FNCE Problem Set 1

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Problem 1

a) Consumer maximizes

$$U = E_0 \left[\sum_{t=0}^{\infty} \beta^t (u(a + \theta_t)c_t - bc_t^2) \right]$$

subject to $c_t = Rk_t + y_t - k_{t+1}$. We get the Lagrangian

$$\mathcal{L} = u(c) + \beta E \left[V(k', y', R') \right] + \lambda \left[Rk + y - c' - k' \right] \tag{1}$$

First order conditions imply

$$\begin{aligned} u_c(c) - \lambda &= 0 \\ \beta E_{y,R} \left[V_k(k', y', R') \right] - \lambda &= 0 \\ V_k(k, y, R) &= R\lambda \end{aligned} \tag{2}$$

The FOCs with respect to c and k imply the Euler equation

$$u_c(c) = \beta E_{y,r} V_k(k', y', R) \tag{3}$$

Iterating forward one time period on the FOC with respect to k gives

$$\begin{aligned} u_c(c) &= \beta E_{y,R} R u_c(c') \\ &= E_y u_c(c') \end{aligned} \tag{4}$$

where $u_c(c) = (a + \theta) + 2bc$, so our Euler equation is $a + \theta_t - 2bc_t = E_t \left[a + \theta_{t+1} - 2bc_{t+1} \right]$.

b) The Euler equation implies

$$\begin{aligned} c_t &= E_t c_{t+1} + (\frac{(1 - \phi)\theta_t}{2b}) \\ &= E_t \left[c_{t+s} \right] + \frac{\theta_t}{2b} \end{aligned} \tag{5}$$

The lifetime budget constraint is