GOVERNMENT COLLEGE OF ENGINEERING, AMRAVATI
(An Autonomous Institute of Government of Maharashtra)

I Sem. B.Tech.(All Branches)

Winter 2018

Course Code: SHU101

Course Name: Engineering Mathematics-I

Time:2 hrs.30 min. **Max Marks:60**

Instructions to candidate:

1) **All questions are compulsory.**
2) **Assume suitable data wherever necessary and clearly state the assumptions made.**
3) **Diagrams/sketches should be given wherever necessary.**
4) **Use of logarithmic table, drawing instruments and nonprogrammable calculators is permitted.**
5) **Figures o the right indicate full marks.**

1. **Attempt any Three:** **12**

(a) Find the eigen values and eigen vectors of the matrix

$$A = \begin{bmatrix} 7 & -2 & 0 \\ -2 & 6 & -2 \\ 0 & -2 & 5 \end{bmatrix}$$

(b) Reduce the following matrix to echelon form and find its rank

$$\begin{bmatrix} 1 & 2 & 3 & -1 \\ -2 & -1 & -3 & -1 \\ 1 & 0 & 1 & 1 \\ 1 & 1 & 1 & -1 \end{bmatrix}$$

(c) Find the eigen values of the matrix A and also A^{-1}. Show that the eigen values of A^{-1} are reciprocals of eigen values of A.

Cont

$$A = \begin{bmatrix} 3 & -3 & 4 \\ 2 & -3 & 4 \\ 0 & -1 & 1 \end{bmatrix}$$

(d) Find the characteristic equation of the matrix A where

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & -1 & 4 \\ 3 & 1 & -1 \end{bmatrix}$$

Also show that the matrix A satisfies the

2. characteristic equation and hence find A^{-1}. **12**

(a) **Attempt any Three:**

If $z = -1 + i\sqrt{3}$ and n is an integer, prove that $z^{2n} + 2^n z^n + 2^{2n} = 0$ if n is not multiple of 3.

(b)

If $1+2i$ is a root of the equation $x^4 - 3x^3 + 8x^2 - 7x + 5 = 0$, find all other roots.

(c)

Show that $\tan^{-1} i\left(\dfrac{x-a}{x+a}\right) = \dfrac{i}{2}\log\dfrac{x}{a}$

(d)

Find the principal value of $i^{\log(1+i)}$ and show that its real part is $e^{-\pi^2/8} \cdot \cos\left(\dfrac{\pi}{4}\log 2\right)$.

3. **12**

(a) **Attempt any Three:**

Find the n^th derivative of $y = \cos x \, \cos 2x \, \cos 3x$.

(b)

If $y = \tan^{-1}\left(\dfrac{\sqrt{1+x^2}-1}{x}\right)$, prove that

$y_n = \dfrac{1}{2}(-1)^{n-1}(n-1)! \sin^n\theta \sin n\theta$, where

(c)

$\theta = \tan^{-1}(1/x)$.

If $y^{1/m} + y^{-1/m} = 2x$, show that

(d) $(x^2-1)y_{n+2} + (2n+1)xy_{n+1} + (n^2-m^2)y_n = 0$

4. **12**

Using Taylor's theorem find $\sqrt{25.15}$.

(a)

Attempt any Three:

$x+y+z=u,\ y+z=uv,\ z=uvw$ prove that

(b) $\dfrac{\partial(x,y,z)}{\partial(u,v,w)}=u^2v\cdot$

If $x=p\cos\alpha-q\sin\alpha,\ y=p\sin\alpha+q\cos\alpha$ and u is a function of x,y prove that

(c) $$\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}=\frac{\partial^2 u}{\partial p^2}+\frac{\partial^2 u}{\partial q^2}$$

If $\dfrac{x^2}{a^2+u}+\dfrac{y^2}{b^2+u}+\dfrac{z^2}{c^2+u}=1$, where u is a homogeneous function of degree n in x,y,z, show that

(d) $$\left(\frac{\partial u}{\partial x}\right)^2+\left(\frac{\partial u}{\partial y}\right)^2+\left(\frac{\partial u}{\partial z}\right)^2=2nu\cdot$$

If $u=\operatorname{cosec}^{-1}\left(\sqrt{\dfrac{x^{1/2}+y^{1/2}}{x^{1/3}+y^{1/3}}}\right)$,prove that

5. (a) $$x^2\frac{\partial^2 u}{\partial x^2}+2xy\frac{\partial^2 u}{\partial x\partial y}+y^2\frac{\partial^2 u}{\partial y^2}=\frac{\tan u}{12}\left[\frac{13}{12}+\frac{1}{12}\tan^2 u\right]$$ **12**

Attempt the following:

(b) Find A,B,C such that $\cos A\cdot\cos B\cdot\cos C$ is maximum for a ΔABC.

(c) If u,v,w are the roots of the equation $(\lambda-x)^3+(\lambda-y)^3+(\lambda-z)^3=0$ in λ,find $\dfrac{\partial(u,v,w)}{\partial(x,y,z)}$

If $y=(\sin^{-1}x)^2$,prove that

$y_n(0)=0$ *if n is odd and*

$y_n(0)=2.2^2.4^2.6^2....(n-2)^2$ *if* $n\neq 2$ *and n is even.*

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