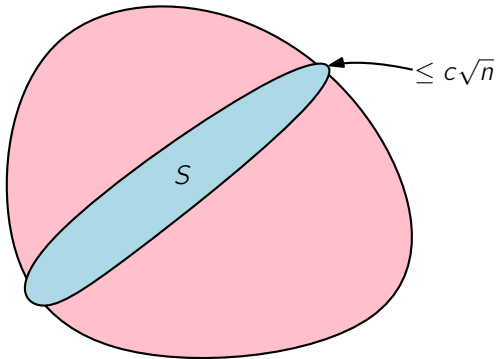
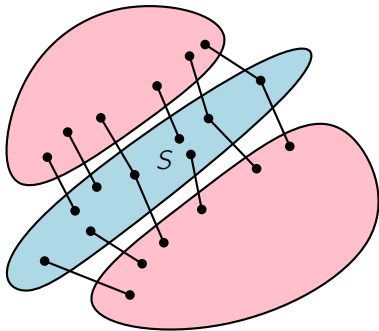
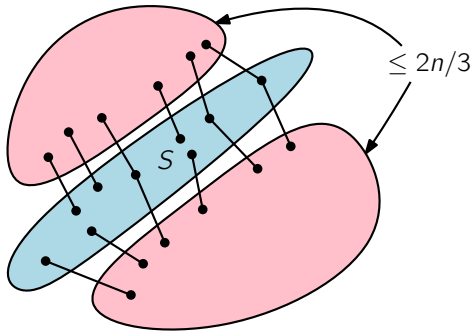
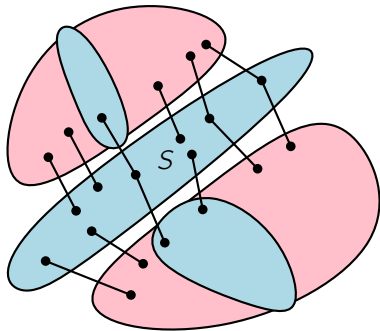
A large, light pink oval with a thin black border is centered on a white background. Inside the oval, the text " $n$ -vertex planar graph  $G$ " is written in a black, sans-serif font.

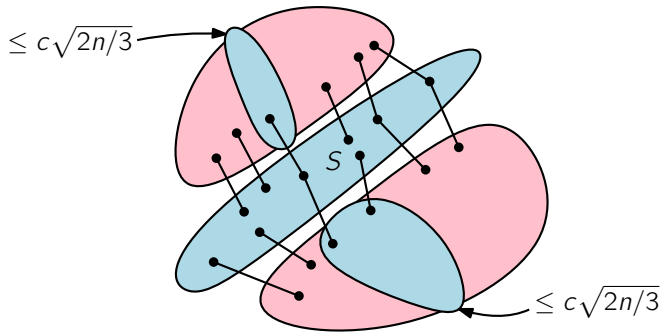
$n$ -vertex  
planar graph  $G$

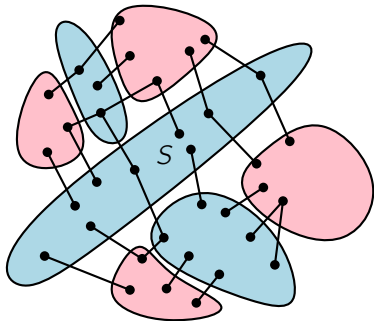


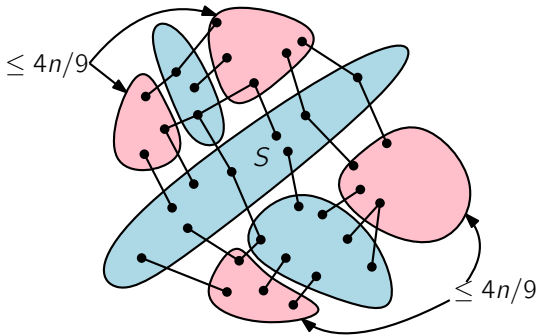




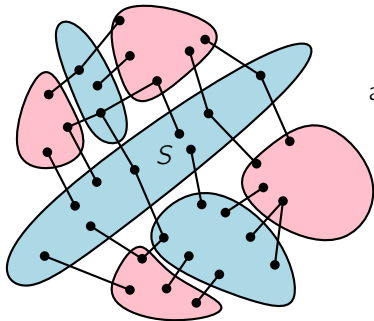




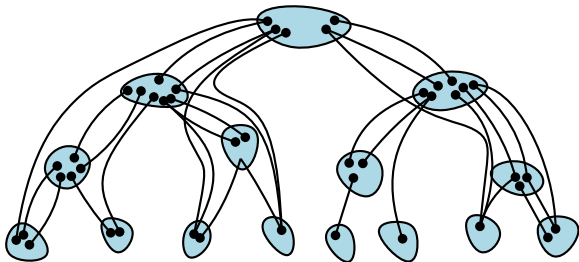




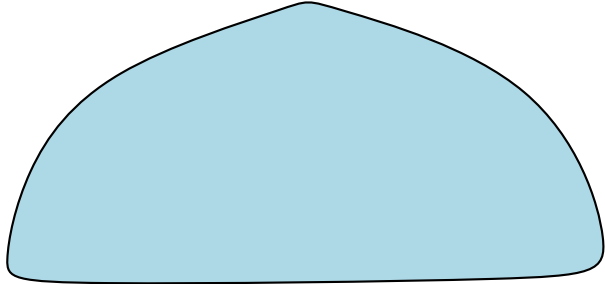




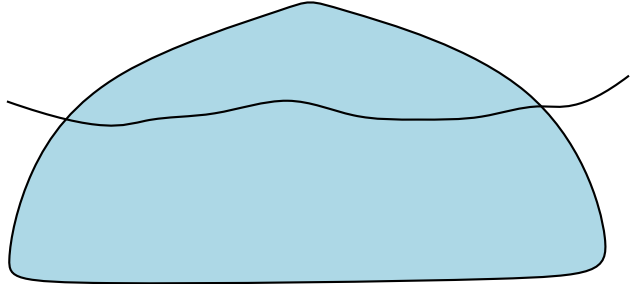
and so on...



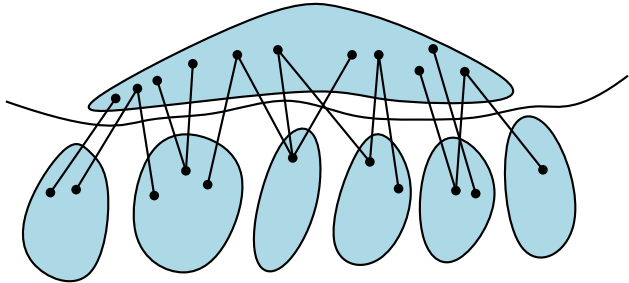
separator "tree"

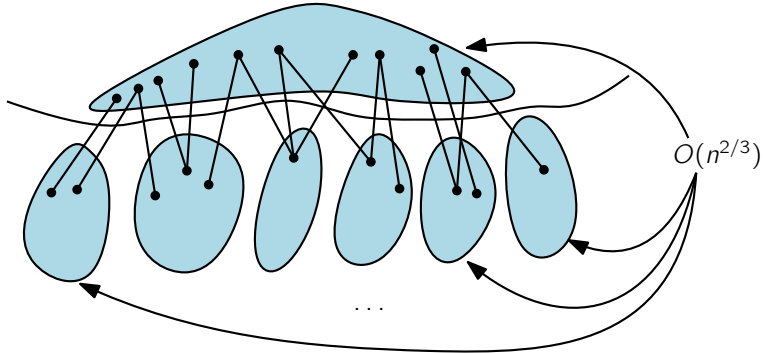


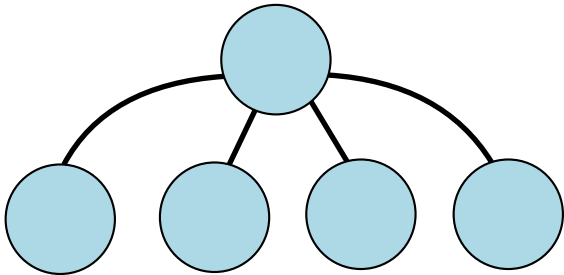
separator "tree"

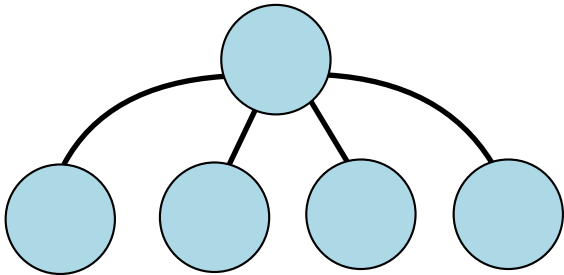


separator "tree"



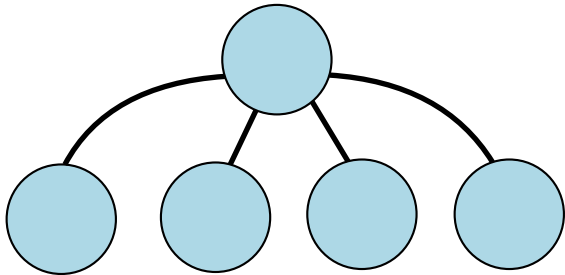




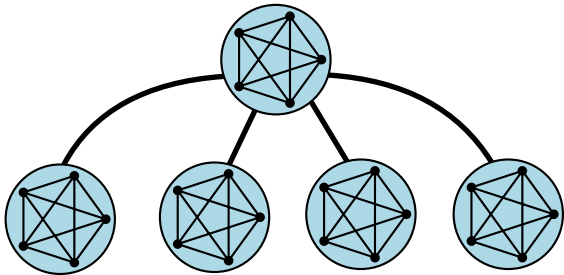


$G$  has a **star partition** of width  $O(n^{2/3})$

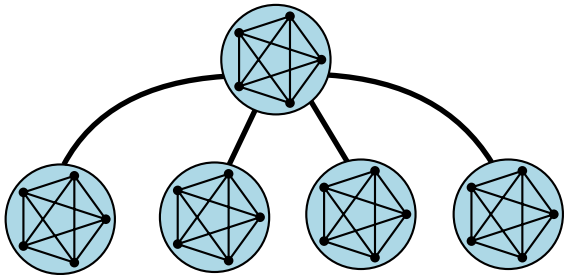




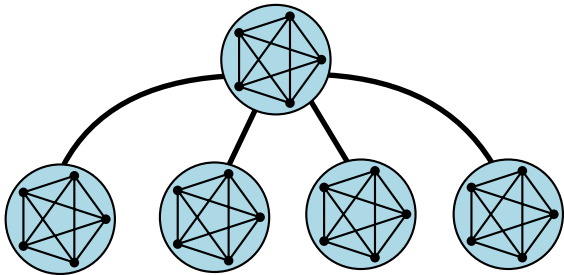
$G$  has a vertex partition  $\mathcal{P}$  into sets of size  $O(n^{2/3})$  s.t.  $G/\mathcal{P}$  is a star



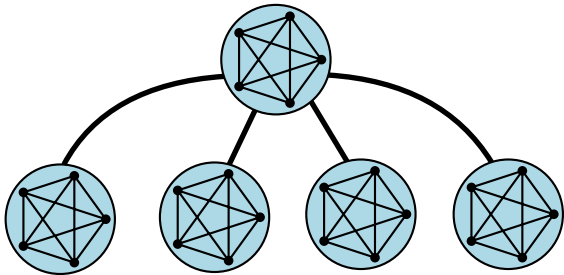
$G \subseteq S \boxtimes K_{O(n^{2/3})}$  where  $S$  is a star



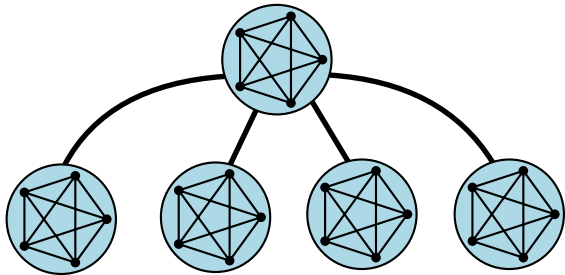
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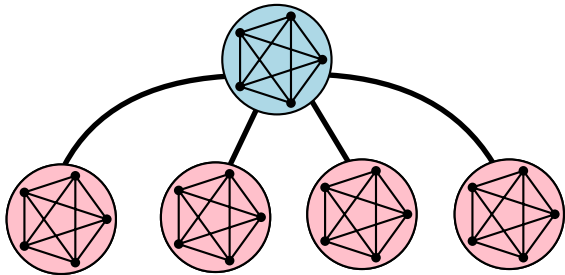
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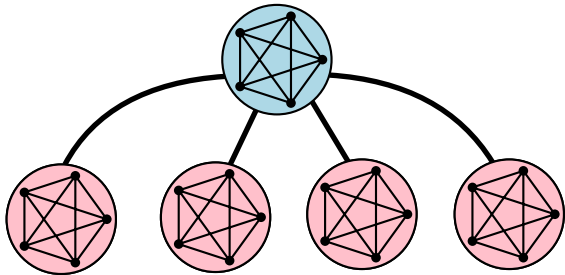
What can we do with parts of size  $\tilde{O}(\sqrt{n})$ ?



**Bad News (Linial-Matousek-Sheffet-Tardos 2008):** There exists planar  $G$  such that every 2-coloring of  $G$  has a monochromatic component of size  $\Omega(n^{2/3})$



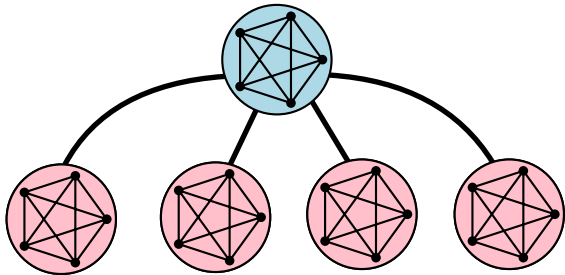
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$G \subseteq H \boxtimes K_{o(n^{2/3})} \Rightarrow H$  is not 2-colorable.





**Bad News (Linial-Matousek-Sheffet-Tardos 2008):** There exists planar  $G$  such that every 2-coloring of  $G$  has a monochromatic component of size  $\Omega(n^{2/3})$

$G \subseteq H \boxtimes K_{o(n^{2/3})} \Rightarrow H$  is not 2-colorable  $\Rightarrow H$  is not a tree.

