Government College of Engineering, Amravati
(An Autonomous Institute of Government of Maharashtra)

Third Semester B. Tech. (CS/IT)

Summer Term-2016

Course Code: SHU304

Course Name: Engineering Mathematics-III

Time: 2 hrs.30 min. **Max Marks: 60**

Instructions to candidate:
1) All questions are compulsory.
2) Assume suitable data wherever necessary and clearly state the assumptions made.
3) Diagrams/sketches should be given wherever necessary.
4) Use of logarithmic table, drawing instruments and nonprogrammable calculators is permitted.
5) Figures to the right indicate full marks.

1. **Attempt any three** **12**

(a) Solve: $(D^2-6D+9)y=\frac{e^x}{x}$

(b) Solve: $(D^2-4D+4)y=8x^2e^{2x}\sin x$

(c) Solve: $(2x+3)^2\frac{d^2y}{dx^2}+(2x+3)\frac{dy}{dx}-2y=24x^2$

(d) Solve: $\frac{d^2y}{dx^2}-y=\cos x\cosh x+3^x$

2. **Attempt any three** **12**

(a) Solve: $x(y^2-z^2)p+y(z^2-x^2)q+z(y^2-x^2)=0$

(b) Solve: $(1-y^2)xq^2+y^2p=0$

(c) Solve: $q(p^2z+q^2)=4$

(d) Find the solution of the equation $\frac{\partial^2u}{\partial x^2}=\frac{\partial u}{\partial y}+2u$ in the form $u=f(x)g(y)$ subject to the conditions

Cont.

$u = 0$ & $\frac{\partial u}{\partial x} = 1 + e^{-3y}$, when x=0 and for all y.

3. **Attempt any three** 12

(a) State convolution theorem and verify it for $f(t) = t$ and $g(t) = \sin at$.

(b) Find Laplace transform of $\frac{1-\cos t}{t^2}$ using division by t property.

(c) Evaluate $L^{-1}\left\{\frac{2s^2 - 6s + 5}{s^3 - 6s^2 + 11s - 6}\right\}$

(d) Solve $y''' - 3y'' + 3y' - y = t^2 e^t$ using transform method. Given that $y(0) = 1,\ y'(0) = 8, y''(0) = -2$

4. **Attempt any three:** 12

(a) Verify whether $\overline{F} = 2xyz^2\bar{i} + (x^2z^2 + z\cos(yz))\bar{j} + (2x^2yz + y\cos(yz))\bar{k}$ is irrotational. If so find corresponding scalar potential ϕ.

(b) If $\overline{F} = (y+z)\bar{i} + (z+x)\bar{j} + (x+y)\bar{k}$. Show that $curl\ curl\ curl\ curl\ \overline{F} = \nabla^4 \overline{F}$.

(c) Find the D.D. of $\phi = 4xz^3 - 3x^2y^2z$ at point $(2,-1,1)$ along the line equally inclined with coordinate axes.

(d) Find the work done by $\overline{F} = x^2 i + yzj + zk$ in moving a particle along the straight line segment from (1,2,2) to (3,4,4).

5. **Attempt the following:**

(a) Solve $\frac{d^3y}{dx^3} + \frac{dy}{dx} = \tan x$ by method of variation of parameters. 06

(b) **Fill in the blanks:** **06**

i) Using Laplace transform the value of integral

$$\int_0^\infty t\,e^{-2t}\sin 3t\,dt = \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots$$

ii) If $\bar{a}$ is a constant vector and $\bar{r} = xi + yj + zk$ then curl of $\bar{a}\times\bar{r} = \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots$

iii) The solution of partial differential equation

$$\frac{z}{pq} = \frac{c\sqrt{1+p^2+q^2}}{pq} + \frac{x}{q} + \frac{y}{p} \text{ is} \ldots\ldots\ldots\ldots\ldots\ldots\ldots$$

Cont.

Government College of Engineering, Amravati

(An Autonomous Institute of Government of Maharashtra)

Third Semester B. Tech. (CS / IT)

Winter – 2014

Course Code: SHU304

Course Name: Engineering Mathematics – III

Time: 2 Hrs. 30 Min. **Max. Marks: 60**

Instructions to Candidate

1) All questions are compulsory.
2) Assume suitable data wherever necessary and clearly state the assumptions made.
3) Diagrams/sketches should be given wherever necessary.
4) Use of logarithmic table, drawing instruments and non-programmable calculators is permitted.
5) Figures to the right indicate full marks.

1. Attempt any three: **12**

(a) Prove that Particular integral of $(D-m)y = f(x)$ is $e^{mx}\int e^{-mx} f(x)\,dx$ and hence find the same of the differential equation $(D^2+2D+1)y = e^{-x}$

(b) Solve : $(D^4 - 2D^3 - 3D^2 + 4D + 4)y = x^2 e^x$

(c) Solve: $x^2\dfrac{d^2y}{dx^2} - 2x\dfrac{dy}{dx} - 4y = x^2 + 2\log x$

(d) Solve by method of variation of parameters the equation $y'' - 2y' + 2y = e^x \tan x$.

2. Attempt any three: **12**

(a) i) Solve $p + q = \sin x + \sin y$

Cont.....

ii) Solve $z = px + qy + \sqrt{1 + p^2 + q^2}$

(b) Solve: $(x-y)(px-qy) = (p-q)^2$

(c) Using method of separation of variables, solve

$$3\frac{\partial u}{\partial x} + 2\frac{\partial u}{\partial y} = 0,\ u(x,0) = 4e^{-x}$$

(d) Solve $(x^2 - yz)p + (y^2 - zx)q = z^2 - xy$

3. **Attempt any three:** **12**

(a) Using first shifting theorem prove that

$$L\{t\cos at\} = \frac{s^2 - a^2}{(s^2 + a^2)^2}$$

(b) Evaluate

i) $\int_0^\infty te^{-2t}\sin t\,dt$ ii) $L\left\{\int_0^t \frac{e^t \sin t}{t}\,dt\right\}$

(c) Find the inverse Laplace transform of

i) $\dfrac{2s^2 - 6s + 5}{s^3 - 6s^2 + 11s - 6}$ ii) $\tan^{-1}(2/s^2)$

(d) Define Unit impulse function, Heaviside unit step function and find the relation between them

(e) State Convolution theorem and verify it for the pair of functions $f(t) = t,\ g(t) = e^{at}$.

4. **Attempt any three:** **12**

(a) If directional derivative of $\phi = ax^2y + by^2z + cz^2x$ at $(1,1,1)$ has maximum magnitude 15 in the direction parallel to $\frac{x-1}{2} = \frac{y-3}{-2} = \frac{z}{1}$. Find a,b,c.

(b) Show that $\bar{F} = (6xy + z^3)i + (3x^2 - z)j + (3xz^2 - y)k$ is irrotational. Find scalar ϕ such that $\bar{F} = \nabla\phi$.

(c) Show that $\overline{F} = \dfrac{\overline{a} \times \overline{r}}{r^n}$ is solenoidal field, where $r = |\overline{r}|$, $\overline{r} = xi + yj + zk$ and $\overline{a}$ be the constant vector.

(d) Find the work done in moving a particle once round the ellipse $\dfrac{x^2}{25} + \dfrac{y^2}{16} = 1,\ z = 0$ under the field of force given by
$\overline{F} = (2x - y + z)i + (x + y - z^2)j + (3x - 2y + 4z)k$
Is the field conservative?

5. **Attempt the following:**

(a) If $\overline{r} \times \dfrac{d\overline{r}}{dt} = 0$, show that $\overline{r}$ has a constant direction. **04**

(b) The small oscillations of a certain system with two degrees of freedom are given by the equations

$$D^2x + 3x - 2y = 0$$
$$D^2y + D^2x - 3x + 5y = 0$$

Where $D = \dfrac{d}{dt}$. If $x = 0, y = 0, Dx = 3, Dy = 2$ when $t = 0$. Find x and y when $t = \pi/2$ **06**

(c) Given $L\{J_0(t)\} = \dfrac{1}{\sqrt{s^2+1}}$, then

i) $\int_0^\infty J_0(t)dt = $ -----

ii) $\int_0^\infty e^{-t} J_0(t)dt = $ ----- **02**

Government College of Engineering, Amravati

(An Autonomous Institute of Government of Maharashtra)

Third Semester B. Tech. (CS / IT)

Winter – 2016

Course Code: SHU304

Course Name: Engineering Mathematics-III

Time: 2 Hrs. 30 Min. **Max. Marks: 60**

Instructions to Candidate

1) All questions are compulsory.
2) Assume suitable data wherever necessary and clearly state the assumptions made.
3) Diagrams/sketches should be given wherever necessary.
4) Use of logarithmic table, drawing instruments and non-programmable calculators is permitted.
5) Figures to the right indicate full marks.

1. **Attempt any three:** **12**

(a) State the formula for particular integral of the differential equation $\phi(D)y = e^{ax}\,V$ and hence use it to find the same for $(D^2+2D-3)y = e^x \cos x$

(b) Solve : $\frac{d^2y}{dx^2} - 4\frac{dy}{dx} + 4y = \sin^2 x$

(c) Solve: $x^2\frac{d^2y}{dx^2} - 2x\frac{dy}{dx} - 4y = x^2 + 2\log x$

(d) Solve by method of variation of parameters the equation $y'' - y = e^{-2x}\cos(e^{-x})$.

2. **Attempt any three:** **12**

(a) i) Solve $p^2 + q = x + \sin y$

Cont.

ii) Solve $\sqrt{1+p^2-q^2}=0$

(b) Solve: $pqz = p^2(xq+p^2)+q^2(yp+q^2)$

(c) Using method of separation of variables, solve
$3\frac{\partial u}{\partial x}+2\frac{\partial u}{\partial y}=0,\ u(x,0)=4e^{-x}$

(d) Solve $(x^2-yz)p+(y^2-zx)q=z^2-xy$

3. **Attempt any three:** **12**

(a) State first shifting theorem. Find the Laplace transform of $\sinh 3t\cos^2 t$

(b) Find the Laplace transform of $\frac{1-\cos t}{t}$ and hence find $L\left(\frac{1-\cos t}{t^2}\right)$.

(c) Find the Laplace transform of the triangular wave given by
$f(t)=\begin{cases} t, & 0<t<a \\ 2a-t, & a<t<2a \end{cases}$ where $f(t+2a)=f(t)$.

(d) Find the inverse Laplace transform of
i) $\frac{s^2-10s+13}{(s+1)(s^2-5s+6)}$ ii) $\log\left(1-\frac{a^2}{s^2}\right)$

4. **Attempt any three:** **12**

(a) Show that
$\overline{F}=(y^2\cos x+z^3)i+(2y\sin x-4)j+(3xz^2+2)k$
is a conservative vector field and find a function ϕ such that $\overline{F}=\nabla\phi$.

(b) Find the directional derivative of
$\phi=x^2y+2y^2z+3z^2x$ at (1,1,1) in the direction

parallel to $\frac{x}{2} = \frac{y}{-2} = \frac{z}{1}$.

(c) Show that $\nabla^2\left[\nabla \cdot \left(\frac{\bar{r}}{r^4}\right)\right] = \frac{-12}{r^6}$

(d) Find the work done in moving a particle once around the circle C in the x-y plane if the circle has centre at the origin and radius 3 and the force field $\bar{F} = (2x - y + z)i + (x + y - z^2)j + (3x - 2y + 4z)k$

5. **Attempt the following:** **12**

(a) Prove that $\bar{r}.\frac{d\bar{r}}{dt} = 0$ iff $\bar{r}$ has a constant magnitude.

(b) Solve the following differential equation using Laplace transform

$\frac{d^2y}{dt^2} - 6\frac{dy}{dt} + 9y = t^2e^{3t}$ *with* $y(0) = 2,\ y'(0) = 6$

(c) If r and $\bar{r}$ have their usual meanings then prove that

i) $\nabla \cdot (r^n\bar{r}) = (n+3)r^n$

ii) $\nabla^2 r^n = n(n+1)r^{n-2}$

Government College of Engineering, Amravati
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Third Semester B.Tech. (CS/IT)

Winter – 2017

Course Code: SHU304

Course Name: Engineering Mathematics-III

Time: 2 Hrs.30 Min **Max Marks: 60**

Instructions to Candidate:

1) All questions are compulsory.
2) Assume suitable data wherever necessary.
3) Diagrams/sketches should be given wherever necessary.
4) Use of logarithmic table, drawing instruments and non-programmable calculator is permitted.
5) Figures to the right indicate full marks.

1. Attempt any Three: **12**

(a) Solve: $\frac{d^2y}{dx^2}-6\frac{dy}{dx}+9y=e^{3x}\operatorname{cosec}^2 x+5^x$

(b) Solve by method of variation of parameters

$$\frac{d^2y}{dx^2}-4\frac{dy}{dx}+4y=e^{2x}\sec^2 x$$

(c) Solve:

$$(3x+2)^2\frac{d^2y}{dx^2}+3(3x+2)\frac{dy}{dx}-36y=3x^2+4x+1$$

(d) Solve: $(D^3-4D)y=2\cosh^2 2x$

2. Attempt any Three: **12**

(a) Solve $z(p^2+q^2)=x^2+y^2$

(b) Solve $(x+y)(p+q)^2+(x-y)(p-q)^2=1$

(c) Find a solution of the equation $\frac{\partial^2 u}{\partial x^2}=\frac{\partial u}{\partial y}+2u$ in

Contd..

the form $u = f(x)g(y)$ subject to the conditions
$u = 0$,
$\frac{\partial u}{\partial x} = 1 + e^{-3y}$ when $x = 0$ for all values of y.

(d) Solve: $(\cos(x+y))\, p + (\sin(x+y))q = z$

3. **Attempt any Three :** 12

(a) Use convolution theorem to find inverse Laplace transform of

$$\bar{f}(s) = \frac{s}{s^4 + 5s^2 + 4}$$

(b) Solve using Laplace transform,

$$\frac{dx}{dt} - y = e^t, \quad \frac{dy}{dt} + x = \sin t.$$

Given that $x(0) = 1$, $y(0) = 0$

(c) Solve $\frac{dy}{dt} + 2y + \int_0^t y\,dt = \sin t$, given $y(0) = 1$

(d) Define Heaviside's unit step function and use it to find the Laplace transform of the function

$$f(t) = \begin{cases} t-1, \text{ if } 1 < t < 2 \\ 3-t, \text{ if } 2 < t < 3 \end{cases}$$

4. **Attempt any Three:** 12

(a) For what values of n the vector $r^{(n+1)}\bar{r}$ is solenoidal and irrotational? $\nabla \times \bar{r}$ = irrot

(b) If $\bar{r} = xi + yj + zk$ and $r = |\bar{r}|$,prove that div $\bar{r}$ = solid

$\nabla^2 f(r) = f''(r) + \frac{2}{r} f'(r)$ where and hence find the value of $\nabla^2(\log r)$

(c) Find the workdone in moving a particle once round the ellipse $\frac{x^2}{16}+\frac{y^2}{4}=1,\ z=0$ under the field of force

$\overline{F}=(2x-y+z)i+(x+y-z^2)j+(3x-2y+4z)k$

(d) The water level at a point (x, y, z) in the ground is given by $W(x, y, z)=x^2+y^2-z$. A machine located at $(1, 1, 1)$ desires to move in such a direction that it will get water maximum. In what direction should it move? What should be the maximum water level?

5. **Attempt the following:** 12

(a) Prove that

$$y=c_1\cos 3x+c_2\sin 3x+\frac{1}{3}\left[x\sin 3x-\frac{1}{3}\cos 3x\cdot\log(\sec 3x)\right]$$

is the general solution of the differential equation $(D^2+9)y=\sec 3x$.

(b) Prove that i) $\nabla^2\left[\nabla\cdot\left(\frac{\overline{r}}{r^4}\right)\right]=-\frac{12}{r^6}$

ii) Solve: $(1-y^2)xq^2+y^2p=0$

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Government College of Engineering, Amravati

(An Autonomous Institute of Government of Maharashtra)

Third Semester B.Tech. (CS/IT)

Winter – 2017

Course Code: SHU304

Course Name: Engineering Mathematics-III

Time: 2 Hrs.30 Min **Max Marks: 60**

Instructions to Candidate:

1) All questions are compulsory.
2) Assume suitable data wherever necessary.
3) Diagrams/sketches should be given wherever necessary.
4) Use of logarithmic table, drawing instruments and non-programmable calculator is permitted.
5) Figures to the right indicate full marks.

1. Attempt any Three: **12**

(a) Solve: $\frac{d^2y}{dx^2}-6\frac{dy}{dx}+9y=e^{3x}\cos ec^2x+5^x$

(b) Solve by method of variation of parameters

$$\frac{d^2y}{dx^2}-4\frac{dy}{dx}+4y=e^{2x}\sec^2 x$$

(c) Solve:

$$(3x+2)^2\frac{d^2y}{dx^2}+3(3x+2)\frac{dy}{dx}-36y=3x^2+4x+1$$

(d) Solve: $(D^3-4D)y=2\cosh^2 2x$

2. Attempt any Three: **12**

(a) Solve $z(p^2+q^2)=x^2+y^2$

(b) Solve $(x+y)(p+q)^2+(x-y)(p-q)^2=1$

(c) Find a solution of the equation $\frac{\partial^2 u}{\partial x^2}=\frac{\partial u}{\partial y}+2u$ in

Contd..

the form $u = f(x)g(y)$ subject to the conditions
$u = 0$,
$\frac{\partial u}{\partial x} = 1 + e^{-3y}$ when $x = 0$ for all values of y.

(d) Solve: $(\cos(x+y))\, p + (\sin(x+y))q = z$

3. **Attempt any Three :** **12**

(a) Use convolution theorem to find inverse Laplace transform of

$$\bar{f}(s) = \frac{s}{s^4 + 5s^2 + 4}$$

(b) Solve using Laplace transform,

$$\frac{dx}{dt} - y = e^t, \quad \frac{dy}{dt} + x = \sin t.$$

Given that $x(0) = 1,\ y(0) = 0$

(c) Solve $\frac{dy}{dt} + 2y + \int_0^t y\,dt = \sin t$, given $y(0) = 1$

(d) Define Heaviside's unit step function and use it to find the Laplace transform of the function

$$f(t) = \begin{cases} t-1, \text{ if } 1 < t < 2 \\ 3-t, \text{ if } 2 < t < 3 \end{cases}$$

4. **Attempt any Three:** **12**

(a) For what values of n the vector $r^{(n+1)}\bar{r}$ is solenoidal and irrotational?

(b) If $\bar{r} = xi + yj + zk$ and $r = |\bar{r}|$,prove that $\nabla^2 f(r) = f''(r) + \frac{2}{r} f'(r)$ where and hence find the value of $\nabla^2(\log r)$

(c) Find the workdone in moving a particle once round the ellipse $\frac{x^2}{16}+\frac{y^2}{4}=1$, $z=0$ under the field of force $\overline{F}=(2x-y+z)i+(x+y-z^2)j+(3x-2y+4z)k$

(d) The water level at a point (x, y, z) in the ground is given by $W(x, y, z)=x^2+y^2-z$. A machine located at $(1, 1, 1)$ desires to move in such a direction that it will get water maximum. In what direction should it move? What should be the maximum water level?

5. **Attempt the following:** 12

(a) Prove that

$$y=c_1\cos 3x+c_2\sin 3x+\frac{1}{3}\left[x\sin 3x-\frac{1}{3}\cos 3x\cdot\log(\sec 3x\right.$$

is the general solution of the differential equation $(D^2+9)y=\sec 3x$.

(b) Prove that i) $\nabla^2\left[\nabla\cdot\left(\frac{\overline{r}}{r^4}\right)\right]=-\frac{12}{r^6}$

ii) Solve: $(1-y^2)xq^2+y^2p=0$

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Government College of Engineering, Amravati

(An Autonomous Institute of Government of Maharashtra)

Third Semester B. Tech. (CS / IT)

Summer – 2016

Course Code: SHU304

Course Name: Engineering Mathematics III

Time: 2 hrs.30 min. **Max Marks: 60**

Instructions to candidate:

1) **All questions are compulsory.**
2) **Assume suitable data wherever necessary and clearly state the assumptions made.**
3) **Diagrams/sketches should be given wherever necessary.**
4) **Use of logarithmic table, drawing instruments and nonprogrammable calculators is permitted.**
5) **Figures o the right indicate full marks.**

1. **Attempt any three:** 12

(a) Solve: $(D^3+1)y = \sin 3x - \cos^2 \frac{x}{2}$

(b) Solve: $\frac{d^2y}{dx^2} + 4y = x\sin x$

(c) Solve: $\frac{d^2y}{dx^2} + y = -\cot x$ using method of variation of parameters.

(d) Solve: $x^2 \frac{d^3y}{dx^3} + 3x\frac{d^2y}{dx^2} + \frac{dy}{dx} - x^{-1}y = 3 - 7x^{-1}$

2. **Attempt any three:** 12

(a) Solve: $pq = x^m y^n z^{2l}$

(b) Solve: $(1-y^2)xq^2 + y^2 p = 0$

(c) Solve: $(2z-y)p + (x+z)q + (2x+y) = 0$

(d) Use method of separation of variable and show that the solution of the heat equation

Cont.

$\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}=0$ which satisfies the conditions $u(0,y)=u(l,y)=u(x,0)=0$ is

$u(x,y)=C\sin\frac{n\pi x}{l}\left(e^{\frac{n\pi y}{l}}-e^{\frac{-n\pi y}{l}}\right)$. Use $k=-n^2$

3. **Attempt any two:** 09

(a) If $\bar{f}(s)=\dfrac{s^2-3}{(s+2)(s-3)(s^2+2s+5)}$ find inverse Laplace transform.

(b) Using convolution theorem, find

$$L^{-1}\left\{\frac{(s+2)^2}{(s^2+4s+8)^2}\right\}$$

(c) If $f(t)=\begin{cases}\dfrac{t}{a}, & 0<t<a\\ \dfrac{1}{a}(2a-t), & a<t<2a\end{cases}$

and $f(t)=f(t+2a)$ .Find $L\{f(t)\}$.

4. **Attempt any three:** 15

(a) A particle moves on the curve $x=2t^2$, $y=t^2-4t$, $z=3t-5$, where t is the time. Find the component of velocity and acceleration at time t=1 in the direction i-3j+2k.

(b) Find the angle between the surfaces $x^2+y^2+z^2=9$, $x^2+y^2-z=3$ at point $(2,-1,2)$.

(c) Define solenoidal field and conservative field. If $\overline{F}=grad(x^3+y^3+z^3-3xyz)$ .Prove that $\overline{F}$ is solenoidal vector at origin and it is conservative everywhere in the space.

(d) A vector field is given by $\overline{F}=(\sin y)i+x(1+\cos y)j$. Evaluate the line integral over a circular path $x^2+y^2=a^2,\ z=0$

5. **Fill in the blanks:** 12

09 (a) A vector $\overline{F}=(x+2y+az)i+(bx-3y-z)j+(4x+cy+2z)k$ is irrotational. Then constants $a=-----,\quad b=--------,\quad c=-----$

(b) The solution of partial differential equation $pq=p+q$ is given as---------------

(c) The solution of partial differential equation $p^2+q^2=z$ is given as---------------

(d) Laplace transform of f(t) be $\log\left\{\frac{(s+b)}{(s+a)}\right\}$. Then function f(t)= ------------------

15 (e) If $1*1=t$ then $1*1*1*-----n\ times$ results --------

(f) The particular integral of the differential equation $(D^2+6D+9)y=x^{-3}e^{-3x}$ is--------------------