University of Pennsylvania
The Wharton School

Professor Gomes — Problem Set 1

Finance 924

**PART I: Consumption Choices**

(1) Consider the problem of a consumer that maximizes:

$$U = E_0\left[\sum_{t=0}^{\infty}\beta^t((a+\theta_t)c_t - bc_t^2)\right]$$
$$s.t. \qquad k_{t+1} + c_t = Rk_t + y_t$$

where $R = 1/\beta$ and the shock $\theta$ captures taste shocks to the marginal utility of consumption

(a) Derive the first order conditions for this problem.

(b) Assuming $\theta_t$ follows an AR(1) process with mean 0, characterize the process for optimal consumption.

(c) What is the process followed by consumption when $\theta_t$ is a random walk?

(d) Now suppose $R < 1/\beta$, and $\theta$ is a random walk. What would be the process of the optimal consumption?

(2) Consider the optimization problem of an individual, that, at any point in time, can invest in her stock of human capital, $h_t$, to get a constant rate of return, $A_t$, in units of consumption good:

$$V = E_0\left[\sum_{t=0}^{\infty}\beta^t \log(c_t)\right]$$
$$s.t. \qquad h_{t+1} + c_t = A_t h_t$$

(a) Suppose $A_t$ follows a Markov process. Write down the recursive formulation of this problem. Hint: The relevant state variables are $h_t$ and $A_t$.

(b) Guess/verify the following functional form for the value function:

$$V(h, A) = \alpha \log(h) + v(A)$$

where $\alpha$ is a constant and $v(\cdot)$ is an increasing function. What is the value of $\alpha$? What is the process for $v(.)$?

(c) What are the optimal consumption and human capital investment choices?

(d) Suppose now $\log(A)$ is i.i.d. normal, with mean $\mu_a$ and standard deviation $\sigma$. What is the expected growth rate of human capital? What is the process of consumption growth? (Note: Work with the gross growth rates of a variable $x$: $\gamma_x = \log(x'/x)$)

## PART II: Numerical Methods

Consider the following problem

$$\begin{aligned} U &= E_0\left[\sum_{t=0}^{\infty}\beta^t u(c_t)\right] \\ s.t \quad c_t &= y_t + Rk_t - k_{t+1}\ \forall t \\ k_0 &> 0 \end{aligned}$$

(1) Assume that $\log(y)$ follows

$$\log(y_{t+1}) = 0.05 + 0.95\log(y_t) + 0.1\varepsilon$$

where $\varepsilon$ is an i.i.d shock that follows a truncated standardized normal distribution in [-4,4]

(a) Construct a 5 and a 9 point discrete Markov Chain approximation to the process for $y$ process, using a grid with equally spaced that is centered around the long run mean.

(b) Simulate your Markov chains. Compute the long run mean, serial correlation and volatility of this simulated process and check the accuracy of your approximation after 1000, 5000 and 10000 periods.

(2) Suppose that preferences are CES/CRRA with an IES of 0.5. Assume $\beta = 0.95$ and $R = 1.04$. For what follows use a grid of 9 points for $y$.

(a) Compute the value function for this problem assuming a discrete grid of 51 and then 101 equally spaced points for $k$.

(b) Plot the value function for this problem as a function of current $k$, holding $y$ at its mean. Plot the same value function when $y$ is one standard deviation above the mean.

(c) Plot optimal consumption, $c$ and saving, $k'$, decisions as a function of $k$ holding $y$ at its mean. Plot the same functions when $y$ is one standard deviation above the mean.

(d) Simulate this economy for 5500 and 10500 periods. Drop the first 500 observations in each simulation. Compare the time series average, standard deviation and persistence of consumption growth across the two simulations.