

[Section-B]

[Answer - 6]

Solⁿ:-

Equivalence Relation \Leftrightarrow Reflexive
Symmetric
Transitive

i) Reflexive - If $\delta(q_1, a) = \delta(q_1, a)$
then
 $q_1 R q_2 \quad \forall a \in \Sigma$
 $\therefore R$ is a reflexive

ii) Symmetric - If $\delta(q_1, a) = \delta(q_2, a)$
 $\delta(q_2, a) = \delta(q_1, a)$
then
 $q_1 R q_2 = q_2 R q_1$
 $\therefore R$ is a symmetric

iii) Transitive - if $\delta(q_1, a) = \delta(q_2, a) \rightarrow \textcircled{i}$
 $\delta(q_2, a) = \delta(q_3, a) \rightarrow \textcircled{ii}$
from \textcircled{i} & \textcircled{ii}
 $\delta(q_1, a) = \delta(q_3, a)$
 $\therefore q_1 R q_3, q_1 R q_2 \text{ \& } q_2 R q_3$

Therefore, R is a transitive
hence,

R is a equivalence Relation.

[Answer - 8]

$S \rightarrow aAa$, $A \rightarrow Sb/bCC/DaA$, $C \rightarrow abb/DD$
 $E \rightarrow aC$, $D \rightarrow aDA$

Solⁿ Step-1(a): Construction of V_N

$$\omega_1 = \{C\} \text{ as } C \rightarrow abb$$

$$\omega_2 = \{\omega_1\} \cup \{E, A\} \text{ as } E \rightarrow aC, A \rightarrow bCC$$

$$\omega_2 = \{C\} \cup \{E, A\}$$

$$\omega_2 = \{A, E, C\}$$

$$\omega_3 = \omega_2 \cup \{S\}$$

$$\omega_3 = \{S, A, E, C\}$$

$$\omega_4 = \omega_3 \cup \phi$$

$$V_N = \{S, A, E, C\}$$

Step 1(b) - Construction of P'

$$P' = \{A \rightarrow \cancel{a}A\}$$

$$P' = \{S \rightarrow aAa, A \rightarrow bCC/Sb, C \rightarrow abb, E \rightarrow aC\}$$

$$\therefore G' = (\{S, A, E, C\}, \{a, b\}, P', S)$$

Step 2: $\omega_1 = \{S\}$ as $S \rightarrow aAa$

$$\omega_2 = \omega_1 \cup \{A, a\} \text{ as } A \rightarrow Sb/bCC$$

$$\omega_2 = \{S, A, a\}$$

$$\omega_3 = \{S, A, a\} \cup \{S, b, C\}$$

$$\omega_3 = \{S, A, a, b, C\}$$

$$\omega_4 = \omega_3 \cup \phi = \omega_3$$

$$\therefore V_N'' = \{S, A, C\}$$

$$\Sigma' = \{a, b\}$$

$$P'' = \{S \rightarrow aAa, A \rightarrow sb/bcc, C \rightarrow abb\}$$

Therefore $G'' = (\{S, A, C\}, \{a, b\}, P'', S)$ is reduced grammar.

[Answer-9]

$$S \rightarrow aS / AB, A \rightarrow \Lambda, B \rightarrow \Lambda, D \rightarrow b$$

Soln-
?

Step 1: - Construct of the set ω of all null variables -

$$\omega_1 = \{A \in V_N : A \rightarrow \Lambda \text{ is a production of } P\}$$

$$\omega_1 = \{A, B\} \text{ as } A \rightarrow \Lambda \text{ \& } B \rightarrow \Lambda$$

$$\omega_2 = \{A, B\} \cup \{S\} \text{ as } S \rightarrow AB$$

$$\omega_2 = \{S, A, B\}$$

$$\omega_3 = \omega_2 \cup \phi = \omega_2$$

$$\text{Thus, } \omega = \omega_2 = \{S, A, B\}$$

Step-2 - Construction of P'

i) $D \rightarrow b$ is included in P'

ii) $S \rightarrow aS$ gives rise to $S \rightarrow aS$ & $S \rightarrow a$

iii) $S \rightarrow AB$ gives rise to $S \rightarrow A$ & $S \rightarrow B$

here, the required grammar without nulls

$$G_1 = (\{S, A, B, D\}, \{a, b\}, P', S)$$

where

$$P' = \{S \rightarrow aS, S \rightarrow a, S \rightarrow AB, S \rightarrow A, S \rightarrow B, D \rightarrow b\}$$

[Answer - 10]

$S \rightarrow AB, A \rightarrow a, B \rightarrow C/b \rightarrow c \rightarrow D, D \rightarrow E, E \rightarrow a$

Step 1 : $\omega_0(S) = \{S\}$
 $\omega_1(S) = \{S\} \cup \phi$
 hence $\omega(S) = \{S\}$

Similarly

$\omega_0(A) = \{A\}$
 $\omega_0(B) = \{B\}$
 $\omega_1(B) = \omega_0(B) \cup \{C\}$
 $\omega_1(B) = \{B, C\}$
 $\omega_2(B) = \{B, C, D\}$
 $\omega_3(B) = \{B, C, D, E\}$

Similarly

$\omega_0(C) = \{C\}$
 $\omega_1(C) = \{C, D\}$
 $\omega_2(C) = \{C, D, E\}$
 $\omega_0(D) = \{D\}$
 $\omega_1(D) = \{D, E\}$
 $\omega_0(E) = \{E\}$

Similarly

Step 2 : The grammar without unit production

$G' = \{S, A, D, B, C, E\}, \{a\}, P', S]$

where

$P' = \{S \rightarrow AB, A \rightarrow a, E \rightarrow a, B \rightarrow b/a, C \rightarrow a, D \rightarrow a\}$

but if we can see,

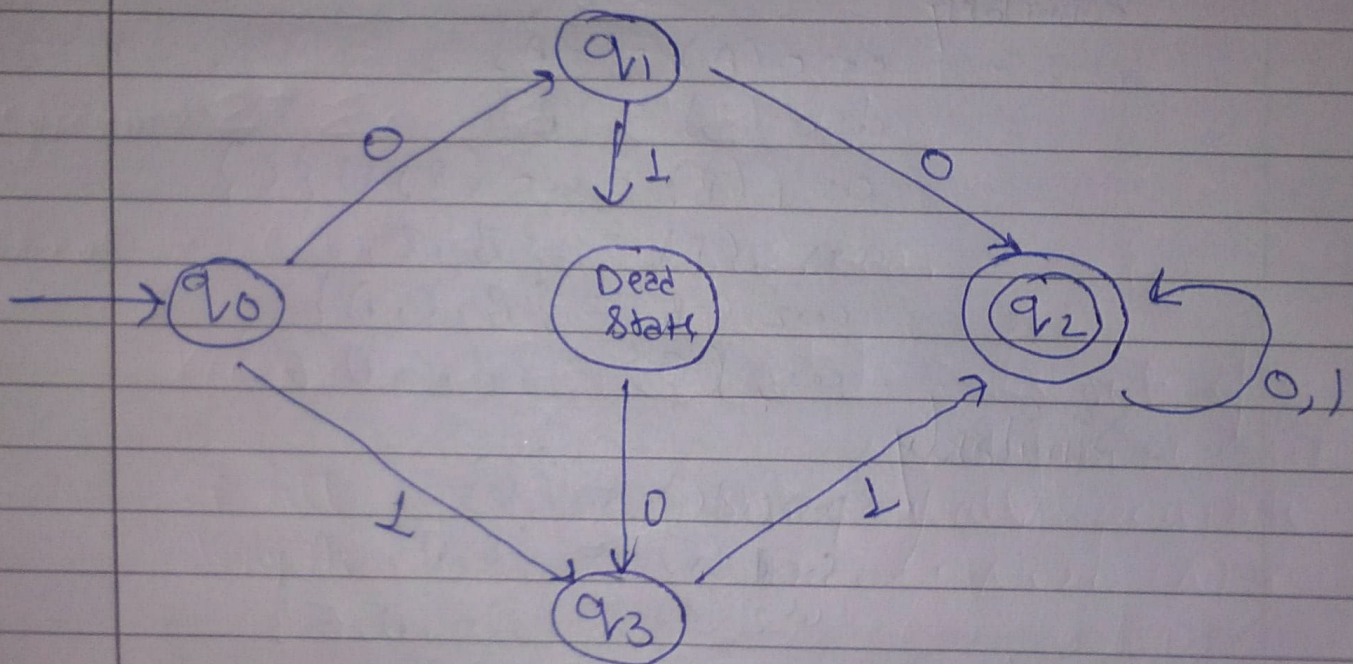
$C \rightarrow D, D \rightarrow E \text{ \& } E \rightarrow a$ production are useless

$P' = \{S \rightarrow AB, A \rightarrow a, B \rightarrow b/a\}$

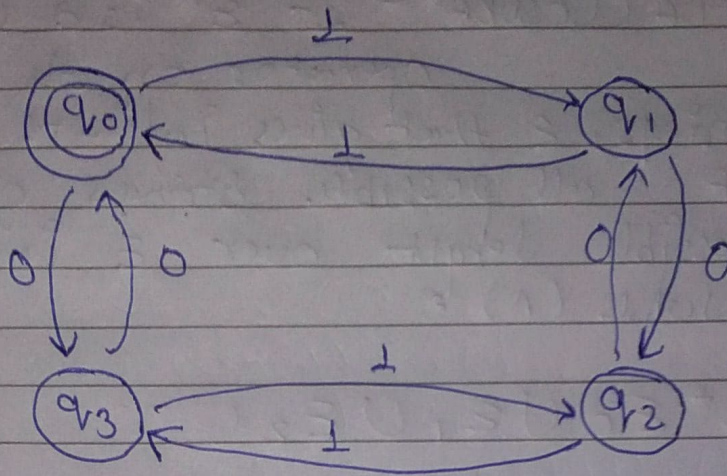
{ Section - A }

[Answer - 1]

00 or 11 over $\{0, 1\}$



[Answer - 2]



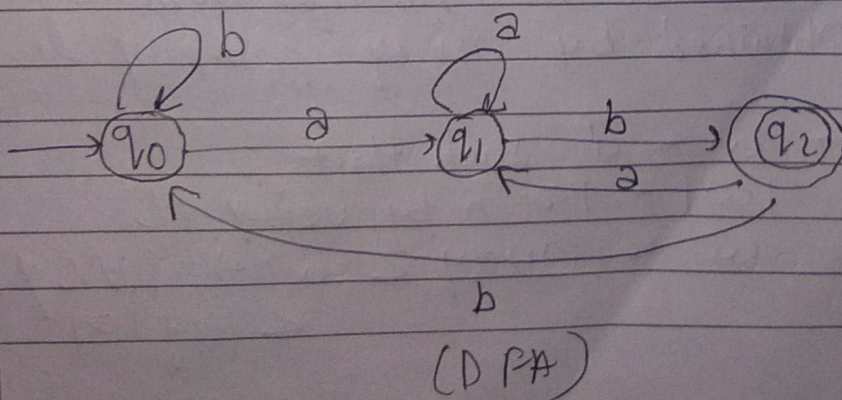
Suppose the string is 1010

$$\begin{aligned}
 \delta(q_0, 1010) &= \delta(q_1, 010) \\
 &= \delta(q_2, 10) \\
 &= \delta(q_3, 0) \\
 &= \delta(q_0, \Lambda) \\
 &= q_0 = \text{final state.}
 \end{aligned}$$

[Answer - 3]

The length of string = 26

The minimum num of states required = 3



[Ans - 4]

KLEENE CLOSURE* - It ϵ^* is a unary operator on the set of strings, ϵ that gives infinite set of all possible strings of all possible length over ϵ including NULL (Λ).

Represent $\epsilon^* = \epsilon_0 \cup \epsilon_1 \cup \epsilon_2 \cup \dots$

Ex: $(a)^* = \{\Lambda, a, aa, aaa, \dots\}$

KLEENE closure Plus $[\epsilon^+]$ - The set of ϵ^+ is infinite set of all possible string of all possible length over ϵ without NULL (Λ).

Representation - $\epsilon^+ = \epsilon^* - \{\Lambda\}$

[Answer - 5]

$L = \{a^p\}$ p is prime is not.

Step 1: Let L is context free
Let n be the natural num
Obtained by using pumping lemma

Step 2: Let $w = a^p$ such that
 $|w| = p > n$ by using PL
 $w = uvxy$ with $|vxy| \leq n$
& $|vy| \geq 1$

Now $UV^i x y^j z = a^{p-r-s-t-m} a^r a^s a^t a^m$

$U = a^{p-r-s-t-m}$, $V = a^r$, $x = a^s$, $y = a^t$, $z = a^m$

Step 3: for $j = p+1$

$$\begin{aligned}
 UV^j x y^j z &= |UV x y z| + |V^{j-1}| + |y^{j-1}| \\
 &= |a^p| + |a^{r(j-1)}| + |a^{t(j-1)}| \\
 &= p + r(j-1) + t(j-1) \\
 &= p + r(p+1-1) + t(p+1-1) \\
 &= p + rp + tp \\
 &= p(1+r+t)
 \end{aligned}$$

Since,

p is prime but $p(1+r+t)$ is not a prime.