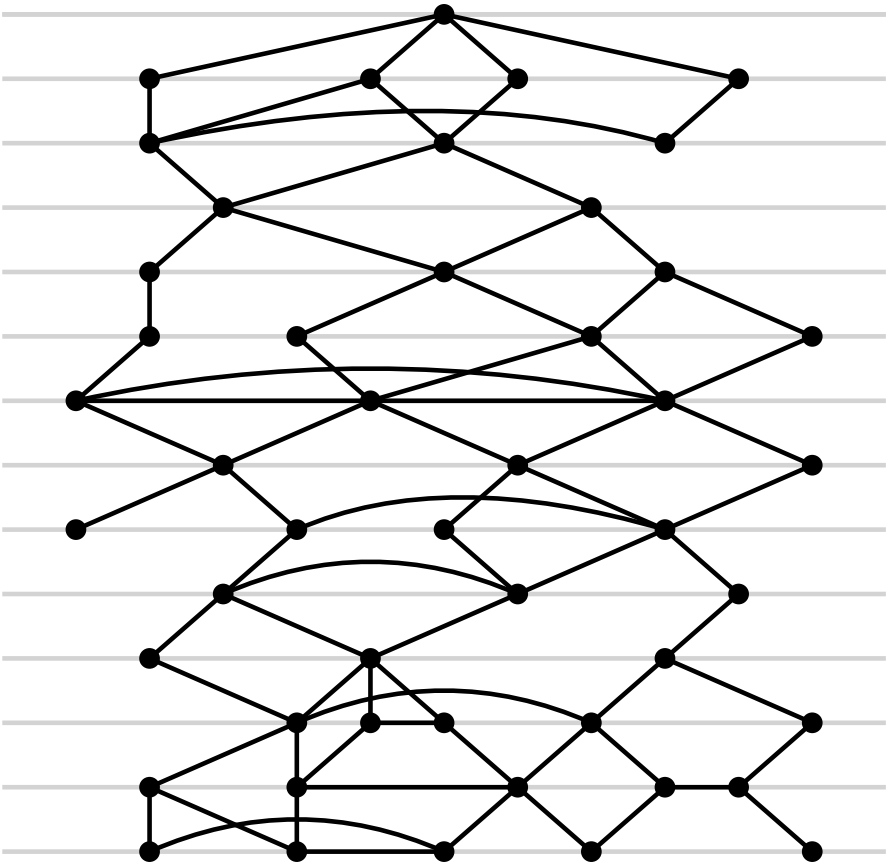
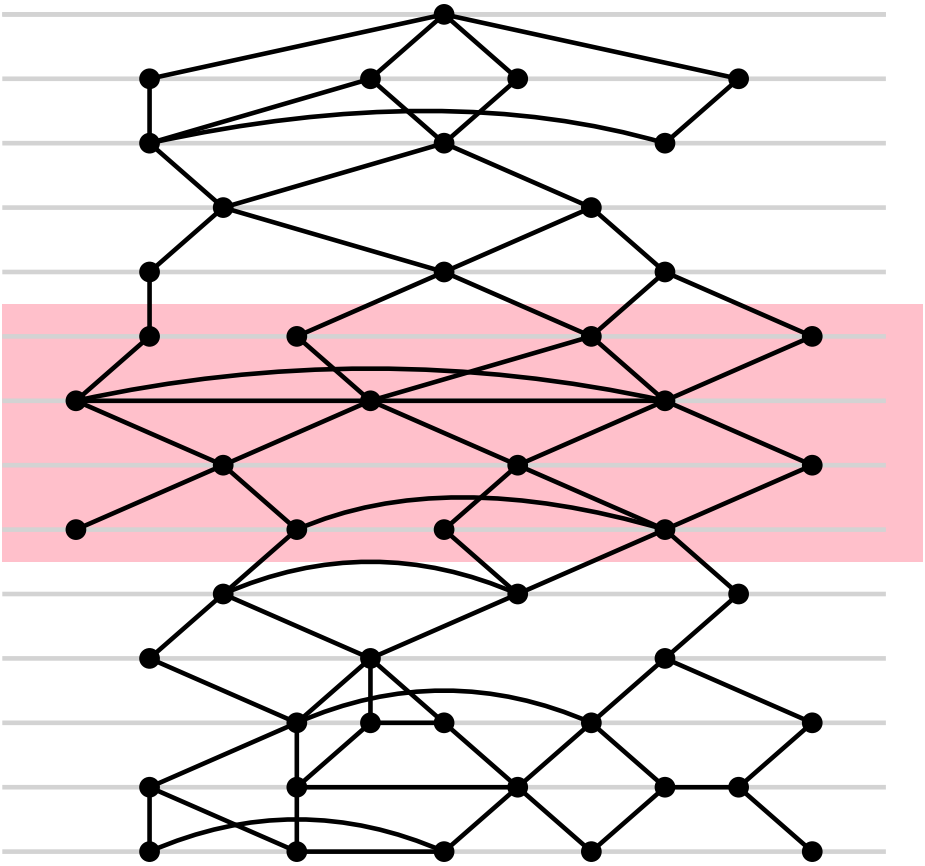
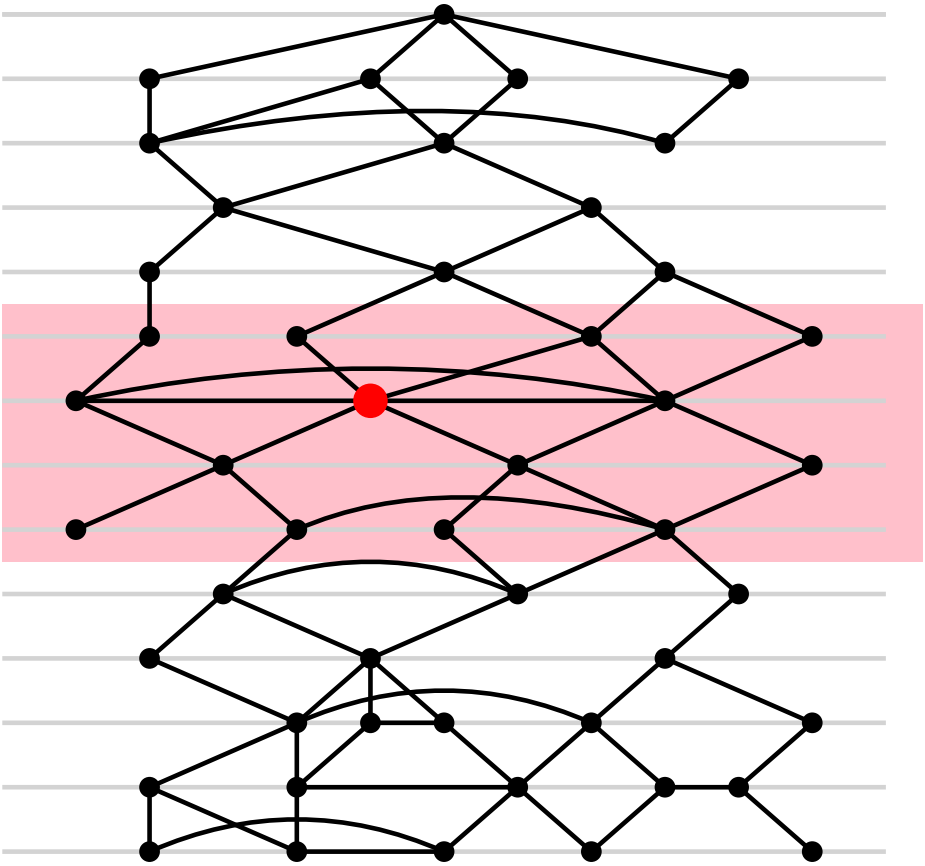
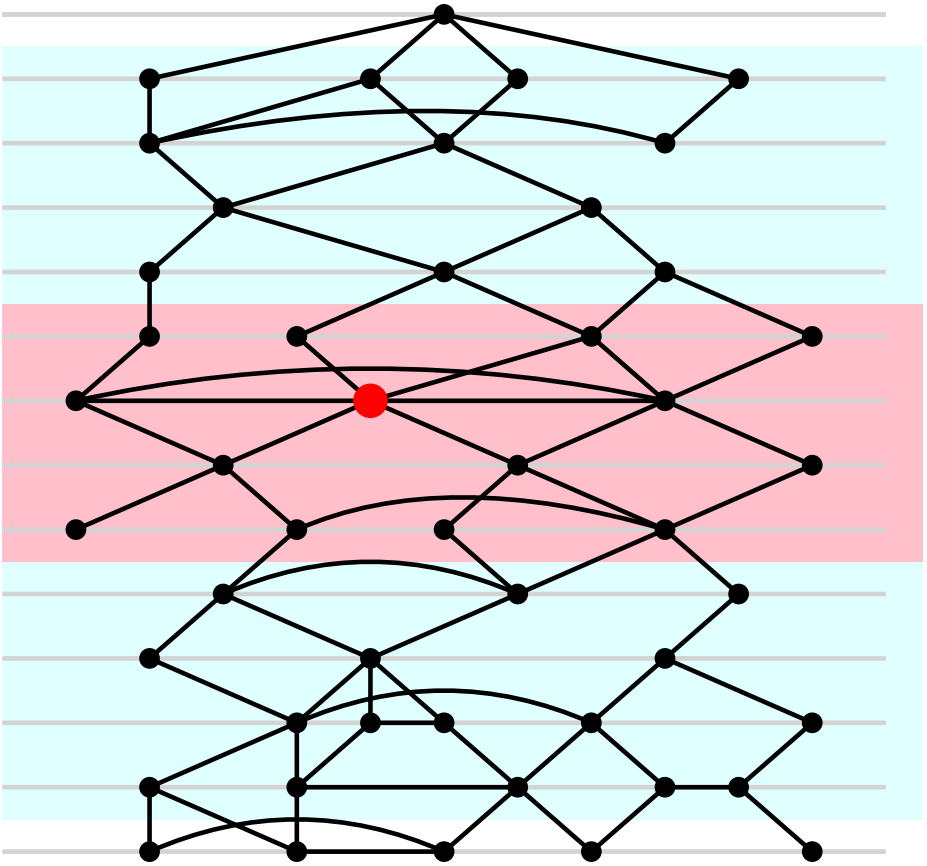




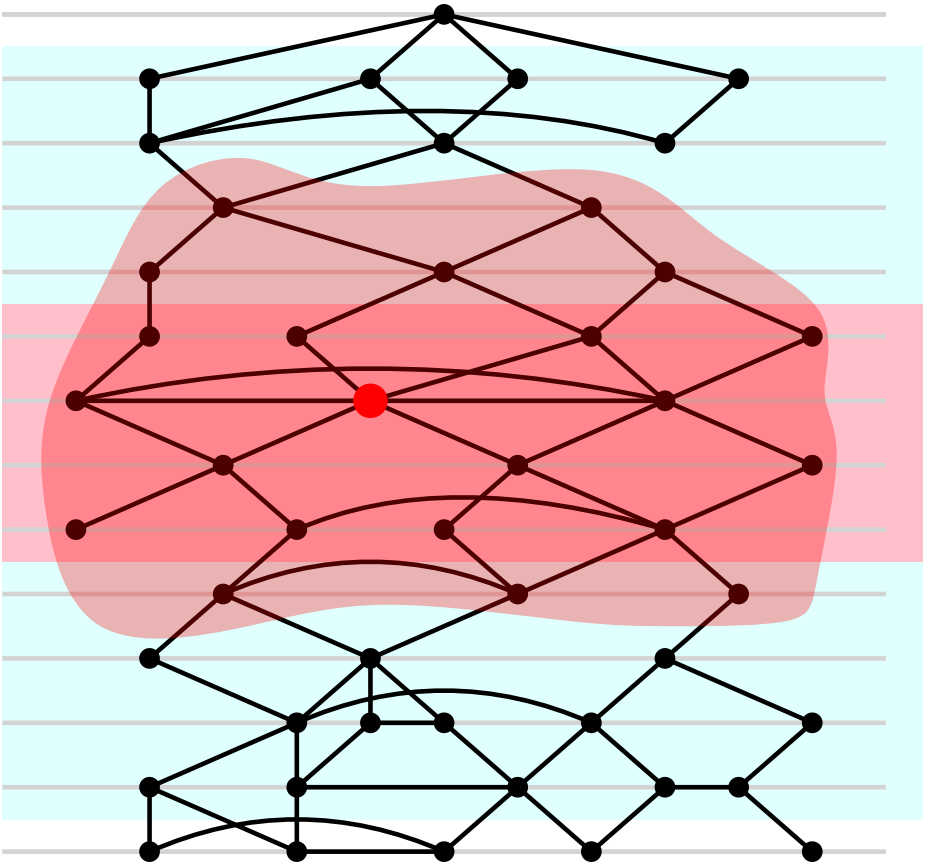
Deal with radius-$[\frac{1}{2}r, r]$ balls centered in some height r strip



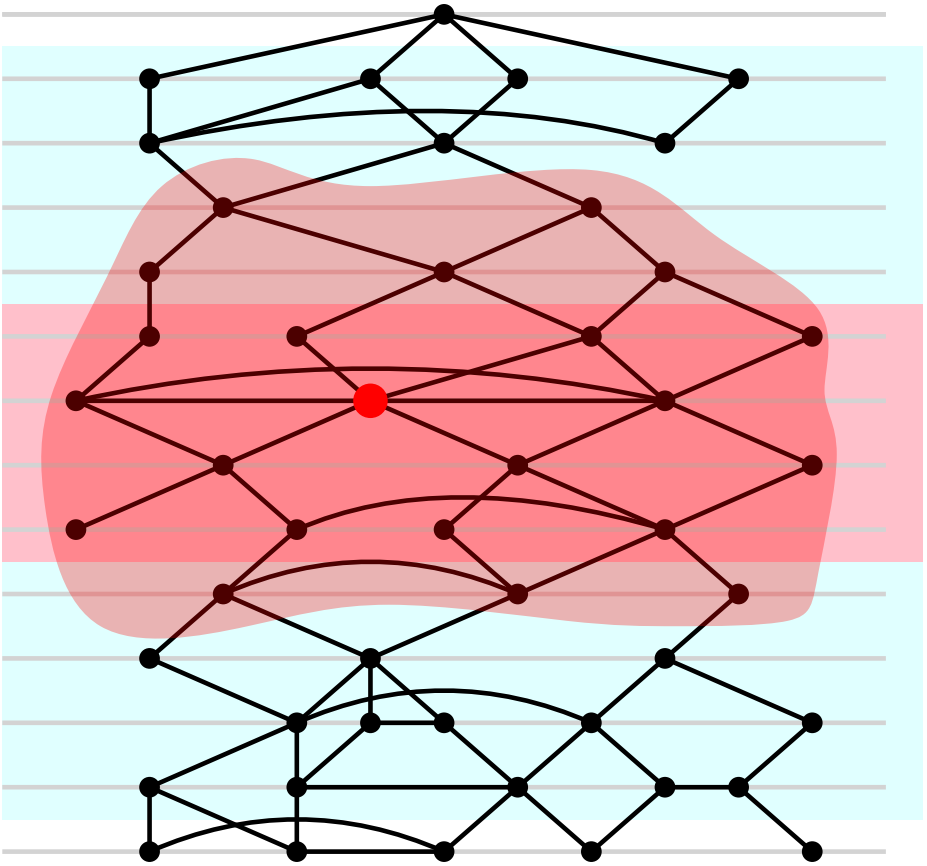
Deal with radius-$[\frac{1}{2}r, r]$ balls centered in some height r strip



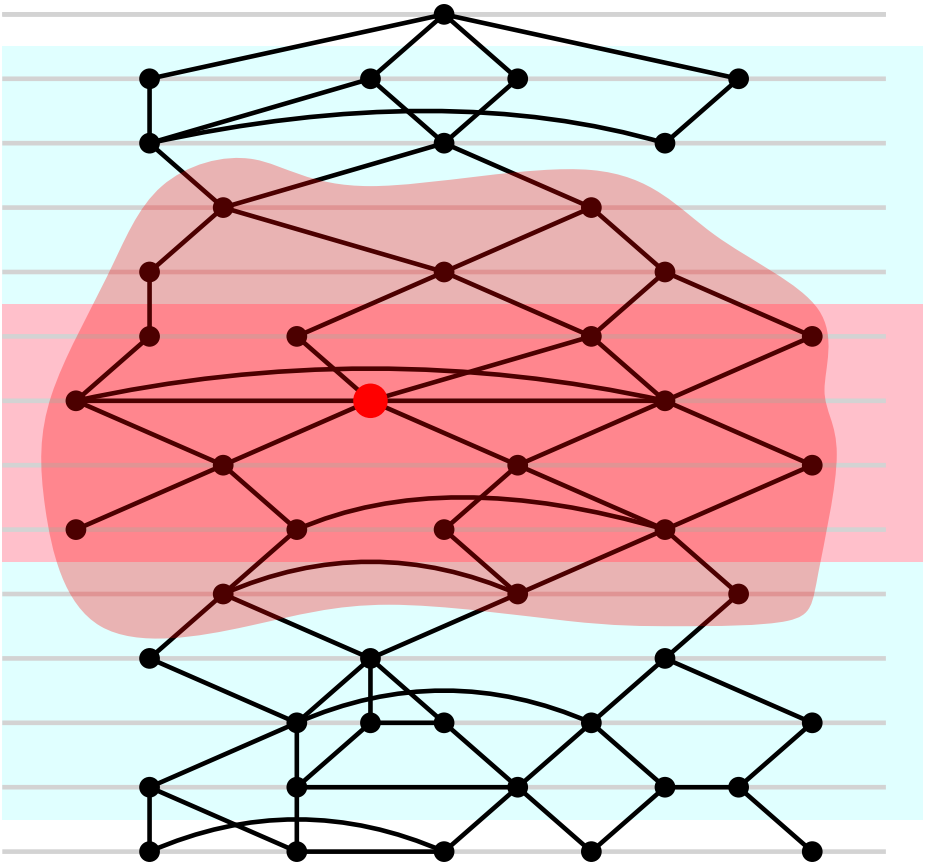
Deal with radius-$[\frac{1}{2}r, r]$ balls centered in some height r strip



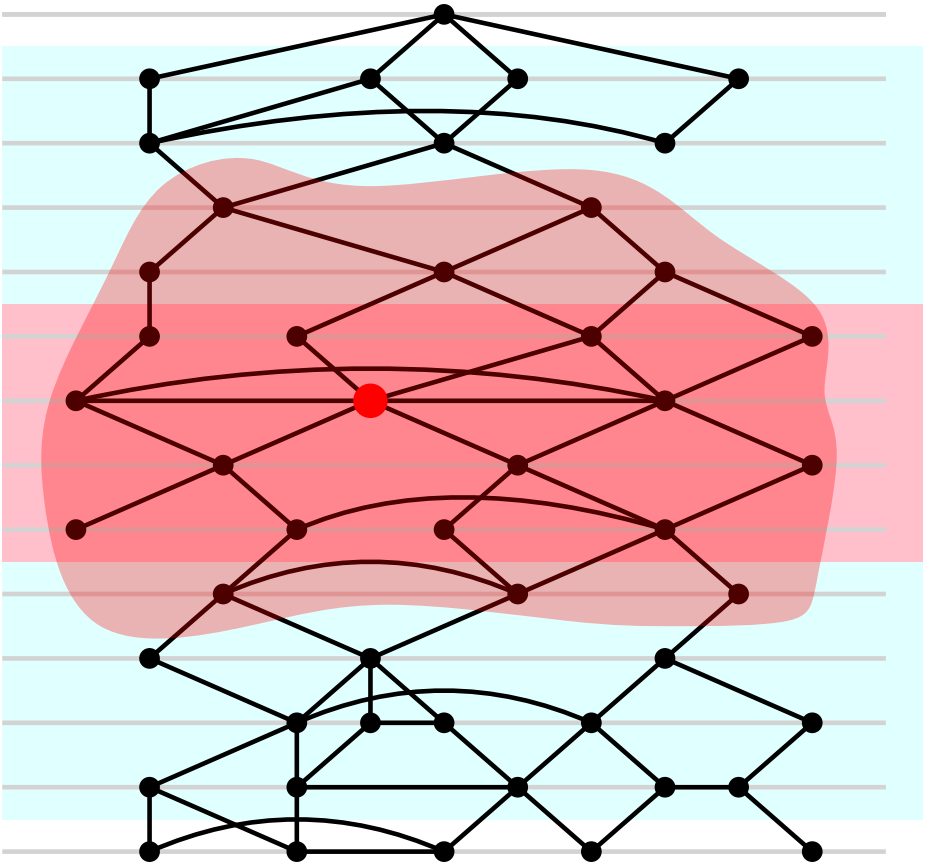
Deal with radius-$[\frac{1}{2}r, r]$ balls
centered in some height r strip



$3r$ layers have treewidth $O(r)$

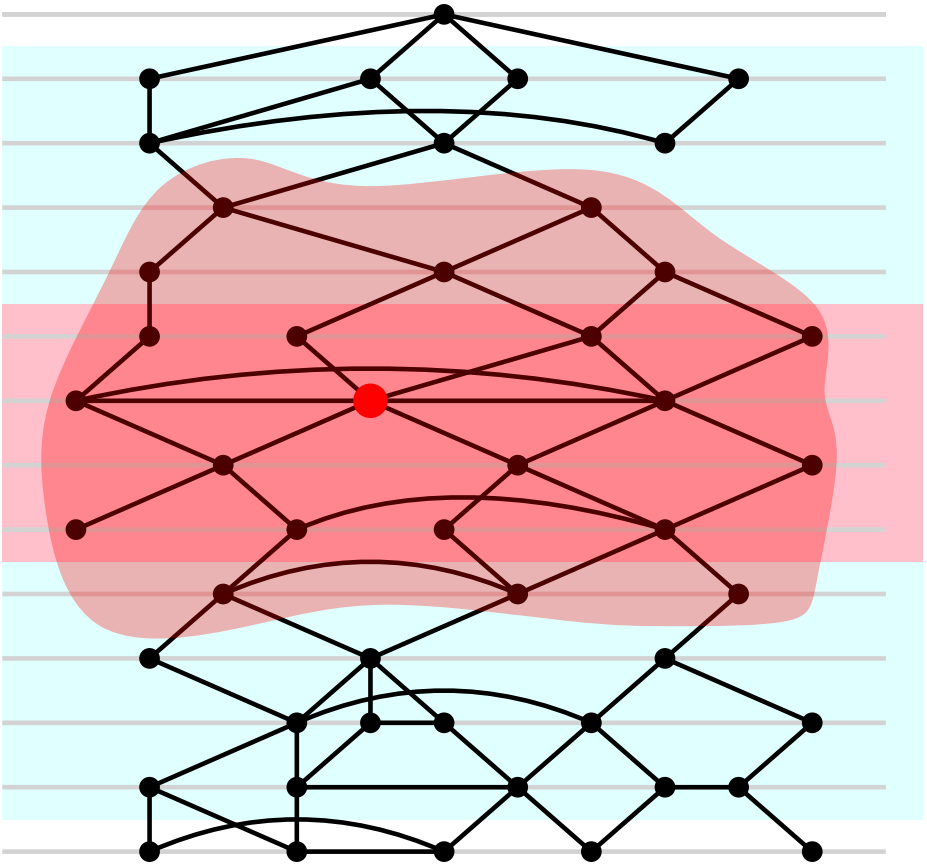


separate using $O(n'/\sqrt{n})$ vertices
into components of size $O(r\sqrt{n})$



separate using $O(n'/\sqrt{n})$ vertices
into components of size $O(r\sqrt{n})$

$\sum n' \leq 3n$, so $\sum O(n'/\sqrt{n}) = O(\sqrt{n})$



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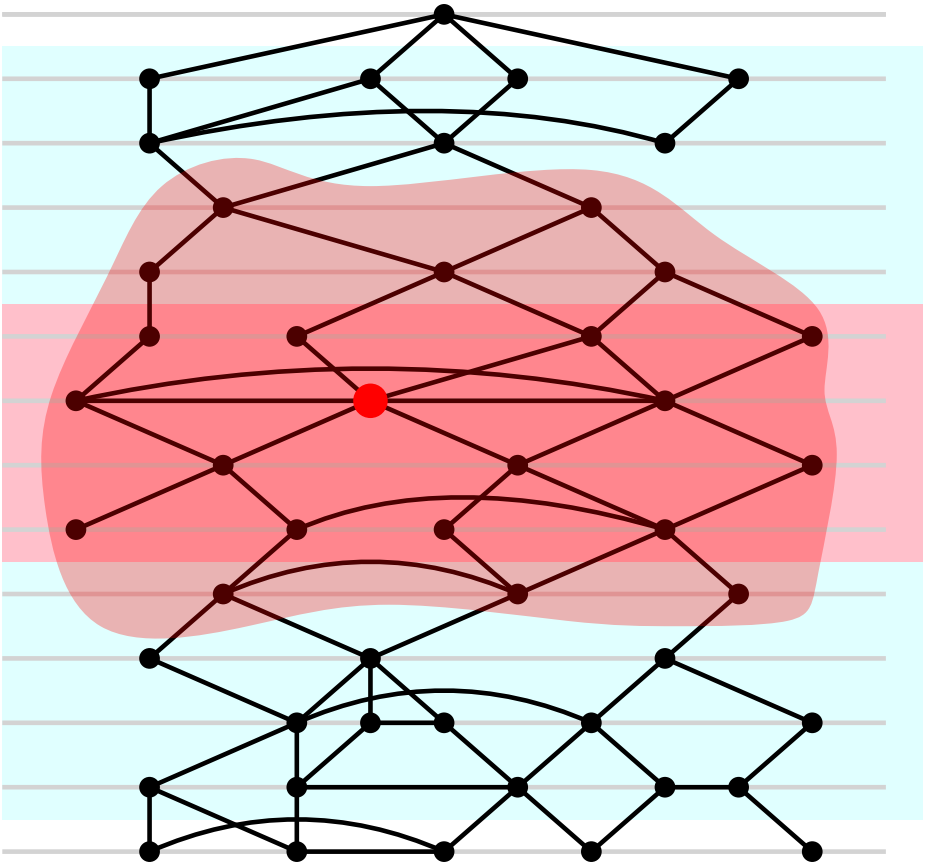
Repeat for $r = 1, 2, 4, 8, \ldots, 2^{\lceil \log n \rceil}$



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QED