Government College of Engineering, Amravati

(An Autonomous Institute of Government of Maharashtra)

Fourth Semester B. Tech. (EE / ET / IN)

Summer – 2014

Course Code: SHU401

Course Name: Engineering Mathematics-IV

Time: 2 Hrs. 30 Min. **Max. Marks: 60**

Instructions to Candidate

1) All questions are compulsory.
2) Assume suitable data wherever necessary and clearly state the assumptions made.
3) Diagrams/sketches should be given wherever necessary.
4) Use of logarithmic table, drawing instruments and non-programmable calculators is permitted.
5) Figures to the right indicate full marks.

1. ATTEMPT ANY FOUR **12**

a) Prove that $\int_0^{2\pi} \frac{\sin^2\theta}{a+b\cos\theta}\,d\theta = \frac{2\pi}{b^2}\left(a-\sqrt{a^2-b^2}\right)$, where $0 < b < a$.

b) State residue theorem. Also find the sum of the residues of the function $f(z) = \frac{\sin z}{z\cos z}$ at its poles inside the circle $|z| = 2$.

c) Find all possible Laurentz series of $f(z) = \frac{1}{z^2+1}$, about all its singular points. Also determine the region of convergence.

Contd..

Contd..

d) Evaluate $\int_C Re(z)dz$, where C is
(i) the shortest path from $z = 1 + i$ to $z = 3 + 2i$ (ii) along the straight line from $(1, 1)$ to $(3, 1)$ and then from $(3, 1)$ to $(3, 2)$.
Are the two integrals in (i) and (ii) are equal.? If not give reason.

e) Write Taylor's series (theorem). Use it to find the expansion of $f(z) = \frac{1}{4-3z}$ about the point $z_0 = 1 + i$.
Also determine the region of convergence.

2. ATTEMPT ANY FOUR **12**

a) Let a mapping $T: V_2 \to V_2$ be defined by $T(x, y) = (x', y')$, where $x' = x\cos\theta - y\sin\theta$ and $y' = x\sin\theta + y\cos\theta$. Show that T is a linear map.

b) Given that $\{(1, 0, 1), (1, 1, 0), (1, 1, 1)\}$ is a basis of $V_3(R)$. Find out the coordinates of $(\alpha, \beta, \gamma) \in V_3(R)$ with respect to the above basis.

c) Given that $S = \{(1,-1, 0), (1, 0, 2)\}$ and $T = \{(0,1, 0), (0, 1, 2)\}$. Determine the subspaces $S \cap T$ and $S + T$. Find dimension of $S \cap T$ and $S + T$.

d) Prove that $B = \{(1, 0, 0), (1, 1, 0), (1, 1, 1)\}$ is a basis of V_3.

e) Define Inner product. Use it to prove that $(u, \alpha v + \beta w) = \bar{\alpha}(u, v) + \bar{\beta}(u, w)$, where V is a inner product space, $u, v \in V$ and $\alpha, \beta \in F$.

3. ATTEMPT ANY TWO **12**

a) In $F^{(n)}$ define for

$u = (\alpha_1, \alpha_2, ---, \alpha_n)$, $v = (\beta_1, \beta_2, ---, \beta_n)$
and $(u, v) = (\alpha_1 \overline{\beta_1}, \alpha_2 \overline{\beta_2}, ---, \alpha_n \overline{\beta_n})$.
Show that this defines an inner product.

b) Prove that M the set of all $m \times n$ matrices of real numbers (or complex numbers) is a vector space over R (or C) under addition of matrices and scalar multiplication to a matrix.

c) Find a linear transformation T from V_2 to V_2 such that $T(1, 0) = (1, 1)$ and $T(0, 1) = (-1, 2)$. Prove that T maps the square with vertices $(0, 0)$, $(1, 0)$, $(1, 1)$ and $(0, 1)$ into a pallelogram.

12

12

4. ATTEMPT ANY THREE

a) Five defective bulbs are accidently mixed with twenty good ones. It is not possible to just look at a bulb and tell whether or not it is defective. Find the probability distribution of the number of defective bulbs, if four bulbs are drawn at random from this lot.

b) Out of 800 families with 4 children each, how many families would be expected to have (i) 2 boys and 2 girls (ii) at least one boy (iii) no girl (iv) at most two girls ? Assume equal probabilities for boys and girls.

c) Data was collected over a period of 10 years, showing number of deaths from horse kicks in each of the 200 army corps. The distribution of deaths was as follows:

No. of deaths	0	1	2	3	4	Total
freq	109	65	22	3	1	200

d) A sample of 100 dry battery cells tested to find the length of life produced the following results :

$$\bar{x} = 12 \text{ hours}, \ \sigma = 3 \text{ hours}.$$

Assuming the data to be normally distributed, what percentage of battery cells are expected to have life

(i) more than 15 hours

(ii) less than 6 hours

(iii) between 10 and 14 hours?

5. ATTEMPT ALL.

12

a) Show that when $|z+1| < 1$,

$$z^{-2} = 1 + \sum_{n=1}^{\infty}(n+1)(z+1)^n.$$

b) Let $S = \{x_1, x_2, x_3\} / x_1, x_2 = 0$ be a subset of V_3. Is it a subspace of V_3 ? Justify.

c) If $\{v_1, v_2, \text{----} v_n\}$ is an orthonormal set in V and if $w \in V$, then $u = w - (w, v_1)v_1 - (w, v_2)v_2 - \dots -(w, v_i)v_i - \dots -(w, v_n)v_n$ is orthogonal to each of $v_1, v_2, \text{----} v_n$.

Government College of Engineering, Amravati

(An Autonomous Institute of Government of Maharashtra)

Fourth Semester B. Tech. (EE / ET / IN)

Summer – 2013

Course Code: SHU401

Course Name: Engineering Mathematics – IV

Time: 2 Hrs. 30 Min. **Max. Marks: 60**

Instructions to Candidate

1) All questions are compulsory.
2) Assume suitable data wherever necessary and clearly state the assumptions made.
3) Diagrams/sketches should be given wherever necessary.
4) Use of logarithmic table, drawing instruments and non-programmable calculators is permitted.
5) Figures to the right indicate full marks.

1. **ATTEMPT ANY THREE** **12**

a) Prove that $\int_0^{\infty} \frac{dx}{x^4+1} = \frac{\pi}{4}\sqrt{2}$.

b) State residue theorem. Also find the sum of the residues of the function $f(z) = \frac{\sin z}{z \cos z}$ at its poles inside the circle $|z| = 2$.

c) Find all possible Laurentz series of $f(z) = \frac{1}{z^2+1}$, about all its singular points. Also determine the region of convergence.

d) Evaluate $\int_C f(z)dz$, where $f(z) = \begin{cases} 4y, & y > 0 \\ 1, & y < 0 \end{cases}$, and C is the arc from $z = -1 - i$ to $z = 1 + i$ of the cubical curve $y = x^3$.

2. **ATTEMPT ANY FOUR** **12**

a) Let a mapping $T: V_2 \to V_2$ be defined by

A sample ... life produced ...

$T(x, y) = (x + y, x)$. Show that T is a linear map.

b) Given that $\{(1, 0, 1), (1, 1, 0), (1, 1, 1)\}$ is a basis of $V_3(R)$ .Find out the coordinates of $(\alpha, \beta, \gamma) \in V_3(R)$ with respect to the above basis.

c) Let $S = \{(1,-1, 0), (1, 0, 2)\}$ and $T = \{(0,1, 0), (0, 1, 2)\}$. Determine the subspaces $S \cap T$ and $S + T$. Find dimension of $S \cap T$ and $S + T$.

d) Prove that $B = \{(1, 1, 1), (1,-1, 1), (0, 1, 1)\}$ is a basis of V_3 .

e) Define Inner product space. Using it prove that
$$(\alpha u + \beta v, \alpha u + \beta v) = \alpha\bar{\alpha}(u, u) + \alpha\bar{\beta}(u, v) + \bar{\alpha}\beta(v, u) + \beta\bar{\beta}(v, v),$$ where V is a inner product space and $u, v \in V$ and $\alpha, \beta \in F$.

3. **ATTEMPT ANY TWO** 12

a) In $F^{(2)}$ define for
$u = (\alpha_1, \alpha_2)$ and $v = (\beta_1, \beta_2)$ and
$$(u, v) = (2\alpha_1\overline{\beta_1} + \alpha_1\overline{\beta_2} + \alpha_2\overline{\beta_1} + \alpha_2\overline{\beta_2}) .$$
Show that this defines an inner product.

b) Prove that M the set of all $m \times n$ matrices of real numbers (or complex numbers) is a vector space over R (or C) under addition of matrices and scalar multiplication to a matrix.

c) Using Gram-Schmidt process orthonormalise the set of vectors $(1, 0, 1, 0), (1, 1, 3, 0), (0, 2, 0, 1)$ of V_4.

4. **ATTEMPT ANY THREE** 12

a) A random variable X has the following probability function:

X	0	1	2	3	4	5	6	7
P	0	k	2k	2k	3k	k^2	$2k^2$	$7k^2+k$

$P(X \leq x) > \frac{1}{2}$.

b) Assume that on the average one telephone number out of fifteen called between 2 P M and 3 P M on week-days is busy. What is the probability that if 6 randomly selected telephone numbers are called

(i) not more than three

(ii) at least three of them will be busy.

c) A car-hire firm has two cars, which it hires out day by day. The number of demands for a car on each day is distributed as a Poisson distribution with mean 1.5. Calculate the proportion of days on which neither car is used and the proportion of days on which some demand is refused. ($e^{-1.5} = 0.2231$)

d) A sample of 100 dry battery cells tested to find the length of life produced the following results :

$\bar{x} = 12$ hours, $\sigma = 3$ hours.

Assuming the data to be normally distributed, what percentage of battery cells are expected to have life

(i) more than 15 hours

(ii) less than 6 hours

(iii) between 10 and 14 hours?

5. **ATTEMPT ALL** 12

a) If $\{u_1, u_2, \ldots, u_n\}$ is an orthonormal set and if $x = \sum_{i=1}^{n} \alpha_i u_i$, $\alpha_i \in C$, then $\|x\|^2 = \sum_{i=1}^{n} |\alpha_i|^2$.

b) Define continuous random variable , Discrete random variable and discrete probability distribution.

c) Show that when $|z+1| < 1$,

$z^{-2} = 1 + \sum_{n=1}^{\infty} (n+1)(z+1)^n$.

d) State Cauchy's Integral theorem and apply it to evaluate the integral $\oint_C \frac{dz}{z-2}$ around

(i) Rectangle with vertices at $3 \pm 2i, -2 \pm 2i$

(ii) Triangle with vertices at $(0,0), (1,0), (0,1)$

e) Find Laurentz series of $\frac{\sinh 3z}{z^3}$ for $0 < |z| < \infty$.

small chance of 1 in 500 tyres to be defective. The tyres are supplied in lots of 10. Using Poisson distribution calculate the approximate number of lots containing no defective, one defective and two defective tyres, respectively, in a consignment of 10,000 lots.

c) The probability that a man aged 60 will live to be 70 is 0·65. What is the probability that out of 10 men, now 60, at least 7 will live to be 70?

5 ATTEMPT ALL.

a) Let P be the set of all polynomials in t over the real field R and $p(t) = \sum_{i=1}^{n-1} a_i t^i$ be any element of P. If a transformation T is defined as $T\{p(t)\} = \sum_{i=1}^{n-1} \frac{a_i}{i+1} t^{i+1}$, prove that T is a linear transformation.

b) Evaluate the following integrals by using Cauchy's integral formula

i) $\int_C \frac{\sin \pi z^2 + \cos \pi z^2}{(z-1)(z-2)} dz$ $\qquad \frac{\sin \pi z^2 + \cos \pi z^2}{(z-1)(z-2)}$

ii) $\frac{1}{2\pi i} \int_C \frac{e^{zt}}{z^2+1} dz,\ t > 0$ $\qquad \frac{1}{2\pi t} \int_c \frac{e^{2t}}{z^2+1} dz$

where C is the circle $|z| = 3$.

ATTEMPT ANY THREE. **12**

a) Determine whether or not the given system of vectors is linearly dependent

$\alpha_1 = \begin{pmatrix} 1 & -2 & 4 \\ 3 & 0 & -1 \end{pmatrix}$, $\alpha_2 = \begin{pmatrix} 2 & -4 & 8 \\ 6 & 0 & -2 \end{pmatrix}$.

b) Prove that the set $S = \{(1,2,1), (2,1,0), (1,-1,2)\}$ forms a basis of $V_3 R$.

c) If V is finite dimensional and T is a linear transformation of V into itself which is not onto, prove that there is some $\alpha \neq 0$ in V such that $T(\alpha) = 0$.

d) Let V be the vector space of 2 X 2 matrices over R. Find the co-ordinate vector of the matrix $A \in V$ relative to the basis

$\left\{ \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}, \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}, \begin{bmatrix} 1 & -1 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} \right\}$ where $A = \begin{bmatrix} 2 & 3 \\ 4 & -7 \end{bmatrix}$

ATTEMPT ALL. **12**

a) In a sample of 1000 cases, the mean of a certain test is 14 and standard deviation is 2.5. Assuming the distribution to be normal, find

i) how many students score between 12 and 15?

ii) how many students score above 18?

iii) how many students score below 8?

iv) how many students score 16?

b) In a certain factory producing cycle tyres, there is

Semester B. Tech. (EE/ET/IN)

Summer– 2015

de: SHU 401

me: Engineering Maths IV

rs. 30 Min.* **Max. Marks: 60***

ns to Candidate

ll questions are compulsory.

ssume suitable data wherever necessary and clearly state the ssumptions made.

Diagrams/sketches should be given wherever necessary.

Use of logarithmic table, drawing instruments and non-programmable calculators is permitted.

Figures to the right indicate full marks.

ATTEMPT ANY THREE. **12**

a) Evaluate $\int_C (z^2 + 3z + 2)dz$ where C is the arc of the cycloid $x = a(\theta + \sin\theta)$, $y = a(1 - \cos\theta)$ between the points (0, 0) and (πa,2a).

z^2+3z+2

b) Integrate z^2 along the straight line OM and also along the path OLM consisting of two straight line segments OL and OM where O is the origin, L is the point $z = 3$ and M the point $z = 3 + i$.

Contd.

Hence show that the integral of z^2 along the closed path OLMO is zero.

c) If $f(z) = \frac{z^2+5z+6}{z-2}$ does Cauchy's theorem apply

i) when the path of integration C is a circle of radius 3 with origin as centre. $z^2+5z+6/z-2$

ii) when C is a circle of radius 1 with origin as centre.

d) Evaluate $\int_0^{2\pi} \frac{\cos 2\theta\, d\theta}{5-4\cos\theta}$ $\frac{\cos 2\theta}{5+4\cos\theta}$

2. ATTEMPT ANY THREE. **12**

a) Apply the calculus of residues to evaluate

$\int_{-\infty}^{\infty} \frac{\cos x\, dx}{(x^2+a^2)(x^2+b^2)}$, $(a > b > 0)$ $\int_{-\infty}^{\infty} \frac{\cos x\, dx}{(x^2+a^2)(x^2+b^2)}$.

b) Show that the mapping T: $V_2(R)$ onto $V_3(R)$ defined by $T(p, q) = (p, q, 0)$ is a linear transformation from $V_2(R)$ onto $V_3(R)$.

c) Write the vector $\alpha = (1, -2, 5)$ as a linear combination of vectors $\alpha_1 = (1, 1, 1)$, $\alpha_2 = (1, 2, 3)$, $\alpha_3 = (2, -1, 1)$ in the vector space $V_3(R)$.

d) Is the vector $\alpha = \begin{bmatrix} 3 & -1 \\ 1 & -2 \end{bmatrix}$ in a vector space of 2 X 2 matrices a linear combination of

$\alpha_1 = \begin{bmatrix} 1 & 1 \\ 0 & -1 \end{bmatrix}$, $\alpha_2 = \begin{bmatrix} 1 & 1 \\ -1 & 0 \end{bmatrix}$, $\alpha_3 = \begin{bmatrix} 1 & -1 \\ 0 & 0 \end{bmatrix}$.

15004056

Government College of Engineering, Amravati

(An Autonomous Institute of Government of Maharashtra)

Forth Semester B. Tech. (EE/ET/IN)

Summer – 2017

Course Code: SHU 401 April 10

Course Name: ENGINEERING MATHEMATICS-IV

Time: 2 ½ Hrs. Max. Marks: 60

Instructions to Candidate

1) All questions are compulsory.
2) Assume suitable data wherever necessary and clearly state the assumptions made.
3) Diagrams/sketches should be given wherever necessary.
4) Use of logarithmic table, drawing instruments and non-programmable calculators is permitted.
5) Figures to the right indicate full marks.

1. (a) **Attempt any three:** 12

Evaluate the following integral $\int_c \mathrm{Re}(z^2)dz$,

from 0 *to* $2+4i$ along the i) line segment joining the points $(0,0)$ & $(2,4)$.

ii) x-axis from 0 to 2, and then vertically to $2+4i$.

(b) Compute the residues at all the singular points of $f(z)=\frac{z^2}{(z^n-1)}$, n any positive integer.

(c) Obtain the Taylor series expansion of

$$f(z)=\frac{1}{z^2+(1+2i)z+2i}$$ about the point $z=o$.

(d) Evaluate: $\int_0^\infty \frac{dx}{x^6+1}$.

2. **Attempt any three:** 12

2

(a) Determine whether the following set of vectors form a basis in R^3, where $u=(2,2,0), v=(3,0,2), w=(2,-2,2)$.

(b) If vector space V is the set of real valued continuous functions over R, then to show that the set W of solutions of differlential equation $2\frac{d^2y}{dx^2}-9\frac{dy}{dx}+2y=0$ is a subspace of V. 14

(c) Let $T:V_3\to V_2$ be defined by $T(x_1,x_2,x_3)=(x_1-x_2,\ x_1+x_3)$. Verify whether T is a linear map $V_3\ to\ V_2$.

(d) Let V be the vector space of 2×2 symmetric matrices over R. Find the coordinate vector of matrix $A=\begin{bmatrix}4 & -11\\ -11 & -7\end{bmatrix}$ relative to the basis $\left\{\begin{bmatrix}1 & -2\\ -2 & 1\end{bmatrix},\begin{bmatrix}2 & 1\\ 1 & 3\end{bmatrix},\begin{bmatrix}4 & -1\\ -1 & -5\end{bmatrix}\right\}$ (4, -2, 1)

3. **Attempt any three:** 12

(a) State and prove triangle inequality.

(b) Apply Gram-Schmidt process to orthonormalized the set $\{(1,0,1,1),(-1,0,-1,1),(0,-1,1,1)\}$.

(c) i) Let $y = \begin{bmatrix} 7 \\ 6 \end{bmatrix}$ & $u = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$. Find the orthogonal projection of y onto u.

ii) Compute the distance between the vectors $\bar{u} = (7,1)$ & $\bar{v} = (3,2)$.

(d) Let V be an inner product space then for arbitrary vectors u and v in V and scalar α,

i) $\|\alpha u\| = |\alpha| \, \|u\|$

ii) $\|u\| \geq 0$ & $\|u\| = 0 \text{ iff } u = 0$

4. **Attempt the following:** 12

(a) Five defective bulbs are accidentally mixed with twenty good ones. It is not possible to just look at a bulb and tell whether or not it is defective. Find the probability distribution of the number of defective bulbs, if four bulbs are drawn at random from this lot.

(b) Out of 800 families with 4 children each, how many families would be expected to have

i) 2 boys and 2 girls

ii) at least one boy

iii) no girl.

(c) Six coins are tossed 6400 times. Using the Poisson distribution, determine the approximate probability of getting six heads x times.

5. Attempt the following 12

(a) Show that an inner product can be determined on V_2 by

$$(x_1, x_2)(y_1, y_2) = \frac{(x_1 - x_2)(y_1 - y_2)}{4} + \frac{(x_1 + x_2)(y_1 + y_2)}{4}.$$

(b) Let R^+ be a set of all positive real numbers we define vector addition and scalar multiplication on R^+ as follows

i) $u + v = u \cdot v, \quad \forall u, v \in R^+$

ii) $\alpha u = u^{\alpha}, \quad \forall u \in R^+, \alpha$ be any scalar

Show that $(R^+, +, \cdot)$ is a vector space.