Unit - 5

VHF, UHF, and Micro Wave Antennas - II

Introduction:-

The antennas operating in the frequency ranged (3000 - 30000 MHz) are called SHF (Super High frequency) Antennas.

The typical Antennas used in the SHF band are Parabolic reflector, Horn Antenna, Lens Antenna etc;

In SHF Antennas, the wavelength is in Centimeter range. Hence it is practical to construct an antenna with such a dimensions, such antennas radiates power in a narrow beams. Such characteristics of aperture Antennas are used extensively for radar applications, and Point to Point radio Communications.

In such SHF Antenna the feeding to antenna is provided through a waveguide instead of using coaxial cable.

The co-axial cable is preffered for VHF and UHF which gives less attenuation compared with two wire transmission line. But such co-axial is unbalanced type of transmission line.

(i) Babinet's Principle And Complementary Antennas:-

To determine the relation b/w wire Antenna and aperture Antenna, is introduced by the babinets principle of optics.

Babinets Principle of optics States:-

"When the field behind a screen with an opening is added to the field of a complementary structure, the sum is equal to the field when there is no screen".

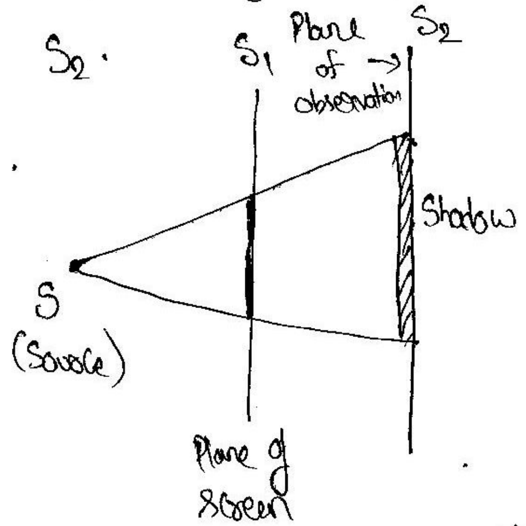
Babinet Principle in optics doesnot Consider Polarization. It deals Primarily with absorbing screens.

By introduction of babinet's Principle many of the Problems of slot antenna can be reduced to situation involving Complementary linear antennas.

Babinet Principle is Explained by Considering following fig:- with three Cases.

Let a source and two imaginary Planes are arranged in which the first plane is a plane of screen S_1 and the plane is a plane of observation S_2.

Now 3-Cases.



→ Case 'I':-

~~Case (I)~~

Case-I:-

Let a Perfectly absorbing screen be placed S_1, then in plane S_2, there is a region of shadow.

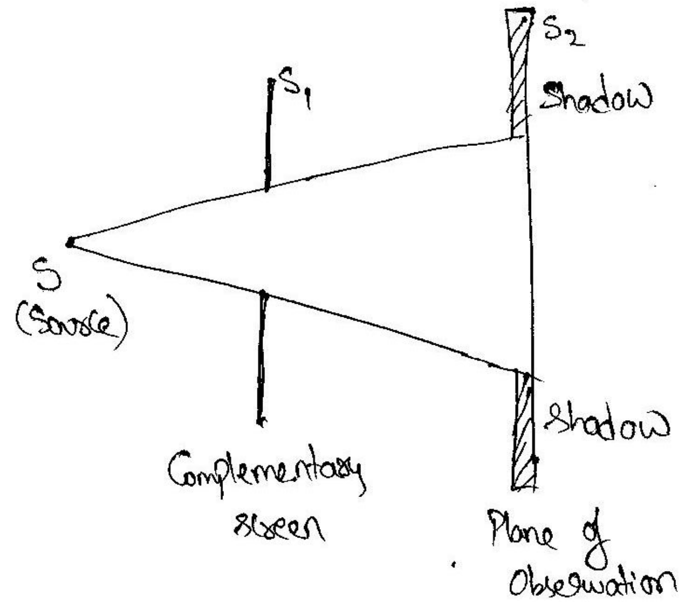
Let the field behind this screen be same function of $f_1(xyz)$ ie

$$\boxed{F_1 = f_1(xyz)} \rightarrow (a)$$

Case-II:-

Let the first screen S_1 be replaced by Complementary screen and the field behind it is

$$\boxed{F_2 = f_2(xyz)} \rightarrow (b)$$



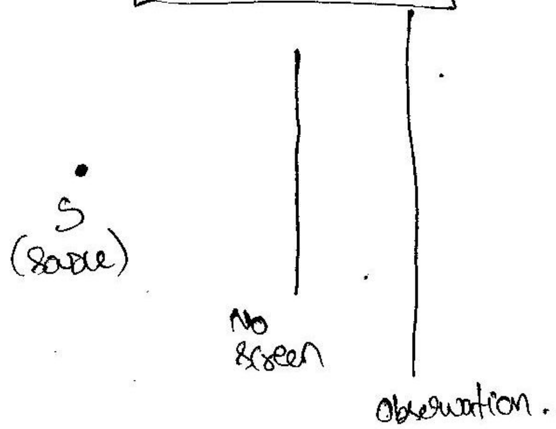
Case-III :- Let there is no screen present, then field is given by

$$F_3 = f_3(x\,y\,z)$$

∴ Babinents principle states that at same point $(x\,y\,z)$

$$F_3(x\,y\,z) = F_1(x\,y\,z) + F_2(x\,y\,z)$$

$$\boxed{F_3 = F_1 + F_2} \rightarrow (c)$$



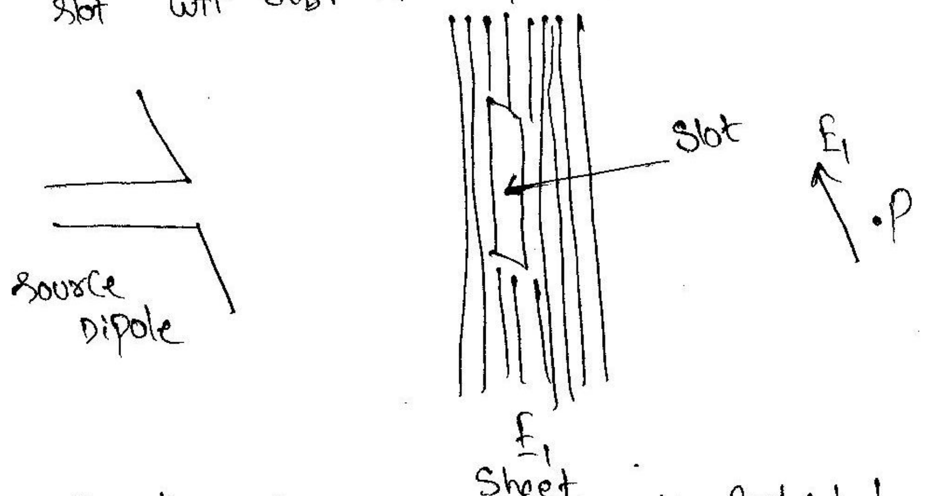
The principle not only applies to points in the plane of observation 'S_2' but also to any point behind screen S_1. The principle is obvious enough for shadow (Case I) it is also true when diffraction is taken in to accounts.

The Eq of (c) is verified easily for simple cases of complementary screens consisting of semi-infinite absorbing planes.

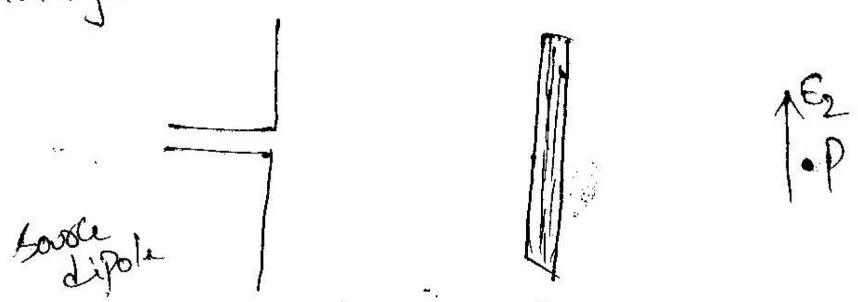
In Electromagnetics at radio frequencies, thin Perfectly absorbing screens are not aviable, Even approximately and one is Concerned with Conducting screens and vector fields for which Polarization Plays important role

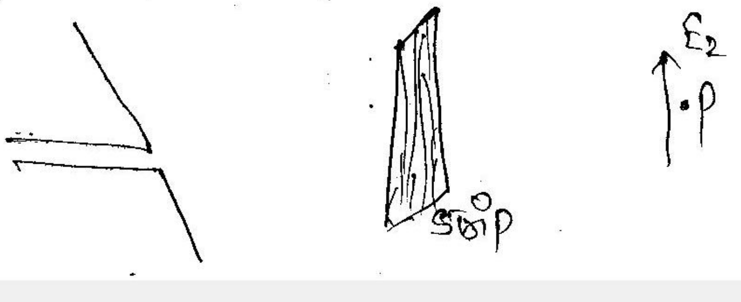
By Extention Principle, valid for Conducting screens and polarized fields Let us Consider 3-Cases following with a short dipole.

<u>Case I</u>:- The dipole is horizontal and original screen is an infinite Perfectly Conducting Plane infinitesimally thin sheet with a vertical slot cutt out. At a point 'P' behind the screen the field is 'E_1'



<u>Case - II</u>:- In this Case original screen is replaced by the Complementary screen Consisting of a Perfectly Conducting plane infinitesimally thin strip of the same dimensions as the slot in the original screen. besides dipole is source and is turned vertical so that E and H are interchanged. At the same point P, beside screen E_2



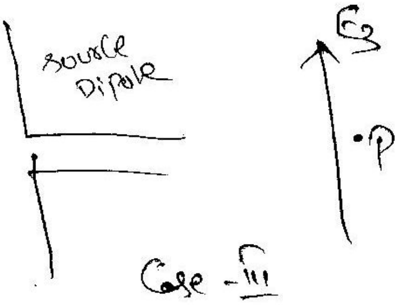


Case - III :-

In this no screen is placed and field at point P is E_3

According to babinets Principle $E_1 + E_2 = E_3$

$$\boxed{\frac{E_1}{E_3} + \frac{E_2}{E_3} = 1}$$



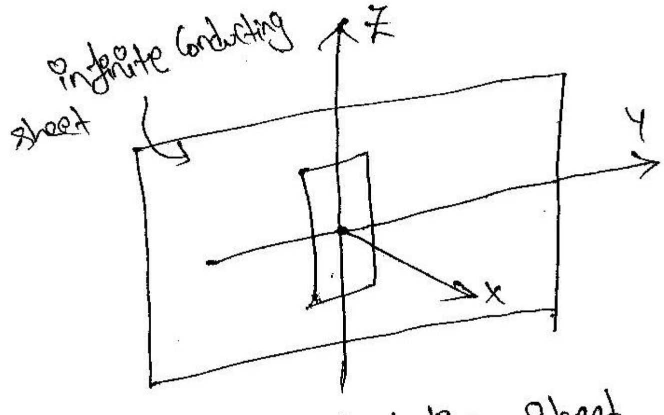
Case - III

Impedance $\boxed{Z_S Z_C = \frac{\eta^2}{4}}$

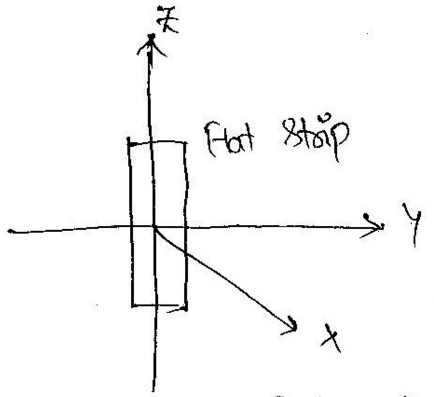
Z_S (screen) impedance
Z_C (Complementary) "
η = intrinsic impedance

*) SLOT Antennas :-

Slot Antennas are most extensively used like in high speed air crafts. Basically a slot antenna is nothing but an aperture of any size and shape made in a Conducting metallic sheet, forming a suitable radiator above 300MHz. Every slot antenna has its Complementary dipole formed in the form of wire (or) strip such that information about the Pattern and impedance of slot can be Easily obtained from the same Properties of Complementary dipole.



Metallic Conducting sheet



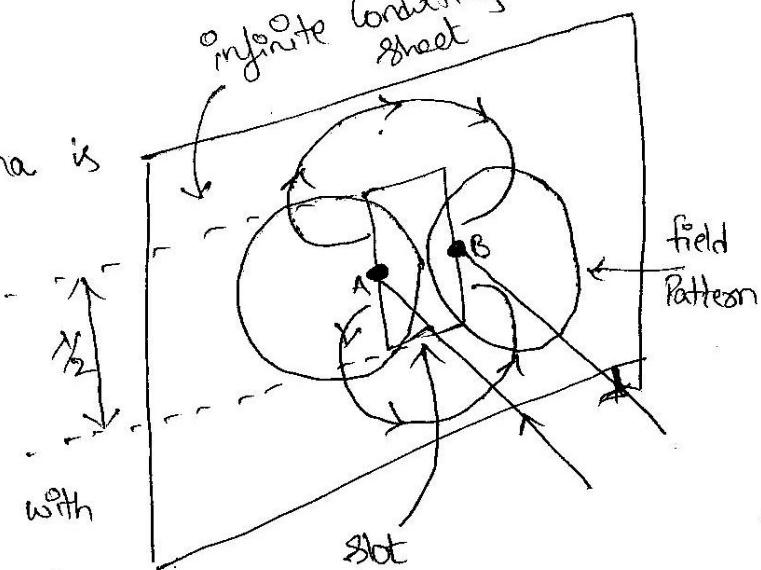
Complementary Flat strip

Now Consider that a slot of λ/2 is Cut in a large Conducting sheet we get Complementary dipole Antenna. In general, the slot antenna is fed by either a generator (or) transmission line connected across it.

In Case of wave guides the slot Antenna is fed with the guided wave incident on slot.

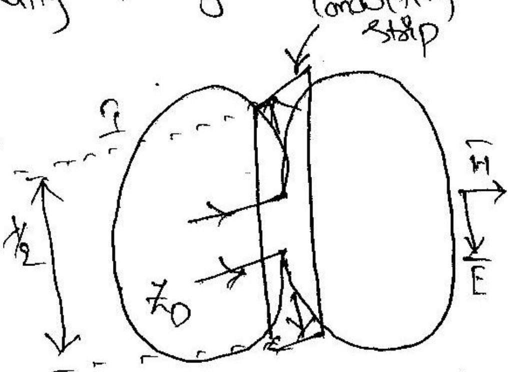
*) Consider that the slot Antenna is fed with a transmission line Connected across point A & B.

As the Antenna is fed with a transmission line, the slot antenna is the dipole.



*) For the Complementary of the slot Antenna is the dipole the regions with Conducting sheet and air are interchanged.

* A/c to R.G. Booker's theory, the field pattern of the slot is Exactly identical in shape as that of half dipole with $\bar{E}$ and $\bar{H}$ interchanged. That means for the slot, the Electric field $\bar{E}$ will be horizontally polarized, while for the dipole it is vertically polarized.



Complementary dipole Antenna

If Z_S and Z_D are the terminal impedance of the slot and the dipole respectively, then both can be related to each other interm of instric impedance of free space ie η_0

$$Z_D \cdot Z_S = \frac{\eta_0^2}{4} = \frac{(377)^2}{4} = 35530$$

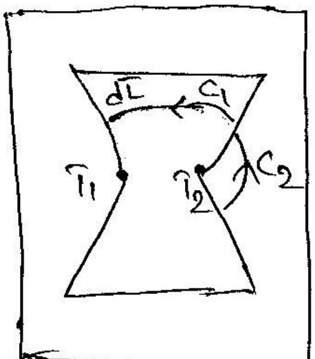
Hence terminal impedance of slot antenna is

$$\boxed{Z_S = \frac{35530}{Z_D}} \quad \text{or} \quad \boxed{Z_S = 35530\, Y_D}$$

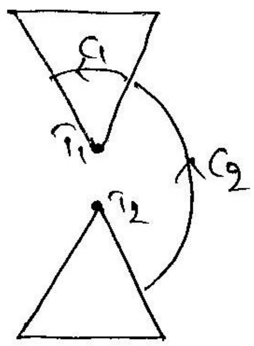
Impedance of Slot Antenna:-

Consider a Slot Antenna and a Complementary Antenna. The terminals of both antennas are indicated by T_1, T_2. Let us assume that both these terminals are seperated by infinitesimal distance in Each Case. Both these antennas are the Part of infinitesimaly thin, Plane and Perfectly Conducting sheet.

Let us assume that voltage V_s to current I_s ratio is the driving Point impeadence of slot antenna denoted by Z_s. Let E_s and H_s be the Electric and magnetic fields of the slot respectively at any Point P. There are two Closed Paths C_1 and C_2, where C_1 is inside slot while C_2 is outside slot.



Slot Antenna



Complementary dipole Antenna.

The voltage V_s at the terminals of the slot is given by

$$V_s = \lim_{C_1 \to 0} \int_{C_1} \bar{E}_s \cdot dL \rightarrow (1)$$

The current I_s at the terminals of slot Antenna is

$$I_s = 2 \cdot \lim_{C_2 \to 0} \int_{C_2} \bar{H}_s \cdot d\bar{L} \rightarrow (2)$$

Note:- Closed Path C_2 is outside the slot. In integral terms only half of closed integral is Consider so that '2' is used in Eq(2).

Now assume that $\bar{E}_D$ and $\bar{H}_D$ be the electric and magnetic fields of the dipole. Then the voltage V_D at the terminal of dipole is

$$V_D = \lim_{C_1 \to 0} \int_{C_2} \bar{E}_D \cdot d\bar{L} \rightarrow (3)$$

Similarly $$I_D = 2 \lim_{C_2 \to 0} \int_{C_1} \bar{H}_D \cdot d\bar{L} \rightarrow (4)$$

Let the characteristics impedance of surrounding medium be Z_0. Then we can write.

$$\lim_{C_2 \to 0} \int_{C_2} \bar{E}_D \cdot dL = Z_0 \lim_{C_2 \to 0} \int_{C_2} \bar{H}_S \cdot d\bar{L} \rightarrow (5)$$

Also $$\lim_{C_1 \to 0} \int_{C_1} \bar{H}_S \cdot d\bar{L} = \frac{1}{Z_0} \lim_{C_1 \to 0} \int_{C_1} \bar{E}_S \cdot d\bar{L} \rightarrow (6)$$

using eq (2) and (3) in eq (5)

$$V_D = Z_0 \left(\frac{I_S}{2}\right) = \frac{Z_0}{2} I_S \rightarrow (7)$$

Similarly using eq (1) and (4) in eq (6)

$$\frac{I_D}{2} = \frac{1}{Z_0} V_S$$

i.e $$V_S = Z_0\left(\frac{I_D}{2}\right) = \frac{Z_0}{2} I_D \rightarrow (8)$$

Multiplying eq (7) and (8) we get

$$V_S \cdot V_D = \left(\frac{Z_0}{2} \cdot I_S\right) \cdot \left(\frac{Z_0}{2} \cdot I_D\right)$$

$$\left(\frac{V_S}{I_S}\right)\left(\frac{V_D}{I_D}\right) = \left(\frac{Z_0^2}{4}\right)$$

$$\boxed{Z_S \cdot Z_D = \frac{Z_0^2}{4}} \rightarrow (9)$$

if surrounding medium is free space $Z_0 = 377\Omega$

$$\therefore Z_s = \frac{(377)^2}{4 \cdot Z_D} = \frac{35532}{Z_D}$$

Thus impedance of slot is directly proportional to intrinsic impedance of surround medium, while inversly proportional to impedance of Complementary dipole Antenna.

suppose Z_D is complex in nature given by

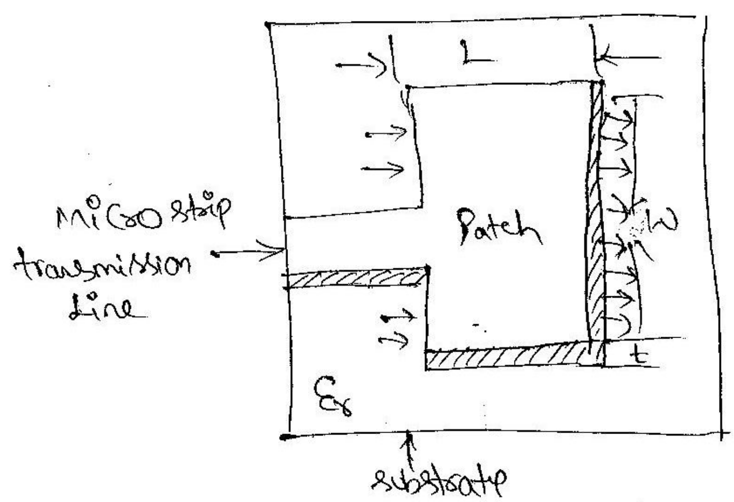
$$Z_D = R_D + JX_D \text{ then}$$

$$Z_s = \frac{35532}{R_D + JX_D} = \frac{35532\,(R_D - JX_D)}{(R_D + JX_D)(R_D - JX_D)} = \frac{35532\,(R_D - JX_D)}{(R_D^2 + X_D^2)}$$

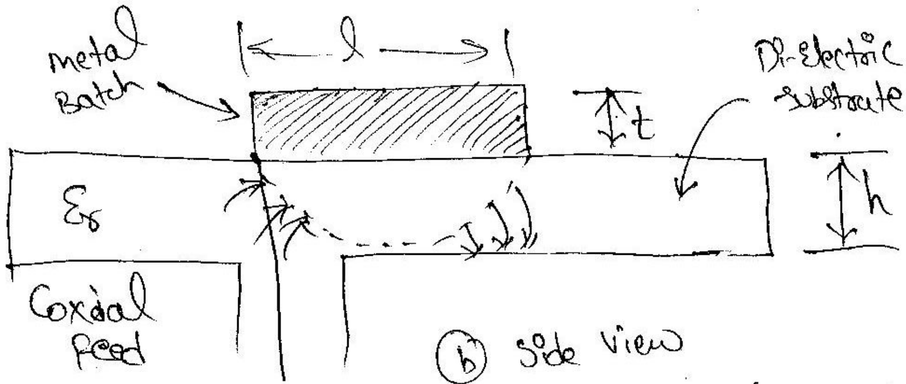
*) Micro Strip (or) Patch Antenna :-

In space crafts (or) air crafts applications, where size weight, cost performance, ease of installation, and aerodynamic profile are constraints, low profile antennas are required. In order to meet these specification microstrip (or) patch antennas are used.

Micro strip antenna is also referred as patch Antenna which is narrow band, wide beam Antenna. It consists of a substrate which is sandwiched b/w metallic antenna patch and ground plane.



(a) Top view.



(b) side view

from figure (a) it indicates that 'L' is length and 'W' is the width of the patch. Microstrip antenna is fed by microstrip transmission line.

From fig (b) the horizontal component of electric field at the edges are in same directions. Thus a linearly polarized radiation with maximum broad side to the patch is obtained.

The patch antenna acts as a resonant $\lambda/2$ parallel plate microstrip transmission line with characteristics impeadence equal to reciprocal of number of n of parallel field cell transmission line.

Each field transmission line has a characteristics impeadance 'Z_o' equal to intrinsic impeadence of medium.

$$Z_o = \eta_i = \sqrt{\frac{\mu}{\varepsilon}} = \sqrt{\frac{\mu_o}{\varepsilon_o}} \cdot \sqrt{\frac{\mu_r}{\varepsilon_r}} \rightarrow (1)$$

$$\left[\because \mu = \mu_o \mu_r \ \& \ \varepsilon = \varepsilon_o \varepsilon_r\right]$$

for air $\mu_r = \varepsilon_r = 1$ and $Z_i = Z_o = 377\,\Omega$

from fig (b), the cross section has 10 fields cells transmission lines. Hence the characteristics impeadence can be obtained for $\varepsilon_r = 2$.

$$Z_c = \frac{Z_o}{n\sqrt{\varepsilon_r}} = \frac{377}{10\sqrt{2}} = 26.63 \text{ Ohms}$$

then the above Eq can be be written by substituting $n = \frac{W}{t}$ as

$$Z_c = \frac{Z_o t}{\sqrt{\varepsilon_r}}$$

The radiation of microstrip antenna is wide (or) broad.

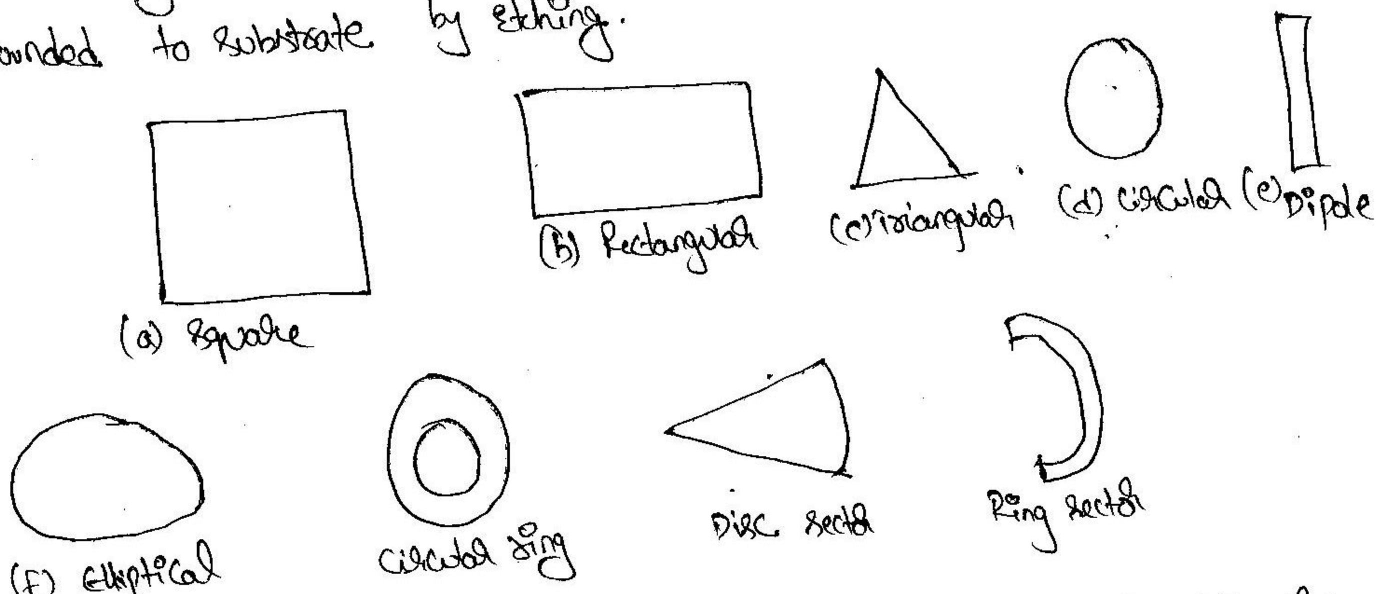
Typically the beam area Ω_A is about π steradians

$$\text{Directivity 'D'} = \frac{4\pi}{\Omega_A} = \frac{4\pi}{\pi} = 4$$

$$D_{dB} = 10 \log_{10}(4) = 10 \times 0.6021 = 6.021\,dB$$

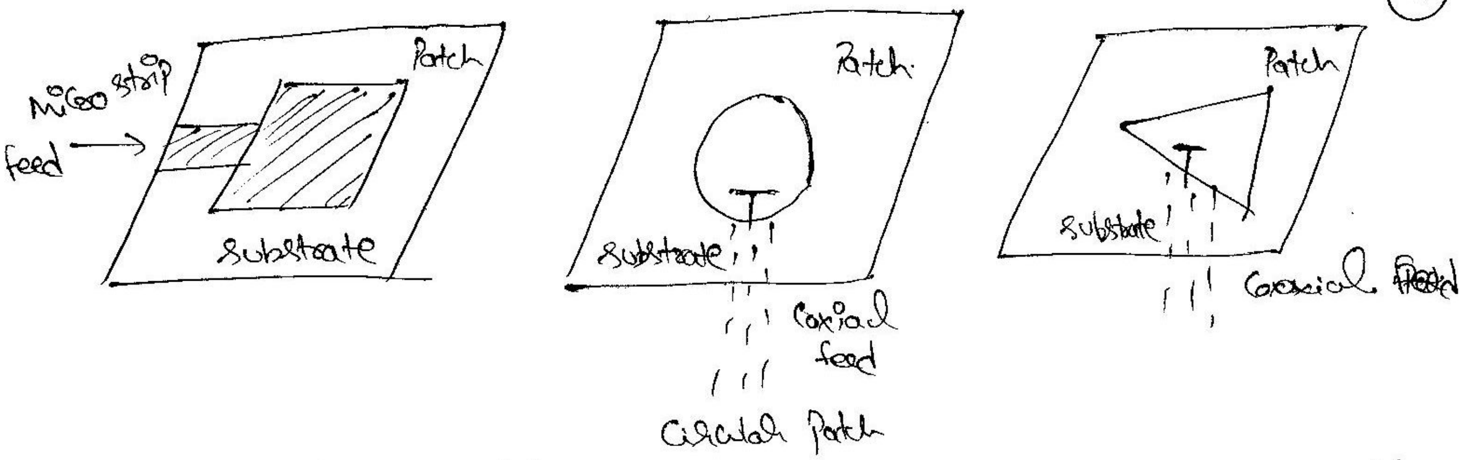
* Microstrip Antennas (MSA) (or) Patch Antenna Types:-

Radiating Elements are also refered as Patches. Patches are bonded to substrate by etching.



Out of these shapes square, rectangular, triangular, & circular are the most commonly used shapes for the patch because of ease in fabrication. Beside this these shapes are useful for low cross polarization radiation.

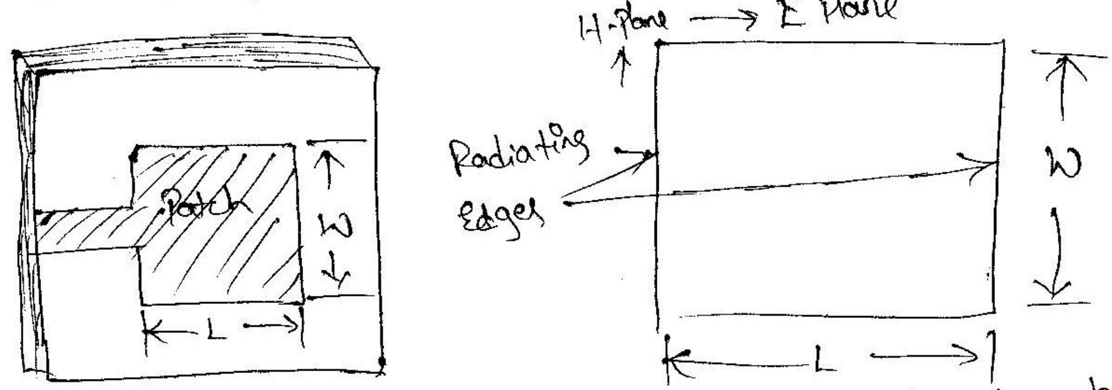
Microstrip dipole have large bandwidth and these are small in shape, occupy less space. Thus array of dipole can provide greater bandwidth. single element (or) array of microstrip antenna can be employed to obtain linear and circular polarization.



Rectangular Microstrip Antenna:-

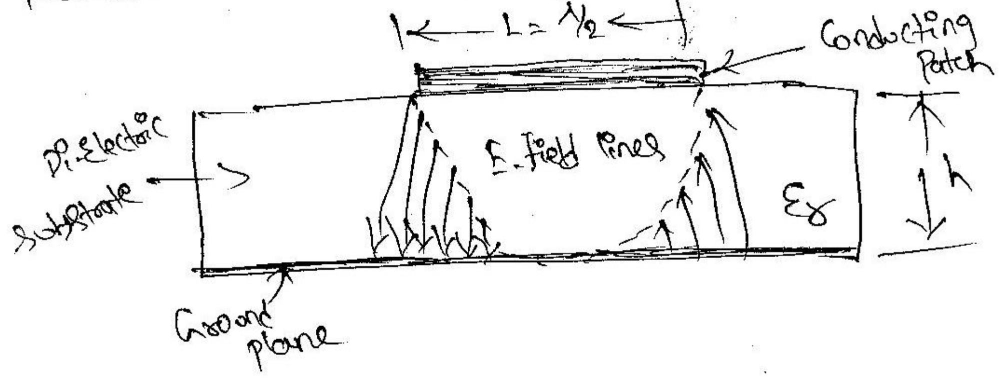
most Commonly used micro strip antenna ; simple to manufacture rectangular shaped Patch

Consider a microstrip antenna of rectangular shaped Patch



The dimensions 'L' (length) of a Patch is always greater than dimensions 'W' (width) of a Patch. The edges with 'L' dimensions Causes resonance at its half wavelength frequency. At the End of L-dimensions, there are radiating Edges which give single Polarization. At the End of W-dimension there are non-radiating Edges with very less radiation which gives Cross Polarization.

When a Patch length is half of wavelength $(L = \lambda/2)$ the Electric field Produced below the Edges of L-dimension are of opposite Polarity.



For Effective radiation of microstrip antenna

(i) The structure has to be half wavelength resonator ($L = \lambda/2$)

(ii) The di-electric substrate should be sufficiently thicker and with low di-electric constant.

(iii) The height of substrate should be limited to a fraction of wavelength.

∴ The critical (or) centre frequency of operation of an antenna approximately given by

$$f_c = \frac{c}{2L\sqrt{\varepsilon_r}}$$ where c = velocity of light $= \dfrac{1}{\sqrt{\varepsilon_0 \mu_0}}$

$$\boxed{\therefore\ f_c = \frac{1}{2L\sqrt{(\varepsilon_0 \varepsilon_r)\mu_0}}} \longrightarrow (1)$$

To obtain frequency operation of patch antenna accurately, we should consider dimension 'W' also

$$f_{r,nm} = \frac{c}{2\sqrt{\varepsilon_{r,eff}}}\left[\left\{\frac{n}{L+2\Delta L}\right\}^2 + \left\{\frac{m}{W+2\Delta W}\right\}^2\right]^{1/2} \longrightarrow (2)$$

for dominant dipole mode (with $n=1$; $m=0$) the freq of operation

$$f_{r,nm} = \frac{c}{2(L+2\Delta L)\sqrt{\varepsilon_{r,eff}}}$$

The width 'W' of the patch is very important parameter as it controls the i/p impedance of an antenna. For typical square shape patch antenna ($L = W$) the i/p impedance is typically 300Ω. When the width is increased, the i/p impedance decreases

*) Feed Methods of Microstrip Antenna :-

The microstrip antennas are narrow band antennas typically. In most of the applications it is observed that the bandwidth is limited further due to mismatch of the impedance of antenna with feeding circuitary. So selecting proper feed technique and feed location, a resonant resistance of antenna is matched with that of the feeding circuit and thus impedance matching is done over a limited bandwidth.

Feeding Techniques :-

(i) Contacting Feed :- In this method, the patch is directly fed with R.F. Power using the connecting element such as microstrip line (or) Co-axial line.

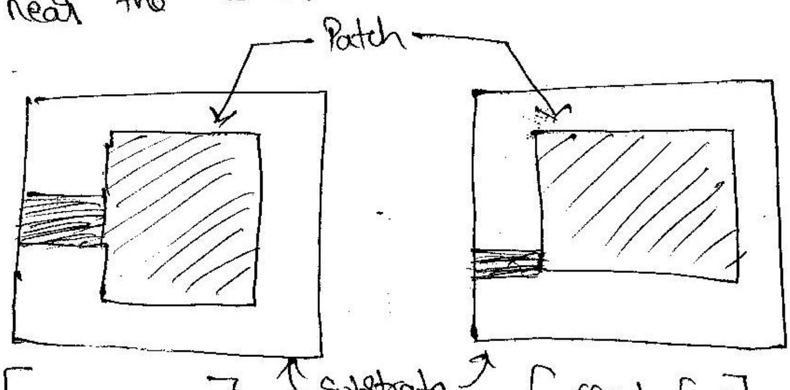
(ii) Non-Contacting Feed :- In this method the patch is not directly fed with RF Power but instead power is transfered to the path from the feed line through electromagnetic coupling. The most commonly used non-contacting feed methods are aperture coupled feed, proximity coupled feed.

(A) Microstrip feed :- In this method feed line is directly connected to the edge of the patch on a substrate. The advantage of using microstrip feed line is that feed line can be etched directly on the same substrate and where patch is also etched.

These are divided in to 4-main classes

*) Centre Feed :- The microstrip line is etched exactly at the centre of the patch at the edge of patch.

*) Offset Feed :- The microstrip line is not at the centre of patch but in general near the corner.

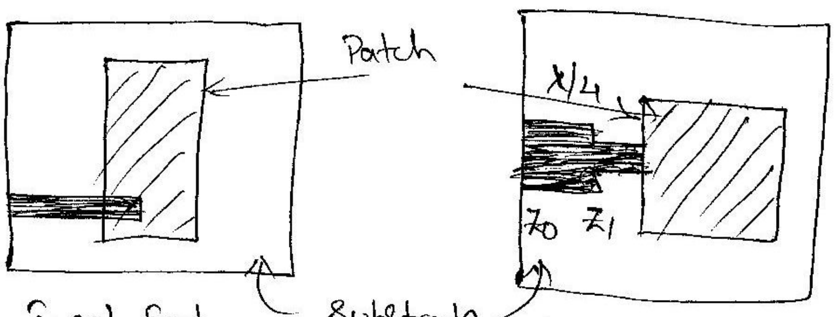


[Centre Feed] [Offset Feed]

*) <u>Inset feed</u> :- The main advantage of feeding antenna at the centre (or) at the corner end is that current increases from edge to the centre and hence the i/p impedance of antenna increases which further creates the impedance matching problems. To reduce the i/p impedance, the line extends inside the patch by appropriate distance from the edge, so that proper impedance matching is done.

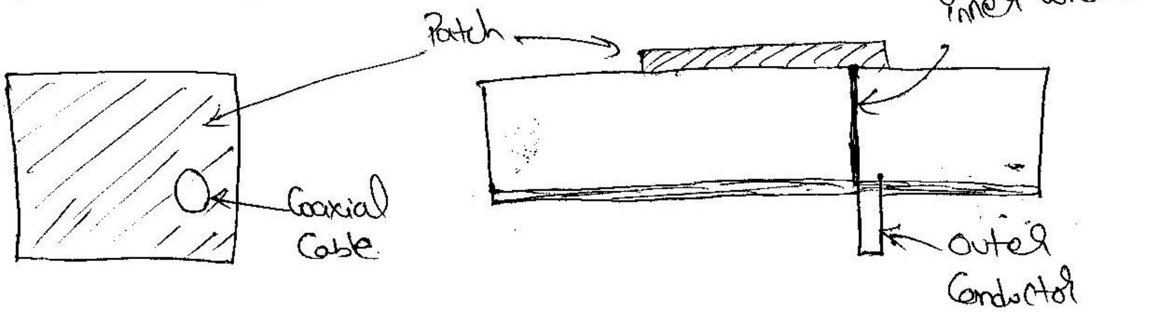
*) <u>Quarter Wave line feed</u> :- In this method, the transmission line impedance is matched with the antenna impedance with the help of quarter wave section of line with characteristic impedance Z_1. It acts as a quarter wave transformer and i/p impedance at the quarter wave

$$Z_0 = \frac{Z_1^2}{Z_A}$$



Inset feed Quarter wave line feed.

(B) <u>Co-axial feed</u> :- The technique is also probe feed. The co-axial cable has two conductors inner conductor and outer conductor. The inner conductor is extended up to radiating patch through the substrate and it is soldered to the metallic patch. While the outer conductor is connected to the ground plane.



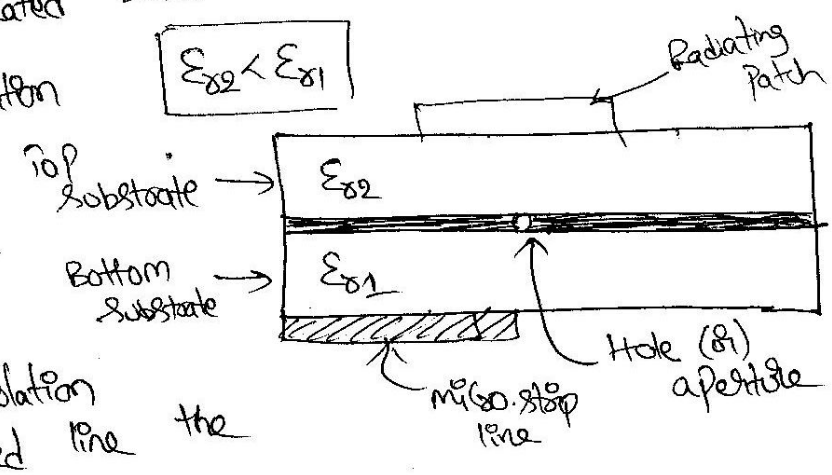
For impedance Matching, i/p impedance of the microstrip antenna can be adjusted by properly selecting feed position.

The main advantage of this techniques is that the co-axial feed can be placed at any location inside path to achieve impedance matching.

The disadvantage of this method is that bandwidth is very narrow. more over a hole is to be drilled through substrate it creates difficulty in modeling an antenna

(c) Aperture Coupled feed :- This technique uses an Electromagnetic coupling b/w a patch and a microstrip line. The feed circuitry is isolated from the antenna by a conducting plane with aperture (or) hole located below the patch at centre.

$\boxed{\varepsilon_{r2} < \varepsilon_{r1}}$

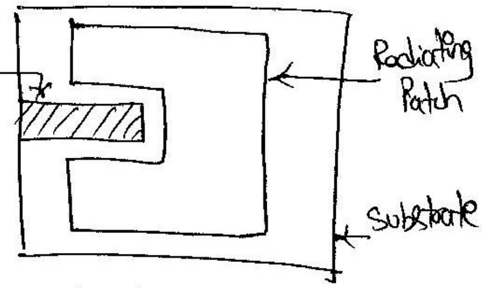


Because of the cross polarization is reduced. The amount of coupling varies with size shape and location of aperture. Due to the isolation of radiating patch and feed line the spurious radiations are lowered.

The Major advantage of di-electric layer need to be properly aligned during fabrication. Also due to two substrates, the overall thickness considerably increases.//

(D) Proximit Coupled feed (Indirect feed) :-

This method is called indirect feed in which similar to the inset feed method the line is etched inside the Patch but it is not touched to the edge of the Patch but is not touched to the Edge of Patch.



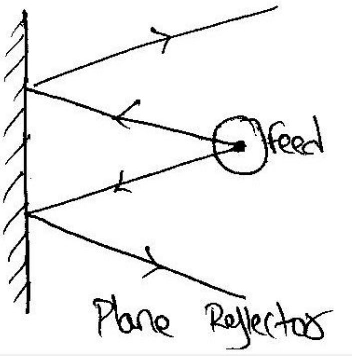
The gap between microstrip line and the radiating Probe Provides a Capacitance but it can cancel the inductance added by increasing length of the Co-axial Cable in the Co-axial feed method.

*) MicroWave Antennas :-

UHF and SHF are respectively (300 – 3000 MHz) and (3000 – 30,000 MHz) but microwave region starts from (1000 MHz to Extended 100,000 MHz) The corresponding wavelength is in Centimeters (10 – 1cm) and less. the transmitting and receiving antennas for use in microwave spectrum (1000 – 100,000 MHz) tend to be directive i.e high gain and narrow beam- width in horizontal and vertical planes.

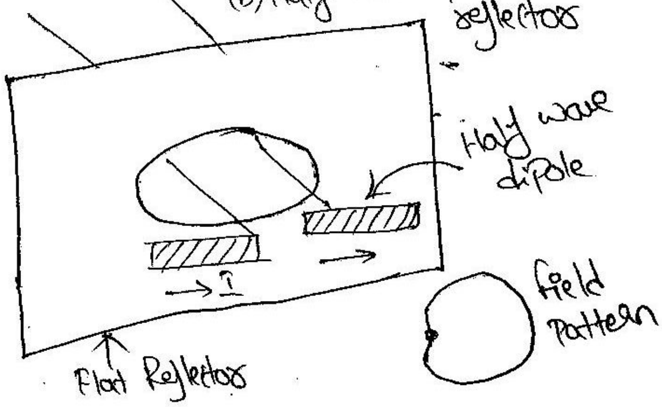
*) Plane Reflectors (or) Flat Sheet Reflectors :-

The Plane reflector is the simplest form of the reflector antenna. When the Plane reflector is kept infront of feed, the Energy is radiated in the desired direction.

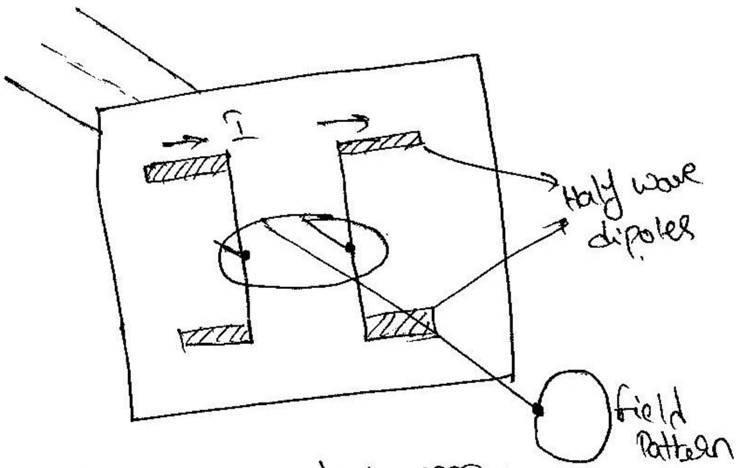


Plane Reflector

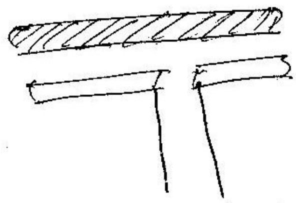
(b) Half wave dipole with Plane reflector



Flat Reflector



(c) Half wave dipole array with plane reflector.



(d) Half wave dipole with reflector element

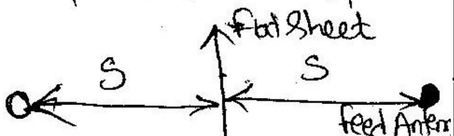
The directivity of this reflector is less, so to increase the directivity
we use a large flat sheet reflector because some of the radiation
Pattern is not covered by plane reflector. The main advantage of
this plane reflector is that for the dipole backward radiations are
reduced and the gain in the forward direction increases. To increase
directivity further, we can use array of two half wave dipoles
infront of a flat ~~plate~~ plane reflector.

It is observed that the flat sheet is less frequency
sensitive than thin element. Hence only reflector element can be
done with the help of method of images. In this method, reflector is
replaced by image of an antenna at a distance 2S from feed antenna
for an infinite plane reflector, assuming zero reflector losses, the
gain of an $\lambda/2$ dipole antenna at a distance 'S' is given by

$$G_f(\phi) = 2\sqrt{\frac{R_{11}+R_{loss}}{R_{11}+R_{loss}-R_{12}}}\,\left|\sin\left(S_r\cos\phi\right)\right| \rightarrow (1)$$

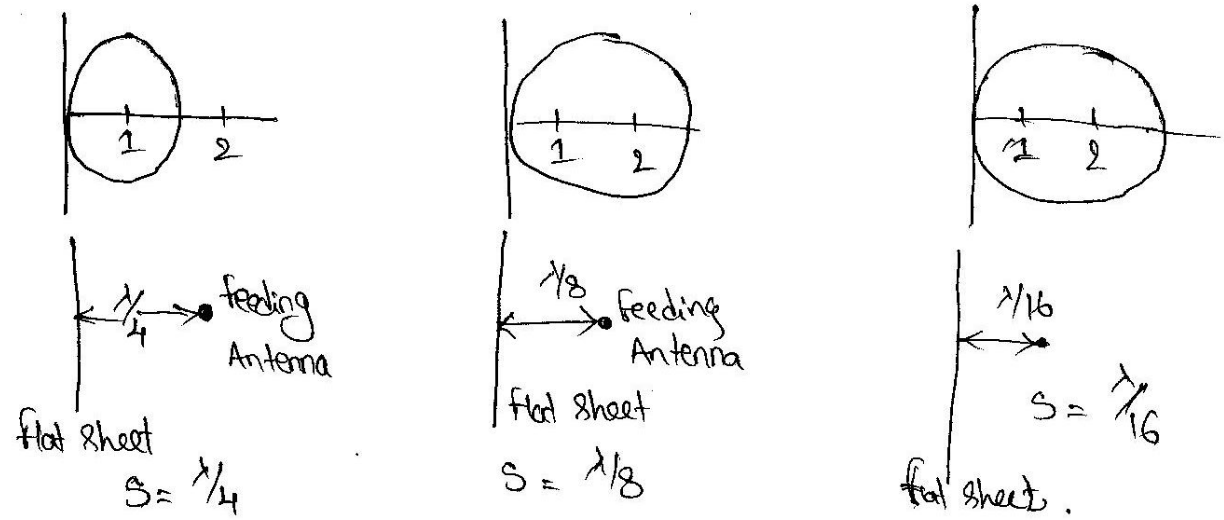
And

$$S_r = \left(\frac{2\pi}{\lambda}\right)\cdot S \rightarrow (2)$$



Eq (1) is gain reflector relative to half wave dipole antenna.
is a function of spacing between flat sheet and half wave dipole Antenna

When the spacing b/w the half wave dipole and infinite sheet decreases, the gain increases for decreasing spacing the gain in field intensity in the free space.

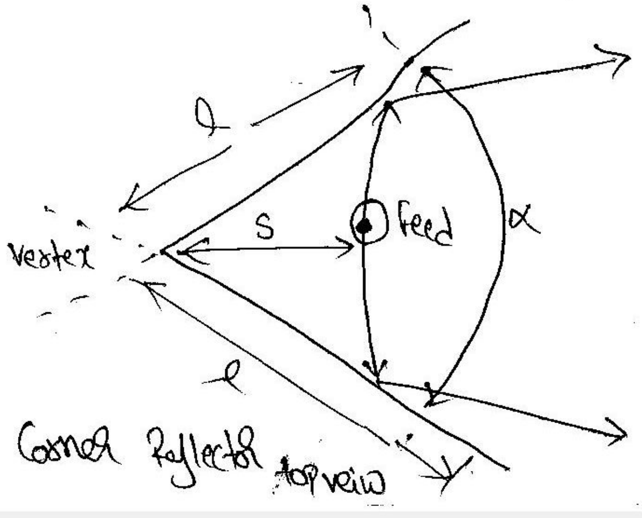


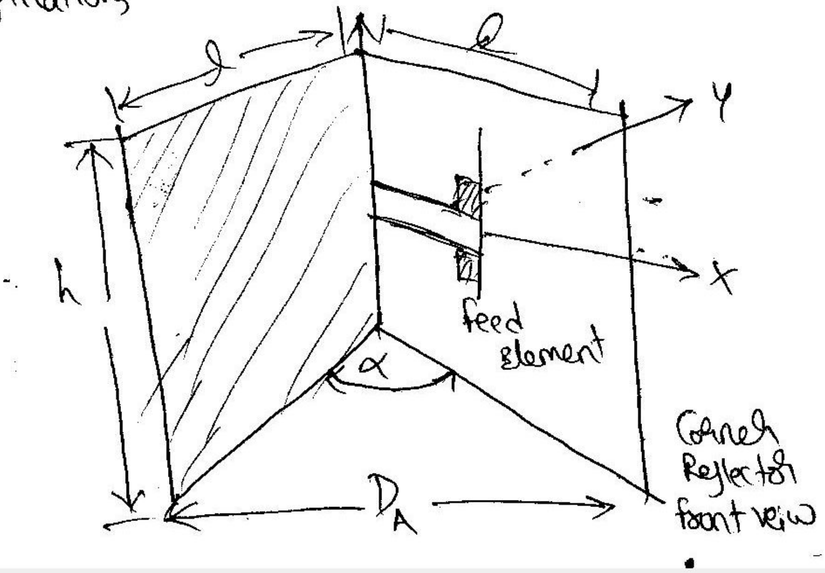
* Corner Reflector :-

To overcome the drawbacks of Plane reflector the shape of plane reflector is modified so that radiation is in the forward direction only and we dont get losses in the side radiations.

The corner reflector can be formed by joining 2-plane reflectors with some angle the reflector is known as corner reflector.

The angle at which 2-plane reflectors are joined is called corner reflector (or) corner angle represented by α and also called as included angle. for Practical applications 'α' value is 90°.





In most of the corner reflectors, the feed element is either a dipole or array of collinear dipoles placed parallel to the vertex at a distance 'S'. To increase the bandwidth, instead of thin wires as feed element, the biconical (or) cylindrical dipoles are preferred.

In mathematical Analysis dimensions specified for corner reflector of Aperture:- Aperture of corner reflector (D_A); length of reflector (l) height (h).

*) The dimensions of aperture reflector (D_A) is selected b/w one & two wavelengths ($\lambda < D_A < 2\lambda$)

*) The spacing b/w vertex of reflector and feed element is selected as a fraction of wavelength ($\lambda/3 < S < 2\lambda/3$)

*) Length of reflector is typically selected as twice the spacing b/w feed and vertex (i.e $l = 2d$) for included angle of 90°.

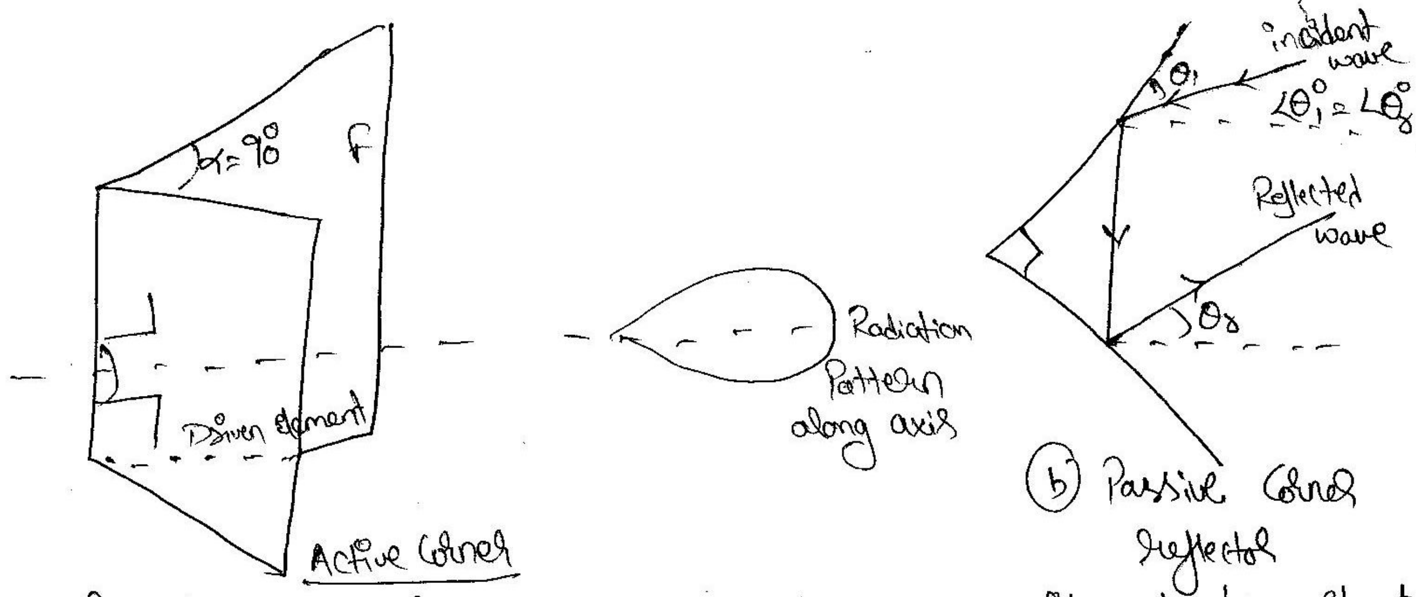
→ The radiation resistance is the function of spacing b/w the feed and reflector. If the spacing is too large the unwanted multiple lobes are formed hence directivity of antenna is lost.

→ If spacing is very small, the radiation resistance decreases. The losses in system increased as the decreased radiation resistance becomes comparable with the loss resistance of antenna.

→ The height of reflector (h) is generally selected as about 1.2 to 1.5 times greater than the total length of feed element.

A corner reflector with two flat conducting sheet at a corner angle 'α' and a driven antenna is called active corner reflector antenna (or) simply corner reflector antenna.

If the corner reflector antenna consists only two flat conducting sheet at a corner angle 'α' without any driven element then it is called as Passive Corner reflector antenna.

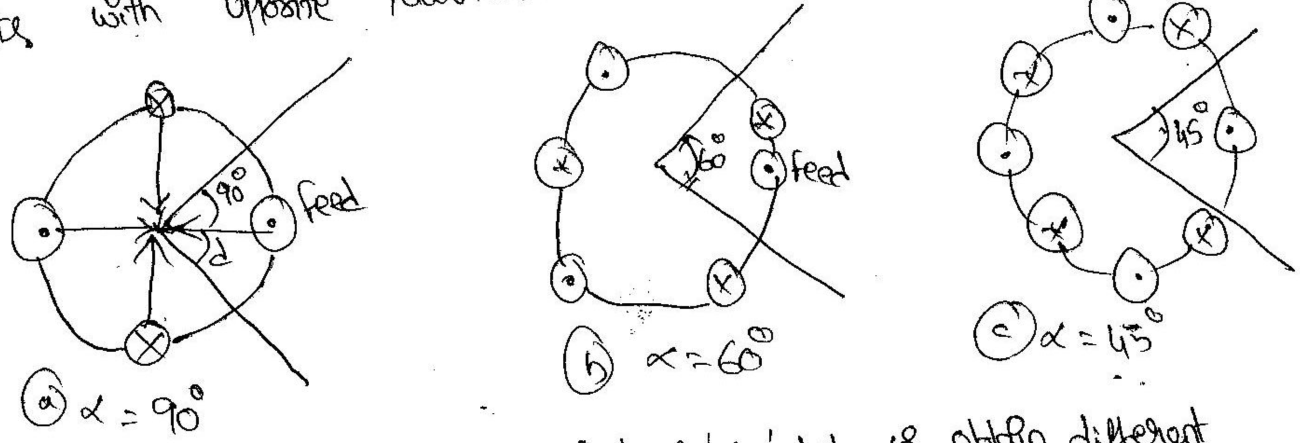


vertical Corner Reflector with field Pattern along the axis

(b) Passive Corner reflector with out driven Element

The analysis for the radiated field of the source with the corner reflectors is found to be useful with included angle $\alpha = \frac{\pi}{N}$ where N is integer.

By applying the image Principle we can show that field Produced b/w the two reflector Planes is same as they would ~~field Produced~~ ~~b/w the~~ result from an array of 2N current Elements spaced Equidistant on a circular Path with radius 'S' with adjacent elements with opposite Polarities.



(a) $\alpha = 90^\circ$ (b) $\alpha = 60^\circ$ (c) $\alpha = 45^\circ$

The corner angle α depends on integer 'N'. Let us obtain different Conditions of the corner angle α with different value of N.

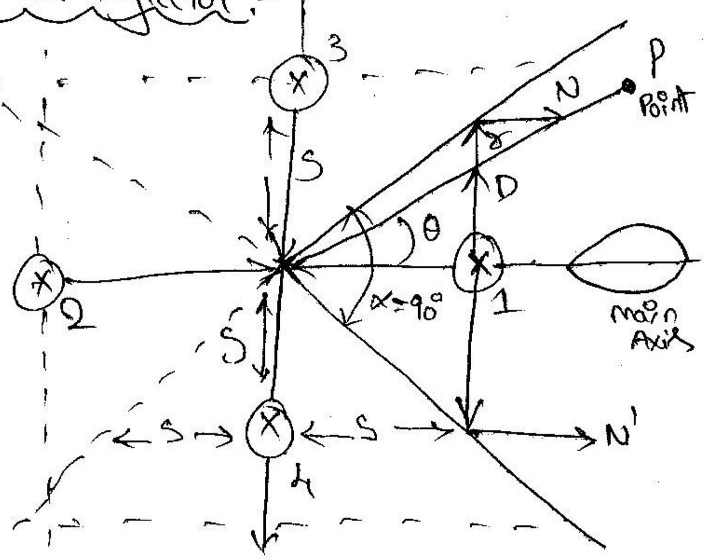
(22) if $N = 1$ $\quad \alpha = \frac{\pi}{1}$ rad $= \pi$ rad (or) 180° → flat sheet (or) Plane reflector

if $N = 2$ $\quad \alpha = \frac{\pi}{2}$ rad $= 90^\circ$ → Square Corner reflector

if $N = 3$ $\quad \alpha = \frac{\pi}{3}$ rad $= 60^\circ$ → Corner reflector with Corner angle $\alpha = 60^\circ$

if $N = 4$ $\quad \alpha = \frac{\pi}{4}$ rad $= 45^\circ$ → " " " Corner angle $\alpha = 45^\circ$.

Method of Images for Square Corner Reflector:-



Consider a Square Corner reflector with Corner angle $\alpha = 90^\circ$. This reflector Consists one driven Element represent by 'D' which is located at Point (1).

Corresponding to one driven Element 3-images represented by (2), (3) & (4)

⊙ → Real (or) Driven Element

(X) → Imaginary Element

The driven Element along with all three images Carry Current of same magnitude.

But Currents in driven antenna and its images (2) are in Phase while the Currents in image Elements (3) & (4) are out of Phase by 180°.

The Electric field in horizontal Plane at distance 'r' from antenna is given by

$$E_\phi(\theta) = k' I_1 \left[\cos(\alpha S \cos\theta) - \cos(\alpha S \sin\theta)\right] \rightarrow (1)$$

I_1 → Current in Each Element

k' → Constant incorporating distance 'r'.

S → distance of driven Element from corner along main-axis

The terminal voltage at centre of driven element is

$$V_1 = I_1 Z_{11} + I_1 Z_{12} - I_1 Z_{13} - I_1 Z_{14}$$

Z_{11} → self impedance of driven element (which is $\lambda/2$ dipole) $= 73\Omega$

Z_{12} → Mutual impedance b/w ① & ②

$Z_{13} = Z_{14}$ = mutual impedance b/w elements ① & ③ and elements ① & ④

$$V_1 = (Z_{11} + Z_{12} - 2Z_{14}) I_1 \rightarrow ②$$

$$\frac{V_1}{I_1} = Z_{11} + Z_{12} - 2Z_{14} \Rightarrow R \quad (\because \frac{V}{I} = R)$$

Power supplied to driven element and images then

$$P = I_1^2 R \longrightarrow ③$$

$$I_1 = \sqrt{\frac{P}{R}}$$

$$\therefore I_1 = \sqrt{\frac{P}{(R_{11} + R_{12} - 2R_{14})}} \rightarrow ④$$

Substitute I_1 in Eq ①

$$E_\phi(\theta) = K' \sqrt{\frac{P}{(R_{11} + R_{12} - 2R_{14})}} \left[\cos(\alpha S\cos\theta) - \cos(\alpha S\sin\theta)\right] \rightarrow ⑤$$

let the reflector is removed so we dont get images ②, ③, ④ and no reflection takes place

$$R_{12} = R_{13} = R_{14} = 0$$

with out images $E_\phi(\theta) = K' \sqrt{\frac{P}{R_{11}}} \rightarrow ⑥$

$\therefore$ Gain is divided by ⑤/⑥

$$\text{Gain} = \frac{\cancel{K'} \sqrt{\frac{P}{R_{11} + R_{12} - R_{13} - R_{14}}} \left[\cos(\alpha S\cos\theta) - (\cos(\alpha S\sin\theta))\right]}{\cancel{K'} \sqrt{\frac{P}{R_{11}}}}$$

$$\text{Gain} = \sqrt{\frac{R_{11}}{R_{11}+R_{12}-2R_{1M}}}\left[\cos(\alpha d \cos\theta) - \cos(\alpha d \sin\theta)\right]$$

Design Equations for Corner Reflector :-

The dimensions in corner reflector antenna

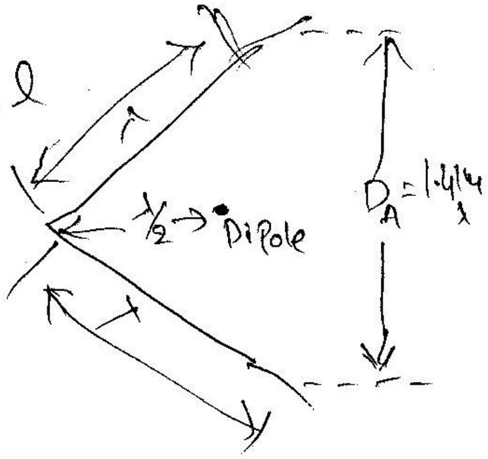
D_A = Dimension of aperture ; S = Distance b/w feed & vertex

l = Side length of reflector sheet

The side length 'l' is selected equal to twice the distance 'S'.

$$S = \frac{l}{2} \quad (\text{or}) \quad \boxed{l = 2S}$$

$$D_A = \sqrt{l^2 + l^2} = \sqrt{2l^2} = \sqrt{2}\, l = 1.414\, l$$



But $l = 2S$

$$D_A = 1.414(2S) = 2.828\,S$$

$$\boxed{D_A = 1.414\, l = 2.828\, S}$$

*) Principle of Parabolic Reflector :-

To improve the overall radiation characteristics of reflector antenna, the parabolic structure is oftenly used. Basically a parabola is a locus of a point which moves in such a way that the distance of point from fixed point ~~which~~ called focus plus the distance from the straight line called directrix.