UNIT - II

Multirate Digital Signal Processing:-

1) Introduction:-

In many practical applications of DSP, one is faced with problem of changing the sampling rate of signal, either increasing it or decreasing it by some amount.

Eg:- In telecommunication system that transmits & receive different types of signals (Eg:- teletype, facsimile, video, speech, etc). there is requirement to process the various signals at different rates commensurate with the corresponding bandwidths of signals.

→ This process of converting a signal from a given rate to a different rate is called sampling rate conversion.

→ The multiple sampling rates in processing of digital signals are called multirate digital signal processing systems.

→ Sampling rate conversion of digital signal done in 2-methods

(a) 1st method is to pass the digital signal through D/A converter, filter it if necessary and then resample resulting analog signal at desired rate (i.e to pass the analog signal through an A/D converter).

(b) 2nd method is to perform sampling rate conversion entirely in digital domain.

*) The process of sampling rate conversion in the digital domain can be viewed as linear filtering operation.

→ The i/p signal $x(n)$ is characterized by the sampling rate

$$F_x = \frac{1}{T_x}$$

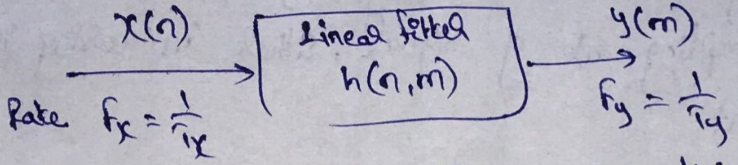
The o/p signal $y(m)$ is characterized by sampling rate $F_y = \frac{1}{T_y}$,

where T_x & T_y are corresponding sampling intervals.

→ The ratio F_y/F_x is $\boxed{\frac{F_y}{F_x} = \frac{I}{D}}$.

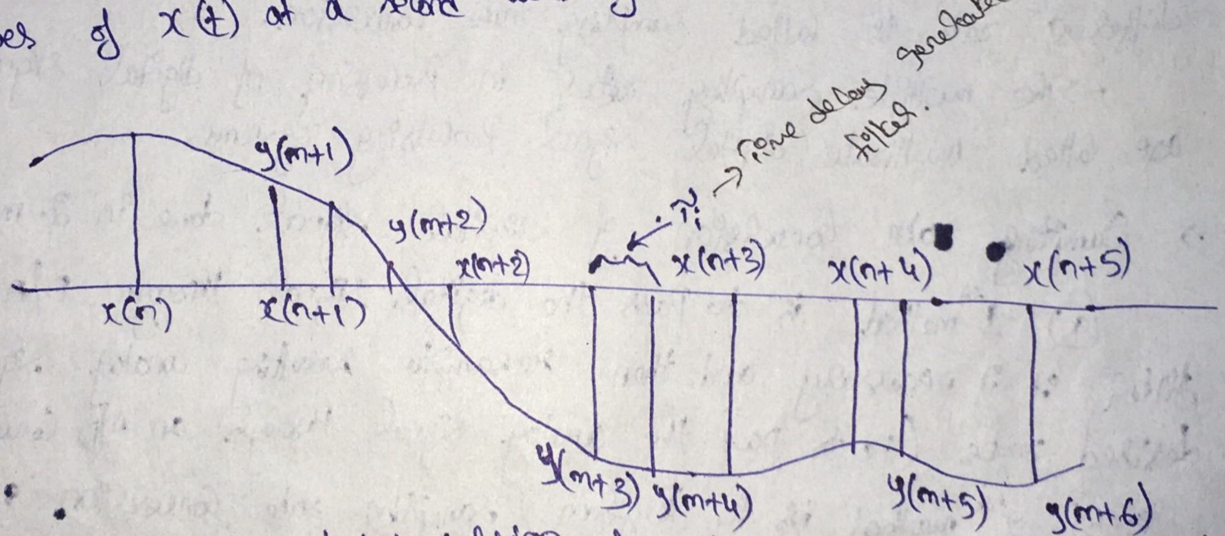
where D & I are relatively Prime integer.

→ The linear filter is characterized by a time variant impulse response denoted as $h(n, m)$. hence i/p $x(n)$ and o/p $y(m)$ are related Convolution summation for time variant systems.



→ The sampling rate Conversion Process Can also be understood from the Point of view of digital resampling of same analog signal.

Let $x(t)$ be analog signal that is sampled at first rate F_x to generate $x(n)$. the goal of rate Conversion is to obtain another sequence $y(m)$ directly from $x(n)$, which is Equally to sampled values of $x(t)$ at a second rate F_y.



$y(m)$ is a time shifted version of $x(n)$. such a time shift Can be realized by using a linear filter that has a flat magnitude response and linear phase response

i.e frequency response of $e^{-j\omega\tau_i}$ where τ_i is time delay generated by filter

→ If the two sampling rates are not Equal the required amount of time shifting will vary from sample to sample.

Thus the rate Converter Can be implemented using a set of linear filter that have same flat magnitude response but generate different time delays.

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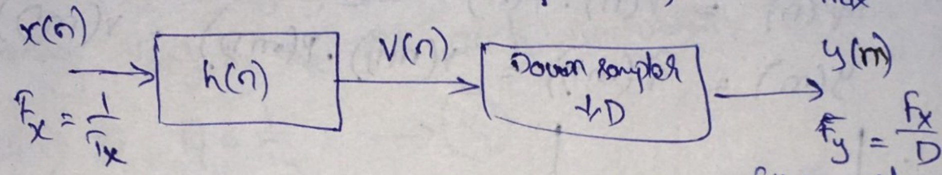
Decimation by a factor 'D' :- (reducing the sampling rate) 3

Let assume signal $x(n)$ with spectrum $X(\omega)$ is to be down sampled by an integer 'D'.

→ The spectrum $X(\omega)$ is assumed to be non zero in freq interval $0 \le |\omega| \le \pi$ or equivalently $|F| \le F_x/2$.

We now reduce sampling rate by simply selecting every D^{th} value of $x(n)$

To avoid aliasing we reduce bandwidth of $x(n)$ to $F_{max} = \frac{F_x}{2D}$ (or equivalently to $\omega_{max} = \frac{\pi}{D}$.



The sequence $x(n)$ is passed through a lowpass filter, characterized by impulse response $h(n)$ and frequency $H_D(\omega)$, which satisfies

$$H_D(\omega) = \begin{cases} 1 & |\omega| \le \frac{\pi}{D} \\ 0 & \text{otherwise.} \end{cases} \rightarrow (1)$$

∴ Filter eliminates the spectrum of $X(\omega)$ in the range $\frac{\pi}{D} \le \omega < \pi$ of order the implementation is that only the freq components of $x(n)$ in the range $|\omega| \le \frac{\pi}{D}$ are of interest in further processing of signals

The o/p of filter is a sequence $v(n)$ given as

$$v(n) = \sum_{k=0}^{\infty} h(k).x(n-k) \rightarrow (2)$$

where $x(n)$ is linear and time invariant the downsampling operations in combination with the filtering results in a time variant system.

$$\boxed{y(m) = v(mD)} = \sum_{k=0}^{\infty} h(k) \cdot x(mD-k) \rightarrow (3)$$

∴ $x(n)$ produces $y(m)$ and $x(n-m_0)$ does not imply $y(n-m_0)$ unless m_0 is a multiple of 'D'.

The freq domain characteristics of o/p sequence $y(m)$ can be obtained by relating the spectrum of $y(m)$ to spectrum of i/p sequence $x(n)$

First it is Convinient to define a sequences $\tilde{v}(n)$ as

$$\tilde{v}(n) = \begin{cases} v(n) & n = 0, \pm D, \pm 2D \\ 0 & \text{otherwise} \end{cases} \rightarrow (4)$$

→ Clearly $\tilde{v}(n)$ (n) can be viewed as a sequence obtained by multiplying $v(n)$ with a periodic train of impulses $p(n)$ with period D.

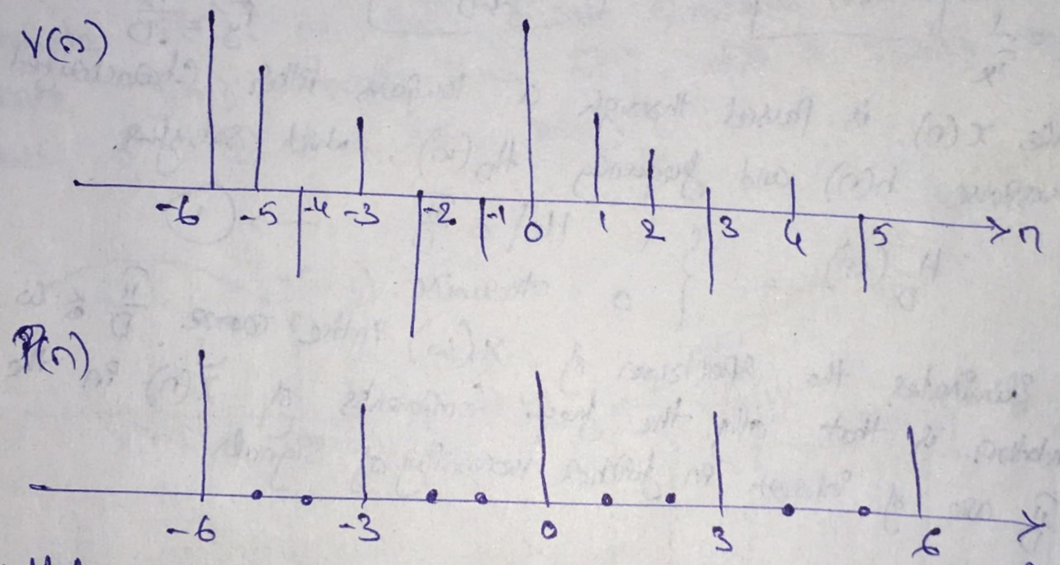
∴ DF series representation of $p(n)$ is

$$p(n) = \frac{1}{D} \sum_{k=0}^{D-1} e^{j2\pi kn/D} \rightarrow (5)$$

hence

$$\tilde{v}(n) = v(n) \cdot p(n) \text{ and } \rightarrow (6)$$

$$y(m) = \tilde{v}(mD) = v(mD) \cdot p(mD) = v(mD) \rightarrow (7)$$



Multiplication of $v(n)$ with periodic impulse train $p(n)$ with period D=3.

Now:- Z-transform of o/p sequence $y(m)$ is

$$Y(z) = \sum_{m=-\infty}^{\infty} y(m) \cdot z^{-m}$$

$$= \sum_{m=-\infty}^{\infty} \tilde{v}(mD) \cdot z^{-m}$$

$$Y(z) = \sum_{m=-\infty}^{\infty} \tilde{v}(m) \cdot z^{-m/D} \rightarrow (8)$$

The fact that $\tilde{v}(m) = 0$, Except at multiples of D. by making use of relations Eq (5), and Eq (6) in Eq (8)

$\therefore$ we get

$$Y(z) = \sum_{m=-\infty}^{\infty} v(m) \left[\frac{1}{D} \sum_{k=0}^{D-1} e^{j2\pi mk/D} \right] . z^{-m/D} \rightarrow (9)$$

$$= \frac{1}{D} \sum_{k=0}^{D-1} \sum_{m=-\infty}^{\infty} v(m) \left(e^{-j2\pi k/D} . z^{1/D} \right)^{-m} \rightarrow (10)$$

$$= \frac{1}{D} \sum_{k=0}^{D-1} V\left(e^{-j2\pi k/D} . z^{1/D} \right) \rightarrow (11)$$

$$= \frac{1}{D} \sum_{k=0}^{D-1} H_D\left(e^{-j2\pi k/D} . z^{1/D} \right) . X\left(e^{-j2\pi k/D} . z^{1/D} \right) \rightarrow (12)$$

$$\therefore V(z) = H_D(z) . X(z) . \rightarrow (13)$$

By evaluating $Y(z)$ in the unit circle, we obtain spectrum of o/p signal $y(m)$.

$\because y(m)$ is $F_y = \frac{1}{T_y} \rightarrow (14)$

The frequency variable denote as ω_y in radians

$$\omega_y = \frac{2\pi F}{F_y} = 2\pi F T_y \rightarrow (15)$$

$\because$ sampling rate are related by $F_y = \frac{F_x}{D} \rightarrow (16)$

it follows frequency variable ω_y and $\omega_x = \frac{2\pi F}{F_x} = 2\pi F T_x . \rightarrow (17)$

are related by $\boxed{\omega_y = D\omega_x} \rightarrow (18)$

thus frequency range $0 \leq |\omega| \leq \frac{\pi}{D}$ is stretched into frequency range $0 \leq |\omega_y| \leq \pi$ by the ground sampling process.

The spectrum $Y(\omega_y)$ is obtained by evaluating eq (11) on unit circle and can be expressed as.

$$Y(\omega_y) = \frac{1}{D} \sum_{k=0}^{D-1} H_D\left(\frac{\omega_y - 2\pi k}{D} \right) X\left(\frac{\omega_y - 2\pi k}{D} \right) \rightarrow (19)$$

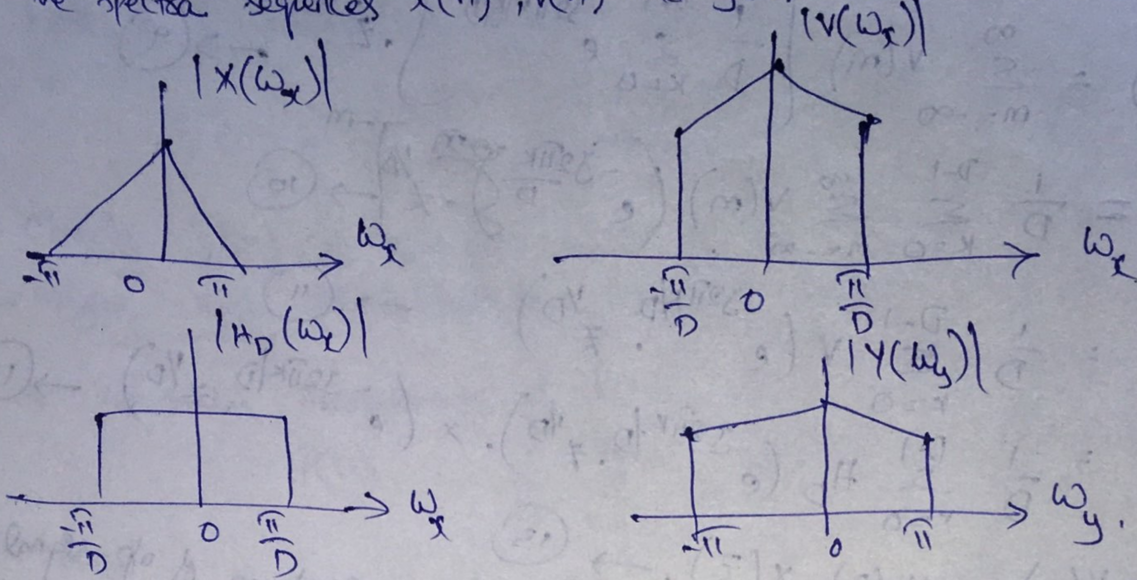
By proper design of filter $H_D(\omega)$ aliasing is eliminated and the 1st term in eq (19) gets vanished.

$$\therefore Y(\omega_y) = \frac{1}{D} H_D\left(\frac{\omega_y}{D} \right) X\left(\frac{\omega_y}{D} \right)$$

$$= \frac{1}{D} X\left(\frac{\omega_y}{D} \right) s. \rightarrow (20)$$

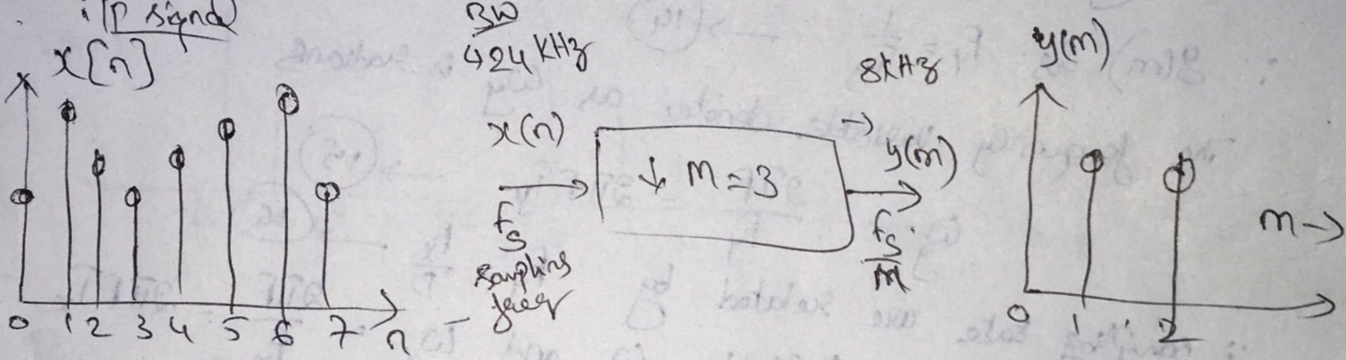
for $0 \leq |\omega_y| \leq \pi$.

The spectra sequences $x(n)$, $v(n)$ and $y(m)$ are given as



Spectra of signals in decimation of $x(n)$ by factor D:

Eg:-



if $M=3$ we should just take every third sample of $x(n)$

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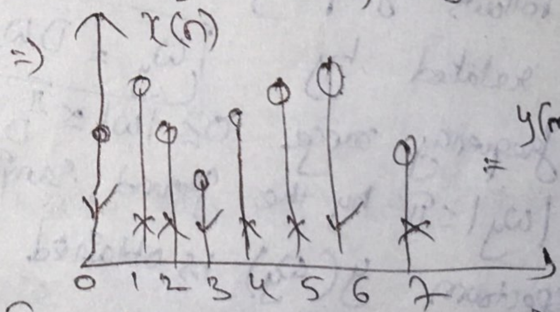


$y(m) = \{0, 3, 6, \}$

Eg:- $x[n] = \{1, 2, 4, 3, 5, -6, -8, -2, -3, 2\}$

down sample by 2 then

o/p $\Rightarrow y(m) = \{1, 4, 5, -8, -3\}$.

Problem:-

For the given data sequence $x(n) = \{1,4,6,8,10,12,13,2,3,15,5\}$ find o/p sequence which is down-sampled version of $x(n)$ by (i) 2 (ii) 3 (iii) 4

Sol:- $x(n) = \{1,4,6,8,10,12,13,2,3,15,5\}$

$y_1(n) = (\downarrow 2)\, x(n)$

$y_1(n) = \{1,6,10,13,3,5\}$

The down-sampler simply keeps every second sample and discards the others

(ii) $y_2(n) = (\downarrow 3)\, x(n)$

$y_2(n) = \{1,8,13,15\}$

The down sampler keeps every 3^{rd} sample and discard others

(iii) $y_3(n) = (\downarrow 4)\, x(n)$

$y_3(n) = \{1,10,3\}$

Problem 2:- For the sequence $x(n) = \{5,6,8,4,\underset{\uparrow}{2},1,3,12,10,7,11\}$ find o/p sequence $Y(z)$ which is down sampled version of $x(n)$ by 2.

Sol:- $x(n) = \{5,6,8,4,\underset{\uparrow}{2},1,3,12,10,7,11\}$

$$X(z) = \{5z^4 + 6z^3 + 8z^2 + 4z + 2 + z^{-1} + 3 \cdot z^{-2} + 12 \cdot z^{-3} + 10 \cdot z^{-4} + 7z^{-5} + 11 \cdot z^{-6}\}$$

$Y(z) = (\downarrow 2)\, X(z)$

$Y(z) = \{5z^4 + 8z + 3z^{-1} + 10z^{-2} + 11z^{-3}\}$

$Y(z)$ can be obtained by considering $X(z)$ and $X(-z)$ let us consider another

Eg: where in

$x(n) = \{9,7,5,3,\underset{\uparrow}{1},2,4,6,8,10\}$

$$X(z) = \{9z^{+4} + 7 \cdot z^3 + 5 \cdot z^2 + 3z + 1 + 2 \cdot z^{-1} + 4 \cdot z^{-2} + 6 \cdot z^{-3} + 8 \cdot z^{-4} + 10 \cdot z^{-5}\}$$

$$X(-z) = \{9 \cdot z^4 - 7z^3 + 5z^{-2} - 3z + 1 - 2z^{-1} + 4 \cdot z^{-2} - 6z^{-3} + 8 \cdot z^{-4} - 10 \cdot z^{-5}\}$$

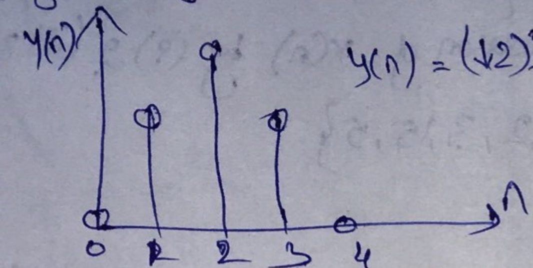
$$X(z) + X(-z) = 2\{9z^4 + 5 \cdot z^2 + 1 + 4z^{-2} + 8 \cdot z^{-4}\}$$

$$X(z) + X(-z) = 2Y(z^2)$$

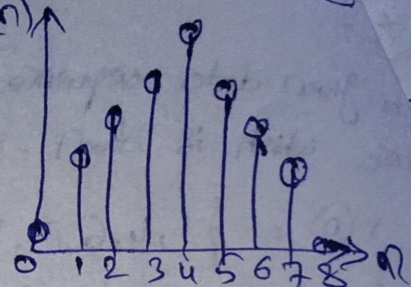
$$Y(z) = \frac{X(z^{1/2}) + X(-z^{1/2})}{2} = Z\left[(\downarrow 2)\, x(n)\right].$$

Problem:- Demonstrate the sampling rate reduction by factor of $M=2$ for the given signal.

Soln:-



$$y(n) = (\downarrow 2)x(n)$$

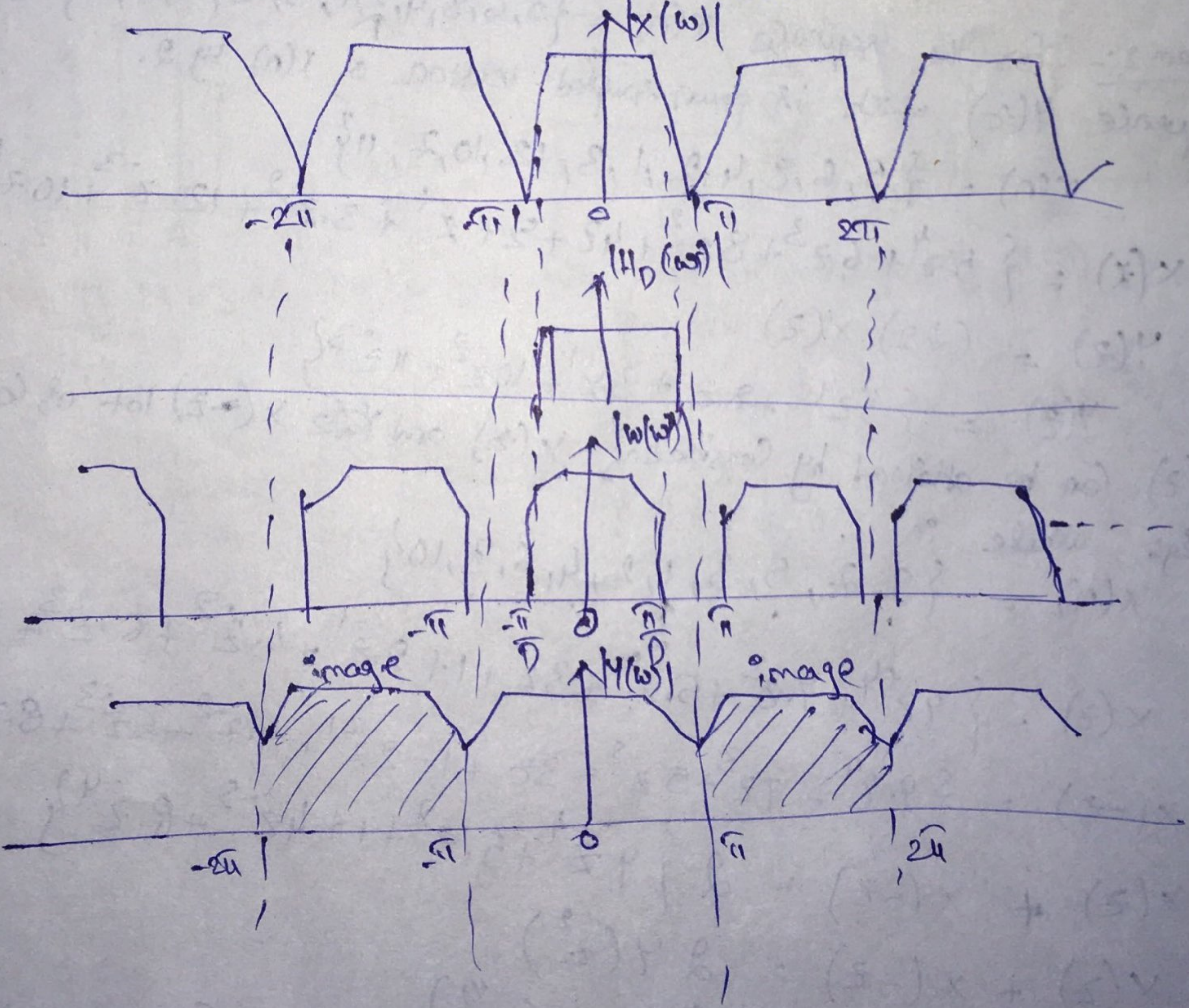


Let us assume signal $x(n)$ is downsampled by factor M. $X(\omega)$ be spectrum of $x(n)$ then spectrum $X(\omega)$ is assumed to exit range $0 \le |\omega| \le \pi$ frequency is defined nyquist criterion $f_x \ge 2$ (or) $F \le \frac{F_x}{2}$

$$x(n) \longrightarrow \boxed{\text{Anti aliasing filter } h(k)} \xrightarrow{w(n)} \boxed{\text{Down sampler } \downarrow M} \xrightarrow{y(m)} f_y = \frac{F_x}{M}$$

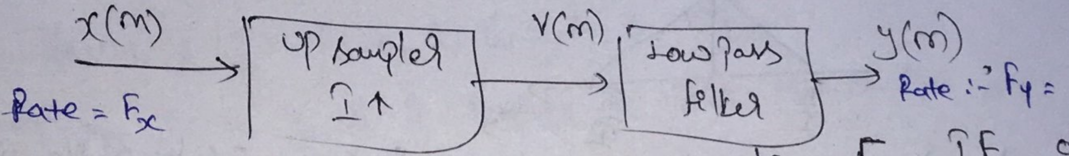
Basically an antialiasing filter is LPF whose freq response H_{LPF} is

$$H_{LPF}(\omega) = \begin{cases} 1 & |\omega| \le \frac{\pi}{M} \\ 0 & \text{otherwise} \end{cases}$$



Interpolation by a factor I :- (increasing sampling rate)

An increase in the sampling rate by an integer factor of I can be accomplished by interpolating I-1 few samples b/w successive values of the signal.



Let V(m) denotes a sequence with a rate $F_y = I F_x$ obtained from x(m) by adding I-1 zeros b/w successive values of x(n).

$$\therefore \; V(m) = \begin{cases} x\left(\frac{m}{I}\right) & m = 0, \pm I, \pm 2I, \dots \\ 0 & \text{otherwise} \end{cases} \rightarrow (1)$$

and its sampling rate is identical to the rate of y(m).

This sequences has Z-transform.

$$V(z) = \sum_{m=-\infty}^{\infty} V(m) \cdot z^{-m}$$

$$= \sum_{m=-\infty}^{\infty} x(m) \cdot z^{mI} \rightarrow (2)$$

$\therefore$ The corresponding sepectrum of v(m) is obtained by evaluating eq (2) on unit circle. The;

$$z = r \cdot e^{j\omega}$$

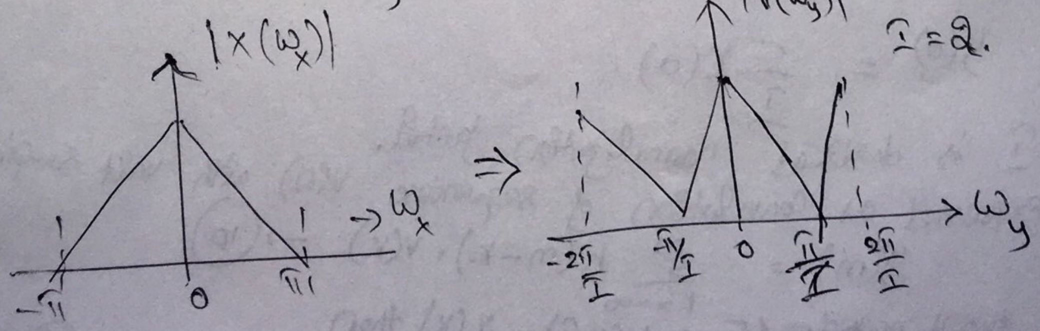
↳ W.K.T

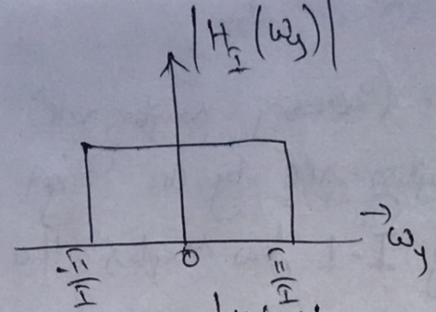
$$\begin{cases} X(z) = \sum_{n=-\infty}^{\infty} x(n) \cdot z^{-n} \\ X(\omega) = X(z)\Big|_{z=e^{j\omega}} = \sum_{n=-\infty}^{\infty} x(n) \cdot e^{-j\omega n} \end{cases} \rightarrow (3)$$

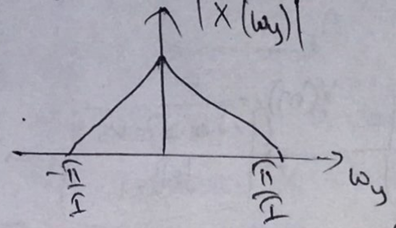
Thus $V(\omega_y) = X(\omega_y I) \rightarrow (4)$

where ω_y denotes frequency variable relative to new sampling rate F_y i.e

$$\omega_y = \frac{2\pi F}{F_y} \quad \text{and.} \quad \omega_y = \frac{\omega_x}{I}. \rightarrow (5)$$







$\therefore$ only the frequency component of $x(n)$ in range $0 \le \omega_y \le \pi/I$ are unique. the images of $X(\omega)$ above $\omega_y = \frac{\pi}{I}$ should be rejected by passing the sequence $v(m)$ to low pass filter with frequency response.

$$\text{freq response} \rightarrow H_I(\omega_y) = \begin{cases} C, & 0 \le |\omega_y| \le \frac{\pi}{I} \\ 0 & \text{otherwise.} \end{cases} \rightarrow (6)$$

where 'C' is a scale factor to normalise the o/p sequence $y(n)$.

$$Y(\omega_y) = \begin{cases} CX(\omega_y I) & 0 \le |\omega_y| \le \frac{\pi}{I} \\ 0 & \text{otherwise} \end{cases} \rightarrow (7)$$

The scale factor 'C' is selected so that o/p $y(m) = x(m/I)$ for $m = 0, \pm I, \pm 2I \dots$

we select the point $m = 0$

$$y(0) = \frac{1}{2\pi}\int_{-\pi}^{\pi} Y(\omega_y).d\omega_y$$

$$= \frac{C}{2\pi}\int_{-\frac{\pi}{I}}^{\frac{\pi}{I}} X(\omega_y).d\omega_y. \rightarrow (8).$$

since $\omega_y = \frac{\omega x}{I}$ then

$$y(0) = \frac{C}{I}\frac{1}{2\pi}\int_{-\pi}^{\pi} X(\omega_y).d\omega_y \rightarrow (9)$$

$$y(0) = \frac{C}{I}x(0)$$

$\therefore$ $C = I$ is desired normalization factor.

since $y(m)$ expressed as convolution of sequence $v(n)$ with unit sample

$$y(m) = \sum_{k=-\infty}^{\infty} h(m-k).v(k) \rightarrow (10)$$

$\because$ $v(k) = 0$ except multiple of I; $v(kI) = x(k)$ then

$$y(m) = \sum_{k=-\infty}^{\infty} h(m-kI).x(k) \rightarrow (11)$$

(5) for the given data sequence $x(n) = \{1, 4, 6, 8, 10\}$ find o/p sequence which is up-sampled version of $x(n)$ by (i) 2 (ii) 3 (11)

Sol:- $x(n) = \{1, 4, 6, 8, 10\}$

(i) $y_1(n) = (\uparrow 2) \cdot x(n)$

$y_1(n) = \{1, 0, 4, 0, 6, 0, 8, 0, 10\}$

upsampling by a factor-2 inserts a zero b/w samples

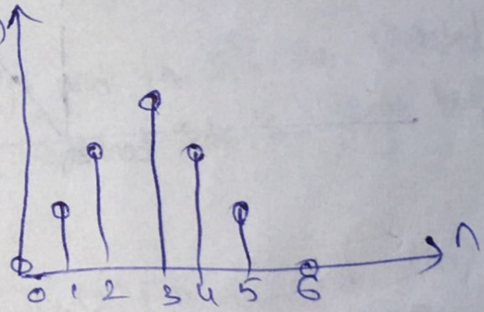
(ii) $y_2(n) = (\uparrow 3) \cdot x(n)$

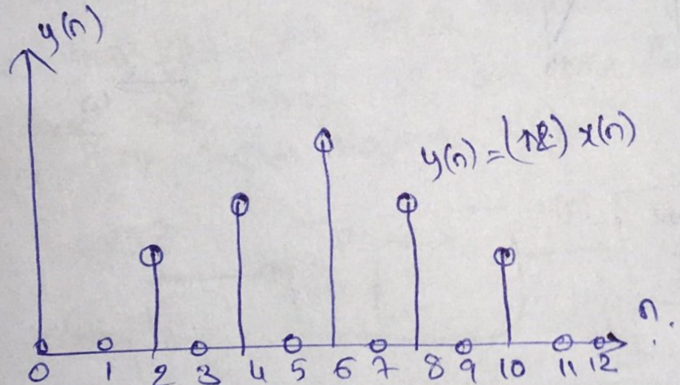
$y_2(n) = \{1, 0, 0, 4, 0, 0, 6, 0, 0, 8, 0, 0, 10\}$

upsampling by a factor-3 inserts 2 zeros b/w samples

Problem:- Demonstrate the increase of sampling rate by a factor of $L = 2$ for given signal.

Sol:-

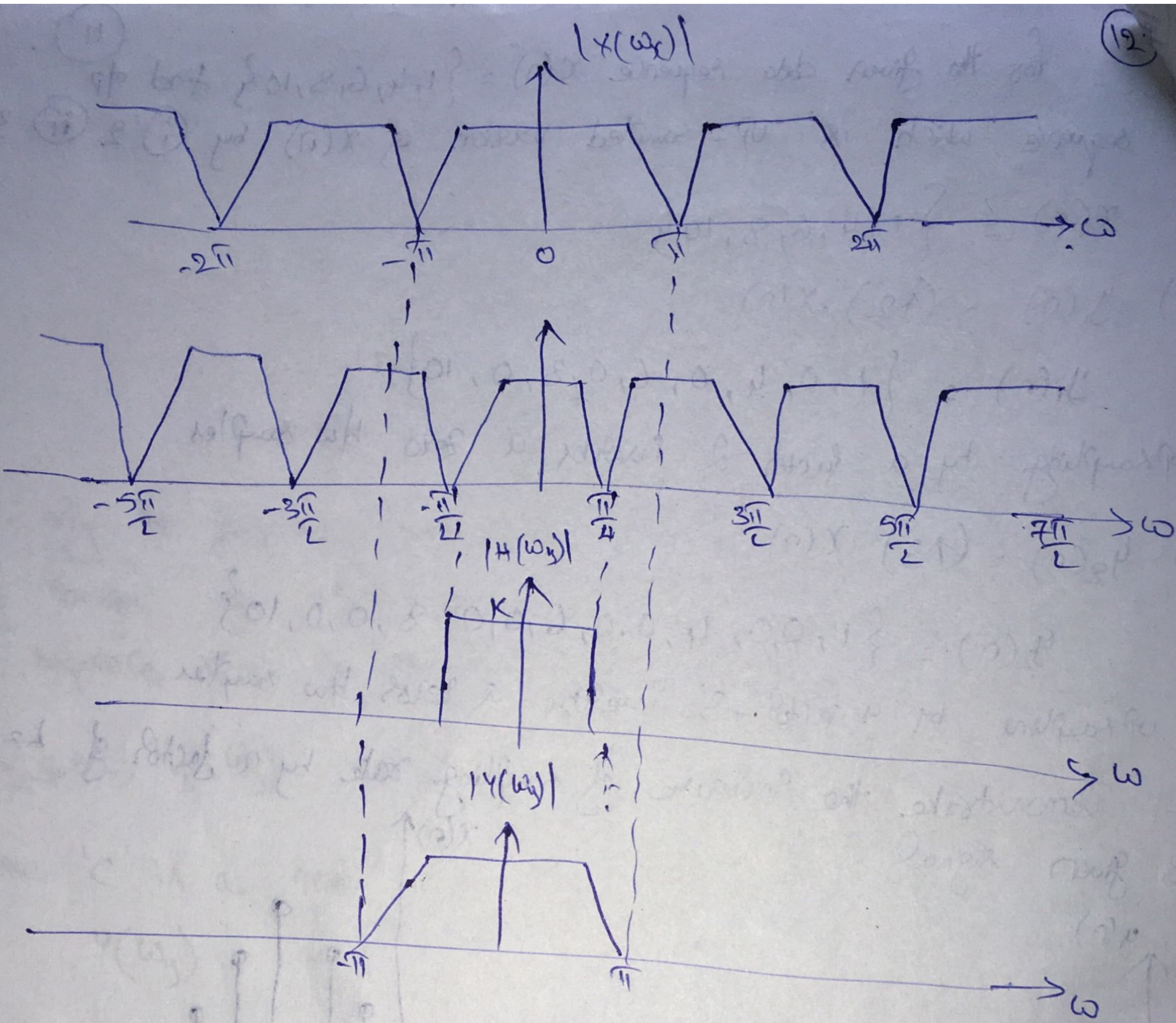




The o/p rate $F_y = I \cdot F_x$ which obtained by adding $L-1$ zeros b/w successive values of $x(n)$

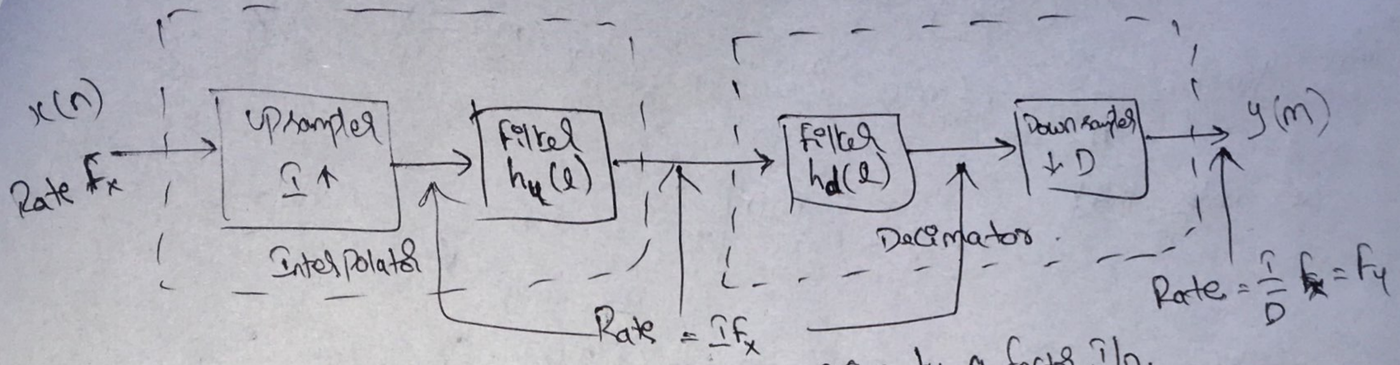
Let us define a term $w(n)$ such that

$$w(n) = \begin{cases} x\left(\frac{m}{L}\right), & m = 0, \pm L, \pm 2L, \ldots \\ 0 & \text{otherwise} \end{cases}$$


|X(ωx)|
-2π
-π
0
π
2π
ω
-5π/2
-3π/2
3π/2
5π/2
7π/2
ω
|H(ωy)|
K
ω
|Y(ωy)|
-π
π
ω

Sampling rate Conversion by a rational factor I/D :-

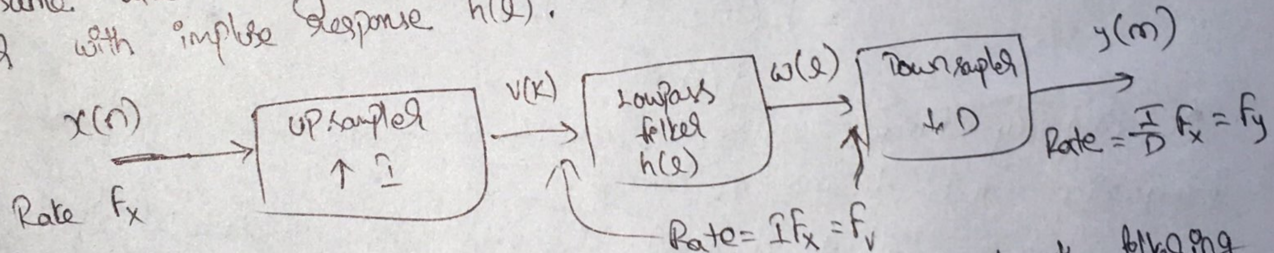
Achived the sampling rate conversion by first performing interpolation by the factor I and then decimating the o/p of interpolator by factor D.



Method for sampling rate conversion by a factor I/D.

Interpolation is performed first and decimation is performed second to preserve the desired spectral characteristics of $x(n)$.

The two filters with impulse response $\{h_u(l)\}$ and $\{h_d(l)\}$ are operated at the same rate namely IF_x and hence can be combined into a single lowpass filter with impulse response $h(l)$.



The frequency response $H(\omega_v)$ of combined filter must incorporate the filtering operation for both interpolation and decimation

$$H(\omega_v) = \begin{cases} I, & 0 \le |\omega_v| \le \min\left(\frac{\pi}{D}, \frac{\pi}{I}\right) \\ 0 & \text{otherwise.} \end{cases} \rightarrow (1)$$

In the time domain the o/p of up-sampler is the sequence

$$v(l) = \begin{cases} x(l/I), & l = 0, \pm I, \pm 2I \\ 0 & \text{otherwise.} \end{cases} \rightarrow (2)$$

and o/p of linear time invariant filter is

$$\omega(l) = \sum_{k=-\infty}^{\infty} h(l-k) \cdot v(k)$$

$$= \sum_{k=-\infty}^{\infty} h(l-kI) \cdot x(k) \rightarrow (3).$$

The o/p of sampling rate converter is sequence $\{y(m)\}$ which is obtained by downsampling the sequence $\{\omega(l)\}$ by factor of D.

$$y(m) = \omega(mD)$$
$$= \sum_{k=-\infty}^{\infty} h(mD - kI)\cdot x(k) \rightarrow (4)$$

let $k = \left\lfloor \frac{mD}{I} \right\rfloor - n \rightarrow (5)$

then $$y(m) = \sum_{n=-\infty}^{\infty} h\left(mD - \frac{mD}{I}\cdot I + nI\right) x\left(\frac{mD}{I} - n\right) \rightarrow (6)$$

we note that $$mD - \left\lfloor \frac{mD}{I} \right\rfloor I = mD \text{ modulo } I. \rightarrow (7)$$
$$= \langle mD \rangle_I$$

$$\therefore y(m) = \sum_{n=-\infty}^{\infty} h(nI + (mD)_I)\cdot x\left(\left\lfloor \frac{mD}{I} \right\rfloor - n\right) \rightarrow (8)$$

the o/p is obtained by passing the sequence $x(n)$ through a time variant filter with impulse response

$$g(n, m+kI) = h(nI + (mD + kDI)_I)$$
$$= h(nI + (mD)_I)$$
$$= g(n,m). \longrightarrow (9).$$

The frequency domain relationship can be obtained on combining results of interpolation and decimation process.

-> The spectrum at o/p of linear filter with impulse response $h(l)$ is

$$V(\omega_v) = H(\omega_v)\, X(\omega_v I).$$
$$= \begin{cases} I\, X(\omega_v I) & , 0 \le |\omega_v| \le \min\left(\frac{\pi}{D}, \frac{\pi}{I}\right) \\ 0 & \text{otherwise} \end{cases} \rightarrow (10)$$

finally spectrum o/p sequence

$$Y(\omega_y) = \begin{cases} \frac{I}{D} X\left(\frac{\omega_y}{D}\right) & 0 \le |\omega_y| \le \min\left(\pi, \frac{\pi D}{I}\right) \\ 0 & \text{otherwise}. \end{cases} \rightarrow (11).$$

i.e $$Y(\omega_y) = \frac{1}{D}\sum_{k=0}^{D-1} V\left(\frac{\omega_y - 2\pi k}{D}\right) //.$$

4) Filter Design of Implementation for Sampling rate Conversion :-

Sampling rate Conversion by a factor $\left(\frac{I}{D}\right)$ Can be achived by increasing the sampling rate by I accomplished by inserting $(I-1)$ 0's b/w successive values of i/p signals $x(n)$ followed by linear filtering of the resulting sequence to eliminate the unwanted images of $X(\omega)$ & finally by downsampling the filtered signal by factor 'D'.

(a) Direct form FIR filter Structures :-

In principle simplest realization of the filter is direct form FIR structure with system function

$$H(z) = \sum_{k=0}^{M-1} h(k) \cdot z^{-k} \rightarrow (1)$$

$h(k)$ is unit sample response of FIR filter.

→ the Lowpass filter (LPF) Can be designed to have linear phase, a specific passband ripple and stopband attenuation.

∴ the filter parameters $h(k)$ Can be used to implement FIR filter directly

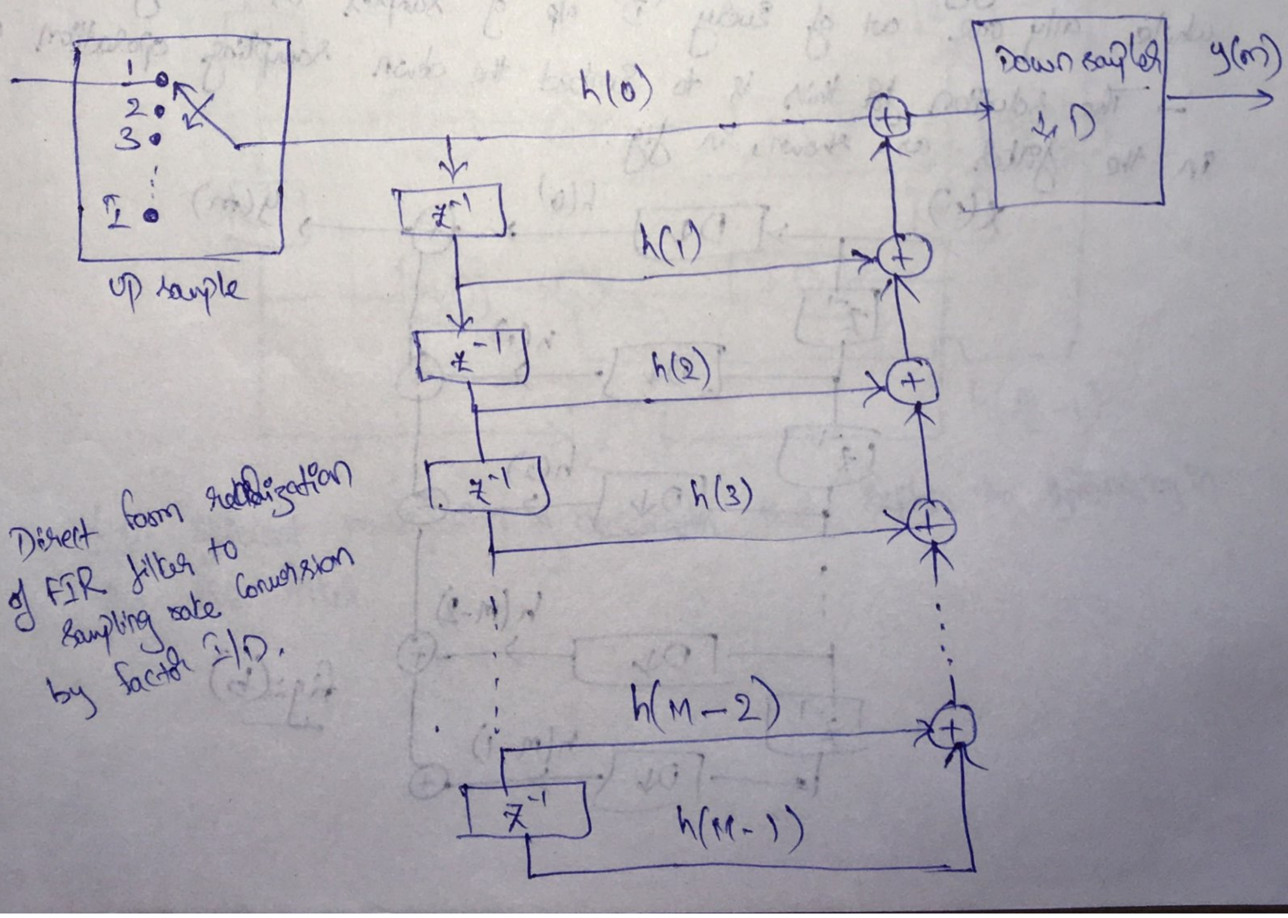


fig:- Direct form realization of FIR filter to Sampling rate Conversion by factor I/D.

Although the direct form FIR filter realisation is simple it is also inefficient.

The inefficiency results from the fact that the upsampling process introduces $(I-1)$ 0's b/w successive points of the i/p signal. If I is large, most of the signal components in the FIR filter are zero.

→ To develop a more efficient filter structure, let us begin with decimator that reduces the sampling rate by factor 'D'.

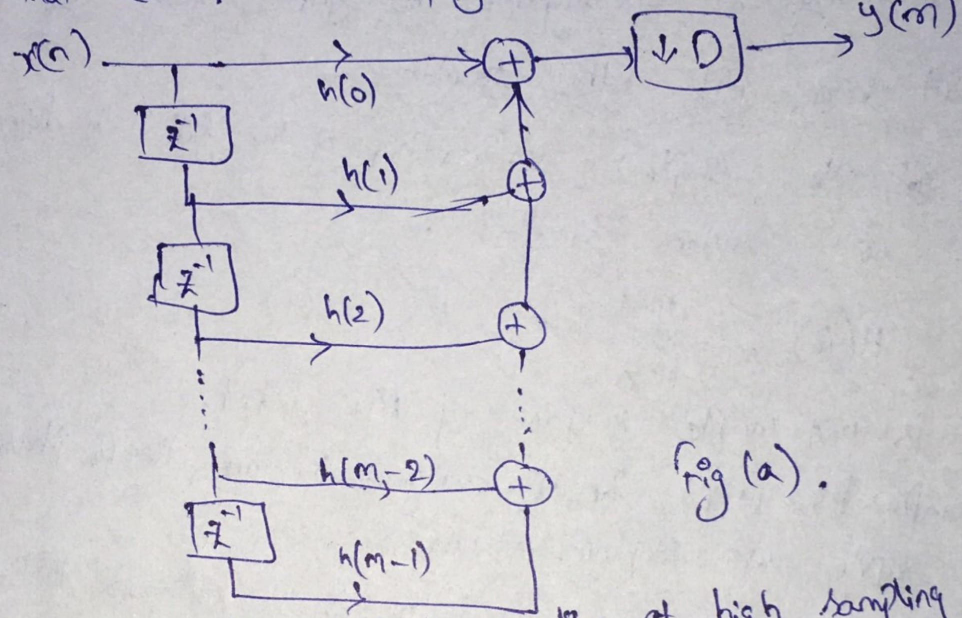


fig (a).

In this configuration, the filter is operating at high sampling rate f_x while only one out of every 'D' o/p of samples is actually needed.

→ The solution for this is to embed the down sampling operation with in the filter as shown in fig.

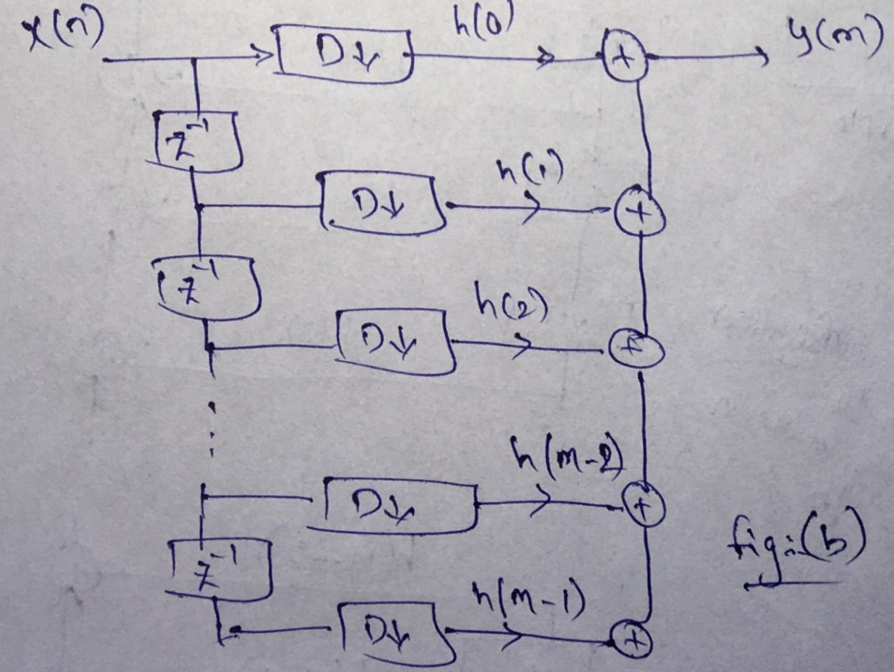


fig:(b)

In this filter structure all Mul & add are Performed at lower sampling rate $\frac{F_x}{D}$. Thus we have archived the desired ~~frequen~~ Efficiency.

→ Additional reduction in Computation Can be achived by exploiting the symmetry characteristics of h(k).

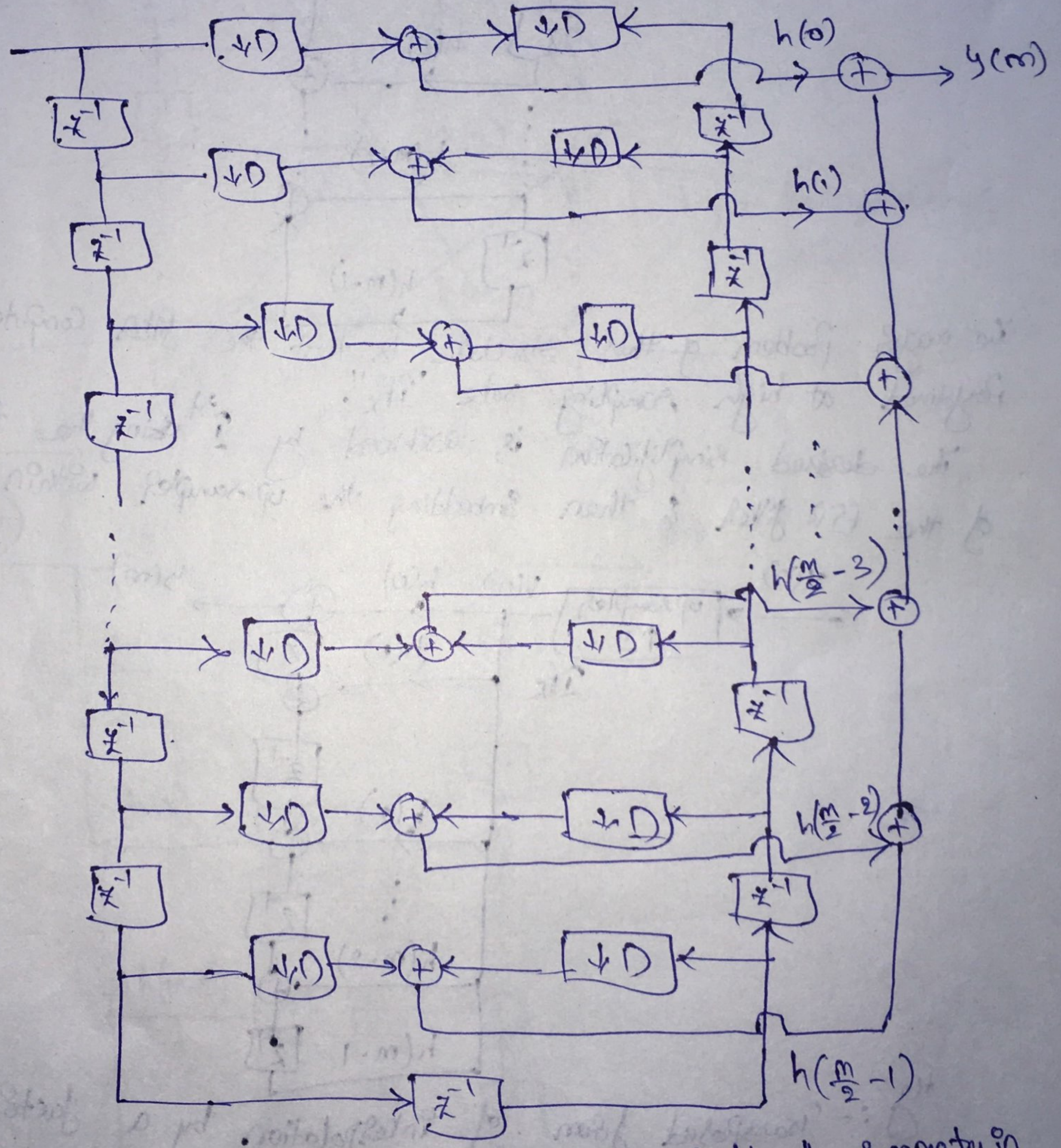
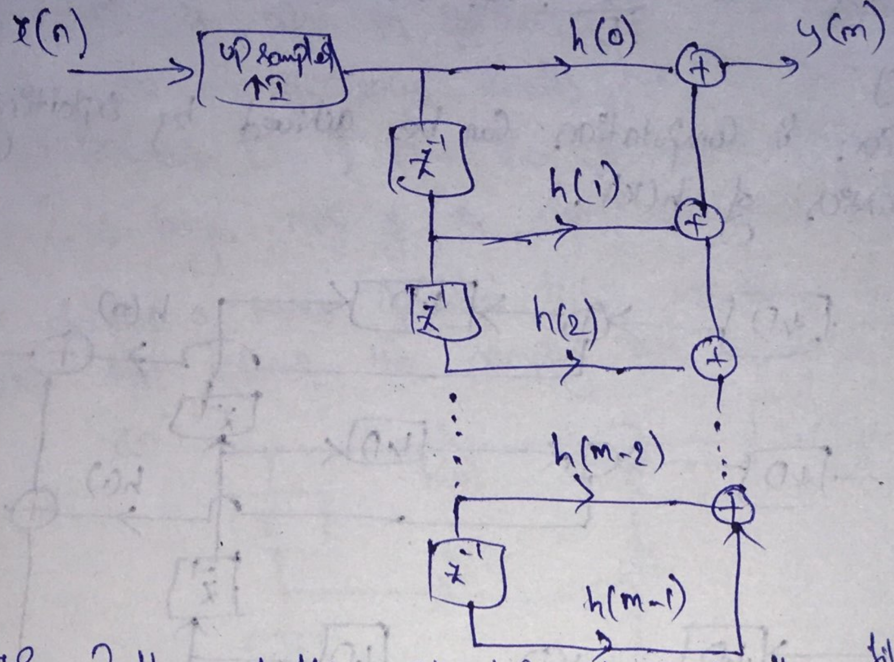


fig:- Efficient realization of decemator that exploits the symmetry in the FIR filter.

Consider the implementation of an interpolator.



The major problem of this structure is that the filter computations are performed at high sampling rate "If_x".

The desired simplification is archived by I^{st} using the transposed form of the FIR filter & then embedding the up sampler within the filter.

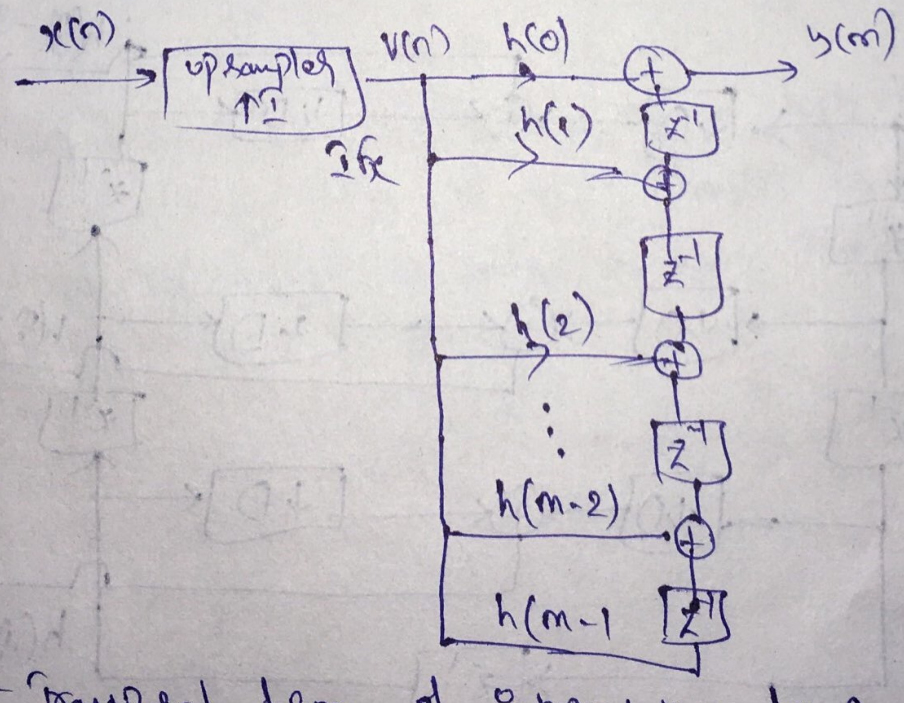
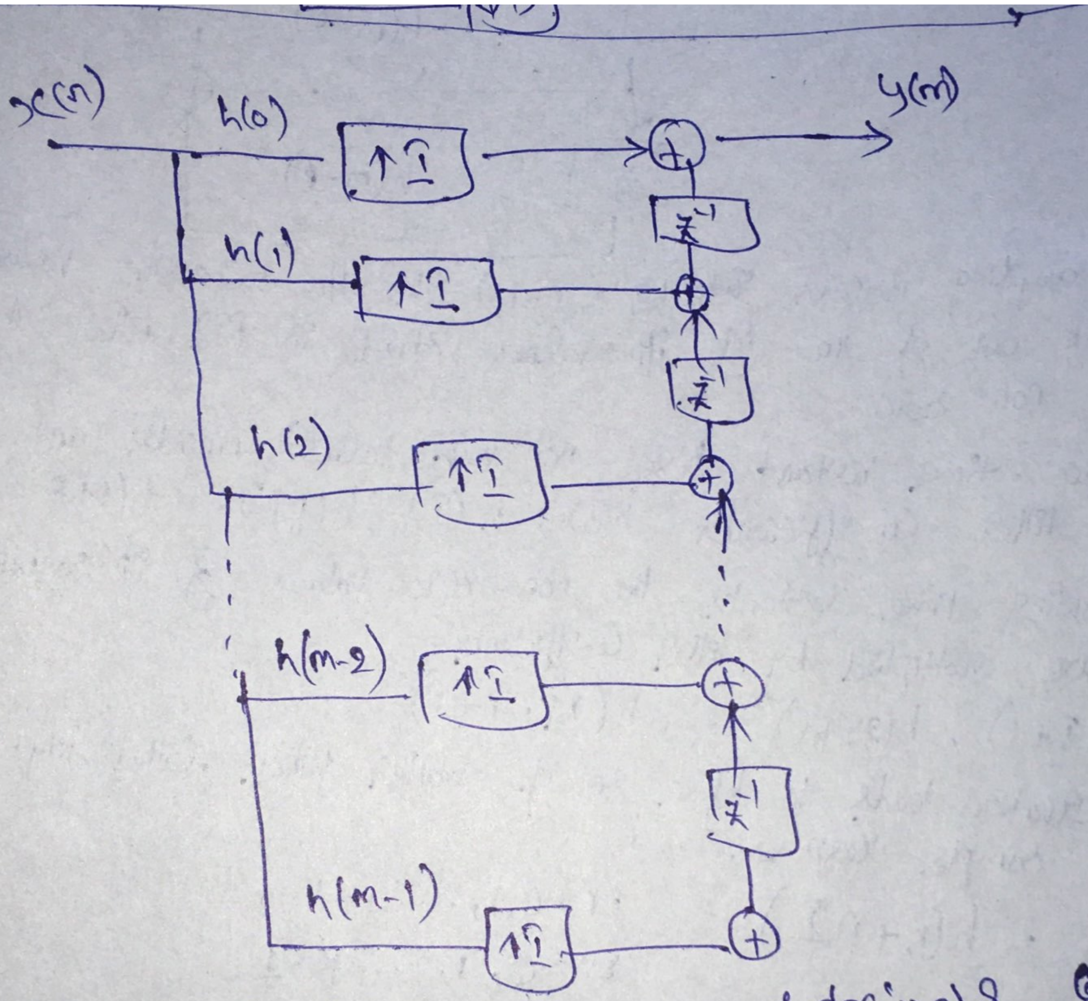


fig :- transposed form of interpolation by a factor I.



It is observed that the response of decimator on interpolator and vice versa.

b) Polyphase filter structures:-

A computational efficiency of filter structure is also archeived by reducing the larger FIR filter of length 'M' into set of smaller filters of length $K = \frac{M}{I}$. where M is selected multiple of I

Let us consider an interpolator given as fig.

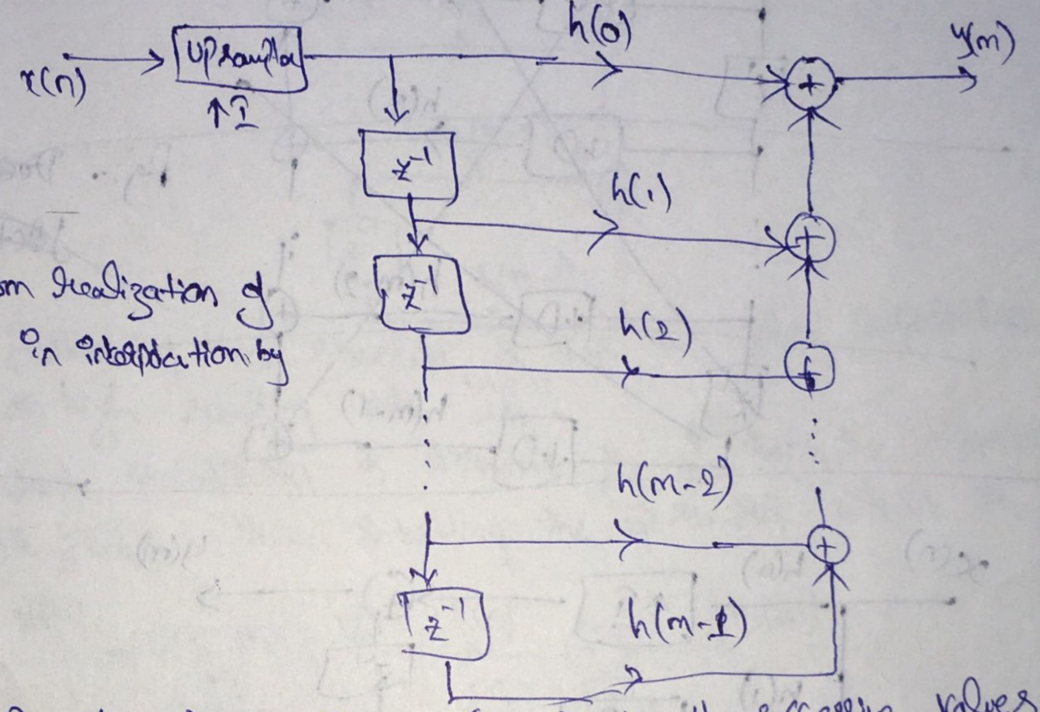


Fig:- Direct form realization of FIR filter in interpolation by factor I.

∵ Since upsampling process inserts $(I-1)$ 0's b/w successive values of $x(n)$ only K out of the M i/p values stored in FIR filter at any one time are non-zero.

At one time instant these non zero values coincide and are multiplied by filter co-efficients $h(0), h(I), h(2I) \ldots h(M-I)$ the non-zero values of i/p sequence.

∴ The following time instants the non-zero values of filter co-efficients coincide and are multiplied by

$h(1), h(I+1), h(2I+1) \ldots h(M-I+1)$

∴ The observation leads to define set of smaller filters, called Poly-Phase filters with unit sample response.

$$P_K(n) = h(k+nI) \quad ; k = 0, 1, \ldots I-1$$
$$n = 0, 1, 2 \ldots K-1$$

where $K = \frac{M}{I}$ is an integer.

$M = K \cdot I$

ie if $K = 0, 1$ & $n = 0, 1, 2, 3$

for $K = 0$

$P_0(0) = h(0)$

$P_0(1) = h(I)$

$P_0(2) = h(2I)$

$P_0(3) = h(3I)$

for $K = 1$

$P_1(0) = h(1)$

$P_1(1) = h(1+I)$

$P_1(2) = h(2+I)$

$P_1(3) = h(3+I)$

The rotation of commutator is clockwise begins with $m = 0$. Thus the polyphase filters perform the computations at low sampling rate F_x and rate of conversion results from the fact I o/p samples are generated one from each filter for each i/p sample.

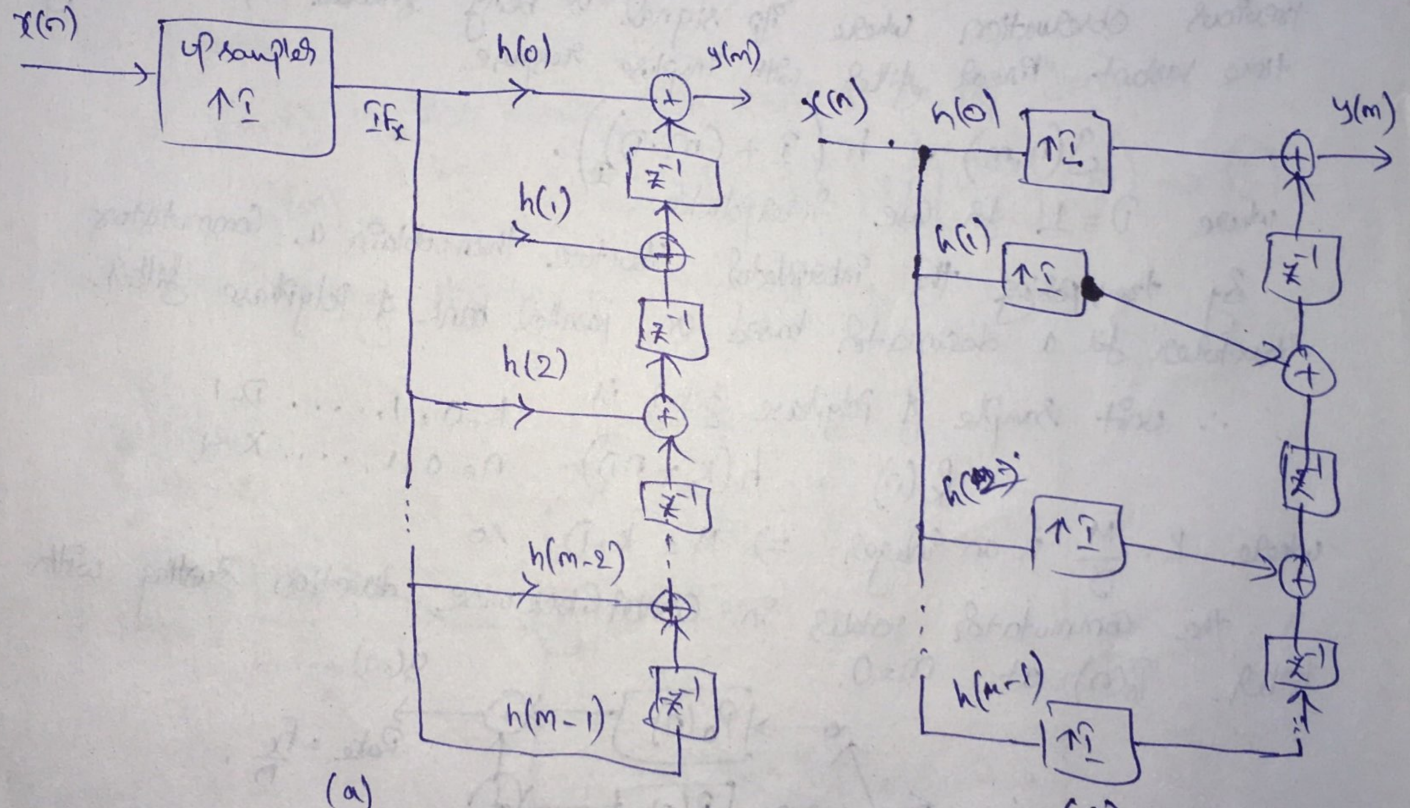


fig:- efficient realization of an interpolator.

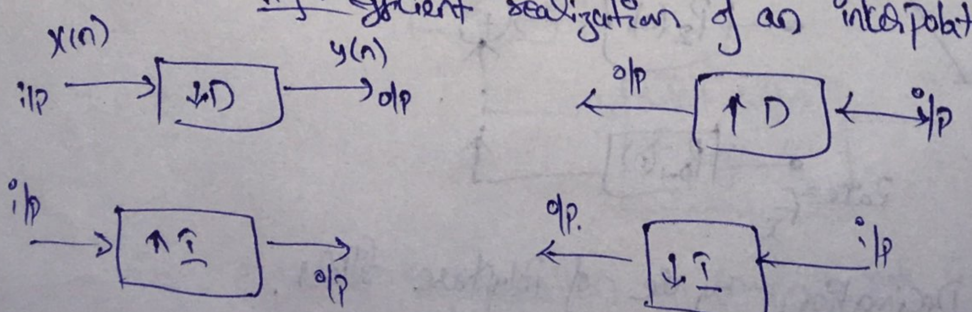


fig:- Duality relationship obtained through transposition.

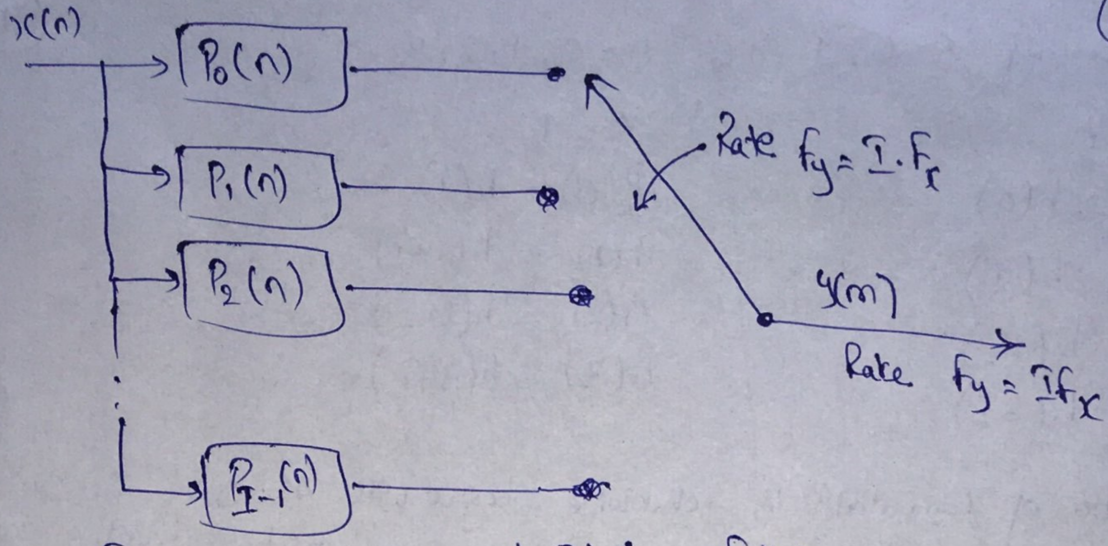


Fig:- Interpolation by use of Polyphase filter.

→ The decomposition of $\{h(k)\}$ into the set of I - sub filters with impluse response $P_k(n)$; $k = 0, 1, \ldots I-1$, is consistent only with our previous observation where i/p signal is being filtered by periodically time variant linear filter with impluse response.

$$g(n,m) = h\left(I + (m \cdot D)\right)_I .$$

where $D = 1$ for case interpolator.

By transposing the interpolator structure then obtain a commutator structurer for a decimator based on parallel bank of polyphase filter.

∴ unit sample of polyphase filter is

$$P_k(n) = h(k + nD) \qquad \begin{aligned} k &= 0, 1, \ldots D-1 \\ n &= 0, 1, \ldots K-1 \end{aligned}$$

where $K = \frac{M}{D}$ is an integer ⇒ $M = K \cdot D$ so

the commutator rotates in counter clock wise direction starting with filter $P_0(n)$ at $m = 0$.

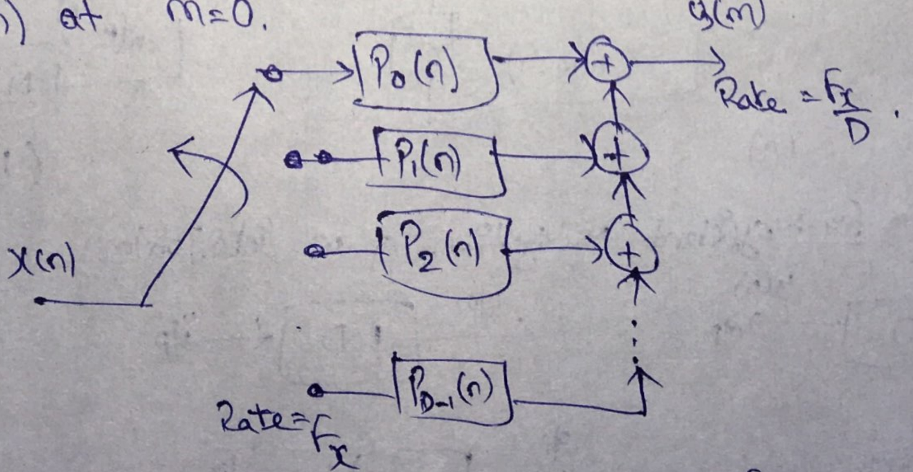


Fig:- Decimation by use of Polyphase filters.

If commutator rotates in clock wise direction then the polyphase filters will have impulse responses as.

$$P_k(n) = h(nI - k) \quad ; k = 0, 1, 2, \ldots I-1.$$

$$P_k(n) = h(nD - k) \quad ; k = 0, 1, 2 \ldots D-1$$

(c) Time variant filter techniques:-

In the sampling rate conversion by a factor I/D the filtering can be accomplished by means of linear time variant filter described by response function

$$g(n,m) = h(mI + (mD)_I) \rightarrow (1)$$

where $h(n)$ — impulse response of low pass FIR filter. Contains the set of co-efficients $\{g(n,m)\}$ for each $m = 0, 1, 2 \ldots I-1$ K-elements

$\because g(n,m)$ is also periodic with period I the o/p $y(m)$ is

$$y(m) = \sum_{n=0}^{K-1} g\left(n, m - \left\lfloor \frac{m}{I} \right\rfloor \cdot I\right) x\left(\left\lfloor \frac{mD}{I} \right\rfloor - n\right) \rightarrow (2)$$

The block processing algorithm for computing Eq (2) is

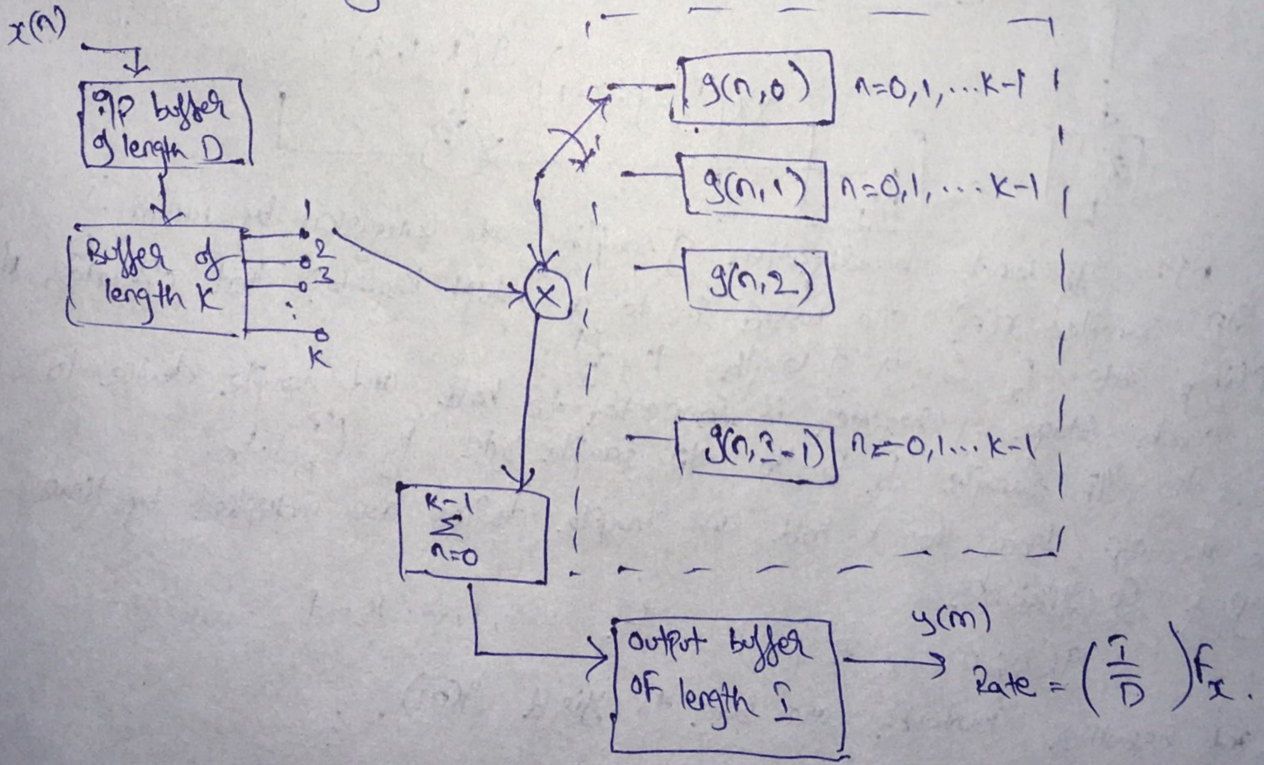


fig:- Efficient implementation of sampling rate conversion by block processing.

A block of D - i/p samples is buffered and shifted into a second buffer of length K one - sample at a time.

- For each o/p sample $y(m)$, the sample from the second buffer are multiplied by corresponding set of filter Co-efficients $g(n, l)$ for $n = 0, 1, \ldots K-1$ and the K Product are accumulated to give $y(l)$ for $l = 0, 1, \ldots I-1$.

Thus this Computation Produces I - o/p's

Alternative method for Computing Eq ② by means of a FIR filter structure with Periodically varying filter Co-efficients.

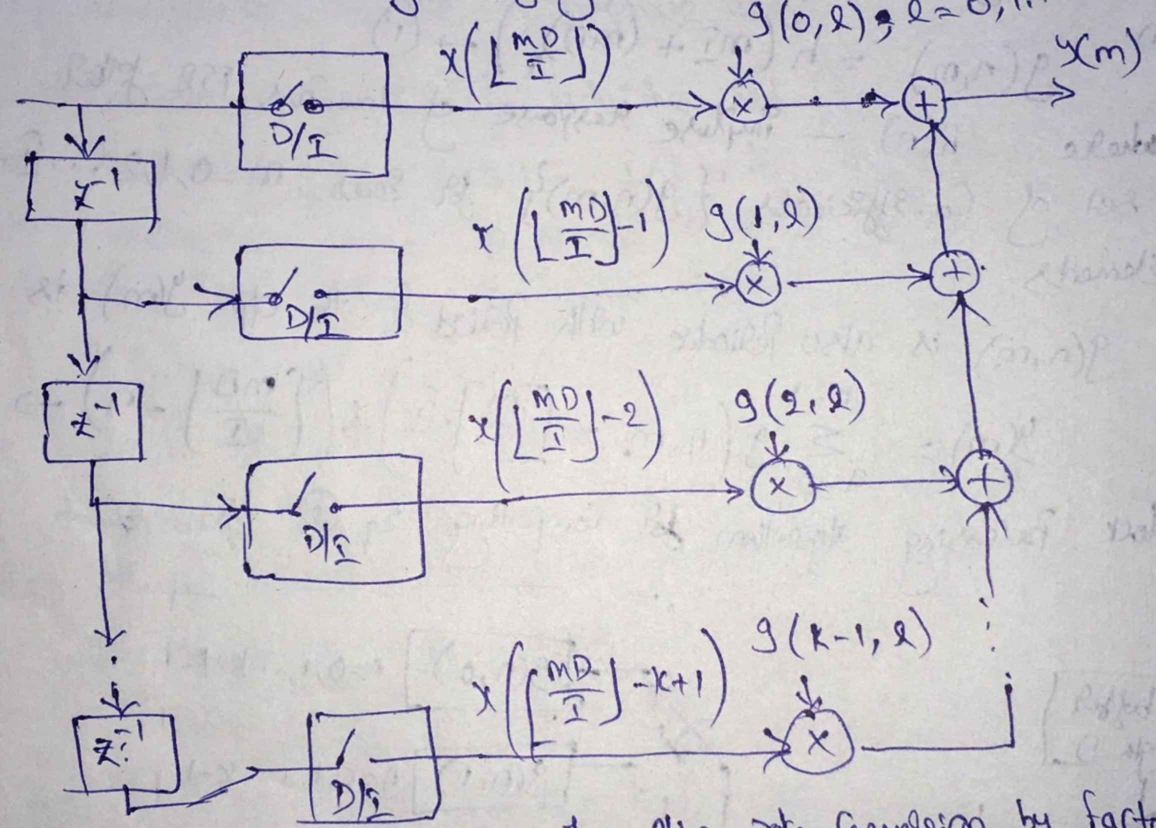


Fig :- Efficient realization of sampling rate Conversion by factor $\frac{I}{D}$.

The i/p samples $x(n)$ are Passed in to a Shift Register that operates at sampling rate F_x & is of length $K = \frac{M}{I}$.

Each Stage of register is Connected to hold and sample device to Couple the i/p sample F_x to the o/p sample rate $F_y = \left(\frac{I}{D}\right) \cdot F_x$.

The K o/p from the K - hold and sample devices are multiplied by time varying Co-efficients

$$g\left(n, m - \left\lfloor \frac{m}{I} \right\rfloor I\right) \cdot \text{ for } n = 0, 1, \ldots K-1$$

and resulting Products are sum to yield $y(m)$. //