(b) Existential quantifier

→ The symbol '$\exists$' (there exist) is used to represent existential quantifier and is read in either of the following ways:-

- $\exists x \in A : P(x)$
- $\exists x \; P(x)$
- $\exists x \in A \; P(x)$
- $P(x)$ is true for every $x \in A$.

Examples

Q→ Given that -

(a) $P(x) : x > 0$ (b) $q(x) : x$ is odd

(c) $r(x) : x$ is a perfect square

(d) $S(x) : x$ is divisible by 3.

(e) $t(x) : x$ is divisible by 2.

- Write the following statements in symbolic form -

(i) At least one integer is odd.

$\exists x \; q(x)$

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(ii) There exists a positive integer that is odd.

$$\exists x \{ P(x) \wedge q(x) \}.$$

(III) If x is odd, then x is not divisible by 2.

$$\forall x \{ q(x) \rightarrow \sim t(x) \}.$$

(IV) No odd integer is divisible by 2.

$$\forall x \{ \sim q(x) \wedge t(x) \}.$$

or, $\forall x \{ q(x) \rightarrow \sim t(x) \}.$

(V) There exists an odd integer divisible by 2.

$$\exists x \{ q(x) \wedge t(x) \}.$$

(VI) If x is odd and x is perfect square then divisible by 3

$$\forall x \{ (q(x) \wedge r(x)) \rightarrow S(x) \}$$

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* Negation of quantified statements

→ Negation of a quantified statement change the quantifier and negate the original statement as mentioned below:-

- $\sim \forall x : P(x) \equiv \exists x : \sim P(x)$
- $\sim \exists x : P(x) \equiv \forall x : \sim P(x)$
- $\sim [\forall x : \sim P(x)] \equiv \exists x \sim\sim P(x)$
- $\sim [\exists x : \sim P(x)] \equiv \forall x : \sim\sim P(x)$.

* Properties of quantifiers.

(i) $\exists x \, (P(x) \rightarrow q(x)) \equiv \forall x \, P(x) \rightarrow \exists x \, q(x)$

(ii) $\exists x \, P(x) \rightarrow \forall x \, q(x) \equiv \forall x \, (P(x) \rightarrow q(x))$

(iii) $\exists x \, (P(x) \vee q(x)) \equiv \exists x \, P(x) \vee \exists x \, q(x)$

$\forall x \, [P(x) \wedge q(x)] \equiv [\forall x \, P(x) \wedge \forall x \, q(x)]$

$[\forall x \, P(x) \vee \forall x \, q(x)] \equiv \forall x \, [P(x) \vee q(x)]$.

(iv) $\sim (\exists x \sim P(x)) \equiv \forall x \, P(x)$

Note:- In general, the negation of a quantified predicate is logically equivalent to the proposition obtained by replacing each $\forall$ by $\exists$ and vice versa, as well as replacing the predicate itself by its negative.

Statement	Negation
→ all true $\forall x\, P(x)$	$\exists x [\sim P(x)]$ at least one false
→ at least one false $\exists x (\sim P(x))$	$\forall x\, P(x)$ all true
→ all false $\forall x (\sim P(x))$	$\exists x\, P(x)$ at least one is true.
→ at least one is true $\exists x\, P(x)$	$\forall x [\sim P(x)]$ all false.