m5 | ..'+y'+z | M6

Minterms and Maxterms in Boolean Algebra

Minterms and Maxterms form very basic fundamentals in analysis and designing of digital functions/ ckts.

Minterms:- A Product term which contains each of the n-variable in either complemented / non- complemented form.

OR

Each individual term in SOP (Sum of the product) is called as minterm. It can be Represented as $\sum m(a,b)$

Maxterm:- A Sum term which Contains each of n-variable in either Complemented / non - complemented form

OR

Each individual term in POS (Product of Sum) is called as maxterm. It Can be Represented as $\prod M(a,b)$

for example

For 2-variable f^n:-

	A	B	Minterm SoP	Maxterm PoS
0	0	0	$\bar{A}\bar{B} = m_0$	$A+B \rightarrow M_0$
1	0	1	$\bar{A}B \rightarrow m_1$	$A+\bar{B} \rightarrow M_1$
2	1	0	$A\bar{B} \rightarrow m_2$	$\bar{A}+B \rightarrow M_2$
3	1	1	$AB \rightarrow m_3$	$\bar{A}+\bar{B} \Rightarrow M_2$

for three variables A,B,C

	A	B	C	Minterms SoP	Maxterm PoS
0	0	0	0	$\bar{A}\bar{B}\bar{C} \rightarrow m_0$	$A+B+C \rightarrow M_0$
1	0	0	1	$\bar{A}\bar{B}C \rightarrow m_1$	$A+B+\bar{C} \rightarrow M_1$
2	0	1	0	$\bar{A}B\bar{C} \rightarrow m_2$	$A+\bar{B}+C \rightarrow M_2$
3	0	1	1	$\bar{A}BC \rightarrow m_3$	$A+\bar{B}+\bar{C} \rightarrow M_3$
4	1	0	0	$A\bar{B}\bar{C} \rightarrow m_4$	$\bar{A}+B+C \rightarrow M_4$
5	1	0	1	$A\bar{B}C \rightarrow m_5$	$\bar{A}+B+\bar{C} \rightarrow M_5$
6	1	1	0	$AB\bar{C} \rightarrow m_6$	$\bar{A}+\bar{B}+C \rightarrow M_6$
7	1	1	1	$ABC \rightarrow m_7$	$\bar{A}+\bar{B}+\bar{C} \rightarrow M_7$

Variable in Complemented or Uncomplemented form known as LITERAL.

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Represent a function in Minterm & Maxterm:-

Minterm

① $f(A,B) = AB + \bar{A}B$

$AB \to 11$, $\bar{A}B \to 01$

Binary to decimal

$3 \to m_3$ $\quad 1 = m_1$

$f(A,B) = \sum m(1,3)$

By this f^n we can generalize the SOP form also

Illy $f(A,B) = \sum m(1,3)$

$= m_1 + m_3$

$= 01 + 11$

$= \bar{A}B + AB$

② $f(A,B,C) = m_2 + m_3 + m_5$

$= 010 + 011 + 101$

$= \bar{A}B\bar{C} + \bar{A}BC + A\bar{B}C$

③ $f(A,B,C) = \sum m(0,1,4,5)$

$= m_0 + m_1 + m_4 + m_5$

$(\bar{A}\bar{B}\bar{C}) + (\bar{A}\bar{B}C) + (A\bar{B}\bar{C}) + (A.\bar{B}C)$

Maxterm

$f(A,B) = (A+B)(\bar{A}+B)$

$(A+B) \to 00 \to 0(M_0)$ $\quad (\bar{A}+B) \to 10 \to 2(M_2)$

$f(A,B) = \pi M[0,2]$

Binary,

POS form $\left\{ \begin{matrix} 0,0 & 1\ 0 \\ (A+B) & (\bar{A}+B) \end{matrix} \right\}$

Can also generalize them.

Illy $f(A,B) = \pi M[0,2]$

$M_0 \cdot M_2$

$(A+B) \cdot (\bar{A}+B)$

② $f(A,B,C) = M_3 \cdot M_4 \cdot M_5 \cdot M_6$

$011 \quad 100 \quad 101 \quad 110$

$(A+\bar{B}+\bar{C})\ (\bar{A}+B+C)\ (\bar{A}+B+\bar{C})\ (\bar{A}+\bar{B}+C)$

③ $f(A,B,C) = \pi(M)[0,1,4,5]$

$= M_0 \cdot M_1 \cdot M_4 \cdot M_5$

$(A+B+C)\ (A+B+\bar{C})\ (\bar{A}+B+C)\ (\bar{A}+B+\bar{C})$

Ques:

(i) $f(A,B,C) = \sum m(0,2,3,6)$

(ii) $f(A,B,C) = \pi M(1,4,5,7)$

(iii) $f(A,B,C,D) = \sum m(1,2,9,13,14)$

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Minimal to Canonical form Conversion (SOP)

Ques. $f = \bar{A} + \bar{B}\bar{C}$

$= \bar{A}\cdot 1 + \bar{B}\bar{C}\cdot 1$

$= \bar{A}(B+\bar{B}) + \bar{B}\bar{C}(A+\bar{A})$

$= \bar{A}B\cdot 1 + \bar{A}\bar{B}\cdot 1 + \bar{B}\bar{C}A + \bar{B}\bar{C}\bar{A}$

$= \bar{A}B(C+\bar{C}) + \bar{A}\bar{B}(C+\bar{C}) + \bar{B}\bar{C}A + \bar{B}\bar{C}\bar{A}$

$= \bar{A}BC + \bar{A}B\bar{C} + \bar{A}\bar{B}C + \underline{\bar{A}\bar{B}\bar{C}} + \bar{B}\bar{C}A + \underline{\bar{B}\bar{C}\bar{A}}$

$= \bar{A}BC + \bar{A}B\bar{C} + \bar{A}\bar{B}C + A\bar{B}\bar{C} + \bar{A}\bar{B}\bar{C}$

$= \quad (3) \qquad (2) \qquad (1) \qquad (4) \qquad (0)$ $\quad [\because A+A=A]$

Decimal equivalent $\rightarrow$ to Binary

$f = \Sigma m(0, 1, 2, 3, 4)$

$= m_0 + m_1 + m_2 + m_3 + m_4$ } minterm

$f = \pi M(5, 6, 7)$

$= M_5 \cdot M_6 \cdot M_7$ } Maxterm

Minimal to canonical form Conversion for POS :-

Ques. $f = (A+B+\bar{C})(\bar{A}+C)$

$= (A+B+\bar{C})(\bar{A}+B+C)(\bar{A}+\bar{B}+C)$

$[\because (\bar{A}+C) = \bar{A}+C+B\bar{B}$

$= (\bar{A}+C+B)(\bar{A}+C+\bar{B})$ by Distributed Law$]$

$= (A+B+\bar{C})(\bar{A}+B+C)(\bar{A}+\bar{B}+C)$

$= \quad 0\ 0\ 1 \qquad 1\ 0\ 0 \qquad 1\ 1\ 0$

$f = \pi M(1, 4, 6)$

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KARNAUGH MAP

K-MAP Representation of logic functions:-

It is a graphical technique for simplifying or reducing a function used for any no. of variables.

Disadvantage: It has no unique sol^n

\# In n variable K-map there are 2^n cells

\# The ordering of variable is done in such a way that only one variable changes at a time b/w adjacent cells.

\# Overlapping and rolling the map is possible.

\# Top-bottom, first & last columns are taken as continuous

2-variable Map:-

$2^n = 2^2 = 4$ minterms $\therefore$ map consists 4 cells

A \ B	0	1
0	00 (0)	01 (1)
1	10 (2)	11 (3)

3-variable Map:- $2^3 = 8$

A \ BC	00	01	11	10
0	0	1	3	2
1	4	5	7	6

AB \ C	0	1
00	0	1
01	2	3
11	6	7
10	4	5

4-variable Map:- $2^4 = 16$ cells

AB \ CD	00	01	11	10
00	0	1	3	2
01	4	5	7	6
11	12	13	15	14
10	8	9	11	10

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5-Variable Map

$2^5 = 32$

AB \ CDE	000	001	011	010	110	111	101	100
00	0	1	3	2	6	7	5	4
01	8	9	11	10	14	15	13	12
11	24	25	27	26	30	31	29	28
10	16	17	19	18	22	23	21	20

Implicants:- The group of 1's is always called implicants

Prime Implicants :- The Largest possible group of 1's

Essential Prime Implicant:- It is single 1 which can't be combined in any other way

Ques:- $f(A,B,C) = \Sigma m(1, 3, 5, 7)$ — $m_1\ m_3\ m_5\ m_7$

$\downarrow$ SOP

① Find out no. of variables. $n = 3 \{A, B, C\}$

② No. of cells $= 2^n = 2^3 = 8$

A \ BC	00	01	11	10
0	0	1 (cell 1)	1 (cell 3)	2
1	4	1 (cell 5)	1 (cell 7)	6

→ Implicants.

$f = I$

$\therefore \boxed{F = C}$

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Ques:- 2

$f(A,B,C) = \Sigma m(0,1,2,4,7)$

A \ BC	00	01	11	10
0	1	1		1
1	1		1	

$f = \mathrm{I} + \mathrm{II} + \mathrm{III} + \mathrm{IV}$

$\bar{B}\bar{C} + \bar{A}\bar{B} + \bar{A}\bar{C} + BCA$

Ques:- 3

$f(A,B,C) = \Sigma m(1,3,6,7)$

A \ BC	00	01	11	10
0		1	1	
1			1	1

III (Redundant

$F = \mathrm{I} + \mathrm{II} + \mathrm{III}$ → neglect

$AB + \bar{A}C + BC$

∵ By Redundant theorem (Each variable repeated 2 times keep the variable in comp. & neglect others.

Ques:- 4

$f(A,B,C) = \Sigma m(0,1,5,6,7)$

A \ BC	00	01	11	10
0	1	1		
1		1	1	1

$f = \mathrm{I} + \mathrm{II} + \mathrm{III}$

$= AB + \bar{A}\bar{B} + AC$

Ques:- 5

$f(A,B,C) = \Sigma m(2,3,4,5) + \Sigma d(6,7)$

A \ BC	00	01	11	10
0			1	1
1	1	1	X	X

$f = \mathrm{I} + \mathrm{II}$

$= \bar{A}B + A\bar{B}$

Consider Don't care (X) to reduce the function by taking max. variable.

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A \ BC	00	01	11	10
0			1	1
1	1	1	1	1

→ quad 1

→ quad 2

$f =$ quad 1 + quad 2

$f = B + A$ Ans.

Ques: $f = A + \overline{B}C$ find the no. of minterms in canonical.

$$
\begin{aligned}
F &= A + \overline{B}C \\
&= A \cdot 1 + \overline{B}C \cdot 1 \\
&= A(B+\overline{B}) + \overline{B}C(A+\overline{A}) \\
&= AB \cdot 1 + A\overline{B} \cdot 1 + \overline{B}CA + \overline{B}C\overline{A} \\
&= AB(C+\overline{C}) + A\overline{B}(C+\overline{C}) + A\overline{B}C + \overline{A}\overline{B}C \\
&= ABC + AB\overline{C} + \underline{A\overline{B}C} + A\overline{B}\,\overline{C} + \underline{A\overline{B}C} + \overline{A}\overline{B}C \\
&= ABC + AB\overline{C} + A\overline{B}\,\overline{C} + \overline{A}\overline{B}C + A\overline{B}C
\end{aligned}
$$

So no. of minterms → 5

Ques: Simplify the following Boolean expression using K-map.

$$f(A,B,C,D) = \Sigma m(4,5,6,7,8,9,10,11,12,13,14,15)$$

Soln

AB \ CD	00	01	11	10
00	(0)	(1)	(3)	(2)
01	1 (4)	1 (5)	1 (7)	1 (6)
11	1 (12)	1 (13)	1 (15)	1 (14)
10	1 (8)	1 (9)	1 (11)	1 (10)

→ I

→ II

$2^4 = 16$

$f = B + A$

or

$f = \underline{A+B}$

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Ques:- Simplify $f(A,B,C) = \bar{A}BC + B\bar{C} + AB\bar{C} + A\bar{B}C$ Using K-Map in SOP & POS form.

Sol^n

$A \backslash BC$	$\bar{B}\bar{C}$	$\bar{B}C$	BC	$B\bar{C}$
$\bar{A}$			1	1
A		1		1

I, II, III

$f = \mathrm{I} + \mathrm{II} + \mathrm{III}$

$= B\bar{A} + B\bar{C} + A\bar{B}C$ — SOP

$f = \Sigma m(2,3,5,6)$

$f = \Pi M[0,1,4,7]$ — POS

$A \backslash BC$	$\bar{B}\bar{C}$	$\bar{B}C$	BC	$B\bar{C}$
$\bar{A}$	0 (0)	0 (1)	3	2
A	0 (4)	5	0 (7)	6

I, II, III

$\bar{f} = \mathrm{I} + \mathrm{II} + \mathrm{III}$

$\bar{f} = \bar{A}\bar{B} + \bar{B}\bar{C} + ABC$

$\bar{\bar{f}} = \overline{\bar{A}\bar{B} + \bar{B}\bar{C} + ABC}$

$\therefore f = (A+B)\cdot(B+C)+(\bar{A}+\bar{B}+\bar{C})$

[Using D-Morgan's Law]

Ques:- $f(A,B,C,D) = \Sigma m(0,1,2,5,7,8,9,10,13,15)$

$AB \backslash CD$	00	01	11	10
00	1 (0)	1 (1)	3	1 (2)
01	4	1 (5)	1 (7)	6
11	12	1 (13)	1 (15)	14
10	1 (8)	9	11	1 (10)

I, II, III

$f = \mathrm{I} + \mathrm{II} + \mathrm{III}$

$= DB + \bar{C}D + \bar{D}B$

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