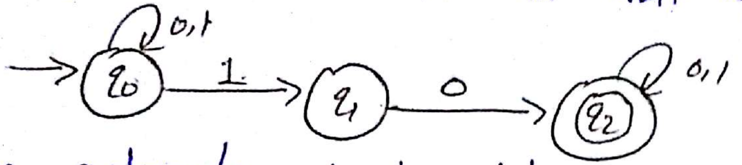
Non-Deterministic Finite Automata :-

Non-deterministic finite automata can reside in multiple states at the same time. The concept of non-deterministic finite automata is being explained with the help of transition diagram:



(Transition Diagram for NFA)

If the automata is in state q_0, and the next input symbol is 1. then the next state will be either:

① q_0 ② q_1

Thus move from q_0 on input 1 cannot be determined uniquely. Such machines ~~form~~ are called Non-deterministic automata.

Definition of NFA :-

A Non-deterministic finite automata is a 5-tuple.

$$M = (Q, \Sigma, \delta, q_0, F)$$

where. Q = A finite set of states

Σ = A finite set of i/p

δ = A transition function from $Q \times \Sigma$ to the power set of Q i.e. to 2^Q.

q_0 = $q_0 \in Q$ is the start | initial state.

F = $F \subseteq Q$ is a set of final | accepting state.

The NFA for fig. can be formally represented as

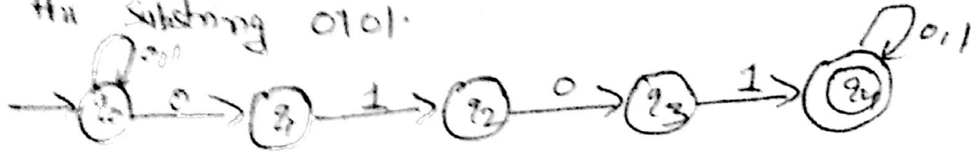
$(\{q_0, q_1, q_2\}, \{0,1\}, \delta, q_0, \{q_2\})$

where the transition function δ is given below

	0	1
$\rightarrow q_0$	$\{q_0\}$	$\{q_0, q_1\}$
q_1	$\{q_2\}$	$\{\phi\}$
q_2 (final)	$\{q_2\}$	$\{q_2\}$

Ques:- Draw a non-deterministic automata to accept strings containing the substring 0101.

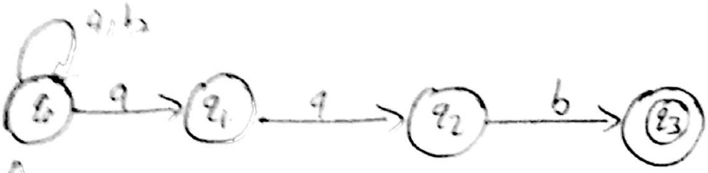
Sol:-



Ques:- Construct a NFA that accepts any positive no of occurrences of various string from the following language L given by.

L = {x ∈ {a,b} | x end with abb

Sol:-



The language accepted by the NFA shown in fig. consisting of string ending with aab. A string of length n, ending in abb can be accepted by the NFA if:

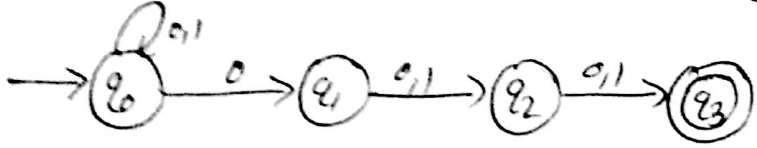
① The machine remains in q_0 for first n-3 inputs

② The last three inputs aabb the machine makes the transition as

$$q_0 \xrightarrow{a} q_1 \xrightarrow{a} q_2 \xrightarrow{a} q_3.$$

Ques:- Draw a NDFA to accepts string over alphabet {0,1} such that the third symbol from the right end is 0.

Sol:-

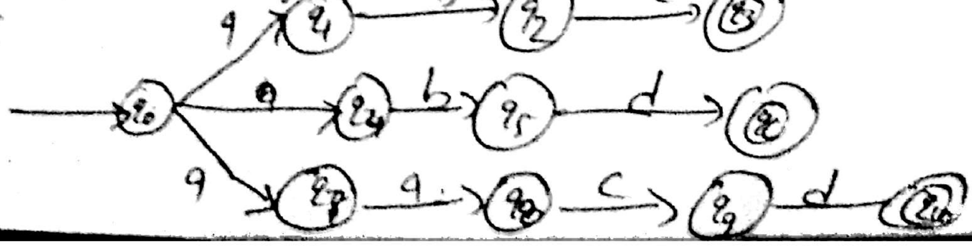


=> for the first n-3 symbols, the machine can remain in the start state q_0.

=> On seeing the third symbol from the right end as 0 (NFA can guess), it makes towards the final state q_3.

Ques:- Design a NFA to recognize the following sets of strings: abc, abd, aacd, Assume that alphabet is {a,b,c,d} Give the transition table and transition diagram.

Sol:-

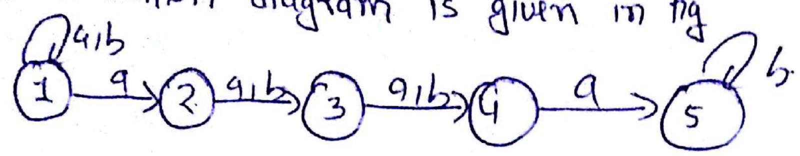


Ques: An NFA with state 1-5 and input alphabet {a,b} has following transition table.
(a) Draw a transition diagram (b) Calculate $\delta^*(1,ab)$
(c) Calculate $\delta^*(1,abaab)$

q	$\delta(q,a)$	$\delta(q,b)$
1	{1,2}	{1}
2	{3}	{3}
3	{4}	{4}
4	{5}	ϕ
5	ϕ	{5}

Solu:-

(a) Transition diagram is given in fig

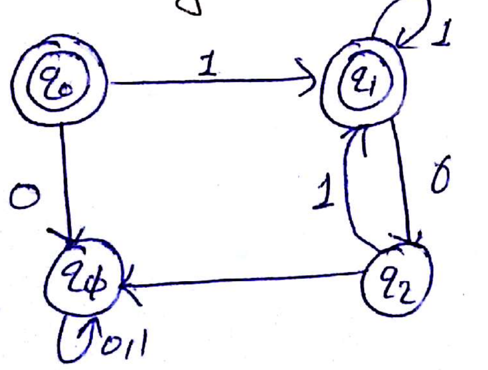


(b) $\delta^*(1,ab) = \delta((\delta(1,a)),b)$
$= \delta(\{1,2\},b) = \delta(1,b) \cup \delta(2,b)$
$= \{1\} \cup \{3\} = \{1,3\}$

(c) $\delta^*(1,abaab) = \delta^*(\{1,2\},baab)$
$= \delta^*(\{1,3\},aab)$
$= \delta^*(\{1,2,4\},ab) = \delta(\{1,2,3,5\},b)$
$= \{1,2,3,4\}$

Ques:- Construct a FA for accepting L over {0,1} such that each 0 is immediately preceded and immediately followed by 1.

Solu:-



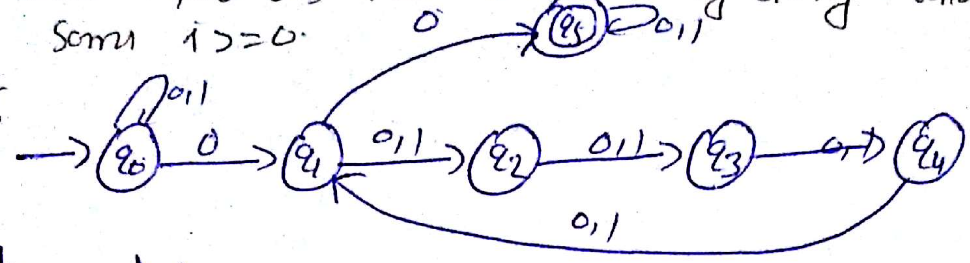
⇒ If the starting symbol is 0, string is rejected by entering the failure state q_ϕ.

⇒ A loop through $q_1 \rightarrow q_2 \rightarrow q_1$ ensure that every 0 is preceded and succeeded by 1

⇒ An input 0 in state q_2 will mean two consecutive 0's. The string is rejected by entering the failure state q_ϕ.

Ques:- Construct NFA that accepts the set of string in $(0+1)^*$ such that some two 0's are sepreted by string whose length is $4i$ for some $i >= 0$.

Solu:-

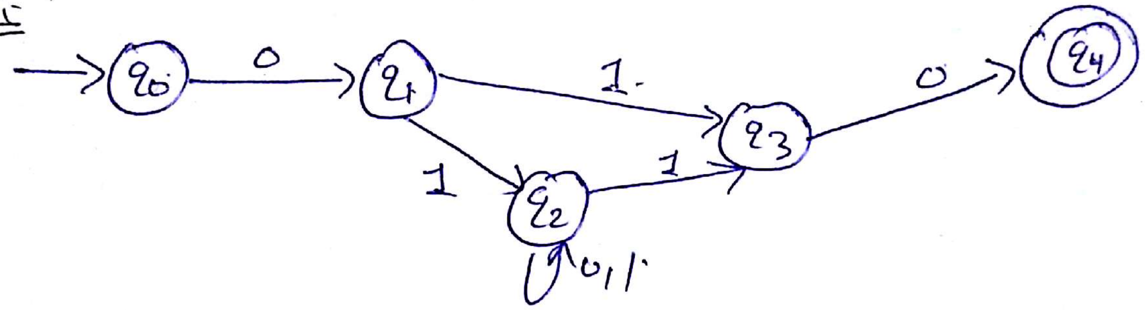


⇒ The loop $q_1 \to q_2 \to q_3 \to q_4 \to q_1$ can absorb a string of length $4i$.

⇒ On receiving the 0 of the portion of the string containing "two 0's sepreted by string of length $4i$, the machine makes a move to q_1 from q_0. Then it loop i times in $q_1 \to q_2 \to q_3 \to q_4 \to q_1$. Then it gets a 0 and makes a moves from q_1 to q_5.

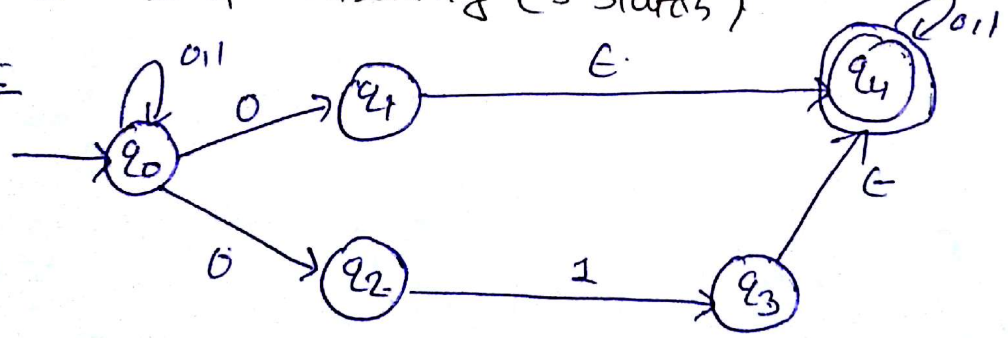
Ques:- Design NFA for the set of strings on the alphabet $\{0,1\}$ that start with 01 and end with 10.

Solu:-



Ques:- Design a NFA over the alphabet $\{0,1\}$ such that 0 or 01 as a substring (5 states)

Solu:-



⇒ Any string containing 0 as a substring can be accepted through the path
$q_0 \to q_1 \to q_4$.

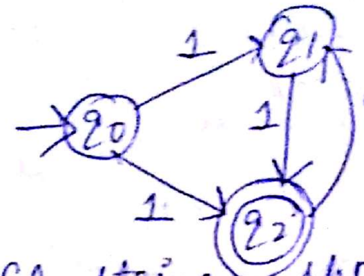
⇒ Any string containing 01 as a substring can be accepted through the path
$q_0 \to q_2 \to q_3 \to q_4$

Book: "Udit Agarwal"

Non-Deterministic finite Automata (NFA)

Q1 Determine an NFA accepting all strings over $\{0,1\}$ which end in 1 but does not contain the substring 00.

solution:



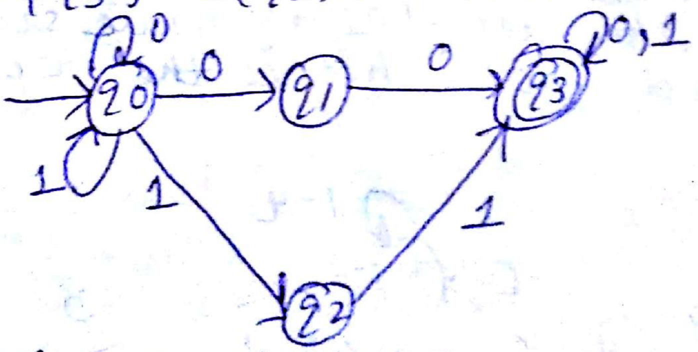
In this NFA, string should end in a 1 and it should not contain 00.

Q2 Sketch the NFA state diagram for $M = (\{q_0, q_1, q_2, q_3\}, \{0,1\}, \delta, q_0, \{q_3\})$ with the state table as

solution:

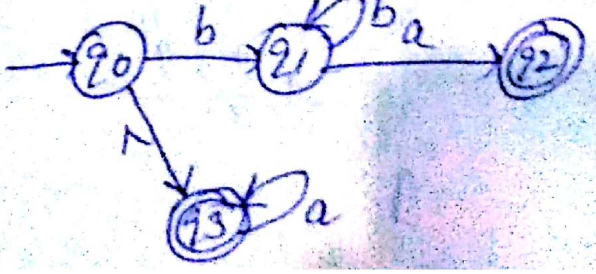
state	input 0	input 1
$\to q_0$	q_0, q_1	q_0, q_2
q_1	q_3	ϕ
q_2	ϕ	q_3
q_3	q_3	q_3

here $\delta(q_0,0) = \{q_0,q_1\}$
$\delta(q_0,1) = \{q_0,q_2\}$ $\delta(q_3,0) = \{q_3\}$ $\delta(q_1,0) = \{q_3\}$
$\delta(q_3,1) = \{q_3\}$ $\delta(q_2,1) = \{q_3\}$

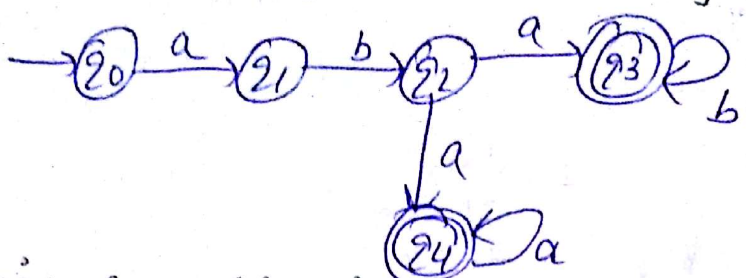


Q Find an NFA with four states for
$L = \{a^n : n \geq 0\} \cup \{b^n a : n \geq 1\}$

solution:

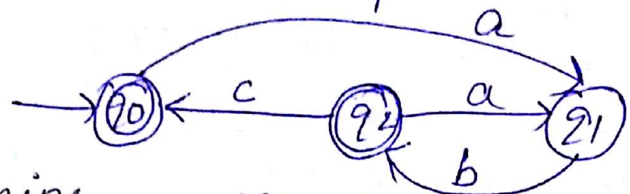


Q4 Design an NFA with no more than 5 states for the set : $L = \{abab^n : n \geq 0\} \cup \{aba^n : n \geq 0\}$



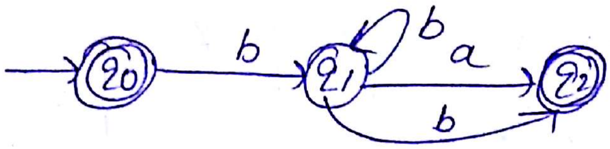
Q5. Design an NFA with three states that accepts the language $\{ab, abc\}^*$

Sol: It accepts "ab" or "abc" in first step and this is looped with initial states so that any combination of "ab" and "abc" can be accepted.



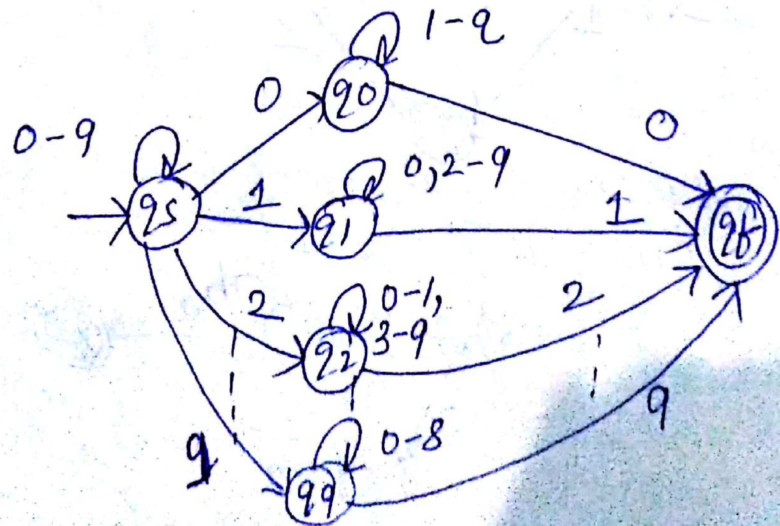
Q6. Determine an NFA that accepts the language $L = (bb^*(a+b))$.

Solution :



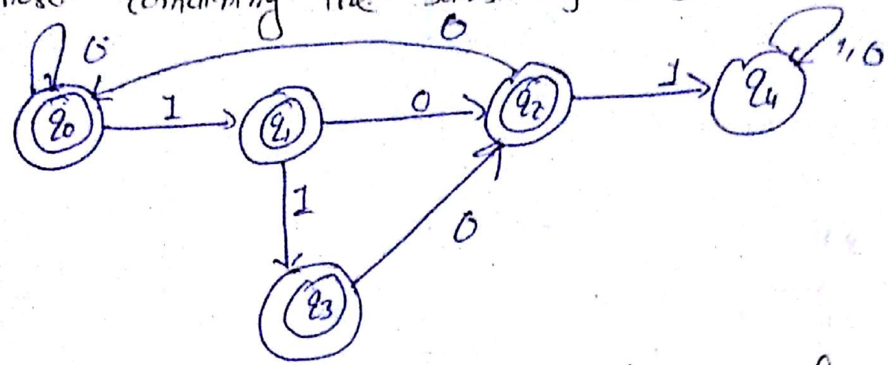
Q7. Give the NDFA to accept the following language: "The set of strings over alphabet $\{0, 1, 2, 3, \ldots, 9\}$ such that the final digit has appeared before".

Solution: The idea is to use a state q_i for $i = 0, 1, 2, \ldots 9$ to represent the fact that we have seen an input i and guessed that this is the repeated digit at the end.



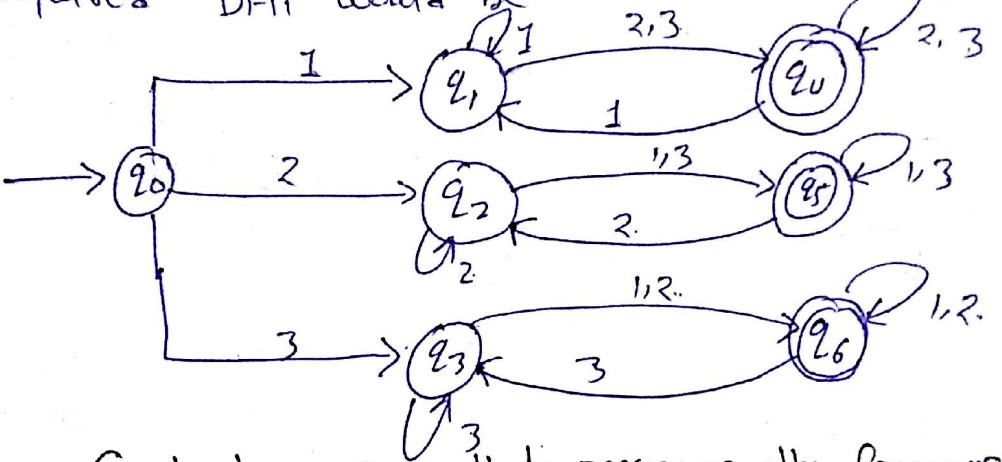
Ques: Construct a DFA that accepts all the string on $\{0,1\}$ except those containing the substring 101.

Solu:-



Ques: Construct a DFA that accepts the following language $L = \{x \in \{1,2,3\}^* \}$ x begin and end with different symbols.

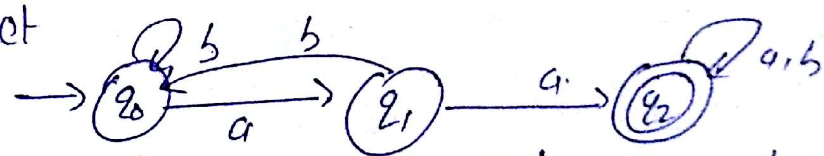
Solu:- We will have three different transition from the start state, for each input symbol. But the ending state transition must be with the symbols other than the starting symbols. Hence the required DFA would be.



Ques:- Construct DFA that recognize the language over $\{a,b\}$ given as follows:

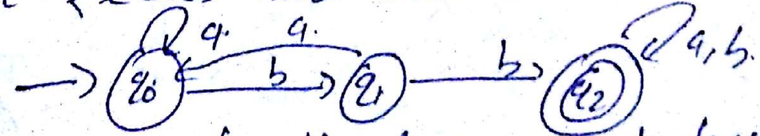
$L = \{x \in \{a,b\}^* \mid (x$ contains (at least) two consecutive a's$)\}$ and $\{(x$ does not contain two consecutive b's$\}$.

Solu let

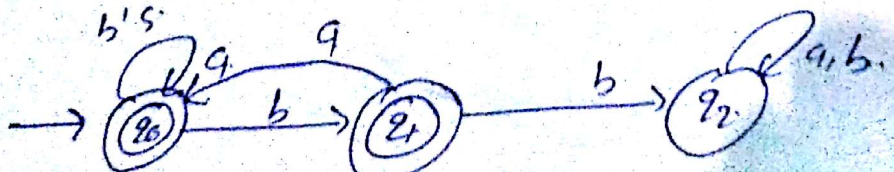


$L = \{x \in \{a,b\}$ (x contains (at least two consecutive a's)$\}$.

and $L = \{x$ does not contains two consecutive b's$\}$..



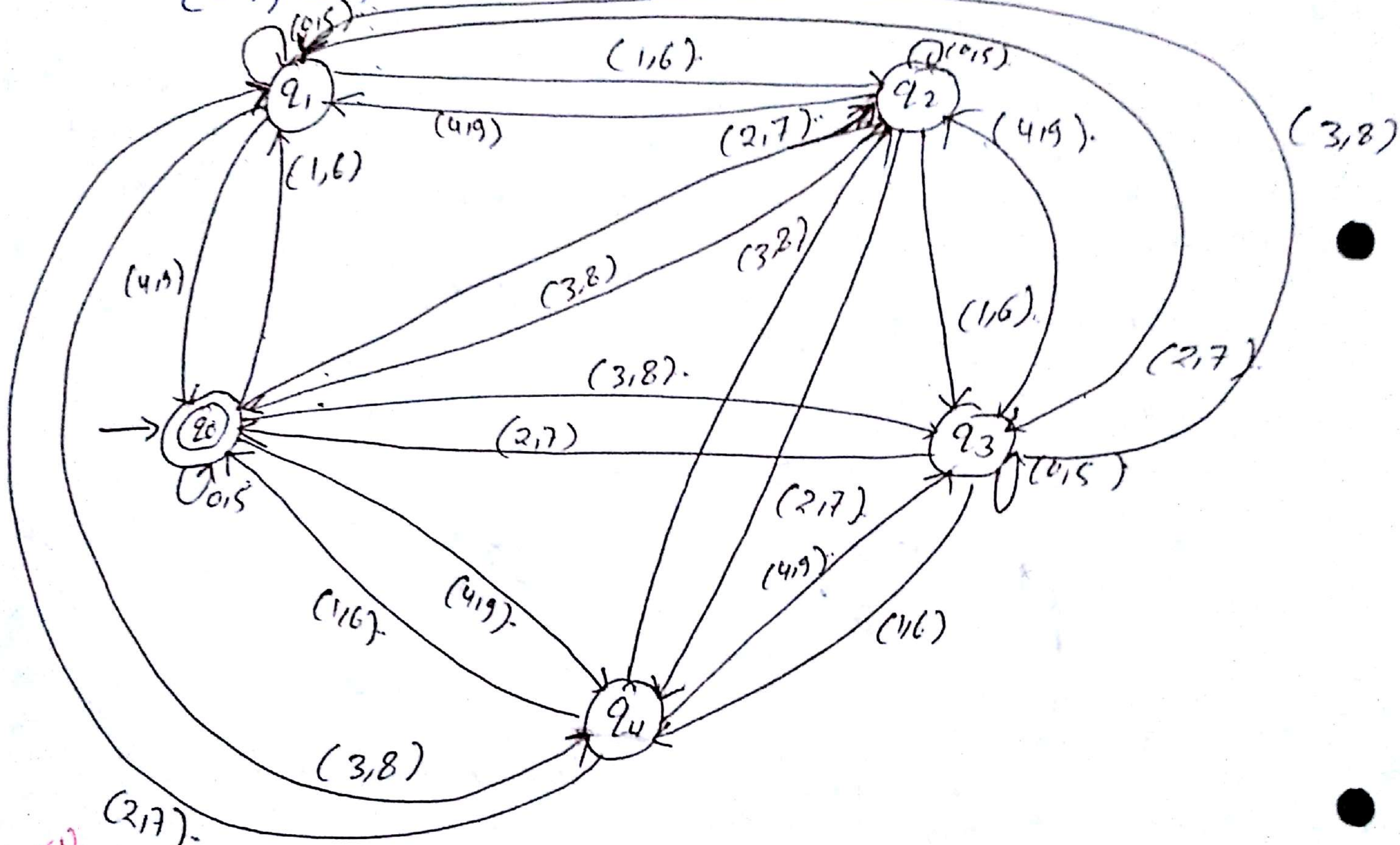
Now to DFA for the language which does not contain two consecutive b's.



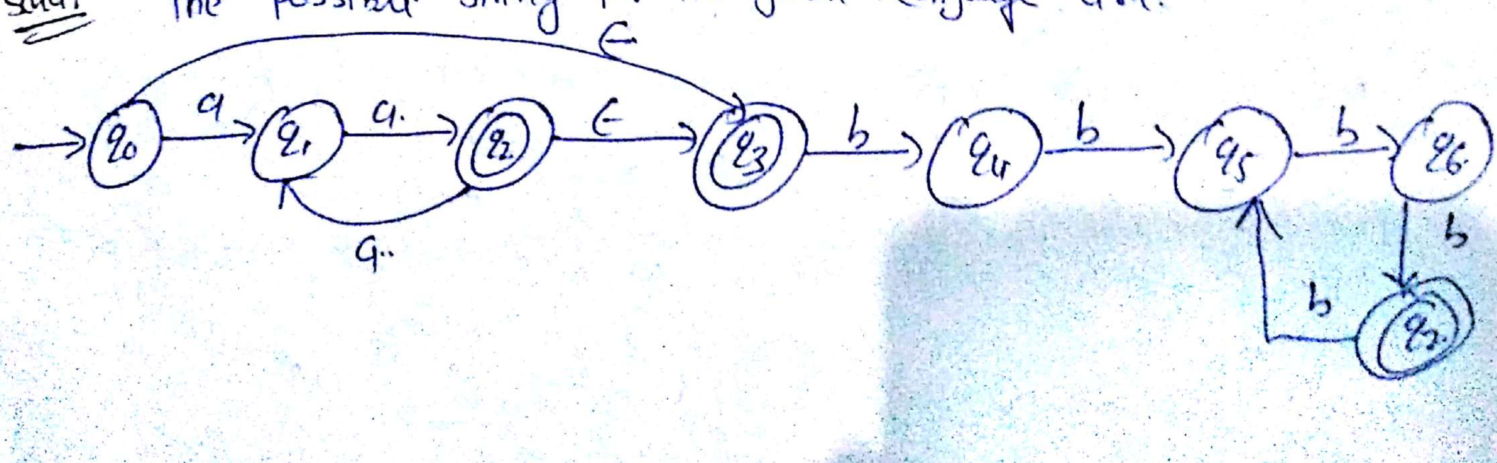
UPTU 07/08 (P-20) (50)

Ques: Design a DFA which accepts the set of string over alphabet $\Sigma = \{1,2,3,4\}$ such that string when interpreted as decimal no. sum of their digit are divisible by 5.

Solu:- (0,5) → remainder 0 → q_0 state
(1,6) → remainder 1 → q_1 state
(2,7) → remainder 2 → q_2 state
(3,8) → remainder 3 → q_3 state
(4,9) → remainder 4 → q_4 state

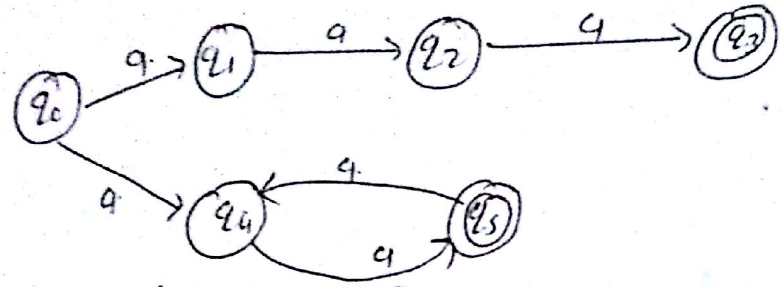


UPTU
Ques! Construct DFA for the following language;
$\{a^m b^n \mid m$ is divisible by 3 and n is divisible by 4$\}$
Solu: The possible string for the given language are.



51 P-21

Ques:- Find the DFA that accepts the language defined by the NFA in fig

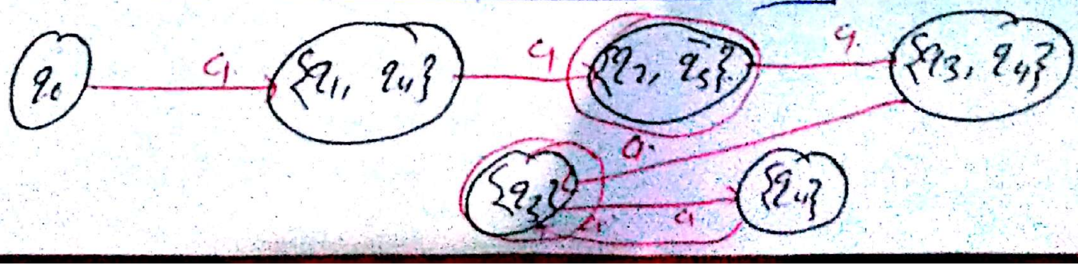


Soln Transition table for given NDFA

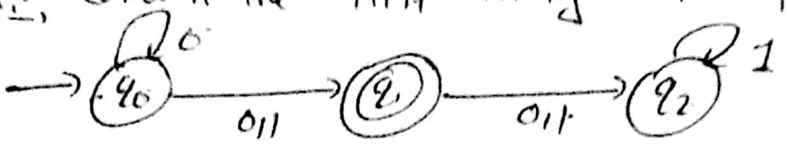
P.S.	Next State a	Next State b
$\rightarrow q_0$	$\{q_1, q_4\}$	$\{\wedge\}$
q_1	$\{q_2\}$	$\{\wedge\}$
q_2	$\{q_3\}$	$\{\wedge\}$
q_3	$\{\wedge\}$	$\{\wedge\}$
q_4	$\{q_5\}$	$\wedge$
q_5	$\{q_4\}$	$\wedge$

Transition table for DFA;

P.S.	N.S. a
$\rightarrow q_0$	$\{q_1, q_4\}$
$\{q_1, q_4\}$	$\{q_2, q_5\}$
$\{q_2, q_5\}$	$\{q_3, q_4\}$
$\{q_3, q_4\}$	$\{q_5\}$
$\{q_5\}$	$\{q_4\}$
$\{q_4\}$	$\{q_5\}$



Ques:-, Convert the NFA in fig into equivalent DFA

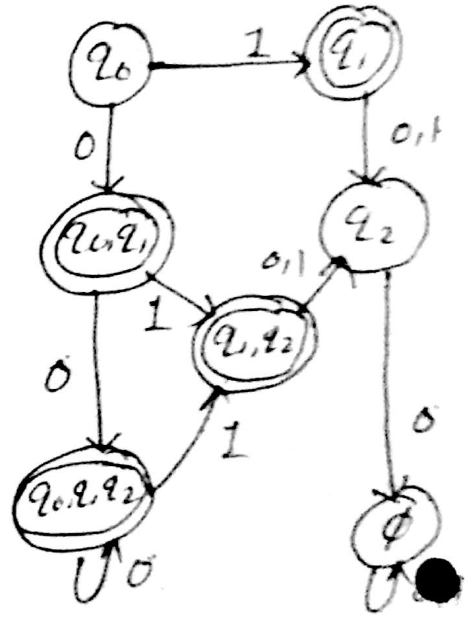


Solu:- Transition table for NFA

I/P / Present state	N.S 0	1
q_0	q_0, q_1	q_1
q_1	q_2	q_2
q_2	$\wedge$	q_2

Transition table for DFA (equivalent)

I/P / P.S	N.S 0	1
$\rightarrow q_0$	$\{q_0, q_1\}$	$\{q_1\}$
$\{q_0, q_1\}$	$\{q_0, q_1, q_2\}$	$\{q_1, q_2\}$
$\{q_1\}$	$\{q_2\}$	$\{q_2\}$
$\{q_0, q_1, q_2\}$	$\{q_0, q_1, q_2\}$	$\{q_1, q_2\}$
$\{q_1, q_2\}$	$\{q_2\}$	$\{q_2\}$
$\{q_2\}$	$\wedge$	$\{q_2\}$

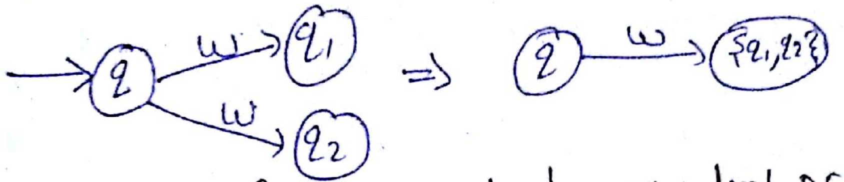


Equivalence of NFA and DFA or Conversion of NFA to DFA:

Conversion of NFA to DFA

Theorem: If a language L is accepted by an NFA then there is a DFA that accept L.

Proof 1: The proof of the theorem is constructive and uses the subset construct. If there are two w paths, the corresponding state from a common state in the DFA will be.



(An NFA and its equivalent DFA).

Proof 2: Let $M = (Q, \Sigma, \delta, q_0, F)$ be an NFA accepting the language L. We construct DFA M as follows.

$$M' = (Q', \Sigma, \delta', q_0, F')$$

where $Q' = 2^Q$ (any state in Q' is represented by $\{q_1, q_2, q_3, \ldots q_i\}$
where $q_1, q_2, q_3 \ldots q_i \in Q$.

$$q_0' = q_0$$

F' = set of all subsets of Q containing an element of F.

Let us consider the construction of Q', q_0 and F'. NFA M is initially at q_0, the initial state. By applying an input symbol 'a', M can reach any of the states in $\delta(q_0, a)$. To describe M, we require all the possible states that M can reach after the application of a. So M' has to remember all these possible states at any instance of time. Hence the states of M' may be defined as subset of Q. As q_0 is initial states so q_0' is defined as $[q_0]$. If the string w is accepted by M, then M reaches a final states on processing w. Therefore a final state ~~of~~ in DFA M' is any subset of Q & having some final state of NFA M.

Steps:

Let $M = (Q, \Sigma, \delta, q_0, F)$ is a NFA which accepts the language $L(M)$. There should be equivalent DFA denoted by $M' = (Q', \Sigma', \delta', q_0', F')$ such that $L(M) = L(M')$

The conversion method will follow following steps.

① The ~~state~~ start state of NFA M will be the start for DFA M'. Hence add q_0 of NFA (start state) to Q'. Then find the transition from this start state.

② For each state $[q_1, q_2, \ldots q_i]$ in Q', the transition for each input symbol Σ can be obtained as.

1) $\delta'([q_1, q_2 \ldots q_i], a) = \delta(q_1, a) \cup \delta(q_2, a) \cup \ldots \delta(q_i, a)$
$= [q_1, q_2 \ldots q_k]$ may be some state

(ii) Add the state $[q_1, q_2 \cdots q_k]$ to DFA if it is not already added in Q'.
(iii) Then find the transitions for every input symbol from Σ for state $[q_1, q_2 \cdots q_k]$. If we get some state $[q_1, q_2, \cdots q_n]$ which is not in Q' DFA then add this state to Q'.

③ For the state $[q_1, q_2, \cdots q_n] \in Q'$ of DFA if any one state q_i is a final state of NFA then $[q_1, q_2 \cdots q_n]$ becomes a final state. Thus the set of all the final states $\in F'$ of DFA.

Numerical:

Ques:- Let $M = (\{q_0, q_1\}, \{0,1\}, \delta, q_0, \{q_1\})$ be NFA where $\delta(q_0, 0) = \{q_0, q_1\}$, $\delta(q_0, 1) = \{q_1\}$, $\delta(q_1, 0) = \phi$, $\delta(q_1, 1) = \{q_0, q_1\}$. Construct its equivalent DFA.

Solu:- Let us draw transition table for δ function for given NFA.

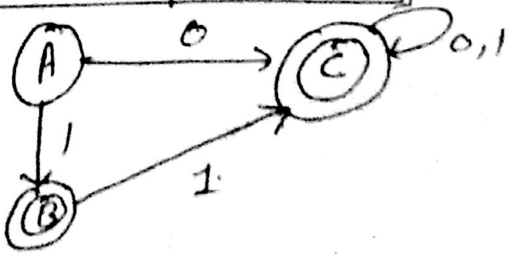
Present State \ I/P	N.S 0	N.S 1
$\rightarrow q_0$	$\{q_0, q_1\}$	$\{q_1\}$
q_1	ϕ	$\{q_0, q_1\}$

from the transition table we can compute that there are $[q_0]$, $[q_1]$, $[q_0, q_1]$ states for its equivalent DFA.

transition table for DFA:-

Present State \ I/P	Next State 0	Next State 1
$\rightarrow q_0$	$\{q_0, q_1\}$	$\{q_1\}$
q_1	$\{\cancel{q_0 q_1}\ q_1\}$	$\{q_0, q_1\}$
q_0, q_1	$\{q_0, q_1\}$	$\{q_0, q_1\}$

$A = [q_0]$
$B = [q_1]$
$C = [q_0, q_1]$



Ques:- Convert the given NFA to DFA

Input / P.State	N.S 0	1
$\rightarrow q_0$	$\{q_0, q_1\}$	q_0
q_1	q_2	q_1
q_2	q_3	q_3
q_3	ϕ	q_2

Transition table for DFA

Present state	Next state I/P 0	1
$\rightarrow q_0$	$\{q_0, q_1\}$	$\{q_0\}$
$\{q_0, q_1\}$	$\{q_0, q_1, q_2\}$	$\{q_0, q_1\}$
$\{q_0, q_1, q_2\}$	$\{q_0, q_1, q_2, q_3\}$	$\{q_0, q_1, q_3\}$
$\{q_0, q_1, q_2, q_3\}$	$\{q_0, q_1, q_2, q_3\}$	$\{q_0, q_1, q_2, q_3\}$
$\{q_0, q_1, q_3\}$	$\{q_0, q_1, q_2\}$	$\{q_0, q_1, q_3\}$

$\delta'([q_0, q_1], 0) = \delta(q_0, 0) \cup \delta(q_1, 0)$

$= \{q_0, q_1\} \cup \{q_2\}$

$= \{q_0, q_1, q_2\}$

$\delta'([q_0, q_1], 1) = \delta(q_0, 1) \cup \delta(q_1, 1)$

$= \{q_0, q_1\}$

$A = \{q_0\}$, $B = \{q_0, q_1\}$, $C = \{q_0, q_1, q_2\}$

$D = \{q_0, q_1, q_2, q_3\}$, $E = \{q_0, q_1, q_3\}$

Present state	Next state 0	1
A	B	A
B	C	B
C	D	E
D	D	D
E	C	E

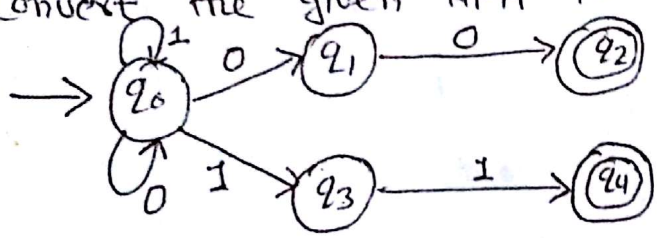
Ques:- Construct DFA equivalent to the given NFA

Input / State	0	1
$\rightarrow p$	$\{p, q\}$	p
q	r	r
r	s	$-$
s	s	s

The equivalent DFA will be

i/p State	Next state 0	Next state 1
→ P	{P, q}	{P}
{P, q}	{P, q, r}	{P, r}
{P, q, r}	{P, q, r, s}	{P, r}
{P, r}	{P, q, s}	{P}
{P, q, s}	{P, q, r, s}	{P, r, s}
{P, q, r, s}	{P, q, r, s}	{P, r, s}
{P, r, s}	{P, q, s}	{P, s}
{P, s}	{P, q, s}	{P, s}

Ques:- Convert the given NFA to its equivalent



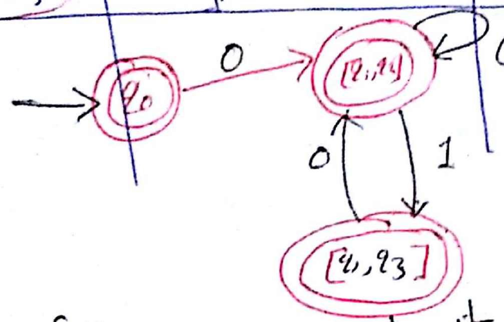
Transition table for NDFA

Solution:-

i/p Present state	Next state 0	Next state 1
→ q_0	{q_0, q_1}	{q_0, q_3}
q_1	{q_2}	φ
q_2	φ	φ
q_3	φ	{q_4}
q_4	φ	φ

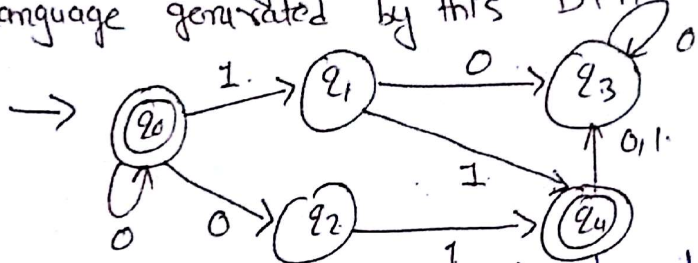
Transition table for equivalent DFA.

Input / Present state	Next state: 0	1
$\rightarrow q_0$	$\{q_1, q_2\}$	$\{\phi\}$
$\{q_1, q_2\}$	$\{q_1, q_2\}$	$\{q_1, q_3\}$
$\{q_1, q_3\}$	$\{q_1, q_2\}$	$\{\phi\}$



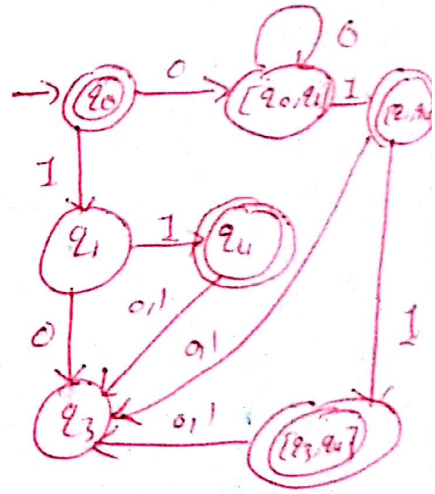
Ques:- Convert the following NFA into its equivalent DFA. Also recognize the language generated by this DFA.

Sol:-



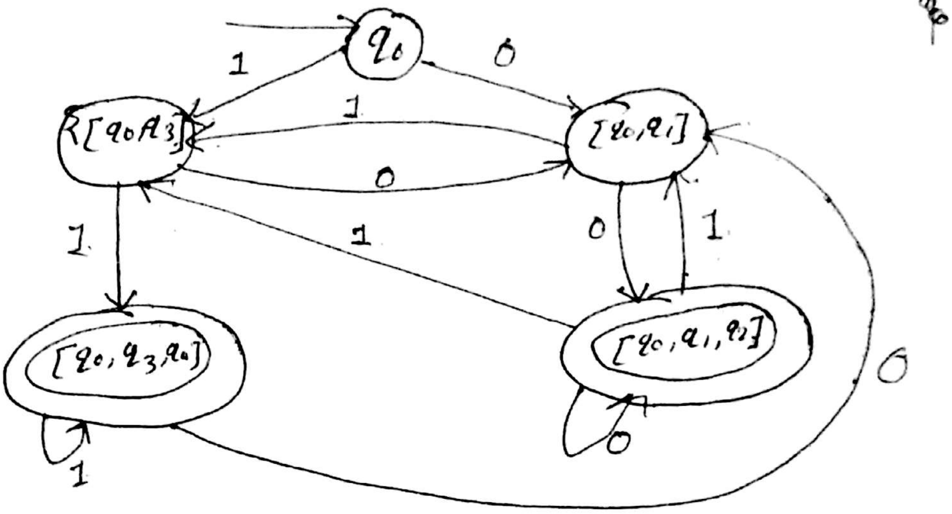
Solui:- we will first doing the transition table from given transition diagram.

I/P / State	N.S: 0	1
q_0	$\{q_0, q_2\}$	q_1
q_1	q_3	q_4
q_2	ϕ	q_4
q_3	q_3	ϕ
q_4	q_3	q_3

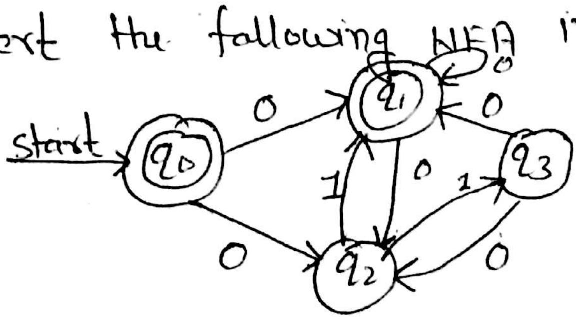


I/P / Present States	Next State: 0	1
$\rightarrow q_0$	$\{q_0, q_2\}$	$\{q_1\}$
$\{q_0, q_2\}$	$\{q_0, q_2\}$	$\{q_1, q_4\}$
$\{q_1\}$	$\{q_3\}$	$\{q_4\}$
$\{q_1, q_4\}$	$\{q_3\}$	$\{q_3, q_4\}$
$\{q_3\}$	$\{q_3\}$	ϕ
$\{q_4\}$	q_3	q_3
$\{q_3, q_4\}$	q_3	q_3

Present state \ Input	Next state 0	1
$\rightarrow q_0$	$\{q_0, q_1\}$	$\{q_0, q_3\}$
$\{q_0, q_1\}$	$\{q_0, q_1, q_2\}$	$\{q_0, q_3\}$
$\{q_0, q_3\}$	$\{q_0, q_1\}$	$\{q_0, q_3, q_4\}$
$\{q_0, q_3, q_4\}$	$\{q_0, q_1\}$	$\{q_0, q_3, q_4\}$
$\{q_0, q_1, q_2\}$	$\{q_0, q_1, q_2\}$	$\{q_0, q_3\}$



Ques:- Convert the following NFA into DFA



Solution:-

States \ Input	Next-State 0	1
$\rightarrow q_0$	$\{q_1, q_2\}$	ϕ
q_1	$\{q_1, q_2\}$	ϕ
q_2	ϕ	$\{q_1, q_3\}$
q_3	$\{q_1, q_2\}$	ϕ

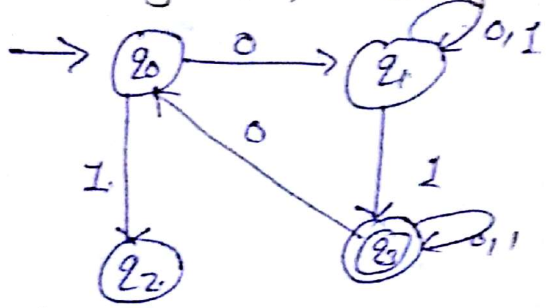
P-89 45

Ques:- For the following state transition table draw the state transition diagram find its equivalent machine. For the string abbaaab test whether both give same results or not. q_0 is initial and q_3 is final state.

Soln:-

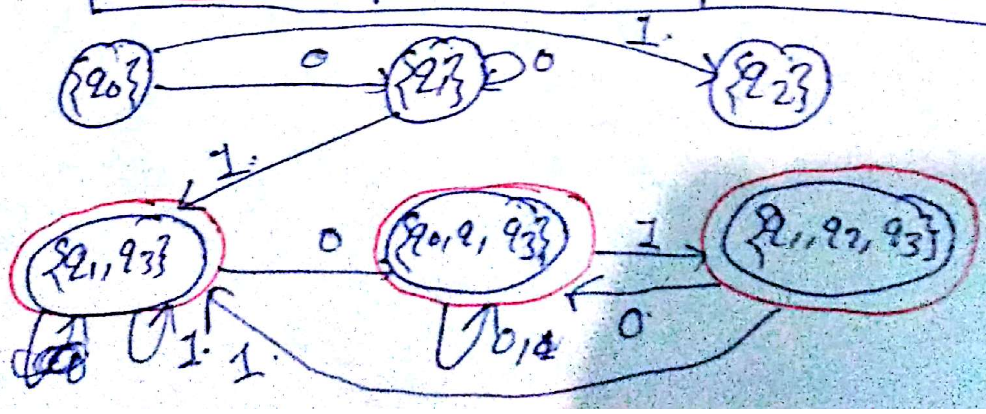
q/Σ	N.S	
	0	1
q_0	q_1	q_2
q_1	q_1	q_1, q_3
q_2	ϕ	ϕ
q_3	q_0, q_3	q_3

Soln The transition diagram for the given transition table can be



Transition table for DFA:-

Σ / Present State	N.S	
	0	1
$\{q_0\}$	$\{q_1\}$	$\{q_2\}$
$\{q_1\}$	$\{q_1\}$	$\{q_1, q_3\}$
$\{q_2\}$	$\{\phi\}$	$\{\phi\}$
$\{q_1, q_3\}$	$\{q_0, q_1, q_3\}$	$\{q_1, q_3\}$
$\{q_0, q_1, q_3\}$	$\{q_0, q_1, q_3\}$	$\{q_1, q_2, q_3\}$
$\{q_1, q_2, q_3\}$	$\{q_0, q_1, q_3\}$	$\{q_1, q_3\}$



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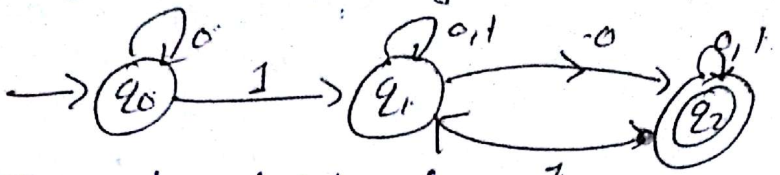
46

Question:- Convert the given NFA to DFA

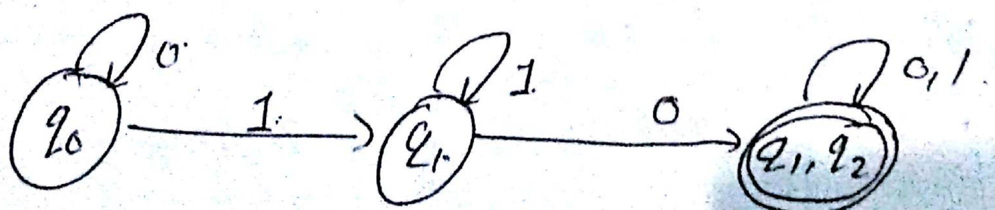


Soln:- Transition table for NFA

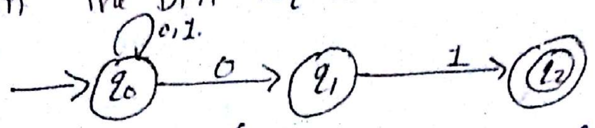
State \ Σ	0	1
$\rightarrow q_0$	q_0	q_1
q_1	$\{q_1, q_2\}$	q_1
q_2	$\{q_2\}$	$\{q_1, q_2\}$

Transition table for DFA

P. State \ Σ	N.S 0	N.S 1
$\rightarrow q_0$	$\{q_0\}$	$\{q_1\}$
$\{q_1\}$	$\{q_1, q_2\}$	$\{q_1\}$
$\{q_1, q_2\}$	$\{q_1, q_2\}$	$\{q_1, q_2\}$



Ques: Obtain the DFA equivalent to the following NFA



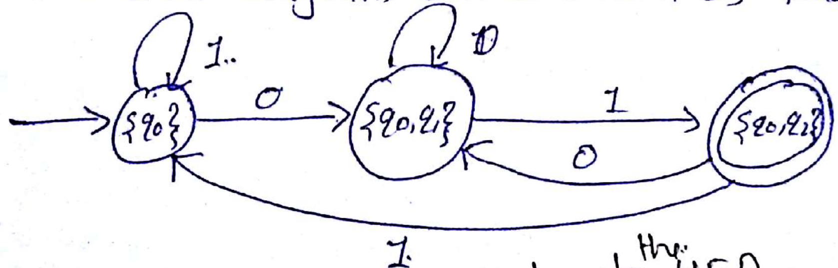
Solu: The transition table for given NFA can be

States / Input → / Present state	Next state 0	1
$\rightarrow q_0$	$\{q_0, q_1\}$	q_0
q_1	$\wedge$	q_2
(q_2)	$\wedge$	$\wedge$

- Transition table for DFA

Input / Present state	N.S 0	1
q_0	$\{q_0, q_1\}$	q_0
$\{q_0, q_1\}$	$\{q_0, q_1\}$	$\{q_0, q_2\}$
$(\{q_0, q_2\})$	$\{q_0, q_1\}$	$\{q_0\}$

Transition diagram can be drawn as follows

- 

Ques: Construct DFA equivalent to the NFA
$M = (\{p, q, r\}, \{0, 1\}, \delta, p, \{q, s\})$.

Present state	N.S. 0	1
$\{p\}$	$\{q, s\}$	$\{q\}$
$\{q\}$	$\{r\}$	$\{q, r\}$
r	$\{s\}$	$\{p\}$
s	—	$\{p\}$

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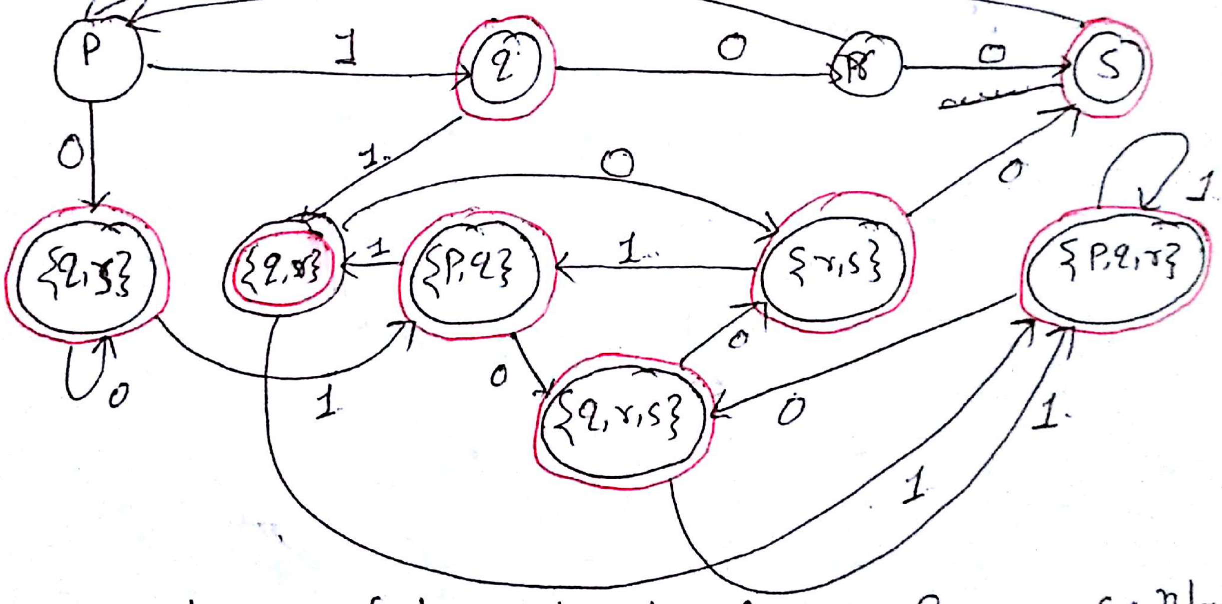
Transition table for equivalent DFA:→

Present state \ I/P	Next State 0	Next State 1
{p}	{q,s}	{q}
{q}	{r}	{q,r}
{q,s}	{q,s}	{p,q}
{r}	{s}	{p}
{q,r}	{r,s}	{p,q,r}
{p,q}	{q,r,s}	{q,r}
{s}	∧	{p}
{r,s}	{s}	{p,q}
{p,q,r}	{q,r,s}	{p,q,r}
{q,r,s}	{r,s}	{p,q,r}

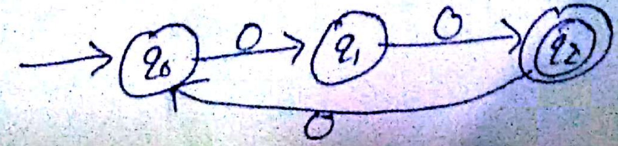
Transition diagram



Ques: Construct a finite automata for the language $\{0^n \mid n \bmod 3 = 0,\ n \geq 0\}$.

Solu: while testing the divisibility by three ~~then~~ we group the input as.

q_0 remainder 0 state.
q_1 " 1 state
q_2 " 2 state



(P-93)

Ques:- Convert to a NFA the following DFA (UPTU - 2004-05) 2013-2014.

	0	1
{p}	{q,s}	{q}
q*	{r}	{q,r}
r	{s}	{p}
s*	ϕ	{p}

Solu:-

Step1:- {p} is taken as the first subset.

0. Successor of {p} = $\delta(\{p\},0) = \{q,s\}$

1. Successor of {p} = $\delta(\{p\},1) = \{q\}$.

	0	1
→{p}	{q,s}	{p}

Step2:- Two new subsets {q,s} and {q} have been generated. Successors of {q,s} and {q} are calculated.

$\delta(\{q,s\},0) = \delta(q,0) \cup \delta(s,0) = \{r\} \cup \phi = \{r\}$

$\delta(\{q,s\},1) = \delta(q,1) \cup \delta(s,1) = \{q,r\} \cup \{p\} = \{p,q,r\}$.

$\delta(\{q\},0) = \{r\}$

$\delta(\{q\},1) = \{q,r\}$

	0	1.
→p	{q,s}	{p}
{q,s}	{r}	{p,q,r}
{q}	{r}	{q,r}

Step3:- Three new ~~state~~ subsets {r}, {p,q,r} and {q,r} are generated. Their successors are

$\delta(\{r\},0) = \{s\}$

$\delta(\{r\},1) = \{p\}$

$\delta(\{p,q,r\},0) = \delta(p,0) \cup \delta(q,0) \cup \delta(r,0)$
$= \{q,s\} \cup \{r\} \cup \{s\}$
$= \{q,r,s\}$

$\delta(\{p,q,r\},1) = \delta(p,1) \cup \delta(q,1) \cup \delta(r,1)$
$= \{q\} \cup \{q,r\} \cup \{p\}$
$= \{p,q,r\}$

$\delta(\{q,r\},0) = \delta(q,0) \cup \delta(r,0)$
$= \{r\} \cup \{s\}$
$= \{r,s\}$.

$\delta(\{q,r\},1) = \delta(q,1) \cup \delta(r,1)$
$= \{q,r\} \cup \{p\}$
$= \{p,q,r\}$

P.S \ I/S	N.S 0	N.S 1
$\{p\}$	$\{q,s\}$	$\{q\}$
$\{q,s\}$	$\{r\}$	$\{p,q,r\}$
$\{q\}$	$\{r\}$	$\{q,r\}$
$\{r\}$	$\{s\}$	$\{p\}$
$\{q,r\}$	$\{r,s\}$	$\{p,q,r\}$
$\{p,q,r\}$	$\{q,r,s\}$	$\{p,q,r\}$

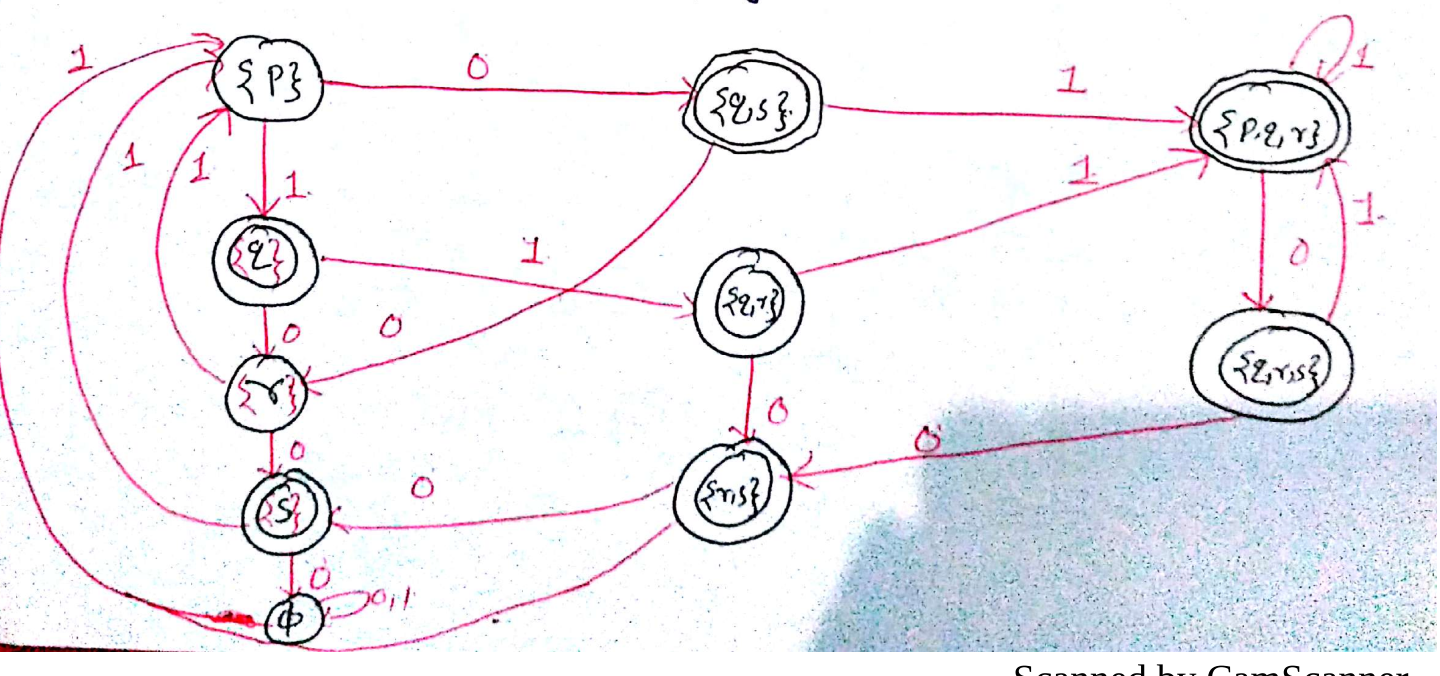
Step4:- Three news subsets $\{s\}$, $\{r,s\}$ and $\{q,r,s\}$ are generated. Their successors are calculated.

$\delta(\{s\},0) = \phi$
$\delta(\{s\},1) = p$
$\delta(\{r,s\},0) = \{s\}$
$\delta(\{r,s\},1) = \{p\}$
$\delta(\{q,r,s\},0) = \{r,s\}$
$\delta(\{q,r,s\},1) = \{p,q,r\}$

P.S \ I/S	N.S 0	N.S 1
$\rightarrow \{p\}$	$\{q,s\}$	$\{q\}$
$\{q,s\}$	$\{r\}$	$\{p,q,r\}$
$\{q\}$	$\{r\}$	$\{q,r\}$
$\{r\}$	$\{s\}$	$\{p\}$
$\{q,r\}$	$\{r,s\}$	$\{p,q,r\}$
$\{p,q,r\}$	$\{q,r,s\}$	$\{p,q,r\}$
$\{s\}$	ϕ	$\{p\}$
$\{r,s\}$	$\{s\}$	$\{p\}$
$\{q,r,s\}$	$\{r,s\}$	$\{p,q,r\}$

Step5:-

Every subset containing q or to be declared as a final state. Final DFA is shown in fig.



Ques:- Convert the following NFA to a DFA and informally describe the language it accepts.

P.S \ Σ	N.S 0	1
→p	{p,q}	{p}
q	{r,s}	{t}
r	{p,r}	{t}
s*	φ	φ
t*	φ	φ

Soln:-

Step 1:- {p} is taken as the first subset.

0- Successor of {p} = δ({p},0) = {p,q}

1- Successor of {p} = δ({p},1) = {p}

Step 2:- The new subset {p,q} is generated. Successors of {p,q} are calculated.

$$\delta(\{p,q\},0) = \delta(p,0) \cup \delta(q,0) = \{p,q\} \cup \{r,s\} = \{p,q,r,s\}$$

$$\delta(\{p,q\},1) = \delta(p,1) \cup \delta(q,1) = \{p\} \cup \{t\} = \{p,t\}$$

Step 3:- Two new subset {p,q,r,s} and {p,t} are generated. Their successors are calculated.

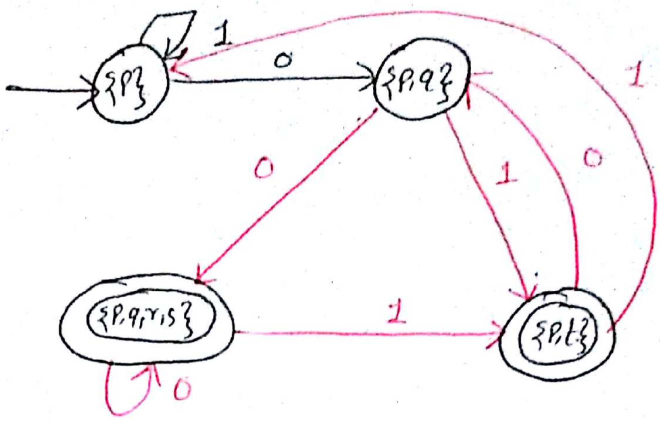
$$\delta(\{p,q,r,s\},0) = \delta(p,0) \cup \delta(q,0) \cup \delta(r,0) \cup \delta(s,0) = \{p,q\} \cup \{r,s\} \cup \{p,r\} \cup \phi = \{p,q,r,s\}$$

$$\delta(\{p,q,r,s\},1) = \delta(p,1) \cup \delta(q,1) \cup \delta(r,1) \cup \delta(s,1) = \{p\} \cup \{t\} \cup \{t\} \cup \phi = \{p,t\}$$

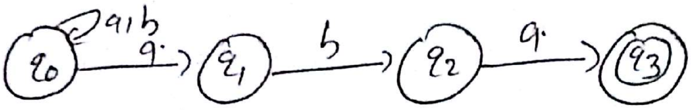
$$\delta(\{p,t\},0) = \delta(p,0) \cup \delta(t,0) = \{p,q\} \cup \phi = \{p,q\}$$

$$\delta(\{p,t\},1) = \delta(p,1) \cup \delta(t,1) = \{p\} \cup \phi = \{p\}.$$

P.S \ N.S	0	1
→{p}	{p,q}	{p}
{p,q}	{p,q,r,s}	{p,t}
{p,q,r,s}*	{p,q,r,s}	{p,t}
{p,t}*	{p,q}	{p}



Ques:-



Construct a NFA that accepts a set of all string over {a,b} ending in aba. Use this NFA to construct DFA accepting the same set of strings.

Solu:- NFA are shown in given up of question.

NFA to DFA Conversion:-

Transition Table for NFA

P.S \ Σ	N.S a	b
q_0	$\{q_0, q_1\}$	q_0
q_1	$\{\phi\}$	q_2
q_2	q_3	ϕ
q_3	ϕ	ϕ

Step1:- $\{q_0\}$ is taken as first subset

a- successor of $\{q_0\} = \delta(q_0, a) = \{q_0, q_1\}$

b- successor of $\{q_0\} = \delta(q_0, b) = \{q_0\}$

Step2:- A new subset $\{q_0, q_1\}$ is generated successors of $\{q_0, q_1\}$ are calculated.

$\delta(\{q_0, q_1\}, a) = \delta(q_0, a) \cup \delta(q_1, a)$

$= \{q_0, q_1\} \cup \phi = \{q_0, q_1\}$

$\delta(\{q_0, q_1\}, b) = \delta(q_0, b) \cup \delta(q_1, b)$

$= \{q_0\} \cup \{q_2\} = \{q_0, q_2\}$

Step3:- A new subset $\{q_0, q_2\}$ is generated. Successor of $\{q_0, q_2\}$ are calculated.

$\delta(\{q_0, q_2\}, a) = \delta(q_0, a) \cup \delta(q_2, a)$

$= \{q_0, q_1\} \cup \{q_3\}$

$= \{q_0, q_1, q_3\}$

$\delta(\{q_0, q_2\}, b) = \delta(q_0, b) \cup \delta(q_2, b) = \{q_0\} \cup \phi = \{q_0\}$

Step4:- A new subset $\{q_0, q_1, q_3\}$ is generated. Successor of $\{q_0, q_1, q_3\}$ are calculated.

$\delta(\{q_0, q_1, q_3\}, a) = \delta(q_0, a) \cup \delta(q_1, a) \cup \delta(q_3, a)$

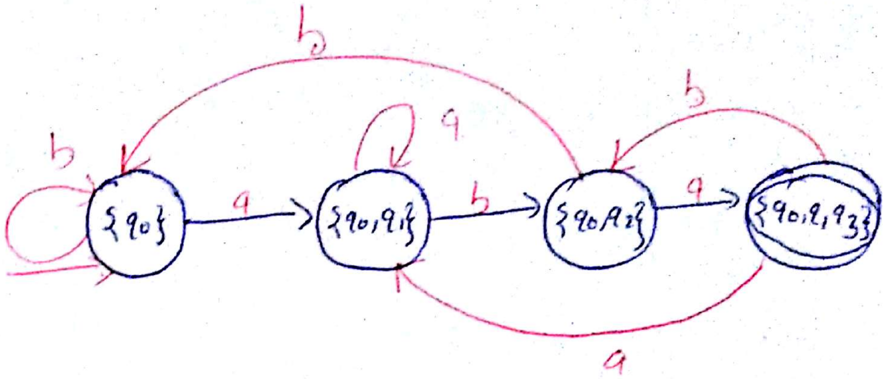
$= \{q_0, q_1\} \cup \phi \cup \phi = \{q_0, q_1\}$

$\delta(\{q_0, q_1, q_3\}, b) = \delta(q_0, b) \cup \delta(q_1, b) \cup \delta(q_3, b)$

$= \{q_0\} \cup \{q_2\} \cup \phi$

$= \{q_0, q_2\}$

P.S \ N.S	0	1
→ {q_0}	{q_0,q_1}	{q_0}
{q_0,q_1}	{q_0,q_1}	{q_0,q_2}
{q_0,q_2}	{q_0,q_1,q_3}	{q_0}
{q_0,q_1,q_3}	{q_0,q_1}	{q_0,q_2}



Ques. Let $M = (\{q_1,q_2,q_3\}, \{0,1\}, \delta, \{q_1\}, \{q_3\})$ is NFA where δ is given as:

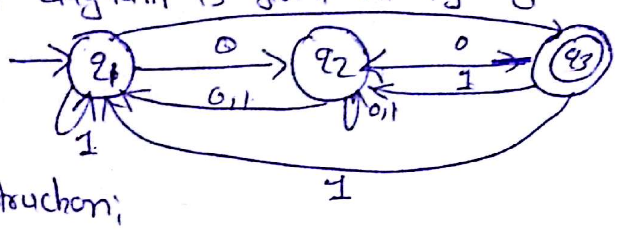
$\delta(q_1,0) = \{q_2,q_3\}$ $\qquad$ $\delta(q_2,1) = \{q_1,q_2\}$

$\delta(q_1,1) = \{q_1\}$ $\qquad$ $\delta(q_3,0) = \{q_2\}$

$\delta(q_2,0) = \{q_1,q_2\}$ $\qquad$ $\delta(q_3,1) = \{q_1,q_2\}$

Construct the transition diagram corresponding to NFA and find and draw its equivalent DFA, show all intermediate step also.

Soln:- Transition diagram is given in fig.



Subset Construction;

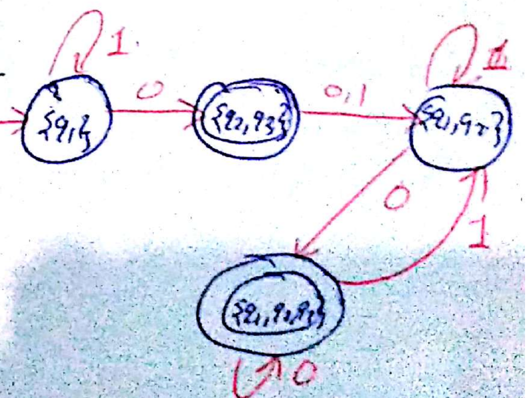
Step 1:

Transition Table for NFA

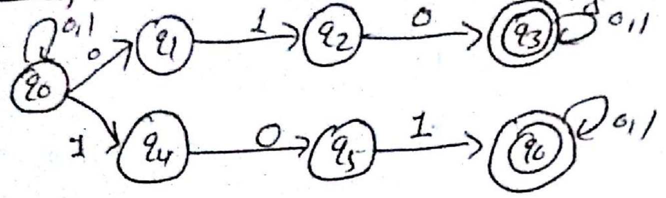
P.S \ Σ	0	1
q_1	{q_2,q_3}	{q_1}
q_2	{q_1,q_2}	{q_1,q_2}
q_3	{q_2}	{q_1,q_2}

Transition Table for DFA

P.S \ Σ	0	1
{q_1}	{q_2,q_3}	{q_1}
{q_2,q_3}	{q_1,q_2}	{q_1,q_2}
{q_1,q_2}	{q_1,q_2,q_3}	{q_1,q_2}
{q_1,q_2,q_3}	{q_1,q_2,q_3}	{q_1,q_2}

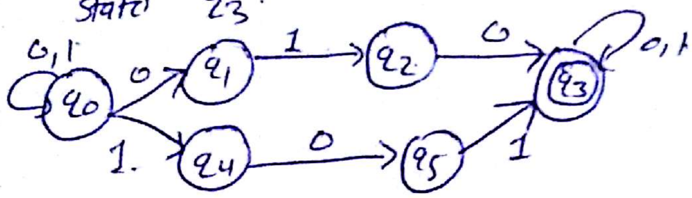


Ques:- Convert the NFA shown in fig to its corresponding DFA



Soln:-

Step 1:- q_3 and q_6 are identical NFA and they can be merged into a single state q_3.

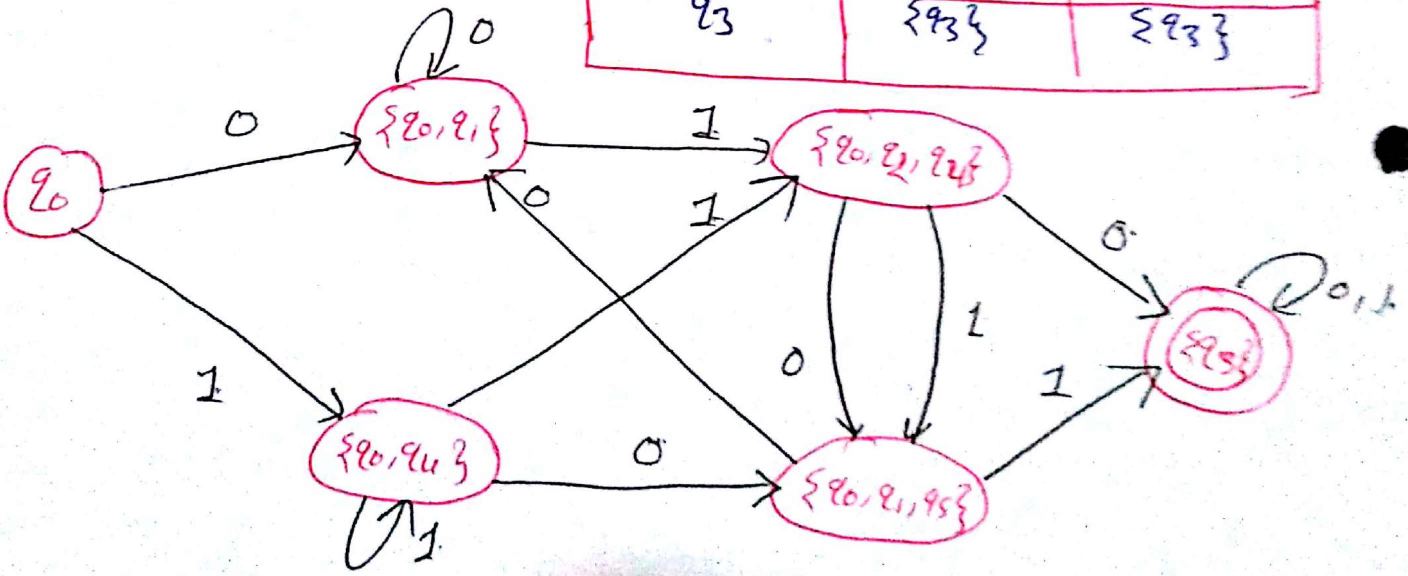


Transition Table for NFA:-

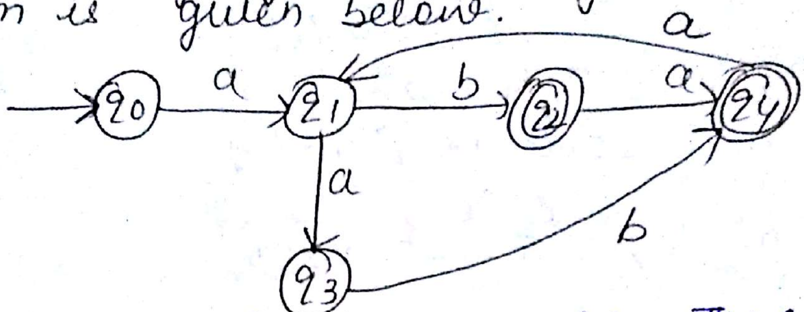
Σ / P.S	N.S 0	N.S 1
q_0	$\{q_0, q_1\}$	$\{q_4\}$
q_1	$\{-\}$	$\{q_2\}$
q_2	$\{q_3\}$	$\{-\}$
q_3	$\{q_3\}$	$\{q_3\}$
q_4	$\{q_5\}$	$\{-\}$
q_5	$\{-\}$	$\{q_3\}$

Transition Table for Corresponding DFA

Σ / P.S	Next State 0	Next State 1
q_0	$\{q_0, q_1\}$	$\{q_4\}$
$\{q_0, q_1\}$	$\{q_0, q_1\}$	$\{q_2, q_4\}$
$\{q_4\}$	$\{q_5\}$	$\{-\}$
$\{q_2, q_4\}$	$\{q_3, q_5\}$	–
$\{q_5\}$	$\{-\}$	$\{q_3\}$
$\{q_3, q_5\}$	$\{q_3\}$	$\{q_3\}$
q_3	$\{q_3\}$	$\{q_3\}$



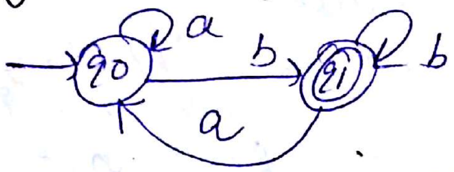
Q8. Find the language accepted by NFA whose state diagram is given below.



Sol: There are two final state q_2 & q_4. The substring ending at q_2 is 'ab and substrings ending at q_4 are 'aba' and 'aab'.

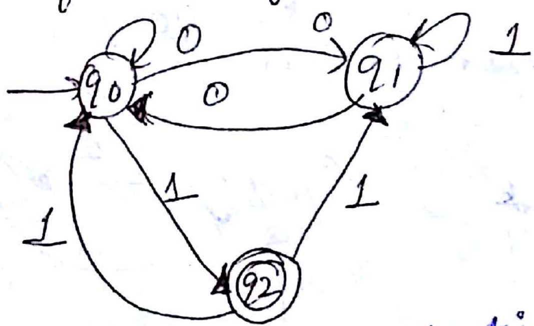
Thus language accepted by this NFA is: $(ab+aab+aba)^*$

Q9. Construct the NFA for language: $L = \{x \in \{a,b\}^* : x$ contains any number of a's followed by at least one b$\}$.



Conversion from NFA to DFA

Q1. Convert the following NFA to its equivalent DFA;

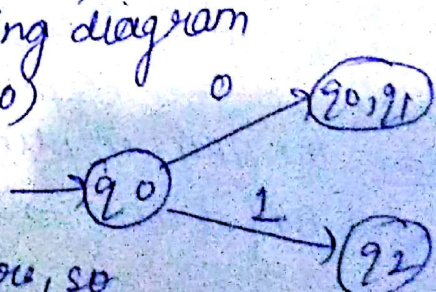


Solution:

Initially, we have q_0 as starting state, so create start state as $[q_0]$

$$\delta([q_0],0) = \{q_0, q_1\}$$
$$\delta([q_0],1) = \{q_2\}$$

Since these do not already exist, so create two new states as shown in adjoining diagram



$$\delta([q_0, q_1],0) = \delta(q_0,0) \cup \delta(q_1,0)$$
$$= \{q_0, q_1\} \cup \{q_0\}$$
$$= \{q_0, q_1\}$$

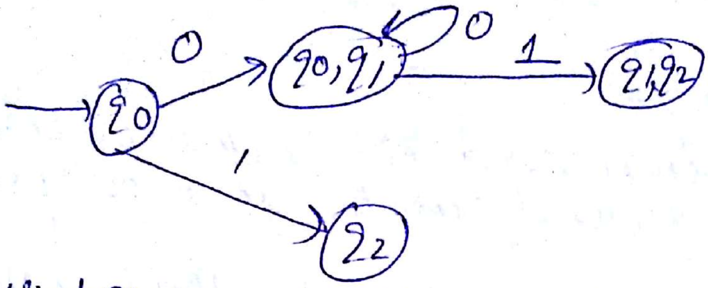
The state $[q_0, q_1]$ is already there, so we draw self loop.

(P-100)

4

$\delta([q_0, q_1], 1) = \delta(q_0, 1) \cup \delta(q_1, 1)$
$= \{q_2\} \cup \{q_1\}$
$= \{q_1, q_2\}$

Create new state as $[q_1, q_2]$

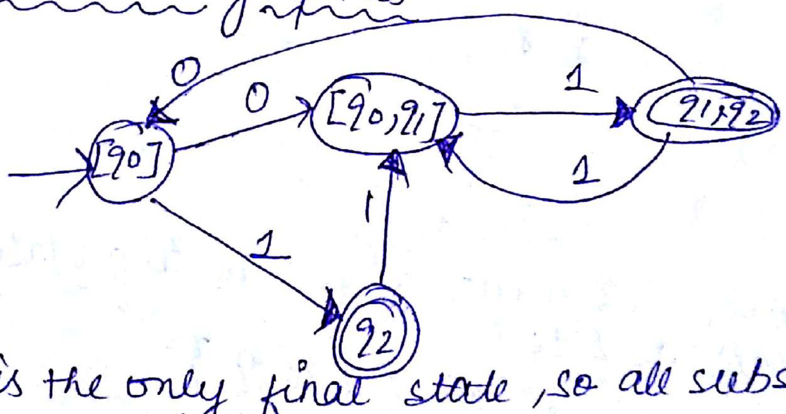


Now, for vertex $[q_2]$, we have,
$\delta([q_2], 0) = \phi$
$\delta([q_2], 1) = \{q_0, q_1\}$ i.e. state $[q_0, q_1]$

for vertex $[q_1, q_2]$, we have
$\delta([q_1, q_2], 0) = \delta(q_1, 0) \cup \delta(q_2, 0)$
$= \{q_0\} \cup \phi = \{q_0\}$
$= [q_0]$ state

and $\delta([q_1, q_2], 1) = \delta(q_1, 1) \cup \delta(q_2, 1)$
$= \{q_1\} \cup \{q_0, q_1\} = \{q_0, q_1\}$
$= [q_0, q_1]$ state

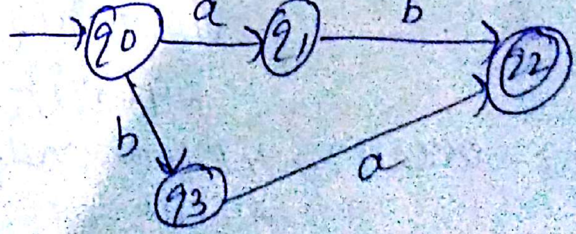
final transition graph is



In NFA, q_2 is the only final state, so all subsets containing q_2 as final state i.e. state $[q_2]$ and $[q_1, q_2]$ are final states in DFA.

Q2. Determine a NFA accepting $\{ab, ba\}$ and use it to find a DFA accepting it.

solution: The NFA accepting $\{ab, ba\}$ is as :→

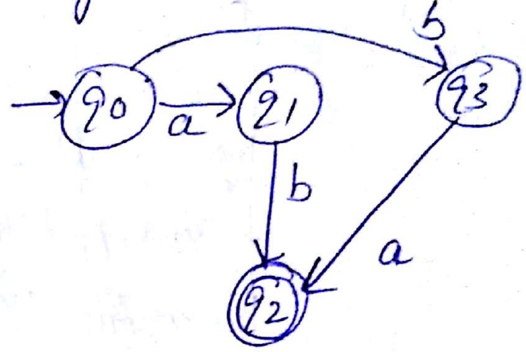


state table of NFA is

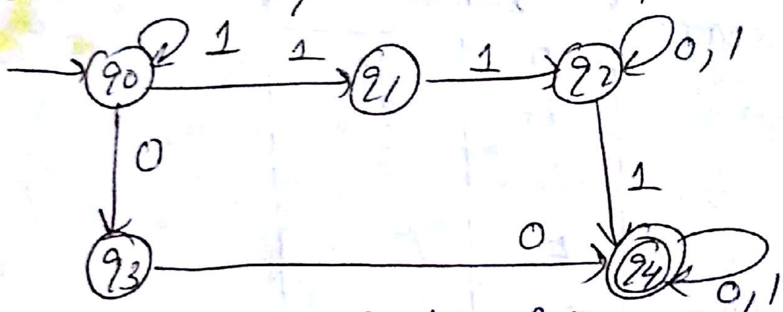
state/Σ	a	b
→q_0	q_1	q_3
q_1	∅	q_2
q_2	∅	∅
q_3	q_2	∅

now, the state table corresponding to DFA:

state/Σ	a	b
q_0	q_1	q_3
q_1	∅	q_2
q_2	∅	∅
q_3	q_2	∅



Q3. construct a DFA equivalent to following NFA.



solution: The given NFA is $M = \{Q, \Sigma, \delta, q_0, F\}$

state	input 0	input 1
→q_0	q_3	q_1, q_0
q_1		q_2
q_2	q_2	q_2, q_4
q_3	q_4	
q_4	q_4	q_4

The transition table for equivalent DFA will be

	state	input 0	input 1
A →	→ [q0]	[q3]	[q1, q0]
B →	[q3]	[q4]	
C →	[q1, q0]	[q3]	[q0, q1, q2]
D →	[q4]	[q4]	[q4]
E →	[q0, q1, q2]	[q2, q3]	[q0, q1, q2, q4]
F →	[q2, q3]	[q2, q4]	[q2, q4]
G →	[q0, q1, q2, q4]	[q2, q3, q4]	[q0, q1, q2, q4]
H →	[q2, q4]	[q2, q4]	[q2, q4]
I →	[q2, q3, q4]	[q2, q4]	[q2, q4]

state	input 0	input 1
→ A	B	C
B	D	—
C	B	E
D	D	D
E	F	G
F	H	H
G	I	G
H	H	H
I	H	H