Government College of Engineering, Amravati

(An Autonomous Institute of Government of Maharashtra)

Second Semester B. Tech.

Summer – 2017

Course Code: SHU201

Course Name: Engineering Mathematics II

Time: 2 Hrs. 30 Min. **Max. Marks: 60**

Instructions to Candidate

1) All questions are compulsory.
2) Assume suitable data wherever necessary and clearly state the assumptions made.
3) Diagrams/sketches should be given wherever necessary.
4) Use of logarithmic table, drawing instruments and non-programmable calculators is permitted.
5) Figures to the right indicate full marks.

1. ATTEMPT ANY THREE. **12**

a) By differentiating under integral sign find $\frac{dF}{d\alpha}$ and $\frac{d^2F}{d\alpha^2}$ if $F(\alpha)=\int_{1/\alpha}^{2/\alpha}\frac{\sin\alpha x}{x^2}\,dx,\ \alpha\neq 0.$

b) Trace the polar curve $r^2=a^2\sin 2\theta$ with full justification.

c) Evaluate $\int_{-1}^{1}(1+x)^{p-1}(1-x)^{q}\,dx$ by using beta and gamma function.

d) Show that

Contd..

$$\int_a^b (x-a)^m (b-x)^n \, dx = (b-a)^{m+n+1} \beta(m+1,, n+1)$$

12

2. **ATTEMPT ANY THREE.**

a) Show that the area of the loop of the curve $r = a\cos n\theta$ is $\pi a^2/4n$. Also find the total area in case if n is odd and n is even.

b) Change the order of integration $\int_0^1 \int_{x^2}^{\sqrt{2-x^2}} f(x, y)\, dx\, dy$

c) Evaluate the double integration

$$\int_0^{a\sqrt{3}} \int_0^{\sqrt{x^2+a^2}} \frac{x}{a^2+x^2+y^2}\, dx\, dy$$

d) Change the integral $\int_0^1 \int_0^x (x,+y)\, dy\, dx$ to polar and hence evaluate.

12

3. **ATTEMPT ALL.**

a) Obtain a Fourier series to represent $x+x^2$ from $x=-\pi$ to $x=\pi$.

b) Obtain a Fourier series to represent $f(x)=e^{-x}$ from $x=-l$ to $x=l$.

c) Find a Fourier cosine series for the function $f(x)=\sin x$ for $0<x<\pi$.

12

4. **ATTEMPT ANY THREE.**

a) Find the integrating factor for the differential equation $\frac{dy}{dx}+x\sin 2y = x^3\cos^2 y$. Use it to solve the differential equation.

b) Show that the family of parabolas $x^2 = 4a(y+a)$ is self orthogonal.

c) Solve $y(xy+2x^2y^2)dx + x(xy-x^2y^2)dy = 0.$

d) Show that the family of cardioids $r = a(1-\cos\theta)$ is not self orthogonal. **12**

5. **ATTEMPT ALL.**

a) The equations of electromotive force in terms of current i for an electrical circuit having resistance R and a condenser of capacity C, in series is $E = Ri + \int \frac{i}{C} dt$ Find the current i at any time t,
when $E = E_0 \sin \omega t$

b) Evaluate the integral

$$\int_0^1 \int_0^{\sqrt{1-x^2}} \int_0^{\sqrt{1-x^2-y^2}} \frac{dzdydx}{\sqrt{1-x^2-y^2-z^2}}$$ by changing to spherical polar coordinates.

c) Trace the polar curve $x^{2/3} + y^{2/3} = a^{2/3}$ with full justification.

Government College of Engineering, Amravati

(An Autonomous Institute of Government of Maharashtra)

Second Semester B. Tech. (All Branches)

Summer Term – 2017

Course Code: SHU 201

Course Name: Engineering Mathematics II

Time: 2 Hrs: 30 Min. Max. Marks: 60

Instructions to Candidate

1) All questions are compulsory.
2) Assume suitable data wherever necessary and clearly state the assumptions made.
3) Diagrams/sketches should be given wherever necessary.
4) Use of logarithmic table, drawing instruments and non-programmable calculators is permitted.
5) Figures to the right indicate full marks.

1. ATTEMPT ANY THREE. 12

a) Evaluate $\int_0^{\infty} e^{-\left(x^2+\frac{a^2}{x^2}\right)} dx$ by differentiating under integral sign and using $\int_0^{\infty} e^{-x^2} dx = \frac{\sqrt{\pi}}{2}$ if necessary.

b) Define asymptotes.

Trace the Cartesian curve $y^2(2a-x)=x^3$ with full justification.

Contd..

c) Show that $\int_0^{\pi/2} \sin^p x\, dx. \int_0^{\pi/2} \sin^{p+1} x\, dx = \frac{\pi}{2(p+1)}$.

d) Show that $y.\beta(x+1, y) = x\beta(x,, y+1)$

2. ATTEMPT ANY THREE. 12

a) Find the area inside the cardioids $r = a\,(1+\cos\theta)$ and outside the circle $r = 2a\cos\theta$.

b) Express the following integral as a single integral and hence evaluate

$$\int_0^{a/\sqrt{2}} \int_0^{x} x\, dx\, dy + \int_{a/\sqrt{2}}^{a} \int_0^{\sqrt{a^2-x^2}} x\, dx\, dy.$$

c) Evaluate $\iint_R x^2 dxdy$ where R is the region in the first quadrant bounded by the hyperbola $xy = 16$ and the lines $y = x,\ y = 0,\ x = 8$.

d) Change the integral $\int_0^{2} \int_0^{\sqrt{2x-x^2}} (x^2 + y^2)\, dx\, dy$ to polar and hence evaluate.

3. ATTEMPT ALL. 12

a) Obtain a Fourier series to represent $x - x^2$ from $x = -\pi$ to $x = \pi$.

b) Obtain a Fourier series to represent the periodic

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function $f(x)=0$ when $0<x<\pi$ and $f(x)=1$ when $\pi<x<2\pi$.

c) Find a Fourier sine series for the function $f(x)=e^{ax}$ for $0<x<\pi$ where a is a constant.

12

4. ATTEMPT ANY THREE. 12

a) Find Integrating factor for the differential equation $(xy^3+y)dx+2(x^2y^2+x+y^4)dy=0$ Use it to solve the differential equation for $x=0,\ y=0$.

b) Find differential equation for the family of parabolas $x^2=4a(y+a)$ and its orthogonal trajectory. Hence show that the family of parabolas $x^2=4a(y+a)$ is self orthogonal.

c) Solve the differential equation $(x+2y)(dx-dy)=(dx+dy)$ by using reducible to homogeneous method.

d) Solve the differential equation

$$y-\cos x\frac{dy}{dx}=y^2(1-\sin x)\cos x$$, given that

$y=2$

when $x=0$.

Contd..

5. ATTEMPT ALL. 12

a) Show that the differential equation for current i in an electrical circuit having resistance R and a inductance L in series and acted on by an electromotive forcs $E\sin\omega t$ satisfies the equation $L\dfrac{di}{dt}+Ri=E\sin\omega t$. Find the current i at any time t , if initially there is no current in the circuit.

b) Find the volume of the sphere of radius a by using triple integration.

c) Trace the curve $r=a\left(\sec\theta+\tan\theta\right)$ with full justification.

Government College of Engineering, Amravati
(An Autonomous Institute of Government of Maharashtra)

First Year B. Tech.

Summer – 2016

Course Code: SHU201

Course Name: Engineering Mathematics II

Time: 2 Hrs. 30 Min. **Max. Marks: 60**

Instructions to Candidate

1) All questions are compulsory.
2) Assume suitable data wherever necessary and clearly state the assumptions made.
3) Diagrams/sketches should be given wherever necessary.
4) Use of logarithmic table, drawing instruments and non-programmable calculators is permitted.
5) Figures to the right indicate full marks.

1. Attempt any three of the following. **(12)**

a) For $c > 1$, Prove that $\int_0^\infty \frac{x^c}{c^x} dx = \frac{\Gamma(c+1)}{(\log c)^{c+1}}$.

b) Prove that $\Gamma n \Gamma(1-n) = \frac{\pi}{\sin n\pi}$, if $\int_0^\infty \frac{x^{n-1}}{1+x} dx = \frac{\pi}{\sin n\pi}$.

Hence deduce that $\Gamma \frac{1}{2} = \sqrt{\pi}$.

c) Trace the curve $r = a \sin 3\theta$.

d) Trace the curve $x^3 + y^3 = 3axy$.

e) If $\left|\frac{b}{a}\right| < 1$, Show that

$$\int_0^{\pi/2} \frac{1}{\sin\theta} \log\left(\frac{a+b\sin\theta}{a-b\sin\theta}\right) d\theta = \pi \sin^{-1}\left(\frac{b}{a}\right).$$

2. Attempt any three of the following. **(12)**

a) Evaluate $\int_0^1 \int_0^{\sqrt{2-y^2}} \frac{y\,dx\,dy}{\sqrt{(2-x^2)(1-x^2y^2)}}$.

b) Evaluate $\iint_R \frac{\left(x^2+y^2\right)^2}{x^2y^2} dxdy$ where R is common to $x^2+y^2=ax$, $x^2+y^2=by$, $a>b>0$.

c) Find the volume of solid bounded by the sphere $x^2+y^2+z^2=4$ and the paraboloid $x^2+y^2=3z$.

d) Evaluate $\int_0^\infty \int_0^\infty \int_0^\infty \frac{dxdydz}{(1+x^2+y^2+z^2)^2}$.

3. Solve any three differential equations. **(12)**

a) Evaluate $\frac{dy}{dx} = \frac{-(x^2y+y^4)}{x(2x^2+4y^3)}$.

b) Evaluate $\frac{dy}{dx} + \frac{(2x+3y)}{(y+2)} = 0$.

c) Evaluate $(xy^{2}+e^{-1/x^{3}})dx - x^{2}ydy = 0$

d) Evaluate $(x+\tan y)dy = \sin 2y dx$

e) Find the constant n such that $(x+y)^{n}$ is an integrating factor of $(4x^{2}+2xy+6y)dx+(2x^{2}+9y+3x)dy=0$ & hence solve the equation.

4. Attempt any two of the following. (12)

a) Find the Orthogonal Trajectory of $x\sin x\cosh y - y\sinh y\cos x = 0$.

b) In a circuit containing inductance L, resistance R & voltage E, the current I is given by $E = RI + L\frac{dI}{dt}$. Given L=640 H, R=250Ω & E=500 V. Current being zero when $t=0$. Find the time that elapses, before it reaches 90% of its maximum value.

c) A constant EMF E Volts is applied to a circuit containing a constant resistance R Ohms in a series & a constant inductance L henries. If the initial current is zero, Show that the current builds up to its theoretical maximum is $L\log 2/R$ seconds.

5. Attempt any two of the following. (12)

a) Determine the Fourier series for the function $f(x)=\sqrt{1-\cos x}$ in the interval $0 \le x \le 2\pi$ & hence deduce that $\sum_{n=1}^{\infty}\frac{1}{4n^{2}-1}=\frac{1}{2}$.

b) A sinusoidal voltage $E\sin\omega t$ is passed through a half wave rectifier which clips the negative portion of the wave. Find the Fourier series of the resulting periodic function

$$f(t) = \begin{cases} 0 & , -T/2 < t < 0 \\ E\sin\omega t & , 0 < t < T/2 \end{cases} \qquad \text{where } T = \frac{2\pi}{\omega}.$$

c) Find the half range sine & cosine series.

$$\text{If } f(t) = \begin{cases} t & ,0 < t < 1 \\ 2-t & ,1 < t < 2 \end{cases}.$$

Government College of Engineering, Amravati
(An Autonomous Institute of Government of Maharashtra)

First Year B. Tech.

Summer Term – 2014

Course Code: SHU201

Course Name: Engineering Mathematics – II

Time: 2 Hrs. 30 Min. **Max. Marks: 60**

Instructions to Candidate

1) All questions are compulsory.
2) Assume suitable data wherever necessary and clearly state the assumptions made.
3) Diagrams/sketches should be given wherever necessary.
4) Use of logarithmic table, drawing instruments and non-programmable calculators is permitted.
5) Figures to the right indicate full marks.

1 Attempt any Three **12**

a) Show that $\dfrac{2^n \Gamma\left(n+\frac{1}{2}\right)}{\sqrt{\pi}} = 1.3.5.........(2n-1)$

b) Using Beta and Gamma function evaluate

$$\int_0^{2a} x^m \sqrt{2ax - x^2}\, dx$$

c) Trace the curve $y^2\left(a^2 - x^2\right) = a^3 x$ with full justification.

d) Show that $\displaystyle\int_0^\infty \frac{\tan^{-1} ax}{x\left(1+x^2\right)} dx = \frac{\pi}{2}\log\left(1+a\right)$

Contd..

2 Attempt any Three 12

a) Change the order of integration and evaluate $\int_0^{\infty}\int_x^{\infty}\frac{e^{-y}}{y}dxdy$

b) Express into polar form and evaluate $\int_0^2 dx \int_0^{\sqrt{2x-x^2}}\left(x^2+y^2\right)dy$

c) Evaluate $\iiint_V z\,dxdydz$ where V is the cylinder bounded by

$z=0, z=1\, and\, x^2+y^2=4$

d) Evaluate $\int_0^{\infty}\int_0^{\infty}\int_0^{\infty}\frac{dxdydz}{\left(1+x^2+y^2+z^2\right)^2}$

3 Attempt Any Three 12

a) Find the orthogonal trajectory of the curves $x\sin x\cosh y - y\sinh y\cos x = c$

b) Solve $e^{-y}\sec^2 y\,dy = dx + x\,dy$

c) Solve $y(xy+1)dx + x\left(1+xy+x^2y^2\right)dy = 0$

d) A resistance of 100 ohms, an inductance of 0.5 henries are connected in series with battery of 20 volts . find the current in the circuit as a function of time.

4 Attempt any Three 12

a) Obtain the Fourier series for the function $f(x)=x-x^2, -1<x<1$.

b) Expand $\pi x - x^2$ in a half range sine series in the interval $(0,\pi)$ upto the first three terms.

c) show that in the interval $0<x<\pi$

$$\frac{e^{ax}-e^{-ax}}{e^{a\pi}-e^{-a\pi}}=\frac{2}{\pi}\left[\frac{\sin x}{a^2+1}-\frac{2\sin 2x}{a^2+4}+\frac{3\sin 3x}{a^2+9}-\dots\right]$$

d) Develop $\sin\left(\frac{\pi x}{l}\right)$ in half range cosine series in $0<x<l$.

5 Attempt the following

a) If R is the region in the first quadrant bounded by $xy=1, xy=2, y=x$ and $y=2x$ then prove that

$$\iint_R f(xy)dxdy=\frac{1}{2}\log 2\int_1^2 f(u)du$$

b) If

$$f(x)=\frac{\pi}{3},\ 0\le x\le\frac{\pi}{3}$$
$$=0,\ \frac{\pi}{3}\le x\le\frac{2\pi}{3}$$
$$=\frac{-\pi}{3},\ \frac{2\pi}{3}\le x\le\pi$$

Then prove that

$$f(x)=\frac{2}{\sqrt{3}}\left[\cos x-\frac{1}{5}\cos 5x+\frac{1}{7}\cos 7x+\dots\right]$$

and also

$$f(x)=\sin 2x+\frac{1}{2}\sin 4x+\frac{1}{10}\sin 10x+\dots$$

Government College of Engineering, Amravati

(An Autonomous Institute of Government of Maharashtra)

First Year B. Tech.

Winter – 2014

Course Code: SHU201

Course Name: Engineering Mathematics II

Time: 2 Hrs. 30 Min. **Max. Marks: 60**

Instructions to Candidate

1) All questions are compulsory.
2) Assume suitable data wherever necessary and clearly state the assumptions made.
3) Diagrams/sketches should be given wherever necessary.
4) Use of logarithmic table, drawing instruments and non-programmable calculators is permitted.
5) Figures to the right indicate full marks.

1. **ATTEMPT ANY THREE** 12

a) Solve $\left(1+e^{x/y}\right)dx - e^{x/y}\left(1-x/y\right)dy = 0.$

b) Solve $(x+1)\frac{dy}{dx} - y = e^{3x}(x+1)^2.$

c) Solve $\frac{dy}{dx} - \frac{y\cos x - \sin y + y}{\sin x - x\cos y - x} = 0.$

d) Solve $(3y+2x-4)dx - (4x-6y-5)dy = 0.$

2. **ATTEMPT ANY TWO** 12

a) Obtain the Fourier series for $f(x) = e^{-x}$ in the interval $0 < x < 2\pi$.

b) Obtain the Fourier series to represent $x - x^2$ in the interval $-\pi < x < \pi$.

c) Express $f(x) = x$ as a half-range sine series in the interval $0 < x < 2$.

3. ATTEMPT ALL. 12

a) Change the order of integration in

$$\int_{-a}^{a}\int_{0}^{\sqrt{a^2-y^2}} f(x,y)\,dx\,dy.$$

b) Calculate $\iint r^3\,dr\,d\theta$ over the area included between the circles $r = 2\sin\theta$ and $r = 4\sin\theta$.

c) Evaluate $\int_0^1\int_0^{\sqrt{1-x^2}}\int_0^{\sqrt{1-x^2-y^2}} xyz\,dx\,dy\,dz$.

4. ATTEMPT ALL. 12

a) Find the value of $\int_0^1 \frac{x^2\,dx}{\sqrt{1-x^4}} \times \int_0^1 \frac{dx}{\sqrt{1-x^4}}$.

b) Prove that $\beta(m, {}^1/_2) = 2^{2m-1}\,\beta(m, m)$.

c) Trace the curve with full justification $r = a(\sec\theta - \tan\theta)$.

5. ATTEMPT ANY TWO. 12

a) Find the volume of the portion of the sphere $x^2 + y^2 + z^2 = a^2$ lying inside the cylinder $x^2 + y^2 = ax$.

b) Obtain the Fourier series for the function f given by

$$f(x) = 1 + {}^{2x}/_{\pi}, \quad -\pi \le x \le 0$$
$$= 1 - {}^{2x}/_{\pi}, \quad 0 \le x \le \pi.$$

c) Solve $\frac{dy}{dx} = \frac{y+x-2}{y-x-4}$.

Government College of Engineering, Amravati

(An Autonomous Institute of Government of Maharashtra)

Second Semester B. Tech.
Winter – 2013

Course Code: SHU201

Course Name: Engineering Mathematics – II

Time: 2 Hrs. 30 Min. **Max. Marks: 60**

Instructions to Candidate

1) All questions are compulsory.
2) Assume suitable data wherever necessary and clearly state the assumptions made.
3) Diagrams/sketches should be given wherever necessary.
4) Use of logarithmic table, drawing instruments and non-programmable calculators is permitted.
5) Figures to the right indicate full marks.

1. **Attempt any Three:** 12

(a) Solve: $(2x+y+1)dx+(4x+2y-1)dy=0$

(b) Solve: $x(1-x^2)\dfrac{dy}{dx}+(2x^2-1)y=x^3$

(c) Solve: $y(xy^2+1)dx+2(x^2y^2+x+y^4)dy=0$

imp **(d)** Solve: $(xy^2+2x^2y^3)dx+(x^2y-x^3y^2)dy=0$

2 **Attempt any Three:** 12

(a) Trace the following curve with full justification
$x^3+y^3=3axy$, a >0

(b) Trace the curve $r=2+3\cos\theta$ with full justification:

(c) Verify the rule of differentiation under the integral sign for the integral $\int_0^{a^2} \tan^{-1}\frac{x}{a}\,dx$

(d) Evaluate $\int_0^{\pi} x\sin^5 x\cos^4 x\,dx$

3 **Attempt any Three:** **12**

(a) Prove that $\int_0^1 \frac{x^{m-1}(1-x)^{n-1}}{(a+bx)^{m+n}}dx = \frac{\beta(m,n)}{a^n(a+b)^m}$

(b) Show the region of integration and Change the order of integration $\int_0^b \int_{-\sqrt{b^2-y^2}}^{a\sqrt{1-\frac{y^2}{b^2}}} f(x,y)\,dxdy,\ a>b$

(c) Express the following integral in polar coordinates showing the region of integration and evaluate

$$\int_0^{2a}\int_0^{\sqrt{2ax-x^2}} (x^2+y^2)\,dxdy$$

(d) Evaluate $\iiint z^2 dxdydz$ taken throughout the volume common to the sphere $x^2+y^2+z^2=a^2$ and the cylinder $x^2+y^2=ax$.

4 **Attempt any Three:** **12**

(a) Obtain Fourier series expansion for the function defined as follows

$$f(x)=\begin{cases}\frac{2x}{\pi}+1, & -\pi\le x\le 0\\ 1-\frac{2x}{\pi}, & 0\le x\le \pi\end{cases}$$

Hence deduce that $\frac{1}{1^2}+\frac{1}{3^2}+\frac{1}{5^2}+\ldots=\frac{\pi^2}{8}$

(b) Obtain Fourier series expansion for the function

$$f(x)=\begin{cases}\pi x, & 0\le x\le 1\\ \pi(2-x), & 1\le x\le 2\end{cases}$$

(c) Find the half range sine series for

$$f(x)=\begin{cases}\frac{1}{4}-x, & 0<x<\frac{1}{2}\\ x-\frac{3}{4}, & \frac{1}{2}<x<1\end{cases}$$

a_0 a_n

(d) Find the orthogonal trajectories of the family of curves $\frac{x^2}{a^2}+\frac{y^2}{b^2+\lambda}=1$, where λ is a parameter.

5 **Attempt all of the following:**

(a) Evaluate $\int_0^1\int_0^{\sqrt{1-x^2}}\int_0^{\sqrt{1-x^2-y^2}} \frac{dzdydx}{\sqrt{1-x^2-y^2-z^2}}$ by changing to spherical polar coordinates.

(b) i) The equation of the asymptote to the curve $y^2(x+a)=x^2(3a-x)$ is-------

ii) The Evaluation of $\iint r\,dr\,d\theta$ over the upper half part of cardioid gives-----------

iii) If $x^h y^k$ be an integrating factor of the differential equation $(2x^2y^2+y)dx+(3x-x^3y)dy=0$ then the values of h and k will be -------

12
4
16

$u=0$
$y^2a=0$
$y^2=0$
$y=0$

Government College of Engineering, Amravati

(An Autonomous Institute of Government of Maharashtra)

Second Semester B. Tech.

Summer – 2014

Course Code: SHU201

Course Name: Engineering Mathematics – II

Time: 2 Hrs. 30 Min. **Max. Marks: 60**

Instructions to Candidate

1) All questions are compulsory.
2) Assume suitable data wherever necessary and clearly state the assumptions made.
3) Diagrams/sketches should be given wherever necessary.
4) Use of logarithmic table, drawing instruments and non-programmable calculators is permitted.
5) Figures to the right indicate full marks.

1. **Attempt any Three:** **12**

(a) Evaluate $\int_0^{\pi/2} \frac{d\theta}{\sqrt{1-(1/2)\sin^2\theta}}$

(b) Prove that $\Gamma m\,\Gamma(m+1/2) = \frac{\sqrt{\pi}}{2^{2m-1}}\Gamma(2m)$

(c) Prove that $\int_0^{\infty} \frac{e^{-x}-e^{-ax}}{x\sec x}dx = \frac{1}{2}\log\left(\frac{a^2+1}{2}\right),\ a>0$ 4

(d) Trace the following curve with full justification $x(x^2+y^2)=a(x^2-y^2)$,a >0

2 **Attempt any Three:** **12**

(a) Solve: $(xy^2+2x^2y^3)dx+(x^2y-x^3y^2)dy=0$

(b) Solve: $y(x^3e^{xy}-y)dx+x(y+x^3e^{xy})dy=0$

4

(c) Solve: $\dfrac{dy}{dx}=\dfrac{y-x}{y-x+2}$

(d) Solve: $2xy\dfrac{dy}{dx}=y^2-2x^3,\ y(1)=2$

3 Attempt any Three: 12

(a) Obtain Fourier series expansion for the function defined as follows

$$f(x)=\begin{cases}\sin x, & 0\le x\le\pi\\ 0, & \pi\le x\le 2\pi\end{cases}$$

Hence deduce that $\dfrac{1}{1.3}+\dfrac{1}{3.5}+\dfrac{1}{5.7}+\ldots=\dfrac{1}{2}$

(b) Define even and odd function and use it to find the Fourier series expansion for the function $f(x)=9-x^2$ in the range $(-3,3)$.

(c) Find the half range sine series and half range cosine series to represent $f(x)=x-x^2$ in the range $(0,1)$

(d) Obtain the Fourier expansion of $\sqrt{1-\cos x}$ in the interval $0\le x\le 2\pi$.

4 Attempt any Three: 15

(a) Evaluate $\displaystyle\int_0^\infty\int_x^\infty \frac{e^{-y}}{y}\,dydx$

(b) Calculate the area included between the curve $r=a(\sec\theta+\cos\theta)$ and its asymptote.

(c) Find the volume bounded by the cylinder $x^2+y^2=4$ and the planes $y+z=4$ and $z=0$.

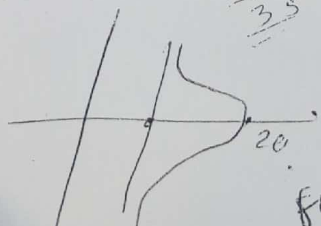
(d) By using the transformation $x+y=u,\ y=uv$, show that $\displaystyle\int_0^1\int_0^{1-x} e^{y/(x+y)}\,dydx=\frac{1}{2}(e-1)$

5 Attempt all of the following:

(a) Trace the curve $r^2=a^2\sin 2\theta$ with full justification.

(b) When a switch is closed in a circuit containing a battery E, a resistance R and an inductance L, the current i builds up at rate given by $L\dfrac{di}{dt}+Ri=E$. Find i as a function of t. How long will it be before the current has reached one half of its maximum value if $E=6\ Volts$, $R=100\ Ohms$ and $L=0.1\ henry$?

y=2
x=1



Government College of Engineering, Amravati

(An Autonomous Institute of Government of Maharashtra)

First Year B. Tech. (All)

Summer Term-2016

Course Code: SHU201

Course Name: Engineering Mathematics-II

Time: 2 Hrs. 30 Min. **Max. Marks: 60**

Instructions to Candidate

1) All questions are compulsory.
2) Assume suitable data wherever necessary and clearly state the assumptions made.
3) Diagrams/sketches should be given wherever necessary.
4) Use of logarithmic table, drawing instruments and non-programmable calculators is permitted.
5) Figures to the right indicate full marks.

1 Attempt any Three 12

A Show that $\frac{2^{2m-1}}{\sqrt{\pi}}\frac{\Gamma m}{\Gamma 2m}=\frac{1}{\Gamma\left(m+\frac{1}{2}\right)}$

B Evaluate $\int_0^{\pi/2}\frac{d\theta}{\sqrt{1-\frac{1}{2}\sin^2\theta}}$

C Trace the curve $r=a\left(1+\cos\theta\right)$

D Using differentiation under integral sign, evaluate $\int_0^\infty \frac{e^{-x}-e^{-ax}}{x\sec x}\,dx,\ a>0$

2 Attempt any Three 12

A Change the order of integration and evaluate $\int_0^2 \int_0^{x^2/4} xy\,dx\,dy$

B Evaluate $\iint xy(x+y)\,dx\,dy$ over the area bounded by parabolas $x^2=y$ and $y^2=-x$

C Find, by triple integration, the volume of a sphere of a radius a

D Evaluate $\int_0^1 \int_0^{\sqrt{1-x^2}} \int_0^{\sqrt{1-x^2-y^2}} \frac{dx\,dy\,dz}{\sqrt{1-x^2-y^2-z^2}}$

3 Attempt any Three 12

A Find the orthogonal trajectories of $x^2+y^2=2ax$.

B Solve $(y^3-2yx^2)\,dx+(2xy^2-x^3)\,dy=0$

C Solve $(2x-y+1)\,dx+(2y-x-1)\,dy=0$

D Solve $\frac{(1-x^2)}{xy}\frac{dy}{dx}=(1+x^2y^2)$

4 Attempt any Three 12

A Find the fourier series for $f(x)$ given by

$$f(x)=1+\frac{2x}{\pi},\quad -\pi<x<0$$
$$=1-\frac{2x}{\pi},\quad 0<x<\pi$$

Deduce that $\frac{1}{1^2}+\frac{1}{3^2}+\ldots=\frac{\pi^2}{8}$.

B Obtain the fourier series of $f(x)=e^{-x}$ in $-l<x<l$.

C If $f(x)=mx,\quad 0\le x\le\frac{\pi}{2}$
$$=m(\pi-x),\quad \frac{\pi}{2}\le x\le\pi$$
show that $f(x)=\frac{4m}{\pi}\left(\frac{\sin x}{1^2}-\frac{\sin 3x}{3^2}+\frac{\sin 5x}{5^2}+---\right)$

D Obtain the fourier series of $f(x)=x+x^2$ in $-\pi<x<\pi$.

5 Attempt the following 12

A Trace the curves (i) $a^2x^2=y^3(2a-y)$, (ii) $xy^2=a^2(a-x)$

B Find, by the triple integraton, the volume of the paraboloid of revolution $x=\sqrt{4z-y^2}$ cut by the plane $z=4$

Government College of Engineering, Amravati

(An Autonomous Institute of Government of Maharashtra)

First Year B. Tech. (All Branches)

Winter – 2016

Course Code: SHU201

Course Name: Engineering Mathematics – II

Time: 2 Hrs. 30 Min. **Max. Marks: 60**

Instructions to Candidate

1) All questions are compulsory.
2) Assume suitable data wherever necessary and clearly state the assumptions made.
3) Diagrams/sketches should be given wherever necessary.
4) Use of logarithmic table, drawing instruments and non-programmable calculators is permitted.
5) Figures to the right indicate full marks.

1. **Attempt any three :** **12**

(a) A resistance of 100 ohms, an inductance of 0.5 henry are connected in series with battery of 20 volts. Find the current in the circuit as a function of time.

(b) Solve :- $\frac{dy}{dx} = \frac{1+y^2+3x^2y}{1-2xy-x^3}$

(c) Find the orthogonal trajectory of $\frac{l}{r} = 1 + \cos\theta$, where l is the parameter.

(d) Solve $x^2 \frac{dy}{dx} = 3x^2 - 2xy + 1$

2. **Attempt the following :** 12

(a) Obtain the fourier series to represent

$f(x) = \frac{1}{4}(\pi - x)^2$,

$0 \le x \le 2\pi$

(b) Find a Fourier series to represent $f(x) = 9 - x^2$,

$-3 \le x \le 3$

12

3. **Attempt any three :**

(a) Evaluate $\int_0^1 \int_0^{\sqrt{1+x^2}} \frac{1}{1+x^2+y^2}\, dy\, dx$

(b) Change the order of integration and evaluate

$\int_0^3 \int_{y^2/9}^{\sqrt{10-y^2}} dy\, dx$

(c) Find the volume cut off from the sphere $x^2 + y^2 + z^2 = a^2$ by the cone $z^2 = x^2 + y^2$ Pg-11·35

(d) Using the transformations $u = x^2/y$, $v = y^2/x$ find $\iint xy\, dx\, dy$ over the area bounded by the four parabolas

$y^2 = 4ax,\ y^2 = 4bx,\ x^2 = 4cy,\ x^2 = 4dy$

4. **Attempt any three :**

(a) Using Gamma function evaluate $\int_0^1 (x \log x)^2\, dx$

(b) Evaluate $\int_0^{\pi/2} \frac{d\phi}{\sqrt{1 - \frac{1}{2}\sin^2\phi}}$

(c) Evaluate $\int_0^\infty \frac{e^{-\beta x}\sin \alpha x}{x} dx$ and hence deduce that

$\int_0^\infty \frac{\sin \alpha x}{x} dx = \frac{\pi}{2}$

(d) Trace the curve $y^2 a^2 = x^2(a^2 - x^2)$.

5. **Attempt the following :**

(a) Find by double integration the area included between the curves $y = -2x^2 + 4x + 7$ and $y = 3x^2 - x - 3$

(b) Find the Fourier series for $f(x) = x + x^2, -\pi < x < \pi$ Pg- 3-45 -unsolved

And hence deduce that

$\frac{\pi^2}{6} = 1 + \frac{1}{2^2} + \frac{1}{3^2} + \text{-------}$

Government College of Engineering, Amravati

(An Autonomous Institute of Government of Maharashtra)

Second Semester B. Tech.

Summer – 2017

Course Code: SHU201

Course Name: Engineering Mathematics II

Time: 2 Hrs. 30 Min. **Max. Marks: 60**

Instructions to Candidate

1) All questions are compulsory.
2) Assume suitable data wherever necessary and clearly state the assumptions made.
3) Diagrams/sketches should be given wherever necessary.
4) Use of logarithmic table, drawing instruments and non-programmable calculators is permitted.
5) Figures to the right indicate full marks.

ATTEMPT ANY THREE. **12**

a) By differentiating under integral sign find $\frac{dF}{d\alpha}$ and

$$\frac{d^2F}{d\alpha^2} \text{ if } F(\alpha) = \int_{1/\alpha}^{2/\alpha} \frac{\sin \alpha x}{x^2}\, dx, \ \alpha \neq 0.$$

b) Trace the polar curve $r^2 = a^2 \sin 2\theta$ with full justification.

c) Evaluate $\int_{-1}^{1} (1+x)^{p-1}(1-x)^{q}\, dx$ by using beta and gamma function.

d) Show that 1+x=

$$\int_a^b (x-a)^m (b-x)^n \, dx = (b-a)^{m+n+1} \beta(m+1,, n+1)$$

2. **ATTEMPT ANY THREE.** 12

a) Show that the area of the loop of the curve $r = a\cos n\theta$ is $\pi a^2/4n$. Also find the total area in case if n is odd and n is even.

b) Change the order of integration $\int_0^1 \int_{x^2}^{\sqrt{2-x^2}} f(x, y)\, dx\, dy$

c) Evaluate the double integration

$$\int_0^{a\sqrt{3}} \int_0^{\sqrt{x^2+a^2}} \frac{x}{a^2+x^2+y^2}\, dx\, dy$$

d) Change the integral $\int_0^1 \int_0^x (x,+y)\, dy\, dx$ to polar and hence evaluate.

3. **ATTEMPT ALL.** 12

a) Obtain a Fourier series to represent $x+x^2$ from $x=-\pi$ to $x=\pi$.

b) Obtain a Fourier series to represent $f(x)=e^{-x}$ from $x=-l$ to $x=l$.

c) Find a Fourier cosine series for the function $f(x)=\sin x$ for $0<x<\pi$.

4. **ATTEMPT ANY THREE.** 12

a) Find the integrating factor for the differential equation $\frac{dy}{dx}+x\sin 2y = x^3\cos^2 y$. Use it to solve the differential equation.

Diued Cos²y

b) Show that the family of parabolas $x^2 = 4a(y+a)$ is self orthogonal.

c) Solve $y(xy+2x^2y^2)dx + x(xy-x^2y^2)dy = 0$.

d) Show that the family of cardioids $r = a(1-\cos\theta)$ is not self orthogonal.

5. **ATTEMPT ALL.** 12

a) The equations of electromotive force in terms of current i for an electrical circuit having resistance R and a condenser of capacity C, in series is $E = Ri + \int \frac{i}{C} dt$ Find the current i at any time t, when $E = E_0 \sin \omega t$

b) Evaluate the integral $\int_0^1 \int_0^{\sqrt{1-x^2}} \int_0^{\sqrt{1-x^2-y^2}} \frac{dz\,dy\,dx}{\sqrt{1-x^2-y^2-z^2}}$ by changing to spherical polar coordinates.

c) Trace the polar curve $x^{2/3}+y^{2/3}=a^{2/3}$ with full justification.

ATTEMPT ALL. 12

a) Show that the differential equation for current i in an electrical circuit having resistance R and a inductance L in series and acted on by an electromotive forcs $E\sin\omega t$ satisfies the equation $L\dfrac{di}{dt}+R\,i=E\sin\omega t$. Find the current i at any time t , if initially there is no current in the circuit.

b) Find the volume of the sphere of radius a by using triple integration.

c) Trace the curve $r=a\left(\sec\theta+\tan\theta\right)$ with full justification.

$\iiint_{0}^{a} dr.d\theta.d\phi$

8.

Government College of Engineering, Amravati

(An Autonomous Institute of Government of Maharashtra)

Second Semester B. Tech. (All Branches)

Summer Term – 2017

Course Code: SHU 201

Course Name: Engineering Mathematics II

Time: 2 Hrs. 30 Min. Max. Marks: 60

Instructions to Candidate

1) All questions are compulsory.
2) Assume suitable data wherever necessary and clearly state the assumptions made.
3) Diagrams/sketches should be given wherever necessary.
4) Use of logarithmic table, drawing instruments and non-programmable calculators is permitted.
5) Figures to the right indicate full marks.

1. ATTEMPT ANY THREE. 12

a) Evaluate $\int_0^\infty e^{-\left(x^2+\frac{a^2}{x^2}\right)}dx$ by differentiating under integral sign and using $\int_0^\infty e^{-x^2}dx=\frac{\sqrt{\pi}}{2}$ if necessary.

b) Define asymptotes.
Trace the Cartesian curve $y^2(2a-x)=x^3$ with full justification.

Contd..

c) Show that $\int_0^{\pi/2} \sin^p x \, dx . \int_0^{\pi/2} \sin^{p+1} x \, dx = \frac{\pi}{2(p+1)}$.

d) Show that $y.\beta(x+1,\ y) = x\beta(x,,\ y+1)$

2. ATTEMPT ANY THREE. 12

a) Find the area inside the cardioids $r = a\ (1+\cos\theta)$ and outside the circle $r = 2a\cos\theta$.

b) Express the following integral as a single integral and hence evaluate

$$\int_0^{a/\sqrt{2}} \int_0^{x} x\, dx\, dy + \int_{a/\sqrt{2}}^{a} \int_0^{\sqrt{a^2-x^2}} x\, dx\, dy.$$

c) Evaluate $\iint_R x^2 dxdy$ where R is the region in the first quadrant bounded by the hyperbola $xy = 16$ and the lines $y = x,\ y = 0,\ x = 8$.

d) Change the integral $\int_0^2 \int_0^{\sqrt{2x-x^2}} (x^2+y^2)\, dx\, dy$ to polar and hence evaluate.

3. ATTEMPT ALL. 12

a) Obtain a Fourier series to represent $x - x^2$ from $x = -\pi$ to $x = \pi$.

b) Obtain a Fourier series to represent the periodic

$f(x) = 1$ when $\pi < x < 2\pi$.

c) Find a Fourier sine series for the function $f(x) = e^{ax}$ for $0 < x < \pi$ where a is a constant.

4. ATTEMPT ANY THREE. 1

a) Find Integrating factor for the differential equation $(xy^3 + y)dx + 2(x^2y^2 + x + y^4)dy = 0$ Use it to solve the differential equation for $x = 0,\ y = 0$.

b) Find differential equation for the family of parabolas $x^2 = 4a\ (y+a)$ and its orthogonal trajectory. Hence show that the family of parabolas $x^2 = 4a\ (y+a)$ is self orthogonal.

c) Solve the differential equation $(x+2y)(dx-dy) = (dx+dy)$ by using reducible to homogeneous method.

d) Solve the differential equation

$$y - \cos x \frac{dy}{dx} = y^2(1-\sin x)\cos x$$, given that

$y = 2$

when $x = 0$.

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Second Semester B. Tech.

Summer – 2017

Course Code: SHU201

Course Name: Engineering Mathematics II

Time: 2 Hrs. 30 Min. **Max. Marks: 60**

Instructions to Candidate

1) All questions are compulsory.
2) Assume suitable data wherever necessary and clearly state the assumptions made.
3) Diagrams/sketches should be given wherever necessary.
4) Use of logarithmic table, drawing instruments and non-programmable calculators is permitted.
5) Figures to the right indicate full marks.

1. ATTEMPT ANY THREE. 12

a) By differentiating under integral sign find $\frac{dF}{d\alpha}$ and $\frac{d^2F}{d\alpha^2}$ if $F(\alpha)=\int_{1/\alpha}^{2/\alpha}\frac{\sin\alpha x}{x^2}\,dx,\ \alpha\neq 0.$

b) Trace the polar curve $r^2=a^2\sin 2\theta$ with full justification.

c) Evaluate $\int_{-1}^{1}(1+x)^{p-1}(1-x)^{q}\,dx$ by using beta and gamma function.

d) Show that

Contd..

$$\int_a^b (x-a)^m (b-x)^n \, dx = (b-a)^{m+n+1} \beta(m+1,, n+1)$$

12

2. **ATTEMPT ANY THREE.**

a) Show that the area of the loop of the curve $r = a \cos n\theta$ is $\pi a^2 / 4n$. Also find the total area in case if n is odd and n is even.

b) Change the order of integration $\int_0^1 \int_{x^2}^{\sqrt{2-x^2}} f(x, y) \, dx \, dy$

c) Evaluate the double integration

$$\int_0^{a\sqrt{3}} \int_0^{\sqrt{x^2+a^2}} \frac{x}{a^2+x^2+y^2} \, dx \, dy$$

d) Change the integral $\int_0^1 \int_0^x (x,+y) \, dy \, dx$ to polar and hence evaluate.

12

3. **ATTEMPT ALL.**

a) Obtain a Fourier series to represent $x+x^2$ from $x=-\pi$ to $x=\pi$.

b) Obtain a Fourier series to represent $f(x)=e^{-x}$ from $x=-l$ to $x=l$.

c) Find a Fourier cosine series for the function $f(x)=\sin x$ for $0<x<\pi$.

12

4. **ATTEMPT ANY THREE.**

a) Find the integrating factor for the differential equation $\frac{dy}{dx}+x\sin 2y = x^3\cos^2 y$. Use it to solve the differential equation.

b) Show that the family of parabolas $x^2 = 4a(y+a)$ is self orthogonal.

c) Solve $y(xy+2x^2y^2)dx + x(xy-x^2y^2)dy = 0$.

d) Show that the family of cardioids $r = a(1-\cos\theta)$ is not self orthogonal.

12

5. **ATTEMPT ALL.**

a) The equations of electromotive force in terms of current i for an electrical circuit having resistance R and a condenser of capacity C, in series is $E = Ri + \int \frac{i}{C} dt$ Find the current i at any time t,

when $E = E_0 \sin \omega t$

b) Evaluate the integral

$$\int_0^1 \int_0^{\sqrt{1-x^2}} \int_0^{\sqrt{1-x^2-y^2}} \frac{dz\,dy\,dx}{\sqrt{1-x^2-y^2-z^2}}$$ by changing to spherical polar coordinates.

c) Trace the polar curve $x^{2/3}+y^{2/3}=a^{2/3}$ with full justification.

b) A sinusoidal voltage $E\sin\omega t$ is passed through a half wave rectifier which clips the negative portion of the wave. Find the Fourier series of the resulting periodic function

$$f(t) = \begin{cases} 0 & , -T/2 < t < 0 \\ E\sin\omega t & , 0 < t < T/2 \end{cases} \quad \text{where } T = \frac{2\pi}{\omega}.$$

c) Find the half range sine & cosine series.

If $f(t) = \begin{cases} t & , 0 < t < 1 \\ 2-t & , 1 < t < 2 \end{cases}$.

Bharat Makhija's Coaching Classes
Batches for B.E. / B-Tech
M1 M2 M3 TOC
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Government College of Engineering, Amravati

(An Autonomous Institute of Government of Maharashtra)

First Year B. Tech.

Summer – 2016

Course Code: SHU201

Course Name: Engineering Mathematics II

Time: 2 Hrs. 30 Min. **Max. Marks: 60**

Instructions to Candidate

1) All questions are compulsory.
2) Assume suitable data wherever necessary and clearly state the assumptions made.
3) Diagrams/sketches should be given wherever necessary.
4) Use of logarithmic table, drawing instruments and non-programmable calculators is permitted.
5) Figures to the right indicate full marks.

1. **Attempt any three of the following.** (12)

a) For $c > 1$, Prove that $\int_0^\infty \frac{x^c}{c^x}dx = \frac{\Gamma(c+1)}{(\log c)^{c+1}}$.

b) Prove that $\Gamma n \Gamma(1-n) = \frac{\pi}{\sin n\pi}$, if $\int_0^\infty \frac{x^{n-1}}{1+x}dx = \frac{\pi}{\sin n\pi}$.

Hence deduce that $\Gamma \frac{1}{2} = \sqrt{\pi}$.

c) Trace the curve $r = a\sin 3\theta$.

d) Trace the curve $x^3 + y^3 = 3axy$.

e) If $\left|\frac{b}{a}\right| < 1$, Show that

$$\int_0^{\pi/2} \frac{1}{\sin\theta} \log\left(\frac{a+b\sin\theta}{a-b\sin\theta}\right) d\theta = \pi \sin^{-1}\left(\frac{b}{a}\right).$$

2. Attempt any three of the following. (12)

a) Evaluate $\int_0^1 \int_0^{\sqrt{2-y^2}} \frac{y\,dx\,dy}{\sqrt{(2-x^2)(1-x^2y^2)}}$.

b) Evaluate $\iint_R \frac{(x^2+y^2)^2}{x^2y^2} dx\,dy$ where R is common to $x^2+y^2 = ax$, $x^2+y^2 = by$, $a > b > 0$.

c) Find the volume of solid bounded by the sphere $x^2+y^2+z^2 = 4$ and the paraboloid $x^2+y^2 = 3z$.

d) Evaluate $\int_0^\infty \int_0^\infty \int_0^\infty \frac{dx\,dy\,dz}{(1+x^2+y^2+z^2)^2}$.

3. Solve any three differential equations. (12)

a) Evaluate $\frac{dy}{dx} = \frac{-(x^2y+y^4)}{x(2x^2+4y^3)}$.

b) Evaluate $\frac{dy}{dx} + \frac{(2x+3y)}{(y+2)} = 0$.

c) Evaluate $(xy^2 + e^{-1/x^3})dx - x^2y\,dy = 0$

d) Evaluate $(x+\tan y)dy = \sin 2y\,dx$

e) Find the constant n such that $(x+y)^n$ is an integrat of $(4x^2+2xy+6y)dx + (2x^2+9y+3x)dy = 0$ & he the equation.

4. Attempt any two of the following.

a) Find the Orthogonal Trajectory of $x\sin x\cosh y - y\sinh y\cos x = 0$.

b) In a circuit containing inductance L, resistance R voltage E, the current I is given by $E = RI + L\frac{d}{d}$ L=640 H, R=250Ω & E=500 V. Current being ze $t = 0$. Find the time that elapses, before it reache its maximum value.

c) A constant EMF E Volts is applied to a circuit c a constant resistance R Ohms in a series & a cons inductance L henries. If the initial current is zero that the current builds up to its theoretical maxim $L\log 2/R$ seconds.

5. Attempt any two of the following.

a) Determine the Fourier series for the function $f(x) = \sqrt{1-\cos x}$ in the interval $0 \le x \le 2\pi$ & deduce that $\sum_{n=1}^{\infty} \frac{1}{4n^2-1} = \frac{1}{2}$.