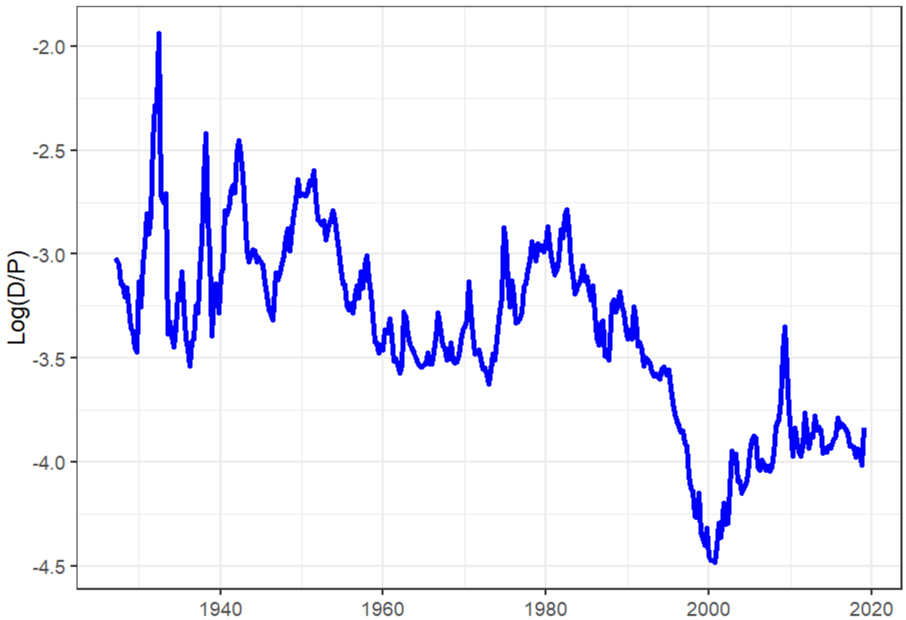

Figure 1: Dividend yield (sum of past years worth of dividends divided by current portfolio price)

1 Problem 6

a) We first run two regressions separately of the form

$$r_{t,t+\tau} = \alpha_r + \beta_{r,\tau}(d_t - p_t) + \epsilon^r_{t+\tau}$$
$$\Delta d_{t,t+\tau} = \alpha_{\Delta_d} + \beta_{\Delta d,\tau}(d_t - p_t) + \epsilon^d p_{t+\tau}$$

the results of which are shown in Table (1). The results suggest all variation in market price-dividend ratios corresponds to changes in expected variation in risk premiums and none about news to future dividend growth.[1]

Next, we calculate the implied regression coefficients from our VAR and the Campbell Shiller linearization of a return

$$r_{t+1} \approx \rho + \kappa(p_{t+1} - d_{t+1}) + (d_{t+1} - d_t) - (p_t - d_t)$$

The implied coefficient on returns is given by

$$\beta_{r,\tau} = \frac{Cov(r_{t,t+\tau} - \tau\bar{r}, e_1' X_t)}{Var(e_1' X_t)}$$

[1]Recall, we can decompose the log price dividend ratio, denoted as z_t here, as

$$Var(z_t) = \sum_{j=0}^{\infty} \kappa_1^j \left[Cov(g_{t+1+j}, z_t) - Cov(r_{t+1+j}, z_t)\right]$$

Thus, if both returns and dividends are unforecastable, the price divdend ratio should be constant, which it clearly is not in Figure 1. We cannot ask "are returns forecastable?", rather we must ask "which of dividend growth or returns are forecastable?" A null hypothesis that specifies returns are not forecastable must also that dividend growth is forecastable. That is we need to test the joint distribution of return and dividend-growth forecastability.

τ	b	t(b)	r2
1	0.078	1.611	0.029
3	0.211	3.050	0.092
5	0.329	5.056	0.173

τ	b	t(b)	r2
1	0.005	0.135	0.0003
3	-0.012	-0.201	0.001
5	-0.005	-0.072	0.0001

Table 1: Direct return regressions (left panel) and dividend growth regressions (right panel)

We can solve for $r_{t+k} - \bar{r}$ as follows

$$\begin{aligned} r_{t+k} &= [-\kappa \quad 1 \quad 0]\,(X_{t+k}) + e_1(X_{t+k-1} - \bar{X}) \\ &= [-\kappa \quad 1 \quad 0]\,(A(X_{t+k-1}) + \epsilon_{t+k}) + e_1(X_{t+k-1}) \\ &= ([-\kappa \quad 1 \quad 0]A + e_1)X_{t+k-1} + [-\kappa \quad 1 \quad 0]\epsilon_{t+k} \\ &= [-\kappa \quad 1 \quad 0]A^k X_t + \sum_{j=0}^{k-1}[-\kappa \quad 1 \quad 0]A^j \epsilon_{t+k-j} + [-\kappa \quad 1 \quad 0]\epsilon_{t+k} \end{aligned} \tag{1}$$

where the last equality holds by backwards iteration and using our VAR. Thus, $r_{t,t+\tau} - \tau\bar{r}$ is given by

$$\begin{aligned} r_{t,t+\tau} - \tau\bar{r} &= \sum_{k=1}^{\tau}(r_{t+k} - \bar{r}) \\ &= \sum_{k=1}^{\tau}\left([-\kappa \quad 1 \quad 0]A^k(X_t) + \sum_{j=0}^{k-1}[-\kappa \quad 1 \quad 0]A^j \epsilon_{t+k-j}\right) \end{aligned}$$

Using equation (1), we can solve for our coefficient of interest.

$$\begin{aligned} \beta_{r,\tau} &= \frac{Cov(r_{t,t+\tau} - \tau\bar{r}, e_1'X)}{Var(e_1'X)} \\ &= \frac{Cov(\sum_{k=1}^{\tau}[-\kappa \quad 1 \quad 0]A^k(X_t), e_1'X)}{Var(e_1'X)} \end{aligned}$$

Similarly, our coefficient for dividend growth is given by

$$\begin{aligned} \beta_{\Delta d,\tau} &= \frac{Cov(\Delta d_{t+\tau}, e_1'X)}{Var(e_1'X)} \\ &= \frac{Cov(e_2'A^\tau X, e_1'X)}{Var(e_1'X)} \end{aligned}$$

where the equality holds, because dividend growth over τ periods implied by the VAR will just be $e_2'A^\tau X$.