University of Pennsylvania
The Wharton School

Professor Gomes — Problem Set 2

Finance 924

PART I: Asset Prices with Complete Markets

Consider the Lucas (1978) asset pricing model and assume consumers have CRRA preferences with risk aversion γ and discount rate β. Suppose that income/divided/endowment growth obeys

$$\log(y_{t+1}/y_t) = g + \epsilon_{t+1}$$

where ϵ_{t+1} is i.i.d. $N(0, \sigma)$.

- Compute the market price, Q_1^b of a risk free bond that is purchased in period t and pays exactly one unit of consumption in period $t+1$.
- Next, compute the market price, Q_j^b of a risk free bond that is purchased in period t and pays exactly one unit of consumption in period $t+j$ for any maturity j.
- Define the implied annual yield on this bond as $(R_j^b)^{-j} = Q_j^b$. Characterize the term structure of these yields at time t. Specifically, is this increasing or decreasing in j?
- How can we change the model to match the fact that this term structure varies over time (often but not always increasing in time horizon)?

PART II: Computing Asset Prices in Incomplete Markets

Consider an economy populated by 5000 households with identical CRRA preferences with risk aversion equal to 2 and discount factor equal to $\beta = 0.97$. Suppose financial markets are incomplete in the sense that each household can only invest in a one period risk free bond, that pays gross interest R. The budget constraint for each household i is given by

$$k_{t+1} + c_t = y_t + Rk_t \qquad \forall t$$

where $k_{t+1} \geq -1$ so no household is allowed to borrow too much. Assume also that household income follows the process

$$\log y_t = \rho \log y_{t-1} + \sigma(1-\rho^2)^{1/2}\epsilon_t$$

with $\rho = 0.6, \sigma = 0.35$ and ϵ is normal iid with mean 0 and variance 1.

- Suppose $R = 1.01$ Solve the problem of each household numerically using your favorite approach. Plot the optimal policies for consumption and capital accumulation against the current level of capital for two different levels of current income, y.
- Simulate the economy for 2500 periods. After dropping the first 500 observations compute the aggregate demand for assets in the economy in each period by summing over the optimal choices $k'(k, y)$.
- If the (average) aggregate demand does not add up to 0, readjust the value of the initial guess for R as necessary until you determine the *competitive equilibrium* in this economy. Hint: a good idea is to employ a simple bisection method.
- [extra crdit!] Take your solution to R in the previous part as given, but replace the borrowing constraint with $k_{t+1} \geq -2$. What happens to aggregate demand? Is it positive or negative? Discuss the intuition.

PART III: Investment and Asset Prices

The profit function, adjustment cost and capital accumulation equations for an infinitely lived firm are given by the linear homogeneous expressions:

$$\begin{aligned}
\Pi_t &= A_t K_t, \\
\Phi_t &= \gamma \left[\frac{I_t}{K_t} - \eta_t \right]^2 K_t, \\
I_t &= K_{t+1} - (1 - \delta) K_t
\end{aligned}$$

(a) Compute the one period gross return to the accumulation of physical capital, $R^K_{t,t+1}$. Note: this can be constructed by reworking the optimal first order condition for capital accumulation by the firm to obtain the standard asset pricing equation:

$$1 = \mathrm{E}_t \left[M_{t,t+1} R^K_{t,t+1} \right]$$

where $M_{t,t+1}$ is the stochastic discount factor.

(b) Recall the expression for firm value is derived from the fact that $\mathrm{E}_t \left[M_{t,t+1} R^S_{t,t+1} \right] = 1$, where $R^S_{t,t+1}$ is the return to the firm's equity. Show that, under the assumptions of linear homogeneity above, we can further establish that $R^S_{t,t+1} = R^K_{t,t+1}$.