Dichotomous Predictors, Non-Linearity, and Data Transformations

Dr. Michael Fix
mfix@gsu.edu

Georgia State University

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Variable Types Revisited

- Four types of variables:
  1. Nominal ("Factors")
  2. Ordinal
  3. Interval
  4. Ratio
- In the context of OLS: Which work as DVs? Which work as IVs?

Dummy Variables

- A term that gets used a lot to mean many things...

- Naturally dichotomous things
- Simplified categorizations
- "Factor" variables
- Ordinal variables (treated as "factors")

Dummy Variable Coding

- The term "dummy" variable is associate with a $\{0{,}1\}$ coding scale
- e.g.

$$\texttt{woman} = \begin{cases} 0 \text{ if man} \\ 1 \text{ if woman} \end{cases}$$

- Why $\{0{,}1\}$?

Dummy Variable Coding

- Two reasons:
  1. Math (will talk about this in a minute)
  2. Software
- Theoretically, as this variables have no meaningful ordering among their values, the assigned numbers do not matter
- **However**, you should always *name* the variable to correspond outcome of interest and set that outcome equal to 1.

Bivariate Regression with Dichotomous Xs

The Math

- For

$$Y_i = \beta_0 + \beta_1 D_i + u_i$$

- we have

$$\mathrm{E}(Y|D=0) = \beta_0$$

- and

$$\mathrm{E}(Y|D=1) = \beta_0 + \beta_1.$$

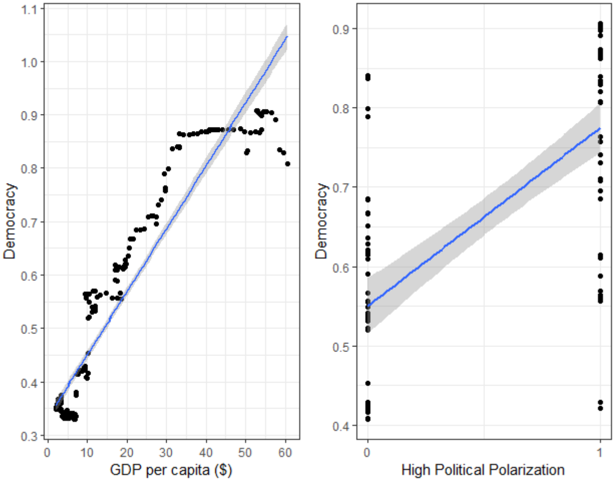
Bivariate Regression with Dichotomous Xs

The Intuition

- Intuitively, we think of OLS as “fitting a line”
- This breaks down with a dummy IV:

Bivariate Regression with Dichotomous Xs

The Intuition



Regression with Dichotomous and Continuous X

The Math

- For,

$$Y_i = \beta_0 + \beta_1 D_i + \beta_2 X_i + u_i$$

- we have

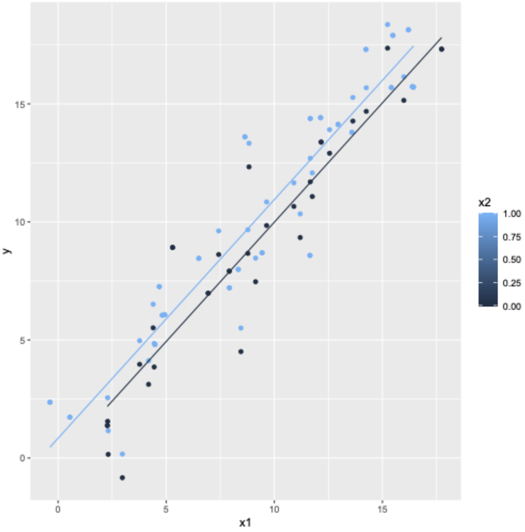
$$\mathrm{E}(Y|X, D = 0) = \beta_0 + \beta_2 X_i$$

- and

$$\mathrm{E}(Y|X, D = 1) = (\beta_0 + \beta_1) + \beta_2 X_i$$

Regression with Dichotomous and Continuous X

The Intuition



Regression with Dichotomous and Continuous X

The Intuition

- As the prior slide shows, effectively the dummy variable represents an intercept shift.
- The estimated effect of X_i on Y_i (β_2) determines the slope of the regression line and is unchanged based on the value of D_i.
- BUT, the intercept of the regression line shifts based on the value of D_i
  - When $D_i = 0$, the intercept is β_0
  - When $D_i = 1$, the intercept is $(\beta_0 + \beta_1)$

Multiple Dummies

The Math

- For

$$Y_i = \beta_0 + \beta_1 D_{1i} + \beta_2 D_{2i} + ... + \beta_\ell D_{\ell i} + u_i$$

- We have

$$\mathrm{E}(Y|D_k = 0)\,\forall\, k \in \ell = \beta_0$$

- Otherwise,

$$\mathrm{E}(Y) = \beta_0 + \sum_{k=1}^{\ell} \beta_k \,\forall\, k\; s.t.\; D_k = 1$$

Multiple Dummies

An Important Note

- Where the D_ℓ are *mutually exclusive and exhaustive*:
  - This is usually the case for so called "factor" variables
  - The expected values are the same as the within-group means.
  - Identification requires that we either
    - omit a "reference category," or
    - omit β_0.

Multiple Dummies

Ordinal Variables: A Special Case

- Suppose we have:

$$\texttt{party} = \begin{cases} -2 = \text{Strong Democrat} \\ -1 = \text{Weak Democrat} \\ 0 = \text{Independent} \\ 1 = \text{Weak Republican} \\ 2 = \text{Strong Republican} \end{cases}$$

Multiple Dummies

Ordinal Variables: A Special Case

- We could estimate:

$$Y_i = \beta_0 + \beta_1(\texttt{party}_i) + u_i$$

- Effectively treating an ordinal variable as if it was continuous

Multiple Dummies

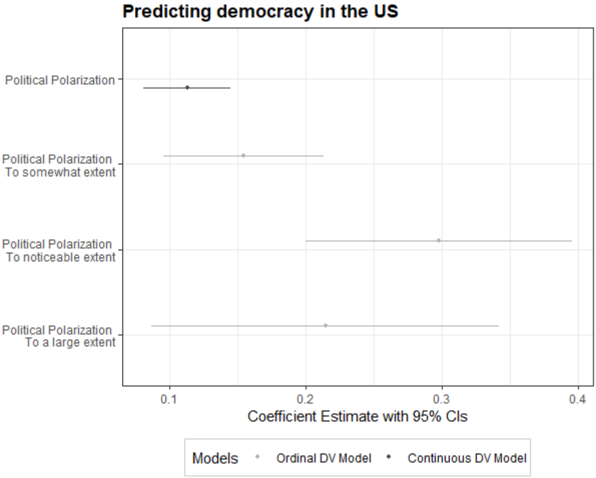
Ordinal Variables: A Special Case

- Alternatively, we could convert it to a series of dummies

$$Y_i = \beta_0 + \beta_1(\texttt{strongdem}_i) + \beta_2(\texttt{weakdem}_i) + \\ \beta_3(\texttt{weakgop}_i) + \beta_4(\texttt{stronggop}_i) + u_i$$

- Note the excluded "reference category" as the outcomes are mutually exclusive and exhaustive

Ordinal Variables: A Comparison



Why Transform Variables?

- Normality (of u_is)
- Linearity
- Additivity
- Interpretation / Model Specification

Note: John Fox has some really helpful slides online that you might find useful for more depth on various transformations.

Monotonic Transformations

"Family of Powers and Roots"

Transformation	p	$f(X)$	Fox's $f(X)$
Cube	3	X^3	$\frac{X^3-1}{3}$
Square	2	X^2	$\frac{X^2-1}{2}$
(None/Identity)	(1)	(X)	(X)
Square Root	$\frac{1}{2}$	$\sqrt{X}$	$2(\sqrt{X}-1)$
Cube Root	$\frac{1}{3}$	$\sqrt[3]{X}$	$3(\sqrt[3]{X}-1)$
Log	0 (sort of)	$\ln(X)$	$\ln(X)$
Inverse Cube Root	-$\frac{1}{3}$	$\frac{1}{\sqrt[3]{X}}$	$\frac{\left(\frac{1}{\sqrt[3]{X}}-1\right)}{-\frac{1}{3}}$
Inverse Square Root	-$\frac{1}{2}$	$\frac{1}{\sqrt{X}}$	$\frac{\left(\frac{1}{\sqrt{X}}-1\right)}{-\frac{1}{2}}$
Inverse	-1	$\frac{1}{X}$	$\frac{\left(\frac{1}{X}-1\right)}{-1}$
Inverse Square	-2	$\frac{1}{X^2}$	$\frac{\left(\frac{1}{X^2}-1\right)}{-2}$
Inverse Cube	-3	$\frac{1}{X^3}$	$\frac{\left(\frac{1}{X^3}-1\right)}{-3}$

A General Rule

Using higher-order power transformations (e.g. squares, cubes, etc.) "inflates" large values and "compresses" small ones; conversely, using lower-order power transformations (logs, etc.) "compresses" large values and "inflates" (or "expands") smaller ones.

Nonmonotonicity

Simple solution: Polynomials

- Second-order / quadratic:

$$Y_i = \beta_0 + \beta_1 X_i + \beta_2 X_i^2 + u_i$$

- Third-order / cubic:

$$Y_i = \beta_0 + \beta_1 X_i + \beta_2 X_i^2 + \beta_3 X_i^3 + u_i$$

- pth-order:

$$Y_i = \beta_0 + \beta_1 X_i + \beta_2 X_i^2 + \beta_3 X_i^3 + \ldots + \beta_p X_i^p + u_i$$

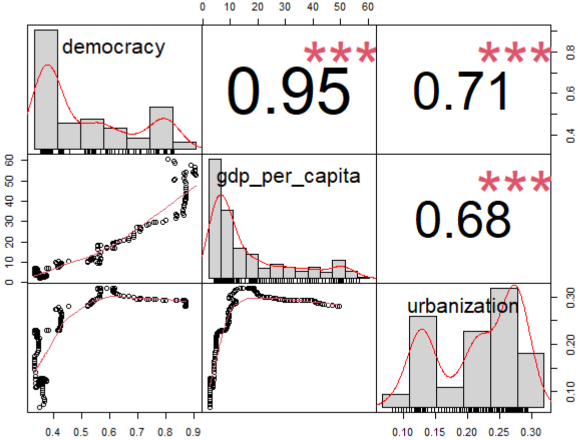
How Do You Know?

Plots are your best friend!

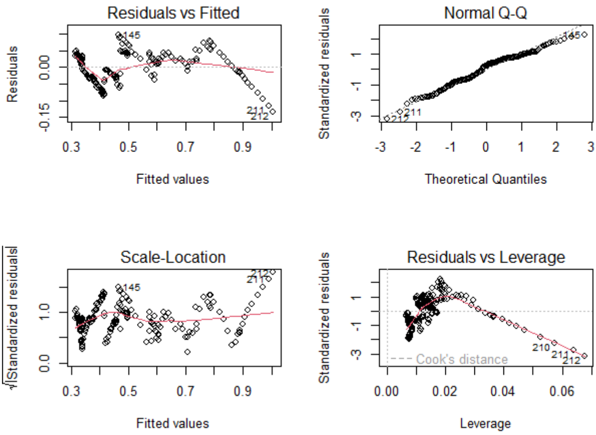
How Do You Know? Toy Model Example

```
## Load your data ----
my_data <- readRDS("data/vdem12.rds")

# Let's change names of some of these variables for the sake of simplicity
# I am also subsetting it to only US
us_data <- my_data |>
  filter(country_name == "United States of America") |>
  rename(democracy = v2x_polyarchy,
         gdp_per_capita = e_gdppc,
         urbanization = e_miurbani,
         regime = v2x_regime,
         polarization = v2cacamps,
         polarization_ordinal = v2cacamps_ord) |>
  mutate(regime_binary = ifelse(regime %in% c(2,3), 1, 0),
         high_polarization = ifelse(polarization >= -1, 1, 0))

# Use correlation matrix to see the relationship between variables
chart.Correlation(us_data |> select(democracy, gdp_per_capita, urbanization))

# This is our toy model
multiple <- lm(democracy ~ gdp_per_capita + urbanization, data = us_data)

# Use plot() to get diagnostics
plot(multiple)
```

First, check your variables



Model diagnostics using *plot()*



Residual distribution and density

```r
# Residual plot with histogram
hist(multiple$residuals, freq = F, xaxt = "n", xlab = "", ylab = "", main = "")
par(new = T) # sets graphical parameters so that I can plot histogram and density plots
plot(density(resid(multiple)))
```