FNCE 924 — Problem Set 2

Felix Nockher and Patrick Shultz

April 16, 2019

Problem a)

a1) Average growth rate γ

We use the following series:

Y — GDPC1: Real GDP Seasonally Adjusted Annual Rate

I — GPDIC1: Real Gross Private Domestic Investment Seasonally Adjusted Annual Rate

C — PCECC96: Real Personal Consumption Expenditures Seasonally Adjusted Annual Rate

The average from 1955-Q1 over these three series produces an annual $\gamma_a = 1.0331$ and a quarterly $\gamma_q = \sqrt{\gamma_a} = 1.0082$.

a2) Average capital depreciation rate δ

We use the following equation:

$$\frac{i}{k} = \frac{(1-\alpha)(\gamma - (1-\delta))}{\frac{1}{\beta} - (1-\delta)}$$

which is derived from the FOC w.r.t. capital accumulation in steady state from the detrended problem:
detrended

$$\mathcal{L} = \sum_{t=0}^{\infty} (\beta^\star)^t \{ E_0 \left[u(c,H) + \lambda (Ak^{1-\alpha} H^\alpha - c - k'\gamma + (1-\delta)k) \right]$$

where $\lambda = \frac{\Lambda}{\gamma_N}$ and $\beta^\star = \beta \gamma_N$.
FOCs

$$\begin{aligned}
[c] \qquad & u_c(c,H) = \lambda \\
[H] \qquad & -u_H(c,H) = \lambda \alpha A k^{1-\alpha} \\
[k'] \qquad & 1 = \beta^\star E_0 \left[\frac{\lambda'}{\lambda} \left((1-\alpha) A k'^{-\alpha} H'^\alpha + 1 - \delta \right) \right] \\
[\lambda] \qquad & Ak'^{1-\alpha} H^\alpha = c + k' - k(1-\delta)
\end{aligned}$$

Then from the Euler equation $[k']$ where in steady state $y = y'$ and $k = k'$ we get

$$\frac{y}{k} = \frac{\frac{1}{\beta} - (1-\delta)}{1-\alpha}$$

Note that we are back to β since in the FOC we have $beta^\star/\gamma$.

a3) Time series for quarterly number of hours worked

We use the series AWHMAN (Average Weekly Hours of Production and Nonsupervisory Employees: Manufacturing) from FRED. We multiply the weekly figure by four to get monthly values and then add the respective 3 months to generate corresponding quarterly values. The time series mean is 488.3 hours a quarter.

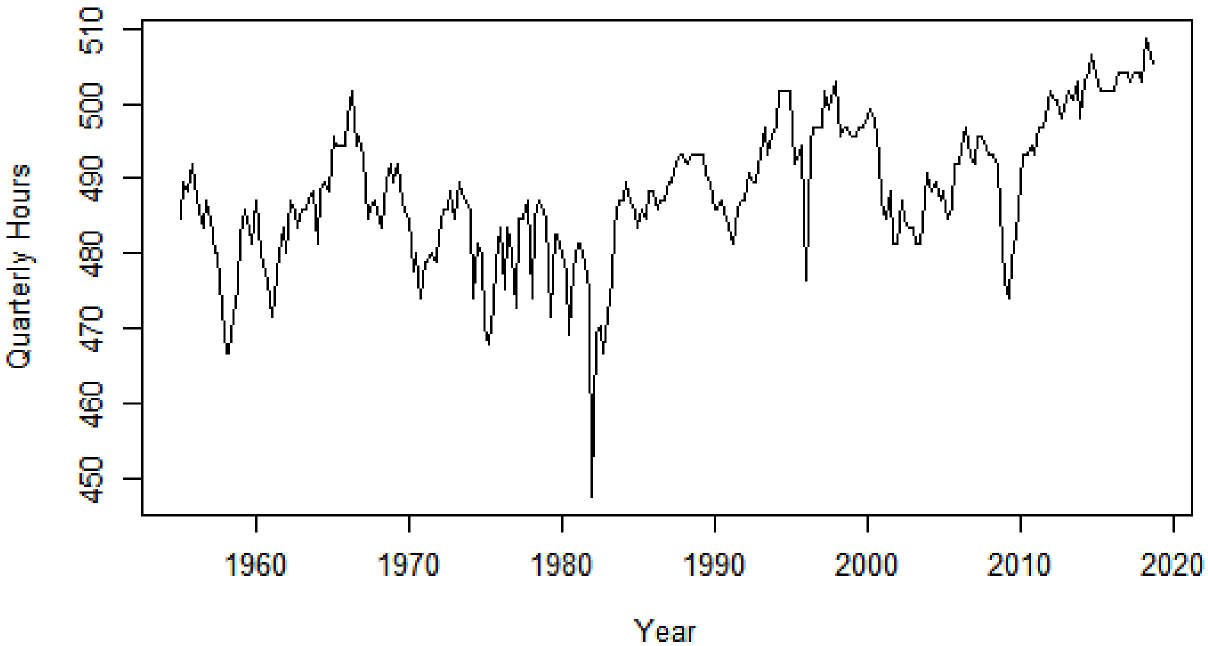

Figure 1: Quarterly Number of Hours Worked, H

a4) Capital stock

We use the following equation (perpetual inventory method): $K_{t+1} = I_t + (1 - \delta)K_t$:

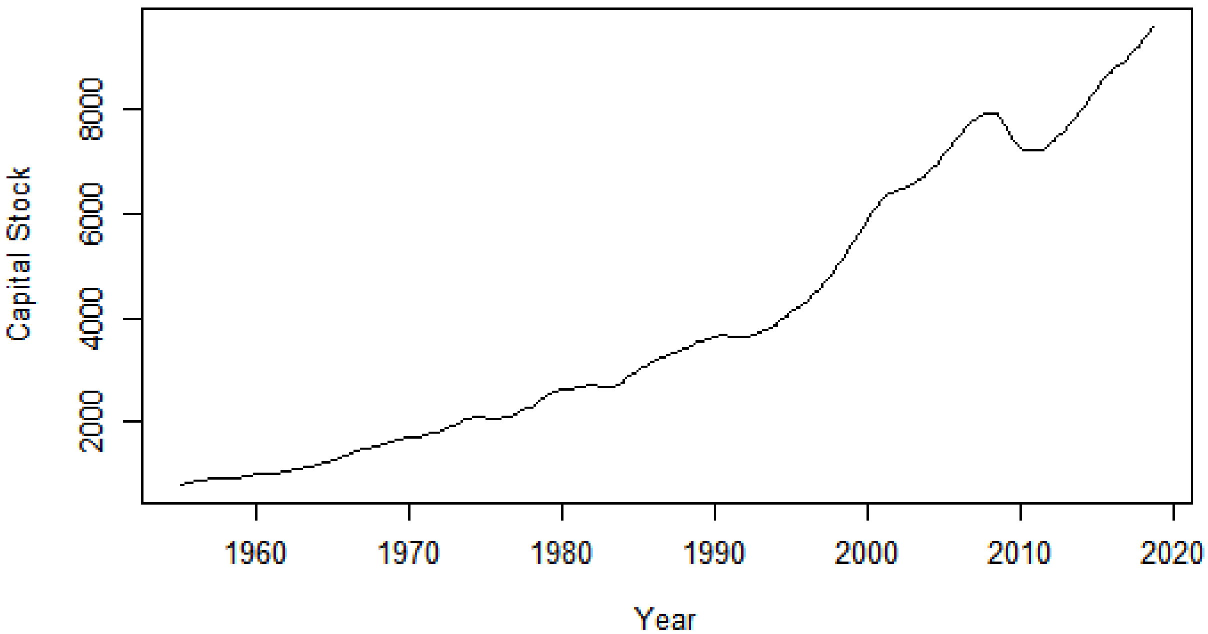

Figure 2: Capital Stock, K

a5) Solow residual (TFP)

We use CRS with $\alpha = 2/3$ and the following equation:

$$log(A) = log(Y) - (1 - \alpha)log(K) - \alpha log(H)$$

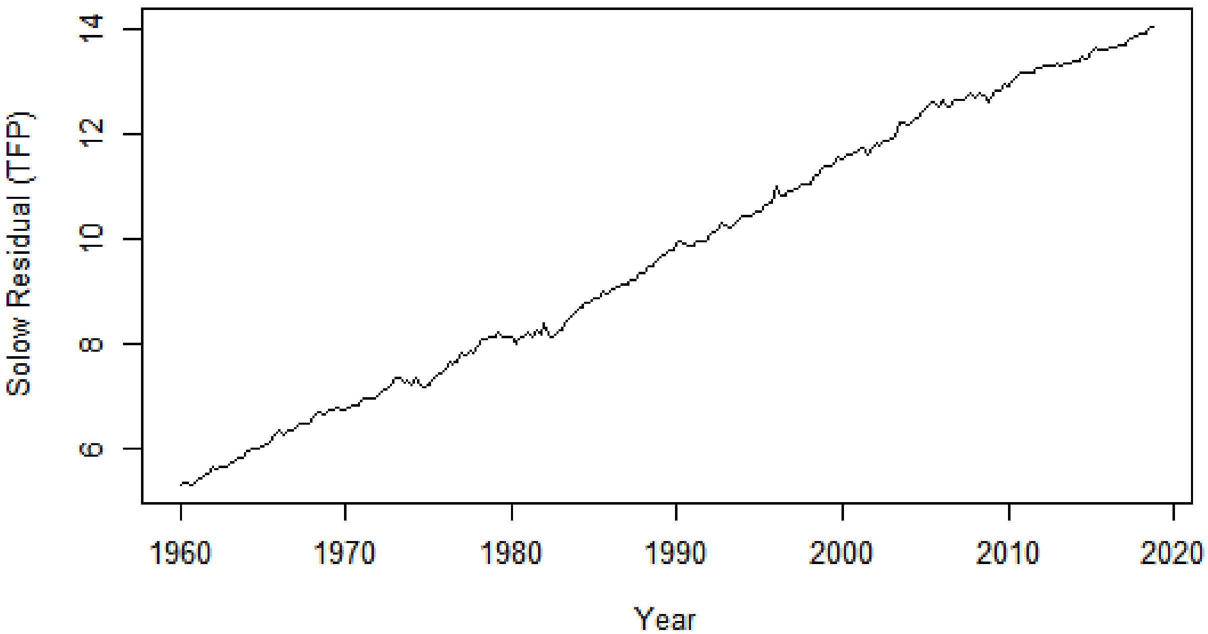

Figure 3: Solow Residual, A

Problem b)

We detrended the log of the series of Y, I, C, K, and H via the HP filter and a frequency of 1600. Note that we also used the filter for the H series to get the cycle component only even though the series has no actual trend.

b1) Second moments

For the filtered series we get:

Table 1: Comparing Second Moments

	y	c	i	h
Sigma(.)	1.44658277806168	1.176	6.541	1.044
Sigma(.)/Sigma(y)		0.813	4.521	0.722
Rho(.,y)		0.871	0.901	0.670

b2/b3) AR(2)/AR(1) process for the Solow Residual

We see that we are very close to rejecting the null of $\varphi_2 = 0$ to the 5% level with a t-stat of 1.95. We reject the null that the $varphi_1$ is zero to the 1% level for both suggested processes. The standard deviation for the AR(1) innovation, σ_a is approx. 0.6%.

Table 2: Autoregression

	Coeff	tstat	Rho_a	sigma_a
AR(2)-1	0.526	8.108		
AR(2)-2	0.126	1.953		
AR(1)-1	0.601	11.591	0.601481835416286	0.00662892932563865

Problem c)

c1) FOCs for KPR preferences

Note that the general outline of the detrended problem can be found in Equation (**??**). We plug in the respective derivatives w.r.t. c and H of $u(c, H) = log(c) + \psi log(1 - H)$:

$$
\begin{aligned}
&[c] && \frac{1}{c} = \lambda \\
&[H] && -(-\frac{1}{1-H}\psi) = \lambda \alpha A k^{1-\alpha} \\
&[k'] && 1 = \beta^{\star} E_0 \left[\frac{\lambda'}{\lambda} \left((1-\alpha) A k'^{-\alpha} H'^{\alpha} + 1 - \delta \right) \right] \\
&[\lambda] && A k'^{1-\alpha} H^{\alpha} = c + k' - k(1-\delta)
\end{aligned}
$$

c2) Calibrate for ψ

We use the FOC w.r.t. hours worked, H, and replace λ via the FOC w.r.t. c and note that $Ak^{1-\alpha}H^{\alpha-1}\alpha = \frac{y}{H}\alpha$:

$$\psi = \frac{1-H}{H}\frac{Y}{H} = \frac{1-H}{H}\frac{y}{c}\alpha$$

Note that we first calculated the daily average of H by dividinng it by $(3months * 4weeks * 7days * 24h)$ such that is a percentage number of hours worked per day. We then calculate both fractions over time and then plug in the average over time to get $psi = 3.24$.

e) GHH preferences

The partials of the GHH utility are:

$$
\begin{aligned}
u_C &= \frac{1}{C - \psi \frac{H^{1+\nu}}{1+\nu}} \\
u_H &= \frac{-\psi H^{\nu}}{C - \psi frac H^{1+\nu} 1 + \nu}
\end{aligned}
$$

As in Subsection we then use the FOC of the problem w.r.t. hours worked, this time not debased to a percentage figure, and replace λ via the result of the FOC w.r.t. consumption:

$$\psi = \frac{1}{H^{\nu}}\frac{Y}{H}\alpha$$

We detrend Y over time by dividing it by population (both in the same units, i.e. Y upscaled from billions), deploying $\nu = 0.5$, and taking the time series average of the ratio to get $\psi = 3.65$.

Table 3: Comparing Second Moments

	y	c	i	h
Sigma(.)	1.44658277806168	1.176	6.541	1.044
Sigma(.)/Sigma(y)		0.813	4.521	0.722
Rho(.,y)		0.871	0.901	0.670

d) Use Dynare to solve the model

We define the equations in our model as

1. $\frac{1}{c_t} = \lambda_t$
2. $\gamma = \beta^\star E_t \left[\frac{c_t}{c_{t+1}} \left((1-\alpha) A_{t+1} k_{t+1}^{-\alpha} H_{t+1}^{\alpha} + (1-\delta) \right) \right]$
3. $\frac{\psi}{1-H_t} = \frac{1}{c_t} \left(\alpha A_t k_t^{1-\alpha} H_t^{\alpha-1} \right)$
4. $A_t k_t^{1-\alpha} H_t^{\alpha} + (1-\delta) k_t - k_{t+1}\gamma = c_t$
5. $\log A_{t+1} = \rho A_t + \sigma_A \epsilon$
6. $y_t = A_t k_t^{1-\alpha} H_t^{\alpha}$
7. $y_t = c_t + i_t$
8. $w_t = \alpha A_t k_t^{1-\alpha} H_t^{\alpha-1}$
9. $r_t = (1-\alpha) A_t k_t^{\alpha}$

We then solve for the steady states of nine unknowns: $r, i, y, c, w, \bar{a}, \psi, \lambda, k$. We can also use the equations above to solve for the deterministic steady state values that will be used as initial values in dynare. Figure 4 displays the impulse response functions for output, consumption, investment, and hours to a positive TFP shock.

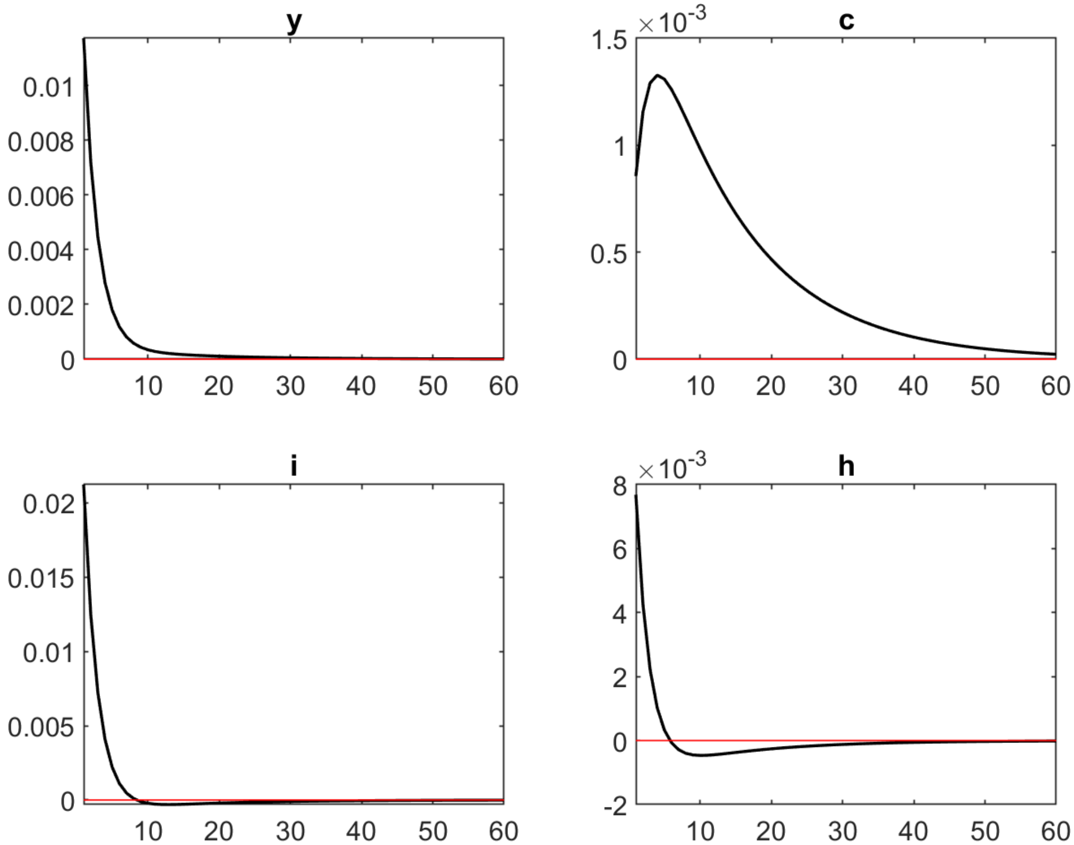

Figure 4: Impulse responses of output, consumption, investment, and hours to a positive TFP shock using KPR preferences

Variable	$\sigma(\cdot)$	$\sigma(\cdot)/\sigma(y)$
y	0.0126	1
i	0.0229	1.82
c	0.0018	0.14
h	0.0084	0.66
w	0.0044	0.35

Table 4: Standard deviations of relevant variables

Comparing to Table 3, we see that the model matches the variance of output and hours well. However, the model fails to capture the volatiltiy of investment and consumption. When looking at the ratio of the standard deviations relative to output, we see that the model captures the relative variances of hours worked and investment, but misses the fact that consumption is less volatile than output.

Order	1	2	3	4	5
a	0.4653	0.1538	-0.0220	-0.1153	-0.1587
y	0.4699	0.1596	-0.0167	-0.1113	-0.1563
i	0.4612	0.1486	-0.0268	-0.1189	-0.1610
k	0.8978	0.7029	0.4797	0.2639	0.0738
c	0.8755	0.6747	0.4539	0.2444	0.0619
h	0.4547	0.1404	-0.0343	-0.1246	-0.1644
r	0.4536	0.1389	-0.0356	-0.1256	-0.1650
w	0.5390	0.2473	0.0634	-0.0508	-0.1191
lambda	0.8755	0.6747	0.4539	0.2444	0.0619

Table 5: Autocorrelations

Variables	a	y	i	k	c	h	r	w	λ
a	1.0000	0.9996	0.9995	0.3797	0.4625	0.9946	0.9820	0.9609	-0.4625
y	0.9996	1.0000	0.9983	0.4059	0.4876	0.9912	0.9762	0.9684	-0.4876
i	0.9995	0.9983	1.0000	0.3514	0.4353	0.9973	0.9873	0.9520	-0.4353
k	0.3797	0.4059	0.3514	1.0000	0.9958	0.2815	0.1981	0.6211	-0.9958
c	0.4625	0.4876	0.4353	0.9958	1.0000	0.3678	0.2866	0.6900	-1.0000
h	0.9946	0.9912	0.9973	0.2815	0.3678	1.0000	0.9963	0.9269	-0.3678
r	0.9820	0.9762	0.9873	0.1981	0.2866	0.9963	1.0000	0.8912	-0.2866
w	0.9609	0.9684	0.9520	0.6211	0.6900	0.9269	0.8912	1.0000	-0.6900
λ	-0.4625	-0.4876	-0.4353	-0.9958	-1.0000	-0.3678	-0.2866	-0.6900	1.0000

Table 6: Correlations

e)

We now assume that preferences take the form

$$u(C,L) = \log\left(C - \psi\frac{H^{1+\nu}}{1+\nu}\right)$$

Our model becomes

1. $\frac{1}{c_t - \psi\frac{H_t^{1+\nu}}{1+\nu}} = \lambda_t$
2. $\gamma = \beta^\star E_t\left[\frac{\lambda_{t+1}}{\lambda_t}\left((1-\alpha)A_{t+1}k_{t+1}^{-\alpha}H_{t+1}^{\alpha} + (1-\delta)\right)\right]$
3. $\psi H_t^{\nu} = \alpha A_t k_t^{1-\alpha}H_t^{\alpha-1}$
4. $A_t k_t^{1-\alpha}H_t^{\alpha} + (1-\delta)k_t - k_{t+1}\gamma = c_t$
5. $\log A_{t+1} = \rho A_t + \sigma_A\epsilon$
6. $y_t = A_t k_t^{1-\alpha}H_t^{\alpha}$
7. $y_t = c_t + i_t$
8. $w_t = \alpha A_t k_t^{1-\alpha}H_t^{\alpha-1}$
9. $r_t = (1-\alpha)A_t k_t^{\alpha}$

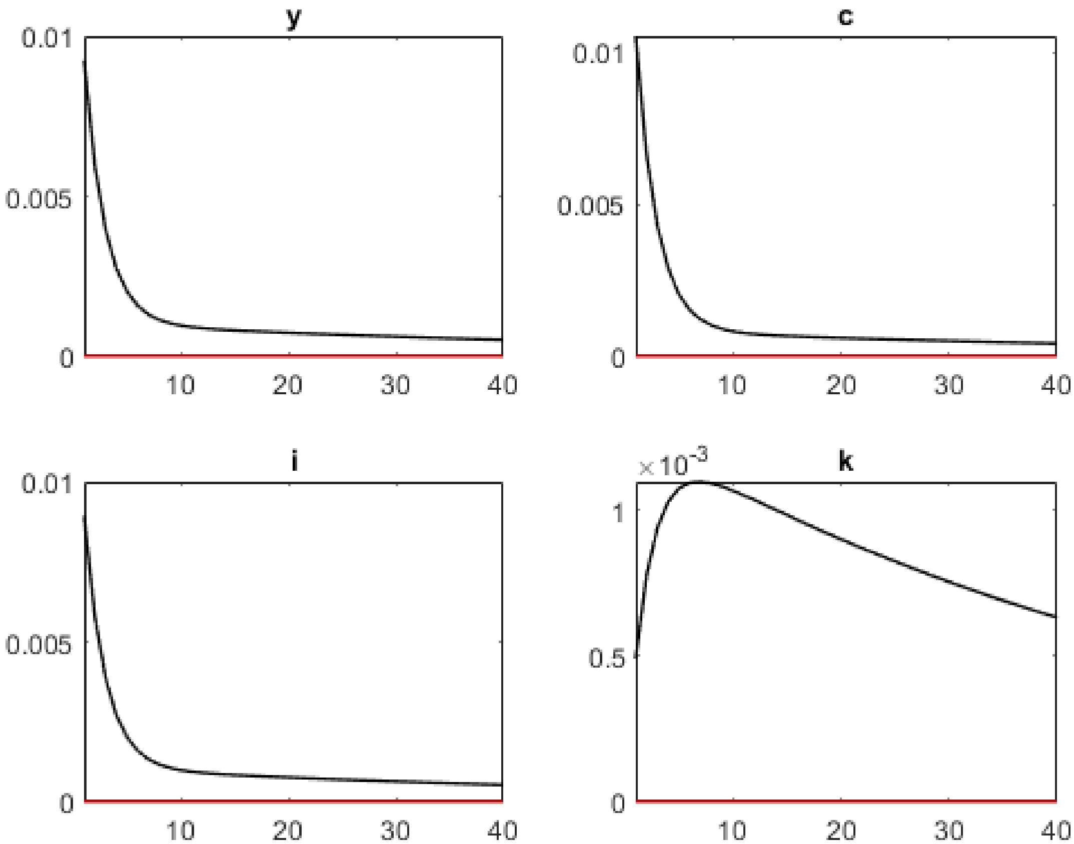

Figure 5: Impulse responses of output, consumption, investment, and hours to a positive TFP shock using GHH preferences

Variable	$\sigma(\cdot)$	$\sigma(\cdot)/\sigma(y)$
y	0.00982	1
i	0.00948	0.965
c	0.011	1.1
h	0.0085	0.86
w	0.0042	0.43

Table 7: Standard deviations of relevant variables

Variables	a	y	i	k	c	h	r	w	λ
a	1.0000	0.9952	0.9944	0.2572	0.9975	0.9984	0.9993	0.9984	-0.0165
y	0.9952	1.0000	1.0000	0.3503	0.9996	0.9992	0.9909	0.9992	-0.1141
i	0.9944	1.0000	1.0000	0.3578	0.9994	0.9988	0.9898	0.9988	-0.1220
k	0.2572	0.3503	0.3578	1.0000	0.3245	0.3116	0.2211	0.3116	-0.9705
c	0.9975	0.9996	0.9994	0.3245	1.0000	0.9999	0.9942	0.9999	-0.0867
h	0.9984	0.9992	0.9988	0.3116	0.9999	1.0000	0.9956	1.0000	-0.0732
r	0.9993	0.9909	0.9898	0.2211	0.9942	0.9956	1.0000	0.9956	0.0207
w	0.9984	0.9992	0.9988	0.3116	0.9999	1.0000	0.9956	1.0000	-0.0732
λ	-0.0165	-0.1141	-0.1220	-0.9705	-0.0867	-0.0732	-0.0732	0.3156	0.1208

Table 9: Correlations

Order	1	2	3	4	5
a	0.4653	0.1538	-0.0220	-0.1153	-0.1587
y	0.4732	0.1640	-0.0125	-0.1079	-0.1539
i	0.4743	0.1653	-0.0113	-0.1070	-0.1533
k	0.9041	0.7192	0.5050	0.2951	0.1075
c	0.4701	0.1600	-0.0162	-0.1108	-0.1558
h	0.4689	0.1584	-0.0178	-0.1120	-0.1566
r	0.4646	0.1529	-0.0229	-0.1160	-0.1592
w	0.4689	0.1584	-0.0178	-0.1120	-0.1566
λ	0.9260	0.7474	0.5313	0.3156	0.1208

Table 8: Autocorrelations

f) Government expenditure

f1) Time series average G/Y

We use the quarterly FRED series W068RCQ027SBEA (Government total expenditures - Seasonally adjusted annual rate). We get a time series average of $\frac{G}{Y} = 20.0\%$ for the data since inception, i.e. 1960-Q1.

f2) AR(1) of the HP filtered series

Table 10: Government Autoregression

rho_g	t_stat_g	sigma_g
0.785	19.407	0.010

Appendix - Code

Data preparation