Comparison of control laws for magnetic detumbling

Mirko Leomanni*

I   Aim of the study

Several control techniques have been developed to detumble a spacecraft immediately after release, considering either fully actuated systems or underactuated systems. While three-axis control is typically available using thrusters, magnetic actuators can only provide two-axis control torques. In spite of this limitation, there is a growing interest in the application of magnetic torquers for detumbling, especially of small LEO satellites, due to the significant savings in terms of overall spacecraft mass and complexity. In this report, we analyze the performance of the traditional b-dot control law, as described in [1] and [2], in comparison with a recently proposed alternative formulation [3], based on angular velocity feedback, for magnetic detumbling.

II   Detumbling control laws

Detumbling consists of damping the angular velocity of a spacecraft to zero. The dynamic equation, which describes the evolution of the angular velocity of a body reference frame centered at the spacecraft center of mass and aligned with the principal axes of inertia, is given by:

$$\dot{\boldsymbol{\omega}} = \mathbf{I}^{-1}(\boldsymbol{\tau} - \boldsymbol{\omega} \times \mathbf{I}\,\boldsymbol{\omega}) \tag{1}$$

where $\boldsymbol{\omega}$ is the angular velocity of the reference frame with respect to an inertial frame, $\boldsymbol{\tau}$ is the sum of external torques, including control torques, and $\mathbf{I}$ is the spacecraft inertia matrix. All quantities in (1) are expressed in the body frame, and the spacecraft is assumed to be a rigid body. Since no external disturbance is considered in this study, $\boldsymbol{\tau}$ represents the control torque delivered by magnetic actuators:

$$\boldsymbol{\tau} = \mathbf{m} \times \mathbf{b} \tag{2}$$

where $\mathbf{m}$ is the commanded magnetic dipole moment and $\mathbf{b}$ is the local geomagnetic field vector. A suitable approximation of the geomagnetic field for LEO circular orbits can be found in [4], while more accurate models, such as the IGRF, can be considered

---

*PhD, University of Siena, leomanni@dii.unisi.it.

for general purpose scenarios. From eq. (2), it is clear that the lever arm is maximized for $\mathbf{m} \perp \mathbf{b}$ and that no torque is produced when $\mathbf{m} \parallel \mathbf{b}$. Another useful consideration for the development of a stabilizing control law is that the control torque has to be proportional to $-\boldsymbol{\omega}$, in order to decrease the kinetic energy of the spacecraft. However, it is not trivial to prove that the kinetic energy steadily decreases for underactuated systems.

The b-dot contol law takes advantage of the fact that the derivative of the magnetic field vector $\dot{\mathbf{b}}$ is both perpendicular to $\mathbf{b}$ and proportional to $\boldsymbol{\omega}$, hence the commanded magnetic dipole can be expressed as:

$$\mathbf{m} = -k\,\dot{\mathbf{b}} \tag{3}$$

where k is a positive gain. In order to speed up the spacecraft spin rate decay, it is also possible to employ a bang-bang solution of the form:

$$\mathbf{m} = -m_{\max}\,\mathrm{sgn}(\dot{\mathbf{b}}) \tag{4}$$

where $m_{\max}$ is the maximum torquer dipole. Since $\dot{\mathbf{b}}$ can be estimated from onboard measurements of the magnetic field, no information about the angular velocity of the satellite is required by eqs. (3) and (4). On the other hand, attitude information is typically available from onboard sensors, and the following alternative control law can be used:

$$\mathbf{m} = -\frac{k_\omega}{\|\mathbf{b}\|^2}\,(\mathbf{b} \times \boldsymbol{\omega}) \tag{5}$$

where k_ω is a positive gain and $(\mathbf{b} \times \boldsymbol{\omega}) \approx \dot{\mathbf{b}} \propto \boldsymbol{\omega}$. Avanzini and Giulietti pointed out that, with the application of this control law in the presence of a time-varying magnetic field, the kinetic energy is strictly decreasing, which means that it approaches zero monotonically, so that global asymptotic stability is guaranteed. This is a stronger property than what Stickler and Alfriend demonstrated for their b-dot law, proving that the time derivative of the kinetic energy is $\dot{\mathcal{T}} \leq 0$, but showing only empirically that the residual motion about the direction of the magnetic field is almost canceled by the magnetic field rotation over time. The following section presents a comparative analysis on the performance of these control laws.

III   Simulation results

An example detumbling maneuver, performed by a small LEO satellite equipped with three orthogonal magnetic coils, has been simulated using the following most relevant parameters:

- Near circular orbit with a radius of 7021 km.

- Inclination of 65 deg to ensure faster convergence.

- Inertia matrix $\mathbf{I} = \mathrm{diag}([0.33\,,\ 0.37\,,\ 0.35])$ Kg·m^2.

- Maximum magnetic dipole moment $m_{\max} = 2$ A·m^2 per axis.

- Initial angular velocity $\boldsymbol{\omega}_0 = [0.604\,,\ 0.760\,,\ 0.384]^T$.

- Control sampling rate of 10 Hz.

A reasonable value of the control gain k_ω has been estimated, according to [3], as:

$$k_\omega = 2\, n\, (1 + sin\,\xi)\, \mathbf{I}_{\min} \tag{6}$$

where n is the mean motion, ξ is the inclination of the orbit plane with respect to the geomagnetic equator and $\mathbf{I}_{\min}$ is the minimum moment of inertia of the spacecraft. To provide a fair comparison, the following static and time-varying gains have been chosen for the proportional b-dot control law:

$$\bar{k} = \frac{k_\omega}{\|\bar{\mathbf{b}}\|^2} \ ; \qquad k = \frac{k_\omega}{\|\mathbf{b}\|^2} \tag{7}$$

where $\|\bar{\mathbf{b}}\|^2$ is the averaged squared norm of the magnetic field over one revolution. Moreover, perfect knowledge of $\dot{\mathbf{b}}$ is assumed through an analytic solution.

A preliminary investigation has been carried out, using the angular velocity feedback law defined by eq. (5), to verify proper operation of the control system.
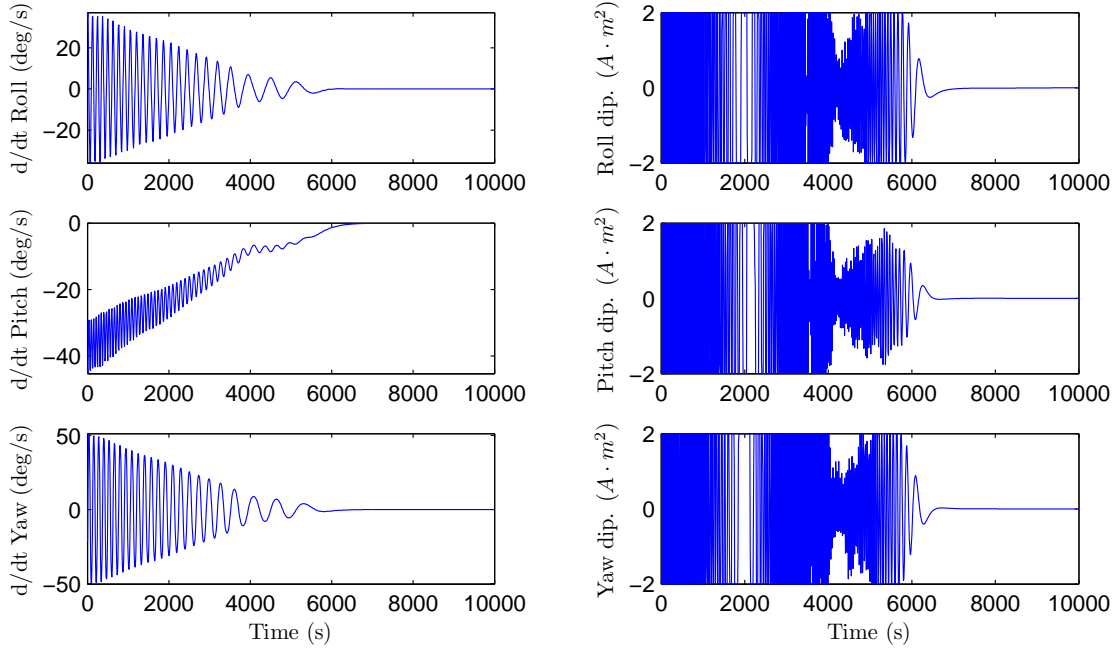

Figure 1: Detumbling operation

3

The results are depicted in fig. 1, where the angular velocity components approach a zero steady-state error, according to the stability properties mentioned in sec. II, while the commanded magnetic dipole components do not exceed the maximum deliverable value, as expected.

Then, the proportional b-dot control law defined by eq. (3) has been evaluated under the same initial conditions, using the static and time-varying gains of eq. (7), together with the bang-bang solution given by eq. (4). The performance of the control laws is compared, in fig. 2, in terms of convergence of the spacecraft absolute angular velocity to zero.
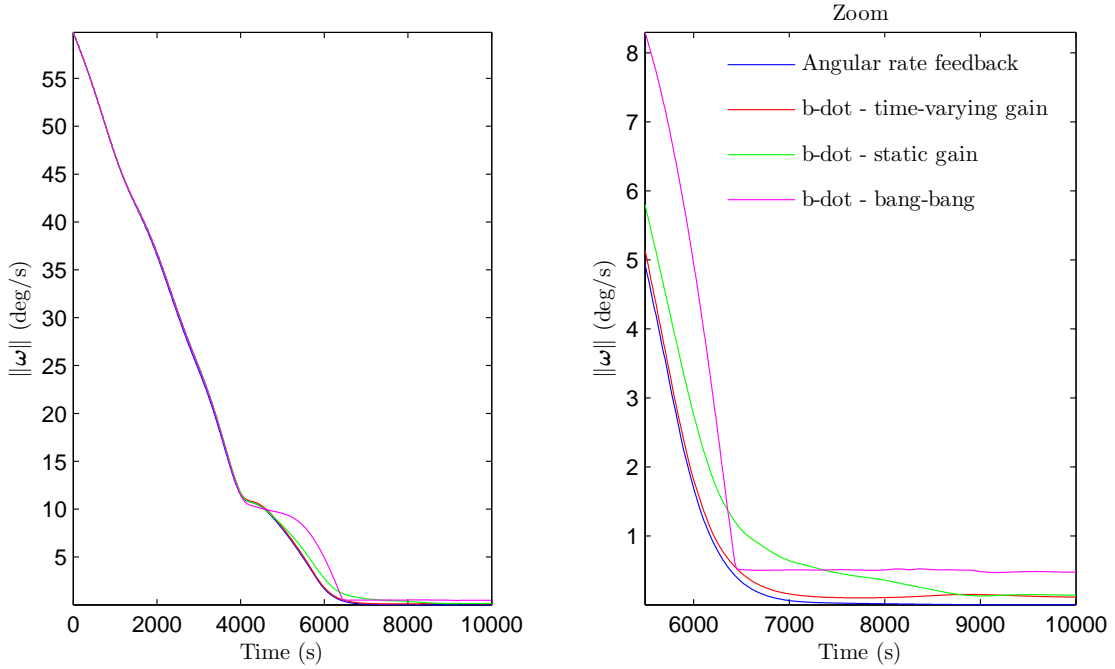

Figure 2: Comparison of detumbling control laws

The initial deceleration transient is identical, as magnetic coils saturate. Differences are visible in the final part (enlarged), where the angular velocity feedback law allows for asymptotic convergence to a zero steady-state absolute angular rate, while a limit cycle featuring two oscillations per orbit around a mean value of twice the orbit rate is reached with the proportional b-dot law. Also, note that the bang-bang b-dot law provides the minimum convergence time, at the price of a higher final error[1] and a decreased power efficiency, while a time-varying gain allows for a faster convergence of the proportional b-dot law to the steady-state limit cycle.

As a final remark observe that, when a practical implementation of the b-dot command law is concerned, it is not possible to obtain an analytic solution for the time derivative

---

[1] The expected behavior of the system in this scenario is a fast transient followed by a very slow convergence.

of the magnetic field, as in this study, without knowing the spacecraft angular velocity. In such cases, a state variable filter is typically used, to provide an estimate of the form:

$$\dot{\hat{\mathbf{b}}} = (\hat{\mathbf{b}} - \mathbf{b})\,\omega_c \tag{8}$$

where the cut frequency ω_c should be chosen approximately 5 to 10 times as high as the fastest change of $\mathbf{b}$. The error resulting from this approximation in a noise-free environment is indicated, in fig. 3, by the difference between the ideal tracking error and the one obtained using an estimate of the magnetic field derivative.
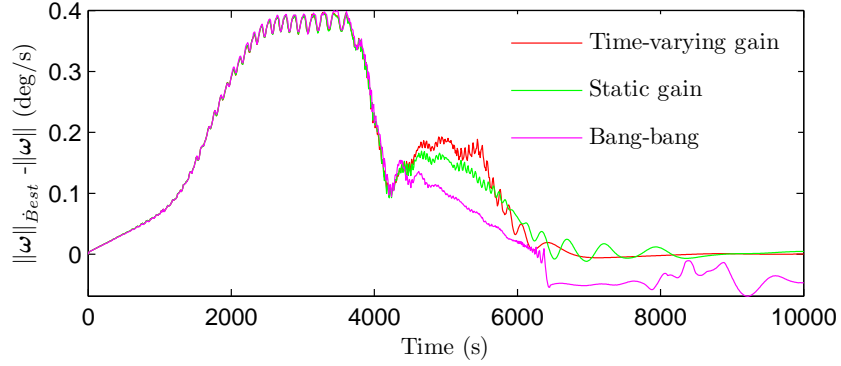

Figure 3: Performance gap between estimated and ideal b-dot

It can be seen that the performance gap is limited, since the magnitude of the relative angular rate error is 25 times lower than the absolute angular rate error at maximum.

References

[1] Stickler, A. and Alfriend, K., "Elementary Magnetic Attitude Control System," *Journal of Spacecraft and Rockets*, Vol. 13, 1976, pp. 282.

[2] Flatley, T., Morgenstern, W., Reth, A., and Bauer, F., "A B-Dot Acquisition Controller for the RADARSAT Spacecraft," *NASA conference publication*, NASA, 1997, pp. 79–90.

[3] Avanzini, G. and Giulietti, F., "Magnetic Detumbling of a Rigid Spacecraft," *Journal of guidance, control, and dynamics*, Vol. 35, No. 4, 2012, pp. 1326–1334.

[4] Hablani, H., "Comparative stability analysis and performance of magnetic controllers for bias momentum satellites," *Journal of Guidance, Control, and Dynamics*, Vol. 18, No. 6, 1995, pp. 1313–1320.

A Matab code

The following code has been developed, using Matlab, for the simulation.

Simulation interface

```
%SIMULATION PARAMETERS
sc.inertia=diag([0.33,0.37,0.35]); %Spacecraft inertia (kg/m^2)
mmax=2.0;                          %Maximum magnetic moment (A*m^2)
rc=7021;                           %Orbit radius (km)
Mt=7.8379e6;                       %Mangetic moment Earth T*km^3
Torb=5855;                         %Orbital period (s)
Gta=11.44*pi/180;                  %Geomagnetic tilt angle (rad)
In=65*pi/180;                      %Inclination (rad)
Ts=10000;                          %Simulation time (sec)
kmg=Mt/rc^3;                       %Dipole Magnitude (T)
wo=2*pi/Torb;                      %Orbital speed    (rad/s)
we=7.2921150e-5;                   %Earth rotation speed (rad/s)
tcamp=10;                          %Plot sampling time
tint=0.1;                          %Fixed integration step
%GAIN CALCULATION
MeanB2=1.0131e-009;
Beta1=0;
cosepsm=cos(In)*cos(Gta)+sin(In)*sin(Gta)*cos(Beta1);
nim=atan2(-sin(Gta)*sin(Beta1),...
sin(In)*cos(Gta)-cos(In)*sin(Gta)*cos(Beta1));
if sin(nim)==0;
sinepsm=sin(In)*cos(Gta)-cos(In)*sin(Gta)*cos(Beta1)/cos(nim);
else
sinepsm=-sin(Gta)*sin(Beta1)/sin(nim);
end
kw=2*wo*(1+sinepsm)*min(diag(sc.inertia));
%INITIAL CONDITION
q0=[-0.062;0.925;-0.007;0.375];
w0=[0.604;-0.760;-0.384];
Yt=[q0' w0' zeros(1,3);zeros(round(Ts/tcamp),10)];
Yt1=Yt;
Yt2=Yt;
tt=zeros(round(Ts/tcamp)+1,1);
for Js=1:round(Ts/tcamp)
%SIMULATION WITH FIXED INTEGRATON STEP (10 Hz)
%agular velocity
Ys=ode4(@(t,Y)AttDyn(t,Y,In,wo,we,kmg,Gta,sc,mmax,kw,tint,0,0),...
```

```
tt(Js):tint:(tt(Js)+tcamp),Yt(Js,:)');
Yt(Js+1,:)=Ys(end,:);
%Modified B.dot
Ys1=ode4(@(t,Y)AttDyn(t,Y,In,wo,we,kmg,Gta,sc,mmax,kw,tint,1,0),...
tt(Js):tint:(tt(Js)+tcamp),Yt1(Js,:)');
Yt1(Js+1,:)=Ys1(end,:);
%Standard B.dot
Ys2=ode4(@(t,Y)AttDyn(t,Y,In,wo,we,kmg,Gta,sc,mmax,kw,tint,1,...
MeanB2),tt(Js):tint:(tt(Js)+tcamp),Yt2(Js,:)');
Yt2(Js+1,:)=Ys2(end,:);
%Bang-Bang B.dot
Ys3=ode4(@(t,Y)AttDyn(t,Y,In,wo,we,kmg,Gta,sc,mmax,kw,tint,2,...
0),tt(Js):tint:(tt(Js)+tcamp),Yt3(Js,:)');
Yt3(Js+1,:)=Ys3(end,:);
tt(Js+1)=tt(Js)+tcamp;
end
%GET INPUT FOR PLOTTING (Only for angular speed feedback)
usim=zeros(length(tt),3);
b=zeros(length(tt),3);
for i=1:length(tt)
b(i,:)=(quat2dcm([Yt(i,4) Yt(i,1:3)])*magField(wo*tt(i),In,Gta,...
we*tt(i),kmg))';
usim(i,:)=getMC(kw,b(i,:)',Yt(i,5:7),mmax)';
end
```

Attitude dynamic model

```
function dydt=AttDyn(t,Y,In,wo,we,kmg,Gta,sc,mmax,kw,tint,bdoton,stgain)
TBO=quat2dcm([Y(4) Y(1:3)']); %orbit->body direction cosine matrix
Beta1=we*t;
b=TBO*magField(wo*t,In,Gta,Beta1,kmg);
%Orbital->body frame speed
wr=Y(5:7)-TBO*[0;-wo;0];
%d/dt quaternion orbital->body frame
dydt(1:3,1)=1/2*(Y(4)*wr-cross(wr,Y(1:3)));
dydt(4,1)=-1/2*wr'*Y(1:3);
%bdot estimation
bdot=(b-Y(8:10,1))/tint;
dydt(8:10,1)=bdot;
%analytic solution (must know w!)
bdot1=TBO*kmg*BdotAn(In,Gta,we,wo,t)-cross(wr,b);
%choose control strategy
if bdoton==0;
Md=getMC(kw,b,Y(5:7),mmax);
```

```
elseif bdoton==1
if stgain==0
Md=getMC1(kw/norm(b)^2,bdot1,mmax);
%Md=getMC1(kw/norm(b)^2,bdot,mmax);
else
Md=getMC1(kw/stgain,bdot1,mmax); %Md=getMC1(kw/stgain,bdot,mmax);
end
else %BANG-BANG b-dot
Md=-mmax*sign(bdot1); %Md=-mmax*sign(bdot);
end
%d/dt angular speed body frame (wrt inertial)
dydt(5:7,1)=sc.inertia\(cross(Md,b)-cross(Y(5:7),sc.inertia*Y(5:7)));
```

Angular velocity feedback law

```
function u=getMC(kw,b,w,mmax)
u=-kw/norm(b)*cross(b/norm(b),w);
for j=1:length(u)
if abs(u(j))>mmax
u(j)=sign(u(j))*mmax;
end
end
```

B-dot control law

```
function u=getMC1(kw,bdot,mmax)
u=-kw*bdot;
for j=1:length(u)
if abs(u(j))>mmax
u(j)=sign(u(j))*mmax;
end
end
```

Magnetic field model

```
function b=magField(wot,In,Gta,Beta1,kmg)
cosepsm=cos(In)*cos(Gta)+sin(In)*sin(Gta)*cos(Beta1);
nim=atan2(-sin(Gta)*sin(Beta1),sin(In)*cos(Gta)-...
cos(In)*sin(Gta)*cos(Beta1));
if sin(nim)==0;
 sinepsm=sin(In)*cos(Gta)-cos(In)*sin(Gta)*cos(Beta1)/cos(nim);
else
 sinepsm=-sin(Gta)*sin(Beta1)/sin(nim);
end
b=kmg*[sinepsm*cos(wot-nim);-cosepsm;2*sinepsm*sin(wot-nim)];
```