Interaction Terms

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Intro to Interaction Terms

- The use of interaction terms in applied political science research is quite common
- Unfortunately, the misuse of interaction terms is nearly as common
- We need to understand:
    - What is an interaction (or multiplicative term)?
    - What are some of the most common problems with the use of interaction terms?
    - How do we properly model and interpret interaction terms?

Intro to Interaction Terms

- As with most things, theory is a good guide for when to use interaction terms
- If we are going to model an interaction term, some key points to remember:
  1. Include all constitutive terms
  2. Do not incorrectly interpret constitutive terms
  3. Coefficient estimates for interaction terms rarely tell the whole story

Why Use Interaction Terms

- Assume that rather than the direct effect of X_1 on Y, you are interested in some *conditional* relationship
    - Perhaps you hypothesize that X_1 will have an impact on Y when some condition is absent but not if that condition is present
    - Or you hypothesize that the impact of X_1 on Y will be different across a set of conditions

Why Use Interaction Terms

- Generally, anytime we are interested in conditional relationship, then we must model an interaction term to properly test our hypotheses
- Failure to do so would result in an underspecified model unless we account for this in some other way (e.g. subsetting our data).

Why Use Interaction Terms

Always follow theory!

Problem 1

Excluding Constitutive Terms

Assume the following (correctly specified) regression equation:

$$Y_i = \beta_0 + \beta_1 X_{1i} + \beta_2 X_{2i} + \beta_3 X_{1i} X_{2i} + u_i$$

- Even if we are only interested in the interaction $X_{1i}X_{2i}$, we still **MUST** include both constitutive terms X_{1i} and X_{2i}
- For example, modeling the above as simply $Y_i = \beta_0 + \beta_3 X_{1i} X_{2i} + u_i$ would be incorrecly specified

Problem 2

Interpreting Constitutive Terms

Given the correct specification::

$$Y_i = \beta_0 + \beta_1 X_{1i} + \beta_2 X_{2i} + \beta_3 X_{1i} X_{2i} + u_i$$

- We must remember that we cannot interpret the coefficient on the constitutive terms as unconditional effect
- In the above example, we cannot interpret β_1 as the effect of X_{1i} on Y_i

Problem 3

Limits of Interpreting Coefficients

- Even when properly specified, interpreting the coefficient on interaction terms can be less than straightforward
- The best way to present meaningful quantities of interest from interactive models is through graphs

Modeling Interaction Effects

$$Y_i = \beta_0 + \beta_1 X_{1i} + \beta_2 X_{2i} + \beta_3 X_{1i} X_{2i} + u_i$$

$$\begin{aligned} \mathrm{E}(Y_i) &= \beta_0 + \beta_1 X_{1i} + \beta_2 X_{2i} + \beta_3 X_{1i} X_{2i} \\ &= \beta_0 + \beta_2 X_{2i} + (\beta_1 + \beta_3 X_{2i}) X_{1i} \\ &= \beta_0 + \beta_2 X_{2i} + \psi_1 X_{1i} \end{aligned}$$

where $\psi_1 = \beta_1 + \beta_3 X_{2i}$. This means:

$$\frac{\partial \mathrm{E}(Y_i)}{\partial X_1} = \beta_1 + \beta_3 X_{2i}.$$

Modeling Interaction Effects

Similarly:

$$\begin{aligned} \mathrm{E}(Y_i) &= \beta_0 + \beta_1 X_{1i} + (\beta_2 + \beta_3 X_{1i}) X_{2i} \\ &= \beta_0 + \beta_1 X_{1i} + \psi_2 X_{2i} \end{aligned}$$

which implies:

$$\frac{\partial \mathrm{E}(Y_i)}{\partial X_2} = \beta_2 + \beta_3 X_{1i}.$$

"Direct Effects"

If $X_2 = 0$, then:

$$\begin{aligned} \mathrm{E}(Y_i) &= \beta_0 + \beta_1 X_{1i} + \beta_2(0) + \beta_3 X_{1i}(0) \\ &= \beta_0 + \beta_1 X_{1i}. \end{aligned}$$

Similarly, for $X_1 = 0$:

$$\begin{aligned} \mathrm{E}(Y_i) &= \beta_0 + \beta_1(0) + \beta_2 X_{2i} + \beta_3(0) X_{2i} \\ &= \beta_0 + \beta_2 X_{2i} \end{aligned}$$

Types of Interactions: Dichotomous Xs

For

$$Y_i = \beta_0 + \beta_1 D_{1i} + \beta_2 D_{2i} + \beta_3 D_{1i} D_{2i} + u_i$$

we have:

$$\begin{aligned}
\mathrm{E}(Y|D_1 = 0, D_2 = 0) &= \beta_0 \\
\mathrm{E}(Y|D_1 = 1, D_2 = 0) &= \beta_0 + \beta_1 \\
\mathrm{E}(Y|D_1 = 0, D_2 = 1) &= \beta_0 + \beta_2 \\
\mathrm{E}(Y|D_1 = 1, D_2 = 1) &= \beta_0 + \beta_1 + \beta_2 + \beta_3
\end{aligned}$$

Dichotomous and Continuous Xs

$$Y_i = \beta_0 + \beta_1 X_i + \beta_2 D_i + \beta_3 X_i D_i + u_i$$

gives:

$$\mathrm{E}(Y|X, D = 0) = \beta_0 + \beta_1 X_i$$
$$\mathrm{E}(Y|X, D = 1) = (\beta_0 + \beta_2) + (\beta_1 + \beta_3) X_i$$

Four possibilities:

- $\beta_2 = \beta_3 = 0$
- $\beta_2 \neq 0$ and $\beta_3 = 0$
- $\beta_2 = 0$ and $\beta_3 \neq 0$
- $\beta_2 \neq 0$ and $\beta_3 \neq 0$

Two Continuous Xs

$$Y_i = \beta_0 + \beta_1 X_{1i} + \beta_2 X_{2i} + \beta_3 X_{1i} X_{2i} + u_i.$$

Implies

$$\beta_3 = 0 \;\rightarrow\; \frac{\partial E(Y)}{\partial X_1} = \beta_1 \,\forall\, X_2 \text{ and } \frac{\partial E(Y)}{\partial X_2} = \beta_2 \,\forall\, X_1$$

Multiplicative Interactions in R

```
### Two Dummies

```{r}
model_dd <- lm(democracy ~  gdp_per_capita + high_polarization*corrupt, data_2000)
```

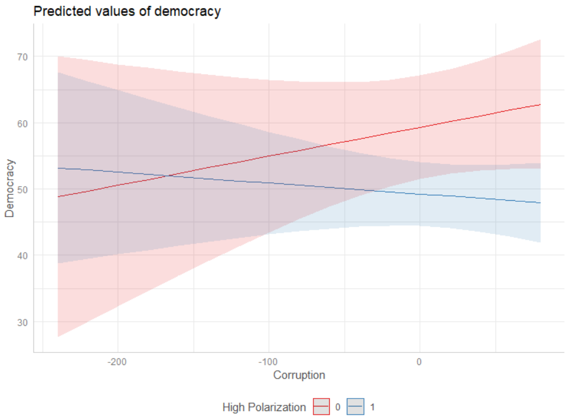
A Dummy and Continuous

```{r}
model_dc <- lm(democracy ~  gdp_per_capita + high_polarization*corrupt_cont, data_2000)
```
```

	Dependent variable:	
	democracy	
	(1)	(2)
gdp_per_capita	0.562***	0.573***
	(0.145)	(0.145)
high_polarization1	-4.028	-10.064**
	(8.659)	(4.793)
corrupt1	6.821	
	(8.158)	
high_polarization1:corrupt1	-9.093	
	(9.867)	
corrupt_cont		0.044
		(0.041)
high_polarization1:corrupt_cont		-0.060
		(0.049)
Constant	47.404***	51.829***
	(7.653)	(4.898)
Observations	160	160
R2	0.165	0.168
Adjusted R2	0.143	0.146
Residual Std. Error (df = 155)	25.400	25.349
F Statistic (df = 4; 155)	7.631***	7.820***
Note:	*p<0.1; **p<0.05; ***p<0.01	

Prediction Plot with *ggpredict* (dummy and continuous)

```{r}
ggpredict(model_dc, terms = c("corrupt_cont", "high_polarization")) |>
  plot() +
  labs(x = "Corruption", y = "Democracy", color = "High Polarization") +
  theme(legend.position = "bottom")
```



Prediction Tables with *ggpredict* (two dummies)

```{r}
ggpredict(model_dd, terms = c("high_polarization", "corrupt"))
```

```
# Predicted values of democracy

corrupt: 0

high_polarization | Predicted |       95% CI
----------------------------------------------
0                 |     54.75 | 40.95, 68.54
1                 |     50.72 | 41.22, 60.22

corrupt: 1

high_polarization | Predicted |       95% CI
----------------------------------------------
0                 |     61.57 | 52.44, 70.69
1                 |     48.45 | 42.96, 53.93

Adjusted for:
* gdp_per_capita = 13.06
```