BOOLEAN ALGEBRA :- The mathematician GEORGE BOOLE developed rules for manipulation of binary variables. The boolean Algebric theorems are :-

i> $A+0=A$ if $A=0$ then $0+0=0 \Rightarrow A$
if $A=1$ then $1+0=1 \Rightarrow A$ ∴ Theorem is proved.

ii> $A.1=A$
iii> $A+1=1$ } Bounded law
iv> $A.0=0$ → Comp

v> $A+A=A$
vi> $A.A=A$ } Identity

vii> $A+\bar{A}=1$
viii> $A.\bar{A}=0$ } Complement Law

ix> $A.(B+C)=AB+AC$ } Distr
x> $A+BC=(A+B)(A+C)$

xi> $A+AB=A$
xii> $A(A+B)=A$
xiii> $A+\bar{A}B=(A+B)$
xiv> $A(\bar{A}+B)=AB$
xv> $AB+A\bar{B}=A$ } Redundance Law

xvi> $(A+B).(A+\bar{B})=A$
xvii> $AB+\bar{A}C=(A+C)(\bar{A}+B)$
xviii> $(A+B)(\bar{A}+C)=AC+\bar{A}B$
xix> $AB+\bar{A}C+BC=AB+\bar{A}C$
xx> $(A+B)(\bar{A}+C)(B+C)=(A+B)(\bar{A}+C)$

xxi> $\overline{A.B.C.\dots}=\bar{A}+\bar{B}+\bar{C}+\dots$
xxii> $\overline{A+B+C+\dots}=\bar{A}.\bar{B}.\bar{C}.\dots$ } De-Morgan's Theorem

De-morgan's

∴ Changes | (from one boolean o/p we can derive other " o/p)

AND → OR
OR → AND
1 → 0
0 → 1
Duality Theorem

$A+\bar{A}=1$
↓ ↓
$A.\bar{A}=0$

$A.1=A$
↓ ↓
$A+0=A$

← i/p	→		← o/p →				← o/p →
A	B	C	B+C	A(B+C)	AB	AC	AB+AC
0	0	0	0	0	0	0	0
0	0	1	1	0	0	0	0
0	1	0	1	0	0	0	0
0	1	1	1	0	0	0	0
1	0	0	0	0	0	0	0
1	0	1	1	1	0	1	1
1	1	0	1	1	1	0	1
1	1	1	1	1	1	1	1

(∴ both are equal) Proved.

ANOTHER WAY to Prove these Theorems:

i> $A+A=A$

$\Rightarrow (A+A).1$ ∵ $(A.1=A)$

$=(A+A).(A+\bar{A})$

$= A+(A.\bar{A})$ ∵ $A+(B.C)=(A+B).(A+C)$

$= A+(0)$ ∵ $A.\bar{A}=0$

$= A$ ∵ $A+0=A$

so $\boxed{A+A=A}$ Proved.

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ii> $A.A = A$

if $A=0$, $0.0=0$

if $A=1$, $1.1=1$

Take L.H.S

$A.A = (A.A)+0 \quad \because A+0=A$

$= (A.A)+(A\bar{A}) \quad \because A\bar{A}=0$

$= A.(A+\bar{A}) \quad \because A(B+C)=AB+AC$

$= A.(1) \quad \because (A+\bar{A}=1)$

$= A$

L.H.S = R.H.S.

iii> $A+1=1$

if $A=0$, $0+1=1$

if $A=1$, $1+1=1$

Take L.H.S

$A+1 = (A+1).1$

$= (A+1)(A+\bar{A})$

$= A+(1.\bar{A})$

$= A+\bar{A}$

$\boxed{A+1=1}$

iv> $A.0=0$

if $A=0$, $0.0=0$

if $A=1$, $1.0=0$

Taking L.H.S $A+1=1$ (proved)

Based on duality

$A+1=1$

$\boxed{A.0=0}$

5> $\bar{\bar{A}} = A$

if $A=0$, $\bar{\bar{0}}=0$

$A=1$, $\bar{\bar{1}}=1$

6> $A+AB=A$

Taking L.H.S

$A+AB = A.1+AB$

$= A(1+B)$

$= A.1 \Rightarrow A$ Proved

7> $A(A+B) = A$

Taking L.H.S $= A(A+B)$

$= (A+A)(A+B) \quad \because A+A=A$ (distributive)

$= A+AB$

$= A(1+B)$

$= A(1)$

$\boxed{A(A+B)=A}$

8> $A+\bar{A}B = A+B$

Taking L.H.S

$= A+\bar{A}B \quad \because A+AB=A$

$= A+AB+\bar{A}B$

$= A+B(A+\bar{A})$

$= A+B(1)$

$= A+B$

$=$ R.H.S Proved.

9> $A(\bar{A}+B) = AB$

Taking L.H.S

$= A(\bar{A}+B)$

$= (A+AB)(\bar{A}+B) \quad \because A+AB=A$

$= A\bar{A}+AB+AB\bar{A}+ABB$

$= 0+AB+0+ABB$

$= AB+ABB$

$\Rightarrow AB(1+B)$

$\Rightarrow AB(1)$

$= AB \Rightarrow$ R.H.S

De-Morgan's Theorems:

i> $\overline{A.B.C....} = \bar{A} + \bar{B} + \bar{C} + ...$

ii> $\overline{A+B+C+....} = \bar{A}.\bar{B}.\bar{C}....$

From truth table, we get relations for two variables:

i/p

A	B	$\bar{A}$	$\bar{B}$	$\overline{AB}$	$\bar{A}+\bar{B}$	$\overline{A+B}$	$\bar{A}.\bar{B}$
0	0	1	1	1	1	1	1
0	1	1	0	1	1	0	0
1	0	0	1	1	1	0	0
1	1	0	0	0	0	0	0

Equal o/p's

Equal o/p's

So,
$$\overline{A.B} = \bar{A} + \bar{B}$$
and
$$\overline{A+B} = \bar{A}.\bar{B}$$

Proved

: Now Consider three variables $\bar{c}$ NAND operation

$$\overline{ABC} = \overline{(AB).C}$$
$$= \overline{(A.B)} + \bar{C}$$
$$\boxed{\overline{A.B.C} = \bar{A} + \bar{B} + \bar{C}}$$

: In similar way, the NOR operation of three variables gives

$$\overline{A+B+C} = \overline{(A+B)+C}$$
$$= \overline{(A+B)}.\bar{C}$$
$$\boxed{\overline{A+B+C} = \bar{A}.\bar{B}.\bar{C}}$$

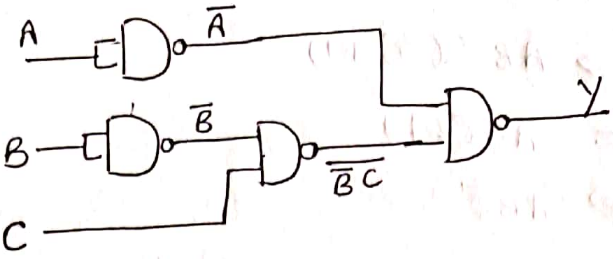
Examples

Realize the given expression by using only NAND gates:–

i> $Y = A + \overline{B}C$

$= \overline{\overline{A + \overline{B}C}}$ $\because \overline{\overline{A}} = A$

$\boxed{Y = \overline{\overline{A} \cdot \overline{\overline{B}C}}}$

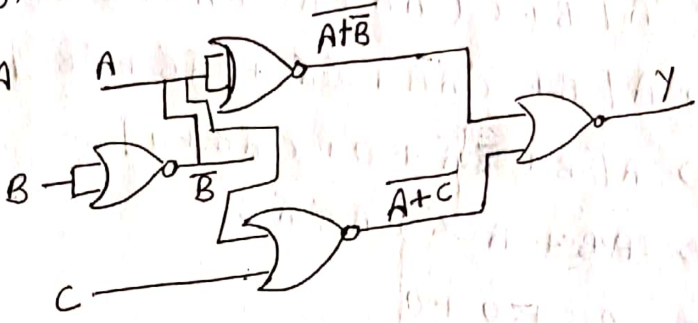


ii> $Y = A + \overline{B}C$ {using only NOR}

$= (A+\overline{B})(A+C)$ $\because$ Distributive Law

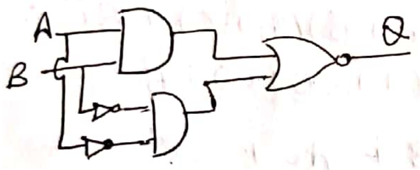
$= \overline{\overline{(A+\overline{B})(A+C)}}$ $\because \overline{\overline{A}} = A$

$\boxed{Y \Rightarrow \overline{\overline{(A+\overline{B})} + \overline{(A+C)}}}$



Write boolean expression for o/p Q.

i>



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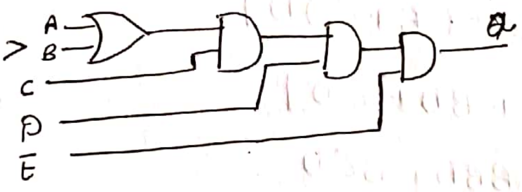

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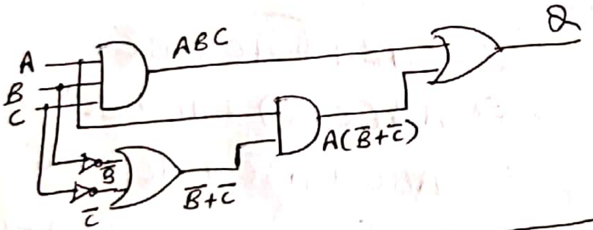

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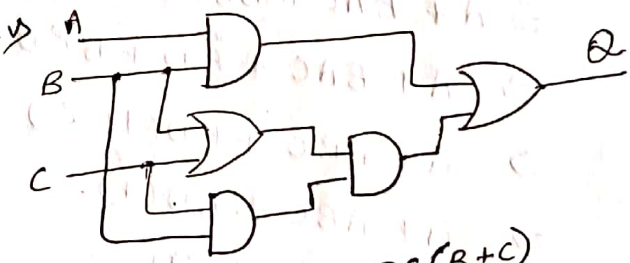


$Q = AB + BC(B+C)$

Reduction in simplest form

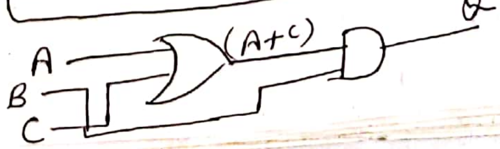
$Q = AB + BC(B+C)$

$= AB + BBC + BCC$ $\because A.A = A$

$= AB + BC + BC$ $\because A + A = A$

$\Rightarrow AB + BC$

$\boxed{Q \Rightarrow B(A+C)}$



Reduce in simplest form:–

i> $\overline{A + BC} + \overline{\overline{A}B}$

$\Rightarrow \overline{\overline{(A+BC)} \cdot \overline{\overline{A\overline{B}}}}$

$\Rightarrow (A+BC)(A\overline{B})$ $\because \overline{\overline{A}} = A$

$\Rightarrow AA\overline{B} + BCA\overline{B}$ $\because$ Distributive

$\Rightarrow AA\overline{B} + 0$ $\because B\overline{B} = 0$

$\Rightarrow A\overline{B}$ $\because AA = A$

Reduce the expressions:

a> $AB\bar{C}\bar{D} + AB\bar{C}$

$\Rightarrow AB\bar{C}(\bar{D}+1)$

$= AB\bar{C}(1)$

$\Rightarrow AB\bar{C}$

b> $A[B+\bar{C}(\overline{AB+A\bar{C}})]$

$= A[B+\bar{C}(\overline{AB}.\overline{A\bar{C}})]$

$= A[B+\bar{C}(\bar{A}+\bar{B})(\bar{A}+C)]$

$= A[B+\bar{C}(\bar{A}\bar{A}+\bar{A}C+\bar{A}\bar{B}+\bar{B}C)]$

$\Rightarrow A[B+\bar{C}\bar{A}\bar{A}+\bar{C}\bar{A}C+\bar{C}\bar{A}\bar{B}+\bar{C}\bar{B}C]$

$\Rightarrow A[B+\bar{C}\bar{A}+0+\bar{C}\bar{A}\bar{B}+0]$

$\Rightarrow AB+A\bar{C}\bar{A}+A\bar{C}\bar{A}\bar{B}$

$= AB+0+0$

$= AB$

c> $A + B[AC + (B+\bar{C})D]$

$= A + B[AC + BD + \bar{C}D]$

$= A + BAC + BBD + B\bar{C}D$

$\Rightarrow A + BAC + BD + B\bar{C}D$

$\Rightarrow A + ABC + BD(1+\bar{C})$

$\Rightarrow A + ABC + BD(1)$

$\Rightarrow A(1+BC) + BD$

$\Rightarrow A(1) + BD$ $\quad \because 1+A=1$

$\Rightarrow A + BD$

d> $(\overline{A+\overline{BC}})(A\bar{B}+ABC)$

$= (\bar{A}.BC)(A\bar{B}+ABC)$

$= \bar{A}BCA\bar{B} + \bar{A}BC.ABC$

$\Rightarrow 0 + 0$

$= 0$

e> $(B+BC)(B+\bar{B}C)(B+D)$

$\because A+AB=A$

$= (B)(B+\bar{B}C)(B+D)$

$= (BB + B\bar{B}C)(B+D)$

$= (B+0)(B+D)$

$\Rightarrow BB + BD$

$\Rightarrow B + BD$

$= B(1+D)$

$= B.(1)$

$= B$

f> Show that

$AB + A\bar{B}C + B\bar{C} = AC + B\bar{C}$

Taking L.H.S

$= AB + A\bar{B}C + B\bar{C}$

$= A(B+\bar{B}C) + B\bar{C}$

$= A(B+\bar{B})(B+C) + B\bar{C}$ $\quad \because$ Distributive

$= A(1)(B+C) + B\bar{C}$

$= AB + AC + B\bar{C}$

$= AB.1 + AC + B\bar{C}$

$\Rightarrow AB(C+\bar{C}) + AC + B\bar{C}$

$= ABC + AB\bar{C} + AC + B\bar{C}$

$= AC(B+1) + B\bar{C}(A+1)$

$\Rightarrow AC(1) + B\bar{C}(1)$

$= AC + B\bar{C}$

$=$ R.H.S $\quad$ Proved

g> Show that

$$A\bar{B}C + B + B\bar{D} + AB\bar{D} + \bar{A}C = B + C$$

Taking L.H.S

$= A\bar{B}C + \bar{A}C + B(1 + \bar{D} + A\bar{D})$

$= C(A\bar{B} + \bar{A}) + B(1 + \bar{D}(1 + A))$

$= C(\bar{A} + A\bar{B}) + B(1 + \bar{D}(1))$

$\Rightarrow C(\bar{A} + A\bar{B}) + B(1 + \bar{D})$

$= C(\bar{A} + A\bar{B}) + B(1)$

$= C(\underbrace{\bar{A} + A}_{1})(\bar{A} + \bar{B}) + B$

$= C(1)(\bar{A} + \bar{B}) + B$

$= \bar{A}C + (\bar{B}C + B)$

$= \bar{A}C + (B + \bar{B})(B + C)$ Distributive law

$= \bar{A}C + (1)(B + C)$

$= \bar{A}C + (B + C)$

$= B + C(1 + \bar{A})$

$= B + C \Rightarrow$ R.H.S

Reduce Expression:>

h> $(\overline{\bar{A} + \overline{A+B}})(\overline{\bar{B} + \overline{B+C}})$

$= \overline{\bar{A} + \overline{A+B}} + \overline{\bar{B} + \overline{B+C}}$

$= A(A+B) + B(B+C)$

$= AA + AB + BB + BC$

$= A + AB + B + BC$

$= A(1+B) + B(1+C)$

$= A(1) + B(1)$

$= A + B$

Reduce the following Boolean Expressions? —

a> $P + Q + P$

b> $P + Q + R + \bar{P}$

c> $P + Q + R + R + \bar{R}$

d> $P + P + P + P$

e> $XY + XY + Y + Y$

f> $X(\bar{X} + YZ)$

g> $X(YZ + \bar{Y}Z)$

h> $AAB(\bar{A}BC + BBC)$

i> $AB + A(B+C) + \bar{B}(B+D)$

j> $(x + y + z)(\bar{x} + \bar{y} + \bar{z})X$

k> $ABC[AB + \bar{C}(BC + AC)]$

l> $A + B + \bar{A}\bar{B}C$

m> $\bar{A}B + \bar{A}B\bar{C} + \bar{A}BCD + \bar{A}B\bar{C}\bar{D}E$

Show that

a> $\overline{\overline{AB} + \overline{A} + AB} = 0$

b> $AB + \overline{AC} + A\overline{B}C(AB + C) = 1$

c> $AB + A(B+C) + B(B+C) = B + AC$

d> $A\overline{B}(C + BD) + \overline{A}\,\overline{B} = \overline{B}C$

e> $\overline{A}\,\overline{B}C + (\overline{A + B + \overline{C}}) + \overline{A}\,\overline{B}\,\overline{C}D = A\overline{B}(C+D)$

f> $ABCD + AB(\overline{C}\,\overline{D}) + (\overline{A}B)CD = AB + CD$

g> $A\overline{B}C + \overline{A}BC + ABC = AC + AB$

h> $A[B + C(\overline{AB + AC})] = AB$

i> $A + \overline{B}C(A + \overline{\overline{B}C}) = A$

j> $\overline{ABC}(\overline{A+B+C}) = ABC$