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Problem Set 4

Problem 1: Stochastic Growth Model

Consider the following problem:

$$\begin{aligned} V(A,K) &= \max_{C,K',L} Nu(C,H) + \beta \mathbb{E}V(A',K'), \\ \text{s.t. } C &= AK^{1-\alpha}(NH)^{\alpha} + (1-\delta)K - K', \\ \log A' &= \rho_a \log A + \sigma_a \epsilon', \ \epsilon \sim N(0,1). \end{aligned}$$

In what follows assume that $\beta = 0.99$

a) Download US quarterly data on gdp, investment, consumption since about 1947 from the FRED website. Also download monthly data for the adult population, employment and average hours worked (this might be available for slightly different periods). Construct:

   a1) the average growth rate of the economy, γ by averaging the growth rates of gdp, consumption and investment between 1955 and 2018.

   a2) the average capital depreciation rate, δ, over the same period, by using the capital accumulation equation in steady-state.

   a3) a time series for the *quarterly* number of hours worked per adult person and report its time series mean

   a4) a time series of the US capital stock since 1955 by using the capital accumulation equation (the perpetual inventory method) and investment data starting in 1947

   a5) a time series of Solow residuals, A, by estimating the coefficient on the production function over the same period. Be sure to impose constant returns to scale.

b) Detrend the data using the HP filter with an appropriate smoothing parameter (don't forget to first take log of all the series). Using this data

   b1) Calculate and report the standard deviations (relative to that of output), cross-correlation with output of consumption, hours and investment.

   b2) Fit an AR(2) process to the Solow residual. Can you reject the hypothesis that the coefficient on the second order term is zero?

   b3) Fit an AR(1) process to the Solow residual and estimate ρ_a and σ_a.

c) Assume that $u(C,L) = \log C + \psi \log(1-H)$ (so called "KPR" preferences).

   c1) Derive the first order conditions for the problem.

   c2) Calibrate the value of ψ so that the steady state value of H matches that in your data.

d) Using Dynare, code the model to solve the problem.

   d1) Report impulse response functions of output, consumption, investment, hours to a positive TFP shock.

   d2) Report the model implied moments. Which moments are especially poorly matched by the model?

e) Now assume that $u(C, L) = \log\left(C - \psi\frac{H^{1+\nu}}{1+\nu}\right)$ ("GHH" preferences, with $\nu = 0.5$). Calibrate ψ to match the average number of hours worked in the data and again solve the problem via Dynare. Report the second moments and the impulse response functions. Describe and explain the main differences in comparison with the model with the KPR utility. Which moments are matched better? What are the most problematic moments in this case?

f) Now download data on government expenditures, G_t and compute

f1) The time series average level of G/Y in your sample.

f2) Using HP filtered data, estimate an AR(1) process for the detrended level of g and report its serial correlation, ρ_g and standard deviation, σ_g.

f3) Solve the expanded model using Dynare and report the impulse response functions to a shock in government expenditures. Explain the response of consumption, investment, output and hours to this shock.

f4) Construct a single table reporting the second moments for the data and each of the model versions you solved this version of the model. Which seems to match the moments in the data better?