Bivariate Regression I: Conceptual Overview and Estimation

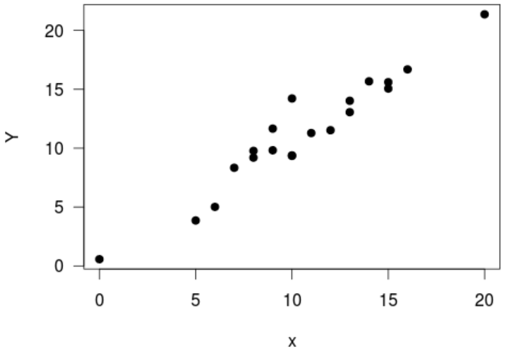
Dr. Michael Fix
mfix@gsu.edu

Georgia State University

3 February 2025

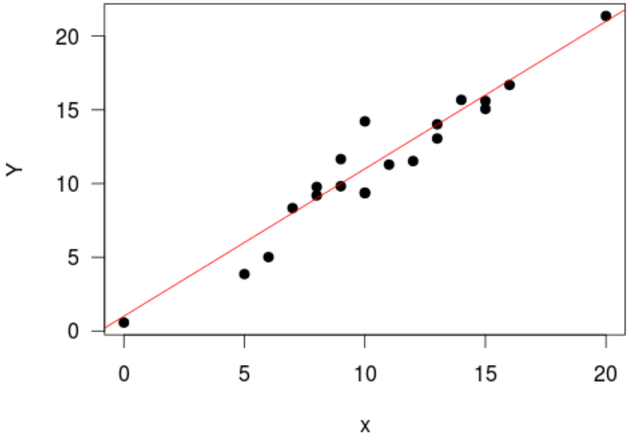
Fundamentals of Regression

- Regression involves the relationship between two (or more) variables:
  - The dependent variable (regressand/response): Y
  - The independent variable (regressor/factor): X
- Graphically, we can represent this with a scatter plot:



Fundamentals of Regression

- Intuitively, we see a line that can be drawn
- How do we get the best line?



Fundamentals of Regression

Least Squares

- The goal is to find a predicted value for Y represented by $\hat{Y}$
- We want to find a line with the basic formula: $\hat{Y} = a + bX$
- Our goal is a line that is the closest to all of the points
- To do this we want to minimize deviation: $d = Y - \hat{Y}$
- Sum this to get the whole and use the square to remove the problem of negatives:

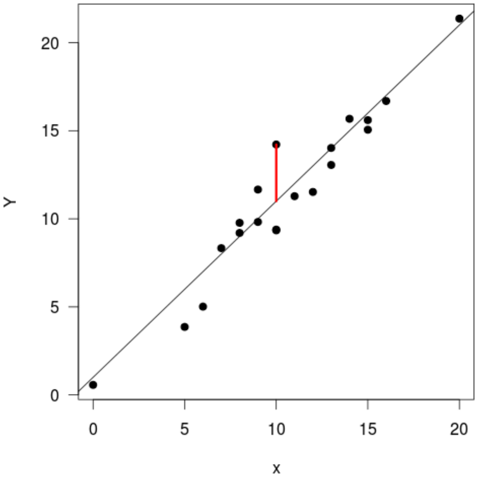
$$\sum d^2 = \sum (Y - \bar{Y})^2 \tag{1}$$

- This method is known as *Ordinary Least Squares (OLS)*

Fundamentals of Regression

Least Squares

- Conceptually we can represent this in graphical form.



Formula for Regression Line

- We need to find the formula for the line that minimizes the sum of squared errors

$$\hat{Y} = a + bX \tag{2}$$

- b indicates the slope of the line
  - This value provides substantive information
  - The change in Y for each unit increase in X
- a indicates the y-intercept of the line
  - This is the value of Y when $X = 0$

Computing OLS Estimates

- b can be calculated from the deviations of X and Y from their respective means:

$$b = \frac{\sum(X - \bar{X})(Y - \bar{Y})}{\sum(X - \bar{X})^2} \tag{3}$$

- a is found by solving equation (2) to get:

$$a = \bar{Y} - b\bar{X} \tag{4}$$

Computing OLS Estimates in R

- OLS is computationally simple enough that in the bivariate case, with a small N, we can hand calculate our estimates
- However, we do not generally do this as it is inefficient and doesn't scale up well

```r
### Load necessary packages ----
# Use install.packages() if you do not have this package
library(tidyverse) # Data manipulation
library(stargazer) # Creates nice regression output tables

### Load your data ----
# We are using V-Dem version 12
my_data <- readRDS("data/vdem12.rds")

# Let's change names of some of these variables for the sake of simplicity
my_data <- my_data |>
  rename(democracy = v2x_polyarchy, gdp_per_capita = e_gdppc)

### Run a bivariate OLS ----
# We are going to use lm() function (which means linear model).
# Always check function help page!
?lm
help(lm)

# Here is how you specify your variables:
# lm(dependent_variable ~ independent_variable(s), data = your_data)
# ~ => this is tilda

# For example:
lm(democracy ~ gdp_per_capita, data = my_data)
```

Regression Output

```r
lm(democracy ~ gdp_per_capita, data = my_data)

# This produces very little info, so we save this output as a list object and then examine it:

my_lm <- lm(democracy ~ gdp_per_capita, data = my_data) # creates a list object called my_lm

summary(my_lm) # gives more detailed output
```

```
> # For example:
> lm(democracy ~ gdp_per_capita, data = my_data)

Call:
lm(formula = democracy ~ gdp_per_capita, data = my_data)

Coefficients:
   (Intercept)  gdp_per_capita
        0.2158          0.0117

> summary(my_lm) # gives more detailed output

Call:
lm(formula = democracy ~ gdp_per_capita, data = my_data)

Residuals:
     Min       1Q   Median       3Q      Max
-2.03380 -0.16797 -0.05647  0.14826  0.58390

Coefficients:
                Estimate Std. Error t value Pr(>|t|)
(Intercept)    0.2158381  0.0018741  115.17   <2e-16 ***
gdp_per_capita 0.0117026  0.0001469   79.68   <2e-16 ***
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Residual standard error: 0.2348 on 21377 degrees of freedom
  (6001 observations deleted due to missingness)
Multiple R-squared:  0.229,	Adjusted R-squared:  0.229
F-statistic:  6349 on 1 and 21377 DF,  p-value: < 2.2e-16
```

Better Regression Output using stargazer()

```r
### Stargazer package ----
# Let's create better looking output using stargazer function
stargazer(my_lm, type = "text") # Change type to latex if you're importing to LaTeX

# Let's make it much better and export it to latex!
stargazer(my_lm,
          type = "latex",
          title = "The relationship between democracy and GDP per capita",
          covariate.labels = c("GDP per capita"),
          dep.var.labels = c("Electoral Democracy Index"),
          ci.level = 0.95,
          star.cutoffs = c(0.05),
          notes.align = "l",
          notes.append = FALSE,
          notes.label = "Notes",
          notes = "*p < 0.05. Standard errors are in parentheses.")
```

Better Regression Output using stargazer()

Table 1: The relationship between democracy and GDP per capita

	Dependent variable:
	Electoral Democracy Index
GDP per capita	0.012*
	(0.0001)
Constant	0.216*
	(0.002)
Observations	21,379
R^2	0.229
Adjusted R^2	0.229
Residual Std. Error	0.235 (df = 21377)
F Statistic	6,349.082* (df = 1; 21377)
Notes	$^{*}p < 0.05$. Standard errors are in parentheses.

Why regression?

	Description	Explanation	Prediction
Task	Summarize data	Correlation/causation	Forecast OOS / future data
Emphasis	Data	Theory / Hypotheses	Outcomes
Focus	Univariate	Multivariate	Multivariate
Typical Application	Summarize / “reduce” data	Discuss marginal associations between predictors and an outcome of interest	Optimize out-of-sample predictive power / minimize prediction error

Where Do We Go From Here?

- How to use OLS for hypothesis testing
- Assumptions of the OLS Estimator
- Model fit
- Beyond the bivariate case

What Won't We Do?

- Multivariate Regression
- Measurement models
- Time series
- Machine Learning