Principles of Communications Lecture 3: Analog Modulation Techniques (1)

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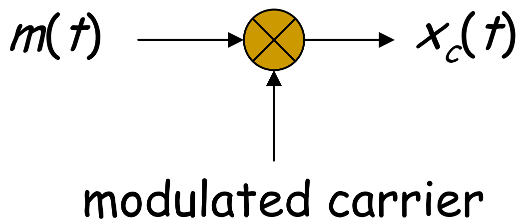
Outlines

- Linear Modulation
- Angle Modulation
- Interference
- Feedback Demodulators
- Analog Pulse Modulation
- Delta Modulation and PCM
- Multiplexing

Types of Modulation



- Analog modulation and Digital modulation
  - A process to translate the information data to a new spectral location depending on the intended frequency for transmission.
- Modulation, historically, is done on the RF transmission system. Thus, the conversion from message signals to RF signals is called modulation.
- Analog modulation: **continuous-wave modulation** and **pulse modulation** (sampled data)
  - Continuous-wave modulation: **linear modulation** (AM) and **angle modulation** (FM)

Linear Modulation

- General form: $\boxed{x_c(t) = A_c(t)\cos\omega_c t}$

  $A_c(t)$: 1-to-1 correspondence to the message $m(t)$

  $\cos(\omega_c t)$: carrier ($\omega_c t$ is fixed)

- **DSB** (Double-Sideband) Suppressed Carrier (SC)

$$x_c(t) = A_C m(t)\cos\omega_c t$$

$$\Leftrightarrow \qquad X_C(f) = \frac{1}{2}A_C M(f+f_C) + \frac{1}{2}A_C M(f-f_C)$$

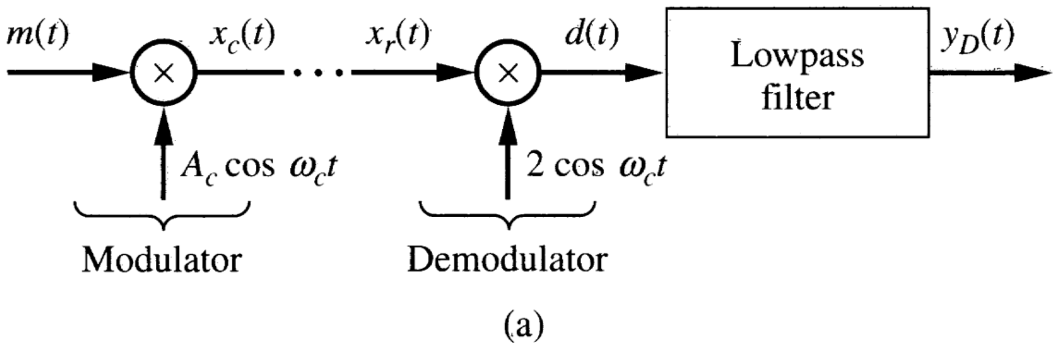
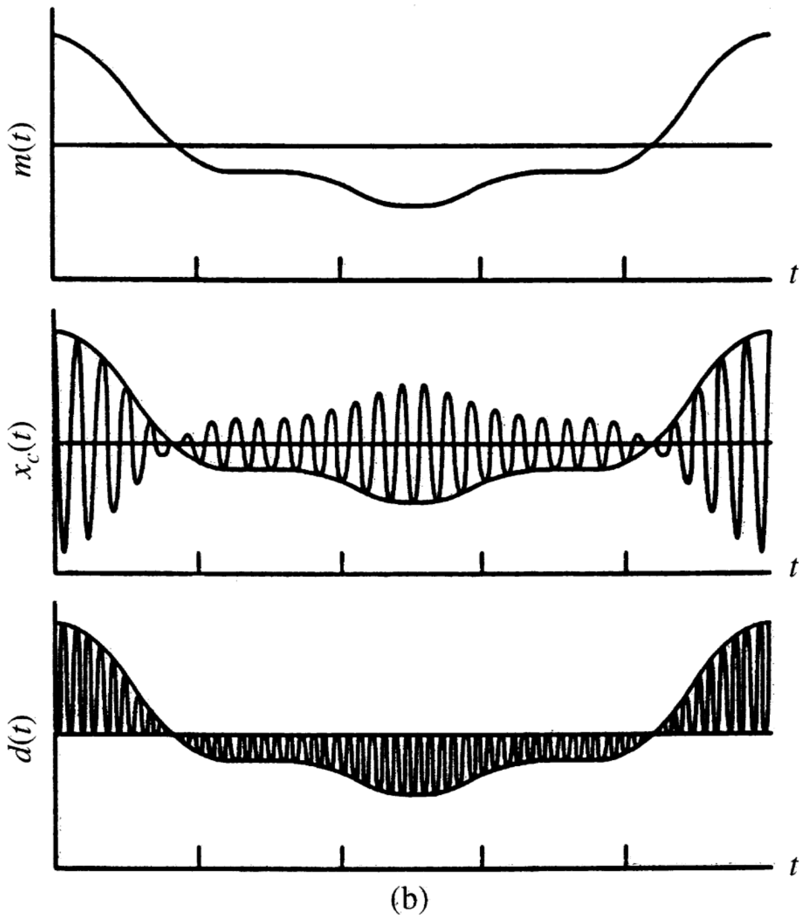
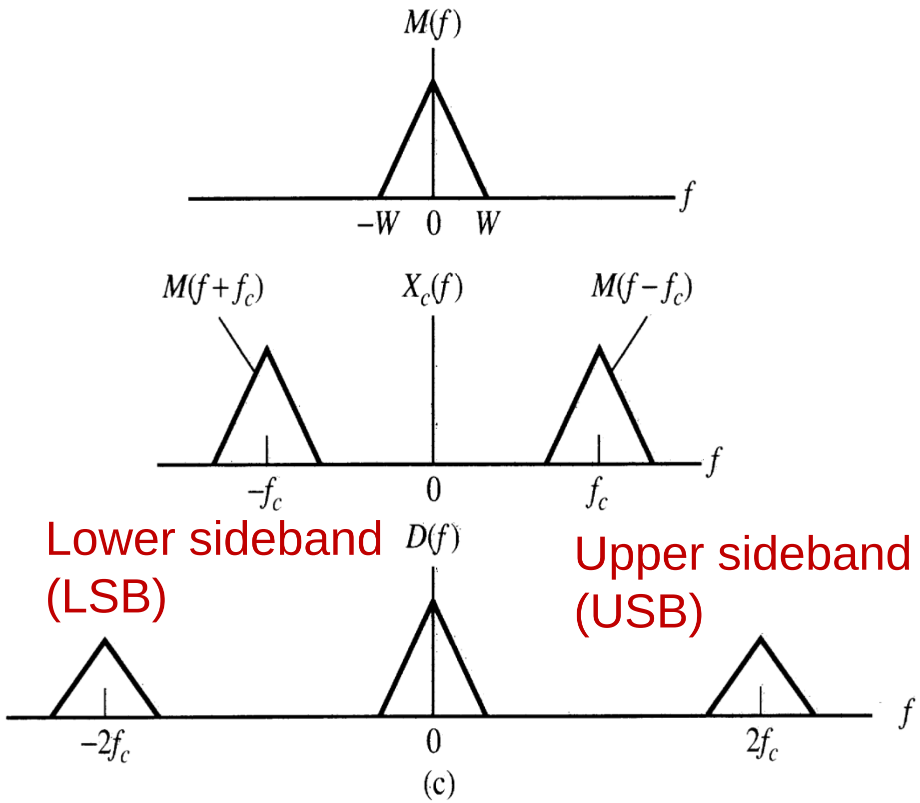


Figure 3.1
Double-sideband modulation.
(a) System. (b) Waveforms.
(c) Spectra.





DSB-SC

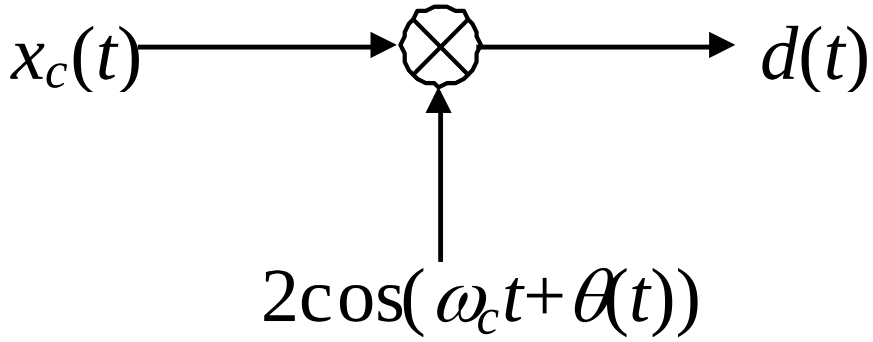
- **Coherent (Synchronous) Demodulator** (Detector): The receiver knows *exactly* the *phase* and *frequency* of the carrier in the received signal.

$$d(t) = x_c(t) \cdot 2\cos\omega_c t = [A_C m(t)\cos\omega_c t] \cdot 2\cos\omega_c t$$
$$= \underbrace{A_C m(t)}_{\text{desired part}} + \underbrace{A_C m(t)\cos 2\omega_c t}_{\text{High freq. noise}}$$

Message *m*(*t*) is recovered!

- What if the receiver reference is not coherent?

  -- A phase error occurs ($\theta(t)$, unknown, random, time-varying, …)



$$d(t) = 2A_C m(t)\cos\omega_c t\cdot\cos(\omega_c t+\theta(t))$$

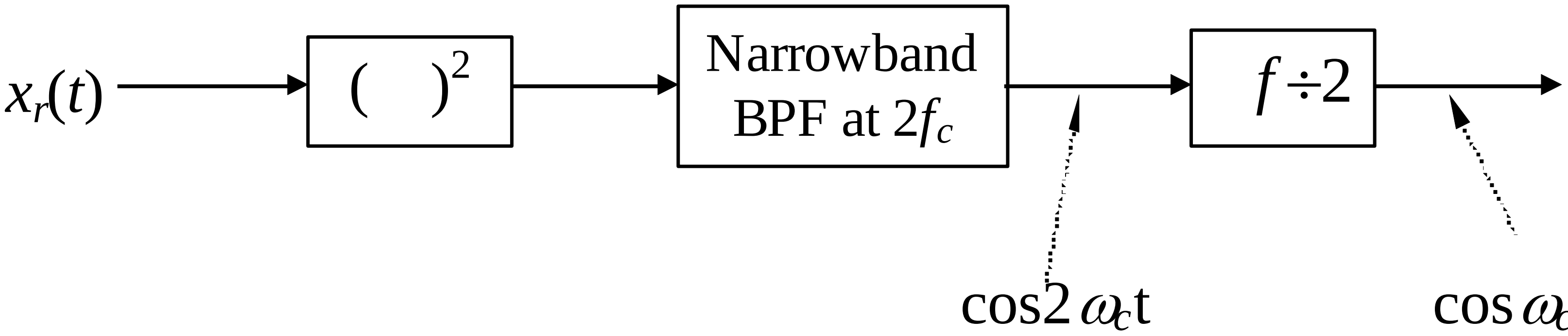
$$= A_C m(t)\cos\theta(t) + A_C m(t)\cos(2\omega_c t+\theta(t))$$

$\Rightarrow$ $$y_D(t) = m(t)\cos\theta(t), \qquad -1\le\cos\theta(t)\le 1$$

It is time-varying !!

Carrier Recovery

- **Carrier recovery:** Regenerate the carrier (f_c and $\theta(t)$) at the receiver site
- Example: Square circuit

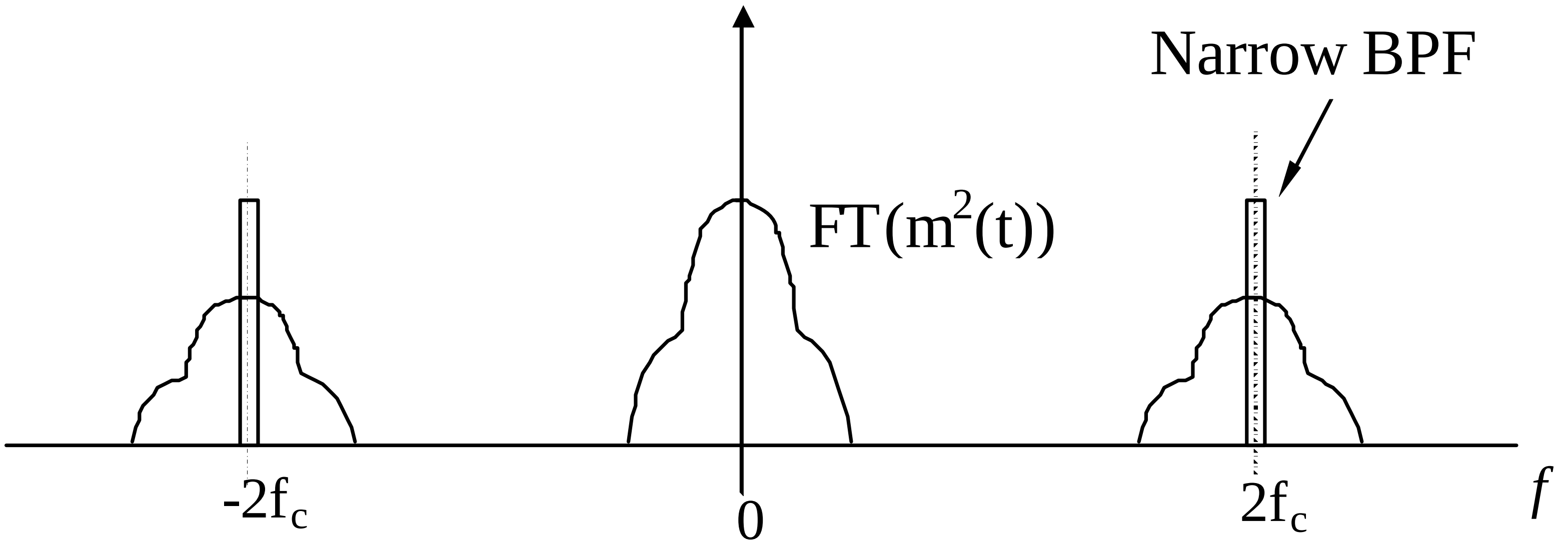


$$x_r^2(t) = A_C^2 m^2(t)\cos^2\omega_c t = \underbrace{\frac{1}{2}A_C^2 m^2(t)}_{\text{DC}} + \underbrace{\frac{1}{2}A_C^2 m^2(t)\cos 2\omega_c t}_{\text{Carrier (2xf)!}}$$

Carrier Recover (2)

- How to extract the carrier? It becomes clearer when we examine it in the frequency domain.

FT of $x_r^2(t)$:



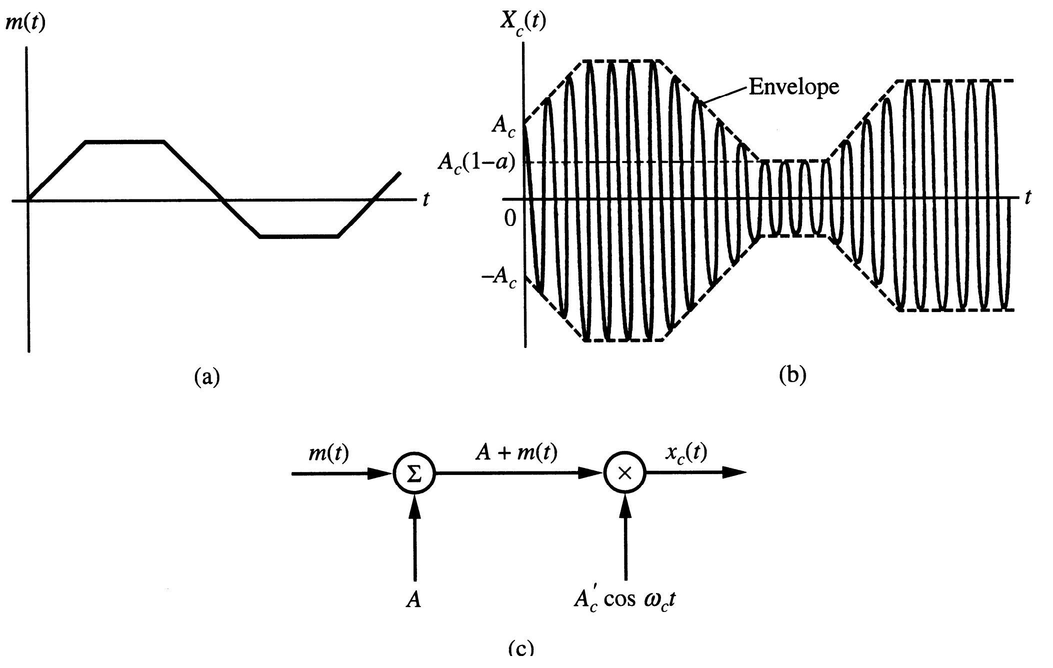
Remarks

- The spectrum of DSB signal does not contain a discrete spectral component at the carrier frequency unless $m(t)$ has a DC component.
- DSB systems with no carrier frequency component present are often referred to as suppressed carrier (SC) systems.
- If the carrier frequency is transmitted along with DSB signal, the demodulation process can be rather simplified.
- Alternatively, let’s see the following amplitude modulation (AM) scheme.

Amplitude Modulation

- A DC bias *A* is added to *m*(*t*) prior to the modulation process
  - The result is that a carrier component is present in the transmitted signal
- Definition

$$x_c(t) = [A + m(t)]A_c' \cos\omega_c t$$
$$= A_c[1 + am_n(t)]\cos\omega_c t$$



AM

- **Amplitude Modulation** (AM): DSB with carrier

$$x_c(t) = A_C[1 + am_n(t)]\cos\omega_c t$$

$$m_n(t) = \frac{m(t)}{\left|\min_t m(t)\right|},$$

$m_n(t)$: the normalized message

$m(t)$: the original message

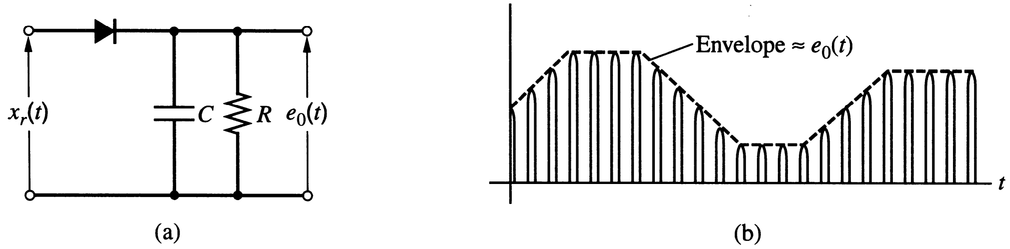
$$a = \frac{\left|\min_t m(t)\right|}{A}$$

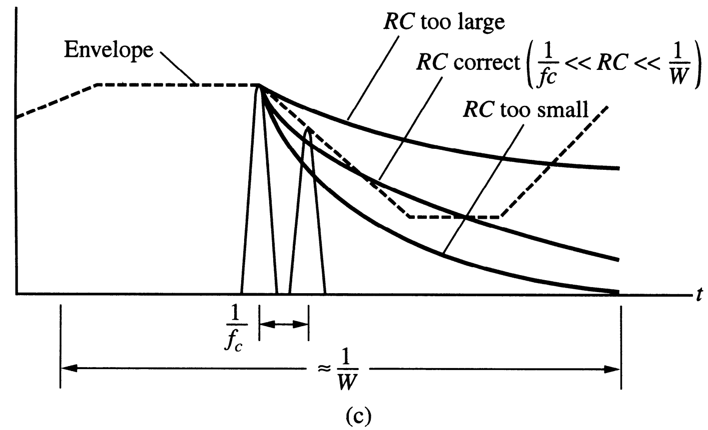
A: the DC bias

a: the modulation index (had better be less than 1)

- Normalized message → $1+m_n(t) >= 0$

Envelope Detection

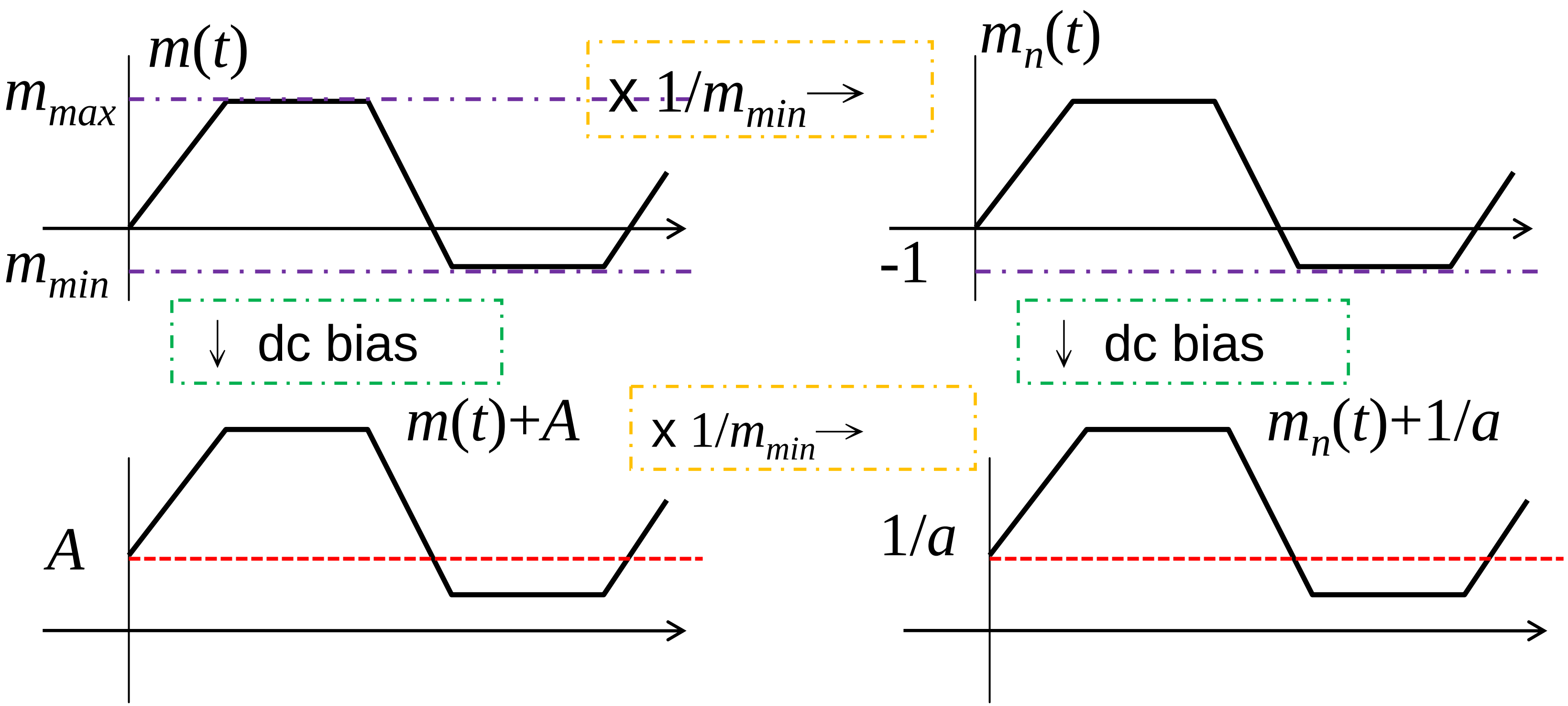




- The modulation index is defined such that if a=1, the minimum value of $A_c[1+am_n(t)]$ is zero
  - a <1, it results in $A_c[1+am_n(t)]$ >0 for all t
- In AM, all the information is just the envelop.
- The envelop detection is a simple and straightforward technique

AM (2)

- **Over-modulation:** modulation index $a > 1$
- **DC bias:** the shifted level of the zero-value message



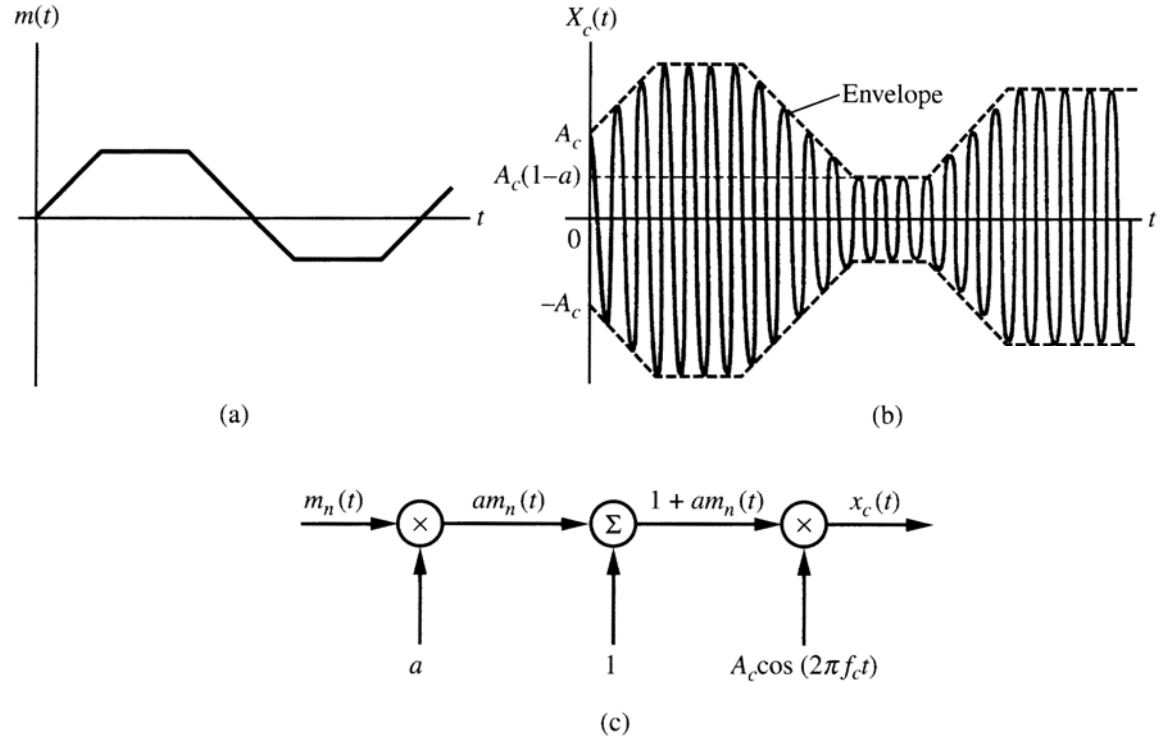
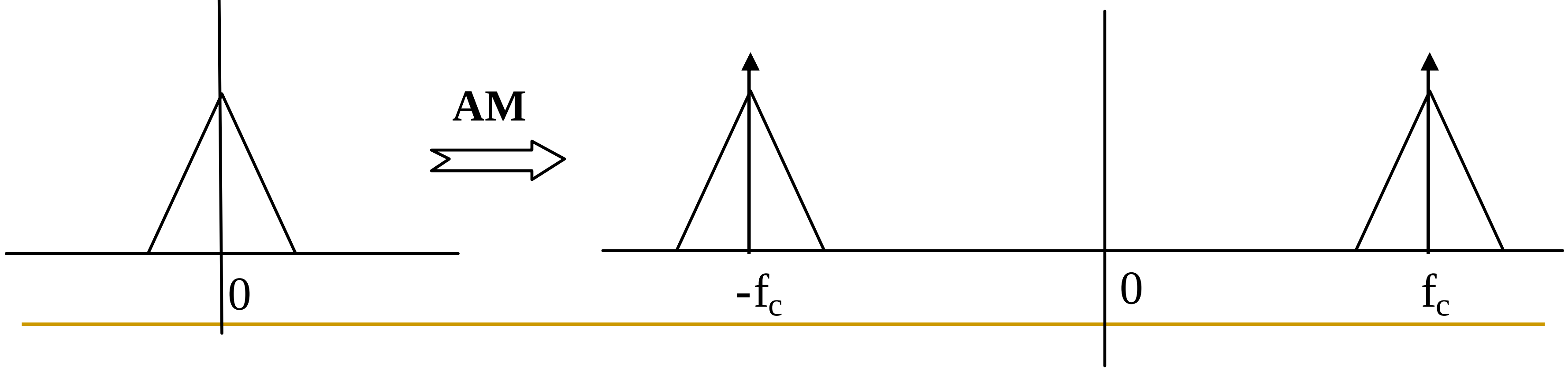


Figure 3.2
Amplitude modulation. (a) Message signal. (b) Modulator output for $a < 1$. (c) Modulator.



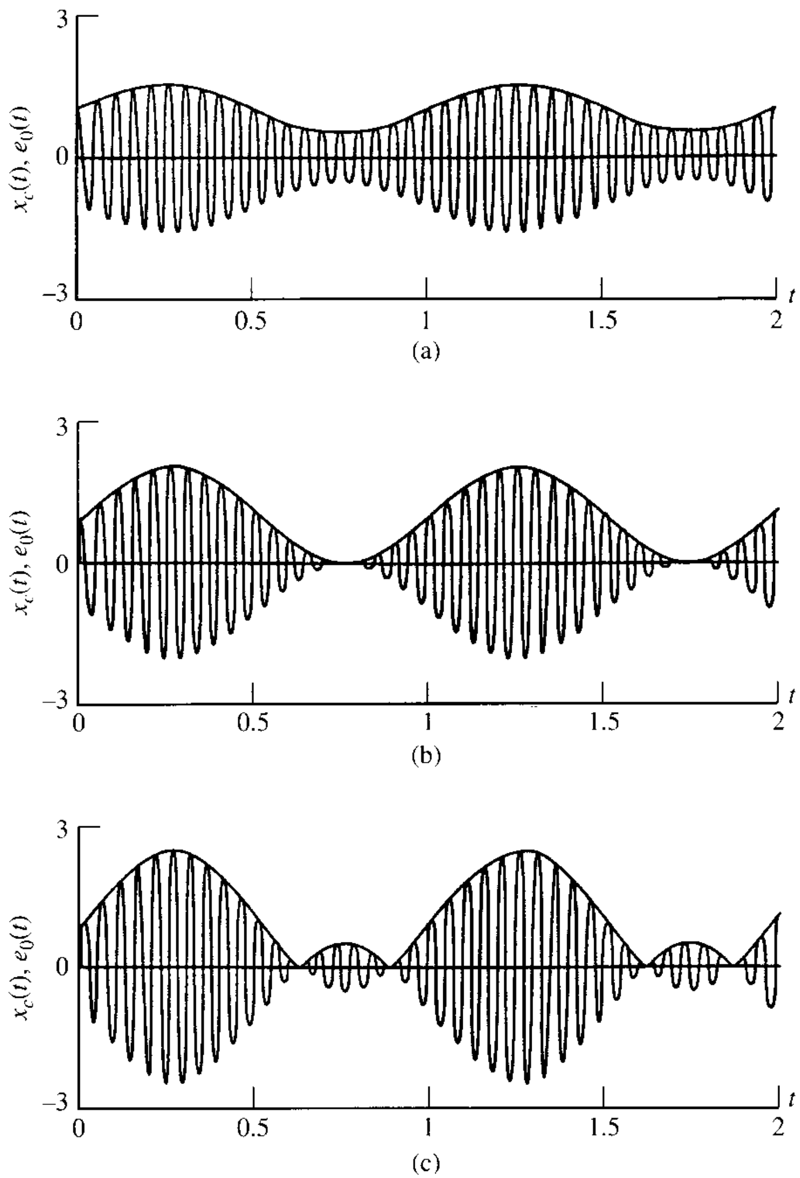


Figure 3.4
Modulated carrier and envelope detector outputs for various values of the modulation index. (a) $a = 0.5$. (b) $a = 1.0$. (c) $a = 1.5$.

AM Demodulation

- **Coherent detection:** precise but requires carrier recovery circuit.
- **Incoherent detection, envelope detection**: simple receiver (LPF) but requires sufficient carrier power ($a < 1$) and $f_c >> W$. (In theory, $f_c > W$ is sufficient, but a "good" LPF is needed.)
- Impulse response of RC circuit:

$$h(t) = \frac{1}{RC} e^{-t/RC} u(t)$$

Remarks

- The time constant RC of the envelop detector is an important design parameter.
- The appropriate RC time constant is related to the carrier frequency f_c and to the bandwidth W of the original signal $m(t)$
  - $1/f_c << RC << 1/W$, between then and must be well separated from both

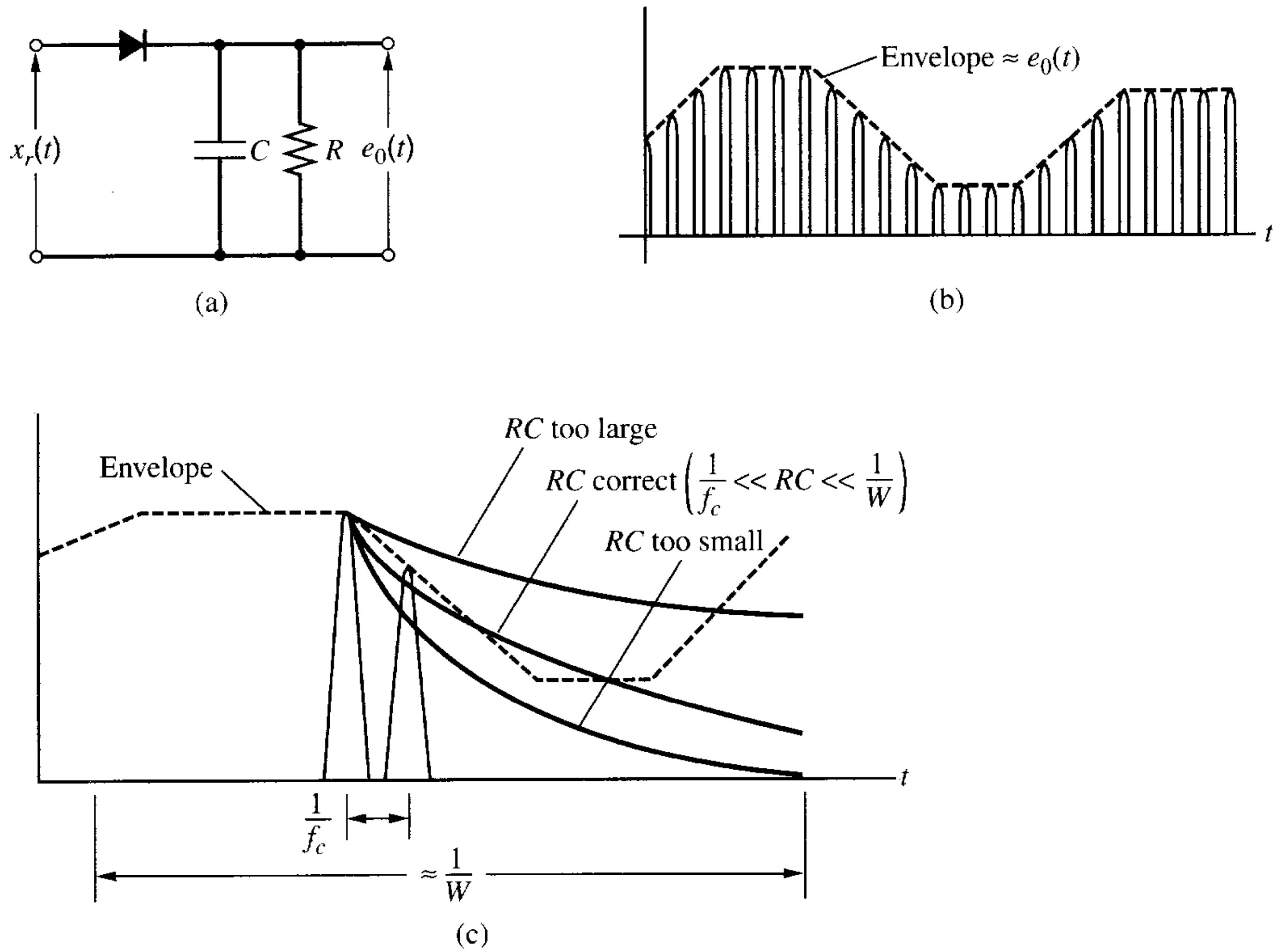


Figure 3.3
Envelope detection. (a) Circuit. (b) Waveforms. (c) Effect of RC time constant.

Power Efficiency of AM

- Suppose that *m*(*t*) has zero mean, then the total power contained in the AM modulator output is

$$\left\langle x_c^2(t)\right\rangle = \left\langle [A+m(t)]^2 (A_C')^2 \cos^2 \omega_c t\right\rangle$$

$\langle\cdot\rangle$ denotes the time average value

$$= \left\langle \frac{1}{2}[A+m(t)]^2 (A_C')^2 \right\rangle + \left\langle \frac{1}{2}[A+m(t)]^2 (A_C')^2 \cos 2\omega_c t\right\rangle$$

$$= \frac{1}{2} A_C'^2 [A^2 + 2A\left\langle m(t)\right\rangle + \left\langle m^2(t)\right\rangle]$$

$$= \frac{1}{2} A_C'^2 [A^2 + \left\langle m^2(t)\right\rangle]$$

- The power efficiency : the power ratio of the input information to the transmitted signal

$$E \equiv \text{Efficiency} \equiv \frac{\left\langle m^2(t)\right\rangle}{A^2 + \left\langle m^2(t)\right\rangle} \times 100\% = \frac{a^2\left\langle m_n^2(t)\right\rangle}{1+a^2\left\langle m_n^2(t)\right\rangle} \times 100\%$$

Power Efficiency Example

$$E \equiv Efficiency \equiv \frac{\langle m^2(t) \rangle}{A^2 + \langle m^2(t) \rangle}(100\%) = \frac{a^2 \langle m_n^2(t) \rangle}{1 + a^2 \langle m_n^2(t) \rangle}(100\%)$$

$$m_n(t) = \frac{m(t)}{\left| \min_t m(t) \right|},$$

- If the signal has symmetrical value, i.e. $|\min m(t)| = |\max m(t)|$, then $|m_n(t)| \leq 1$ and hence $\langle m_n^2(t) \rangle \leq 1$.
  - If $a \leq 1$, the maximum efficiency is 50%, e.g. the square wave-type
  - For a sine wave, $\langle m_n^2(t) \rangle = 1/2$, for a=1, the efficiency is 33.3%
  - If we allow $a > 1$,
    - Efficiency can exceed 50%, ($a \to \infty$, the efficiency=100%)
    - But, the envelope detector is precluded.

Remarks

- The main advantage of AM:
  - A coherent reference is not necessary for demodulation as long as $a \leq 1$
- The disadvantage of AM:
  - The power efficiency
  - The DC value of the message signal $m(t)$ cannot be accurately recovered. (mixed with carrier)

Single Sideband (SSB) Modulation

- Why SSB?
  - In DSB, either the USB and the LSB have equal amplitude and odd phase symmetry about the carrier frequency

  Send only "half" signal (USB & LSB symmetric);

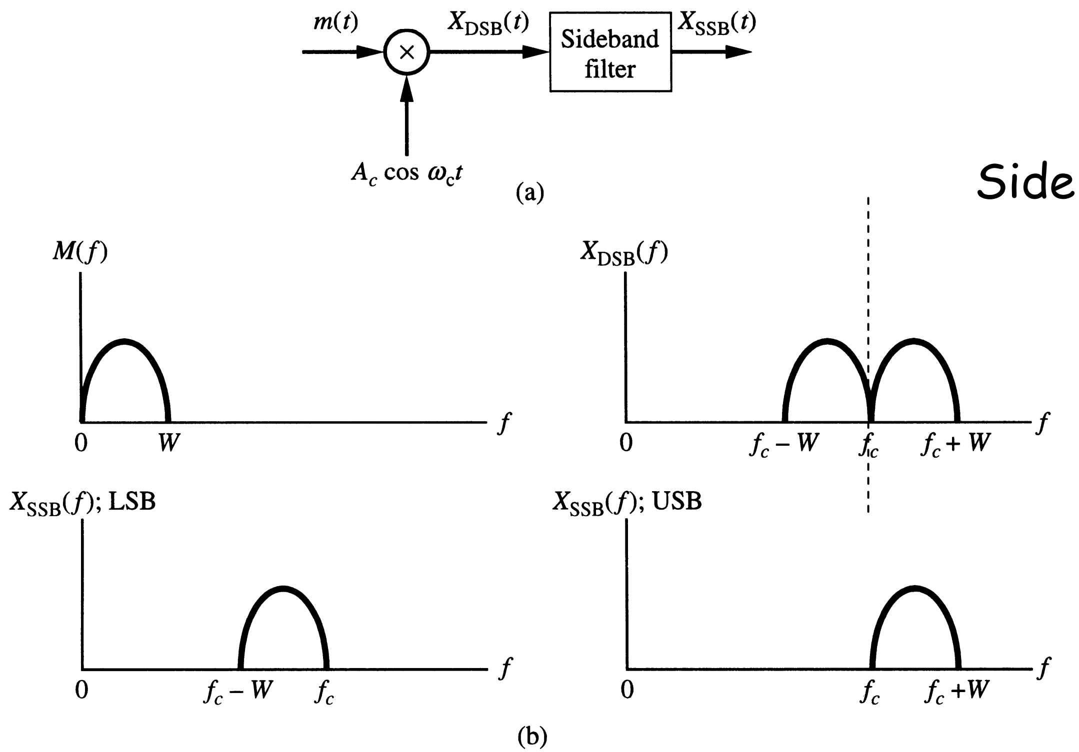
  Good power efficiency; Good bandwidth utilization

  Basis of more advanced modulations

- Methods to generate SSB signals
- Method 1: **Sideband** (BPF) filtering

  Easy to understand, but difficult to implement.
- Method 2: **Phase-shift** modulation

Sideband Filtering



Sideband filtering

- An ideal passband filter is necessary
- The (very) low frequency component will be encapulated

SSB Modulation

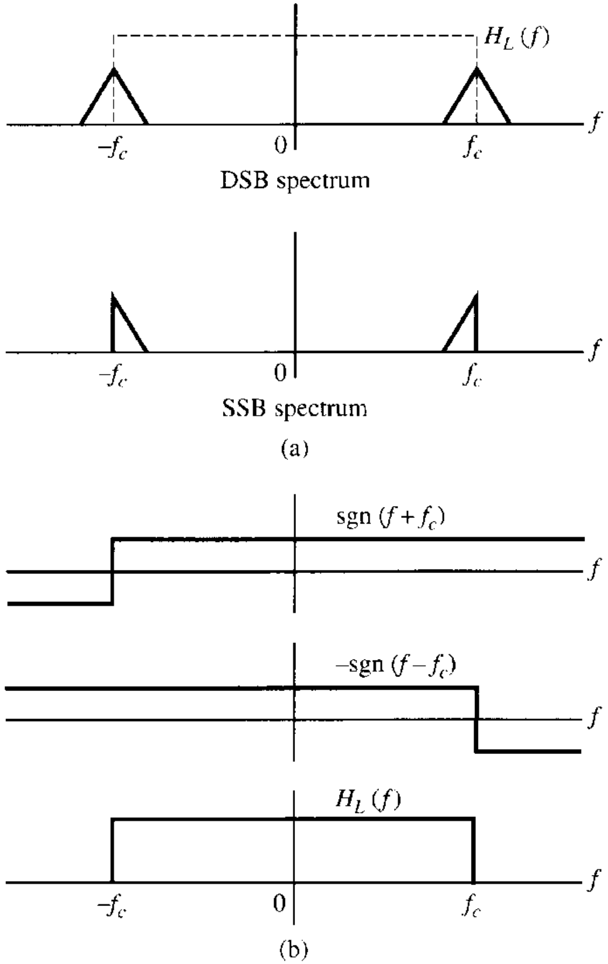
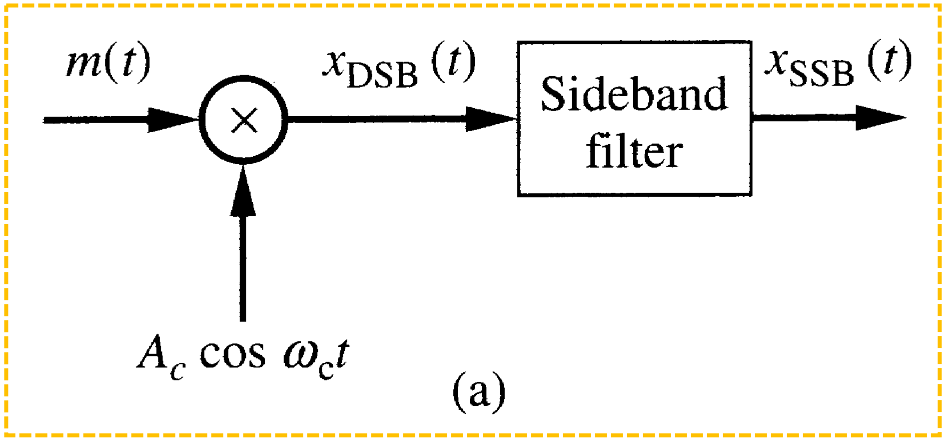


Figure 3.7
Generation of lower-sideband SSB. (a) Sideband filtering process. (b) Generation of lower-sideband filter.



SSB Signal Generation

DSB signal: $X_{DSB}(f) = \frac{A_C}{2} M(f + f_c) + \frac{A_C}{2} M(f - f_c)$

LPF: $H_L(f) = \frac{1}{2}[\mathrm{sgn}(f + f_c) - \mathrm{sgn}(f - f_c)]$

$$
\begin{aligned}
X_c(f) &= X_{DSB}(f) \cdot H_L(f) \\
&= \frac{1}{4} A_C [M(f + f_c)\mathrm{sgn}(f + f_c) + M(f - f_c)\mathrm{sgn}(f + f_c)] \\
&\quad - \frac{1}{4} A_C [M(f + f_c)\mathrm{sgn}(f - f_c) + M(f - f_c)\mathrm{sgn}(f - f_c)] \\
&= \frac{A_C}{4}[M(f - f_c) + M(f + f_c)] && \text{part-A} \\
&\quad + \frac{A_C}{4}[M(f + f_c)\mathrm{sgn}(f + f_c) - M(f - f_c)\mathrm{sgn}(f - f_c)] && \text{part-B}
\end{aligned}
$$

SSB Signal Generation (2)

Part-A $\leftrightarrow$ (FT of) DSB signal: $\dfrac{A_C}{2} m(t)\cos\omega_c t$

Part-B: Let $\hat{m}(t) \equiv \Im^{-1}\{-j\,\mathrm{sgn}(f)\cdot M(f)\}$

- Define **Hilbert Transform**: $\xrightarrow{m(t)} \boxed{-j\mathrm{sgn}(f)} \xrightarrow{\hat{m}(t)}$

Thus, $\hat{M}(f) = -j\,\mathrm{sgn}(f)\cdot M(f)$

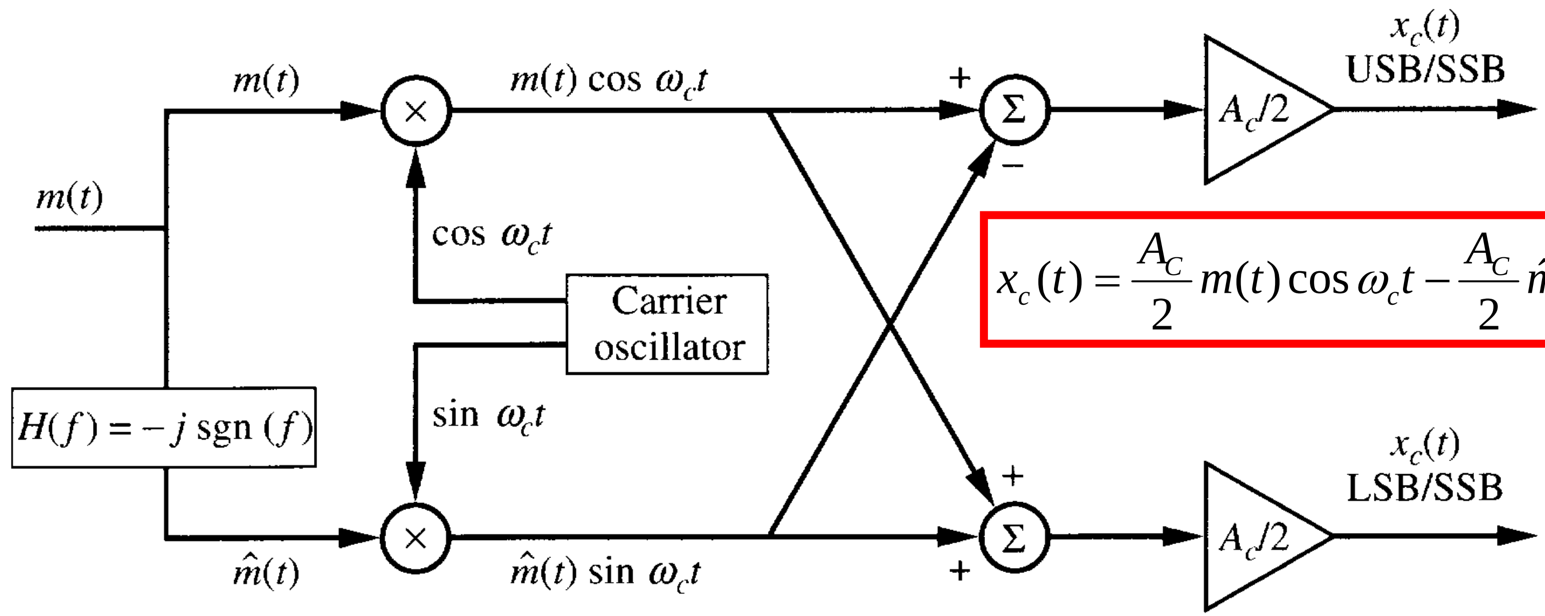
$$\hat{M}(f - f_c) \leftrightarrow \hat{m}(t)e^{j2\pi f_c t}$$

$$\Im^{-1}\{\text{part-B}\} \quad = \frac{A_C}{4}[\,j\hat{m}(t)e^{-j2\pi f_c t} - j\hat{m}(t)e^{j2\pi f_c t}\,]$$

$$= \frac{A_C}{2}\hat{m}(t)[\,j\frac{1}{2}(e^{j2\pi f_c t} - e^{-j2\pi f_c t})] = \frac{A_C}{2}\hat{m}(t)\sin\omega_c t$$

Phase-shift SSB Modulator

Lower-Side Band: $x_c(t) = \frac{A_C}{2} m(t)\cos\omega_c t + \frac{A_C}{2}\hat{m}(t)\sin\omega_c t$



$x_c(t) = \frac{A_C}{2} m(t)\cos\omega_c t - \frac{A_C}{2}\hat{m}(t)\sin\omega_c t$

$x_c(t) = \frac{A_C}{2} m(t)\cos\omega_c t + \frac{A_C}{2}\hat{m}(t)\sin\omega_c t$

Figure 3.8
Phase-shift modulator.

Phase-shift SSB Modulator **(2)**

Upper-Side Band: $x_c(t) = \frac{A_C}{2} m(t) \cos \omega_c t - \frac{A_C}{2} \hat{m}(t) \sin \omega_c t$

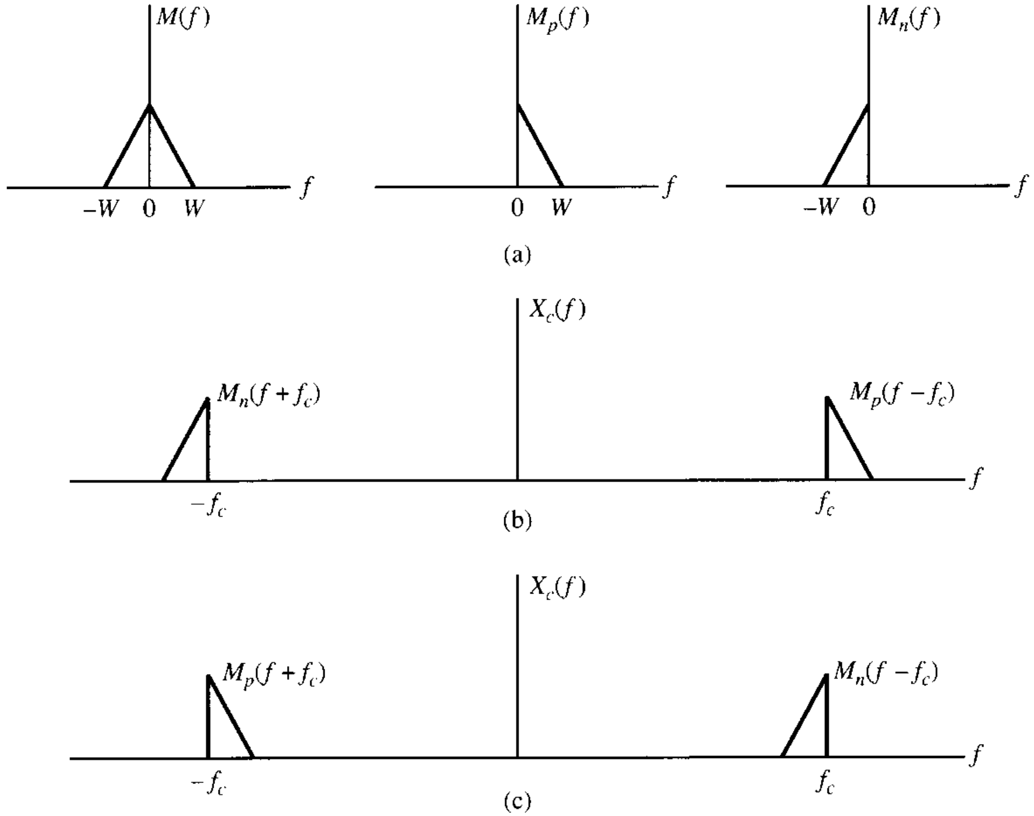


Figure 3.9
Alternative derivation of SSB signals. (a) $M(f), M_p(f)$, and $M_n(f)$. (b) Upper-sideband SSB signal. (c) Lower-sideband SSB signal.