

Government College of Engineering, Amravati
(An Autonomous Institute of Government of Maharashtra)

Third Semester B. Tech. (CS/IT)

Summer Term– 2016

Course Code: CSU303

Course Name: Discrete Mathematics and Graph Theory

Time: 2 hr. 30min.

Max. Marks: 60

Instructions to Candidate

- 1) All questions are compulsory.
- 2) Assume suitable data wherever necessary and clearly state the assumptions made.
- 3) Diagrams/sketches should be given wherever necessary.
- 4) Use of logarithmic table, drawing instruments and non-programmable calculators is permitted.
- 5) Figures to the right indicate full marks.

1. Attempt

- (a) Show the following equivalence without using truth table. 6

$$(i) \neg (P \wedge Q) \rightarrow (\neg P \vee (\neg PVQ)) \Leftrightarrow (\neg PVQ)$$

$$(ii) (P \rightarrow C) \wedge (Q \rightarrow C) \Leftrightarrow (PVQ) \rightarrow C$$

$$(iii) A \rightarrow (PVC) \Leftrightarrow (A \wedge \neg P) \rightarrow C$$

- (b) Write a formulas which are equivalent to the following formulas contains \neg and \wedge only. 6

i) $\neg (P \leftrightarrow (Q \rightarrow (RVP)))$

ii) $((\neg PVQ) \wedge R) \rightarrow (PVR)$

2. Attempt Any Two

- (a) Two equivalence relation R and S are given by their relation matrices M_R and M_S . show that $R \circ S$ is not an equivalence relation. 6

$$M_R = \begin{bmatrix} 1 & 1 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad M_S = \begin{bmatrix} 1 & 1 & 0 \\ 1 & 1 & 1 \\ 0 & 1 & 1 \end{bmatrix}$$

- (b) Let A be the set of factors of a particular positive integer m and let \leq be the relation divides draw Hasse diagram for
(i) $m=60$ (ii) $m=210$ 6
- (c) Given $S = \{1, 2, \dots, 12\}$ and a relation R on S where $R = \{ \langle x, y \rangle \mid x+y=12 \}$ what are the properties of the relation R? 6

3. Attempt

- (a) Which of the following systems satisfy the properties of $\langle I, +, X \rangle$ which are designated by (A-1) to (A-4), (M-1) to (M-3), (D) and (C)? 6
- i) All Positive integers
 - ii) $\langle \mathbb{Z}_6, +_6, X_6 \rangle$
 - iii) $\langle \mathbb{Z}_7, +_7, X_7 \rangle$
- (b) Use the K-Map representation to find a minimal sum of product expressions for following Boolean function. 6
- i) $f(a, b, c) = \sum_m(0, 1, 4, 6)$
 - ii) $f(a, b, c, d) = \sum_m(0, 5, 7, 8, 12, 14)$
 - iii) $f(a, b, c, d) = \sum_m(0, 1, 2, 3, 13, 15)$

4. **Attempt Any Two**

- (a) Explain the following grammars and differentiate with the help of example. 6
(i) Unrestricted grammar (ii) Context sensitive grammar
(iii) Context free grammar (iv) Regular grammar
- (b) Explain: 6
i) Tree ii) Binary tree iii) complete binary tree
iv) In degree and Out degree
- (c) Find the complement of every element of the lattice $\langle S_n, D \rangle$ for $n=75$ and $n=60$. 6

5. **Attempt any two**

- (a) From the adjacency matrix of a simple digraph, how will you determine whether it is a directed tree? If it is directed tree, how will you determine its root and terminal nodes? 6
- (b) Explain 6
1. Deterministic finite automata
2. Non deterministic finite automata
- (c) Find a deterministic finite-state acceptor equivalent to the nondeterministic one given as $M = \langle \{a,b\}, \{q_0, q_1, q_2\}, q_0, \delta, \{q_2\} \rangle$ where δ is given by table 6

Present state	Input symbol	
	a	b
q_0	$\{q_0, q_1\}$	$\{q_2\}$
q_1	$\{q_0\}$	$\{q_1\}$
q_2	$\{q_1\}$	$\{q_0, q_1\}$

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1. Solve.

(a) Obtain Principal Conjunctive & Disjunctive Normal Form of $(\neg P \rightarrow R) \wedge (Q \leftrightarrow P)$. **6**

(b) Show the following equivalence without using truth table. **6**

(i) $\neg (P \wedge Q) \rightarrow (\neg P \vee (\neg P \vee Q)) \Leftrightarrow (\neg P \vee Q)$

(ii) $(P \rightarrow C) \wedge (Q \rightarrow C) \Leftrightarrow (P \vee Q) \rightarrow C$

2. Solve.

- (a) Let $R = \{ \langle 1, 2 \rangle, \langle 3, 4 \rangle, \langle 2, 2 \rangle \}$ and $S = \{ \langle 4, 2 \rangle, \langle 2, 5 \rangle, \langle 3, 1 \rangle, \langle 1, 3 \rangle \}$. Find $R \circ S, S \circ R, R \circ (S \circ R), (R \circ S) \circ R, R \circ R, S \circ S$ and $R \circ R \circ R$ 6
- (b) Let $f: R \rightarrow R$ and $g: R \rightarrow R$, where R is the set of real Numbers. find $f \circ g$ and $g \circ f$, where $f(x) = x^2 - 2$ and $g(x) = x + 4$. state whether these functions are one to one, into and onto. 6

OR

- (c) Write down the composition table for $\langle Z_7, +_7 \rangle$ and $\langle Z_7^*, \times_7 \rangle$ where $Z_7^* = Z_7 - \{ [0] \}$ 6

3. Solve.

- (a) Find the complement of every element of the Lattice $\langle S_n, D \rangle$ for $n = 75$. 6

OR

- (b) Simplify the following Boolean expressions: 6
- (i) $(a * b' * c) \oplus (a * b' * c) \oplus (a * b' * c')$.
- (ii) $(a * c) \oplus c \oplus [(b \oplus b') * c]$.
- (c) Expand the following functions into their canonical sum-of-product form. 6
- (i) $f(a, b, c, d) = xy + yw'z$
- (ii) $f(a, b, c, d) = w + y'z + x'y$

4. Solve.

- (a) The language $L(G) = \{ a^n b^m c^n \mid n, m \geq 1 \}$ is generated by the grammar $G = \langle \{S, A, B, C\}, \{a, b\}, S, \emptyset \rangle$ where \emptyset consist of 6

productions

$S \rightarrow aS$

$S \rightarrow aB$

$B \rightarrow bC$

$C \rightarrow aC$

$C \rightarrow a$

Find the derivation for $a^3 b a^4$

- (b) Explain the following terms with example
(i) degree of node (ii) Acyclic graph
(iii) complete graph (iv) isomorphic graph

6

OR

- (c) Show that in complete binary tree the total number of edges is given by $2(n_t - 1)$, where n_t is the number of terminal nodes.

6

Solve.

- (a) Explain finite state machine with suitable example.
- (b) Consider the finite state machine whose state transition table is

6

6

	0	1
S_0	S_0	S_1
S_1	S_1	S_2
S_2	S_2	S_3
S_3	S_3	S_0

- (i) List the values of the transition function for $w=011001$
- (ii) List the values of the transition function for $w=1110$

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1 Solve:

(a) Obtain the DNF and CNF: **6m**
 $(\sim P \rightarrow R) \wedge (Q \rightarrow P) \wedge (P \rightarrow Q)$

(b) Show that $S \vee R$ is tautologically implied by **6m**
 $(P \vee Q) \wedge (P \rightarrow R) \wedge (Q \rightarrow S)$, by validity using
(i) truth table and (ii) rules of inference

2 Solve any two:

(a) Let $A = \{2, 3\}$, $B = \{4, 6, 9\}$ and R be a relation **6m**
such that $a R b$, if and only if a is divisor of b .
Find domain, range, inverse, compliment, matrix
and digraph of R along with the list of degrees.

(b) In a survey of 260 computer science students, the following data were obtained: 94 like to work in USA, 64 like to work in UK, 58 like to work in UAE, 28 like UK and UAE, 26 USA and UK, 22 USA and UAE, and 14 like all the three places. Find how many students like none of the three countries and how many like only USA? 6m

(c) Let $X=Y=Z=R$ and let $f: X \rightarrow Y$ and $g: Y \rightarrow Z$ are defined by $f(x) = x+1$ and $g(y) = y^2+2$. Find $(g \circ f)$, $(f \circ f)$, $(g \circ g)$ and $(f \circ g)$ 6m

3 Solve:

(a) Obtain SOP & POS form of the given expression in 3 variables: $x_1 * (x_2' + x_3)$. 6m

(b) Minimize the given expression using K-Map and design logical diagram for it: 6m

(i) $F(a,b,c,d) = \sum_m (0,4,5,7,8,12)$

(ii) $F(w,x,y,z) = \sum_m (0,1,2,4,5,6,8,9,12,13)$

4 Solve any two:

(a) Write short note on: Isomorphic graph, bipartite graph and complete graph 6m

(b) Draw tree for the algebraic expression: $((a - b) * (c / d)) + e$ and find its infix, prefix and postfix polish expressions along with their values if $a = 9$, $b = 4$, $c = 6$, $d = 2$ and $e = 5$. 6m

(c) What is minimal spanning tree? Explain any one algorithm of it with a proper example. 6m

5 Solve any two:

(a) Design a Finite state acceptor that will accept the set of natural numbers which are divisible by 5. 6m

- (b) Consider a grammar: 6m
 $G = \langle \{E, T\}, \{a, b, c, +, *\}, E, \Phi \rangle$ with productions
 $E \rightarrow aTb / abEb / ab / a, T \rightarrow bE / aTTb$;
Prove that the G is ambiguous and derive the
string "ababab"
- (c) What is a Turing machine? Elaborate its working 6m
with a suitable example

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- 1 Solve: 6
 - a) Prove the following using truth table:
A] $(P \vee Q) \wedge (P \rightarrow R) \wedge (Q \rightarrow R) \Rightarrow R$
B] $\sim (P \wedge Q) \rightarrow (\sim P \vee (\sim P \vee Q)) \Leftrightarrow \sim P \vee Q$
 - b) Obtain the DNF and CNF of: 6
 $\sim (P \wedge Q) \rightarrow ((P \rightarrow \sim Q) \wedge (\sim Q \rightarrow P))$
- 2 Solve any two: 6
 - a) Let $A = \{1, 3, 4, 6\}$ and $B = \{1, 2, 3\}$ and R be a relation $a R b$, if and only if a is a multiple of b . Find domain, range, inverse, compliment, matrix and digraph of R along with the list of degrees.

Contd..

- b) Mention and explain the set operations in detail 6
- c) Let $f(x) = x + 3$, $g(x) = x - 4$ and $h(x) = 5x$ are functions from $R \rightarrow R$, where R is the set of real numbers. Show that $f(g \circ h) = (f \circ g) \circ h$
- 3 Solve:
- a) Minimize the expression & design its logic diagram: $f(a,b,c,d) = \sum m(0, 1, 2, 3, 5, 8, 10, 12, 13, 14)$ 6
- b) Obtain SOP & POS form of $x_1 + x_2$ and $x_1 * x_2$ expression for 3 variables, respectively 6
- 4 Solve any two:
- a) What is Graph? Explain its special 4 types and matrix representation in detail with example 6
- b) What is minimal spanning tree? Explain prim's algorithm with a proper example 6
- c) What is a tree? Draw tree for $((a-b)*c)+(d/(e+f))$ and find its inorder, preorder and postorder expressions along with their values if $a=10$, $b=6$, $c=2$, $d=20$, $e=7$ and $f=3$ 6
- 5 Solve any two:
- a) What is a grammar? Mention and explain its types in detail 6
- b) Consider a grammar $G = \langle \{S\}, \{p, +\}, S, \Phi \rangle$, with productions $S \rightarrow S+S$, $S \rightarrow p$; Derive " $p+p+p$ " string & show whether it is ambiguous or not. 6
- c) Design a Finite state acceptor that will accept the set of natural numbers which are divisible by 3 6

$$\begin{array}{r} 4 \\ 3 \overline{) 125} \\ \underline{12} \\ 5 \end{array}$$

$$\begin{array}{r} 2 \\ 3 \overline{) 118} \\ \underline{6} \\ 58 \\ \underline{54} \\ 4 \end{array}$$