

Government College of Engineering, Amravati
(An Autonomous Institute of Government of Maharashtra)

Third Semester B. Tech. (CS / IT)

Winter – 2014

Course Code: SHU304

Course Name: Engineering Mathematics – III

Time: 2 Hrs. 30 Min.

Max. Marks: 60

Instructions to Candidate

- 1) All questions are compulsory.
- 2) Assume suitable data wherever necessary and clearly state the assumptions made.
- 3) Diagrams/sketches should be given wherever necessary.
- 4) Use of logarithmic table, drawing instruments and non-programmable calculators is permitted.
- 5) Figures to the right indicate full marks.

1. Attempt any three: 12

(a) Prove that Particular integral of $(D - m)y = f(x)$ is $e^{mx} \int e^{-mx} f(x) dx$ and hence find the same of the differential equation $(D^2 + 2D + 1)y = e^{-x}$

(b) Solve : $(D^4 - 2D^3 - 3D^2 + 4D + 4)y = x^2 e^x$

(c) Solve: $x^2 \frac{d^2y}{dx^2} - 2x \frac{dy}{dx} - 4y = x^2 + 2 \log x$

(d) Solve by method of variation of parameters the equation $y'' - 2y' + 2y = e^x \tan x$. (C)

2. Attempt any three: 12

(a) i) Solve $p + q = \sin x + \sin y$

Cont.

- ii) Solve $z = px + qy + \sqrt{1 + p^2 + q^2}$
- (b) Solve: $(x - y)(px - qy) = (p - q)^2$

- (c) Using method of separation of variables, solve
 $3\frac{\partial u}{\partial x} + 2\frac{\partial u}{\partial y} = 0, u(x, 0) = 4e^{-x}$
- (d) Solve $(x^2 - yz)p + (y^2 - zx)q = z^2 - xy$

3. Attempt any three:

- (a) Using first shifting theorem prove that

$$L\{t \cos at\} = \frac{s^2 - a^2}{(s^2 + a^2)^2}$$

- (b) Evaluate

i) $\int_0^\infty te^{-2t} \sin t dt$ ii) $L\left\{\int_0^t \frac{e^s \sin s}{s} ds\right\}$ ✓ ✓

- (c) Find the inverse Laplace transform of

i) $\frac{2s^2 - 6s + 5}{s^3 - 6s^2 + 11s - 6}$ ii) $\tan^{-1}(2/s)$

- (d) Define Unit impulse function, Heaviside unit step function and find the relation between them

- (e) State Convolution theorem and verify it for the pair of functions $f(t) = t, g(t) = e^{at}$. ✓

4. Attempt any three:

12

- (a) If directional derivative of $\phi = ax^2y + by^2z + cz^2x$ at $(1,1,1)$ has maximum magnitude 15 in the direction parallel to $\frac{x-1}{2} = \frac{y-3}{-2} = \frac{z}{1}$. Find a,b,c.
- (b) Show that $\bar{F} = (6xy + z^3)i + (3x^2 - z)j + (3xz^2 - y)k$ is irrotational. Find scalar ϕ such that $\bar{F} = \nabla\phi$.

(c) Show that $\bar{F} = \frac{\bar{a} \times \bar{r}}{r^n}$ is solenoidal field, where $r = |\bar{r}|$, $\bar{r} = xi + yj + zk$ and \bar{a} be the constant vector.

(d) Find the work done in moving a particle once round the ellipse $\frac{x^2}{25} + \frac{y^2}{16} = 1, z = 0$ under the field of force given by

$$\bar{F} = (2x - y + z)i + (x + y - z^2)j + (3x - 2y + 4z)k$$

Is the field conservative?

Attempt the following:

(a) If $\bar{r} \times \frac{d\bar{r}}{dt} = 0$, show that \bar{r} has a constant direction.

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(b) The small oscillations of a certain system with two degrees of freedom are given by the equations

$$D^2x + 3x - 2y = 0$$

$$D^2y + D^2x - 3x + 5y = 0$$

Where $D = \frac{d}{dt}$. If $x = 0, y = 0, Dx = 3, Dy = 2$

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when $t = 0$. Find x and y when $t = \pi/2$

(c) Given $L\{J_0(t)\} = \frac{1}{\sqrt{s^2 + 1}}$, then

i) $\int_0^\infty J_0(t)dt = \dots$

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ii) $\int_0^\infty e^{-t} J_0(t)dt = \dots$

Government College of Engineering, Amravati
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Third Semester B. Tech. (CS / IT)

Winter – 2016

Course Code: SHU304

Course Name: Engineering Mathematics-III

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Max. Marks: 60

Instructions to Candidate

- 1) All questions are compulsory.
- 2) Assume suitable data wherever necessary and clearly state the assumptions made.
- 3) Diagrams/sketches should be given wherever necessary.
- 4) Use of logarithmic table, drawing instruments and non-programmable calculators is permitted.
- 5) Figures to the right indicate full marks.

1. Attempt any three: 12

- (a) State the formula for particular integral of the differential equation $\phi(D)y = e^{ax} V$ and hence use it to find the same for $(D^2 + 2D - 3)y = e^x \cos x$
- (b) Solve : $\frac{d^2y}{dx^2} - 4\frac{dy}{dx} + 4y = \sin^2 x$
- (c) Solve: $x^2 \frac{d^2y}{dx^2} - 2x \frac{dy}{dx} - 4y = x^2 + 2 \log x$
- (d) Solve by method of variation of parameters the equation $y'' - y = e^{-2x} \cos(e^{-x})$.

2. Attempt any three: 12

- (a) i) Solve $p^2 + q = x + \sin y$

Cont.

- ii) Solve $\sqrt{1+p^2-q^2} = 0$
 (b) Solve: $pqz = p^2(xq + p^2) + q^2(yp + q^2)$

(c) Using method of separation of variables, solve

$$3 \frac{\partial u}{\partial x} + 2 \frac{\partial u}{\partial y} = 0, u(x, 0) = 4e^{-x}$$

- (d) Solve $(x^2 - yz)p + (y^2 - zx)q = z^2 - xy$

3. **Attempt any three:** State first shifting theorem. Find the Laplace transform of $\sinh 3t \cos^2 t$ 12

- (a) Find the Laplace transform of $\frac{1-\cos t}{t}$ and hence

$$\text{find } L\left(\frac{1-\cos t}{t^2}\right).$$

- (c) Find the Laplace transform of the triangular wave given by

$$f(t) = \begin{cases} t, & 0 < t < a \\ 2a-t, & a < t < 2a \end{cases} \quad \text{where } f(t+2a) = f(t).$$

- (d) Find the inverse Laplace transform of

$$\text{i) } \frac{s^2 - 10s + 13}{(s+1)(s^2 - 5s + 6)} \quad \text{ii) } \log\left(1 - \frac{a^2}{s^2}\right)$$

Attempt any three:

12

4.

- (a) Show that

$\bar{F} = (y^2 \cos x + z^3)i + (2y \sin x - 4)j + (3xz^2 + 2)k$ is a conservative vector field and find a function ϕ such that $\bar{F} = \nabla \phi$.

- (b) Find the directional derivative of $\phi = x^2y + 2y^2z + 3z^2x$ at $(1, 1, 1)$ in the direction

parallel to $\frac{x}{2} = \frac{y}{-2} = \frac{z}{1}$.

(c) Show that $\nabla^2 \left[\nabla \cdot \left(\frac{\bar{r}}{r^4} \right) \right] = \frac{-12}{r^6}$

- (d) Find the work done in moving a particle once around the circle C in the x-y plane if the circle has centre at the origin and radius 3 and the force field $\bar{F} = (2x - y + z)i + (x + y - z^2)j + (3x - 2y + 4z)k$

5. **Attempt the following:**

12

- (a) Prove that $\bar{r} \cdot \frac{d\bar{r}}{dt} = 0$ iff \bar{r} has a constant magnitude.

- (b) Solve the following differential equation using Laplace transform

$$\frac{d^2y}{dt^2} - 6\frac{dy}{dt} + 9y = t^2 e^{3t} \text{ with } y(0) = 2, y'(0) = 6$$

- (c) If r and \bar{r} have their usual meanings then prove that

i) $\nabla \cdot (r^n \bar{r}) = (n+3)r^n$

ii) $\nabla^2 r^n = n(n+1)r^{n-2}$

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Max Marks: 60

Instructions to Candidate:

- 1) All questions are compulsory.
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- 4) Use of logarithmic table, drawing instruments and non-programmable calculator is permitted.
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1. Attempt any Three:

12

(a) Solve: $\frac{d^2y}{dx^2} - 6\frac{dy}{dx} + 9y = e^{3x} \csc^2 x + 5^x$

(b) Solve by method of variation of parameters

$$\frac{d^2y}{dx^2} - 4\frac{dy}{dx} + 4y = e^{2x} \sec^2 x$$

(c) Solve:

$$(3x+2)^2 \frac{d^2y}{dx^2} + 3(3x+2) \frac{dy}{dx} - 36y = 3x^2 + 4x + 1$$

(d) Solve: $(D^3 - 4D)y = 2 \cosh^2 2x$

2. Attempt any Three:

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(a) Solve $z(p^2 + q^2) = x^2 + y^2$

(b) Solve $(x+y)(p+q)^2 + (x-y)(p-q)^2 = 1$

(c) Find a solution of the equation $\frac{\partial^2 u}{\partial x^2} = \frac{\partial u}{\partial y} + 2u$ in

Contd..

the form $u = f(x)g(y)$ subject to the conditions
 $u = 0$,

$$\frac{\partial u}{\partial x} = 1 + e^{-3y} \text{ when } x = 0 \text{ for all values of } y.$$

- (d) Solve: $(\cos(x+y))p + (\sin(x+y))q = z$

3. Attempt any Three :

- (a) Use convolution theorem to find inverse Laplace transform of

$$\bar{f}(s) = \frac{s}{s^4 + 5s^2 + 4}$$

- (b) Solve using Laplace transform,

$$\frac{dx}{dt} - y = e^t, \quad \frac{dy}{dt} + x = \sin t.$$

Given that $x(0) = 1, y(0) = 0$

- (c) Solve $\frac{dy}{dt} + 2y + \int_0^t y dt = \sin t$, given $y(0) = 1$

- (d) Define Heaviside's unit step function and use it to find the Laplace transform of the function

$$f(t) = \begin{cases} t-1, & \text{if } 1 < t < 2 \\ 3-t, & \text{if } 2 < t < 3 \end{cases}$$

4. Attempt any Three:

- (a) For what values of n the vector $r^{(n+1)}\bar{r}$ is solenoidal and irrotational?

$$\nabla \times \bar{r} = \{ \dots \}$$

- (b) If $\bar{r} = xi + yj + zk$ and $r = |\bar{r}|$, prove that $\nabla^2 \times \bar{r} = \frac{1}{r^2} \bar{r}$

$$\nabla^2 f(r) = f''(r) + \frac{2}{r} f'(r) \text{ where } \text{and hence find}$$

the value of $\nabla^2(\log r)$

- (c) Find the workdone in moving a particle once

12

12

round the ellipse $\frac{x^2}{16} + \frac{y^2}{4} = 1, z = 0$ under the field of force

$$\vec{F} = (2x - y + z)i + (x + y - z^2)j + (3x - 2y + 4z)k$$

- (d) The water level at a point (x, y, z) in the ground is given by $W(x, y, z) = x^2 + y^2 - z$. A machine located at $(1, 1, 1)$ desires to move in such a direction that it will get water maximum. In what direction should it move? What should be the maximum water level?

5. Attempt the following:

- (a) Prove that

$$y = c_1 \cos 3x + c_2 \sin 3x + \frac{1}{3} \left[x \sin 3x - \frac{1}{3} \cos 3x \cdot \log(\sec 3x) \right]$$

is the general solution of the differential equation

$$(D^2 + 9)y = \sec 3x.$$

- (b) Prove that i) $\nabla^2 \left[\nabla \cdot \left(\frac{\bar{r}}{r^4} \right) \right] = -\frac{12}{r^6} = v^2 + v''$

ii) Solve: $(1 - y^2)xq^2 + y^2 p = 0$

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Max Marks: 60

Instructions to Candidate:

- 1) All questions are compulsory.
- 2) Assume suitable data wherever necessary.
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- 4) Use of logarithmic table, drawing instruments and non-programmable calculator is permitted.
- 5) Figures to the right indicate full marks.

1. Attempt any Three: 12

(a) Solve: $\frac{d^2y}{dx^2} - 6\frac{dy}{dx} + 9y = e^{3x} \cos ec^2 x + 5^x$

(b) Solve by method of variation of parameters

$$\frac{d^2y}{dx^2} - 4\frac{dy}{dx} + 4y = e^{2x} \sec^2 x$$

(c) Solve:

$$(3x+2)^2 \frac{d^2y}{dx^2} + 3(3x+2) \frac{dy}{dx} - 36y = 3x^2 + 4x + 1$$

(d) Solve: $(D^3 - 4D)y = 2 \cosh^2 2x$

2. Attempt any Three: 12

(a) Solve $z(p^2 + q^2) = x^2 + y^2$

(b) Solve $(x+y)(p+q)^2 + (x-y)(p-q)^2 = 1$

(c) Find a solution of the equation $\frac{\partial^2 u}{\partial x^2} = \frac{\partial u}{\partial y} + 2u$ in

Contd..

the form $u = f(x)g(y)$ subject to the conditions

$$u = 0,$$

$$\frac{\partial u}{\partial x} = 1 + e^{-3y} \text{ when } x = 0 \text{ for all values of } y.$$

- (d) Solve: $(\cos(x+y))p + (\sin(x+y))q = z$

3. Attempt any Three : 12

- (a) Use convolution theorem to find inverse Laplace transform of

$$\bar{f}(s) = \frac{s}{s^4 + 5s^2 + 4}$$

- (b) Solve using Laplace transform,

$$\frac{dx}{dt} - y = e^t, \quad \frac{dy}{dt} + x = \sin t.$$

Given that $x(0) = 1, y(0) = 0$

- (c) Solve $\frac{dy}{dt} + 2y + \int_0^t y dt = \sin t$, given $y(0) = 1$

- (d) Define Heaviside's unit step function and use it to find the Laplace transform of the function

$$f(t) = \begin{cases} t-1, & \text{if } 1 < t < 2 \\ 3-t, & \text{if } 2 < t < 3 \end{cases}$$

4. Attempt any Three: 12

- (a) For what values of n the vector $r^{(n+1)}\bar{r}$ is solenoidal and irrotational?

- (b) If $\bar{r} = xi + yj + zk$ and $r = |\bar{r}|$, prove that

$$\nabla^2 f(r) = f''(r) + \frac{2}{r} f'(r) \text{ where and hence find the value of } \nabla^2(\log r)$$

- (c) Find the workdone in moving a particle once

round the ellipse $\frac{x^2}{16} + \frac{y^2}{4} = 1$, $z = 0$ under the

field of force

$$\overline{F} = (2x - y + z)i + (x + y - z^2)j + (3x - 2y + 4z)k$$

- (d) The water level at a point (x, y, z) in the ground is given by $W(x, y, z) = x^2 + y^2 - z$. A machine located at $(1, 1, 1)$ desires to move in such a direction that it will get water maximum. In what direction should it move? What should be the maximum water level?

5. Attempt the following:

12

- (a) Prove that

$$y = c_1 \cos 3x + c_2 \sin 3x + \frac{1}{3} \left[x \sin 3x - \frac{1}{3} \cos 3x \cdot \log(\sec 3x) \right]$$

is the general solution of the differential equation

$$(D^2 + 9)y = \sec 3x.$$

- (b) Prove that i) $\nabla^2 \left[\nabla \cdot \left(\frac{\bar{r}}{r^4} \right) \right] = -\frac{12}{r^6}$

ii) Solve: $(1 - y^2)xq^2 + y^2 p = 0$

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Government College of Engineering, Amravati
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Third Semester B. Tech. (CS / IT)

Summer – 2016

Course Code: SHU304

Course Name: Engineering Mathematics III

Time: 2 hrs.30 min.

Max Marks: 60

Instructions to candidate:

- 1) All questions are compulsory.
- 2) Assume suitable data wherever necessary and clearly state the assumptions made.
- 3) Diagrams/sketches should be given wherever necessary.
- 4) Use of logarithmic table, drawing instruments and nonprogrammable calculators is permitted.
- 5) Figures on the right indicate full marks.

1. Attempt any three: 12

- (a) Solve: $(D^3 + 1)y = \sin 3x - \cos^2 \frac{x}{2}$
- (b) Solve: $\frac{d^2y}{dx^2} + 4y = x \sin x$
- (c) Solve: $\frac{d^2y}{dx^2} + y = -\cot x$ using method of variation of parameters.
- (d) Solve: $x^2 \frac{d^3y}{dx^3} + 3x \frac{d^2y}{dx^2} + \frac{dy}{dx} - x^{-1}y = 3 - 7x^{-1}$

2. Attempt any three: 12

- (a) Solve: $pq = x^m y^n z^{2l}$
- (b) Solve: $(1 - y^2)xq^2 + y^2 p = 0$
- (c) Solve: $(2z - y)p + (x + z)q + (2x + y) = 0$
- (d) Use method of separation of variable and show that the solution of the heat equation

Cont.

$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$ which satisfies the conditions

$u(0, y) = u(l, y) = u(x, 0) = 0$ is

$$u(x, y) = C \sin \frac{n\pi x}{l} \left(e^{\frac{n\pi y}{l}} - e^{-\frac{n\pi y}{l}} \right). \text{ Use } k = -n^2$$

Attempt any two:

3.

- (a) If $\bar{f}(s) = \frac{s^2 - 3}{(s+2)(s-3)(s^2 + 2s + 5)}$ find inverse Laplace transform.

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- (b) Using convolution theorem, find

$$L^{-1} \left\{ \frac{(s+2)^2}{(s^2 + 4s + 8)^2} \right\}$$

- (c) If $f(t) = \begin{cases} \frac{t}{a}, & 0 < t < a \\ \frac{1}{a}(2a-t), & a < t < 2a \end{cases}$

and $f(t) = f(t+2a)$. Find $L\{f(t)\}$.

4.

Attempt any three:

15

- (a) A particle moves on the curve $x = 2t^2$, $y = t^2 - 4t$, $z = 3t - 5$, where t is the time. Find the component of velocity and acceleration at time $t=1$ in the direction $i-3j+2k$.

- (b) Find the angle between the surfaces $x^2 + y^2 + z^2 = 9$, $x^2 + y^2 - z = 3$ at point $(2, -1, 2)$.

- (c) Define solenoidal field and conservative field. If $\bar{F} = \text{grad}(x^3 + y^3 + z^3 - 3xyz)$. Prove that \bar{F} is solenoidal vector at origin and it is conservative

everywhere in the space.

- (d) A vector field is given by
 $\bar{F} = (\sin y)i + x(1 + \cos y)j$. Evaluate the line integral
over a circular path $x^2 + y^2 = a^2$, $z = 0$

5. **Fill in the blanks:** 12

- 09 (a) A vector
 $\bar{F} = (x + 2y + az)i + (bx - 3y - z)j + (4x + cy + 2z)k$
is irrotational. Then constants
 $a = \dots$, $b = \dots$, $c = \dots$
The solution of partial differential equation
(b) $pq = p + q$ is given as-----
- (c) The solution of partial differential equation
 $p^2 + q^2 = z$ is given as-----
Laplace transform of $f(t)$ be $\log \left\{ \frac{(s+b)}{(s+a)} \right\}$. Then
(d) function $f(t) = \dots$

15 If $1 * 1 = t$ then $1 * 1 * 1 * \dots n \text{ times}$ results -----

- (e) The particular integral of the differential equation
(f) $(D^2 + 6D + 9)y = x^{-3}e^{-3x}$ is-----