gamma_{4t} \end{pmatrix} \Rightarrow \quad r_t = C\gamma_t + \eta_t$$

LS Coefficients as Portfolio Returns, cont'd

- The least squares value of γ_t:

$$\gamma_t = (C'C)^{-1}C'r_t = Xr_t$$

or:

$$\gamma_{jt} = \sum_{i=1}^{n} x_{ji}r_{it},\ j = 1, 2, 3, 4$$

- γ_{jt} - are returns on portfolios with weights $\{x_{ji}\}_{i=1}^{n}$ - j^{th} row of X.
- In particular, γ_{2t} - is the return on a portfolio with weights equal to the elements of the 2^{nd} row of X.
- Consider the properties of the elements in the different rows of X:

$$\underline{XC = (C'C)^{-1}C'C = I}$$

- Noting that X is $(4 \times n)$ and C is $(n \times 4)$, can rewrite the above given relationship as:

$$\Rightarrow \sum_{i=1}^{n} x_{ji}c_{ik} = \begin{cases} 1,\ j = k \\ 0,\ j \neq k \end{cases}$$

- In particular for $j = 2$:

$$\begin{cases} \sum_{i=0}^{n} x_{2i} = 0 \\ \sum_{i=0}^{n} x_{2i}\beta_i = 1 \\ \sum_{i=0}^{n} x_{2i}\beta_i^2 = 0 \\ \sum_{i=0}^{n} x_{2i}\sigma(\epsilon_i) = 0 \end{cases}$$

- Hence, γ_{2t} is the return on a zero-investment portfolio that has unit-beta and that zeros out any effects of β_i^2 and $\sigma(\epsilon_i)$ on expected returns.
- Given the above discussion, the expected value of γ_{2t} is $E[R_m] - E[R_0]$.
- Thus, we can use the mean value of the time series of γ_{2t} to test that $E[R_m] - E[R_0] > 0$.

Similarly:

- γ_{1t} is the return on a standard portfolio (a portfolio with weights sum up to one). The effect of β_i, β_i^2 and $\sigma(\epsilon_i)$ on the expected returns on such a portfolio is zero. Thus, the expected value of γ_{1t}, that is $E[\hat{\gamma}_{1,t}]$ is equal to $E[R_0]$, and we can test that $E[R_0] = R_f$ using the average of the time series of γ_{1t}.

- γ_{3t} - is a return on a zero-investment portfolio with the expected value equal to q, a unit exposure to β^2 risk and zero exposure to all other risks –that is – $E[\hat{\gamma}_{1,t}] = q$

- γ_{4t} - is a return on a zero-investment portfolio with the expected value equal to d, a unit exposure to a non-beta risk and zero exposure to all other risks –that is – $E[\hat{\gamma}_{4,t}] = d$

Test

- Thus, we can use the time series of the LS coefficient estimates to test CAPM-implied restrictions:
- $E[r_0] = r_f \quad \text{using} \quad \widehat{E[r_0]} = \frac{1}{T}\sum_{t=1}^{T}\hat{\gamma}_{1t}$
- $E[r_m] - E[r_0] > 0 \quad \text{using} \quad \widehat{E[r_m] - E[r_0]} = \frac{1}{T}\sum_{t=1}^{T}\hat{\gamma}_{2t}$
- $q = 0 \quad \text{using} \quad \hat{q} = \frac{1}{T}\sum_{t=1}^{T}\hat{\gamma}_{3t} \quad \text{and} \quad d = 0 \quad \text{using} \quad \hat{d} = \frac{1}{T}\sum_{t=1}^{T}\hat{\gamma}_{4t}$
- The FM procedure to cross sectional tests can be interpreted as constructing time-series of returns on factor mimicking portfolios (R_0 or $R_m - R_0$) and portfolios that are exposed solely to non-linear beta risk or non-beta risk (q and d), and use mean returns on these portfolios to test the CAPM restrictions.

FM pooled OLS, pure cross section

- If we are interesting in estimating the market price of risk and testing the cross sectional restrictions of the CAPM we can do the following:
  - Use the FM approach (run cross sectional regression for all periods and then take averages)
  - Use standard cross sectional regresison (avg. over time first and then run a cross sectional regression)
  - Use pooled time-series cross sectional OLS
- Are these approaches different?
- FM, Pooled and standard cross section are equivalent if:
  - No variation in betas through time
  - No serial correlation in returns

Estimation Error in Betas

- Note that the true CAPM betas are not known; in empirical work they have to be replaced by $\hat{\beta}$, their estimate
- This substitution induces errors in variables (EIV) problem:
- Solutions:
  - Reduce noise in betas by grouping stocks into portfolios –if estimation error in betas are uncorrelated across assets combining stocks in portfolios will reduce the estimation error.
  - Adjust SEs for the fact that $\hat{\beta}$ are generated regressors (Shanken (1982)).

Data Issues and Early Evidence

- Have to decide on:
  - Time interval for measuring returns (annual, monthly, etc)
  - Time interval for estimation
  - Which assets to include in tests
  - The choice of wealth portfolio
    - CRSP VW or EW indices are most common (NYSE, AMEX, NASDAQ)
    - S & P 500
    - Portfolio of stocks and bonds
- Early tests of the CAPM generally favor the CAPM:
  - Black-Jensen-Scholes (1972)
  - Fama-MacBeth (1973)
- More recent evidence?

Empirical Evidence:FF(1992)

- All nonfinancial firms in the intersection of (i) the NYSE, AMEX, and NASDAQ return files from CRSP and (ii) the merged COMPUSTAT annual industrial files of income statement and balance-sheet data from CRSP
- 1962–1989: pre-1962 data are tilted toward big historically successful firms. Maintain the six-month gap between fiscal yearend and the return tests by matching the accounting data for all fiscal yearends in calendar year $t-1$ with the returns for July of year t to June of $t+1$ (ensure that the accounting variables are known before the returns they are used to explain)
- Use a firm's market equity at the end of December of year $t-1$ to compute its book-to-market, leverage, and earnings-price ratios for $t-1$, and use its market equity for June of year t to measure its size

FF(92)

- Estimate βs for portfolios and then assign a portfolio's beta to each stock in the portfolio. Why?
  - Estimates of market βs for portfolios are more precise for portfolios than for individual stocks
  - Size, E/P, leverage, B/M measured precisely for individual stocks
- In June of each year, all NYSE stocks on CRSP are sorted by size to determine the NYSE decile breakpoints for size. NYSE, AMEX, and NASDAQ stocks that have the required CRSP-COMPUSTAT data are then allocated to 10 size portfolios based on the NYSE breakpoints

FF(1992) -Size Dimension

	1A	1B	2	3	4	5	6	7	8	9	10A	10B
Panel A: Portfolios Formed on Size												
Return	1.64	1.16	1.29	1.24	1.25	1.29	1.17	1.07	1.10	0.95	0.88	0.90
	1.44	1.44	1.39	1.34	1.33	1.24	1.22	1.16	1.08	1.02	0.95	0.90
n(ME)	1.98	3.18	3.63	4.10	4.50	4.89	5.30	5.73	6.24	6.82	7.39	8.44
ln(BE/ME)	-0.01	-0.21	-0.23	-0.26	-0.32	-0.36	-0.36	-0.44	-0.40	-0.42	-0.51	-0.65
ln(A/ME)	0.73	0.50	0.46	0.43	0.37	0.32	0.32	0.24	0.29	0.27	0.17	-0.03
ln(A/BE)	0.75	0.71	0.69	0.69	0.68	0.67	0.68	0.67	0.69	0.70	0.68	0.62
E/P dummy	0.26	0.14	0.11	0.09	0.06	0.04	0.04	0.03	0.03	0.02	0.02	0.01
E(+)/P	0.09	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.09	0.09
Firms	772	189	236	170	144	140	128	125	119	114	60	64

FF(1992) -Pre Beta Dimension

	1A	1B	2	3	4	5	6	7	8	9	10A	10B
Panel B: Portfolios Formed on Pre-Ranking β												
Return	1.20	1.20	1.32	1.26	1.31	1.30	1.30	1.23	1.23	1.33	1.34	1.18
β	0.81	0.79	0.92	1.04	1.13	1.19	1.26	1.32	1.41	1.52	1.63	1.73
ln(ME)	4.21	4.86	4.75	4.68	4.59	4.48	4.36	4.25	3.97	3.78	3.52	3.15
ln(BE/ME)	−0.18	−0.13	−0.22	−0.21	−0.23	−0.22	−0.22	−0.25	−0.23	−0.27	−0.31	−0.50
ln(A/ME)	0.60	0.66	0.49	0.45	0.42	0.42	0.45	0.42	0.47	0.46	0.46	0.31
ln(A/BE)	0.78	0.79	0.71	0.66	0.64	0.65	0.67	0.67	0.70	0.73	0.77	0.81
E/P dummy	0.12	0.06	0.09	0.09	0.08	0.09	0.10	0.12	0.12	0.14	0.17	0.23
E(+)/P	0.11	0.12	0.10	0.10	0.10	0.10	0.10	0.09	0.10	0.09	0.09	0.08
Firms	116	80	185	181	179	182	185	205	227	267	165	291

FF(1992) -Two Dimensional Size-Pre Beta Sort

FF(1992): Two-Dimensional Size–Pre-Beta Sort

	All	Low-β	β-2	β-3	β-4	β-5	β-6	β-7	β-8	β-9	High-β
Panel A: Average Monthly Returns (in Percent)											
All	1.25	1.34	1.29	1.36	1.31	1.33	1.28	1.24	1.21	1.25	1.14
Small-ME	1.52	1.71	1.57	1.79	1.61	1.50	1.50	1.37	1.63	1.50	1.42
ME-2	1.29	1.25	1.42	1.36	1.39	1.65	1.61	1.37	1.31	1.34	1.11
ME-3	1.24	1.12	1.31	1.17	1.70	1.29	1.10	1.31	1.36	1.26	0.76
ME-4	1.25	1.27	1.13	1.54	1.06	1.34	1.06	1.41	1.17	1.35	0.98
ME-5	1.29	1.34	1.42	1.39	1.48	1.42	1.18	1.13	1.27	1.18	1.08
ME-6	1.17	1.08	1.53	1.27	1.15	1.20	1.21	1.18	1.04	1.07	1.02
ME-7	1.07	0.95	1.21	1.26	1.09	1.18	1.11	1.24	0.62	1.32	0.76
ME-8	1.10	1.09	1.05	1.37	1.20	1.27	0.98	1.18	1.02	1.01	0.94
ME-9	0.95	0.98	0.88	1.02	1.14	1.07	1.23	0.94	0.82	0.88	0.59
Large-ME	0.89	1.01	0.93	1.10	0.94	0.93	0.89	1.03	0.71	0.74	0.56

	All	Low-β	β-2	β-3	β-4	β-5	β-6	β-7	β-8	β-9	High-β
Panel B: Post-Ranking βs											
All		0.87	0.99	1.09	1.16	1.26	1.29	1.35	1.45	1.52	1.72
Small-ME	1.44	1.05	1.18	1.28	1.32	1.40	1.40	1.49	1.61	1.64	1.79
ME-2	1.39	0.91	1.15	1.17	1.24	1.36	1.41	1.43	1.50	1.66	1.76
ME-3	1.35	0.97	1.13	1.13	1.21	1.26	1.28	1.39	1.50	1.51	1.75
ME-4	1.34	0.78	1.03	1.17	1.16	1.29	1.37	1.46	1.51	1.64	1.71
ME-5	1.25	0.66	0.85	1.12	1.15	1.16	1.26	1.30	1.43	1.59	1.68
ME-6	1.23	0.61	0.78	1.05	1.16	1.22	1.28	1.36	1.46	1.49	1.70
ME-7	1.17	0.57	0.92	1.01	1.11	1.14	1.26	1.24	1.39	1.34	1.60
ME-8	1.09	0.53	0.74	0.94	1.02	1.13	1.12	1.18	1.26	1.35	1.52
ME-9	1.03	0.58	0.74	0.80	0.95	1.06	1.15	1.14	1.21	1.22	1.42
Large-ME	0.92	0.57	0.71	0.78	0.89	0.95	0.92	1.02	1.01	1.11	1.32

FF(92) Two-Dimensional Size-Pre Beta Sort

- The data reveal that
  - Variation in betas related to size is consistent with variation in average returns
  - Variation in betas unrelated to size is not compensated in average returns
- FF run cross sectional regressions using the FM procedure
  - Each month, the cross-section of firm returns is regressed on beta, size and some additional variables
  - Time series averages of monthly slopes allows us to test what risks or what firm characteristics help explain the variation in mean return in the cross section

FF(92) Cross Sectional Evidence

β	ln(ME)	ln(BE/ME)	ln(A/ME)	ln(A/BE)	E/P Dummy	E(+)/P
0.15 (0.46)						
	−0.15 (−2.58)					
−0.37 (−1.21)	−0.17 (−3.41)					
		0.50 (5.71)				
			0.50 (5.69)	−0.57 (−5.34)		
					0.57 (2.28)	4.72 (4.57)
	−0.11 (−1.99)	0.35 (4.44)				
	−0.11 (−2.06)		0.35 (4.32)	−0.50 (−4.56)		
	−0.16 (−3.06)				0.06 (0.38)	2.99 (3.04)
	−0.13 (−2.47)	0.33 (4.46)			−0.14 (−0.90)	0.87 (1.23)
	−0.13 (−2.47)		0.32 (4.28)	−0.46 (−4.45)	−0.08 (−0.56)	1.15 (1.57)

FF(92) Book/Market Sort

Portfolio	1A	1B	2	3	4	5	6	7	8	9	10A	10B
Panel A: Stocks Sorted on Book-to-Market Equity (BE/ME)												
Return	0.30	0.67	0.87	0.97	1.04	1.17	1.30	1.44	1.50	1.59	1.92	1.83
β	1.36	1.34	1.32	1.30	1.28	1.27	1.27	1.27	1.27	1.29	1.33	1.35
ln(ME)	4.53	4.67	4.69	4.56	4.47	4.38	4.23	4.06	3.85	3.51	3.06	2.65
ln(BE/ME)	-2.22	-1.51	-1.09	-0.75	-0.51	-0.32	-0.14	0.03	0.21	0.42	0.66	1.02
ln(A/ME)	-1.24	-0.79	-0.40	-0.05	0.20	0.40	0.56	0.71	0.91	1.12	1.35	1.75
ln(A/BE)	0.94	0.71	0.68	0.70	0.71	0.71	0.70	0.68	0.70	0.70	0.70	0.73
E/P dummy	0.29	0.15	0.10	0.08	0.08	0.08	0.09	0.09	0.11	0.15	0.22	0.36
E(+)/P	0.03	0.04	0.06	0.08	0.09	0.10	0.11	0.11	0.12	0.12	0.11	0.10
Firms	89	98	209	222	226	230	235	237	239	239	120	117

FF(92) Two-Dimensional Size-B/M Sort

	Book-to-Market Portfolios										
	All	Low	2	3	4	5	6	7	8	9	High
All	1.23	0.64	0.98	1.06	1.17	1.24	1.26	1.39	1.40	1.50	1.63
Small-ME	1.47	0.70	1.14	1.20	1.43	1.56	1.51	1.70	1.71	1.82	1.92
ME-2	1.22	0.43	1.05	0.96	1.19	1.33	1.19	1.58	1.28	1.43	1.79
ME-3	1.22	0.56	0.88	1.23	0.95	1.36	1.30	1.30	1.40	1.54	1.60
ME-4	1.19	0.39	0.72	1.06	1.36	1.13	1.21	1.34	1.59	1.51	1.47
ME-5	1.24	0.88	0.65	1.08	1.47	1.13	1.43	1.44	1.26	1.52	1.49
ME-6	1.15	0.70	0.98	1.14	1.23	0.94	1.27	1.19	1.19	1.24	1.50
ME-7	1.07	0.95	1.00	0.99	0.83	0.99	1.13	0.99	1.16	1.10	1.47
ME-8	1.08	0.66	1.13	0.91	0.95	0.99	1.01	1.15	1.05	1.29	1.55
ME-9	0.95	0.44	0.89	0.92	1.00	1.05	0.93	0.82	1.11	1.04	1.22
Large-ME	0.89	0.93	0.88	0.84	0.71	0.79	0.83	0.81	0.96	0.97	1.18

FF(92) Two-Dimensional Size-B/M Sort

Table VI

Subperiod Average Monthly Returns on the NYSE Equal-Weighted and Value-Weighted Portfolios and Subperiod Means of the Intercepts and Slopes from the Monthly FM Cross-Sectional Regressions of Returns on (a) Size (ln(ME)) and Book-to-Market Equity (ln(BE/ME)), and (b) β, ln(ME), and ln(BE/ME)

Mean is the time-series mean of a monthly return, Std is its time-series standard deviation, and t(Mn) is Mean divided by its time-series standard error.

	7/63–12/90 (330 Mos.)			7/63–12/76 (162 Mos.)			1/77–12/90 (168 Mos.)		
Variable	Mean	Std	t(Mn)	Mean	Std	t(Mn)	Mean	Std	t(Mn)
NYSE Value-Weighted (VW) and Equal-Weighted (EW) Portfolio Returns									
VW	0.81	4.47	3.27	0.56	4.26	1.67	1.04	4.66	2.89
EW	0.97	5.49	3.19	0.77	5.70	1.72	1.15	5.28	2.82
$R_{it} = a + b_{2t}\ln(ME_{it}) + b_{3t}\ln(BE/ME_{it}) + e_{it}$									
a	1.77	8.51	3.77	1.86	10.10	2.33	1.69	6.67	3.27
b_2	−0.11	1.02	−1.99	−0.16	1.25	−1.62	−0.07	0.73	−1.16
b_3	0.35	1.45	4.43	0.36	1.53	2.96	0.35	1.37	3.30
$R_{it} = a + b_{1t}\beta_{it} + b_{2t}\ln(ME_{it}) + b_{3t}\ln(BE/ME_{it}) + e_{it}$									
a	2.07	5.75	6.55	1.73	6.22	3.54	2.40	5.25	5.92
b_1	−0.17	5.12	−0.62	0.10	5.33	0.25	−0.44	4.91	−1.17
b_2	−0.12	0.89	−2.52	−0.15	1.03	−1.91	−0.09	0.74	−1.64
b_3	0.33	1.24	4.80	0.34	1.36	3.17	0.31	1.10	3.67

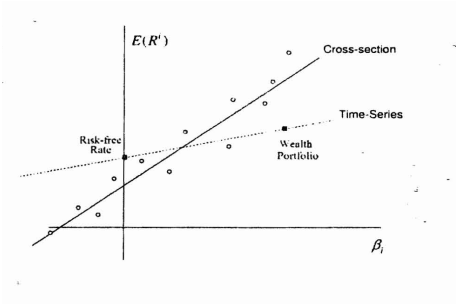
Roll's Critique

- The CAPM says the wealth portfolio is ex-ante efficient
- However, the true wealth portfolio is not observable
- Roll (1977) argue that the CAPM theory is therefore not testable
- Given that no empirical proxy is perfect, if we reject the CAPM with some proxy portfolio how different the true wealth portfolio should be relative to the employed proxy, for the rejection to be overturned?
- Kandel and Stambaugh (1987) show that if the correlation between the proxy and the true wealth return exceeds about 0.7 then the rejection of the CAPM with the proxy would also imply the rejection of the CAPM with the true wealth portfolio

CAPM: Time Series vs. Cross Sectional Tests

- Are time series and cross sectional tests of the CAPM different ? yes in general
- Time series tests:

$$
\begin{aligned}
R^e_{i,t} &= \alpha_i + \beta_i R^e_{w,t} + \epsilon_{i,t} \\
R^e_{w,t} &= 0 + 1 \times R^e_{w,t} + 0 \\
R_{f,t} - R_{f,t} &= 0 + 0 \times R^e_{w,t} + 0
\end{aligned}
$$



CAPM: Time Series vs. Cross Sectional Tests

- Consider the cross sectional implications of the time series regression:

$$\begin{aligned} TS: \quad R^e_{i,t} &= \alpha_i + \beta_i R^e_{w,t} + \epsilon_{i,t} \\ E[TS]: \quad E[R^e_{i,t}] &= \alpha_i + \beta_i E[R^e_{w,t}] \end{aligned}$$

- Compare to the cross sectional regression:

$$E[R^e_{i,t}] = \beta\lambda + \alpha_i$$

- The standard OLS cross sectional regression picks the slope that minimizes the sum of squares of pricing errors across *all* included assets.
- Hence, the cross sectional line implied by the cross sectional regression does not go through $E[R_f]$ and $E[R_w]$.

CAPM: Time Series vs. Cross Sectional Tests

- Estimates
  - The TS estimates of the risk price is just the sample mean of excess returns on the wealth portfolio
  - The XS estimates os constructed to provide the best fit to the whole cross section of mean returns. Thus in general
$$\hat{\lambda}_{XS} \neq \bar{R}_w^e$$
- Consequently,
$$\hat{\alpha}_{TS} \neq \hat{\alpha}_{XS}$$
- Think about the case when TS and XS approaches are equivalent

Cross Sectional Tests

Fama and French (1992): the cross section of expected returns

Size and book-to-market combine to capture the cross-sectional variation in average returns; the relation between beta and average return is flat

Cross Sectional Tests

Fama and French (1992): sample construction

All nonfinancial firms in the intersection of (i) the NYSE, AMEX, and NASDAQ return files from CRSP and (ii) the merged COMPUSTAT annual industrial files of income statement and balance-sheet data from CRSP

1962–1989: pre-1962 data are tilted toward big historically successful firms

Maintain the six-month gap between fiscal yearend and the return tests by matching the accounting data for all fiscal yearends in calendar year $t-1$ with the returns for July of year t to June of $t+1$ (ensure that the accounting variables are known before the returns they are used to explain)

Use a firm's market equity at the end of December of year $t-1$ to compute its book-to-market, leverage, and earnings-price ratios for $t-1$, and use its market equity for June of year t to measure its size

Cross Sectional Tests

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Cross Sectional Tests

Fama and French (1992): beta estimation

Estimate βs for portfolios and then assign a portfolio's beta to each stock in the portfolio. Why?

- Estimates of market βs for portfolios are more precise for portfolios than for individual stocks
- Size, E/P, leverage, B/M measured precisely for individual stocks

In June of each year, all NYSE stocks on CRSP are sorted by size to determine the NYSE decile breakpoints for size. NYSE, AMEX, and NASDAQ stocks that have the required CRSP-COMPUSTAT data are then allocated to 10 size portfolios based on the NYSE breakpoints

- Why use size portfolios?

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- Why use size portfolios?

Cross Sectional Tests

Fama and French (1992): beta estimation, continued

To allow for variation in β unrelated to size, subdivide each size decile into 10 portfolios on the basis of pre-ranking βs for individual stocks

- The pre-ranking βs are estimated on 24 to 60 monthly observations (as available) in the 5 years before July of year t
- Set the β breakpoints for each size decile using only NYSE stocks that satisfy the COMPUSTAT-CRSP data requirements for year $t-1$

After assigning firms to the size-β portfolios in June, calculate the equal-weighted monthly portfolio returns from July to June

Estimate βs using the full sample (330 months) of post-ranking returns on each of the 100 portfolios, with CRSP value-weighted portfolio of NYSE, AMEX, and NASDAQ stocks as proxy for the market

Cross Sectional Tests

Fama and French (1992): beta estimation, continued

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Cross Sectional Tests

Fama and French (1992): beta estimation, continued

Estimate β as the sum of the slopes in the regression of the portfolio return on the current and prior month's market return (nonsynchronous trading)

Allocate the full-period post-ranking β of a size-β portfolio to each stock in the portfolio

Results on β estimates:

- Forming portfolios on size and pre-ranking βs, rather than on size alone, magnifies the range of full-period post-ranking βs
- The β sort is not a refined size sort

The Portfolio Approach

Fama and French (1993): Common factors

Five common risk factors in the returns on stocks and bonds; stock returns have shared variation due to the stock market factors; stock returns are linked to bond returns through bond market factors

Regress stock and bond returns on common risk factors; the slopes measure factor loadings with a clear interpretation as sensitivities

If assets are priced rationally, variables related to average returns must proxy for sensitivity to common risk factors

The Portfolio Approach

Fama and French (1993): Common factors in bond returns

TERM: the difference between the monthly long-term government bond return and the one-month Treasury bill rate measured at the end of previous month

- Proxy for the deviation of long-term bond returns from expected returns due to shifts in interest rates

DEF: the difference between the return on a market portfolio of long-term corporate bonds and the long-term government bond return

- Proxy for default risk in bonds

The Portfolio Approach

Fama and French (1993): Common factors in stock returns

Zero-investment factor mimicking portfolios

In June of each year t, all NYSE stocks are ranked on size (price times shares); the median NYSE size is then used to split NYSE, AMEX, and NASDAQ stocks into two groups, small and big

Break NYSE, AMEX, and NASDAQ stocks into three book-to-market equity groups based on the breakpoints for the bottom 30%, middle 40%, and top 30% of the ranked values of BE/ME for NYSE stocks

- Book equity: the COMPUSTAT book value of stochholder's equity, plus balanced-sheet deferred taxes and investment tax credit, minus the book value of preferred stock
- BE/ME: book equity for the fiscal year ending in calendar year $t-1$ divided by market equity at the end of December of $t-1$.

The Portfolio Approach

Fama and French (1993): Common factors in stock returns

Zero-investment factor mimicking portfolios

In June of each year t, all NYSE stocks are ranked on size (price times shares); the median NYSE size is then used to split NYSE, AMEX, and NASDAQ stocks into two groups, small and big

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- BE/ME: book equity for the fiscal year ending in calendar year $t-1$ divided by market equity at the end of December of $t-1$.

The Portfolio Approach

Fama and French (1993): Common factors in stock returns

Zero-investment factor mimicking portfolios

In June of each year t, all NYSE stocks are ranked on size (price times shares); the median NYSE size is then used to split NYSE, AMEX, and NASDAQ stocks into two groups, small and big

Break NYSE, AMEX, and NASDAQ stocks into three book-to-market equity groups based on the breakpoints for the bottom 30%, middle 40%, and top 30% of the ranked values of BE/ME for NYSE stocks

- Book equity: the COMPUSTAT book value of stochholder's equity, plus balanced-sheet deferred taxes and investment tax credit, minus the book value of preferred stock
- BE/ME: book equity for the fiscal year ending in calendar year $t-1$ divided by market equity at the end of December of $t-1$.

The Portfolio Approach

Fama and French (1993): Common factors in stock returns

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The Portfolio Approach

Fama and French (1993): Common factors in stock returns, continued

Construct six portfolios (S/L, S/M, S/H, B/L, B/M, B/H) from the intersections of the two size and the three B/M groups. Monthly value-weighted returns on the six portfolios are calculated from July of year t to June of $t+1$

SMB is the difference, each month, between the simple average of the returns on the three small portfolios and the simple average of the returns on the three big portfolios

HML is the difference, each month, between the simple average of the returns on the three high portfolios and the simple average of the returns on the three low portfolios

The Portfolio Approach

Fama and French (1993): Common factors in stock returns, continued

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Portfolio Approach

Fama and French (1996)

Except for the continuation of short-term returns, the effects of size, earnings-price ratio, cash flow-price ratio, book-to-market, past sales growth, long-term past return largely disappear in a three-factor model:

$$\mathrm{E}[R_i] - R_f = b_i[\mathrm{E}[R_m] - R_f] + s_i\,\mathrm{E}[\mathrm{SMB}] + h_i\,\mathrm{E}[\mathrm{HML}]$$

ICAPM and/or APT interpretation
See slides for Part 2 of this lecture!

The Portfolio Approach

Fama and French (1996): the factor model explains size and book-to-market effects

Table I—*Continued*

	Book-to-Market Equity (BE/ME) Quintiles									
Size	Low	2	3	4	High	Low	2	3	4	High
Panel B: Regressions: $R_i - R_f = a_i + b_i(R_M - R_f) + s_i SMB + h_i HML + e_i$										
			a					t(a)		
Small	−0.45	−0.16	−0.05	0.04	0.02	−4.19	−2.04	−0.82	0.69	0.29
2	−0.07	−0.04	0.09	0.07	0.03	−0.80	−0.59	1.33	1.13	0.51
3	−0.08	0.04	−0.00	0.06	0.07	−1.07	0.47	−0.06	0.88	0.89
4	0.14	−0.19	−0.06	0.02	0.06	1.74	−2.43	−0.73	0.27	0.59
Big	0.20	−0.04	−0.10	−0.08	−0.14	3.14	−0.52	−1.23	−1.07	−1.17
			b					t(b)		
Small	1.03	1.01	0.94	0.89	0.94	39.10	50.89	59.93	58.47	57.71
2	1.10	1.04	0.99	0.97	1.08	52.94	61.14	58.17	62.97	65.58
3	1.10	1.02	0.98	0.97	1.07	57.08	55.49	53.11	55.96	52.37
4	1.07	1.07	1.05	1.03	1.18	54.77	54.48	51.79	45.76	46.27
Big	0.96	1.02	0.98	0.99	1.07	60.25	57.77	47.03	53.25	37.18
			s					t(s)		
Small	1.47	1.27	1.18	1.17	1.23	39.01	44.48	52.26	53.82	52.65
2	1.01	0.97	0.88	0.73	0.90	34.10	39.94	36.19	32.92	38.17
3	0.75	0.63	0.59	0.47	0.64	27.09	24.13	22.37	18.97	22.01
4	0.36	0.30	0.29	0.22	0.41	12.87	10.64	10.17	6.82	11.26
Big	−0.16	−0.13	−0.25	−0.16	−0.03	−6.97	−5.12	−8.45	−6.21	−0.77
			h					t(h)		
Small	−0.27	0.10	0.25	0.37	0.63	−6.28	3.03	9.74	15.16	23.62
2	−0.49	0.00	0.26	0.46	0.69	−14.66	0.34	9.21	18.14	25.59
3	−0.39	0.03	0.32	0.49	0.68	−12.56	0.89	10.73	17.45	20.43
4	−0.44	0.03	0.31	0.54	0.72	−13.98	0.97	9.45	14.70	17.34
Big	−0.47	0.00	0.20	0.56	0.82	−18.23	0.18	6.04	18.71	17.57
			R^2					s(e)		
Small	0.93	0.95	0.96	0.96	0.96	1.97	1.49	1.18	1.13	1.22
2	0.95	0.96	0.95	0.95	0.96	1.55	1.27	1.28	1.16	1.23
3	0.95	0.94	0.93	0.93	0.92	1.44	1.37	1.38	1.30	1.52
4	0.94	0.92	0.91	0.88	0.89	1.46	1.47	1.51	1.69	1.91
Big	0.94	0.92	0.87	0.89	0.81	1.19	1.32	1.55	1.39	2.15

The Portfolio Approach

Fama and French (1996): the factor model does not explain momentum

Table VII

Three-Factor Regressions for Monthly Excess Returns (in Percent) on Equal-Weight NYSE Portfolios Formed on Past Returns: 7/63–12/93, 366 Months

$$R_i - R_f = a_i + b_i(R_M - R_f) + s_i\text{SMB} + h_i\text{HML} + e_i$$

The formation of the past-return deciles is described in Table VI. Decile 1 contains the NYSE stocks with the lowest continuously compounded returns during the portfolio-formation period (12-2, 48-2, or 60-13 months before the return month). $t()$ is a regression coefficient divided by its standard error. The regression R^2s are adjusted for degrees of freedom. GRS is the F-statistic of Gibbons, Ross, and Shanken (1989), testing the hypothesis that the regression intercepts for a set of ten portfolios are all 0.0. p(GRS) is the p-value of GRS.

	1	2	3	4	5	6	7	8	9	10	GRS	p(GRS)
Portfolio formation months are t-12 to t-2												
a	−1.15	−0.39	−0.21	−0.22	−0.04	−0.05	0.12	0.21	0.33	0.59		
b	1.14	1.06	1.04	1.02	1.02	1.02	1.04	1.03	1.10	1.13		
s	1.35	0.77	0.66	0.59	0.53	0.48	0.47	0.45	0.51	0.68		
h	0.54	0.35	0.35	0.33	0.32	0.30	0.29	0.23	0.23	0.04		
$t(a)$	−5.34	−3.05	−2.05	−2.81	−0.54	−0.93	1.94	3.08	3.88	4.56	4.45	0.000
$t(b)$	21.31	33.36	42.03	51.48	61.03	73.62	68.96	62.67	51.75	35.25		
$t(s)$	17.64	16.96	18.59	20.87	22.06	23.96	21.53	19.03	16.89	14.84		
$t(h)$	6.21	6.72	8.74	10.18	11.86	13.16	11.88	8.50	6.68	0.70		
R^2	0.75	0.85	0.89	0.92	0.94	0.96	0.95	0.94	0.92	0.86		
Portfolio formation months are t-48 to t-2												
a	−0.73	−0.32	−0.09	−0.08	−0.05	−0.00	0.07	0.10	0.15	0.37		
b	1.16	1.12	1.06	1.05	1.02	1.01	1.00	0.99	1.04	1.11		
s	1.59	0.87	0.64	0.52	0.48	0.42	0.41	0.40	0.42	0.49		
h	0.90	0.60	0.44	0.44	0.36	0.31	0.18	0.11	−0.05	−0.26		
$t(a)$	−2.91	−2.79	−0.96	−0.99	−0.67	−0.01	1.08	1.46	2.09	3.60	2.02	0.031
$t(b)$	18.61	39.22	46.55	53.19	57.82	63.78	64.72	58.62	57.02	43.37		
$t(s)$	17.91	21.36	19.68	18.61	19.17	18.51	18.52	16.61	16.22	13.40		
$t(h)$	8.91	12.94	11.93	13.78	12.61	11.87	7.34	4.19	−1.55	−6.35		
R^2	0.73	0.88	0.91	0.92	0.93	0.94	0.95	0.93	0.94	0.90		
Portfolio formation months are t-60 to t-13												
a	−0.18	−0.16	−0.13	−0.07	0.00	0.02	0.06	0.10	−0.07	−0.12		
b	1.13	1.09	1.07	1.04	0.99	1.00	1.00	1.01	1.06	1.15		
s	1.50	0.83	0.67	0.59	0.47	0.38	0.35	0.40	0.45	0.50		
h	0.87	0.54	0.50	0.42	0.34	0.29	0.23	0.13	−0.00	−0.26		
$t(a)$	−0.80	−1.64	−1.69	−0.99	0.02	0.40	0.96	1.43	−0.92	−1.36	1.29	0.235
$t(b)$	20.24	44.40	55.03	61.09	63.79	65.68	62.58	58.26	60.49	53.04		
$t(s)$	18.77	23.63	24.09	24.06	21.21	17.44	15.43	16.18	18.06	16.33		
$t(h)$	9.59	13.67	15.94	15.31	13.46	11.82	8.98	4.46	−0.14	−7.50		
R^2	0.75	0.91	0.93	0.94	0.94	0.94	0.94	0.93	0.94	0.93		

Macroeconomic Factor Models

Chen, Roll, and Ross (1986): macroeconomic variables as factors

The survey conducted by Graham and Harvey (2001) shows that CFOs consider primarily macroeconomic risks in cost of capital calculations

The overall goal is to identify a set of macroeconomic factors associated with returns on financial assets

Chen, Roll, and Ross view an asset's price P_t as a stream of expected cash flow, $\bar{c}_{t+1}, \bar{c}_{t+2}, \ldots,$ discounted at a rate k:

$$P_t = \frac{\bar{c}_{t+1}}{1+k} + \frac{\bar{c}_{t+2}}{(1+k)^2} + \cdots$$

factors affecting price changes (or asset returns) are factors related to changes in expected cash flows or changes in discount rates

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Macroeconomic Factor Models

Chen, Roll, and Ross (1986), macroeconomic factors

This reasoning leads to five factors:

1. IP: monthly growth rate of industrial production
2. EI: change in expected inflation—changes in short-term T-bill rate
3. UI: unexpected inflation—the difference between actual and expected inflation
4. CG: unexpected changes in risk premium—the difference between the returns on corporate Baa bonds and long-term governance bonds
5. GB: unexpected changes in the term premium—the difference between the returns on long- and short-term governance bonds

The returns on the equally weighted portfolio (EWNY) and the value weighted portfolio (VWNY) of NYSE included as the sixth factor

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Chen, Roll, and Ross (1986), test design and main results

Two-stage regressions, 20 size portfolios as the test assets

Significant factors: industrial production, risk premium on bonds, and unanticipated inflation

Market returns are not statistically significant in the multifactor tests

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Macroeconomic Factor Models

Liew and Vassalou (2000), empirical methods

HML and SMB contain significant information about future GDP growth

- Univariate regressions:

$$\text{GDPgrowth}_{t,t+4} = a + b\,\text{FactorRet}_{t-4,t} + e_{t,t+4}$$

- Multiple regressions:

$$\begin{aligned}
\text{GDPgrowth}_{t,t+4} &= a + b\,\text{MKT}_{t-4,t} + c\,\text{HML}_{t-4,t} + d\ \text{SMB}_{t-4,t} + u_{t,t+4} \\
\text{GDPgrowth}_{t,t+4} &= a + b\,\text{MKT}_{t-4,t} + c\,\text{FactorRet}_{t-4,t} + d\ \text{TB}_t \\
&\quad + e\,\text{DY}_t + f\,\text{TERM}_t + g\,\text{IDPgrowth}_{t-4,t} + v_{t,t+4}
\end{aligned}$$

Macroeconomic Factor Models

Liew and Vassalou (2000), empirical results

Table 5

Univariate regressions of GDP growth rates conditional on past four-quarters of factor returns

$$\text{GDPgrowth}_{(t,t+4)} = a + b{*}\text{FactorRet}_{(t-4,t)} + e_{(t,t+4)}$$

In the regression notation, 'FactorRet' stands for MKT, HML, SMB, and WML. MKT is the excess return on the local market index proxied by the Morgan Stanley Capital International index. The regressions use the annually rebalanced HML, SMB, and WML portfolios. HML is the return on a portfolio that is long on high book-to-market stocks and short on low book-to-market stocks, holding the size and momentum characteristics of the portfolio constant. SMB is the return on a portfolio that is long on small capitalization stocks and short on big capitalization stocks, holding the book-to-market and momentum characteristics of the portfolio constant. WML is the return on a portfolio that is long on the best performing stocks of the past year ('winners') and short on the worst performing stocks of the past year ('losers'), holding the book-to-market and size effects of the portfolio constant. GDP growth is calculated as the continuously compounded growth rate in a country's Gross Domestic Product, which is seasonally adjusted. For Japan, we use the seasonally adjusted Gross National Product. All returns are annualized and continuously compounded. All tests are performed in the local currency of each country. T-values are corrected for heteroskedasticity and serial correlation, up to three lags, using the Newey and West (1987) estimator.

Country	Slope coefficients				T-values				Coefficients of determination (%)			
	MKT	HML	SML	WML	MKT	HML	SMB	WML	MKT	HML	SMB	WML
Australia	0.026	– 0.002	0.050	0.022	1.40	– 0.12	3.94	2.09	4.2	– 2.5	37.1	8.6
Canada	0.049	– 0.025	0.075	0.016	2.48	– 0.68	3.48	0.67	11.6	– 0.19	9.6	– 0.2
France	0.011	0.063	0.090	0.001	0.68	2.80	4.04	0.03	– 0.5	20.8	30.7	– 2.1
Germany	0.054	0.266	0.082	0.002	1.10	4.04	1.64	0.01	2.8	– 1.0	10.3	– 4.7
Italy	0.035	0.067	0.064	– 0.022	2.91	3.01	3.07	– 0.98	29.1	30.2	22.7	– 0.4
Japan	0.030	0.036	0.035	0.036	1.98	1.15	1.35	1.16	12.0	0.9	3.8	1.0
Netherlands	0.033	0.030	0.033	0.020	1.74	1.22	1.20	0.83	5.9	2.9	1.8	0.5
Switzerland	– 0.004	0.126	0.079	– 0.107	– 0.14	6.54	5.78	– 2.00	– 2.7	52.5	35.7	9.8
United Kingdom	0.072	0.075	0.088	– 0.083	2.96	1.90	4.51	– 1.58	14.5	8.2	25.4	4.9
United States	0.059	0.046	0.056	– 0.016	2.85	1.53	1.84	– 0.51	12.7	3.5	9.5	– 0.9

Macroeconomic Factor Models

Liew and Vassalou (2000), empirical results, continued

Table 7
Predicting annual Gross Domestic Product (GDP) growth rates conditional on information about the return on the market, HML, and SMB using multivariate regressions

In $GDPgrowth_{(t,\,t+4)} = a + b{*}MKT_{(t-4,\,t)} + c{*}HML_{(t-4,\,t)} + d{*}SMB_{(t-4,\,t)} + u_{(t,\,t+4)}$, MKT is the excess return on the local market portfolio, and it is proxied by the Morgan Stanley Capital International country index. The regressions use the annually rebalanced HML and SMB portfolios. HML is the return on a portfolio that is long on high book-to-market stocks and short on low book-to-market stocks, holding the size and momentum characteristics of the portfolio constant. SMB is the return on a portfolio that is long on small capitalization stocks and short on big capitalization stocks, holding the book-to-market and momentum characteristics of the portfolio constant. GDP growth is calculated as the continuously compounded growth rate in a country's Gross Domestic Product, which is seasonally adjusted. For Japan, we use the seasonally adjusted Gross National Product. All returns are continuously compounded and expressed in the local currency of each country. T-values are corrected for heteroskedasticity and serial correlation, up to three lags, using the Newey and West (1987) estimator. R^2 are adjusted for degrees of freedom.

Country	MKT		HML		SMB		Adjusted R^2 (%)
	Slope	T-value	Slope	T-value	Slope	T-value	
Australia	0.007	0.53	0.014	1.15	0.050	3.93	35.4
Canada	0.051	2.89	− 0.019	− 0.56	0.090	3.19	24.2
France	− 0.008	− 0.48	0.048	1.82	0.073	3.46	38.2
Germany	0.123	2.41	− 0.007	− 0.08	0.206	2.91	41.0
Italy	0.025	2.45	0.023	0.86	0.031	1.68	39.6
Japan	0.029	2.06	0.032	1.04	0.026	1.21	15.4
Netherlands	0.051	3.52	0.038	1.61	0.039	2.06	17.1
Switzerland	− 0.015	− 0.67	0.116	3.72	0.029	1.54	60.3
United Kingdom	0.055	3.04	0.048	1.67	0.067	3.35	35.8
United States	0.069	3.43	0.081	3.31	0.040	1.61	29.7