Propositions

→ True
→ False

It is a declarative Sentence that is either true or False, but not both

Denoted by lowercase letters p, q, r, ----etc.
The truth value of a propostion is true, then it is denoted by T or 1

False proposition is denoted by F or 0

Logical Connections (operations/operators)

These are words or phase or Connections or links used to Combine two or more sentences

1. Disjunctions (OR) $\vee$
2. Conjunctions (AND) $\wedge$
3. Negation (NOT) $\neg$ ($\sim$)
4. Exclusive or (XOR) $\oplus$
5. Implication (if - then) $\rightarrow$ or $\Rightarrow$
6. Biconditional (if and only if / iff) $\longleftrightarrow$ or $\Leftrightarrow$

Truth table can be used to show how these operators can combine propositions to compound propositions.

1. Disjunction (OR) :- ($\vee$ or $\downarrow$)

* Let p and q are two propositions
* The Disjunction of p and q logically denoted by $p \vee q$ (p or q)
* The Disjunction $p \vee q$ is False when p and q both

are False otherwise its true.

Example:-

Let p : Today is Friday
q : It is a raining today

The disjunction is :-
$p \vee q$: Today is Friday or it is raining today

Truth table

$2^1=2$ p	$2^0=1$ q	$p \vee q$	p	q	$p \vee q$
T	T	T	1	1	1
T	F	T	1	0	1
F	T	T	0	1	1
F	F	F	0	0	0

2. Conjunction (AND) :- ($\wedge$ or $\uparrow$)

- ✓ Let p and q be two propositions
- ✓ The Conjunction of p and q denoted by $p \wedge q$ (p & q)
- ✓ The Conjunction $p \wedge q$ is true when both are true Otherwise its false.

Example:-

Let p : Today is Friday
q : It is raining today

The Conjunction is

$p \wedge q$: Today is Friday and it is raining today.

Truth Table

$2^1=2$ p	$2^0=1$ q	$p \wedge q$	p	q	$p \wedge q$
T	T	T	1	1	1
T	F	F	1	0	0
F	T	F	0	1	0
F	F	F	0	0	0

3. Negation (NOT) (~

The Negation of p denoted by $\neg p$ (or $\sim p$)

Example:- p : Today is Friday
q or($\neg p$) : Today is not Friday

Truth table

p	$\neg p$	p	$\neg p$
T	F	1	0
F	T	0	1

4. Exclusive OR (XOR) :-

Let p and q be two propositions
The Exclusive OR (XOR) p and q denoted by $p \oplus q$
The Exclusive OR is true when exactly one of p and q is true otherwise its False

Example:- p : Neha will pass the course of Mathematics
q : Neha will fail the course of Mathematics

The Exclusive OR is
$p \oplus q$: Neha will pass or fail the course of Mathematic

Truth Table

p	q	p ⊕ q
T	T	F
T	F	T
F	T	T
F	F	F

p	q	p ⊕ q
1	1	0
1	0	1
0	1	1
0	0	0

Ques:- Prepare truth tables for the following propositions

ⓐ $p \wedge (\neg q)$ ⓑ $\neg(p \wedge \neg q)$

Solⁿ ⓐ Truth table

p	q	$\neg q$	$p \wedge \neg q$
T	T	F	F
T	F	T	T
F	T	F	F
F	F	T	F

ⓑ Truth table

p	q	$\neg q$	$p \wedge \neg q$	$\neg(p \wedge \neg q)$
T	T	F	F	T
T	F	T	T	F
F	T	F	F	T
F	F	T	F	T

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Ques:- Give truth tables for the following statements :-

ⓐ $\neg p \vee q$ ⓑ $\sim p \wedge \sim q$

ⓐ

p	$\neg p$	q	$\neg p \vee q$
T	F	T	T
T	F	F	F
F	T	T	T
F	T	F	T

p	$\neg p$	q	$\neg q$	$\sim p \wedge \sim q$
T	F	T	F	F
T	F	F	T	F
F	T	T	F	F
F	T	F	T	T

4. Implication (if - Then) ($\longrightarrow$ or $\Rightarrow$)

OR

Conditional operator (if --- then)

- An implication $p \longrightarrow q$ ($p \Rightarrow q$) is the proposition "If p then q"
- It is false if p is true and q is false otherwise true.

Truth table :-

p	q	$p \to q$	p	q	$p \to q$
T	T	T	1	1	1
T	F	F	1	0	0
F	T	T	0	1	1
F	F	T	0	0	1

Example:-

Assume that your father gave you following promise

If you receive marks of 90% or better in the going semester exam then you will get a car.

q – Got a car

Case I :- If Marks > 90% then got car (T T → T)

Case II :- If Marks > 90% then not got car (T F → F)

Case III :- If Marks < 90% then got car (F T → T)

Case IV :- If Marks < 90% then not got car (F F → T)

Example 2 :-

If you try hard your exam, then you will succeed

p : You tried hard for your exam
q : You succeed.

Case I :- You tried hard for your exam and you succeed (T T → T)

Case II :- You tried hard for your exam but you failed (T F → F)

Case III :- You haven't tried hard for exam but you succeed (F T → T)

Case IV :- You haven't tried for exam & you failed (F F → T)

Example 3 :-

If you have connection with seniors then you will get promoted.

Case I :- You have connection with seniors and you got promoted

(T T → T)

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Case II :- You have Connection with seniors but you can't promoted
(T F → F)

Case III :- You haven't Connection with seniors but you got promotion
(F T → T)

Case IV :- You haven't Connection with seniors & you can't promoted

(F F → T)

Example 4 :-
If you get 100 % marks on the final exam, then will be awarded a Trophy

Case I :- You get 100 %. marks and awarded a Trophy (T T → T)

Case II :- You get 100 %. marks but can't awarded Trophy (T F → F)

Case III :- You can't get 100 %. marks but awarded a Trophy (F T → T)

Case IV :- You can't get 100 %. marks but can't awarded (F F → F)

Ques :- Prepare the truth tables for the following statements
(a) $\sim p \Rightarrow \sim q$ (b) $(p \vee q) \Rightarrow p$

Solⁿ

p	q	$\sim p$	$\sim q$	$\sim p \Rightarrow \sim q$	p	q	$p \vee q$	$(p \vee q) \Rightarrow p$
T	T	F	F	T	T	T	T	T
T	F	F	T	T	T	F	T	T
F	T	T	F	F	F	T	T	F
F	F	T	T	T	F	F	F	T

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Ques: Give the truth tables for the following statements

ⓐ $(p \wedge q) \Rightarrow (p \vee q)$ ⓑ $\neg(p \wedge q) \vee \neg(q \Rightarrow p)$

Sol^n ⓐ

p	q	$p \wedge q$ (A)	$p \vee q$ (B)	$A \Rightarrow B$
T	T	T	T	T
T	F	F	T	T
F	T	F	T	T
F	F	F	F	T

ⓑ

p	q	$p \wedge q$	$\neg(p \wedge q)$ (A)	$q \Rightarrow p$	$\neg(q \Rightarrow p)$ (B)	$A \vee B$
T	T	T	F	T	F	T
T	F	F	T	T	F	T
F	T	F	T	F	T	T
F	F	F	T	T	F	T

Ques:- Prepare the truth table for $p \wedge (q \vee r)$

Sol^n

p	q	r	$q \vee r$ (A)	$p \wedge A$
T	T	T	T	T
T	T	F	T	T
T	F	T	T	T
T	F	F	F	F
F	T	T	T	F
F	T	F	T	F
F	F	T	T	F
F	F	F	F	F

Biconditional Law (if and only if or iff)

Statement of the form "if and if" are called biconditional statements.

It is denoted as $p \leftrightarrow q$ and read as "p if and only if q".

The proposition $p \leftrightarrow q$ is true if p and q have the same truth values and is false if p and q don't have the same truth values.

p	q	$p \leftrightarrow q$	p	q	$p \leftrightarrow q$
T	T	T	1	1	1
T	F	F	1	0	0
F	T	F	0	1	0
F	F	T	0	0	1

Precedence of logical operators:-

Precedence of logical operators helps us to decide which operator will get evaluated first in a complicated looking compound proposition

For example:- $p \rightarrow q \uparrow \neg p$

operator	Name	Precedence
$\neg$	Negation	1
$\wedge$ ($\uparrow$)	Conjunction	2
$\vee$ ($\downarrow$) $\oplus$	Disjunction/XOR	3
$\rightarrow$	Implication	4
$\leftrightarrow$	Biconditional	5

$$\underbrace{p \rightarrow \underbrace{q \uparrow \underbrace{\neg p}_{1}}_{2}}_{3}$$

Ques:- Prepare the truth table of each statement

(a) $(p \Leftrightarrow q) \wedge (r \wedge q)$ (b) $(p \Rightarrow q) \wedge \sim q$

(c) $(p \wedge q \Rightarrow p) \Rightarrow (q \wedge \sim q)$ (d) $(p \Rightarrow q \wedge r) \vee (\neg p \vee q)$

(e) $[\sim p \Rightarrow (q \wedge r)] \vee r$ (f) $p \not\Rightarrow q \times r \quad \neg(p \wedge q) \vee (\neg q \Leftrightarrow p)$

(g) $p \Rightarrow (\neg q \vee r) \wedge (q \vee p \Leftrightarrow \neg r)$

(h) $\neg(p \Rightarrow q) \Leftrightarrow q \wedge \sim q$

Solⁿ:- (a)

p	q	r	A $p \Leftrightarrow q$	B $r \wedge q$	$A \wedge B$
T	T	T	T	T	T
T	T	F	T	F	F
T	F	T	F	F	F
T	F	F	F	F	F
F	T	T	F	T	F
F	T	F	F	F	F
F	F	T	T	F	F
F	F	F	T	F	F

(b)

p	q	A $p \Rightarrow q$	$\sim q$	$A \wedge \sim q$
T	T	T	F	F
T	F	F	T	F
F	T	T	F	F
F	F	T	T	T

(c)

p	q	$p \wedge q$	A $p \wedge q \Rightarrow p$	$\sim q$	B $q \wedge \sim q$	$A \Rightarrow B$
T	T	T	T	F	F	F
T	F	F	T	T	F	F
F	T	F	T	F	F	F
F	F	F	T	T	F	F

(d)

p	q	r	A $q \wedge r$	$p \Rightarrow A$	$\neg p$	B $\neg p \vee q$	$A \vee B$
T	T	T	T	T	F	T	T
T	T	F	F	F	F	T	T
T	F	T	F	F	F	F	F
T	F	F	F	F	F	F	F
F	T	T	T	T	T	T	T
F	T	F	F	T	T	T	T
F	F	T	F	T	T	T	T
F	F	F	F	T	T	T	T

(e)

p	q	r	$q \wedge r$ (A)	$\sim p$	$\sim p \Rightarrow A$ (B)	$B \vee r$
T	T	T	T	F	T	T
T	T	F	F	F	T	T
T	F	T	F	F	T	T
T	F	F	F	F	T	T
F	T	T	T	T	T	T
F	T	F	F	T	F	F
F	F	T	F	T	F	T
F	F	F	F	T	F	F

(f)

p	q	$p \wedge q$	$\neg(p \wedge q)$ (A)	$\neg q$	$\neg q \Leftrightarrow p$ (B)	$A \vee B$
T	T	T	F	F	F	F
T	F	F	T	T	T	T
F	T	F	T	F	T	T
F	F	F	T	T	F	T

(h)

p	q	$p \Rightarrow q$	$\neg(p \Rightarrow q)$ (A)	$\neg q$	$q \wedge \neg q$ (B)	$A \Leftrightarrow B$
T	T	T	F	F	F	T
T	F	F	T	T	F	F
F	T	T	F	F	F	T
F	F	T	F	T	F	T

(g)

p	q	r	$\neg q$	$\neg q \vee r$	$p \Rightarrow (\neg q \vee r)$ (A)	$\neg r$	$q \vee p$	$q \vee p \Leftrightarrow \neg r$ (B)	$A \wedge B$
T	T	T	F	T	T	F	T	F	F
T	T	F	F	F	F	T	T	T	F
T	F	T	T	T	T	F	T	F	F
T	F	F	T	T	T	T	T	T	T
F	T	T	F	T	T	F	T	F	F
F	T	F	F	F	T	T	T	T	T
F	F	T	T	T	T	F	F	T	T
F	F	F	T	T	T	T	F	F	F

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Tautologies :-

A proposition P is a tautology if it is true under all circumstances.
It means it contains only T in the final column of its truth table.

Ques:- Prove that the statement $(p \rightarrow q) \leftrightarrow (\sim q \rightarrow \sim p)$ is a tautology.
$p \vee p \leftrightarrow p$

Sol^n

p	q	A: $p \rightarrow q$	$\sim q$	$\sim p$	B: $\sim q \rightarrow \sim p$	$A \leftrightarrow B$
T	T	T	F	F	T	T
T	F	F	T	F	F	T
F	T	T	F	T	T	T
F	F	T	T	T	T	T

p	$p \vee p$	$p \vee p \leftrightarrow p$
T	T	T
F	F	T

As the final column contains all T's, so it is a tautology.

Contradiction:-

A statement that is always false is called a contradiction.

p	$\sim p$	$p \wedge \sim p$
T	F	F
F	T	F

As the final column contains all F's so it is a contradiction.

Contingency (Satisfiable) :-

A statement that can be either true or false depending on the truth values of its variables, is called contingency or satisfiable.

p	q	A $p \to q$	B $p \wedge q$	$A \to B$
T	T	T	T	T
T	F	F	F	T
F	T	T	F	F
F	F	T	F	F

As the final column contains true and false values, so it is a contingency.

Ques:- Check whether each of the following is a tautology, contradiction or contingency :-

ⓐ $(p \Rightarrow q) \wedge (p \vee q)$

ⓑ $p \Rightarrow p \vee q$

ⓒ $(p \wedge q) \wedge \sim(p \vee q)$

ⓓ $[(p \wedge q) \Rightarrow p] \Rightarrow (q \wedge \neg q)$

ⓔ $[((p \Rightarrow q) \vee p) \wedge q] \Rightarrow q$

Sol^n

ⓔ

p	q	$p \Rightarrow q$	A $(p \Rightarrow q) \vee p$	B $A \wedge q$	$B \Rightarrow q$
T	T	T	T	T	T
T	F	F	T	F	T
F	T	T	T	T	T
F	F	T	T	F	T

As the final column contains all T's, so it is a tautology

ⓓ

p	q	$p \wedge q$	A $p \wedge q \Rightarrow p$	$\neg q$	B $q \wedge \neg q$	$A \Rightarrow B$
T	T	T	T	F	F	F
T	F	F	T	T	F	F
F	T	F	T	F	F	F
F	F	F	T	T	F	F

As, the final column contains all F's so it is contradiction

ⓒ

p	q	A $p \wedge q$	$p \vee q$	B $\neg(p \vee q)$	$A \wedge B$
T	T	T	T	F	F
T	F	F	T	F	F
F	T	F	T	F	F
F	F	F	F	T	F

As, the final column contains all F's so, it is contradiction

ⓑ

p	q	$p \vee q$	$p \Rightarrow p \vee q$
T	T	T	T
T	F	T	T
F	T	T	T
F	F	F	T

As, the final column contains all T's so it is a tautology.

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ⓐ $(p \Rightarrow q) \wedge p \vee q$

p	q	A $p \Rightarrow q$	B $p \vee q$	$A \wedge B$
T	T	T	T	T
T	F	F	T	F
F	T	T	T	T
F	F	T	F	F

As, the final column contain true and false both then it is a contingency or satisfiablity.

Ques:- Examine $(p \Rightarrow q) \wedge \neg q$ is a tautology or not

Sol^n Truth table

p	q	A $p \Rightarrow q$	$\neg q$	$A \wedge \neg q$	p	q	A $p \Rightarrow q$	$\neg q$	$A \wedge \neg q$
T	T	T	F	F	1	1	1	0	0
T	F	F	T	F	1	0	0	1	0
F	T	T	F	F	0	1	1	0	0
F	F	T	T	T	0	0	1	1	1

Thus, the statement, $(p \Rightarrow q) \wedge \neg q$ has both True and False entries in the last column.

Hence, it is not a tautology.

Logical Equivalance :-

Two proposition are said to be logical equivalence if the truth values of both the propositions are same.

Note :-

① Let A and B are two propositions then logical equivalence is denoted by $(\equiv)$ and written as $A \equiv B$ and read as "A is logically equivalence to B"

② If A and B are logically equivalent then $A \leftrightarrow B$ is tautology.

Ques:- Let p and q are two propositions then show that the implication $p \rightarrow q$ and its contra-positive $\sim q \rightarrow \sim p$ are logically equivalent.

Sol^n :- To prove that it is logically equivalent we have to construct the truth table

A : $p \longrightarrow q$
B : $\sim q \longrightarrow \sim p$

p	q	$A: p \rightarrow q$	$\sim p$	$\sim q$	$B: \sim q \rightarrow \sim p$	$A \leftrightarrow B$
T	T	T	F	F	T	T
T	F	F	F	T	F	T
F	T	T	T	F	T	T
F	F	T	T	T	T	T

Since $A \leftrightarrow B$ is tautology, hence both propositions are equivalent.

Ques:- Show that $\neg(p \vee q)$ and $\neg p \wedge \neg q$ both are logically Equivalent.

Solⁿ

p	q	$p \vee q$	A $\neg(p \vee q)$	$\neg p$	$\neg q$	B $\neg p \wedge \neg q$
T	T	T	F	F	F	F
T	F	T	F	F	T	F
F	T	T	F	T	F	F
F	F	F	T	T	T	T

Since $A \equiv B$, hence both propositions are equivalent.

Equivalent Forms:-

1. $\neg\neg p \equiv p$
2. $p \vee p \equiv p$
3. $(p \wedge \neg p) \vee q \equiv q$
4. $p \vee \neg p \equiv q \vee \neg q$
5. $p \rightarrow q \equiv \neg p \vee q$
6. $p \rightarrow (q \rightarrow r) \equiv p \rightarrow (\neg q \vee r)$
7. $q \Longleftrightarrow r \equiv (q \rightarrow r) \wedge (r \rightarrow q)$
8. $p \wedge q \equiv \neg(\neg p \vee \neg q)$ De-Morgan's Law
9. $p \vee q \equiv \neg(\neg p \wedge \neg q)$ These can be used to remove either Conjunction or disjunction
10. $p \Rightarrow q \equiv \neg q \Rightarrow \sim p$
11. $q \Rightarrow p \equiv \neg p \Rightarrow \neg q$

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Ques:- If p, q, r are three statements, then prove that

$$p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$$

Solⁿ:- Truth table

p	q	r	$q \wedge r$	A $p \vee (q \wedge r)$	$p \vee q$	$p \vee r$	B $(p \vee q) \wedge (p \vee r)$
T	T	T	T	T	T	T	T
T	T	F	F	T	T	T	T
T	F	T	F	T	T	T	T
T	F	F	F	T	T	T	T
F	T	T	T	T	T	T	T
F	T	F	F	F	T	F	F
F	F	T	F	F	F	T	F
F	F	F	F	F	F	F	F

Since, the corresponding entries under the column $p \vee (q \wedge r)$ and $(p \vee q) \wedge (p \vee r)$ (i.e; $A \equiv B$) are identical, i.e; the truth table of the statements A and B are the same for all the truth values of the component statements p, q, r, therefore $A \Leftrightarrow B$ is a tautology.

Hence, the statements $A \equiv B$

Ques:- Write the verbal sentence for the given statements :-

p = it is cold q = It is raining

(a) $\neg p$ (b) $p \wedge q$ (c) $p \vee q$ (d) $q \vee \sim p$

Solⁿ:-

p = It is Cold

q = It is raining

(a) $\neg p$:- It is not cold

(b) $p \wedge q$:- It is cold and it is raining

It is cold and raining

(c) $p \vee q$:- It is cold or raining

(d) $q \vee \sim p$:- It is raining or not cold

Ques:- Write down the Truth values or truth table

(a) $4+2 = 5$ and $6+5 = 9$

F $\wedge$ F = F

(b) $3+2 = 5$ and $4+7 = 11$

T $\wedge$ T = T

(c) $6+1 = 7$ or $2+6 = 8$

T $\vee$ T = T

Types of Conditional statement:-

Let p and q are any two statements, then some other conditional related to $p \to q$ are given as :-

① $p \to q$:- It is called direct conditional statement

② $q \to p$:- It is called Converse implication
i.e; converse statement of $p \to q$

③ $\sim p \to \sim q$:- It is called inverse or opposite implication
i.e; It is if not p then not q

④ $\sim q \to \sim p$:- It is called Contra-positive implication
i.e; It is said that if not q then not p

Let us consider the following truth table, which clearly explain the different types of conditions as :-

p	q	$\sim p$	$\sim q$	$p \to q$	$q \to p$	$\sim p \to \sim q$	$\sim q \to \sim p$
T	T	F	F	T	T	T	T
T	F	F	T	F	T	T	F
F	T	T	F	T	F	F	T
F	F	T	T	T	T	T	T

Ques:- Consider the conditional statement p, "If the floods destroy my house or fires destroy my house, then my insurance company will pay me".
Write the converse, inverse and contrapositive of the following statements.

Sol^n:- Let us represent the symbolic statements of the given condition as :-

p :- Flood destroy my house

q :- Fires destroy my house

r :- Insurance company will pay me.

Then the above statement in symbolic and mathematical form is written as :-

$$(p \vee q) \rightarrow r \quad ------ (1)$$

(a) Converse Statement of (1) is

$$r \rightarrow p \vee q$$

i.e; "If my insurance company will pay me than flood destroy my house or fire destroy my house".

(b) Inverse Statement :-

$$\sim(p \vee q) \rightarrow \sim r$$

i.e; Note :- $\sim(p \vee q)$ can be written as

$$\sim(p \vee q) \equiv \sim p \wedge \sim q$$

i.e; $\sim p \wedge \sim q \rightarrow \sim r$

i.e; "If flood does not destroy my house and fires does not destroy my house then my insurance company will not pay me"

(c) Contra Positive :-

$$\sim r \rightarrow \sim(p \vee q)$$

or

$$\sim r \rightarrow \sim p \wedge \sim q$$

i.e; "If my insurance company will not pay me, then flood does not destroy my house and fires does not destroy my house."

Ques:- There are two restaurents next to each other one has sign board that says " Good food is not cheap" and other has a sign board that says " Cheap food is not good". Are the sign board saying the same things?

Soln:- Let us represent the statements in mathematical and logical form as :-

p :- Food is good

q :- Food is cheap

Then the statement " Good food is not Cheap" is written as

$$p \rightarrow \sim q \quad ---- \text{(i)}$$

The argument " Cheap food is not good" is written as

$$q \rightarrow \sim p \quad ---- \text{(ii)}$$

Construct the truth table of (i) & (ii) as :-

p	q	$\sim p$	$\sim q$	$p \rightarrow \sim q$	$q \rightarrow \sim p$
T	T	F	F	F	F
T	F	F	T	T	T
F	T	T	F	T	T
F	F	T	T	T	T

In truth table, the last two columns are same, hence we can say that the sign boards are saying the same things.

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Laws of Algebra of propositions :-

1. Idempotent Law :-

(i) $p \vee p \Leftrightarrow p$ (ii) $p \wedge p \Leftrightarrow p$

2. Commutative Law :-

(i) $p \vee q = q \vee p$ (ii) $p \wedge q = q \wedge p$

3. Associative Law :-

(i) $(p \vee q) \vee r \Leftrightarrow p \vee (q \vee r)$
(ii) $(p \wedge q) \wedge r \Leftrightarrow p \wedge (q \wedge r)$

4. Distributive Law :-

(i) $p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$
(ii) $p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$

5. De-Morgan's Law :-

Let p and q be two propositions, the de-morgan's law states that

(i) $\sim (p \vee q) \equiv (\sim p) \wedge (\sim q)$
(ii) $\sim (p \wedge q) \equiv (\sim p) \vee (\sim q)$

6. Identity Law :-

(i) $p \wedge T = T \wedge p = p$
(ii) $p \vee F = F \vee p = p$

7. Complement Law :-

(i) $p \vee (\sim p) = T$ (ii) $p \wedge (\sim p) = F$
(iii) $p \equiv (\sim \sim p)$

Material Implication :- $(p \to q) \cong (\sim p \vee q)$

Material Equivalence. ① $(p \leftrightarrow q) \cong [(p \to q) \wedge (q \to p)]$

② $[p \leftrightarrow q) \cong [(p \to q) \vee (\sim p \wedge \sim q)]$

8. **Absorption Law :-**

(i) $p \wedge (p \vee q) = p$ (ii) $p \vee (p \wedge q) = p$

9. **Dominace Law :-**

(i) $p \wedge F = F \wedge p = F$ (ii) $p \vee T = T \vee p = T$

10. **Negation Law :-**

(i) $p \wedge (\sim p) = F$ (ii) $p \vee (\sim p) = T$

(iii) $\sim(\sim p) = p$, it is called involution Law

11. **Conditional Law :-**

$$p \longrightarrow q \equiv \neg p \vee q$$

12. **Bi-Conditional Law :-**

$$p \Leftrightarrow q \equiv (p \to q) \wedge (q \to p)$$

$$(p \Leftrightarrow q) \equiv \{(p \wedge q) \vee ((\sim p) \wedge (\sim q))\}$$

13. **Exportion Law :-**

$$(p \wedge q) \Rightarrow r \equiv p \Rightarrow (q \Rightarrow r$$

14. **Absurdity Law :-**

$$(p \Rightarrow q) \wedge (p \Rightarrow \neg q) \equiv \neg p$$

15. **Negation of a biconditional statement :-**

$$\neg (p \Leftrightarrow q) = \neg [(p \Rightarrow q) \wedge (q \Rightarrow p)]$$

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Ques:- Prove that the following formula is tautology without using truth table

$$\sim(\sim p \wedge q) \wedge (p \vee q) \equiv p$$

Sol^n L.H.S $\sim(\sim p \wedge q) \wedge (p \vee q)$

\# $\neg(p \vee q) \equiv \neg p \wedge \neg q$ (By Demorgan's Law)

$\equiv (\neg(\neg p) \cancel{\wedge} \neg q) \wedge (p \vee q)$

$\equiv (p \vee \neg q) \wedge (p \vee q)$

\# $p \vee (q \vee r) \equiv (p \vee q) \wedge (p \vee r)$ (Distributed Law)

$\equiv p \vee (\sim q \wedge q)$

$\equiv p \vee F$

$\equiv p$ or T (By identity Law

Ques:- Prove that $\neg(p \vee (\neg p \wedge q)) \equiv \neg p \wedge \neg q$

Sol^n :- L.H.S

$\neg(p \vee (\neg p \wedge q))$

\# $\neg(p \vee q) \equiv \neg p \wedge \neg q$ By De-Morgan's Law

$\neg p \wedge \neg(\neg p \wedge q)$

$\neg p \wedge (\neg(\neg p) \vee \neg q)$ (By De-Morgan's Law)

$\neg p \wedge (p \vee \neg q)$

\# $p \wedge (q \wedge r) \equiv (p \wedge q) \vee (p \wedge r)$ Distributed Law

$(\neg p \wedge p) \vee (\neg p \wedge \neg q)$

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$\equiv F \vee (\neg p \wedge \neg q)$

$\equiv \neg p \wedge \neg q$ (By identity Law)

Ques:- Using Laws of algebra of statements show that $(p \vee q) \wedge (\neg p \wedge \neg q)$ is a Contradiction.

Soln L.H.S $(p \vee q) \wedge (\neg p \wedge \neg q)$

$\equiv (p \vee q) \wedge \neg (p \vee q)$ (By De-Morgan's Law)

$\equiv l \wedge \neg l$ Let $p \vee q = l$

$\equiv F$

Hence, $(p \vee q) \wedge (\neg p \wedge \neg q)$ is a Contradiction.

Ques:- Using the Law algebra of the statement show that

(a) $(p \vee q) \wedge \neg p \equiv \neg p \wedge q$

(b) $p \vee (p \wedge q) \equiv p$

(c) $\neg (p \vee q) \vee (\neg p \wedge q) = \neg p$

(d) $\neg (p \vee (\neg p \wedge q)) = \neg p \wedge \neg q$

Soln (a) L.H.S $(p \vee q) \wedge \neg p$

$p \wedge q) \equiv q \wedge p$ commutative Law

$\equiv \neg p \wedge (p \vee q)$

$p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$ Distributed Law

$\equiv (\neg p \wedge p) \vee (\neg p \wedge q)$

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$\equiv F \vee (\neg p \wedge q)$

$\equiv \neg p \wedge q$ [By identity Law $p \vee F = p$]

H.P

(b) L.H.S $p \vee (p \wedge q)$

$p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$ (Distributed Law)

$\equiv (p \vee p) \wedge (p \vee q)$

$\equiv p \wedge t$

$\equiv p$ (Identity Law)

H.P

(c) L.H.S $\neg (p \vee q) \vee (\neg p \wedge q)$

$\equiv (\neg p \wedge \neg q) \vee (\neg p \wedge q)$

$\equiv \neg p \wedge (\neg q \vee q)$ By Distributed Law

$\equiv \neg p \wedge T$

$\equiv \neg p$ By identity Law

H.P

(d) L.H.S $\neg (p \vee \neg (p \wedge q))$

$\neg (p \vee q) = \neg p \wedge \neg q$ De-Morgan's Law

Compare $p \rightarrow p$, $q \rightarrow \neg (p \wedge q)$

$\equiv (\neg p) \wedge \neg (\neg (p \wedge q))$

$= \neg p \wedge ((\neg (\neg p)) \vee (\neg q))$ Again Using De-Morgan's Law

$= \neg p \wedge (p \vee \neg q)$

$p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$ (Distributed Law)

Compare $p \to \neg p$, $q = p$, $r = \neg q$

$\equiv (\neg p \wedge p) \vee (\neg p \wedge \neg q)$

$\equiv F \vee (\neg p \wedge \neg q)$

$= \neg p \wedge \neg q$ (By identity law)

H.P

Derived Connectors:-

1. NAND:-

It means negation after ANDing of two statements, Assume p and q be two propositions. Nanding of p and q to be a propositions which is false when both p and q are true, Otherwise true. It is denoted by $p \uparrow q$

p	q	$p \uparrow q$
T	T	F
T	F	T
F	T	T
F	F	T

p	q	$p \uparrow q$
1	1	0
1	0	1
0	1	1
0	0	1

2. NOR or Joint Denial:-

It means negation after ORing of two statements, Assume P and Q be two propositions. Noring of p and q to be proposition which is true when both p and q false, otherwise false. It is denoted by $p \downarrow q$

p	q	$p \downarrow q$
T	T	F
T	F	F
F	T	F
F	F	T

p	q	$p \downarrow q$
1	1	0
1	0	0
0	1	0
0	0	1

Ans:- Generate the truth table for following

(i) $A \oplus B \oplus C$ (ii) $A \uparrow B \uparrow C$

Sol^n (i)

A	B	C	$A \oplus B$	$A \oplus B \oplus C$
T	T	T	F	T
T	T	F	F	F
T	F	T	T	F
T	F	F	T	T
F	T	T	T	F
F	T	F	T	T
F	F	T	F	T
F	F	F	F	F

(ii)

A	B	C	$A \uparrow B$	$A \uparrow B \uparrow C$
T	T	T	F	T
T	T	F	F	T
T	F	T	T	F
T	F	F	T	T
F	T	T	T	F
F	T	F	T	T
F	F	T	T	F
F	F	F	T	T

Argument :-

An argument is an assertion ; that a group of propositions called premises, yields another proposition, called the conclusion. Let $P_1, P_2, P_3, \dots, P_n$ is the group of propositions that yields the conclusion Q. Then it is denoted as $P_1, P_2, \dots, P_n \vdash Q$.

OR

An argument is a process which yield a conclusion from a given set of propositions, called "PREMISES".

Let $p_1, p_2, \dots, p_n$ be the set of propositions and let it yields a conclusion q, then it is represented as :-

$$p_1, p_2, \dots, p_n \vdash q$$

OR

$$\begin{array}{l} p_1 \\ p_2 \\ p_3 \\ \vdots \\ p_n \\ \hline \therefore\ q \rightarrow \text{Conclusion} \end{array}$$

Conclusion :-

The conclusion of an argument is the proposition that is asserted on the basis of other proposition of the argument.

Premises :-

The propositions, which are assumed for accepting the conclusion, are called premises of that argument.

Valid Argument :-

An argument $(p_1, p_2 \dots p_n) \vdash q$ is said to be valid if $(p_1 \wedge p_2 \wedge \dots \wedge p_n) \rightarrow q$ is tautology.

Falacy Argument :-

An argument is called Falacy or an invalid argument if it is not a valid argument.

Ques: Show that the following argument is valid

$p \vee q$
$\sim p$
---
q

Sol^n In this question there are two premises $p \vee q$ and $\sim p$ and the conclusion is q.

The ~~follo~~ given argument will be valid if $[(p \vee q) \wedge \sim q] \rightarrow q$ is a tautology.

Let us construct the truth table as :-

p	q	(A) $\sim p$	(B) $p \vee q$	(C) $(A) \wedge (B)$	$C \rightarrow q$
T	T	F	T	F	T
T	F	F	T	F	T
F	T	T	T	T	T
F	F	T	F	F	T

Since all entries in the last column are of 'T' only, therefore $[(p \vee q) \wedge \sim q] \rightarrow q$ is a tautology, hence the given argument is valid.

Ques:- Test the validity of the following argument

If a man is a bachelor, he is worried
If a " " worried, he die's young

Bachelor's die young.

Solⁿ:- Let us represent the premises and its conclusion in logical form as

p : A man is a bachelor
q : Man is worried
r : Man dies young

The symbolic representation of above argument using logical operators is as :-

$$\begin{array}{l} p \to q \\ q \to r \\ \hline p \to r \end{array}$$

Now above argument is valid if
$[(p \to q) \wedge (q \to r)] \to (p \to r)$ is a tautology.

Construct the truth table as :-

p	q	r	a $p \to q$	b $q \to r$	c $a \wedge b$	d $p \to r$	$c \to d$
T	T	T	T	T	T	T	T
T	T	F	T	F	F	F	T
T	F	T	F	T	F	T	T
T	F	F	F	T	F	F	T
F	T	T	T	T	T	T	T
F	T	F	T	F	F	T	T
F	F	T	T	T	T	T	T
F	F	F	T	T	T	T	T

Since all entries in the last column of the table are of 'T' only, hence it is tautology and therefore the given arguments is valid.

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Ques:- Test the validity of the following argument.
"If it rains, then it will be cold.
If it is cold, then I will stay at home.
Since it rains therefore I shall stay at home."

Soln :- Let Argument represent in logical form as:-
p : It rains
q : It will be cold
r : I shall stay at home.

Now, logical form is given as:-

$$\begin{array}{l} p \rightarrow q \\ q \rightarrow r \\ \underline{p} \\ \therefore r \end{array}$$

If above is valid then $[(p \rightarrow q) \wedge (q \rightarrow r) \wedge p] \rightarrow r$ is a tautology.

Ans:- Argument is valid.

Ques: Test the validity of the argument:
"If two sides of a triangle are equal, then the opposite angles are equal.
Two sides of a triangles are not equal.

$\therefore$ The opposite angles are not equal."

Soln :- Represent it as:-
p : Two sides of a triangle are equal
q : " opposite angles are of equal
then the symbolic & logical form is as:-

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$p \rightarrow q$
$\sim p$
$\therefore \sim q$

$\therefore [(p \rightarrow q) \wedge \sim p] \rightarrow \sim q$ is tautology then it is valid otherwise invalid

Ans:: Invalid.

Ques: Test whether the argument given below is valid or not.
I will become famous or I will be writer
I will not be a writer
$\therefore$ I will become famous

Ans: Valid

Ques: Show that the following argument is not valid

$$\begin{array}{l} p \\ \sim q \vee r \\ \sim p \rightarrow q \\ \hline r \end{array}$$

Ques:- Test the validity of the argument:
"If the labour market is perfect then the wages of all persons in a particular employment will be equal. But it is always the case that for such persons are not equal.
Therefore the labour market is not perfect.

Ans:- Valid

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Theory of Inference :-

The rules of inference are criteria for determining the validity of an argument.

The following inference rules are used to derive the validity of an argument, which are given as:

1. Modus Ponens or Rule of Detachment :-

$$\begin{array}{l} p \to q \\ p \\ \hline \therefore q \end{array}$$

2. Hypothetical Syllogism

$$\begin{array}{l} p \to q \\ q \to r \\ \hline \therefore p \to r \end{array}$$

3. Constructive dilemma

$$\begin{array}{l} (p \to q) \land (r \to s) \\ p \lor r \\ \hline \therefore q \lor s \end{array}$$

4. Simplification

$$\begin{array}{l} p \land q \\ \hline \therefore p \end{array}$$

5. Modus tollens

$$\begin{array}{l} p \to q \\ \sim q \\ \hline \therefore \sim p \end{array}$$

6. Disjunctive Syllogism

$$\begin{array}{l} p \lor q \\ \sim p \\ \hline \therefore q \end{array}$$

7. Absorption

$$\begin{array}{l} p \to q \\ \hline \therefore p \to (p \land q) \end{array}$$

8. Conjunction

$$\begin{array}{l} p \\ q \\ \hline \therefore p \land q \end{array}$$

9. Addition

$$\begin{array}{l} p \\ \hline \therefore p \lor q \end{array}$$

10. Resolution

$$\begin{array}{l} p \vee q \\ p \vee r \\ \hline \therefore q \vee r \end{array}$$

Ques: Show that the following rule is valid:

$p \vdash p \vee q$ or $p \quad \therefore p \vee q$

Sol^n We can prove this rule from the truth table

p	q	$p \vee q$
T	T	T
T	F	T
F	T	T
F	F	F

→ line 1

→ line 2

p is true line 1 and 2 and $p \vee q$ is also true in line 1 and 2.

Hence, argument is valid.

Ques: Show that the rule modus ponens is valid

$$\begin{array}{l} p \rightarrow q \\ p \\ \therefore q \end{array}$$

Sol^n The truth table of this rule is a follows

p	q	$p \rightarrow q$
T	T	T
T	F	F
F	T	T
F	F	T

The p is the true in line 1 & 2 and $p \rightarrow q$ and p both are true in line 1 and p, $p \rightarrow q$ and q all are true in line 1. Hence argument is valid.

Ques: Show that the rule of hypothetical syllogism is valid

$$p \to q$$
$$q \to r$$
$$\therefore p \to r$$

Sol^n The truth table of above rule is as:-

p	q	r	$p \to q$	$q \to r$	$p \to r$
T	T	T	T	T	T
T	T	F	T	F	F
T	F	T	F	T	T
T	F	F	F	T	F
F	T	T	T	T	T
F	T	F	T	F	T
F	F	T	T	T	T
F	F	F	T	T	T

$p \to q$ is true in line 1, 2, 5, 6, 7, 8
$q \to r$ is " " " 1, 3, 4, 5, 7, 8
$p \to r$ " " " " 1, 3, 5, 6, 7, 8

Both $p \to q$ and $q \to r$ is true in lines 1, 5, 7, 8
$p \to r$ " " " 1, 5, 7, 8
Hence, argument is valid.

Ques: Show that the rule of disjunctive syllogism is valid

$$p \lor q$$
$$\sim p$$
$$\therefore q$$

Solⁿ:- The truth table of the above rule is as follows:-

p	q	$\sim p$	$p \vee q$
T	T	F	T
T	F	F	T
F	T	T	T
F	F	T	F

$p \vee q$ is true in line 1, 2 & 3, $\sim p$ is true in line 3. Both $p \vee q$ and $\sim p$ is true in line 3.
As q is also true in line 3.
Hence, Argument is valid.

Ques: Show that rule of absorption is valid

$$p \rightarrow q$$
$$\therefore \quad p \rightarrow p \wedge q$$

Ans: Argument is valid.

Proof of validity:-

We can test the validity of any argument by constructing the truth tables. But as the no. of variable statements increase, the truth tables grow unwieldly. So, a more efficient method to test the validity of the argument is to deduce its conclusion from its premises by a sequence of elementary arguments each of which is known to be valid.

Ques: Prove that the argument $p \to \sim q$, $r \to q$, $r \vdash \sim p$ is valid without using truth tables.

Soln
(i) $p \to \sim q$ Given
(ii) $r \to q$ Given
(iii) r Given
(iv) q (Modus Ponen using (ii) & (iii))
(v) $\sim p$ (Modus Tonen " (i) & (iv))

Ques: Prove that the argument $(p \to q) \wedge (r \to s)$, $q \to s$, $(q \to s) \to (p \vee r) \vdash q \vee s$ is valid using deduction method.

Soln
(i) $(p \to q) \wedge (r \to s)$ (Given)
(ii) $q \to s$ (Given)
(iii) $(q \to s) \to (p \vee r)$ (Given)
(iv) $p \vee r$ (Modus ponens using (iii) & (ii))
(v) $q \vee s$ (Constructive dilemma using (i) & (iv))

Ques: Prove that the argument $(p \to q) \wedge (r \to s)$, $(p \vee r) \wedge (q \vee r)$ $\vdash q \vee s$

Sol^n

(i) $(p \to q) \wedge (r \to s)$ (Given)

(ii) $(p \vee r) \wedge (q \vee r)$ (Given)

(iii) $(p \vee r)$ (Simplification using (iii))

(iv) $q \vee s$ (Constructive dilemma using (i) & (iii))

Ques: Prove that the argument $(p \wedge q) \vee (r \to s)$, $(t \to r)$, $\sim(p \wedge q) \vdash t$ is valid without using truth tables.

Sol^n:-

(i) $(p \wedge q) \vee (r \to s)$ (Given)

(ii) $t \to r$ (Given)

(iii) $\sim(p \wedge q)$ (Given)

(iv) $r \to s$ (Disjunctive Syllogism using (i) & (iii))

(v) $t \to s$ (Hypothetical syllogism using (ii) & (iii))

Ques: Prove that the argument $p, q \vdash (p \vee r) \wedge q$ is valid without using truth tables.

Sol^n:-

(i) p (Given)

(ii) q (Given)

(iii) $p \vee r$ (Rule of addition using (i))

(iv) $(p \vee r) \wedge q$ (Rule of Conjunction using (ii) & (iii))

Ques: Prove that the argument $p \vee (q \to p)$, $\sim p \wedge r \vdash \sim q$ is valid without using truth table

Sol^n

(i) $p \vee (q \to p)$

(ii) $\sim(p) \wedge r$

(iii) $\sim p$ (Rule of simplification using (ii))

(iv) $q \to p$ (Disjunctive syllogism using (i) & (iii))

(v) $\sim q$ Modus tollens using (iv) & (iii)

Ques: Prove that the argument $p \to q$, $q \to r$, $r \to s$, $\sim s$, $p \vee t \vdash t$ is valid without using truth table.

Solⁿ
(I) $p \to q$ (Given)
(II) $q \to r$ (Given)
(III) $r \to s$ (Given)
(IV) $\sim s$ (Given)
(V) $p \vee t$ (Given)
(VI) $p \to r$ (Hypothetical Syllogism using (I) & (II))
(VII) $p \to s$ (" " " (VI) + (III))
(VIII) $\sim p$ (Modus tollens using (VII) + (IV))
(IX) t (Disjunctive Syllogism using (V) & (VIII))

Ques: Prove that the argument $p \to (q \vee r)$, $(s \wedge t) \to q$, $(q \vee r) \to (s \wedge t) \vdash p \to q$ is valid without using truth table

Solⁿ
(I) $p \to (q \vee r)$ (Given)
(II) $(s \wedge t) \to q$ (Given)
(III) $(q \vee r) \to (s \wedge t)$ (Given)
(IV) $p \to (s \wedge t)$ (Hypothetical Syllogism using (I) & (III))
(V) $p \to q$ (" " " (II) + (IV))

Ques:- Prove the validity of the following argument using theory of inference rule
"If I get the job and work hard, then I will get promoted. If I get promoted, then I will be happy. I will not be happy. Therefore either I will not get the job or I will not work hard. deduce ~~to~~ without using truth table.

Solⁿ :- p : I will be get the job
q : I will be work hard

r : I will be get promoted
s : I will be happy

Then the above argument in symbolic form can be written as :-

$(p \wedge q) \rightarrow r$
$r \rightarrow s$
$\sim s$
$\therefore \sim p \vee \sim q$

(i) $(p \wedge q) \rightarrow r$
(ii) $r \rightarrow s$
(iii) $\sim s$
(iv) $(p \wedge q) \rightarrow s$ (Hypothetical syllogism using (i) & (ii))
(v) $\sim (p \wedge q)$ (Modus tollens using (iii) & (iv))
(vi) $\sim p \vee \sim q$ (Demorgan's Law)

Hence the argument is Valid.

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Predicate logic

Till now, we have discussed the simple statements and the logical techniques to combine simple sentences into the compound statements.
We can't apply those techniques to arguments of the following form.

"All human are mortal. Ravi is human, Therefore, Ravi is mortal."

The validity of such type of argument depends upon the inner logical structure of simple statement, it contains

OR

p – Every student is brilliant
q – Shyfali is a student

R – Shyfali is brilliant

Representing Simple Facts in FOPL (models world in terms of objects)

Real world Facts can be represented as logical propositions written as well formed formulas in propositional logic.

Symbols → Objects, Properties, Relations } Representation

Symbols are formed of the following :-

(i) Set of all Uppercase alphabets
(ii) Set of digits from 0 to 9
(iii) Underscore Character.

[All students are intelligent] can't be written in propositional logic.
↓
First order Predicate logic

Defining a Sentence :-

Every $\boxed{\text{atomic Sentence}}$ is a sentence
↓
is defined as predicate constant of arity ~~is defined~~ 'n' followed by $t_1, t_2, \ldots, t_n$ terms enclosed in parenthesis and separated by commas.

① If 'S' is a sentence then $\neg S$ is also a sentence.
② If S_1, S_2 are two sentences $\rightarrow$ $S_1 \wedge S_2$ (Conjunction)

③ " " " " " $\rightarrow$ $S_1 \vee S_2$ (Disjunction)

④ " " " " " $\rightarrow$ $S_1 \rightarrow S_2$ (Implication)

⑤ " " " " " $\rightarrow$ $S_1 \equiv S_2$ (Equivalence)

⑥ If x is a variable and S is a sentence then $\forall S$ is a sentence.

⑦ If x is a variable and S is a sentence then $\exists S$ is a sentence.

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Quantifier:-

The restrictions namely "for every, all and some etc." are called quantifiers.

We have two types of quantifiers

① Universal Quantifier
② Existential "

1. Universal Quantifier:-

If $p(x)$ is a proposition over the universe U. Then it is denoted as $\forall x, p(x)$ and read as "for every $x \in U$, $p(x)$ is true". The quantifier $\forall$ is called universal quantifier.

There are several ways to write a proposition, with a universal quantifier.

$\forall x \in A, p(x)$ or $p(x), \forall x \in A$

or $\forall x, p(x$ or $p(x)$ is true for all $x \in A$

2. Existential Quantifier:-

If $p(x)$ is a proposition over the universe U. Then it is denoted as $\exists x\, P(x)$ and read as "There exist at least one value in the universe of variable x such that $p(x)$ is true. The quantifier $\exists$ is called the existential quantifier.

There are several ways to write a proposition, with an existential quantifier i.e

$(\exists x \in A)\, p(x)$ or $\exists x \in A$ such that $p(x)$

or $(\exists x)\, p(x)$ or $p(x)$ is true for som $x \in A$

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Ques:- Let $A(x)$: x has a white colour, $B(x)$: x is a polar bear, $C(x)$: x is found in cold regions, over the universe of animals. Translate the following into simple sentence:

(i) $\exists x\,(B(x) \wedge \sim A(x))$ (ii) $(\exists x)(\sim C(x))$
(iii) $(\forall x)\,(B(x) \wedge C(x) \rightarrow A(x))$

Solⁿ (i) There exist a polar bear whose colour is not white
(ii) There exist an animal that is not found in cold regions
(iii) Every polar bear that is found in cold regions has a white colour

Ques: Let $K(x)$: x is a two-wheeler, $L(x)$: x is a scooter. $M(x)$: x is manufacured by Bajaj
Express the following using quantifiers.

(i) Every two wheeler is a scooter
(ii) There is a two wheeler that is not manufactured by Bajaj
(iii) There is a two wheeler manufacured by Bajaj that is not a scooter.
(iv) Every two wheeler that is a scooter is manufacured by Bajaj.

Solⁿ:- (i) $(\forall x)\,(K(x) \rightarrow L(x))$
(ii) $(\exists x)\,(K(x) \wedge M(x))$
(iii) $(\exists x)\,(K(x) \wedge M(x) \rightarrow \sim L(x))$
(iv) $(\forall x)\,(K(x) \wedge L(x) \rightarrow M(x))$

Negation of Quantified Propositions :-

When we negate a quantified proposition i.e., when a universally quantified proposition is negated we obtain an existentially quantified proposition and when an existentially quantified proposition is negated, we obtain a universally quantified proposition.

The two rules for negation of quantified proposition are as follows. These are also called De Morgan's Law.

(i) $\sim \exists x (p(x)) \cong \forall x \sim p(x)$

(ii) $\sim \forall x (p(x)) \cong \exists x \sim p(x)$

Ques:- Negate each of the following propositions

(i) All boys can run faster than all girls
(ii) Some girls are more intelligent than all boys
(iii) Some students do not live in hostel
(iv) All students pass the semester exams.
(v) Some of the students are absent and the classroom's empty.

Sol^n:- (i) Some boys can run faster than some girls

(ii) All girls are more intelligent than some boys.
(iii) All students live in hostel
(iv) Some students don't pass the semester exams

(v) All students are present and the class-room is full.

Ques:- Negate each of the following propositions

(i) $\forall x\, p(x) \wedge \exists y\, q(y)$

(ii) $\forall x\, p(x) \wedge \forall y\, q(y)$

(iii) $\exists x\, p(x) \vee \forall y\, q(y)$

(iv) $\exists x\, p(x) \vee \exists y\, q(y)$

(v) $(\exists x \in U)\,(x+6 = 25)$

(vi) $(\forall x \in U)\,(x < 25)$

Sol^n

(i) $\sim(\forall x\, p(x) \wedge \exists y\, q(y))$

$\cong \sim\forall x\, p(x) \vee \sim\exists y\, q(y)$ $\quad (\because \sim(p\wedge q) = \sim p \vee \sim q)$

$\cong \exists x \sim p(x) \vee \forall y \sim q(y)$

(ii) $\sim(\forall x\, p(x) \wedge \forall y\, q(y))$

$\cong \sim\forall x\, p(x) \vee \sim\forall y\, q(y)$ $\quad (\because \sim(p\wedge q) = \sim p \vee \sim q)$

$\cong \exists x \sim p(x) \vee \exists y \sim q(y)$

(iii) $\sim(\exists x\, p(x) \vee \forall y\, q(y))$

$\cong \sim\exists x\, p(x) \wedge \sim\forall y\, q(y)$ $\quad \because \sim(p\vee q) = \sim p \wedge \sim q$

$\cong \forall x \sim p(x) \wedge \exists y \sim q(y)$

(iv) $\sim(\exists x\, p(x) \vee \exists y\, q(y))$

$\cong \sim\exists x\, p(x) \wedge \sim\exists y\, q(y)$ $\quad \because \sim(p\vee q) = \sim p \wedge \sim q$

$\cong \forall x \sim p(x) \wedge \forall y \sim q(y)$

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(v) $\sim(\exists x \in U)(x+6)=25$

$\cong \forall x \in U \sim(x+6)=25$

$\cong (\forall x \in U)(x+6 \neq 25)$

(vi) $\sim(\forall x \in U)(x<25)$

$\cong \exists x \in U \sim(x<25)$

$\cong (\exists x \in U)(x \geq 25)$

Ques: Let $K(x)$: x is student, $M(x)$: x is clever, $N(x)$: x is successful

Express the following using quantifiers

(a) There exist a student

(b) Some students are clever

(c) Some students are not successful

Sol^n (a) $\exists x, K(x)$ or $(\exists x) K(x)$

(b) There exist x s.t x is student and x is clever

$\exists x\,(K(x) \wedge M(x))$

(c) There exist x, such that x is student & x is not successful.

$\exists x\,[K(x) \wedge \sim N(x)]$

Propositions with Multiple Quantifiers :-

The proposition having more than one variable can be quantified with multiple quantifiers. The multiple universal quantifiers can be arranged in any order without altering the meaning of the resulting proposition. Also the multiple existential quantifiers can be arranged in any order without altering the meaning of propositions.

The proposition which contains both universal and existential quantifiers, the order of these quantifiers can't be exchanged without altering the meaning of proposition. e.g:-, the proposition $\exists x \forall y\, p(x,y)$ means

"There exists some x such that $p(x,y)$ is true for every y."

Ques: Write the negation for each of the following. Determine whether the resulting statement is true or false. Assume $U = R$

(i) $\forall x \exists m (x^2 < m)$

(ii) $\exists m \forall x (x^2 < m)$

Solⁿ: (i) Negation of $\forall x \exists m (x^2 < m)$ is $\exists x \forall m (x^2 \geq m)$. The meaning of $\exists x \forall m (x^2 \geq m)$ is that there exists some x s.t $x^2 \geq m$, for every m. The statement is true as there is some greatest x s.t $x^2 \geq m$, for every m.

(ii) Negation of $\exists m \forall x (x^2 < m)$ is $\forall m \exists x (x^2 \geq m)$. The meaning of $\forall m \exists x (x^2 \geq m)$ is that for every m, there exists some x such that $(x^2 \geq m)$.

The statement is true as for every m, there exists some greatest x such that $x^2 \geq m$