PROJECT MODE

November 2020

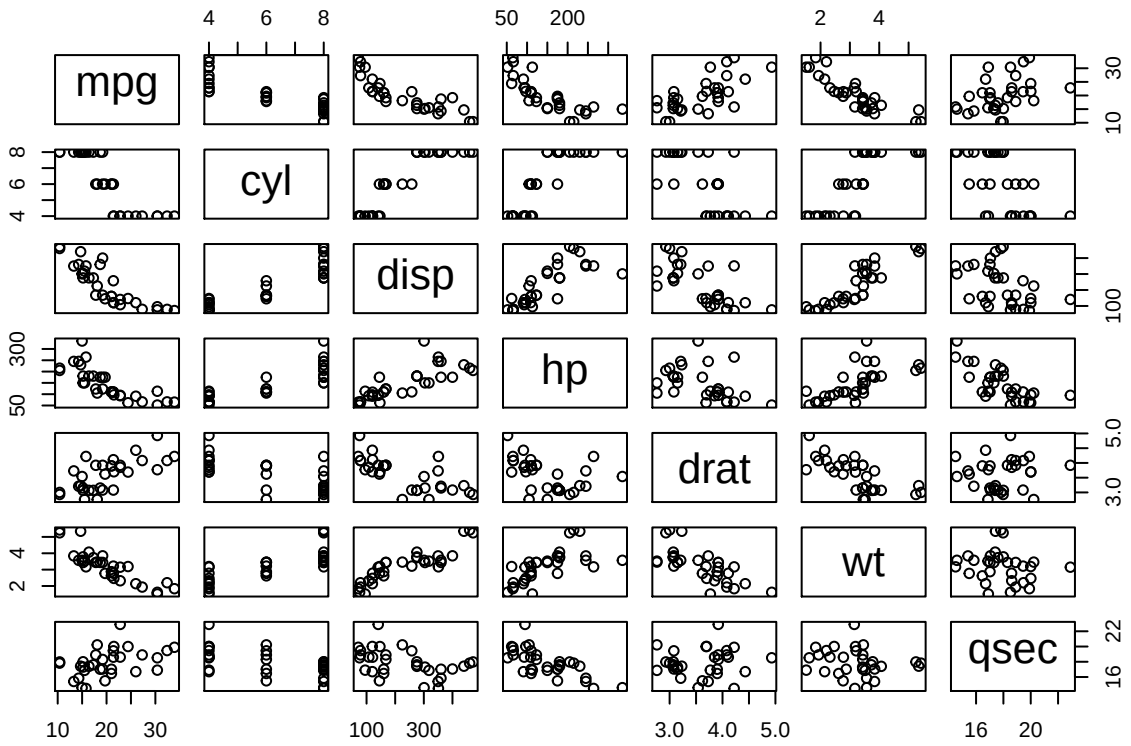
Statistical modeling—Linear model, Analysis of variance (ANOVA), Predictions and Variable Selection

##install.packages("olsrr")

```
### Regression model
### Making a matrix scatter plot sing the Mtcars dataset
mtcars.philant<-mtcars[, 1:7]
mtcars.philant
```

```
##                      mpg cyl  disp  hp drat    wt  qsec
## Mazda RX4           21.0   6 160.0 110 3.90 2.620 16.46
## Mazda RX4 Wag       21.0   6 160.0 110 3.90 2.875 17.02
## Datsun 710          22.8   4 108.0  93 3.85 2.320 18.61
## Hornet 4 Drive      21.4   6 258.0 110 3.08 3.215 19.44
## Hornet Sportabout   18.7   8 360.0 175 3.15 3.440 17.02
## Valiant             18.1   6 225.0 105 2.76 3.460 20.22
## Duster 360          14.3   8 360.0 245 3.21 3.570 15.84
## Merc 240D           24.4   4 146.7  62 3.69 3.190 20.00
## Merc 230            22.8   4 140.8  95 3.92 3.150 22.90
## Merc 280            19.2   6 167.6 123 3.92 3.440 18.30
## Merc 280C           17.8   6 167.6 123 3.92 3.440 18.90
## Merc 450SE          16.4   8 275.8 180 3.07 4.070 17.40
## Merc 450SL          17.3   8 275.8 180 3.07 3.730 17.60
## Merc 450SLC         15.2   8 275.8 180 3.07 3.780 18.00
## Cadillac Fleetwood  10.4   8 472.0 205 2.93 5.250 17.98
## Lincoln Continental 10.4   8 460.0 215 3.00 5.424 17.82
## Chrysler Imperial   14.7   8 440.0 230 3.23 5.345 17.42
## Fiat 128            32.4   4  78.7  66 4.08 2.200 19.47
## Honda Civic         30.4   4  75.7  52 4.93 1.615 18.52
## Toyota Corolla      33.9   4  71.1  65 4.22 1.835 19.90
## Toyota Corona       21.5   4 120.1  97 3.70 2.465 20.01
## Dodge Challenger    15.5   8 318.0 150 2.76 3.520 16.87
## AMC Javelin         15.2   8 304.0 150 3.15 3.435 17.30
## Camaro Z28          13.3   8 350.0 245 3.73 3.840 15.41
## Pontiac Firebird    19.2   8 400.0 175 3.08 3.845 17.05
## Fiat X1-9           27.3   4  79.0  66 4.08 1.935 18.90
## Porsche 914-2       26.0   4 120.3  91 4.43 2.140 16.70
## Lotus Europa        30.4   4  95.1 113 3.77 1.513 16.90
## Ford Pantera L      15.8   8 351.0 264 4.22 3.170 14.50
## Ferrari Dino        19.7   6 145.0 175 3.62 2.770 15.50
## Maserati Bora       15.0   8 301.0 335 3.54 3.570 14.60
## Volvo 142E          21.4   4 121.0 109 4.11 2.780 18.60
```

```r
pairs(mtcars.philant)
```




```r
### Fitting a linear regression model for the above dataset
philant.model1<- lm(mpg~., data = mtcars.philant)
philant.model1
```

```
##
## Call:
## lm(formula = mpg ~ ., data = mtcars.philant)
##
## Coefficients:
## (Intercept)          cyl         disp           hp         drat           wt
##    26.30736     -0.81856      0.01320     -0.01793      1.32041     -4.19083
##        qsec
##     0.40146
```

```r
### This is the same as
philant.model2<- lm(mpg~cyl+disp+hp+drat+qsec, data = mtcars.philant)
philant.model2
```

```
##
## Call:
## lm(formula = mpg ~ cyl + disp + hp + drat + qsec, data = mtcars.philant)
##
```

```
## Coefficients:
## (Intercept)          cyl         disp           hp         drat         qsec
##    38.45215     -1.23333     -0.01028     -0.03105      1.47160     -0.50969
```

```
### Using model fit
summary(philant.model2)
```

```
##
## Call:
## lm(formula = mpg ~ cyl + disp + hp + drat + qsec, data = mtcars.philant)
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -4.0590 -2.0712 -0.5211  1.6405  7.0628
##
## Coefficients:
##             Estimate Std. Error t value Pr(>|t|)
## (Intercept) 38.45215   16.69506   2.303   0.0295 *
## cyl         -1.23333    0.94496  -1.305   0.2033
## disp        -0.01028    0.01150  -0.894   0.3796
## hp          -0.03105    0.01767  -1.757   0.0907 .
## drat         1.47160    1.74248   0.845   0.4061
## qsec        -0.50969    0.51640  -0.987   0.3327
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 3.013 on 26 degrees of freedom
## Multiple R-squared:  0.7904, Adjusted R-squared:  0.7501
## F-statistic: 19.61 on 5 and 26 DF,  p-value: 4.415e-08
```

```
summary(philant.model1)
```

```
##
## Call:
## lm(formula = mpg ~ ., data = mtcars.philant)
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -3.9682 -1.5795 -0.4353  1.1662  5.5272
##
## Coefficients:
##             Estimate Std. Error t value Pr(>|t|)
## (Intercept) 26.30736   14.62994   1.798  0.08424 .
## cyl         -0.81856    0.81156  -1.009  0.32282
## disp         0.01320    0.01204   1.097  0.28307
## hp          -0.01793    0.01551  -1.156  0.25846
## drat         1.32041    1.47948   0.892  0.38065
## wt          -4.19083    1.25791  -3.332  0.00269 **
## qsec         0.40146    0.51658   0.777  0.44436
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 2.557 on 25 degrees of freedom
```

```
## Multiple R-squared:  0.8548, Adjusted R-squared:   0.82
## F-statistic: 24.53 on 6 and 25 DF,  p-value: 2.45e-09
```

```
### Analysis of variance ANOVE
anova(philant.model1)
```

```
## Analysis of Variance Table
##
## Response: mpg
##           Df Sum Sq Mean Sq  F value    Pr(>F)
## cyl        1 817.71  817.71 125.0503 3.209e-11 ***
## disp       1  37.59   37.59   5.7491  0.024287 *
## hp         1   9.37    9.37   1.4331  0.242493
## drat       1  16.47   16.47   2.5183  0.125100
## wt         1  77.48   77.48  11.8481  0.002041 **
## qsec       1   3.95    3.95   0.6040  0.444365
## Residuals 25 163.48    6.54
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

```
anova(philant.model2)
```

```
## Analysis of Variance Table
##
## Response: mpg
##           Df Sum Sq Mean Sq F value   Pr(>F)
## cyl        1 817.71  817.71 90.0652 6.23e-10 ***
## disp       1  37.59   37.59  4.1407  0.05218 .
## hp         1   9.37    9.37  1.0321  0.31902
## drat       1  16.47   16.47  1.8138  0.18968
## qsec       1   8.84    8.84  0.9742  0.33274
## Residuals 26 236.06    9.08
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

```
### The coefficient estimate giving the effect on the response of each covariate taking in account the
model1<-lm(mpg~cyl, data = mtcars.philant)
summary(model1)
```

```
##
## Call:
## lm(formula = mpg ~ cyl, data = mtcars.philant)
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -4.9814 -2.1185  0.2217  1.0717  7.5186
##
## Coefficients:
##             Estimate Std. Error t value Pr(>|t|)
## (Intercept)  37.8846     2.0738   18.27  < 2e-16 ***
## cyl          -2.8758     0.3224   -8.92 6.11e-10 ***
## ---
```

```
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 3.206 on 30 degrees of freedom
## Multiple R-squared:  0.7262, Adjusted R-squared:  0.7171
## F-statistic: 79.56 on 1 and 30 DF,  p-value: 6.113e-10
```

```
model2<-lm(disp~wt, data = mtcars.philant)
summary(model2)
```

```
##
## Call:
## lm(formula = disp ~ wt, data = mtcars.philant)
##
## Residuals:
##    Min     1Q Median     3Q    Max
## -88.18 -33.62 -10.05  35.15 125.59
##
## Coefficients:
##             Estimate Std. Error t value Pr(>|t|)
## (Intercept)  -131.15      35.72  -3.672 0.000933 ***
## wt            112.48      10.64  10.576 1.22e-11 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 57.94 on 30 degrees of freedom
## Multiple R-squared:  0.7885, Adjusted R-squared:  0.7815
## F-statistic: 111.8 on 1 and 30 DF,  p-value: 1.222e-11
```

```
### ANOVA table for for each model. Remember the other covariates does not account for the residuals in
anova(model1)
```

```
## Analysis of Variance Table
##
## Response: mpg
##           Df Sum Sq Mean Sq F value    Pr(>F)
## cyl        1 817.71  817.71  79.561 6.113e-10 ***
## Residuals 30 308.33   10.28
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

```
anova(model2)
```

```
## Analysis of Variance Table
##
## Response: disp
##           Df Sum Sq Mean Sq F value    Pr(>F)
## wt         1 375476  375476  111.85 1.222e-11 ***
## Residuals 30 100709    3357
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

```
### Modeling check using residuals to check linearity, constant variance, independence assumption. this
resid(model1)
```

```
##           Mazda RX4       Mazda RX4 Wag          Datsun 710      Hornet 4 Drive
##           0.3701643           0.3701643          -3.5814159           0.7701643
##   Hornet Sportabout             Valiant          Duster 360            Merc 240D
##           3.8217446          -2.5298357          -0.5782554          -1.9814159
##            Merc 230            Merc 280            Merc 280C           Merc 450SE
##          -3.5814159          -1.4298357          -2.8298357           1.5217446
##          Merc 450SL         Merc 450SLC  Cadillac Fleetwood Lincoln Continental
##           2.4217446           0.3217446          -4.4782554          -4.4782554
##   Chrysler Imperial            Fiat 128         Honda Civic      Toyota Corolla
##          -0.1782554           6.0185841           4.0185841           7.5185841
##       Toyota Corona    Dodge Challenger          AMC Javelin          Camaro Z28
##          -4.8814159           0.6217446           0.3217446          -1.5782554
##    Pontiac Firebird           Fiat X1-9       Porsche 914-2        Lotus Europa
##           4.3217446           0.9185841          -0.3814159           4.0185841
##      Ford Pantera L        Ferrari Dino       Maserati Bora          Volvo 142E
##           0.9217446          -0.9298357           0.1217446          -4.9814159
```

```
resid(model2)
```

```
##           Mazda RX4       Mazda RX4 Wag          Datsun 710      Hornet 4 Drive
##           -3.544307          -32.226232          -21.800865           27.531201
##   Hornet Sportabout             Valiant          Duster 360            Merc 240D
##          104.223620          -33.025943           89.601462          -80.956846
##            Merc 230            Merc 280            Merc 280C           Merc 450SE
##          -82.357720          -88.176380          -88.176380          -50.837608
##          Merc 450SL         Merc 450SLC  Cadillac Fleetwood Lincoln Continental
##          -12.595040          -18.218947           12.638189          -18.933007
##   Chrysler Imperial            Fiat 128         Honda Civic      Toyota Corolla
##          -30.047234          -37.603489           25.196222           -4.148968
##       Toyota Corona    Dodge Challenger          AMC Javelin          Camaro Z28
##          -26.010195           53.225369           48.786010           49.232364
##    Pontiac Firebird           Fiat X1-9       Porsche 914-2        Lotus Europa
##           98.669974           -7.496782           10.745200           56.068992
##      Ford Pantera L        Ferrari Dino       Maserati Bora          Volvo 142E
##          125.592717          -35.416028           30.601462          -60.540809
```

```
### Fitted models
fitted(model1)
```

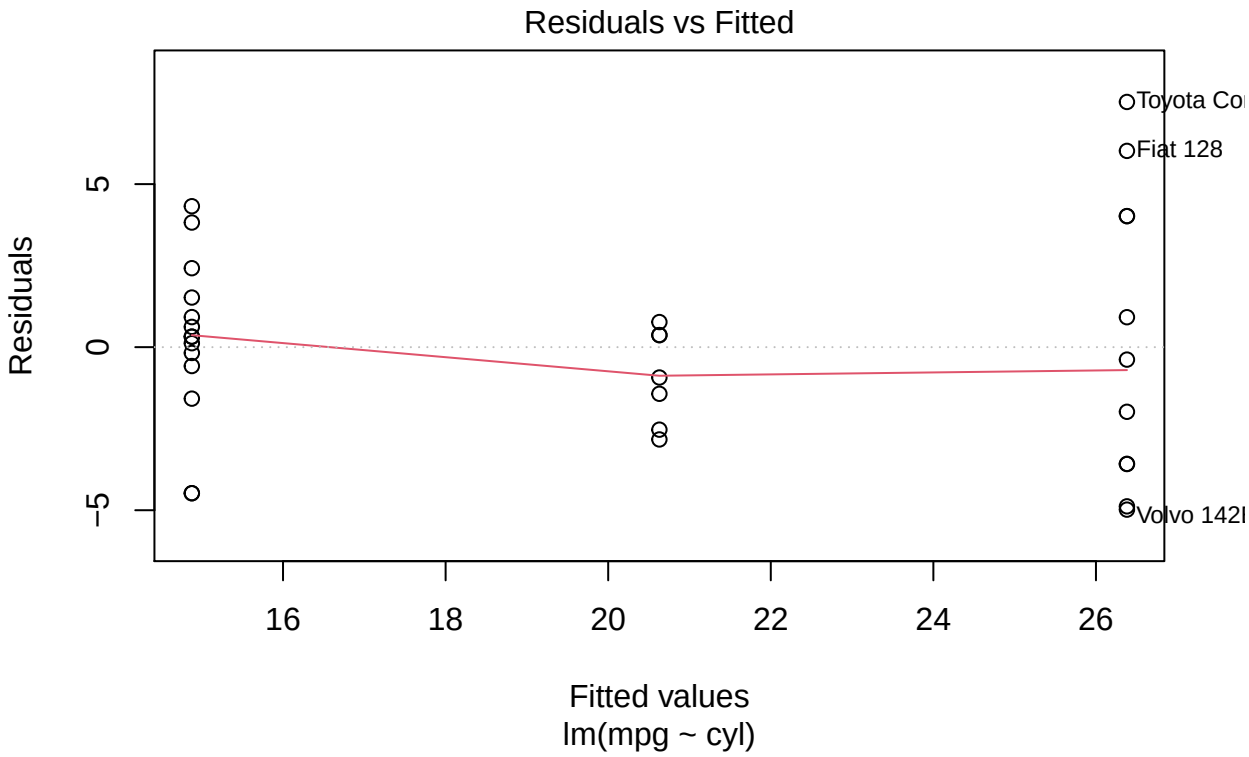
```
##           Mazda RX4       Mazda RX4 Wag          Datsun 710      Hornet 4 Drive
##            20.62984            20.62984            26.38142            20.62984
##   Hornet Sportabout             Valiant          Duster 360            Merc 240D
##            14.87826            20.62984            14.87826            26.38142
##            Merc 230            Merc 280            Merc 280C           Merc 450SE
##            26.38142            20.62984            20.62984            14.87826
##          Merc 450SL         Merc 450SLC  Cadillac Fleetwood Lincoln Continental
##            14.87826            14.87826            14.87826            14.87826
##   Chrysler Imperial            Fiat 128         Honda Civic      Toyota Corolla
##            14.87826            26.38142            26.38142            26.38142
```

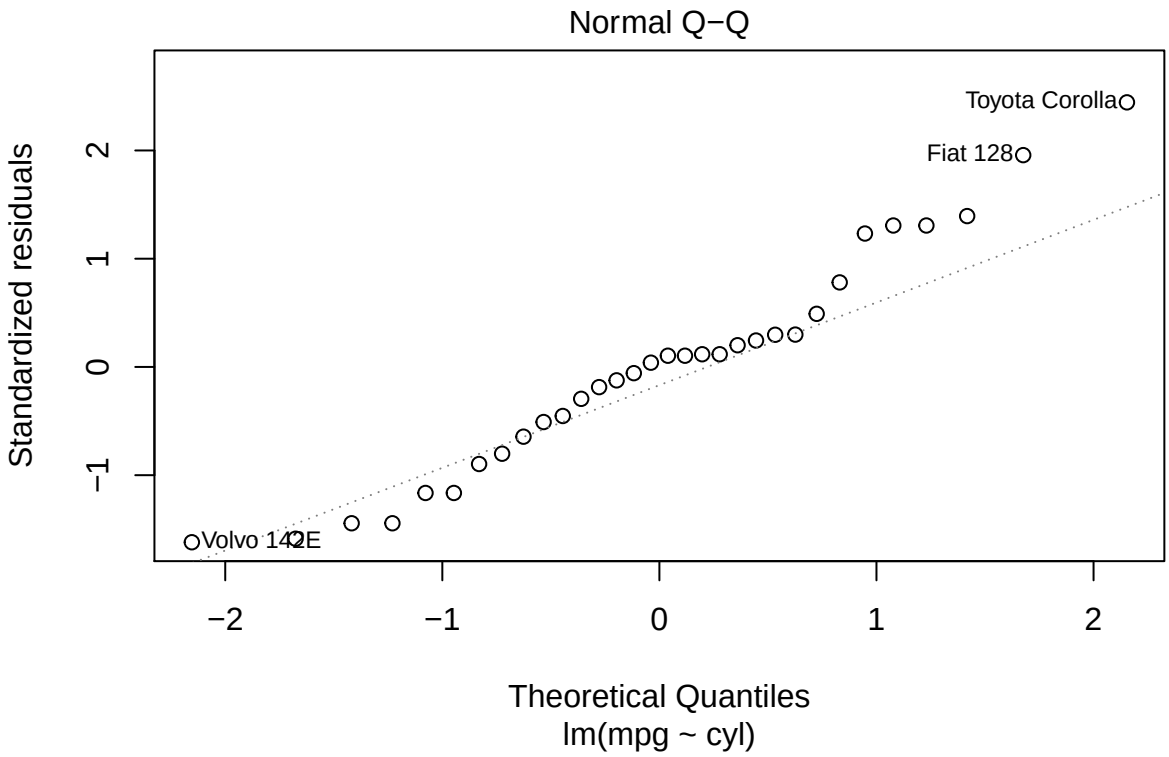
```
##       Toyota Corona    Dodge Challenger         AMC Javelin          Camaro Z28
##            26.38142            14.87826            14.87826            14.87826
##    Pontiac Firebird           Fiat X1-9       Porsche 914-2        Lotus Europa
##            14.87826            26.38142            26.38142            26.38142
##      Ford Pantera L        Ferrari Dino       Maserati Bora          Volvo 142E
##            14.87826            20.62984            14.87826            26.38142
```

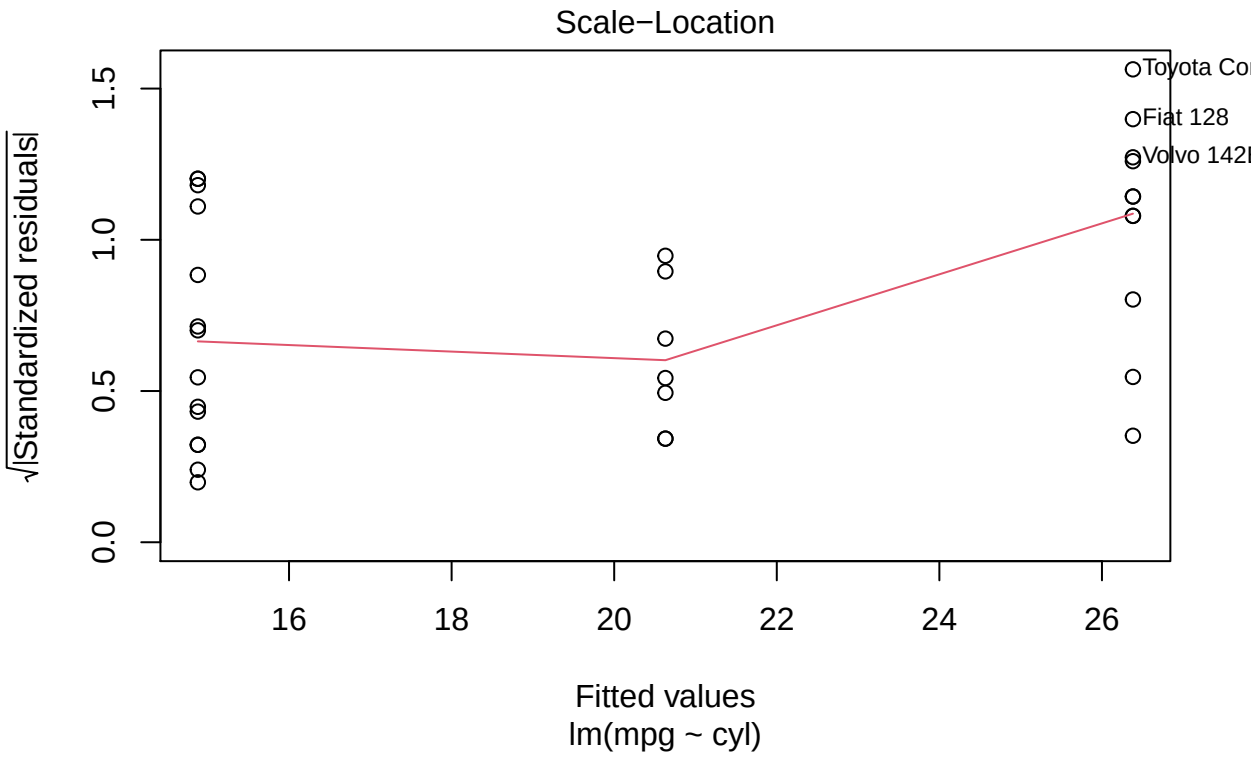
```
fitted(model2)
```

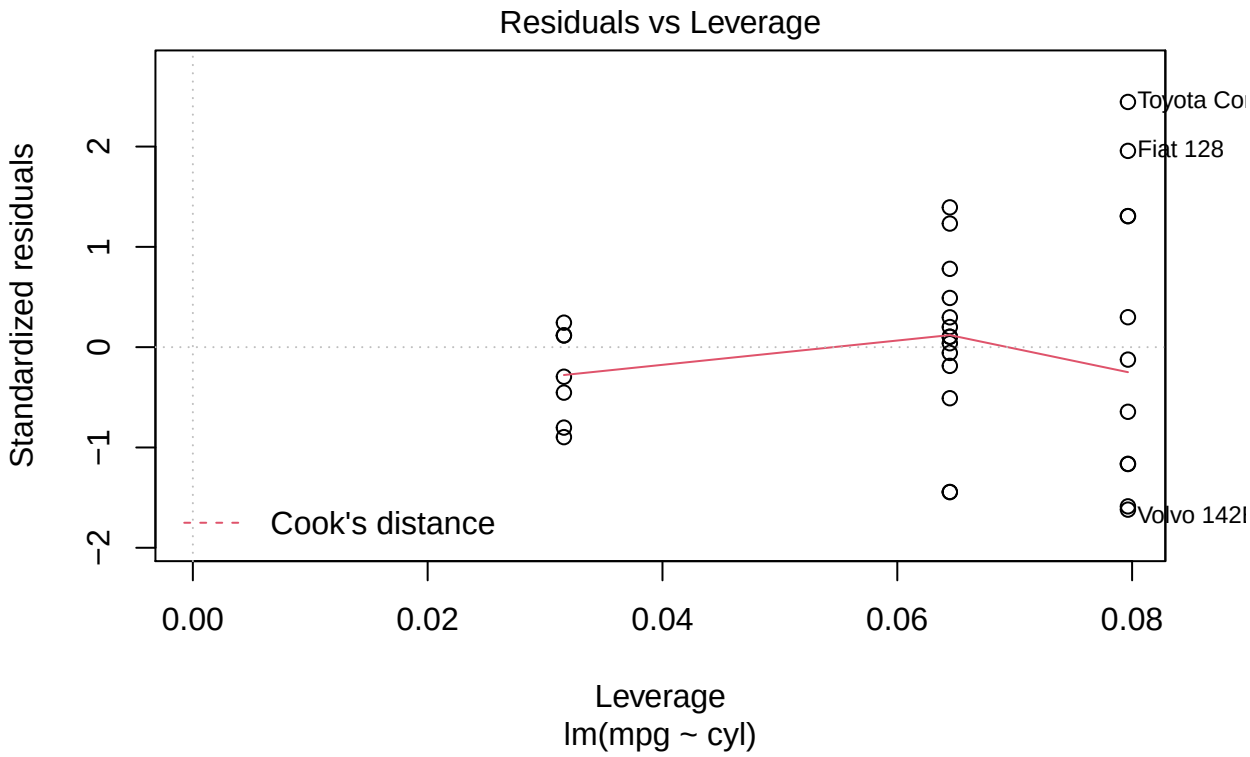
```
##           Mazda RX4       Mazda RX4 Wag          Datsun 710      Hornet 4 Drive
##           163.54431           192.22623           129.80087           230.46880
##   Hornet Sportabout             Valiant          Duster 360           Merc 240D
##           255.77638           258.02594           270.39854           227.65685
##            Merc 230            Merc 280           Merc 280C          Merc 450SE
##           223.15772           255.77638           255.77638           326.63761
##          Merc 450SL         Merc 450SLC  Cadillac Fleetwood Lincoln Continental
##           288.39504           294.01895           459.36181           478.93301
##   Chrysler Imperial            Fiat 128         Honda Civic      Toyota Corolla
##           470.04723           116.30349            50.50378            75.24897
##       Toyota Corona    Dodge Challenger         AMC Javelin          Camaro Z28
##           146.11020           264.77463           255.21399           300.76764
##    Pontiac Firebird           Fiat X1-9       Porsche 914-2        Lotus Europa
##           301.33003            86.49678           109.55480            39.03101
##      Ford Pantera L        Ferrari Dino       Maserati Bora          Volvo 142E
##           225.40728           180.41603           270.39854           181.54081
```

```
plot(model1)
```

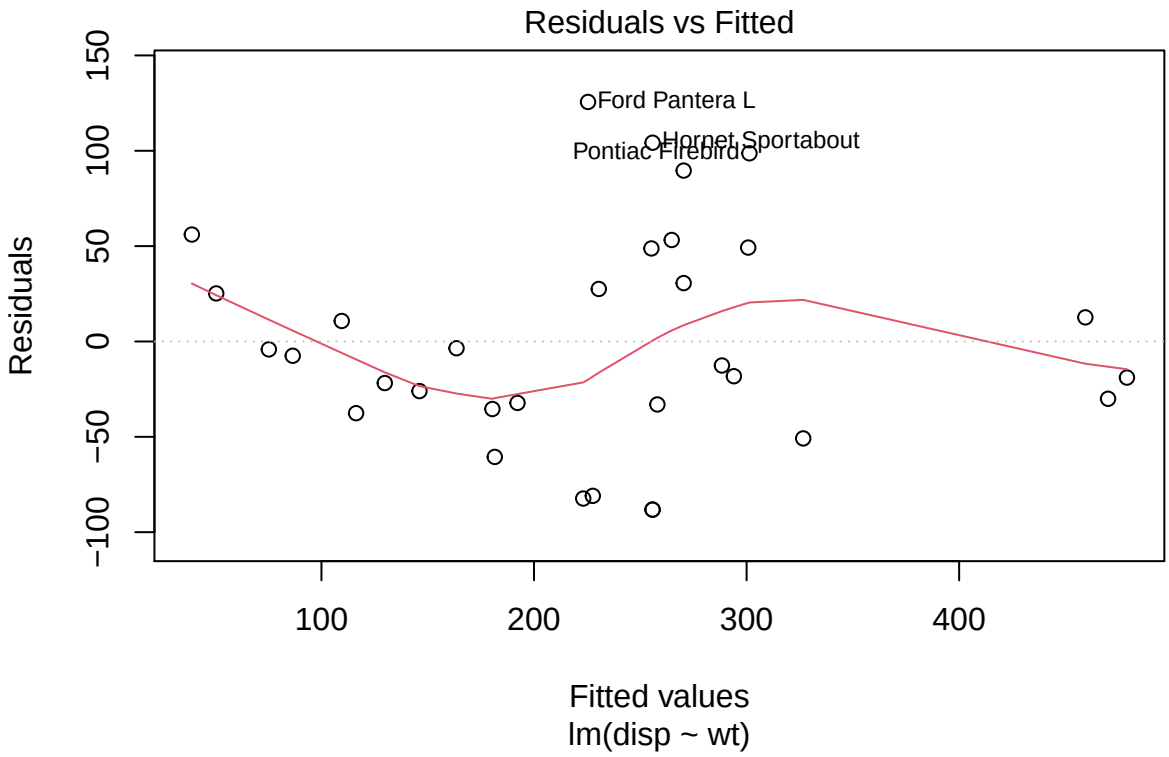

Residuals vs Fitted
Residuals
5
0
−5
16
18
20
22
24
26
Fitted values
lm(mpg ~ cyl)
Toyota Co
Fiat 128
Volvo 142E

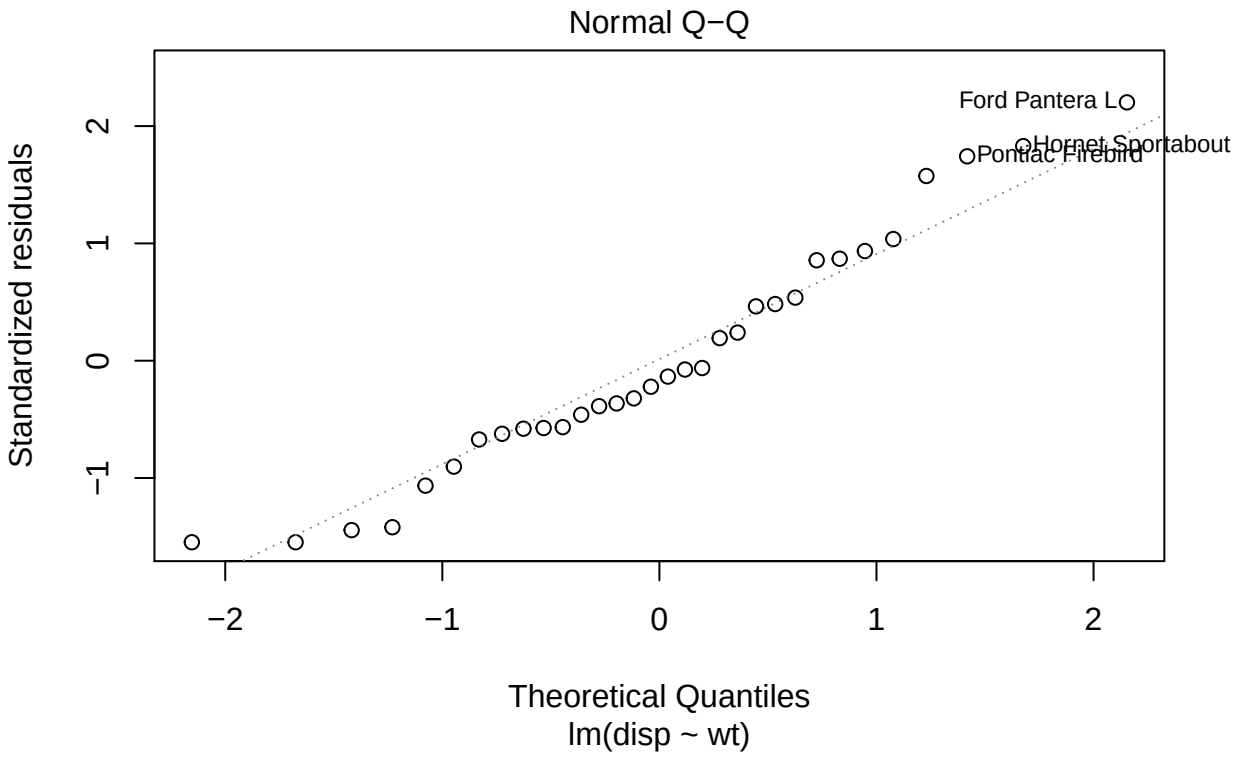

Normal Q−Q
Standardized residuals
Theoretical Quantiles
lm(mpg ~ cyl)
Toyota Corolla
Fiat 128
Volvo 142E
−2
−1
0
1
2

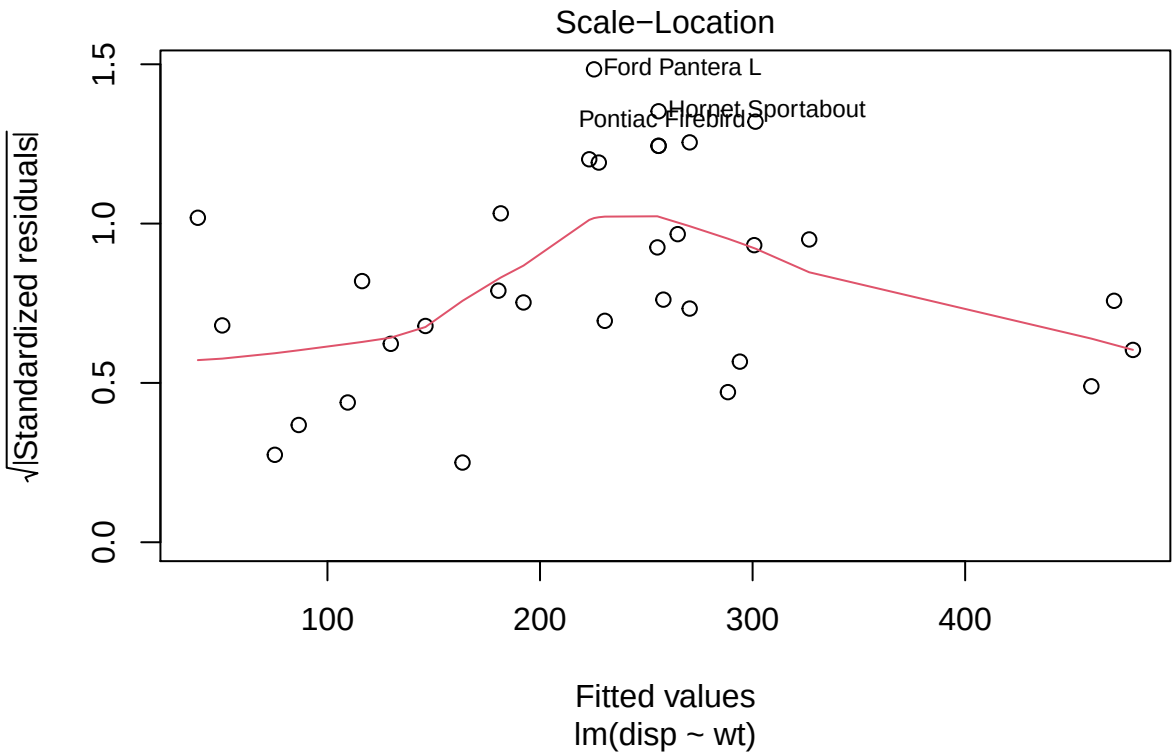

Scale−Location
√|Standardized residuals|
0.0
0.5
1.0
1.5
16
18
20
22
24
26
Fitted values
lm(mpg ~ cyl)
Toyota Co
Fiat 128
Volvo 142I

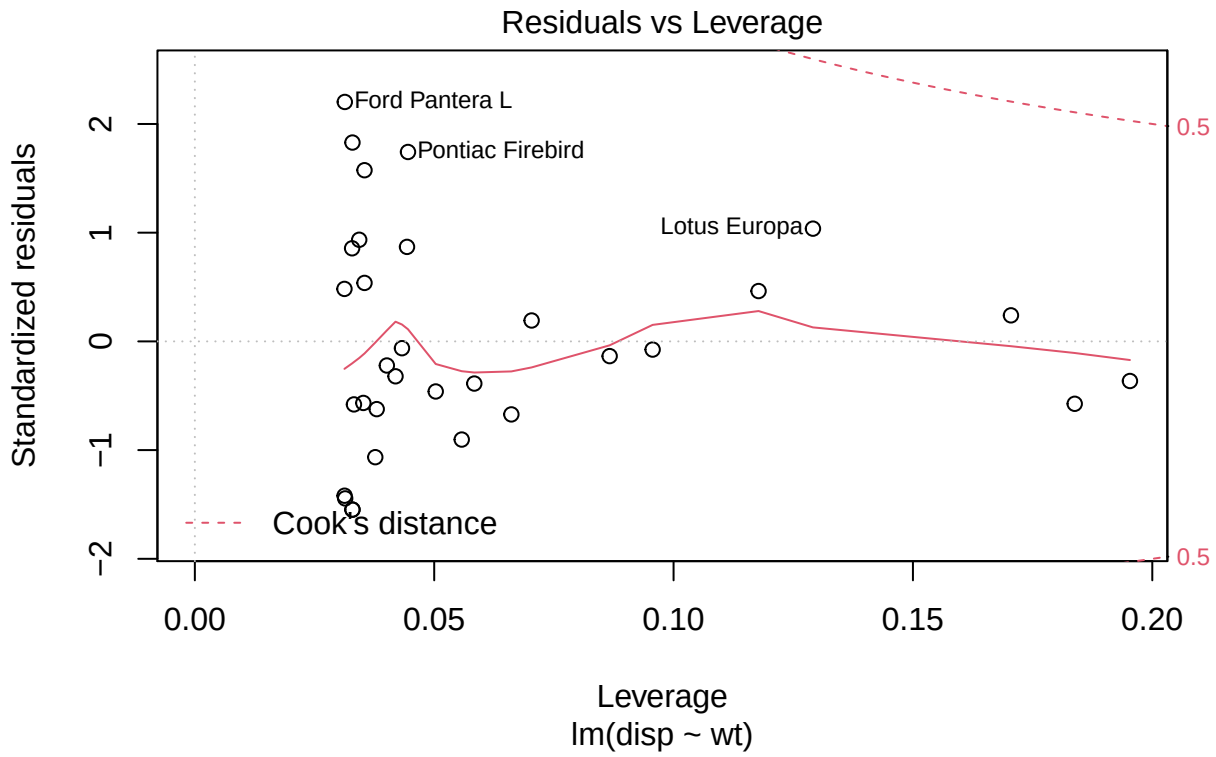

Residuals vs Leverage
Standardized residuals
2
1
0
−1
−2
0.00
0.02
0.04
0.06
0.08
Leverage
lm(mpg ~ cyl)
Cook's distance
Toyota Co
Fiat 128
Volvo 142E

```
plot(model2)
```


Residuals vs Fitted
Ford Pantera L
Hornet Sportabout
Pontiac Firebird
Residuals
150
100
50
0
−50
−100
100
200
300
400
Fitted values
lm(disp ~ wt)


Normal Q−Q
Standardized residuals
Theoretical Quantiles
lm(disp ~ wt)
Ford Pantera L
Hornet Sportabout
Pontiac Firebird
−2
−1
0
1
2


Scale−Location
Ford Pantera L
Hornet Sportabout
Pontiac Firebird
√|Standardized residuals|
0.0
0.5
1.0
1.5
100
200
300
400
Fitted values
lm(disp ~ wt)




```
### Predictions.
predict(model1, newdata = data.frame(cyl=c(5,7)))
```

```
##        1        2
## 23.50563 17.75405
```

```
predict(model1, newdata = data.frame(cyl=c(5,7)),se.fit = FALSE)
```

```
##        1        2
## 23.50563 17.75405
```

```
predict(model2, newdata = data.frame(wt=c(3,9)), se.fit = TRUE)
```

```
## $fit
##        1        2
## 206.2860 881.1548
##
## $se.fit
##        1        2
## 10.49971 62.34849
##
## $df
## [1] 30
##
```

```
## $residual.scale
## [1] 57.93937
```

```
predict(model2, newdata = data.frame(wt=c(3,9)), se.fit = TRUE)
```

```
## $fit
##        1        2
## 206.2860 881.1548
##
## $se.fit
##        1        2
## 10.49971 62.34849
##
## $df
## [1] 30
##
## $residual.scale
## [1] 57.93937
```

```
predict(model2, newdata = data.frame(wt=c(3,9)),interval = "confidence")
```

```
##        fit      lwr       upr
## 1 206.2860 184.8427  227.7293
## 2 881.1548 753.8222 1008.4874
```

```
predict(model1, newdata = data.frame(cyl=c(5,7)),se.fit = FALSE, interval = "prediction")
```

```
##        fit      lwr      upr
## 1 23.50563 16.81097 30.20028
## 2 17.75405 11.08372 24.42437
```

```
### Variable selection
### All possible regression. This test all the possible subsets of day set of potential independent var
library(olsrr)
```

```
##
## Attaching package: 'olsrr'

## The following object is masked from 'package:datasets':
##
##     rivers
```

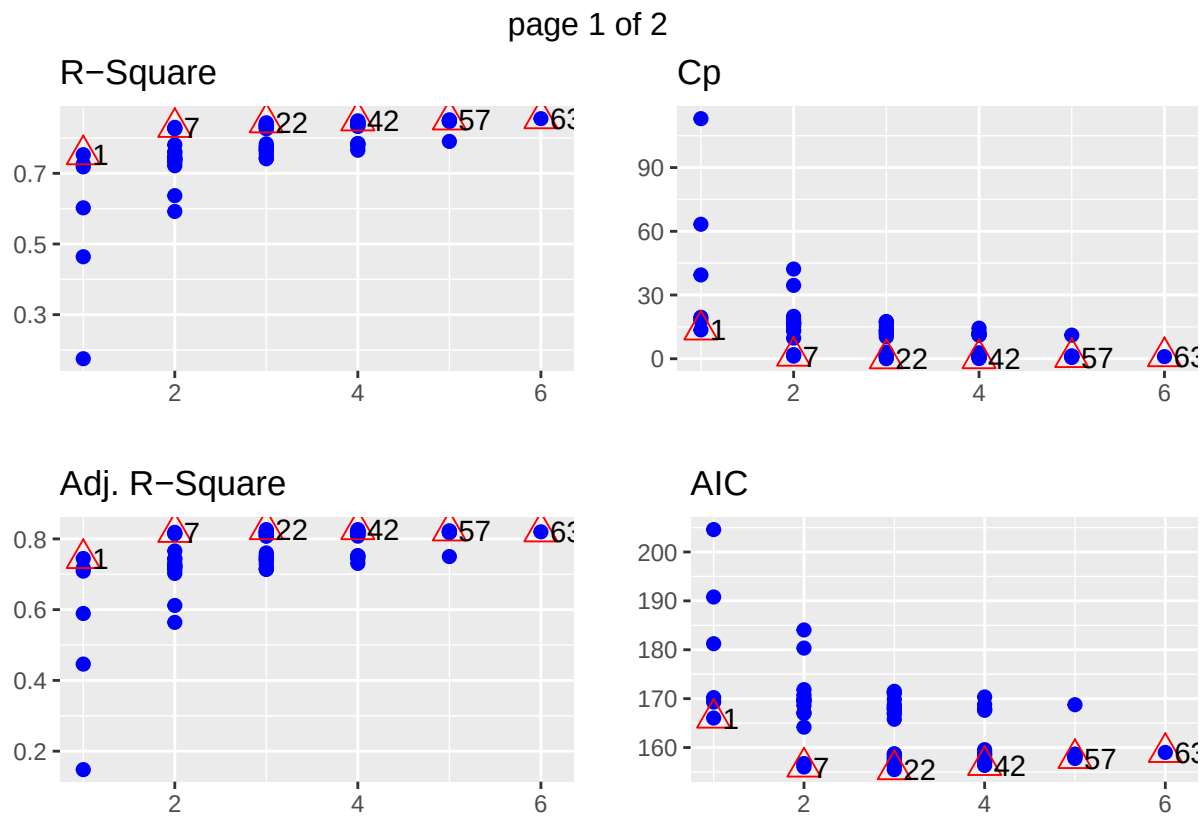
```
ols_step_all_possible(philant.model1, details=TRUE)
```

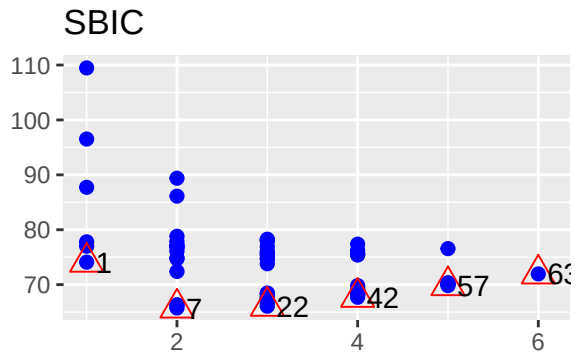
```
##    Index N               Predictors  R-Square Adj. R-Square Mallow's Cp
## 5      1 1                       wt 0.7528328     0.7445939   14.562907
## 1      2 1                      cyl 0.7261800     0.7170527   19.152594
## 2      3 1                     disp 0.7183433     0.7089548   20.502090
## 3      4 1                       hp 0.6024373     0.5891853   40.461438
## 4      5 1                     drat 0.4639952     0.4461283   64.301580
```

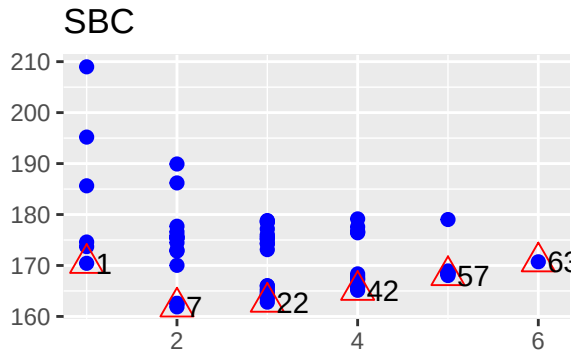
```
## 6      6 1                  qsec 0.1752963     0.1478062  114.016355
## 10     7 2                cyl wt 0.8302274     0.8185189    3.235333
## 17     8 2                 hp wt 0.8267855     0.8148396    3.828045
## 21     9 2               wt qsec 0.8264161     0.8144448    3.891644
## 14    10 2               disp wt 0.7809306     0.7658223   11.724387
## 19    11 2               drat wt 0.7608970     0.7444071   15.174232
## 7     12 2              cyl disp 0.7595658     0.7429841   15.403468
## 12    13 2               disp hp 0.7482402     0.7308774   17.353768
## 16    14 2               hp drat 0.7411716     0.7233214   18.570998
## 8     15 2                cyl hp 0.7407084     0.7228263   18.650758
## 9     16 2              cyl drat 0.7402482     0.7223343   18.730010
## 11    17 2              cyl qsec 0.7373272     0.7192119   19.233010
## 13    18 2             disp drat 0.7310094     0.7124583   20.320958
## 15    19 2             disp qsec 0.7215598     0.7023571   21.948200
## 18    20 2               hp qsec 0.6368769     0.6118339   36.530851
## 20    21 2             drat qsec 0.5921951     0.5640706   44.225178
## 27    22 3             cyl hp wt 0.8431500     0.8263446    3.010026
## 31    23 3           cyl wt qsec 0.8396119     0.8224275    3.619295
## 41    24 3          drat wt qsec 0.8370214     0.8195594    4.065383
## 38    25 3            hp drat wt 0.8368791     0.8194018    4.089899
## 40    26 3            hp wt qsec 0.8347678     0.8170643    4.453469
## 24    27 3           cyl disp wt 0.8326070     0.8146721    4.825552
## 29    28 3           cyl drat wt 0.8302283     0.8120385    5.235177
## 33    29 3            disp hp wt 0.8268361     0.8082829    5.819316
## 37    30 3          disp wt qsec 0.8264170     0.8078189    5.891488
## 35    31 3          disp drat wt 0.7835315     0.7603385   13.276498
## 28    32 3           cyl hp qsec 0.7757451     0.7517177   14.617349
## 32    33 3          disp hp drat 0.7750131     0.7509073   14.743399
## 26    34 3           cyl hp drat 0.7693992     0.7446920   15.710117
## 22    35 3           cyl disp hp 0.7678877     0.7430186   15.970401
## 23    36 3         cyl disp drat 0.7650941     0.7399256   16.451482
## 25    37 3         cyl disp qsec 0.7626594     0.7372300   16.870738
## 34    38 3          disp hp qsec 0.7541953     0.7278591   18.328281
## 39    39 3          hp drat qsec 0.7442512     0.7168495   20.040681
## 30    40 3         cyl drat qsec 0.7424811     0.7148898   20.345503
## 36    41 3        disp drat qsec 0.7412673     0.7135459   20.554521
## 43    42 4         cyl disp hp wt 0.8486348     0.8262103    4.065526
## 56    43 4       hp drat wt qsec 0.8453853     0.8224794    4.625103
## 48    44 4         cyl hp drat wt 0.8451439     0.8222023    4.666661
## 50    45 4         cyl hp wt qsec 0.8443942     0.8213415    4.795768
## 47    46 4       cyl disp wt qsec 0.8439955     0.8208838    4.864420
## 51    47 4       cyl drat wt qsec 0.8419825     0.8185725    5.211076
## 55    48 4      disp drat wt qsec 0.8383592     0.8144124    5.835012
## 52    49 4        disp hp drat wt 0.8376289     0.8135739    5.960779
## 54    50 4        disp hp wt qsec 0.8351443     0.8107212    6.388632
## 45    51 4       cyl disp drat wt 0.8326074     0.8078085    6.825487
## 44    52 4       cyl disp hp qsec 0.7846156     0.7527068   15.089817
## 49    53 4       cyl hp drat qsec 0.7839256     0.7519146   15.208629
## 42    54 4       cyl disp hp drat 0.7825119     0.7502914   15.452087
## 53    55 4      disp hp drat qsec 0.7766318     0.7435402   16.464646
## 46    56 4     cyl disp drat qsec 0.7654719     0.7307270   18.386417
## 57    57 5    cyl disp hp drat wt 0.8513152     0.8227219    5.603955
## 59    58 5    cyl disp hp wt qsec 0.8501969     0.8213886    5.796524
## 62    59 5   disp hp drat wt qsec 0.8489147     0.8198599    6.017318
```

```
## 61    60 5      cyl hp drat wt qsec 0.8478334     0.8185706    6.203520
## 60    61 5    cyl disp drat wt qsec 0.8470572     0.8176451    6.337198
## 58    62 5    cyl disp hp drat qsec 0.7903665     0.7500523   16.099497
## 63    63 6 cyl disp hp drat wt qsec 0.8548224     0.8199798    7.000000
```

```
philant<-ols_step_all_possible(philant.model1)
plot(philant)
```





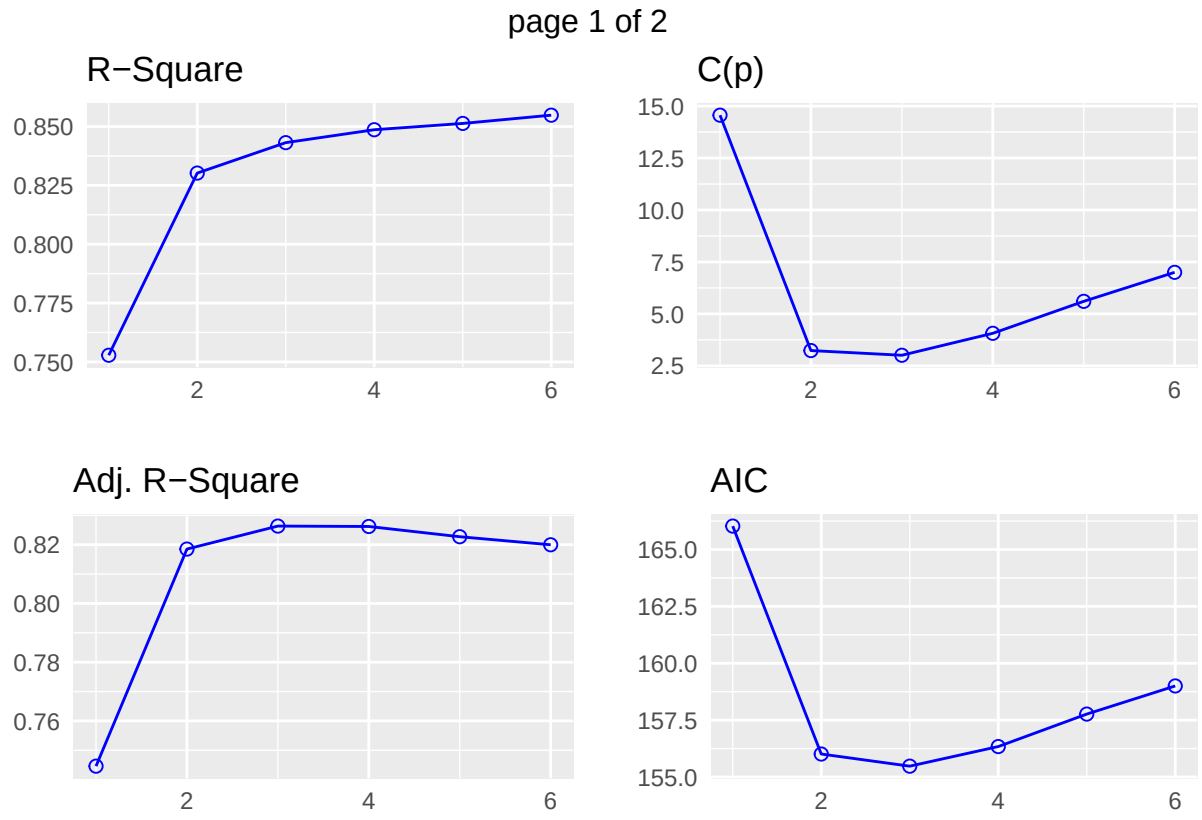


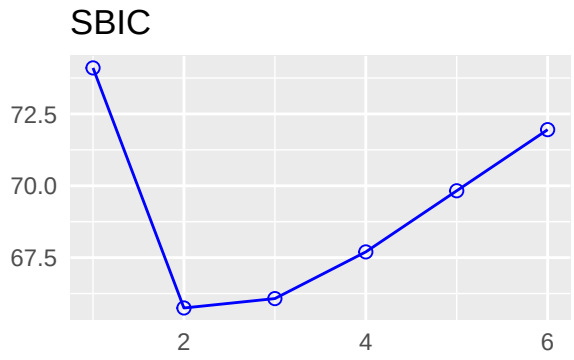

```
### BEST SUBSET REGRESSION. This selects the best subset of predictors that's do the best at meeting so
ols_step_best_subset(philant.model1, details=TRUE)
```

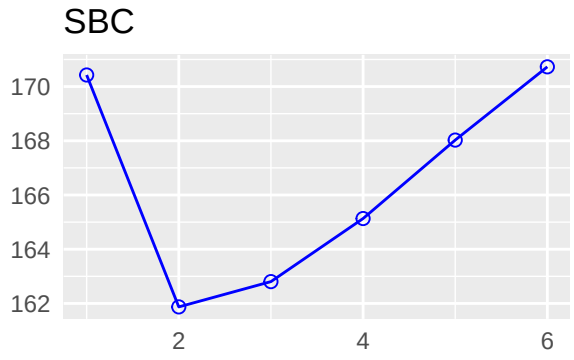
```
##         Best Subsets Regression
## ---------------------------------------
## Model Index    Predictors
## ---------------------------------------
##      1         wt
##      2         cyl wt
##      3         cyl hp wt
##      4         cyl disp hp wt
##      5         cyl disp hp drat wt
##      6         cyl disp hp drat wt qsec
## ---------------------------------------
##
##                                                    Subsets Regression Summary
## ------------------------------------------------------------------------------------------------------------
##                        Adj.        Pred
## Model    R-Square    R-Square    R-Square     C(p)        AIC        SBIC        SBC         MSEP
## ------------------------------------------------------------------------------------------------------------
##   1        0.7528      0.7446      0.7087    14.5629    166.0294    74.1015    170.4266    296.9167
##   2        0.8302      0.8185      0.7904     3.2353    156.0101    65.7475    161.8730    211.2280
##   3        0.8431      0.8263      0.7957     3.0100    155.4766    66.0743    162.8053    202.3777
##   4        0.8486      0.8262      0.7915     4.0655    156.3376    67.6986    165.1320    202.8124
##   5        0.8513      0.8227      0.7803     5.6040    157.7659    69.8267    168.0260    207.1898
##   6        0.8548      0.8200      0.7675     7.0000    159.0020    71.9531    170.7279    210.7318
```

```
## ----------------------------------------------------------------------------------------------------
## AIC: Akaike Information Criteria
##  SBIC: Sawa's Bayesian Information Criteria
##  SBC: Schwarz Bayesian Criteria
##  MSEP: Estimated error of prediction, assuming multivariate normality
##  FPE: Final Prediction Error
##  HSP: Hocking's Sp
##  APC: Amemiya Prediction Criteria
```

```
philant1<-ols_step_best_subset(philant.model1)
plot(philant1)
```








```
### Stepwise forward regression. This put in one variable at a time and check which one is the best, an
ols_step_forward_p(philant.model1, details = TRUE)
```

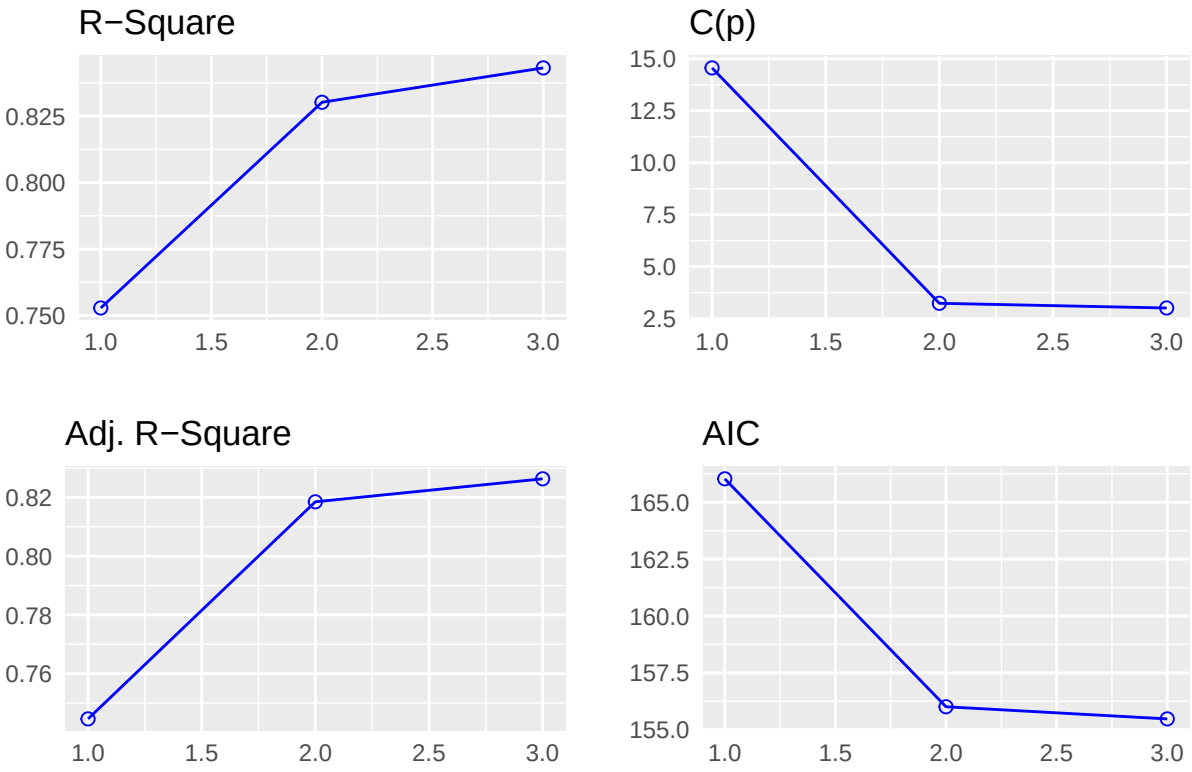
```
## Forward Selection Method
## ---------------------------
##
## Candidate Terms:
##
## 1. cyl
## 2. disp
## 3. hp
## 4. drat
## 5. wt
## 6. qsec
##
## We are selecting variables based on p value...
##
##
## Forward Selection: Step 1
##
## - wt
##
##                         Model Summary
## ----------------------------------------------------------------
## R                       0.868       RMSE                 3.046
## R-Squared               0.753       Coef. Var           15.161
```

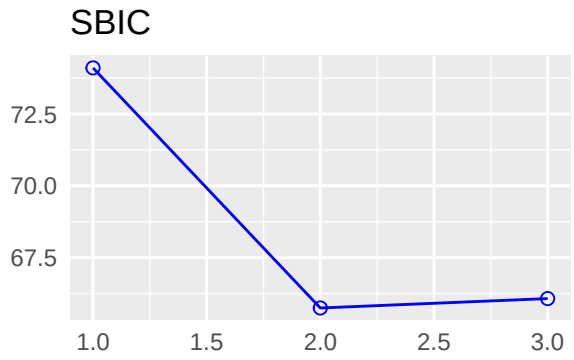
```
## Adj. R-Squared          0.745       MSE                 9.277
## Pred R-Squared          0.709       MAE                 2.341
## ----------------------------------------------------------------
##  RMSE: Root Mean Square Error
##  MSE: Mean Square Error
##  MAE: Mean Absolute Error
##
##                               ANOVA
## --------------------------------------------------------------------
##                Sum of
##               Squares        DF    Mean Square      F         Sig.
## --------------------------------------------------------------------
## Regression     847.725         1        847.725    91.375    0.0000
## Residual       278.322        30          9.277
## Total         1126.047        31
## --------------------------------------------------------------------
##
##                                 Parameter Estimates
## ----------------------------------------------------------------------------------------
##       model      Beta    Std. Error    Std. Beta      t        Sig      lower     upper
## ----------------------------------------------------------------------------------------
## (Intercept)    37.285         1.878                 19.858    0.000    33.450    41.120
##          wt    -5.344         0.559       -0.868    -9.559    0.000    -6.486    -4.203
## ----------------------------------------------------------------------------------------
##
##
##
## Forward Selection: Step 2
##
## - cyl
##
##                          Model Summary
## ----------------------------------------------------------------
## R                       0.911       RMSE                2.568
## R-Squared               0.830       Coef. Var          12.780
## Adj. R-Squared          0.819       MSE                 6.592
## Pred R-Squared          0.790       MAE                 1.921
## ----------------------------------------------------------------
##  RMSE: Root Mean Square Error
##  MSE: Mean Square Error
##  MAE: Mean Absolute Error
##
##                               ANOVA
## --------------------------------------------------------------------
##                Sum of
##               Squares        DF    Mean Square      F         Sig.
## --------------------------------------------------------------------
## Regression     934.875         2        467.438    70.908    0.0000
## Residual       191.172        29          6.592
## Total         1126.047        31
## --------------------------------------------------------------------
##
##                                 Parameter Estimates
## ----------------------------------------------------------------------------------------
```

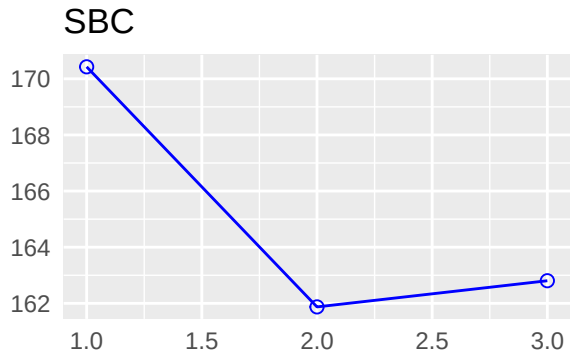
```
##       model      Beta    Std. Error    Std. Beta      t        Sig      lower     upper
## ------------------------------------------------------------------------------------------
## (Intercept)    39.686         1.715                23.141    0.000    36.179    43.194
##          wt    -3.191         0.757       -0.518    -4.216    0.000    -4.739    -1.643
##         cyl    -1.508         0.415       -0.447    -3.636    0.001    -2.356    -0.660
## ------------------------------------------------------------------------------------------
##
##
##
## Forward Selection: Step 3
##
## - hp
##
##                         Model Summary
## ----------------------------------------------------------------
## R                       0.918       RMSE                 2.512
## R-Squared               0.843       Coef. Var           12.501
## Adj. R-Squared          0.826       MSE                  6.308
## Pred R-Squared          0.796       MAE                  1.845
## ----------------------------------------------------------------
##  RMSE: Root Mean Square Error
##  MSE: Mean Square Error
##  MAE: Mean Absolute Error
##
##                                ANOVA
## -------------------------------------------------------------------
##                Sum of
##               Squares        DF    Mean Square      F         Sig.
## -------------------------------------------------------------------
## Regression     949.427         3        316.476    50.171    0.0000
## Residual       176.621        28          6.308
## Total         1126.047        31
## -------------------------------------------------------------------
##
##                                  Parameter Estimates
## ------------------------------------------------------------------------------------------
##       model      Beta    Std. Error    Std. Beta      t        Sig      lower     upper
## ------------------------------------------------------------------------------------------
## (Intercept)    38.752         1.787                21.687    0.000    35.092    42.412
##          wt    -3.167         0.741       -0.514    -4.276    0.000    -4.684    -1.650
##         cyl    -0.942         0.551       -0.279    -1.709    0.098    -2.070     0.187
##          hp    -0.018         0.012       -0.205    -1.519    0.140    -0.042     0.006
## ------------------------------------------------------------------------------------------
##
##
##
## No more variables to be added.
##
## Variables Entered:
##
## + wt
## + cyl
## + hp
##
```

```
##
## Final Model Output
## ------------------
##
##                         Model Summary
## ------------------------------------------------------------
## R                       0.918       RMSE                2.512
## R-Squared               0.843       Coef. Var          12.501
## Adj. R-Squared          0.826       MSE                 6.308
## Pred R-Squared          0.796       MAE                 1.845
## ------------------------------------------------------------
##  RMSE: Root Mean Square Error
##  MSE: Mean Square Error
##  MAE: Mean Absolute Error
##
##                                ANOVA
## -------------------------------------------------------------------
##                 Sum of
##                Squares        DF    Mean Square      F         Sig.
## -------------------------------------------------------------------
## Regression     949.427         3        316.476    50.171    0.0000
## Residual       176.621        28          6.308
## Total         1126.047        31
## -------------------------------------------------------------------
##
##                                 Parameter Estimates
## ----------------------------------------------------------------------------------------------
##       model      Beta    Std. Error    Std. Beta      t        Sig      lower     upper
## ----------------------------------------------------------------------------------------------
## (Intercept)    38.752         1.787                21.687    0.000    35.092    42.412
##          wt    -3.167         0.741       -0.514    -4.276    0.000    -4.684    -1.650
##         cyl    -0.942         0.551       -0.279    -1.709    0.098    -2.070     0.187
##          hp    -0.018         0.012       -0.205    -1.519    0.140    -0.042     0.006
## ----------------------------------------------------------------------------------------------


##
##                            Selection Summary
## ------------------------------------------------------------------------
##         Variable                    Adj.
## Step    Entered     R-Square    R-Square     C(p)        AIC        RMSE
## ------------------------------------------------------------------------
##    1    wt            0.7528      0.7446    14.5629    166.0294    3.0459
##    2    cyl           0.8302      0.8185     3.2353    156.0101    2.5675
##    3    hp            0.8431      0.8263     3.0100    155.4766    2.5115
## ------------------------------------------------------------------------
```

```
philant3<-ols_step_forward_p(philant.model1)
plot(philant3)
```


R−Square
0.825
0.800
0.775
0.750
1.0
1.5
2.0
2.5
3.0
C(p)
15.0
12.5
10.0
7.5
5.0
2.5
1.0
1.5
2.0
2.5
3.0
Adj. R−Square
0.82
0.80
0.78
0.76
1.0
1.5
2.0
2.5
3.0
AIC
165.0
162.5
160.0
157.5
155.0
1.0
1.5
2.0
2.5
3.0






```
### Backwards elimination. this starts with all the variable, check the worst one and remove it till we
ols_step_backward_p(philant.model1, details = TRUE)
```

```
## Backward Elimination Method
## ---------------------------
##
## Candidate Terms:
##
## 1 . cyl
## 2 . disp
## 3 . hp
## 4 . drat
## 5 . wt
## 6 . qsec
##
## We are eliminating variables based on p value...
##
## - qsec
##
## Backward Elimination: Step 1
##
##  Variable qsec Removed
##
##                         Model Summary
## ------------------------------------------------------------------
## R                       0.923       RMSE                 2.538
```

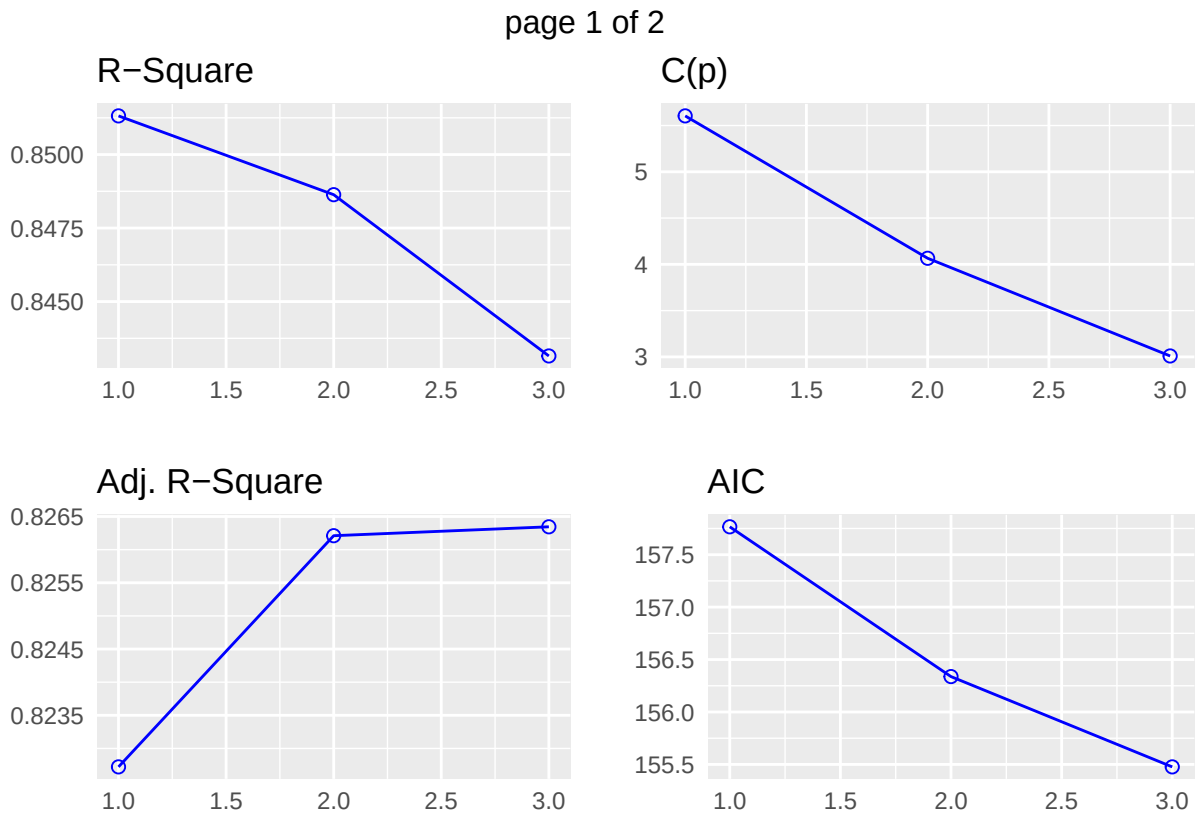
```
## R-Squared               0.851       Coef. Var          12.631
## Adj. R-Squared          0.823       MSE                 6.439
## Pred R-Squared          0.780       MAE                 1.795
## ----------------------------------------------------------------
##  RMSE: Root Mean Square Error
##  MSE: Mean Square Error
##  MAE: Mean Absolute Error
##
##                                ANOVA
## --------------------------------------------------------------------
##                 Sum of
##                Squares        DF    Mean Square      F         Sig.
## --------------------------------------------------------------------
## Regression     958.621         5        191.724    29.773    0.0000
## Residual       167.426        26          6.439
## Total         1126.047        31
## --------------------------------------------------------------------
##
##                                  Parameter Estimates
## ----------------------------------------------------------------------------------------------
##       model      Beta    Std. Error    Std. Beta      t        Sig      lower     upper
## ----------------------------------------------------------------------------------------------
## (Intercept)    36.008         7.571                 4.756    0.000    20.445    51.572
##         cyl    -1.107         0.716       -0.328    -1.547    0.134    -2.579     0.364
##        disp     0.012         0.012        0.254     1.039    0.308    -0.012     0.037
##          hp    -0.024         0.013       -0.273    -1.809    0.082    -0.051     0.003
##        drat     0.952         1.391        0.084     0.685    0.500    -1.907     3.811
##          wt    -3.673         1.059       -0.596    -3.469    0.002    -5.850    -1.496
## ----------------------------------------------------------------------------------------------
##
##
## - drat
##
## Backward Elimination: Step 2
##
##  Variable drat Removed
##
##                          Model Summary
## ---------------------------------------------------------------
## R                       0.921       RMSE                2.513
## R-Squared               0.849       Coef. Var          12.506
## Adj. R-Squared          0.826       MSE                 6.313
## Pred R-Squared          0.791       MAE                 1.771
## ---------------------------------------------------------------
##  RMSE: Root Mean Square Error
##  MSE: Mean Square Error
##  MAE: Mean Absolute Error
##
##                                ANOVA
## --------------------------------------------------------------------
##                 Sum of
##                Squares        DF    Mean Square      F         Sig.
## --------------------------------------------------------------------
## Regression     955.603         4        238.901    37.844    0.0000
```

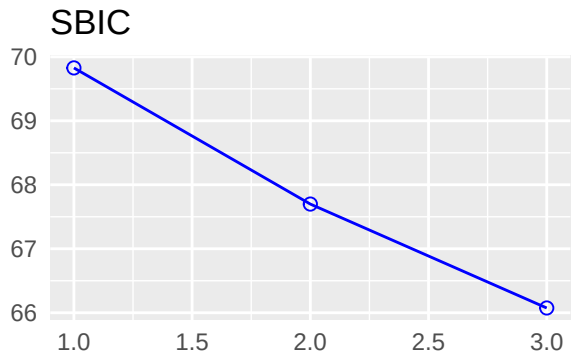
```
## Residual       170.444        27          6.313
## Total         1126.047        31
## ----------------------------------------------------------------------
##
##                                 Parameter Estimates
## ----------------------------------------------------------------------------------------------
##       model      Beta    Std. Error    Std. Beta      t        Sig      lower     upper
## ----------------------------------------------------------------------------------------------
## (Intercept)    40.829         2.757                14.807    0.000    35.171    46.486
##         cyl    -1.293         0.656       -0.383    -1.972    0.059    -2.639     0.052
##        disp     0.012         0.012        0.239     0.989    0.331    -0.012     0.036
##          hp    -0.021         0.012       -0.234    -1.691    0.102    -0.045     0.004
##          wt    -3.854         1.015       -0.626    -3.795    0.001    -5.937    -1.770
## ----------------------------------------------------------------------------------------------
##
##
## - disp
##
## Backward Elimination: Step 3
##
##  Variable disp Removed
##
##                         Model Summary
## ---------------------------------------------------------------
## R                       0.918       RMSE                 2.512
## R-Squared               0.843       Coef. Var           12.501
## Adj. R-Squared          0.826       MSE                  6.308
## Pred R-Squared          0.796       MAE                  1.845
## ---------------------------------------------------------------
##  RMSE: Root Mean Square Error
##  MSE: Mean Square Error
##  MAE: Mean Absolute Error
##
##                                ANOVA
## ----------------------------------------------------------------------
##                 Sum of
##                Squares        DF    Mean Square      F         Sig.
## ----------------------------------------------------------------------
## Regression     949.427         3        316.476    50.171    0.0000
## Residual       176.621        28          6.308
## Total         1126.047        31
## ----------------------------------------------------------------------
##
##                                 Parameter Estimates
## ----------------------------------------------------------------------------------------------
##       model      Beta    Std. Error    Std. Beta      t        Sig      lower     upper
## ----------------------------------------------------------------------------------------------
## (Intercept)    38.752         1.787                21.687    0.000    35.092    42.412
##         cyl    -0.942         0.551       -0.279    -1.709    0.098    -2.070     0.187
##          hp    -0.018         0.012       -0.205    -1.519    0.140    -0.042     0.006
##          wt    -3.167         0.741       -0.514    -4.276    0.000    -4.684    -1.650
## ----------------------------------------------------------------------------------------------
##
##
```

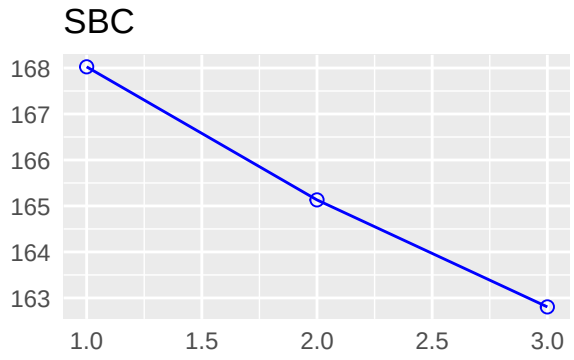
```
##
## No more variables satisfy the condition of p value = 0.3
##
##
## Variables Removed:
##
## - qsec
## - drat
## - disp
##
##
## Final Model Output
## ------------------
##
##                         Model Summary
## ----------------------------------------------------------------
## R                       0.918       RMSE                 2.512
## R-Squared               0.843       Coef. Var           12.501
## Adj. R-Squared          0.826       MSE                  6.308
## Pred R-Squared          0.796       MAE                  1.845
## ----------------------------------------------------------------
##  RMSE: Root Mean Square Error
##  MSE: Mean Square Error
##  MAE: Mean Absolute Error
##
##                               ANOVA
## --------------------------------------------------------------------
##                Sum of
##               Squares        DF    Mean Square      F         Sig.
## --------------------------------------------------------------------
## Regression     949.427         3        316.476    50.171    0.0000
## Residual       176.621        28          6.308
## Total         1126.047        31
## --------------------------------------------------------------------
##
##                                 Parameter Estimates
## ----------------------------------------------------------------------------------------
##       model      Beta    Std. Error    Std. Beta      t        Sig      lower     upper
## ----------------------------------------------------------------------------------------
## (Intercept)    38.752         1.787                 21.687    0.000    35.092    42.412
##         cyl    -0.942         0.551       -0.279    -1.709    0.098    -2.070     0.187
##          hp    -0.018         0.012       -0.205    -1.519    0.140    -0.042     0.006
##          wt    -3.167         0.741       -0.514    -4.276    0.000    -4.684    -1.650
## ----------------------------------------------------------------------------------------


##
##
##                           Elimination Summary
## ------------------------------------------------------------------------
##         Variable                  Adj.
## Step    Removed     R-Square    R-Square     C(p)       AIC        RMSE
## ------------------------------------------------------------------------
##    1    qsec          0.8513      0.8227    5.6040    157.7659    2.5376
##    2    drat          0.8486      0.8262    4.0655    156.3376    2.5125
```

```
##     3     disp           0.8431       0.8263     3.0100     155.4766     2.5115
## ---------------------------------------------------------------------------------
```

```
philant4<-ols_step_backward_p(philant.model1)
plot(philant4)
```








```
### Step-wise regression. In this regression there are no variables left to enter or remove any more. T
ols_step_both_p(philant.model1, details = TRUE)
```

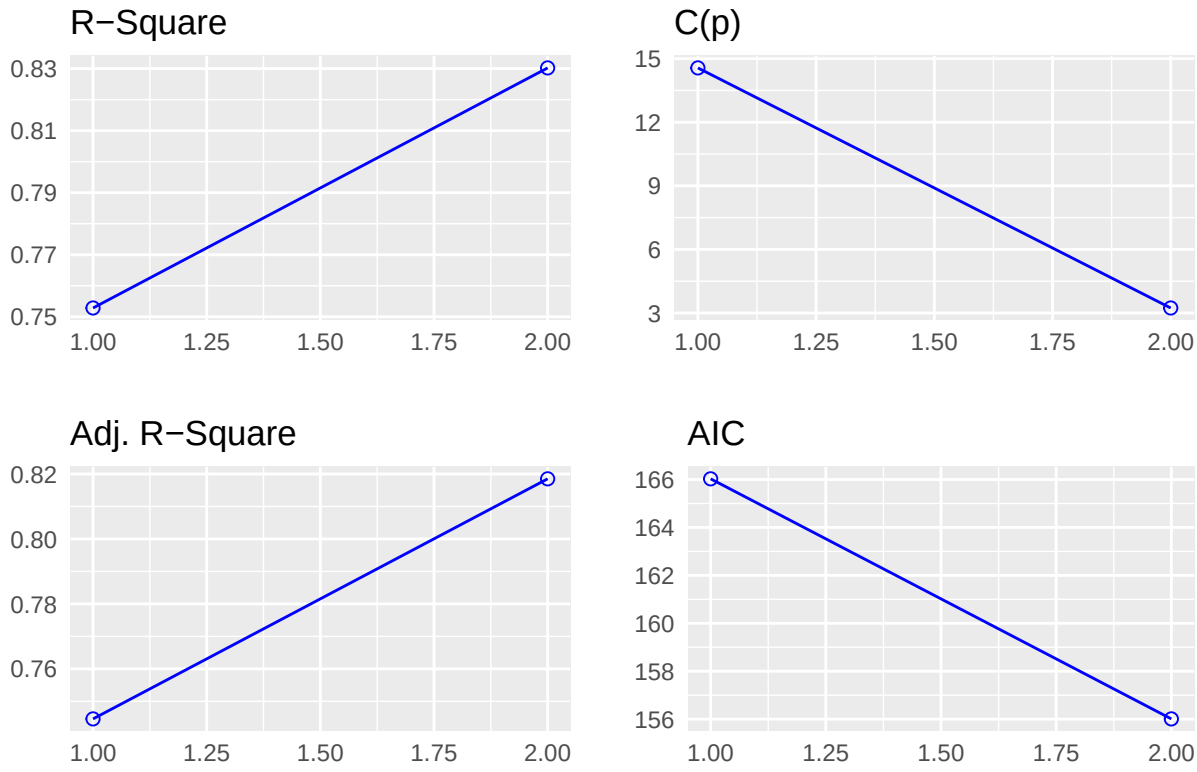
```
## Stepwise Selection Method
## ---------------------------
##
## Candidate Terms:
##
## 1. cyl
## 2. disp
## 3. hp
## 4. drat
## 5. wt
## 6. qsec
##
## We are selecting variables based on p value...
##
##
## Stepwise Selection: Step 1
##
## - wt added
##
##                         Model Summary
## ----------------------------------------------------------------
## R                       0.868       RMSE                 3.046
## R-Squared               0.753       Coef. Var           15.161
```

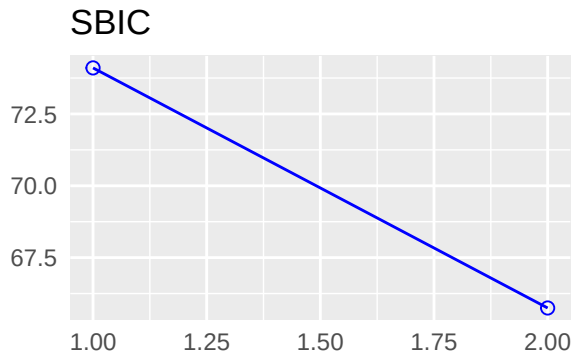
```
## Adj. R-Squared          0.745       MSE                 9.277
## Pred R-Squared          0.709       MAE                 2.341
## ----------------------------------------------------------------
##  RMSE: Root Mean Square Error
##  MSE: Mean Square Error
##  MAE: Mean Absolute Error
##
##                               ANOVA
## --------------------------------------------------------------------
##                Sum of
##               Squares        DF    Mean Square      F         Sig.
## --------------------------------------------------------------------
## Regression     847.725         1        847.725    91.375    0.0000
## Residual       278.322        30          9.277
## Total         1126.047        31
## --------------------------------------------------------------------
##
##                                  Parameter Estimates
## ----------------------------------------------------------------------------------------
##       model      Beta    Std. Error    Std. Beta      t        Sig      lower     upper
## ----------------------------------------------------------------------------------------
## (Intercept)    37.285         1.878                 19.858    0.000    33.450    41.120
##          wt    -5.344         0.559       -0.868    -9.559    0.000    -6.486    -4.203
## ----------------------------------------------------------------------------------------
##
##
##
## Stepwise Selection: Step 2
##
## - cyl added
##
##                         Model Summary
## ----------------------------------------------------------------
## R                       0.911       RMSE                2.568
## R-Squared               0.830       Coef. Var          12.780
## Adj. R-Squared          0.819       MSE                 6.592
## Pred R-Squared          0.790       MAE                 1.921
## ----------------------------------------------------------------
##  RMSE: Root Mean Square Error
##  MSE: Mean Square Error
##  MAE: Mean Absolute Error
##
##                               ANOVA
## --------------------------------------------------------------------
##                Sum of
##               Squares        DF    Mean Square      F         Sig.
## --------------------------------------------------------------------
## Regression     934.875         2        467.438    70.908    0.0000
## Residual       191.172        29          6.592
## Total         1126.047        31
## --------------------------------------------------------------------
##
##                                  Parameter Estimates
## ----------------------------------------------------------------------------------------
```

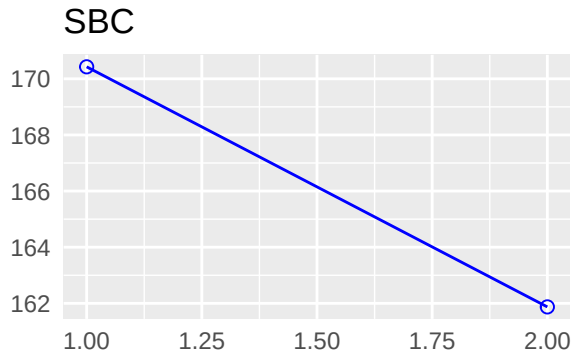
```
##       model      Beta    Std. Error    Std. Beta      t        Sig      lower     upper
## ---------------------------------------------------------------------------------------
## (Intercept)    39.686         1.715                23.141    0.000    36.179    43.194
##          wt    -3.191         0.757       -0.518    -4.216    0.000    -4.739    -1.643
##         cyl    -1.508         0.415       -0.447    -3.636    0.001    -2.356    -0.660
## ---------------------------------------------------------------------------------------
##
##
##
##                          Model Summary
## ----------------------------------------------------------------
## R                        0.911       RMSE                 2.568
## R-Squared                0.830       Coef. Var           12.780
## Adj. R-Squared           0.819       MSE                  6.592
## Pred R-Squared           0.790       MAE                  1.921
## ----------------------------------------------------------------
##  RMSE: Root Mean Square Error
##  MSE: Mean Square Error
##  MAE: Mean Absolute Error
##
##                                ANOVA
## -------------------------------------------------------------------
##                 Sum of
##                Squares        DF    Mean Square      F         Sig.
## -------------------------------------------------------------------
## Regression     934.875         2        467.438    70.908    0.0000
## Residual       191.172        29          6.592
## Total         1126.047        31
## -------------------------------------------------------------------
##
##                                  Parameter Estimates
## ---------------------------------------------------------------------------------------
##       model      Beta    Std. Error    Std. Beta      t        Sig      lower     upper
## ---------------------------------------------------------------------------------------
## (Intercept)    39.686         1.715                23.141    0.000    36.179    43.194
##          wt    -3.191         0.757       -0.518    -4.216    0.000    -4.739    -1.643
##         cyl    -1.508         0.415       -0.447    -3.636    0.001    -2.356    -0.660
## ---------------------------------------------------------------------------------------
##
##
##
## No more variables to be added/removed.
##
##
## Final Model Output
## ------------------
##
##                          Model Summary
## ----------------------------------------------------------------
## R                        0.911       RMSE                 2.568
## R-Squared                0.830       Coef. Var           12.780
## Adj. R-Squared           0.819       MSE                  6.592
## Pred R-Squared           0.790       MAE                  1.921
## ----------------------------------------------------------------
```

```
##  RMSE: Root Mean Square Error
##  MSE: Mean Square Error
##  MAE: Mean Absolute Error
##
##                                ANOVA
## --------------------------------------------------------------------
##                 Sum of
##                Squares        DF    Mean Square      F         Sig.
## --------------------------------------------------------------------
## Regression     934.875         2        467.438    70.908    0.0000
## Residual       191.172        29          6.592
## Total         1126.047        31
## --------------------------------------------------------------------
##
##                                  Parameter Estimates
## ----------------------------------------------------------------------------------------
##       model      Beta    Std. Error    Std. Beta      t        Sig      lower     upper
## ----------------------------------------------------------------------------------------
## (Intercept)    39.686         1.715                 23.141    0.000    36.179    43.194
##          wt    -3.191         0.757       -0.518    -4.216    0.000    -4.739    -1.643
##         cyl    -1.508         0.415       -0.447    -3.636    0.001    -2.356    -0.660
## ----------------------------------------------------------------------------------------

##
##                              Stepwise Selection Summary
## ---------------------------------------------------------------------------------
##                      Added/                   Adj.
## Step    Variable    Removed     R-Square    R-Square     C(p)        AIC        RMSE
## ---------------------------------------------------------------------------------
##    1        wt       addition       0.753       0.745    14.5630    166.0294    3.0459
##    2       cyl       addition       0.830       0.819     3.2350    156.0101    2.5675
## ---------------------------------------------------------------------------------
```

```
philant5<-ols_step_both_p(philant.model1)
plot(philant5)
```


R−Square
0.83
0.81
0.79
0.77
0.75
1.00
1.25
1.50
1.75
2.00
C(p)
15
12
9
6
3
1.00
1.25
1.50
1.75
2.00
Adj. R−Square
0.82
0.80
0.78
0.76
1.00
1.25
1.50
1.75
2.00
AIC
166
164
162
160
158
156
1.00
1.25
1.50
1.75
2.00





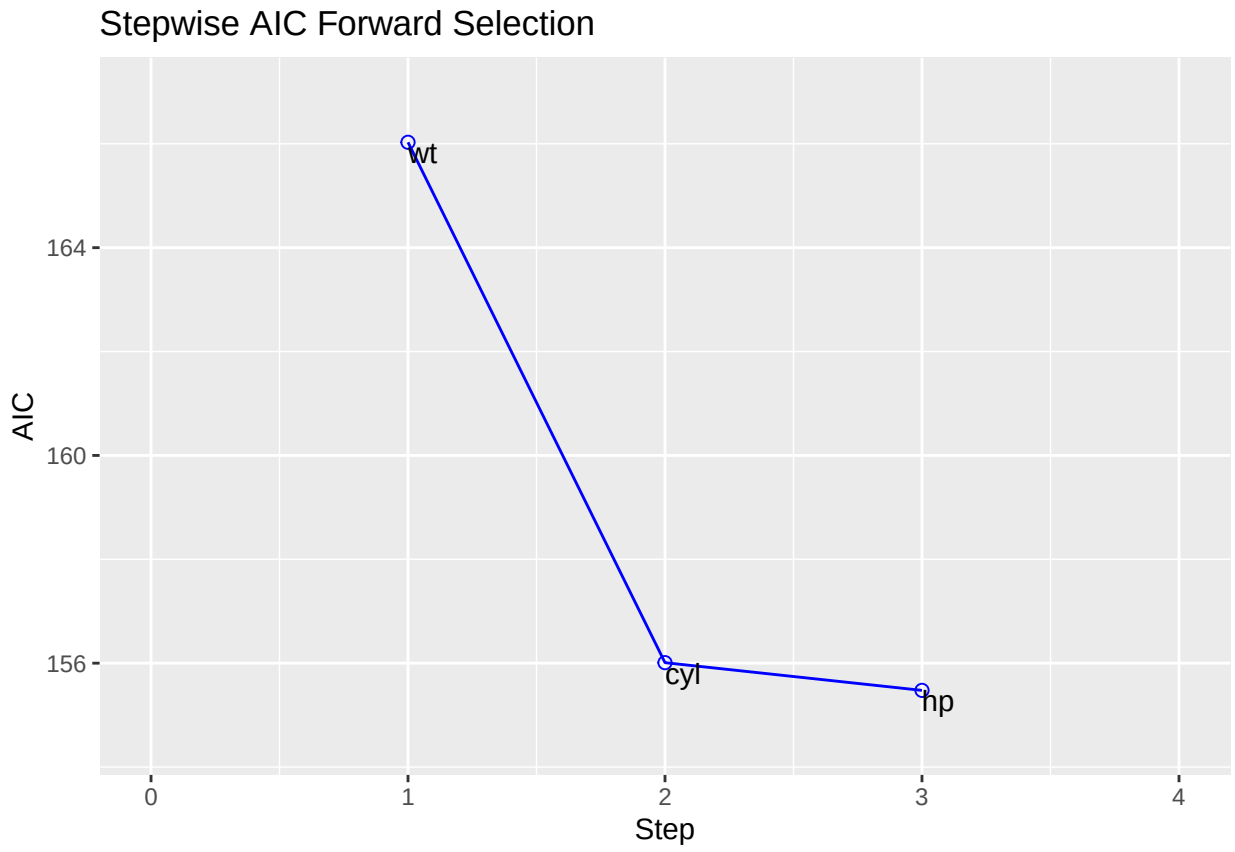

```
### Stepwise AIC forward regression
ols_step_forward_aic(philant.model1, details = TRUE)
```

```
## Forward Selection Method
## -------------------------
##
## Candidate Terms:
##
## 1 . cyl
## 2 . disp
## 3 . hp
## 4 . drat
## 5 . wt
## 6 . qsec
##
##  Step 0: AIC = 208.7555
##  mpg ~ 1
##
## ----------------------------------------------------------------------
## Variable     DF      AIC      Sum Sq       RSS      R-Sq     Adj. R-Sq
## ----------------------------------------------------------------------
## wt            1    166.029    847.725    278.322    0.753        0.745
## cyl           1    169.306    817.713    308.334    0.726        0.717
## disp          1    170.209    808.888    317.159    0.718        0.709
## hp            1    181.239    678.373    447.674    0.602        0.589
## drat          1    190.800    522.480    603.567    0.464        0.446
```

```
## qsec          1    204.588    197.392    928.655    0.175        0.148
## ---------------------------------------------------------------------
##
##
## - wt
##
##
##  Step 1 : AIC = 166.0294
##  mpg ~ wt
##
## ---------------------------------------------------------------------
## Variable     DF      AIC      Sum Sq      RSS      R-Sq     Adj. R-Sq
## ---------------------------------------------------------------------
## cyl           1    156.010    87.150    191.172    0.830        0.819
## hp            1    156.652    83.274    195.048    0.827        0.815
## qsec          1    156.720    82.858    195.464    0.826        0.814
## disp          1    164.168    31.639    246.683    0.781        0.766
## drat          1    166.968     9.081    269.241    0.761        0.744
## ---------------------------------------------------------------------
##
## - cyl
##
##
##  Step 2 : AIC = 156.0101
##  mpg ~ wt + cyl
##
## ---------------------------------------------------------------------
## Variable     DF      AIC      Sum Sq      RSS      R-Sq     Adj. R-Sq
## ---------------------------------------------------------------------
## hp            1    155.477    14.551    176.621    0.843        0.826
## qsec          1    156.190    10.567    180.605    0.840        0.822
## disp          1    157.558     2.680    188.492    0.833        0.815
## drat          1    158.010     0.001    191.171    0.830        0.812
## ---------------------------------------------------------------------
##
## - hp
##
##
##  Step 3 : AIC = 155.4766
##  mpg ~ wt + cyl + hp
##
## ---------------------------------------------------------------------
## Variable     DF      AIC      Sum Sq      RSS      R-Sq     Adj. R-Sq
## ---------------------------------------------------------------------
## disp          1    156.338     6.176    170.444    0.849        0.826
## drat          1    157.067     2.245    174.375    0.845        0.822
## qsec          1    157.222     1.401    175.219    0.844        0.821
## ---------------------------------------------------------------------
##
##
## No more variables to be added.
##
## Variables Entered:
##
```

```
## - wt
## - cyl
## - hp
##
##
## Final Model Output
## ------------------
##
##                         Model Summary
## ----------------------------------------------------------------
## R                       0.918       RMSE                 2.512
## R-Squared               0.843       Coef. Var           12.501
## Adj. R-Squared          0.826       MSE                  6.308
## Pred R-Squared          0.796       MAE                  1.845
## ----------------------------------------------------------------
##  RMSE: Root Mean Square Error
##  MSE: Mean Square Error
##  MAE: Mean Absolute Error
##
##                                ANOVA
## --------------------------------------------------------------------
##                 Sum of
##                Squares        DF    Mean Square      F         Sig.
## --------------------------------------------------------------------
## Regression     949.427         3        316.476    50.171    0.0000
## Residual       176.621        28          6.308
## Total         1126.047        31
## --------------------------------------------------------------------
##
##                                  Parameter Estimates
## ----------------------------------------------------------------------------------------
##       model      Beta    Std. Error    Std. Beta      t        Sig      lower     upper
## ----------------------------------------------------------------------------------------
## (Intercept)    38.752         1.787                 21.687    0.000    35.092    42.412
##          wt    -3.167         0.741       -0.514    -4.276    0.000    -4.684    -1.650
##         cyl    -0.942         0.551       -0.279    -1.709    0.098    -2.070     0.187
##          hp    -0.018         0.012       -0.205    -1.519    0.140    -0.042     0.006
## ----------------------------------------------------------------------------------------


##
##                         Selection Summary
## ------------------------------------------------------------------
## Variable       AIC      Sum Sq       RSS       R-Sq      Adj. R-Sq
## ------------------------------------------------------------------
## wt           166.029    847.725    278.322    0.75283      0.74459
## cyl          156.010    934.875    191.172    0.83023      0.81852
## hp           155.477    949.427    176.621    0.84315      0.82634
## ------------------------------------------------------------------
```

```
philant6<-ols_step_forward_aic(philant.model1)
plot(philant6)
```



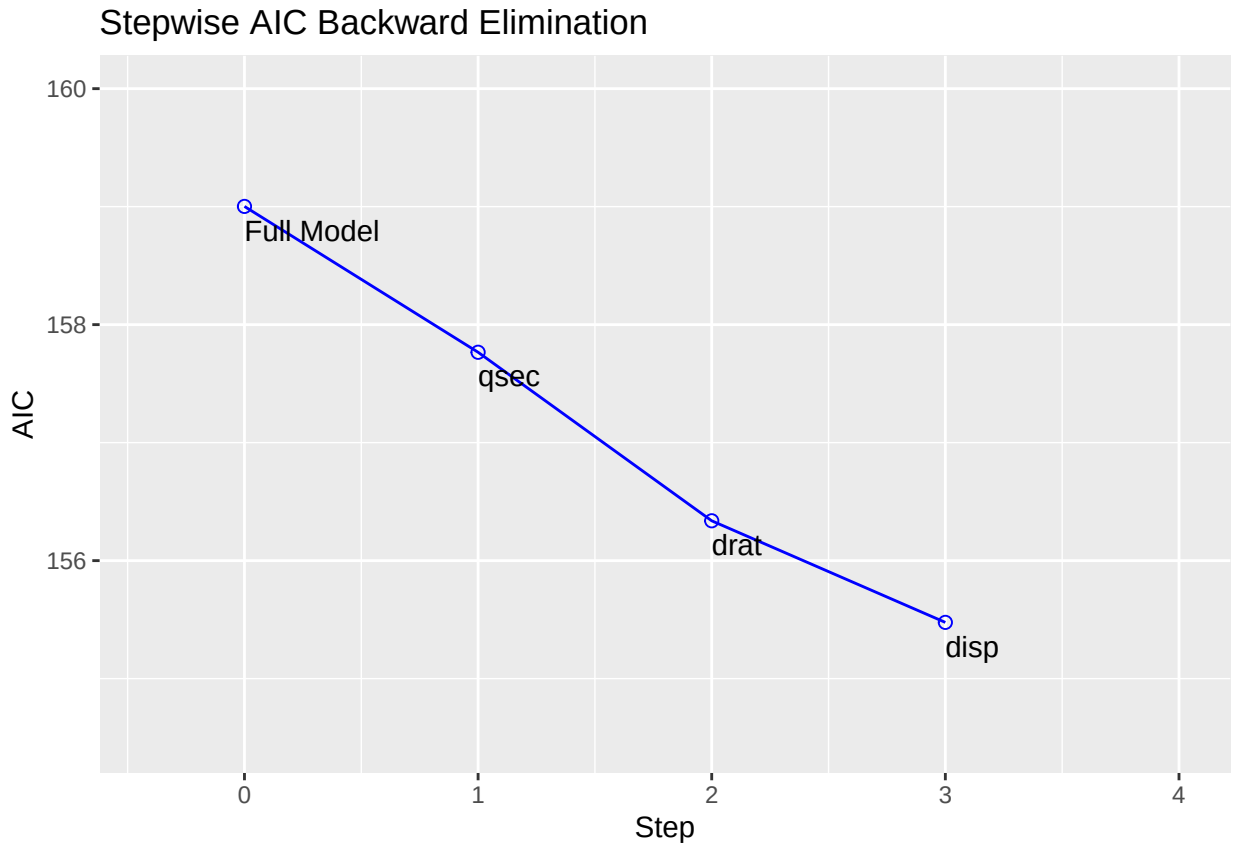

```
### Stepwise AIC backwards regression
ols_step_backward_aic(philant.model1, details = TRUE)
```

```
## Backward Elimination Method
## ---------------------------
##
## Candidate Terms:
##
## 1 . cyl
## 2 . disp
## 3 . hp
## 4 . drat
## 5 . wt
## 6 . qsec
##
##  Step 0: AIC = 159.002
##  mpg ~ cyl + disp + hp + drat + wt + qsec
##
## ----------------------------------------------------------------------
## Variable     DF      AIC      Sum Sq      RSS      R-Sq     Adj. R-Sq
## ----------------------------------------------------------------------
## qsec         1     157.766     3.949    167.426    0.851        0.823
## drat         1     158.006     5.209    168.685    0.850        0.821
## cyl          1     158.278     6.652    170.129    0.849        0.820
## disp         1     158.507     7.870    171.347    0.848        0.819
## hp           1     158.669     8.744    172.221    0.847        0.818
```

```
## wt           1     168.759    72.580    236.057    0.790        0.750
## ---------------------------------------------------------------------
##
##
## Variables Removed:
##
## - qsec
##
##
##   Step 1 : AIC = 157.7659
##  mpg ~ cyl + disp + hp + drat + wt
##
## ---------------------------------------------------------------------
## Variable     DF      AIC      Sum Sq      RSS      R-Sq     Adj. R-Sq
## ---------------------------------------------------------------------
## drat         1     156.338     3.018    170.444    0.849        0.826
## disp         1     157.067     6.949    174.375    0.845        0.822
## cyl          1     158.584    15.411    182.838    0.838        0.814
## hp           1     159.558    21.066    188.492    0.833        0.808
## wt           1     167.936    77.476    244.902    0.783        0.750
## ---------------------------------------------------------------------
##
## - drat
##
##
##   Step 2 : AIC = 156.3376
##  mpg ~ cyl + disp + hp + wt
##
## ---------------------------------------------------------------------
## Variable     DF      AIC      Sum Sq      RSS      R-Sq     Adj. R-Sq
## ---------------------------------------------------------------------
## disp         1     155.477     6.176    176.621    0.843        0.826
## hp           1     157.558    18.048    188.492    0.833        0.815
## cyl          1     158.643    24.546    194.991    0.827        0.808
## wt           1     168.018    90.925    261.369    0.768        0.743
## ---------------------------------------------------------------------
##
## - disp
##
##
##   Step 3 : AIC = 155.4766
##  mpg ~ cyl + hp + wt
##
## ---------------------------------------------------------------------
## Variable     DF      AIC      Sum Sq       RSS      R-Sq     Adj. R-Sq
## ---------------------------------------------------------------------
## hp           1     156.010    14.551    191.172    0.830        0.819
## cyl          1     156.652    18.427    195.048    0.827        0.815
## wt           1     169.562   115.354    291.975    0.741        0.723
## ---------------------------------------------------------------------
##
##
## No more variables to be removed.
##
```

```
## Variables Removed:
##
## - qsec
## - drat
## - disp
##
##
## Final Model Output
## ------------------
##
##                         Model Summary
## ----------------------------------------------------------------
## R                       0.918       RMSE                 2.512
## R-Squared               0.843       Coef. Var           12.501
## Adj. R-Squared          0.826       MSE                  6.308
## Pred R-Squared          0.796       MAE                  1.845
## ----------------------------------------------------------------
##  RMSE: Root Mean Square Error
##  MSE: Mean Square Error
##  MAE: Mean Absolute Error
##
##                               ANOVA
## -------------------------------------------------------------------
##                 Sum of
##                Squares        DF    Mean Square      F         Sig.
## -------------------------------------------------------------------
## Regression     949.427         3        316.476    50.171    0.0000
## Residual       176.621        28          6.308
## Total         1126.047        31
## -------------------------------------------------------------------
##
##                                 Parameter Estimates
## ----------------------------------------------------------------------------------------
##       model      Beta    Std. Error    Std. Beta      t        Sig      lower     upper
## ----------------------------------------------------------------------------------------
## (Intercept)    38.752         1.787                21.687    0.000    35.092    42.412
##         cyl    -0.942         0.551       -0.279    -1.709    0.098    -2.070     0.187
##          hp    -0.018         0.012       -0.205    -1.519    0.140    -0.042     0.006
##          wt    -3.167         0.741       -0.514    -4.276    0.000    -4.684    -1.650
## ----------------------------------------------------------------------------------------


##
##
##                     Backward Elimination Summary
## ---------------------------------------------------------------------
## Variable        AIC        RSS       Sum Sq      R-Sq      Adj. R-Sq
## ---------------------------------------------------------------------
## Full Model    159.002    163.477    962.570    0.85482       0.81998
## qsec          157.766    167.426    958.621    0.85132       0.82272
## drat          156.338    170.444    955.603    0.84863       0.82621
## disp          155.477    176.621    949.427    0.84315       0.82634
## ---------------------------------------------------------------------
```

```
philant7<-ols_step_backward_aic(philant.model1)
plot(philant7)
```




```
### Stepwise AIC regression
ols_step_both_aic(philant.model1, details = TRUE)
```

```
## Stepwise Selection Method
## -------------------------
##
## Candidate Terms:
##
## 1 . cyl
## 2 . disp
## 3 . hp
## 4 . drat
## 5 . wt
## 6 . qsec
##
##  Step 0: AIC = 208.7555
##  mpg ~ 1
##
##
## Variables Entered/Removed:
##
##                           Enter New Variables
```

```
## ----------------------------------------------------------------------
## Variable     DF      AIC      Sum Sq       RSS      R-Sq     Adj. R-Sq
## ----------------------------------------------------------------------
## wt            1    166.029    847.725    278.322    0.753        0.745
## cyl           1    169.306    817.713    308.334    0.726        0.717
## disp          1    170.209    808.888    317.159    0.718        0.709
## hp            1    181.239    678.373    447.674    0.602        0.589
## drat          1    190.800    522.480    603.567    0.464        0.446
## qsec          1    204.588    197.392    928.655    0.175        0.148
## ----------------------------------------------------------------------
##
## - wt added
##
##
##  Step 1 : AIC = 166.0294
##  mpg ~ wt
##
##                           Enter New Variables
## ----------------------------------------------------------------------
## Variable     DF      AIC      Sum Sq       RSS      R-Sq     Adj. R-Sq
## ----------------------------------------------------------------------
## cyl           1    156.010    934.875    191.172    0.830        0.819
## hp            1    156.652    930.999    195.048    0.827        0.815
## qsec          1    156.720    930.584    195.464    0.826        0.814
## disp          1    164.168    879.365    246.683    0.781        0.766
## drat          1    166.968    856.806    269.241    0.761        0.744
## ----------------------------------------------------------------------
##
## - cyl added
##
##
##  Step 2 : AIC = 156.0101
##  mpg ~ wt + cyl
##
##                        Remove Existing Variables
## ----------------------------------------------------------------------
## Variable     DF      AIC      Sum Sq       RSS      R-Sq     Adj. R-Sq
## ----------------------------------------------------------------------
## cyl           1    166.029    847.725    278.322    0.753        0.745
## wt            1    169.306    817.713    308.334    0.726        0.717
## ----------------------------------------------------------------------
##
##                           Enter New Variables
## ----------------------------------------------------------------------
## Variable     DF      AIC      Sum Sq       RSS      R-Sq     Adj. R-Sq
## ----------------------------------------------------------------------
## hp            1    155.477    949.427    176.621    0.843        0.826
## qsec          1    156.190    945.443    180.605    0.840        0.822
## disp          1    157.558    937.555    188.492    0.833        0.815
## drat          1    158.010    934.876    191.171    0.830        0.812
## ----------------------------------------------------------------------
##
## - hp added
##
```

```
##
##  Step 3 : AIC = 155.4766
##  mpg ~ wt + cyl + hp
##
##                        Remove Existing Variables
## ---------------------------------------------------------------------------
## Variable     DF      AIC      Sum Sq       RSS      R-Sq     Adj. R-Sq
## ---------------------------------------------------------------------------
## hp            1    156.010    934.875    191.172    0.830        0.819
## cyl           1    156.652    930.999    195.048    0.827        0.815
## wt            1    169.562    834.073    291.975    0.741        0.723
## ---------------------------------------------------------------------------
##
##                           Enter New Variables
## ---------------------------------------------------------------------------
## Variable     DF      AIC      Sum Sq       RSS      R-Sq     Adj. R-Sq
## ---------------------------------------------------------------------------
## disp          1    156.338    955.603    170.444    0.849        0.826
## drat          1    157.067    951.672    174.375    0.845        0.822
## qsec          1    157.222    950.828    175.219    0.844        0.821
## ---------------------------------------------------------------------------
##
##
## No more variables to be added or removed.
##
## Final Model Output
## ------------------
##
##                          Model Summary
## ----------------------------------------------------------------
## R                       0.918       RMSE                 2.512
## R-Squared               0.843       Coef. Var           12.501
## Adj. R-Squared          0.826       MSE                  6.308
## Pred R-Squared          0.796       MAE                  1.845
## ----------------------------------------------------------------
##  RMSE: Root Mean Square Error
##  MSE: Mean Square Error
##  MAE: Mean Absolute Error
##
##                                ANOVA
## --------------------------------------------------------------------
##                 Sum of
##                Squares        DF    Mean Square      F         Sig.
## --------------------------------------------------------------------
## Regression     949.427         3        316.476    50.171    0.0000
## Residual       176.621        28          6.308
## Total         1126.047        31
## --------------------------------------------------------------------
##
##                                  Parameter Estimates
## ----------------------------------------------------------------------------------------
##       model      Beta    Std. Error    Std. Beta      t        Sig      lower     upper
## ----------------------------------------------------------------------------------------
## (Intercept)    38.752         1.787                21.687    0.000    35.092    42.412
```

```
##          wt    -3.167         0.741       -0.514    -4.276    0.000    -4.684    -1.650
##         cyl    -0.942         0.551       -0.279    -1.709    0.098    -2.070     0.187
##          hp    -0.018         0.012       -0.205    -1.519    0.140    -0.042     0.006
## ---------------------------------------------------------------------------------------
```

```
##
##
##                               Stepwise Summary
## -------------------------------------------------------------------------
## Variable     Method       AIC        RSS      Sum Sq      R-Sq      Adj. R-Sq
## -------------------------------------------------------------------------
## wt          addition    166.029    278.322    847.725    0.75283      0.74459
## cyl         addition    156.010    191.172    934.875    0.83023      0.81852
## hp          addition    155.477    176.621    949.427    0.84315      0.82634
## -------------------------------------------------------------------------
```

```
philant8<-ols_step_both_aic(philant.model1)
plot(philant8)
```