ANTENNA WAVE PROPAGATION

ELECTRONICS & COMMUNICATION ENGINEERING

unit - 2

HAND NOTES

BY:

P.RAJESH M.TECH.,
ASSISTANT PROFFESOR
EMAIL: rajesh.crit@gmail.com
ph: 9989786119
9985786099

CRIT COLLEGE OF ENGG & TECHNOLOGY

ANANTAPURAMU

Unit-3

Antenna Arrays:-

Introduction:- In the Point to Point Communication, it is desired to have most of the Energy radiated in one Particular direction. That means it is desired to have greater directivity in a desired direction Particularly. which is not Possible with single dipole antenna.

Hence to increase the field strength in desired direction by using group of antenna excited simultaneously. such a group of antenna is Called array of Antenna (or) simply Antenna Array.

Thus Antenna array Can be defined as the system of similar antennas directed to get required high directivity in desired direction.

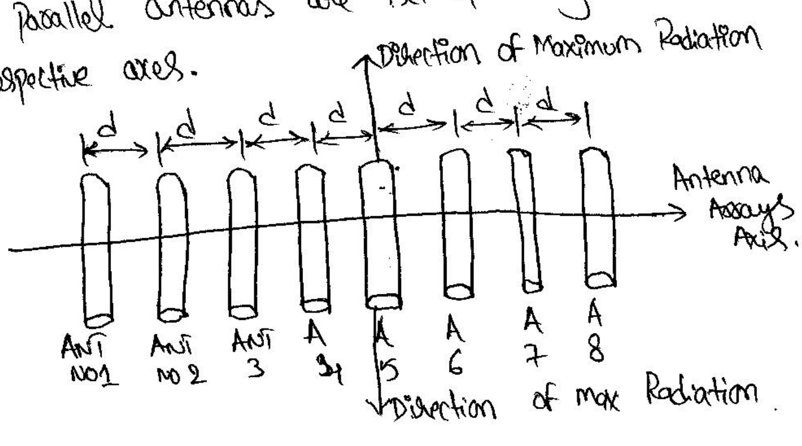
* The Antenna array is said to linear if the Elements of antenna array are Equally Spaced along a straight line.

* The linear Antenna array is said to be uniform linear array if all the Elements are fed with current of Equal magnitude with Progressive uniform phase shift along the line.

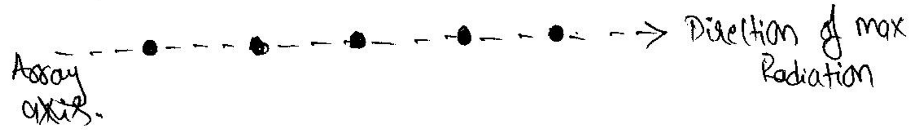
~~2) Array of Point Sources:-~~

2) Various forms of Antenna Arrays:-

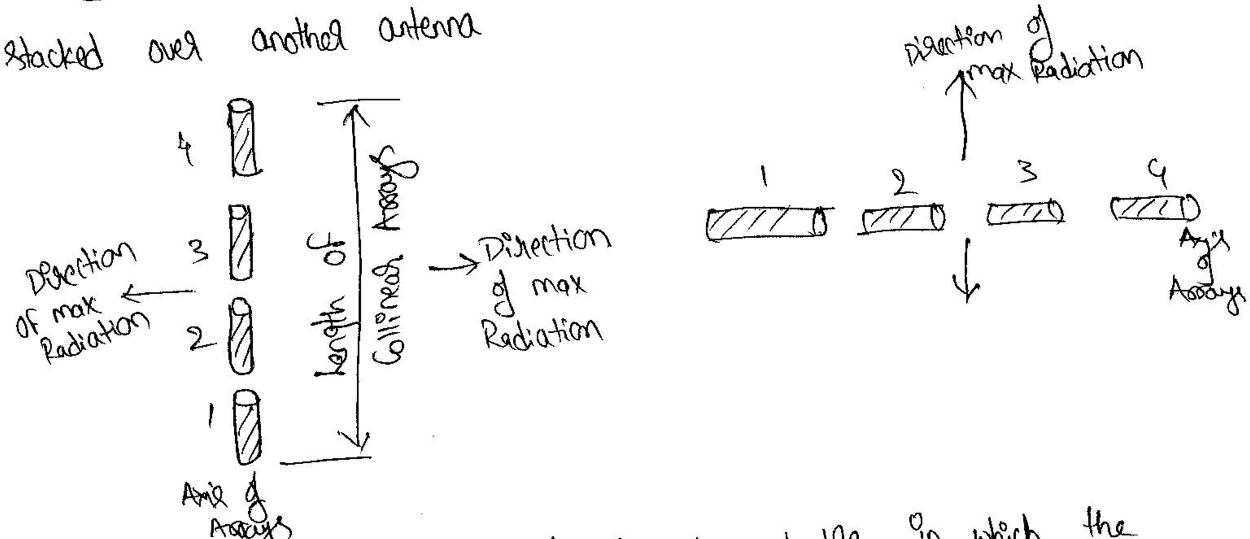
(i) Broad side Array:- This is one of the important antenna arrays used in Practice. Broad side array is one in which a number of identical Parallel antennas are set up along a line drawn perpendicular to their respective axes.



(ii) End Fire Arrays :-
Is nothing but broad side Array except that individual elements are fed in out of Phase (usually 180°). Thus in End fire array individual elements are fed with currents of equal magnitude but their phases varies progressively along the line in such a way as to make the entire arrangement substantially unidirectional.

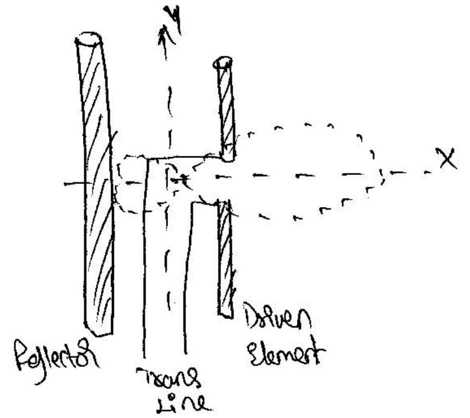


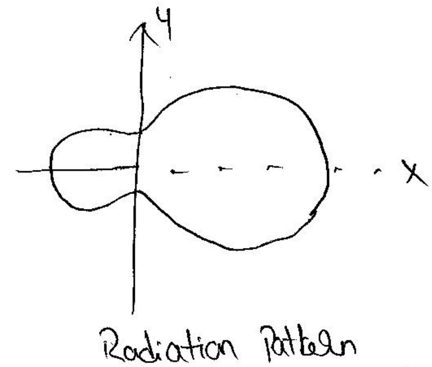
iii) Collinear Arrays :-
The antennas are arranged co-axially i.e antennas are mounted end to end in a single line. In other words antenna is stacked over another antenna



A Collinear arrays is a broad side radiator, in which the direction of max radiation is perpendicular to the line of Antenna.

(iv) Parasitic Arrays :- Parasitic Array is one driven element and one Parasitic element and this may be considered as two elements array. Multi elements having number of Parasitic elements are called Parasitic Arrays.





A Parasitic Array with linear half wave dipole as elements is normally called as Yagi-Uda (or) Yagi Antenna.

2) Arrays of Point Sources :-

Point source regarded as Point Source (or) Volumeless radiator. Point source means a single antenna to radiate the energy.

(i) Arrays of two Point sources :-

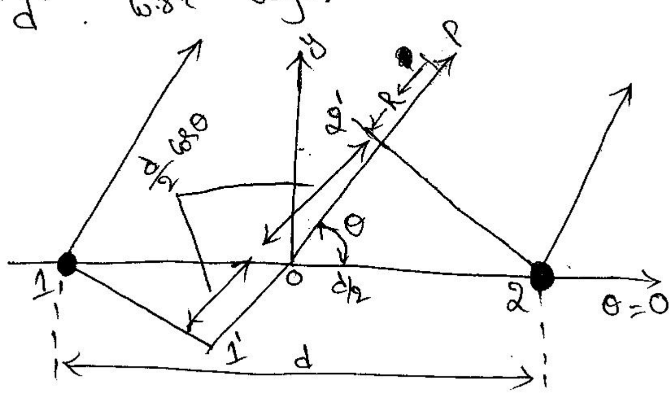
Let us consider two point sources seperated by equal distance and fed with different Phasing conditions and magnitudes.

Under this we have 3-Cases :-

* Equal Amplitude and same phase
* Equal " and opposite Phase
* " " and Any Phase

*→ Equal Amplitude and same Phase :-

Let the two point sources they are seperated by a distance of 'd'. w.r.t origin which are simitrical to origin.



4

Here the total field at point P is the sum of field due to source-1 and source 2 the wave from source to reaches to point 'P'.

Path difference $= (1'2')$ meters $= \left(\frac{d}{2}\cos\theta + \frac{d}{2}\cos\theta\right)$ meters

$= d\cos\theta$ meters

$$\boxed{(1'2')_{meters} = \frac{d}{\lambda}\cos\theta \text{ wavelengths}} \rightarrow (1)$$

Then from optics it is known as

$\boxed{\psi - \text{si}}$

Phase angle $(\psi) = 2\pi$ (Path difference)

$$\psi = 2\pi\left(\frac{d}{\lambda}\cos\theta\right) \text{ radians}$$

$$\psi = \frac{2\pi}{\lambda} d\cos\theta \text{ radians} \qquad \boxed{\because \beta = \frac{2\pi}{\lambda}}$$

$$\boxed{\therefore \psi = \beta d\cos\theta \text{ radians}} \rightarrow (2)$$

Total field strength (E_T):-

Total far field at distant point 'P' in direction 'θ' is given

$$E = E_1 \cdot e^{-j\psi/2} + E_2 e^{+j\psi/2} \rightarrow (3)$$

E_1 = far electric field at distant point P, due to source 1

E_2 = " " " " P " " 2

E = Total electric field at distant point

$\psi = \beta d\cos\theta$. radians

where $E_1 \cdot e^{-j\psi/2} \equiv$ field component due to source 1

$E_2 \cdot e^{+j\psi/2} \equiv$ field component due to source 2.

When $E_1 = E_2 \equiv E_0$ (say)

$$E_T = E_0\left(e^{-j\psi/2} + e^{+j\psi/2}\right) \quad \Big| \quad \because \cos\theta = \frac{e^{j\theta} + e^{-j\theta}}{2} \text{ from trigonometry}$$

$$= 2E_0 \left(\frac{e^{-j\psi/2} + e^{j\psi/2}}{2} \right).$$

$$E_T = 2E_0 \cos\left(\frac{\psi}{2}\right).$$

$$= 2E_0 \cos\left(\frac{\beta d \cos\theta}{2}\right) \qquad \left(\because \beta = \frac{2\pi}{\lambda},\ d = \lambda/2\right)$$

Amp (Phase)

$$E_T = 2E_0 \cos\left(\frac{\frac{2\pi}{\cancel{\lambda}} \times \frac{\cancel{\lambda}}{\cancel{2}} \cdot \cos\theta}{2}\right)$$

$$E_T = 2E_0 \cos\left(\frac{\pi}{2}\cos\theta\right) \longrightarrow (4)$$

The amplitude is max when $2E_0 = 1$

$$E_0 = 1/2$$

$$E_T = 2\left[\frac{1}{2}\right] \cdot \cos\left[\frac{\pi}{2}\cos\theta\right] = \cos\left(\frac{\pi}{2}\cos\theta\right)$$

$$\boxed{E_T = \cos\left(\frac{\pi}{2}\cos\theta\right)} \longrightarrow (5)$$

Maxima Direction of field strength :-

E_T is maximum, when $\cos\left(\frac{\pi}{2}\cdot\cos\theta\right)$ is max and value is $=1$

$\therefore E$ is max when $\cos\left(\frac{\pi}{2}\cos\theta\right) = \pm 1$

$$\frac{\pi}{2}\cos\theta_{max} = \pm n\pi \qquad \text{when } n = 0,1,2\ldots$$

$$\frac{\pi}{2}\cos\theta_{max} = 0 \qquad \text{if } n = 0$$

$$\cos\theta_{max} = 0$$

$$\boxed{\theta_{max} = 90^\circ \text{ and } 270^\circ}$$

Minima Direction :-

E_T is minimum when $\cos\left(\frac{\pi}{2}\cos\theta\right)$ is minimum value is '0'.

$\therefore E_T$ is minimum when $\cos\left(\frac{\pi}{2}\cos\theta\right) = 0$ when $n = 0,1,2\ldots$

$$\left(\frac{\pi}{2}\cdot\cos\theta_{min}\right) = \pm(2n+1)\frac{\pi}{2}$$

$$\cos\theta_{min} = \pm 1 \Rightarrow \theta_{min} = 0^\circ \text{ and } 180^\circ$$

Half Power Point direction :- (HPPD)

It is defined as where total field strength falls $\frac{1}{\sqrt{2}}$ times of its max voltage (or) current.

$$E_T = \cos\left(\frac{\pi}{2}\cos\theta\right)_{HPPD} = \pm\frac{1}{\sqrt{2}}$$

$$\frac{\pi}{2}\cos\theta_{HPPD} = \pm(2n+1)\frac{\pi}{4} \quad \text{where } n = 0,1,2\ldots$$

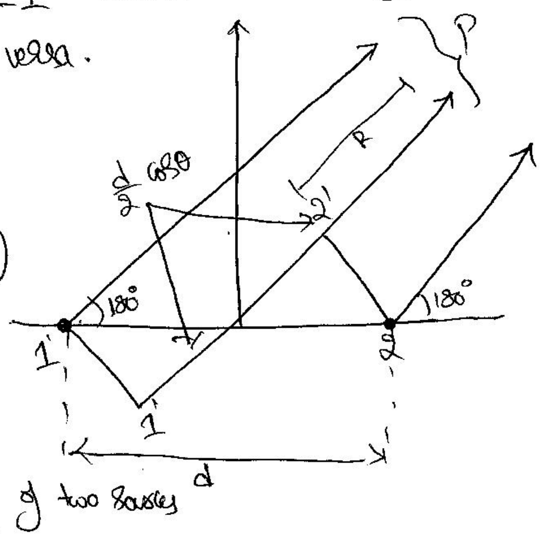
$$\frac{\pi}{2}\cos\theta_{HPPD} = \pm\frac{\pi}{4}$$

$$\cos\theta_{HPPD} = \pm\frac{1}{2}$$

$$\boxed{\theta_{HPPD} = 60^\circ, 120^\circ}$$

* **Equal amplitude and opposite phase :-**

Here 2-points sources are fed with equal in magnitude and opposite in phase i.e if source-1 radiates max energy then source-2 radiates min energy vice versa.



Total Electric field strength (E_T) :-

$$E_T = \left(-E_1 \cdot e^{-j\psi/2}\right) + \left(+E_2\, e^{j\psi/2}\right) \rightarrow (1)$$

because phase of source 1 & source 2 at distant point P is $-\psi/2$ and $+\frac{\psi}{2}$

∵ The reference being at b/w midway of two sources

$$E_1 = E_2 = E_0 \text{ (say)}$$

$$E_T = E_0\, 2j\left(\frac{e^{j\psi/2} - e^{-j\psi/2}}{2j}\right) \rightarrow (2)$$

$$E = 2jE_0 \sin\psi/2$$

$$E = \underbrace{2jE_0}_{\text{Ampl}} \underbrace{\sin\left(\frac{\beta d}{2}\cos\theta\right)}_{\text{Phase}} \rightarrow (3)$$

Let $d = \frac{\lambda}{2}$ and $2E_0 J = 1$

The amplitude is maximum when $2E_0J = 1$

$$E_{norm} = \sin\left(\frac{\pi}{2}\cos\theta\right).$$

maximum Direction of field strength:-

from Eq (3)

$$E_T = 2J\left(\frac{1}{2J}\right)\sin\left(\frac{\beta d\cos\theta}{2}\right)$$

$$E_T = \sin\left(\frac{\beta d\cos\theta}{2}\right)$$

$$= \sin\left(\frac{2\pi}{\lambda}\cdot\frac{\lambda}{2}\cdot\frac{\cos\theta}{2}\right)$$

$$E_T = \sin\left(\frac{\pi}{2}\cos\theta\right) \rightarrow (4)$$

maximum:-

from Eq (4) $\sin\left(\frac{\pi}{2}\cos\theta\right) = \pm 1$

$$\frac{\pi}{2}\cos\theta = \sin^{-1}(\pm 1)$$

$$\frac{\pi}{2}\cos\theta_{max} = \pm(2n+1)\frac{\pi}{2} \quad \text{when } n = 0, 1, 2 \ldots$$

$$\cos\theta_{max} = \pm 1 \text{ if } n = 0$$

$$\theta_{max} = 0^\circ \text{ and } 180^\circ$$

Minima Directions:-

from Eq (4) $\sin\left(\frac{\pi}{2}\cos\theta\right) = 0$

$$\frac{\pi}{2}\cos\theta_{min} = \pm n\pi \quad \text{where } n = 0, 1, 2 \ldots$$

$$\cos\theta_{min} = 0$$

$$\theta_{min} = 90^\circ \text{ and } -90^\circ$$

Half Power Point Direction:- If Radiation intensity falls $\frac{1}{\sqrt{2}}$ times of maximum value

from Eq (4) $E_T = \sin\left(\frac{\pi}{2}\cos\theta\right)$

$$\sin\left(\frac{\pi}{2}\cos\theta\right) = \frac{1}{\sqrt{2}} \quad \left(\because E_T = \frac{1}{\sqrt{2}}\right)$$

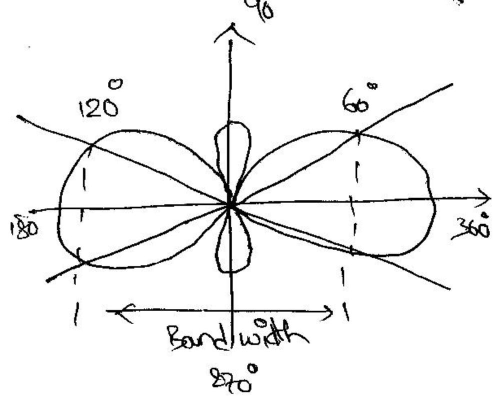
$$\frac{\pi}{2}\cos\theta = \sin^{-1}\left(\pm\frac{1}{\sqrt{2}}\right)$$

$$\frac{\pi}{2}\cos\theta = (2n+1)\frac{\pi}{4} \quad \text{if } n=0$$

$$\cos\theta_{HPPD} = \pm\frac{1}{2}$$

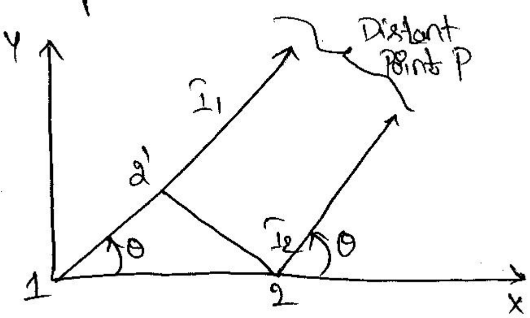
$$\theta_{HPPD} = \cos^{-1}\left(\frac{1}{2}\right)$$

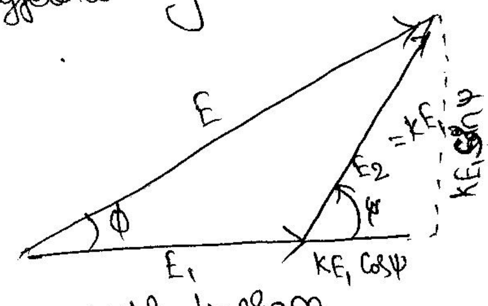
$$\boxed{\theta_{HPPD} = 60^\circ \text{ (or) } 120^\circ}$$



*Unequal amplitude and Any Phase :-

Let us Consider in which amplitudes of two Point sources are not Equal and hence Any Phase difference say 'α'.





vector diagram

Let us also assume at source 1' taken as reference for phase and amplitudes of fields due to source 1' and 2 at a distant point P' is E_1 and E_2 in which E_1 is greater than E_2. Then the total phase difference b/w the radiations of two sources at point P is

$$\psi = \frac{2\pi}{\lambda} d\cos\theta + \alpha \rightarrow (1)$$

where α is the Phase angle.

when $\alpha = 0$ (or) 180° and $E_1 = E_2 = E_0$ then it will Corresponds to above two Cases

The total field at point P is given by

$$E = E_1 \cdot e^{j0} + E_2 \cdot e^{j\psi} = E_1\left(1 + \frac{E_2}{E_1} \cdot e^{j\psi}\right) \quad \Big|\because e^{j0} = e^0 = 1$$

$$\boxed{E = E_1\left(1 + K \cdot e^{j\psi}\right)} \longrightarrow (2)$$

where $K = \frac{E_2}{E_1}$ $\quad \because E_1 > E_2$

$K < 1$

i.e $0 \leq K \leq 1$

from Eq (2)
The magnitude and Phase angle (ϕ) at Point P taken by

Modulus.

$$\therefore E = \left|E_1\left\{1 + K(\cos\psi + j\sin\psi)\right)\right| \longrightarrow (3)$$

$$E_\phi = E_1\sqrt{(1+K\cos\psi)^2 + (K\sin\psi)^2} \angle\phi$$

$$\boxed{\phi = \tan^{-1}\frac{K\sin\psi}{1+K\cos\psi}} \longrightarrow (4)$$

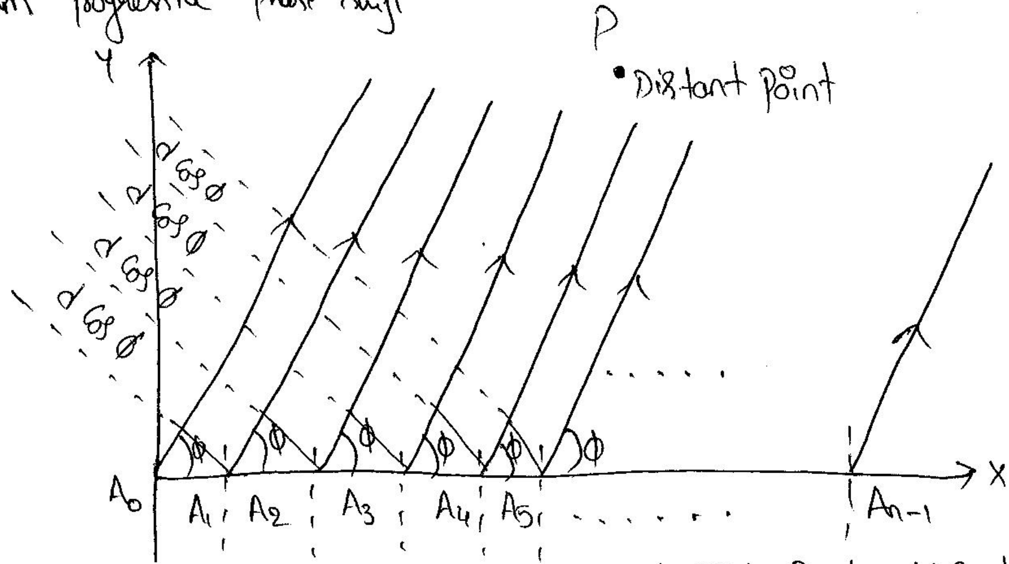
3) n- Elements Uniform linear Arrays :-

At higher frequencies for point to point Commnications it is necessary to have Pattern with single beam radiation. Such highly directive single beam Pattern can be obtained by increasing the point sources.

An array of n-elements is said to be linear array if all the individual elements are spaced equally along a line. An array is said to be uniform array if elements in array is said to be fed with Currents with Equal magnitudes and with uniform progressive Phase shift along the line.

Consider a general n-elements linear and uniform array with all the individual elements spaced equally at distance d from each other and all elements are fed with currents equal in magnitude and

Uniform Progressive phase Shift



The total resultant field at the distant point P is obtained by adding the fields due to 'n' individual sources

$$E_T = E_0 \cdot e^{j0} + E_0 \cdot e^{j\psi} + E_0 \cdot e^{2j\psi} + \dots + E_0 \cdot e^{j(n-1)\psi}$$

$$E_T = E_0 \left[1 + e^{j\psi} + e^{j2\psi} + \dots + e^{j(n-1)\psi} \right] \rightarrow (1)$$

note that $\boxed{\psi = \beta d \cos\phi + \alpha}$ indicates total Phase difference of fields from adjacent sources calculated at P.

α — Progressive phase shift b/w two adjacent point sources.

α — lies b/w 0° and 180°

If $\alpha = 0^\circ$ we get n-element uniform linear broadside array

$\alpha = 180^\circ$ " " " " " End fire array

Multiplying eq (1) by $e^{j\psi}$ we get

$$E_T \cdot e^{j\psi} = E_0 \left\{ e^{j\psi} + e^{j2\psi} + e^{j3\psi} + \dots + e^{jn\psi} \right] \rightarrow (2)$$

Substracting eq (2) from eq (1) we get

$$E_T - E_T \cdot e^{j\psi} = E_0 \left\{ \left[1 + e^{j\psi} + e^{j2\psi} + \dots + e^{j(n-1)\psi} \right] - \left[e^{j\psi} + e^{j2\psi} + \dots + e^{jn\psi} \right] \right\}$$

$$E_T \left(1 - e^{j\psi} \right) = E_0 \cdot \left(1 - e^{jn\psi} \right).$$

$$E_T = E_0\left[\frac{1-e^{Jn\psi}}{1-e^{J\psi}}\right] \rightarrow (3)$$

simplifying mathematically we get

$$E_T = E_0\left[\frac{e^{J\frac{n\psi}{2}}\left(e^{-J\frac{n\psi}{2}}-e^{\frac{Jn\psi}{2}}\right)}{e^{J\frac{\psi}{2}}\left(e^{-J\frac{\psi}{2}}-e^{J\psi/2}\right)}\right]$$

According to trignometric identity

$$e^{-J\theta}-e^{J\theta} = -2J\sin\theta$$

$$\Rightarrow E_T = E_0\left[\frac{\left(-J2\sin\frac{n\psi}{2}\right)\cdot e^{J\frac{n\psi}{2}}}{\left(-J\,2\sin\psi/2\right)\cdot e^{J\psi/2}}\right]$$

$$E_T = E_0\left[\frac{\sin n\frac{\psi}{2}}{\sin\frac{\psi}{2}}\right]\cdot e^{J\left(\frac{n-1}{2}\right)\psi} \rightarrow (4)$$

The magnitude of resultant field is given by

$$E_T = E_0\left[\frac{\sin n\frac{\psi}{2}}{\sin\frac{\psi}{2}}\right] \rightarrow (5)$$

The Phase angle θ of resultant field at Point p is given by

$$\theta = \frac{(n-1)}{2}\psi = \frac{(n-1)}{2}\beta d\cos\phi + \alpha \rightarrow (6)$$

4) Array of n-Elements of Isotropic Point sources with Equal amplitude & Equal spacing (broadside Array) (12)

(or)

Broad side Array Properties :-

An array is said to be broad side array if Phase angle is such that it makes maximum radiation Perpendicular to the line of array i.e 90° (or) 270°

in broad side array sources are in Phase i.e $\alpha = 0$ and $\psi = 0$ for max must be satisfied

$$\therefore \psi = \beta d \cos\theta + \alpha \quad \text{(or)} \quad \beta d \cos\theta = 0$$

$$= \beta d \cos\theta + 0 \qquad \cos\theta = 0$$

$$\theta = \cos^{-1}(0)$$

$$\theta = 90^\circ \text{ (or) } 270^\circ$$

The principal max occur in directions.

(a) Directions of Pattern maxima :- (Major Lobe)

Array factor $$\frac{E_T}{E_0} = \left[\frac{\sin \frac{n\psi}{2}}{\sin \psi/2}\right] \rightarrow (1) \qquad \psi = \beta d \cos\theta_{max} \rightarrow (a)$$

$$E_T = E_0 \left(\frac{\sin \frac{n\psi}{2}}{\sin \frac{\psi}{2}}\right) \rightarrow (2)$$

for maximum

$$\sin \frac{n\psi}{2} = \pm 1$$

$$\frac{n\psi}{2} = \sin^{-1}(\pm 1) = (2n+1)\frac{\pi}{2}$$

integer

$$\frac{n\psi}{2} = (2n+1) \cdot \frac{\pi}{2}$$

$$n\psi = (2n+1) \cdot \pi$$

$$\psi = \frac{(2n+1)}{n} \cdot \pi \rightarrow (3)$$

Substitute Eq (a) in Eq (3)

$$\beta d \cos\theta_{max} = \frac{\pi}{n}(2n+1)$$

$$\cos\theta_{max} = \frac{\pi}{n\beta d}\cdot(2n+1)$$

$$= \frac{\cancel{\pi}}{n\cdot\left(\frac{2\cancel{\pi}}{\lambda}\right)\cdot d}(2n+1)$$

$$\cos\theta_{max} = \frac{\lambda}{2n\cdot d}(2n+1)$$

$$\boxed{\theta_{max} = \cos^{-1}\left(\pm\frac{(2n+1)\cdot\lambda}{2nd}\right)}$$

Eq:- Let $n=4$; $d=\frac{\lambda}{2}$; $N=1$

$$\theta_{max} = \cos^{-1}\left(\pm\frac{(2+1)\cdot\lambda}{\cancel{2}\cdot 4\cdot\frac{\lambda}{\cancel{2}}}\right) = \cos^{-1}\left(\frac{3}{4}\right)$$

$$\theta_{max} = \cos^{-1}\left(\frac{3}{4}\right) = 41.4^\circ$$ //.

(b) Direction of pattern minimum:- (Minor lobe):-
minima of minor lobe in array of n-isotropic sources of equal amplitude and phase is given

$$E_T = E_0\frac{\sin n\psi/2}{\sin\psi/2} = 0$$

$$\sin\frac{n\psi}{2} = 0$$

$$\frac{n\psi}{2} = \sin^{-1}(0)$$

$$\frac{n\psi}{2} = \pm 1\ \underset{\text{integer}}{N}\pi$$

$$n\psi = \pm 2N\pi$$

$$\psi = \pm\frac{2N\pi}{n} \rightarrow (4)$$

Sub ① in ④

$$\psi = \frac{\pm 2N\pi}{n} \rightarrow ④$$

$$\beta d \cos\theta_{min} = \pm\frac{2N\pi}{n}$$

$$\cos\theta_{min} = \pm\frac{2N\pi}{n.\beta d} = \frac{\pm 2N\pi}{n.\frac{2\pi}{\lambda}.d}$$

$$\boxed{\cos\theta_{min} = \frac{\pm N.\lambda}{nd}} \rightarrow ⑤$$

Eg:- Let $n=4$; $N=1$; $d=\lambda/2$

$$\cos\theta_{min} = \pm\frac{1.\cancel{\lambda}}{\cancel{4}^{2}\cancel{\lambda}/\cancel{2}} = \frac{1}{2}$$

$$\theta_{min} = \cos^{-1}\left(\frac{1}{2}\right) = 60° \text{ (or) } 120°$$

ⓒ Beam width of major lobe :- defined as angle b/w the first null (or) double the angle b/w first null and major lobe max directions.

denoted as Complementary angle $\gamma = 90-\theta$

$$\theta = 90-\gamma$$

from Eq ⑤

$$\cos\theta_{min} = \left[\pm\frac{N\lambda}{nd}\right]$$

$$\cos\theta_{min} = \left\{\pm\frac{N\lambda}{nd}\right\}.$$

$$\cos(90-\gamma) = \left[\pm\frac{N\lambda}{nd}\right]$$

$$\sin\gamma = \pm\frac{N\lambda}{nd}$$

where γ is very small $\therefore \sin\gamma = \gamma$

$$\boxed{\gamma = \left[\pm\frac{N\lambda}{nd}\right]}$$

when $N = 1$

$$\gamma = \frac{\lambda}{nd}$$

$N = 2$

$$\gamma = \pm \frac{2\lambda}{nd} \text{ radians} = \frac{2\lambda}{nd} \times 57.3 \text{ degrees}$$

$$\gamma_{BWFN} = \frac{114.6\lambda}{L}$$

wher L = Total length of array in meters $= (n-1)\cdot d \approx nd$.

$$\gamma_{HPBW} = \frac{\gamma_{BWFN}}{2} = \frac{1}{L/\lambda} \text{ radian} = \frac{57.3}{L/\lambda} \text{ degrees}$$

~~Directivity~~

5) Array of n-sources of Equal Amplitude and Spacing (End-fire) Case.

For an array to be End fire, the phase angles is such that makes the maximum radiation in the line of array $\theta = 0$ (or) 180°

Thus for any array to be End fire $\psi = 0$ and $\theta = 0^\circ$ (or) 180°

$$\psi = \beta d \cos\theta + \alpha$$

$$0 = \beta d \cos(0^\circ) + \alpha$$

$$\alpha = -\beta d = -\frac{2\pi d}{\lambda} \quad \left(\because \beta = \frac{2\pi}{\lambda}\right)$$

this indicates that the phase difference b/w the sources of an End fire is retarded progressively by same amount as changing b/w the radians.

For Example if spacing b/w two sources is $\frac{\lambda}{2}$ (or) $\frac{\lambda}{4}$ then the phase angle by which angle by source 2 lags behind source is

$\frac{2\pi}{\lambda} \times \frac{\lambda}{2}$ (or) $\frac{2\pi}{\lambda} \times \frac{\lambda}{4}$ ie π (or) $\frac{\pi}{2}$ radians.

a) Direction of Pattern maxima:-

Array factor $\quad \sin n\psi/2 = 1$ if $\sin \psi/2 \neq 0$

$$n\psi/2 = \pm(2N+1)\cdot\frac{\pi}{2}$$

$$\cancel{2}(n\psi/\cancel{2}) = \pm(2N+1)\cdot\pi$$

$$n\psi = \pm(2N+1)\cdot\pi$$

$$\psi = \frac{\pm(2N+1)\cdot\pi}{n}$$

for end fire case $\alpha = -\beta d$; $\psi = 0$

$$\because \psi = \beta d \cos(\theta_{max}) + \alpha = \pm\frac{(2N+1)}{n}\cdot\pi$$

$$\beta d \cos(\theta_{max}) - \beta d = \pm\frac{(2N+1)\cdot\pi}{n}$$

$$\Rightarrow \beta d\left[\cos(\theta_{max}) - 1\right] = \pm\frac{(2N+1)\pi}{n}$$

$$\cos\theta_{max} - 1 = \pm\frac{(2N+1)}{\beta d n}\cdot\pi$$

$$\boxed{\theta_{max} = \cos^{-1}\left[\pm\frac{(2N+1)\cdot\pi}{\beta d n} + 1\right]}$$

Eg:- if $n = 4$, $d = \frac{\lambda}{2}$; $\alpha = -\pi$

$$\theta_{max} = \cos^{-1}\left[\frac{(2\cdot1+1)\cdot\pi}{2\cdot\frac{\pi}{\lambda}\cdot 4\lambda/2} + 1\right] ; \text{ if } N=1$$

$$= \cos^{-1}\left[\pm\frac{3}{4} + 1\right] = \cos^{-1}\left[\frac{7}{4}, \frac{1}{4}\right] = \cos^{-1}\left[\frac{7}{4}\right],\ \cos^{-1}\left[\frac{1}{4}\right]$$

$\Rightarrow \cos^{-1}[0.25] = 75.5^\circ$ | $\cos^{-1}\left[\frac{7}{4}\right]$ does not exists

(b) Direction of Pattern minima:-

obtained by Putting $\alpha = -\beta d$ in eq

$$(\theta_{min}) = \cos^{-1}\left[\frac{1}{\beta d}\left\{\pm\frac{2N\pi}{n} - \alpha\right\}\right]$$

now:-

$$\beta d \cos(\theta_{min}) + \alpha = \pm\frac{2N\pi}{n}$$

$$\beta d \cos(\theta_{min}) - \beta d = \pm\frac{2N\pi}{n}$$

$$\beta d\left\{(\cos\theta_{min}) - 1\right\} = \pm\frac{2N\pi}{n}$$

$$(\cos\theta_{min} - 1) = \pm\frac{2N\pi}{\beta n d} = \pm\frac{2N\pi}{\frac{2\pi}{\lambda}\cdot nd} = \pm\frac{N\lambda}{nd}$$

(or)

$$\left(1 - 2\sin^2\frac{\theta_{min}}{2} - 1\right) = \pm\frac{N\lambda}{nd} \quad \Big| \because \cos\theta = 1 - 2\sin^2\frac{\theta}{2}.$$

$$-2\sin^2\frac{\theta_{min}}{2} = \pm\frac{N\lambda}{nd}$$

$$-2\sin^2\frac{\theta_{min}}{2} = \pm\frac{N\lambda}{2nd}$$

$$-\cancel{2}\sin\cdot\frac{\theta_{min}}{\cancel{2}} = \pm\sqrt{\frac{N\lambda}{2nd}}$$

$$\frac{\theta_{min}}{2} = \sin^{-1}\left[\pm\sqrt{\frac{N\lambda}{2nd}}\right].$$

$$\theta_{min} = 2\sin^{-1}\left(\pm\sqrt{\frac{N\lambda}{2nd}}\right).$$

(c) Beam width of major lobes:-

It is defined as the average angular width b/w the first null and direction of first nulls (or) double the angle b/w first null and direction of major lobe.

$$BW = 2\theta$$

Beam width = 2 x Angle b/w first nulls and maximum of major lobes.

from above eqr $\theta_{min}^{o} = 2\sin^{-1}\left[\pm\sqrt{\frac{N\lambda}{2nd}}\right]$

$$\sin(\theta_{min}^{o}) \simeq \theta_{min}^{o} = 2\cdot\left(\pm\sqrt{\frac{N\lambda}{2nd}}\right) = \pm\sqrt{\frac{4N\lambda}{2nd}}$$

(or)

$$\theta_{min} = \pm\sqrt{\frac{2N\lambda}{nd}}$$

if array is long of length 'L' then

$$L = (n-1)d$$

$$L \approx nd$$

$$\theta_{min} = \pm\sqrt{\frac{2N\lambda}{nd}} = \pm\sqrt{\frac{2N\lambda}{L}}$$

Beam width b/w first nulls $(BWFN) = 2\times\theta_{min}^{o}$

$$BWFN = 2\times\left(\pm\sqrt{\frac{2N\lambda}{nd}}\right) = \pm 2\sqrt{\frac{2N\lambda}{nd}}$$

$$2\theta_{o} = 2\times\sqrt{\frac{2N}{L/\lambda}} = \pm\sqrt{\frac{2\times 1}{L/\lambda}} \text{ radians } (\text{if } N=1)$$

$$BWFN = \pm 2\sqrt{\frac{2}{L/\lambda}} = \pm 57.3\times 2\sqrt{\frac{2}{L/\lambda}} \text{ degrees}$$

$$\boxed{BWFN = \pm 114.6\sqrt{\frac{2}{L/\lambda}}}$$

Multiplication of Pattern :-

To find the field Pattern of two antenna's (or) two Point sources we have used mathematically method but it is Complex to do the same Procedure for array of antennas.

So an alternative method is available to find the field Pattern Called as Principle of Multiplication Pattern or simply multiplication Pattern.

It is defined as the total field Pattern of an array of the Point source is the multiplication of individual source is the Pattern & Pattern of array of Point sources after the adjustment the phase Pattern is the addition of the Phases of individual sources and Phase of array of Point sources.

Total phase Pattern is the Phase Centre of the array. The Principle of multiplication Pattern is applicable for two and three dimensional Pattern

Let E = Total field

$E_i(\theta,\phi)$ = field Pattern of individual source

$E_a(\theta,\phi)$ = field Pattern of array of isotopic Point sources

$E_{pi}(\theta,\phi)$ = Phase Pattern of individual source

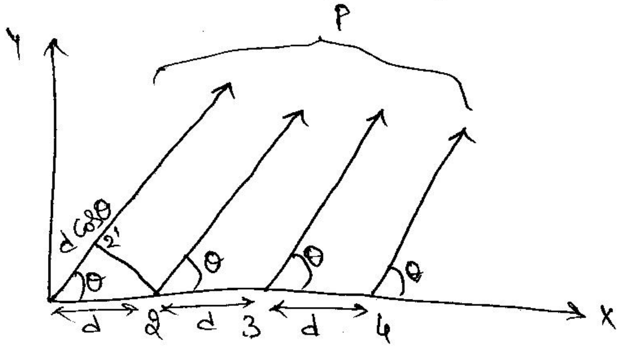
$E_{pa}(\theta,\phi)$ = Phase Pattern of array of isotopic Point sources.

Then total field pattern of an array of non-isotopic but similar source,

$$E = \underbrace{\left\{E_i(\theta,\phi) \times E_a(\theta,\phi)\right\}}_{\text{Multiplication of field Pattern}} \times \underbrace{\left\{E_{pi}(\theta,\phi) + E_{pa}(\theta,\phi)\right\}}_{\text{Addition of Phase Pattern.}}$$

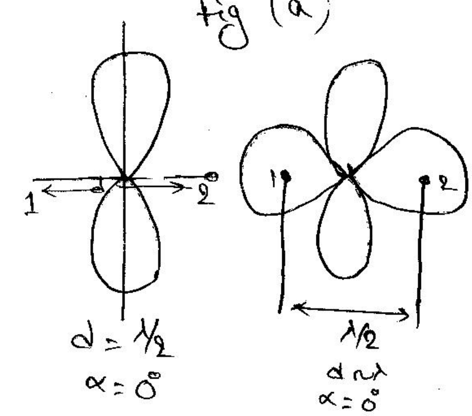
The angle θ, φ represents polar & azimuthal angles.

→ Let 4 - Antennas (point sources) separated by a distance of $\lambda/2$ b/w each and every element and are fed with equal amplitude and same phase $(\alpha = 0^\circ)$.

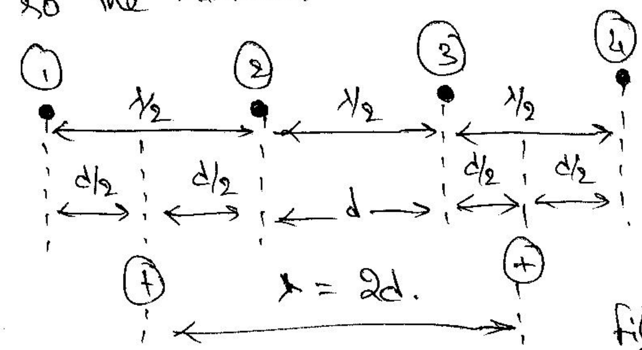


Linear Array 4-isotropic elements spaced $\lambda/2$ apart fed in phase

fig (a)



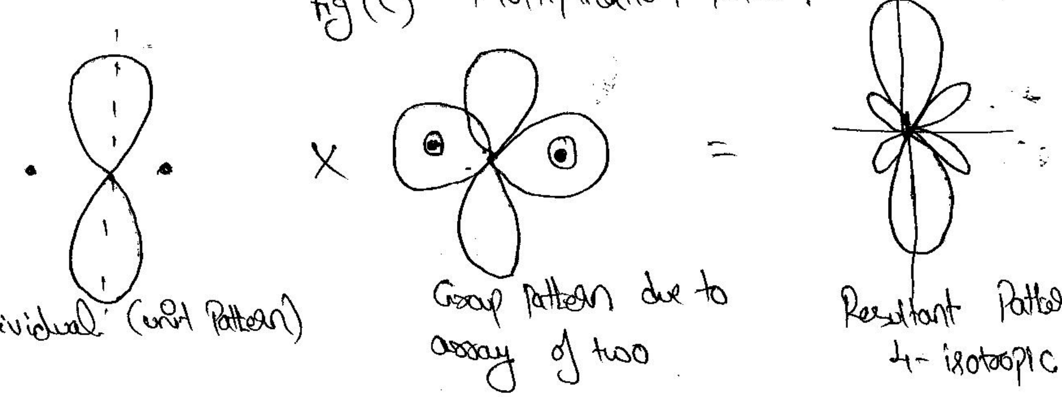
To draw the field pattern we consider first two elements they are spaced with $\lambda/2$ and are with equal amplitude source phase so the maximum radiation is towards 90° and 270°.



Array of 4-identical elements Replacement of array by two single antenna placed at distance 'λ'

fig (b)

fig (c) Multiplication Pattern.



individual (unit pattern) Group pattern due to array of two Resultant pattern of 4- isotropic elements.

Binomial Arrays (or) non-Uniform Amplitude Array:-

In this array the antenna elements are fed with un-equal amplitudes (or) non uniform amplitudes and elements are arranged according to Co-efficient of successive term of binomial series.

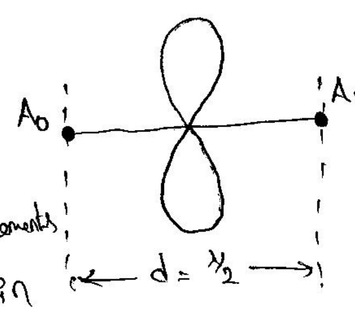
The binomial series given by

$$(a+b)^{n-1} = a^{n-1} + \frac{(n-1)}{1!} a^{n-2} b + \frac{(n-1)(n-2)}{2!} a^{n-3} \cdot b^2 + \frac{(n-1)(n-2)(n-3)}{3!} a^{n-4} \cdot b^3 + \ldots$$

where n = no. of radiating sources in the array.

It is found that this uniform linear array as the array length is increased to increase the directivity the second & minor lobes also appears in the patterns. In some of special applications it is desired to have single main lobe with no minor lobes. That means the minor lobe should be eliminated completely & reduced to minimum level as compared to main lobe.

To achieve such pattern the array is aranged in such a way that broadside array radiate more strongly at centre than that form edges. Let us consider array of two identical in-phase point sources spaced $\lambda/2$ a part. The far-field pattern is



$$F = \cos\left(\frac{\pi}{2}\cos\theta\right) \rightarrow (1)$$

To eliminate the side lobes spacing b/w the elements (1) must not exceed $\lambda/2$ and (2) current amplitudes in radiating sources are proportional to the Co-efficient of successive term of binomial series.

Two conditions satisfied by binomial array and the Co-efficients which corresponds to amplitudes of sources obtained by putting $n = 1, 2, 3 \ldots\ldots$

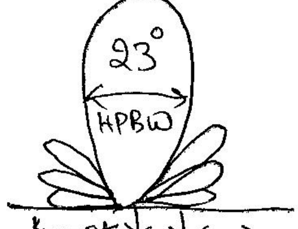
The Co-efficient of any radiating number sources can be obtained from Pascal's triangle.

no of sources	Pascal's triangle
$n = 1 \Rightarrow$	1
$n = 2 \Rightarrow$	1 1
$n = 3 \Rightarrow$	1 2 1
$n = 4 \Rightarrow$	1 3 3 1
$n = 5 \Rightarrow$	1 4 6 4 1
$n = 6 \Rightarrow$	1 5 10 10 5 1

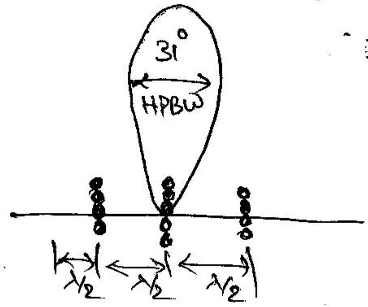
The elemination of secondary lobe takes place at cost of directivity Half power beam width (HPBW) of Binomial array is more than that of uniform array for same length of the array.

For Eg:- For radiating source $n = 5$ spaced $\lambda/2$ apart HPBW of Binomial and uniform arrays are respectively 23° and 31° Thus in uniform array secondary lobe appears but principal lobe is sharp and narrow as binomial array width of beam widens but without secondary lobes.

Uniform linear Array



Binomial array



*8) Fundamentals of Tchebyscheff Polynomials:-

The beam width b/w first nulls is specified, then the side lobe level can be minimised. The current distribution that produces such pattern is called Tchebyscheff distribution.

The distance b/w two successive array elements d is less than (or) equal to $\lambda/2$. Ak to this approach it is practically very difficult to reduce side lobe level without sacrificing the antenna performance in some other respect such as beamwidth, gain (or) directivity.

The Tchebyscheff polynomial with variable x is denoted by $T_m(x)$.

the Tchebyscheff polynomial is defined by equations as

$$\left.\begin{aligned} T_m(x) &= \cos(m\cos^{-1}x) \;;\; -1<x<+1 \rightarrow (a) \\ T_m(x) &= \cosh(m\cosh^{-1}x) \;;\; |x|>1 \rightarrow (b) \end{aligned}\right\} \rightarrow (1)$$

Here m is integer constant with range from 0 to ∞.

Let us assume Tchebyscheff polynomials for different values of m.

Let m = 0. Then from Eq 1-(a)

$$T_0(x) = \cos(m\cos^{-1}x)$$

let $\delta = \cos^{-1}x$ then

$$T_0(x) = \cos(m\delta)$$

$$\boxed{T_0(x) = \cos(0) = 1} \rightarrow (2)$$

Let m = 1 then from Eq 1-(a)

$$T_1(x) = \cos(m\cos^{-1}x)$$

$$T_1(x) = \cos(\cos^{-1}x)$$

$$\boxed{T_1(x) = x} \rightarrow (3).$$

Let $m=2$ then from Eq 1-(a)

$$T_2(x) = \cos(2 \cdot \cos^{-1}x)$$

$$T_2(x) = \cos(2\delta)$$

by trignometric $\cos 2\delta = 2\cos^2\delta - 1$

$$T_2(x) = 2\cos^2\delta - 1$$

Substuting $\delta = \cos^{-1}x$ in above Eq

$$T_2(x) = 2\cos^2(\cos^{-1}x) - 1$$

$$\boxed{T_2(x) = 2x^2 - 1} \rightarrow (4)$$

Let $m=3$ then Eq 1-(a) is

$$T_3(x) = \cos(3\cos^{-1}(x))$$

$$T_3(x) = \cos(3\delta)$$

trignometric Property $\cos 3\delta = 4\cos^3\delta - 3\cos\delta$

$$T_3(x) = 4\cos^3\delta - 3\cos\delta$$

Substituting $\delta = \cos^{-1}x$, we can write

$$\boxed{T_3(x) = 4x^3 - 3x} \rightarrow (5)$$

Let $m=4$ then Eq 1-(a)

$$T_4(x) = \cos(4\cos^{-1}x)$$

$$T_4(x) = \cos 4\delta$$

By trignometric Property $\cos 4\delta = 2\cos^2 2\delta - 1$

$$T_4(x) = 2\cos^2 2\delta - 1$$

But $\cos 2\delta = 2\cos^2\delta - 1$ then

$$T_4(x) = 2(2\cos^2\delta - 1)^2 - 1$$

$$T_4(x) = 2[4\cos^2\delta - 4\cos^2\delta + 1] - 1$$

$$T_4(x) = 8\cos^4\theta - 8\cos^2\theta + 1$$

Substituting $\theta = \cos^{-1}x$ we can

$$\boxed{T_4(x) = 8x^4 - 8x^2 + 1}$$

Now Polynomials of higher values of m can be obtained by using recursive formulae

$$\boxed{T_{m+1}(x) = 2x\,T_m(x) - T_{m-1}(x)}$$

$m = 5$ then

$$T_{4+1}(x) = T_5(x) = 2x\,T_4(x) - T_{4-1}(x)$$

$$T_5(x) = 2x\,T_4(x) - T_3(x)$$

Substituting Polynomials $T_4(x)$ and $T_3(x)$ in above eq

$$T_5(x) = 2x\left[8x^4 - 8x^2 + 1\right] - \left[4x^3 - 3x\right]$$

$$T_5(x) = 16x^5 - 16x^3 + 2x - 4x^3 + 3x$$

$$\boxed{\therefore\ T_5(x) = 16x^5 - 20x^3 + 5x}$$

$m = 6$ then

$$T_6(x) = T_{5+1}(x) = 2x\,T_5(x) - T_{5-1}(x)$$

$$= 2x\,T_5(x) - T_4(x)$$

Substituting Polynomials $T_5(x)$ and $T_4(x)$ in above eq

$$T_6(x) = 2x\left[16x^5 - 20x^3 + 5x\right] - \left[8x^4 - 8x^2 + 1\right]$$

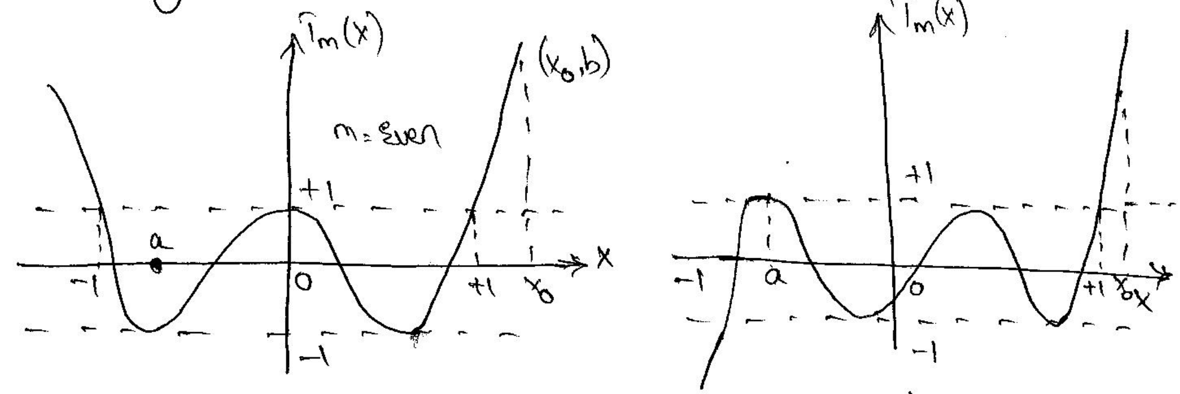
$$T_6(x) = 32x^6 - 40x^4 + 10x^2 - 8x^4 + 8x^2 - 1$$

$$\boxed{T_6(x) = 32x^6 - 48x^4 + 18x^2 - 1}$$

Thus first six Tchebyscheff Polynomials Can be summarized as

$m = 0$	$T_0(x) = 1$
$m = 1$	$T_1(x) = x$
$m = 2$	$T_2(x) = 2x^2 - 1$
$m = 3$	$T_3(x) = 4x^3 - 3x$
$m = 4$	$T_4(x) = 8x^4 - 8x^2 + 1$
$m = 5$	$T_5(x) = 16x^5 - 20x^3 + 5x$
$m = 6$	$T_6(x) = 32x^6 - 48x^4 + 18x^2 - 1$

The value of 'm' Can be either Even (or) Odd.



m = Even   m = odd

Characteristics of Tchebyscheff Polynomial:-

* All the Polynomials oscillate between the values -1 and 1
* In the region $|x| < 1$ then m^{th} order Tchebyscheff Polynomial crosses the axis m-times
* In the region $|x| > 1$, the Tchebyscheff Polynomial go on increasing without any Control. The rate of Polynomial given by x^m.