*) Effective aperture and directivity of short dipole :-

Maximum A_e of Antenna is $= \dfrac{V^2}{4 S R_r}$

V = induced voltage S = Poynting vector

$V = EL$ $S = EH$

Radiation Resistance of short dipole is

$$R_r = 80\pi^2 \left(\frac{L}{\lambda}\right)^2 \text{ ohms}$$

$$A_e = \frac{(EL)^2}{4(EH)\, 80\pi^2 \left(\frac{L}{\lambda}\right)^2}$$

but we know $\dfrac{E}{H} = \eta \ (\approx 377\,\Omega)$

$$\Rightarrow H = \frac{E}{\eta}$$

$$A_e = \frac{E^2 L^2}{4\,\frac{E^2}{\eta} \cdot 80\pi^2 \frac{L^2}{\lambda^2}}$$

$$A_e = \frac{\lambda^2 \times 377}{4 \times 80\pi^2} \quad [\because \eta = 377\,\Omega \text{ in free space}]$$

$$A_e = 0.119\lambda^2$$

$\therefore$ Directivity $D = \dfrac{4\pi}{\lambda^2} A_e \Rightarrow \dfrac{4\pi}{\lambda^2} \times 0.119\lambda^2$

$$\boxed{D \Rightarrow 1.5}$$ //.

2.3 HALF-WAVE DIPOLE

Half wavelength dipole or simply half wave dipole ($\lambda/2$ antenna) is one of the simplest antenna and is frequently employed as an element of a more complex directional system. Example : antenna arrays. A $\lambda/2$ antenna is the fundamental radio antenna of metal rod or tubing or thin wire which has a physical length of half wavelength in free space at the frequency of operation. Otherwise called as half wave doublet.

A dipole antenna may be defined as a symmetrical antenna in which the two ends are at equal potential relative to mid point. Now we will calculate the radiation fields of half wave dipole as shown in figure 2.1.

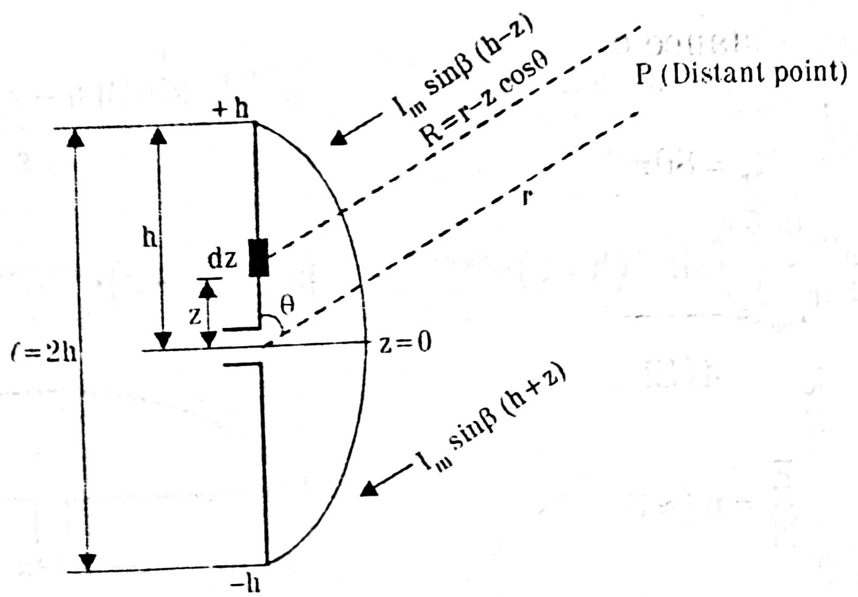


Figure 2.1: Half wave dipole antenna

The dipole is usually fed at the centre having maximum current at the centre i.e., maximum radiation in the plane normal to the axis. The overall specified length is $\ell = 2h$ and vertical antenna of height $h = \frac{\ell}{2}$. The fields originate due to a current element Idz. Since current is assumed sinusoidal asymtotically distributed,

$$I = I_m \sin\beta(h-z) \text{ for } z > 0$$

$$I = I_m \sin\beta(h+z) \text{ for } z < 0 \qquad \text{....(1)}$$

Vector potential at a distant point P due to current element Idz is given by,

$$dA_z = \frac{\mu I dz e^{-j\beta R}}{4\pi R} \qquad \text{... (2)}$$

R = distance between Idz to distant point P.

The total vector potential due to all such current elements at distant point P is given by

$$\int dA_z = \int_{-h}^{0} \frac{\mu I dz e^{-j\beta R}}{4\pi R} + \int_{0}^{h} \frac{\mu I dz e^{-j\beta R}}{4\pi R}$$

$$A_z = \frac{\mu}{4\pi} \int_{-h}^{0} \frac{I_m \sin\beta(h+z) e^{-j\beta R}}{R} dz + \frac{\mu}{4\pi} \int_{0}^{h} \frac{I_m \sin\beta(h-z) e^{-j\beta R}}{R} dz \qquad \text{... (3)}$$

Since the distant point P is at a large distance where the fields are needed so the lines to the point P are assumed to be parallel.

$$R = r - z\cos\theta$$

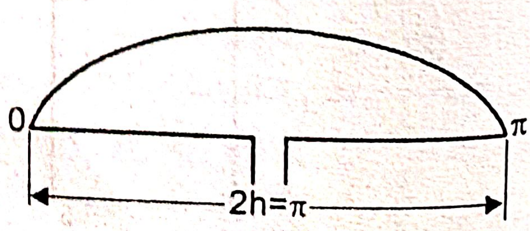
$$R \approx r$$

$$A_z = \frac{\mu}{4\pi} \int_{-h}^{0} \frac{I_m \sin\beta(h+z) e^{-j\beta(r - z\cos\theta)}}{r} dz + \frac{\mu}{4\pi} \int_{0}^{h} \frac{I_m \sin\beta(h-z) e^{-j\beta(r-z\cos\theta)}}{r} dz$$

$$A_z = \frac{\mu I_m e^{-j\beta r}}{4\pi r} \left[\int_{-h}^{0} \sin\beta(h+z) e^{j\beta z\cos\theta} dz + \int_{0}^{h} \sin\beta(h-z) e^{j\beta z\cos\theta} dz \right] \qquad \text{... (4)}$$

$$\ell = 2h = \frac{\lambda}{2}$$

$$\therefore \quad h = \frac{\lambda}{4} = \frac{\pi}{2}$$



[∵ the current distribution is sinusoidal, $2h = \pi$ and $h = \pi/2$]

By changing the limits of first integral and substitute

$\sin\beta(h+z) = \sin\beta\,(\pi/2+z) = \cos\beta z \quad [\because h = \pi/2]$

$\sin\beta(h-z) = \sin\beta(\pi/2-z) = \cos\beta z$

$$A_z = \frac{\mu I_m e^{-j\beta r}}{4\pi r}\left[\int_0^h \cos\beta z e^{-j\beta z\cos\theta}dz + \int_0^h \cos\beta z e^{j\beta z\cos\theta}dz\right]$$

$$= \frac{\mu I_m e^{-j\beta r}}{4\pi r}\int_0^h \cos\beta z\left[e^{j\beta z\cos\theta} + e^{-j\beta z\cos\theta}\right]dz \quad \ldots (5)$$

$$A_z = \frac{\mu I_m e^{-j\beta r}}{4\pi r}\int_0^h \cos\beta z\, 2\cos(\beta z\cos\theta)\, dz$$

But, $2\cos\alpha\cos\beta = \cos(\alpha-\beta) + \cos(\alpha+\beta)$

$$= \frac{\mu I_m e^{-j\beta r}}{4\pi r}\int_0^{\lambda/4}\{\cos(\beta z(1+\cos\theta)) + \cos(\beta z(1-\cos\theta))\}dz \quad [\because h = \lambda/4]$$

$$A_z = \frac{\mu I_m e^{-j\beta r}}{4\pi r}\left[\frac{\sin(\beta z(1+\cos\theta))}{\beta(1+\cos\theta)} + \frac{\sin(\beta z(1-\cos\theta))}{\beta(1-\cos\theta)}\right]_0^{\frac{\lambda}{4}} \quad \ldots (6)$$

$$A_z = \frac{\mu I_m e^{-j\beta r}}{4\pi\beta r}\left[\frac{(1-\cos\theta)\sin\left(\frac{\pi}{2}+\frac{\pi}{2}\cos\theta\right) + (1+\cos\theta)\sin\left(\frac{\pi}{2}-\frac{\pi}{2}\cos\theta\right)}{(1+\cos\theta)(1-\cos\theta)}\right]$$

$$\left[\because \beta\frac{\lambda}{4} = \frac{2\pi}{\lambda}\cdot\frac{\lambda}{4} = \frac{\pi}{2}\right]$$

$$= \frac{\mu I_m e^{-j\beta r}}{4\pi\beta r}\left[\frac{(1-\cos\theta)\cos\left(\frac{\pi}{2}\cos\theta\right) + (1+\cos\theta)\cos\left(\frac{\pi}{2}\cos\theta\right)}{1^2 - \cos^2\theta}\right] \quad \ldots (7)$$

$$= \frac{\mu I_m e^{-j\beta r}}{4\pi\beta r}\cdot\frac{2\cos\left(\frac{\pi}{2}\cos\theta\right)}{\sin^2\theta}$$

$$\boxed{A_z = \frac{\mu I_m e^{-j\beta r}}{2\pi\beta r}\left[\frac{\cos\left(\frac{\pi}{2}\cos\theta\right)}{\sin^2\theta}\right]} \quad \ldots (8)$$

But for a current element along z-axis, from Maxwell's equation,

$$\nabla \times A = \mu H \qquad [\because B = \nabla \times A,\ B = \mu H]$$

If only radiation field is considered, (i.e., variation with respect to r is considered)

$$\mu H_\phi = (\nabla \times A)_\phi = \frac{1}{r}\frac{\partial}{\partial r}(A_0 \cdot r) \qquad \nabla \times A = \begin{vmatrix} a_r & ra_\theta & r\sin\theta a_\phi \\ \frac{\partial}{\partial r} & \frac{\partial}{\partial \theta} & \frac{\partial}{\partial \phi} \\ A_r & rA_\theta & A_\phi r\sin\theta \end{vmatrix} \times \frac{1}{r^2\sin\theta}$$

$$\mu H_\phi = \frac{1}{r}\frac{\partial}{\partial r}\left[(-A_z\sin\theta \cdot r)\right] \quad [\because A_\theta = -A_z\sin\theta]$$

$$= -\frac{1}{r}\sin\theta\frac{\partial(rA_z)}{\partial r} \quad \ldots (9)$$

$$\mu H_\phi = \left(\frac{-\sin\theta}{r}\right)\frac{\mu I_m r e^{-j\beta r}(-j\beta)}{2\pi\beta r}\frac{\cos\left(\frac{\pi}{2}\cos\theta\right)}{\sin^2\theta}$$

$$H_\phi = \frac{jI_m e^{-j\beta r}}{2\pi r}\frac{\cos\left(\frac{\pi}{2}\cos\theta\right)}{\sin\theta} \quad \ldots (10)$$

$$|H_\phi| = \frac{I_m}{2\pi r}\left\{\frac{\cos\left(\frac{\pi}{2}\cos\theta\right)}{\sin\theta}\right\} A/m \quad \ldots (11)$$

This is the required magnetic field intensity expression for a half wave dipole. The electric field expression for the radiation field can be achieved from,

$$\eta = \frac{E_0}{H_\phi} = 120\pi$$

$$|E_\theta| = 120\pi |H_\phi|$$

$$= 120\pi \frac{I_m}{2\pi r}\left\{\frac{\cos\left(\frac{\pi}{2}\cos\theta\right)}{\sin\theta}\right\}$$

$$|E_\theta| = \frac{60 I_m}{r}\left\{\frac{\cos\left(\frac{\pi}{2}\cos\theta\right)}{\sin\theta}\right\}\frac{V}{m} \quad \text{... (12)}$$

The maximum value of Poynting vector is,

$$P_{max} = |E_\theta||H_\phi|$$

$$P_{av} = \frac{E_\theta}{\sqrt{2}}\cdot\frac{H_\phi}{\sqrt{2}} = \frac{1}{2}E_\theta H_\phi = \frac{P_{max}}{2}$$

$$P_{av} = \frac{1}{2}\cdot\frac{60 I_m}{r}\cdot\frac{I_m}{2\pi r}\left\{\frac{\cos\left(\frac{\pi}{2}\cos\theta\right)}{\sin\theta}\right\}^2$$

$$P_{av} = \frac{15 I_m^2}{\pi r^2}\left\{\frac{\cos\left(\frac{\pi}{2}\cos\theta\right)}{\sin\theta}\right\}^2 \frac{W}{m^2} \quad \text{... (13)}$$

2.3.1 Effective aperture and directivity of half wave dipole

The induced current I has sinusoidal distribution and at any point x from the origin, the current I is given by,

$$I = I_m \cos\frac{2\pi x}{\lambda} \quad \text{... (1)}$$

where, I_m is the maximum current and x lies between 0 and $\pm\frac{\lambda}{4}$

For an infinitesimal small current, equation (1) can be written as,

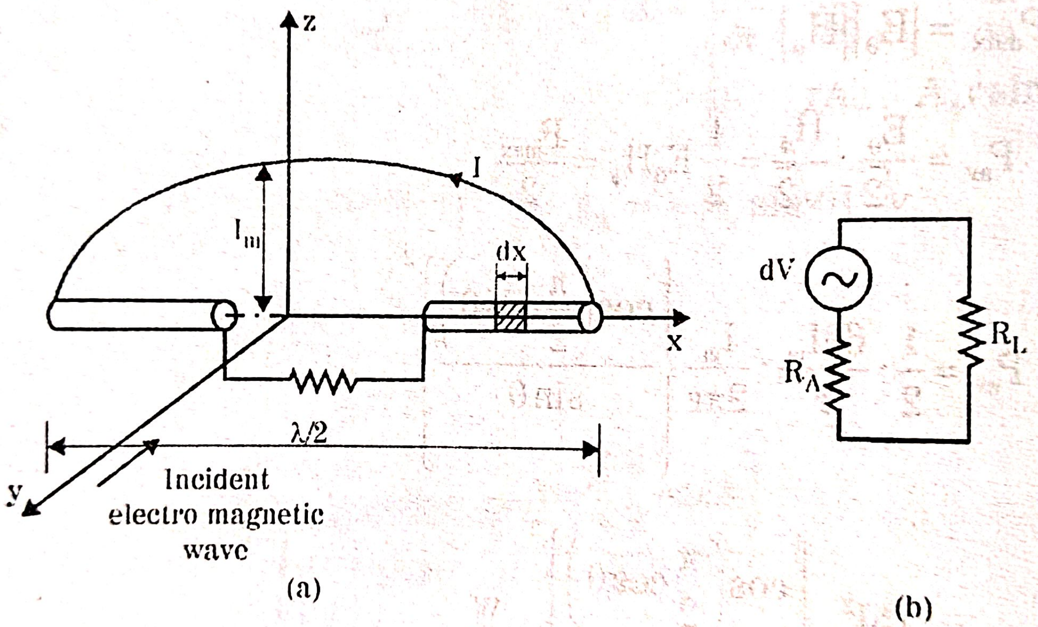
$$dI = dI_m \cos\frac{2\pi x}{\lambda} \quad \text{... (2)}$$

The entire antenna system shown in figure 2.2(a) is replaced by figure 2.2(b) and in terms of infinitesimal small voltage,

$$dv = dv_m \cos\frac{2\pi x}{\lambda} \quad \text{... (3)}$$

Thus, the infinitesimal small voltage dv induced in infinitesimal small element dx, due to incident wave of electric field intensity E is given by,

$$dv = E\,dx \cos\frac{2\pi x}{\lambda} \qquad \because \frac{dv_m}{dx} = E$$



a) λ/2 antenna terminated with load b) equivalent thevenins generator

Figure 2.2

Total induced voltage v is obtained by integrating above equation over the entire length of λ/2.

$$\int dv = \int_0^{\lambda/2} E \cos\frac{2\pi x}{\lambda}\,dx$$

$$v = 2\int_0^{\lambda/4} E \cos\frac{2\pi x}{\lambda}\,dx \quad \text{... (4)}$$

$$= 2E\left[\frac{\sin\frac{2\pi x}{\lambda}}{2\frac{\pi}{\lambda}}\right]_0^{\frac{\lambda}{4}}$$

$$= \frac{2 \cdot E \cdot \lambda}{2\pi}\left[\sin\frac{2\pi}{\lambda}\cdot\frac{\lambda}{4} - \sin 0\right]$$

$$= \frac{E\lambda}{\pi}[1-0]$$

$$V = \frac{E\lambda}{\pi} \qquad \text{... (5)}$$

The power density or Poynting's vector is given by

$$S = EH = \frac{E^2}{\eta} = \frac{E^2}{377} \qquad \text{... (6)}$$

The radiation resistance of $\lambda/2$ antenna is 73Ω

$$A_{em} = \frac{V^2}{4SR_r} \qquad \text{... (7)}$$

$$= \frac{E^2\lambda^2}{\pi^2 \cdot 4 \cdot \frac{E^2}{377} \times 73}$$

$$A_{em} = 0.13\,\lambda^2 \qquad \text{.... (8)}$$

Thus in a half wave dipole antenna, power absorbed from incident wave is over an area of $0.13\lambda^2$ which is passed on to the load resistance

Directivity, $$D = \frac{4\pi}{\lambda^2}\cdot A_{em} = \frac{4\pi}{\lambda^2}\cdot 0.13\lambda^2$$

$$\boxed{D = 1.63} \qquad \text{.... (9)}$$

2.3.2 Power radiated and radiation resistance of half wave dipole

The total power radiated from a $\lambda/2$ antenna is given by the surface integral of the Poynting vector over any surrounding surface, sphere.

$$W = \oint P_{av} ds = \int_0^{\pi} \frac{15 I_m^2}{\pi r^2} \left\{ \frac{\cos^2 \frac{\pi}{2} \cos\theta}{\sin^2\theta} \right\} \cdot ds \text{ (from equation 2.3.13)}$$

$$I_{rms} = \frac{I_m}{\sqrt{2}} \qquad \therefore I_m = \sqrt{2}\, I_{rms}; \qquad ds = 2\pi r^2 \sin\theta d\theta$$

$$W = \int_0^{\pi} \frac{15\left(\sqrt{2} I_{rms}\right)^2}{\pi r^2} \frac{\cos^2 \frac{\pi}{2} \cos\theta}{\sin^2\theta} \cdot 2\pi r^2 \sin\theta d\theta$$

$$= 60\, I_{rms}^2 \int_0^{\pi} \frac{1}{2} \left\{ \frac{1 + \cos(\pi\cos\theta)}{\sin\theta} \right\} d\theta \qquad \because \cos 2\theta = 2\cos^2\theta - 1 \quad \cos^2\theta = \frac{1+\cos 2\theta}{2}$$

The total power radiated,

$$W = 60\, I_{rms}^2 \cdot I$$

where, $$I = \frac{1}{2} \int_0^{\pi} \left\{ \frac{1 + \cos(\pi\cos\theta)}{\sin\theta} \right\} d\theta$$

This is an integral of special type and is quite involved. It can be evaluated either analytical i.e. numerical integration or graphical method or by trapezoidal rules.

The value of integral is,

$I = 1.219$ $\qquad W = 60\, I^2_{rms} \cdot 1.219$ $\qquad W = 73.14\, I^2_{rms}$

$R_r = W/I^2_{rms}$ $\qquad R_r = 73.14\Omega$ $\qquad R_r = 73$ Ohms.

For quarter wave monopole antenna, the radiation resistance is half of the dipoles radiation resistance, i.e., 73.14/2 or 36.57Ω.

The only difference between a λ/2 antenna (Half wave dipole antenna) and a λ/4 antenna (Marconi antenna or quarter wave monopole antenna) is that the dipole radiates power more or less in all directions whereas monopole radiates power in a hemisphere surface and that is why its radiation resistance is half of the dipole.

2.4 MONOPOLE

Basically, the quarter wave monopole antenna consists of one half of a half wave dipole antenna located on a conducting ground plane as in fig 2.3. The monopole antenna is perpendicular to the plane which is usually assumed to be infinite and perfectly conducting. It is fed by a coaxial cable connected to its base.

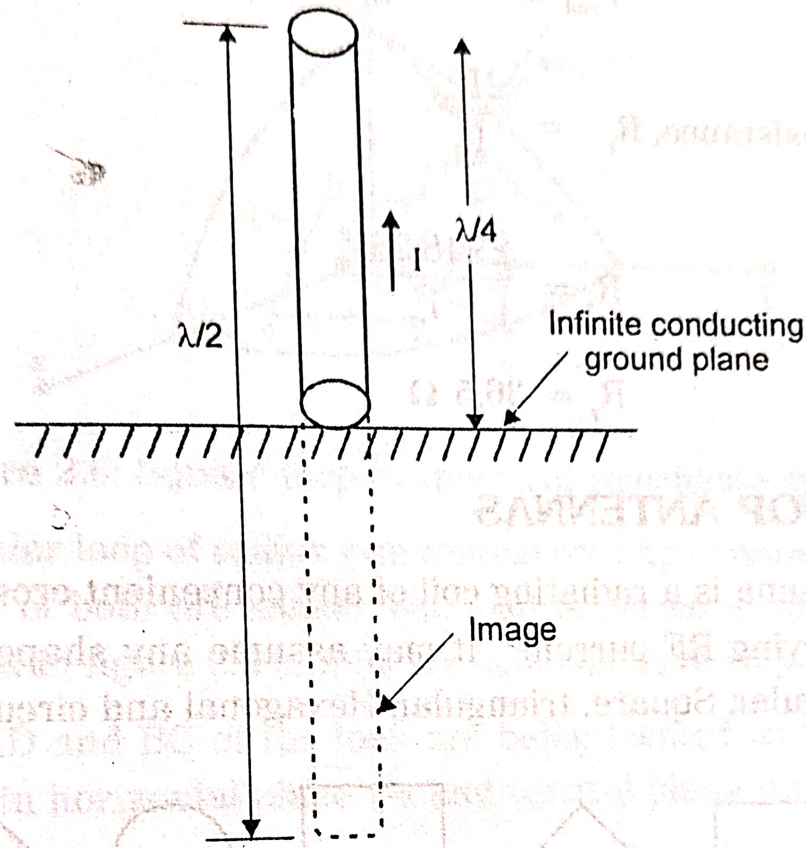


Figure 2.3: The quarter wave monopole antenna

Using the image theory, the infinite, perfectly conducting ground plane is replaced with the image of the monopole. The field produced in the region above the ground plane due to the λ/4 monopole with its image is the same as the field due to λ/2 dipole. Thus equations 2.3.10 and 2.3.12 hold for the λ/4 monopole,

The radiated field components of quarter wave monopole are,

$$H_\phi = \frac{jI_m e^{-j\beta r}\cos(\pi/2\cos\theta)}{2\pi r\sin\theta}$$

$$E_\theta = \eta H_\phi = \frac{j60 I_m e^{-j\beta r}\cos\left(\pi/_2 \cos\theta\right)}{r\sin\theta}$$

However, the integration in equation is only over the hemispherical surface above the ground plane (i.e., $0 \le \theta \le \pi/2$) because the monopole radiates only through that surface. Hence, the monopole radiates only half as much power as the dipole with the same current. Thus for a $\lambda/4$ monopole,

$$P_{rad} \approx 18.28\, I_m^2$$

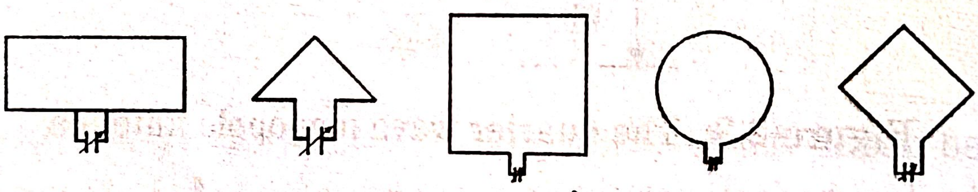
$$\text{Radiation resistance, } R_r = \frac{2P_{rad}}{I_m^2}$$

$$R_r = \frac{2 \times 18.28\, I_m^2}{I_m^2}$$

$$R_r = 36.5\ \Omega$$

2.5 SMALL LOOP ANTENNAS

The loop antenna is a radiating coil of any convenient cross - section of one or more turns carrying RF current. It may assume any shape as in figure 2.4. (Example : Rectangular, Square, triangular, Hexagonal and circular).



(a) Rectangular (b) Triangular (c) Square (d) Circular (e) Square

Figure 2.4: Loop antennas of different shapes

A loop antenna of more than one turn is called as frame. It is used in radio receiver, aircraft receiver, direction finding and UHF transmitters. Currents are of the same magnitude and phase throughout the loop if dimensions are small in comparison to wave length ($a << \lambda$).

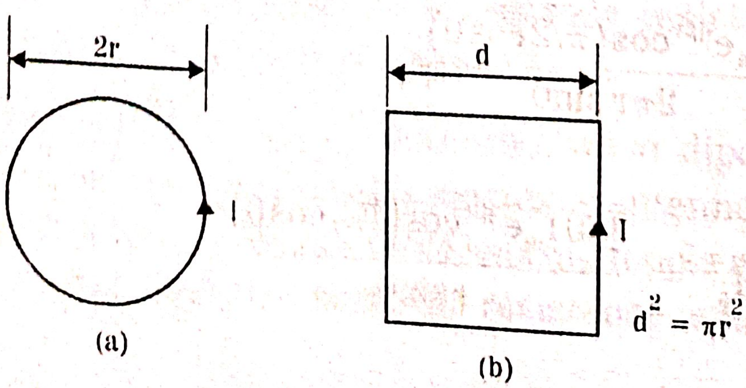


Figure 2.5: Circular loop and square loop of equal area

The radiation efficiency of closed loop antenna is low for transmission purposes.

2.5.1 Radiated fields of small loop antenna

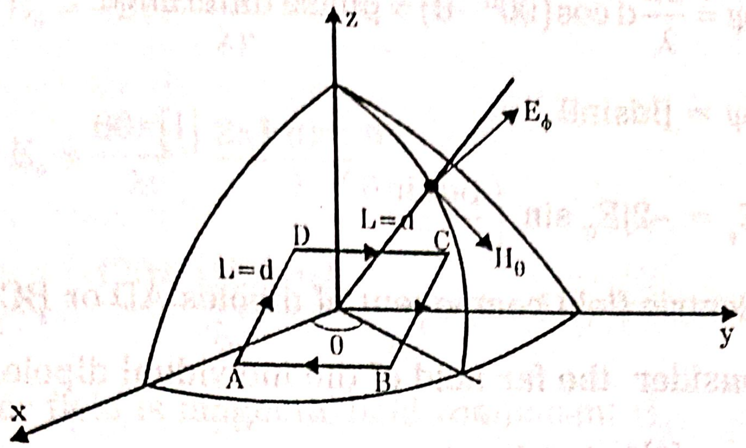


Figure 2.6: Square loop to spherical coordinate system

Let the circular loop of radius r be represented by square loop of side length d such that areas of both are same. The loop is placed at the centre of the co-ordinate system as in figure 2.6 and its far field will have only E_ϕ component.

The sides AD and BC of the loop are being treated as short dipoles, their radiation pattern in horizontal plane x-z and vertical plane y-z in figure 2.7.

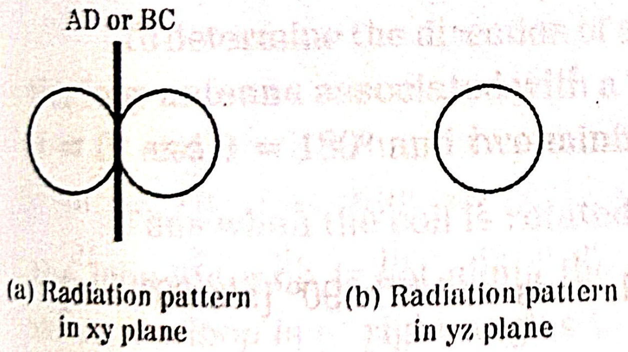


Figure 2.7

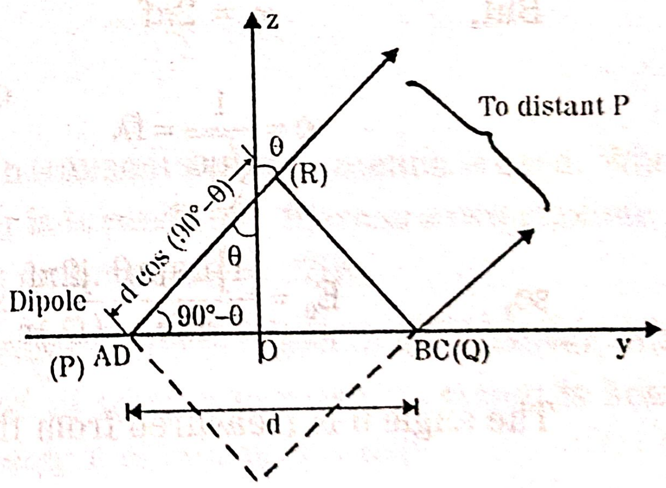


Figure 2.8: Dipole AD and BC in YZ plane

Both the dipoles radiating uniformly in all directions. Individuals dipoles AD and BC will behave like two isotropic point sources in yz plane as in figure 2.8.

E_ϕ = field component due to AD+ field component due to BC

$E_\phi = -E_0\, e^{j\psi/2} + E_0\, e^{-j\psi/2}$ from triangle, $\angle RPQ$ $\cos(90-\theta) = \frac{PR}{PQ} = \frac{PR}{d}$

$$= -E_0\,(e^{j\psi/2} - e^{-j\psi/2})$$

$$E_\phi = -2jE_0 \sin\psi/2.$$

$PR = d\cos(90-\theta)$

$PR = d\sin\theta$

where, $\psi = \frac{2\pi}{\lambda} d\cos(90^\circ - \theta) =$ phase difference

$$\psi = \beta d\sin\theta$$

$$E_\phi = -2jE_0 \sin\left(\frac{\beta d\sin\theta}{2}\right)$$

where, E_0 = Electric field component of dipoles AD or BC.

To find E_0, consider the far field of the individual dipole.

$$E_0 = \frac{[I]L\sin\theta}{4\pi\varepsilon_0}\left(\frac{j\omega}{c^2 r}\right) \quad \text{[From section 1.5 case (i)]}$$

Short dipole was oriented in z direction, where as the dipoles AD and BC are oriented along y direction.

$$E_0 = \frac{[I]L\sin\theta}{4\pi\,\varepsilon_0}\,\frac{j\omega}{c^2 r}$$

But, $\omega = 2\pi f$

$$c = \frac{1}{\sqrt{\mu\varepsilon}} = f\lambda$$

so, $$E_0 = \frac{[I]L\sin\theta}{4\pi\varepsilon}\,\frac{j2\pi f}{(f\lambda)r} \times \sqrt{\mu\varepsilon}$$

The angle θ is measured from the dipole axis and it is 90° [$\sin 90^\circ = 1$]

$$E_0 = \frac{j[I]L\sin 90^\circ}{2\lambda r}\sqrt{\frac{\mu}{\varepsilon}}$$

In freespace

$$\left[\because \sqrt{\frac{\mu_0}{\varepsilon_0}} = 120\pi\right]$$

$$E_0 = \frac{j[I]L120}{2\lambda r}\pi$$

$$\mu_0 = 4\pi \times 10^{-7}\ \text{H/m}$$

$$\varepsilon_0 = 1/36\pi \times 10^{-9}\ \text{F/m}$$

$$E_0 = \frac{j60\pi[I]L}{\lambda r}$$

$$E_\phi = -2j \frac{j60\pi[I]L}{\lambda r} \frac{\beta d \sin\theta}{2}$$

$$E_\phi = \frac{60\pi[I]}{\lambda r} \frac{2\pi L d \sin\theta}{\lambda}$$

$$E_\phi = \frac{120\pi^2 [I] \sin\theta A}{r\lambda^2} \quad [\because A = Ld, \text{ area of the loop}]$$

The other far field is magnetic field component H_θ

$$\eta = \frac{E_\phi}{H_\theta}; \qquad H_\theta = \frac{E_\phi}{120\pi}$$

$$H_\theta = \frac{120\pi^2 [I] \sin\theta \cdot A}{r\lambda^2 \cdot 120\pi}$$

$$H_\theta = \frac{\pi[I]\sin\theta\, A}{r\lambda^2} \ A/m$$