Partially ordered Set (POSET)

Set P
↓
R be a Relation on set P
↓
Partially ordered relation if satisfies
- → R is Reflexive
- → R is Anti-Symmetric
- → Transitive

If given f^n satisfy above three properties then we can say that given set is POSET

Partial ordering Relation :-

A Relation 'R' on a set 'p' is said to be partial ordering (or) partial ordering relation if and only if R is reflexive, Anti-symmetric and transitive.

→ Partial ordering is denoted by the symbol '$\preccurlyeq$'

Partial ordered set on POSET :-

If $\preccurlyeq$ is a partial ordering on a set 'P', Then the order pair $(P, \preccurlyeq)$ is called a "Partial ordered set or "POSET".

Totally ordered Relation :-

Let $(P, \preccurlyeq)$ is a partially ordered set if ~~∀~~ for every two elements $a, b \in P$, we have either $a \preccurlyeq b$ or $b \preccurlyeq a$ (comparable), then '$\preccurlyeq$' is called a simple ordering (or) linear ordering on 'P' and $(P, \preccurlyeq)$ is called a totally ordered set / simply ordered set or a chain.

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Ques:- A Set $S = \{a, b, c\}$ together with the relation of set inclusion $\subseteq$ is a partial order on $P(S)$, where $P(S)$ is the power set of S

Soln Given that $S = \{a, b, c\}$

The power set of S is given as :-

$P(S) = \{\phi, \{a\}, \{b\}, \{c\}, \{a,b\}, \{b,c\}, \{a,c\}, \{a,b,c\}\}$.

then $(P(S), \subseteq)$ is a poset if it satisfies the following Conditions:-

① Reflexivity:- Since $A \subseteq A \ \forall \ A \in P(S)$ hence it is reflexive

② Antisymmetry :- If $A \subseteq B$ and $B \subseteq A \Rightarrow A = B$

Hence, it is antisymmetry.

③ Transitivity :- If $A \subseteq B$, $B \subseteq C \Rightarrow A \subseteq C$

hence it is transitive

$\therefore$ $(P(S), \subseteq)$ is poset.

Ques:- Let $A = \{2, 3, 6, 12, 24, 36\}$ and R be a relation in A which is defined by "a divides b" then R is poset in A.

Soln $A = \{2, 3, 6, 12, 24, 36\}$

① Reflexivity:- Since $a/a \ \forall \ a \in A$

$\therefore$ It is reflexive

② Antisymmetry:- If a/b and $b/a \ \forall \ a, b \in A \Rightarrow a = b$

③ Transitivity:- Let $a, b, c \in A$ then

if $a/b = b/c$ then a/c

$\therefore$ it is transitive

hence $(A, '/')$ is a POSET.

Comparable Two elements a and b in a POSET $(S, \leq)$ are said to be comparable if either $a \leq b$ or $b \leq a$. Otherwise it is called incomparable.

HASSE DIAGRAM:-

Graphical representation of POSET is known as Hasse diagram.

Procedure for Drawing Hasse Diagram:-

① Draw the directed graph of given relation

② Delete all loop at all vertices i.e

③ Eleminate all edges that are implied by the transitive relation

④ Draw the digraph of a partial order with edges pointing upward so that arrows may be omitted from edges.

⑤ Replace the circles representing the vertices by dots.

Ques:- Draw the Hasse Diagram of the following :-

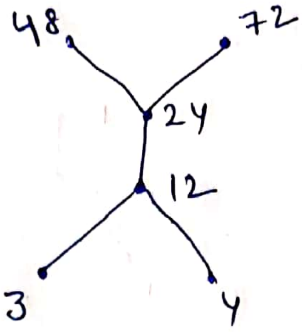
ⓐ Draw the Hasse Diagram of $(A, \leq)$ where $A = \{3, 4, 12, 24, 48, 72\}$ and the relation $\leq$ be s.t $a \leq b$ if a divides b.

ⓑ Draw the Hasse Diagram of the relation S defined all "divides" on the set B where $B = \{2, 3, 4, 6, 12, 36, 48\}$

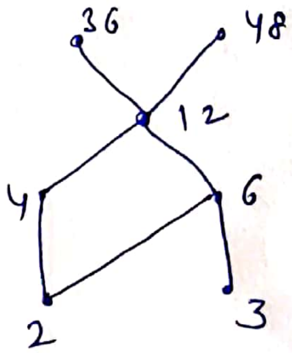
Sol^n The Hasse diagram is given as:-

ⓐ $R = \{\cancel{(3,4)}, (3,12), (3,24), (3,48), (3,72), (4,12), (4,24), (4,48), (4,72), (12,24), (12,48), (12,72), (24,48), (24,72)\}$

$R = \{(3,12), (4,12)\ (12,24), (24,48), (24,72)\}$



11 (b)



Component of Poset :-

① Maximal Element:- a is maximal in $(A, \leq)$ if there is no $b \in A$ s.t $a \leq b$ i.e; (all top elements of the Hass Diagram.

② Minimal Element:- a is minimal in $(A, \leq)$ if there is no $b \in S$ s.t $b \leq a$
i.e; (all the bottom elements of the Hasse Diagram)

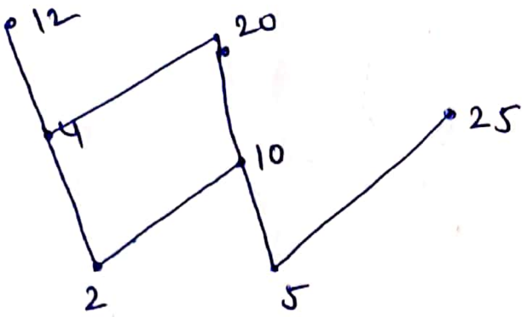
③ Greatest Element:- a is the greaest element of $(A, \leq)$ if $b \leq a$ for all $b \in S$. It must be unique.

\# If more than one maximal elements in a Hasse diagram, then there is no. greatest element in the POSET.

④ Least Element:- a is the least element of $(A, \leq)$ if $a \leq b$ for all $b \in S$. It must be unique.

\# If there is more than one minimal elements in a Hasse Diagram, then there is no least element in POSET

Ques:. which element of POSET are $\{\{2,4,5,10,12,25\}, /\}$ Maximal, minimal, Greatest and least elements.



Maximal = 12, 20, 25

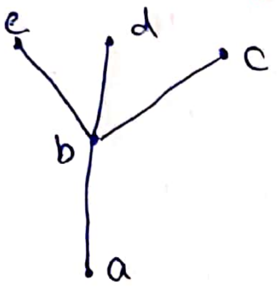
Minimal = 2, 5

Greatest = None

Least = None

Ques: find the maximal, minimal, Greatest & Lowest elements of the following diagram :-

(i)



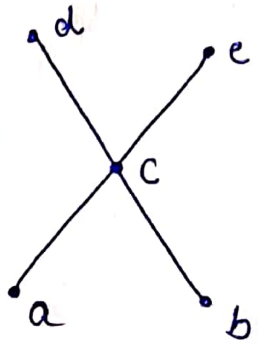
Maximal = b, c, d

Minimal = a

Greatest = None

Least = a

(ii)



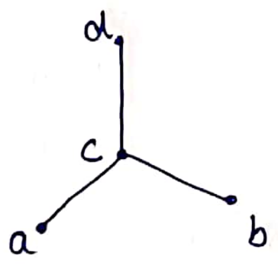
Maximal = d, e

Minimal = a, b

Greatest = None

Least = None

(iii)



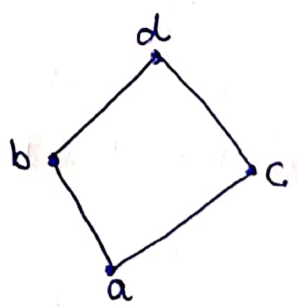
Maximal = d

Minimal = a, b

Greatest = d

Least = None

(iv)



Maximal = d

Minimal = a

Greatest = d

Least = a