Solutions to problems found on the Project Euler website. The problem descriptions are given in the code itself. Although this sheet may contain a restatement of the problems, it is mostly intended for a more thorough and mathematical explanation of the solutions as opposed to bare code. Not all problems may appear.

Problem 1

Multiples of 3 and 5: If we list all the natural numbers below 10 that are multiples of 3 or 5, we get 3, 5, 6 and 9. The sum of these multiples is 23.
Find the sum of all the multiples of 3 or 5 below 1000.

Firstly, the classic summation formula for a series of numbers is $\sum\limits_{i=1}^{n} i = \frac{n(n+1)}{2}$. In general, this can be extended to $\sum\limits_{i=a}^{b} i = \frac{b(a+b)}{2}$ where b is the maximum number and a is the minimum number (proof omitted). Since here the summation is every third or fifth number, we have that the number of terms to sum is not b but $\frac{b}{a}$. For ease of reading, we define $n = \lfloor \frac{b}{a} \rfloor$. Then we have $\sum\limits_{\substack{i=a \\ i=i+m}}^{b} i = \frac{n(a+b)}{2}$ where m is the multiple. With a little bit of algebraic manipulation, we arrive at $\sum\limits_{\substack{i=a \\ i=i+m}}^{b} i = \frac{na(1+\frac{b}{a})}{2} = \frac{na(1+n)}{2}$. While it seems trivial, replacing $(a+b)$ with $a(1+n)$ is necessary to insure that the summation has the correct number of integers and does not accidentally include a number that is too high because of rounding errors. So now we have $\sum\limits_{\substack{i=a \\ i=i+m}}^{b} i = \frac{na(1+n)}{2}$. To find the summation of all multiples of three and five, we can use this formula for $m = 3$ and $m = 5$. However, this double-counts numbers where i is a multiple of both 3 and 5 (a multiple of 15). So we subtract out all multiples of fifteen to go from double-counting them to single-counting. The final mathematics looks like this:

$$\sum_{\substack{i=1 \\ i=i+3}}^{999} i + \sum_{\substack{i=1 \\ i=i+5}}^{999} i = \frac{\lfloor \frac{999}{3} \rfloor (3)(1 + \lfloor \frac{999}{3} \rfloor)}{2} + \frac{\lfloor \frac{999}{5} \rfloor (5)(1 + \lfloor \frac{999}{5} \rfloor)}{2} - \frac{\lfloor \frac{999}{15} \rfloor (15)(1 + \lfloor \frac{999}{15} \rfloor)}{2}$$

Assuming one's button-pushing skills are correct, this expression should evaluate to 233,168.

Problem 2