Unit :- V

Mathematical Induction

Let P(n) be a statement involving the natural no. n
To prove that P(n) is true for all natural numbers,
we proceed as:-

(i) Verify P(n) is true for $n = 1$
(ii) Assume the result is true for $n = k > 1$
(iii) Using (i) & (ii) prove that P(k+1) is true
that is called mathematical induction

Ques:- Show that $1+2+3+\cdots+n = \frac{n(n+1)}{2}$

Sol^n Let $S(n) = 1+2+3+\cdots+n = \frac{n(n+1)}{2}$ —— (i)

For $n = 1$

$$S(1) = 1 = \frac{1(1+1)}{2}$$

$$= 1 \Rightarrow S(n) \text{ is true for } n = 1$$

Let S(n) is true for $n = k$ i.e;

$$S(k) = 1+2+\cdots+k = \frac{k(k+1)}{2} \quad \text{—— (ii)}$$

Now, we have to show that S(n) is true for $n = k+1$. Put $n = k+1$ into S(n) then

$$S(k+1) = 1+2+\cdots+k+(k+1) = \frac{(k+1)(k+2)}{2}$$

Now, L.H.S for $S(k+1) = \underline{1+2+\cdots+k}+(k+1)$

$$\frac{k(k+1)}{2} + k+1 \longrightarrow \text{by (ii)}$$

$$(k+1)\left(\frac{k}{2}+1\right) \Rightarrow \frac{(k+1)(k+2)}{2}$$

L.H.S = R.H.S

therefore S(n) is true for $n = k+1$
So, By Mathematical induction Principle S(n) is true $\forall\ n \in N$

Ques:- Show $1^3+2^3+3^3+\cdots+n^3 = \left(\frac{n(n+1)}{2}\right)^2$

Sol^n Let $P(n) = 1^3+2^3+3^3+\cdots+n^3 = \left(\frac{n(n+1)}{2}\right)^2$

for $n=1$

$P(1) = 1 = \left(\frac{1(1+1)}{2}\right)^2$ —— (1)

$1 = 1$ L.H.S = R.H.S

$P(n)$ is true for $n=1$

Let $P(n)$ is true for $n=k$ i.e.;

$P(k) = 1^3+2^3+3^3+\cdots+k^3 = \left(\frac{k(k+1)}{2}\right)^2$ —— (11)

Now we have to show that $P(n)$ is true for $n=k+1$. Put $n=(k+1)$ into $P(n)$ then

$P(k+1) = 1^3+2^3+3^3+\cdots+k^3+(k+1)^3 = \left(\frac{(k+1)(k+2)}{2}\right)^2$

Now. L.H.S for

$$P(k+1) = \underbrace{1^3+2^3+3^3+\cdots+k^3}+(k+1)^3$$

$$= \frac{k^2(k+1)^2}{4} + (k+1)^3$$

$$= (k+1)^2\left(\frac{k^2}{4}+k+1\right) \Rightarrow (k+1)^2\left(\frac{k^2+4k+4}{4}\right)$$

$$= \frac{(k+1)^2(k+2)^2}{4} \Rightarrow \left(\frac{(k+1)(k+2)}{2}\right)^2$$

L.H.S = R.H.S

therefor $P(n)$ is true for $n=k+1$

So, By Mathematical induction Principle $P(n)$ is true $\forall\, n\in N$.

Ques: Show that for all $n\in N$, 7^n-3^n is divisible by 4.

Sol^n Let $P(n) = 7^n - 3^n$

for $n=1$ $P(1) = 7^1-3^1 = 4 =$ multiple of 4 —— (1)

Let $S(n)$ is true for $n=k$ i.e;

$$S(k) = 7^k - 3^k = 4m \quad \text{— (ii)}$$

Now, we have to show that $S(n)$ is true for $n=k+1$. Put $n=k+1$ into $S(n)$ then

$$S(k+1) = 7^{k+1} - 3^{k+1}$$

$$= 7 \cdot 7^k - 3^{k+1}$$

$$= 7(7^k - 3^k) + 7 \, 3^k - 3^{k+1}$$

$$= 7(4m) + 3^k(7-3) \quad \text{— by (ii)}$$

$$= 7(4m) + 3^k \, 4 \Rightarrow 4(7m + 3^k)$$

$m, k \in N$

therefor $S(n)$ is true for $n=k+1$

So, By Mathematical induction Principle $S(n)$ is true for all $n \in N$.

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