ANTENNA WAVE PROPAGATION

ELECTRONICS & COMMUNICATION ENGINEERING

HAND NOTES

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ANANTAPURAMU

Antenna Wave Propagation :-

unit - I

Antenna Basics :-

*) Introduction :-

i) Communication :- The Process of Conveying information (or) intelligence (or) message from one Place to another is known as Communication

↳ Radio Communication :- When Electromagnetic waves (or) Radio waves used for the Communication Purpose is so Called Radio Commu.

History :-

* → Meaning for "Antenna" in Zoology is "Feeler" i.e a Part of some insects which feel (or) organ of touch.

* → Plural of "Antenna" is "Antennae"

* → Where as Antenna (or) Aerial (or) Radiator in radio sense is a device for receiving (or) sending radio waves and plural of Radio Antenna is "Antennas".

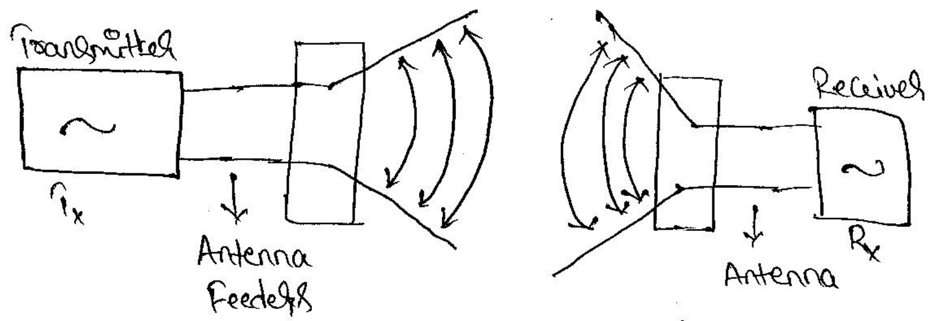
Defination of Antenna :-

An antenna is defined as a "metallic device (as rod (or) wire) for radiating (or) receiving radio waves (Electromagnetic waves).

(or)

An Antenna (or) Aerial is a system of Evaluated Conductors which Couples (or) matches the transmitter (or) receiver to free space.

A transmitting antenna Connected to a transmitter by a transmission line forces Electromagnetic (Radio) waves in to free space which travel in space with velocity of light.



(i) Transmission line :- It is used to carry the Electromagnetic waves

(ii) Antenna Feeders :- They are used to connect transmitter (or) Receiver with the antenna.

Note:- *) → The source of Electromagnetic fields is the charged particles we get the strength of depends on the availability of charged particles electric and magnetic fields.

*) Maxwell's Equations :-
Which gives existence of Electric & magnetic fields.

Two types of them are :-
(i) Point form (or) Differential form
(ii) Integral form.

These maxwell Eqs are based on basic laws Gauss law, Faradays law, Amperes circuits law.

(i) Point form (or) Differential form :-

a) Gauss law :- It is defined as the net flux density comming out of closed surface is equal to charge enclosed by that surface.

from gauss law:- $\nabla \cdot D = \rho_V$

$\nabla \cdot B = 0$

where D = Electric flux density C/m^2

$$E = \frac{F}{Q}$$

$$F = \frac{Q_1 Q_2}{4\pi \epsilon r^2} \quad ; \quad Q_1 = Q_2$$

$$F = \frac{Q_1^2}{4\pi \epsilon r^2}$$

$$\Rightarrow E = \frac{Q^{\cancel{2}}}{4\pi \epsilon r^2 \cancel{Q}} = \frac{Q}{4\pi \epsilon r^2}$$

$\because D = \epsilon E$

$$D = \frac{Q}{4\pi r^2 \cancel{\epsilon}} \cancel{\epsilon} \quad C/m^2$$

ρ_V - charge density

B - magnetic flux density N/m^2

$B = \mu H$

μ = Permeability of medium

$\mu = \mu_0 \mu_r$

$\mu_0 = 4\pi \times 10^{-7}$ H/m^2

$\mu_r = 1$ (air)

$\epsilon_0 = 8.54 \times 10^{-12}$ F/m^2

b) <u>faraday's law</u>:- It gives relation between Electric field intensity and magnetic field intensity.

$$\nabla \times E = \frac{-\partial B}{\partial t}$$

$$\nabla \times E = -\mu \cdot \frac{\partial H}{\partial t}$$

c) <u>Ampere's law</u>:- It gives relation b/w the current density (J) Electric to the Electric field intensity

$$\nabla \times H = J + \frac{\partial D}{\partial t}$$

$$= J + \epsilon \frac{\partial E}{\partial t}$$

$\frac{\partial D}{\partial t}$ = displacement current.

J = Conduction current density

$J = I/A$ A/m^2

(ii) Integral form of maxwell Eq's :-

Divergence theorem gives the relation b/w surface integral & volume integral from this theorem.

$$\iint_S D.dS = \iiint_V \rho_v . dV$$

Stokes law gives relation b/w l & S

$$\iint_S B . dS = 0$$

Faraday's law :-

$$\oint_l E . dl = \iint_S -\frac{\partial B}{\partial t} . dS$$

$$\oint_l E . dl = -\mu \iint_S \frac{\partial H}{\partial t} . dS$$

Ampere's law :-

$$\oint_l H . dl = I_{enc}$$

$$\oint_l H . dl = \iint_S J + \frac{\partial D}{\partial t}$$

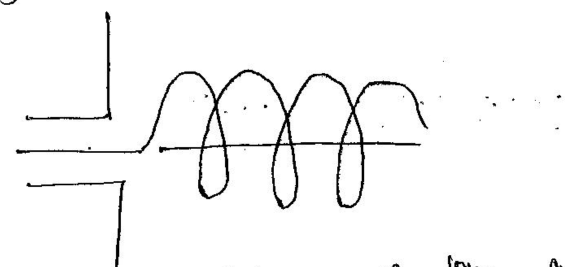
These maxwell Eq are used to find the Electric fields & magnetic fields from different Co-ordinate systems.

*) Types of Antenna's :-

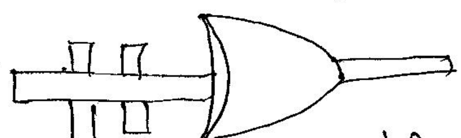
1) Isotropic Antenna
2) Helical Antenna
3) Dipole Antenna
4) Dish Antenna
5) Loop Antenna
6) Two wire Antenna
7) Rhombic Antenna

1) Isotropic Antenna :- means it radiates Energy to all the directions with Equal strength.
→ It is impractical antenna

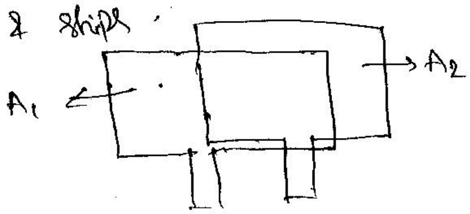
2) Helical Antenna :- The structure of antenna is like a helics & couples Energy from transmitter to free space & free space to receiver



3) Dipole Antenna :- These Antenna are like a bowl structure and is used in communication system i.e to transmission & reception of TV signals

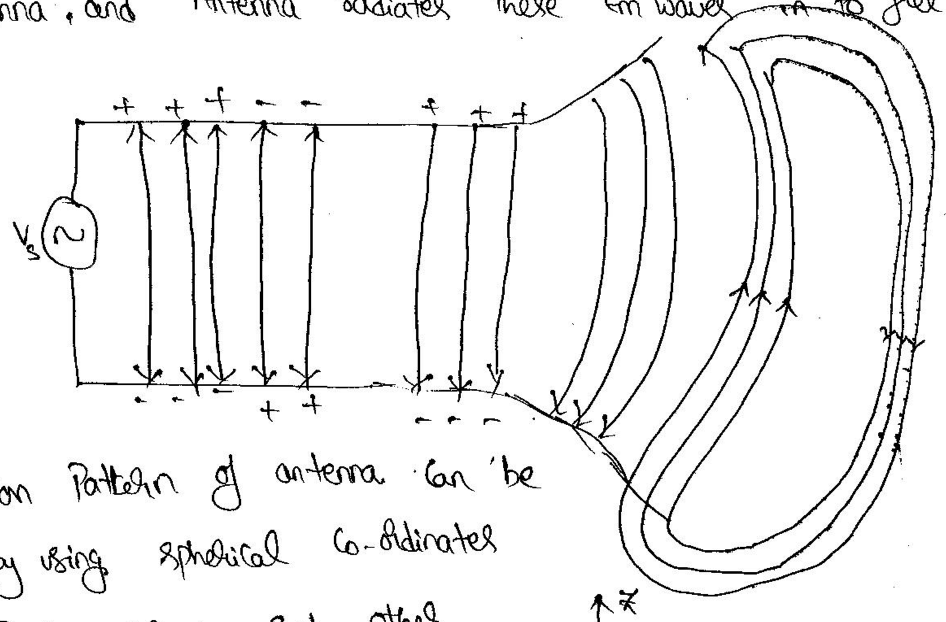


4) Loop Antenna :- structure is like a loop These Antenna are used in aerospace & ships.

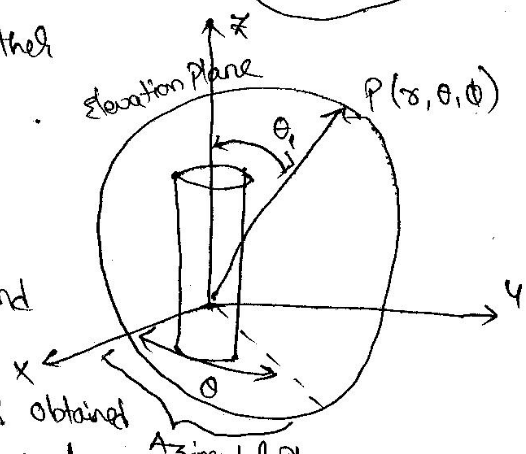


5) Two wire Antenna :-
When External voltage 'V_s' is applied to two wire transmission line as a result Electric field is developed b/w the conductors and the charge carriers move towards opposite charges these Electric field lines are target to the applied field.
* Since the charge carriers are moving as a ~~result~~ magnetic field is developed and which is tangent to generated magnetic field. these Electric and magnetic fields are perpendicular to each other and if the line is terminated with some impedance, the Electromagnetic

Waves are Standing waves will be generated and these are fed to an Antenna, and Antenna radiates these Em waves in to free space



The radiation Pattern of antenna can be measured by using spherical Co-ordinates



X, Y, Z are Perpendicular to Each other

'r' is the radial displacement and

θ, φ are angular displacement

*) where θ, is called Elevation angle it is the angle b/w radius vector and Z-axis

*) φ is called as azumutal angle it is obtained in Z=0 Plane by Extending radius vector from origin. Then the plane is called azmutal Plane.

~~ESD~~

2) Isotropic radiation :-

Isotropic radiator is an ideal antenna radiates energy with equal strength uniformly to all the directions.

* → In practical no antenna can radiate energy in all direction, so it is an impractical antenna.

* → The radiation pattern of isotropic radiator is find by spherical co-ordinate system.

↳ Let us consider an isotropic radiator which is placed in a spherical co-ordinate system with sphere of radius 'r'.

By pointing theorem we can find the amount of energy delivered by isotropic radiator.

Pointing theorem defined as the amount of energy delivered from unit area.

$$\bar{P} = E \times H \quad w/m^2$$

$$P = P_r$$

The total power is given by $P_t = \iint_S P.ds$

$$= \iint_S P_r . ds$$

$$P_t = P_r \iint_S ds$$

$$ds = A = 4\pi r^2$$

$$\therefore P_t = P_r 4\pi r^2$$

$$\boxed{P_r = \frac{P_t}{4\pi r^2} \quad w/m^2}$$

3) Dipole Antenna (or) Short Electric Dipoles :-

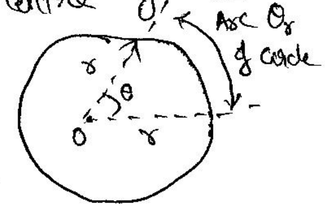
It is a short length antenna & takes less time to travel one pole to another pole by charge carrier. The physical length of dipole antenna is 'λ' and is divided in to 2-halfs (or) poles of each length 'λ/2'.

~~Basic Antenna Parameters:~~

The Electric Potential at two Ends of antenna Equals to midpoint Potential. The radiation resistance of dipole antenna is given by $R_r = 80\pi^2\left(\frac{dl}{\lambda}\right)^2$

where dl is differential length radiation resistance of half wave dipole is $R_r = 73\Omega$

4) **Radian**:- Radian is used to measure the plane angle and it is nothing but the total angle subtended at the centre of the circle with its vertex.

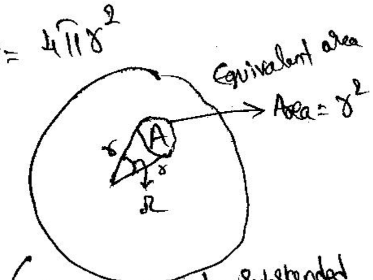


1 radian $= 57.3$ degrees.

The total circumference 'C' of circle with radius 'r' is $C = 2\pi r$

5) **Staradian** :- It is used to measure the solid angle it is the total angle subtended at the centre of the sphere. The solid angle is represented by

$d\Omega = 4\pi$ with surface area(A) $= 4\pi r^2$



$$1\,sr = 1\,rad^2 = \left(\frac{180}{\pi}\right)^2 \cdot (\text{degree})^2$$

$$= 3282.3 \text{ sq.deg}$$

(Ω = solid angle subtended by area A)

$$\boxed{1\,sr = \frac{\text{solid angle}}{4\pi}}$$

The infinitesimal area 'ds' on the surface of sphere with radius 'r' is given by $\boxed{ds = r^2 \sin\theta \cdot d\theta \cdot d\phi \ m^2}$

Hence the Element of solid angle $d\Omega$ of a sphere is

$$\boxed{d\Omega = \frac{ds}{r^2} = \sin\theta \cdot d\theta \cdot d\phi \ \text{steradian}}$$

6) Antenna Parameters (or) Antenna Properties :-

(i) Radiation Pattern
(ii) Radiation Intensity (or) Power radiation Pattern.
(iii) Directive gain & Directivity
(iv) Power gain
(v) Beam width, beam area & beam Efficiency
(vi) Antenna Apertures
(vii) Effective height
(viii) Front to back ratio
(ix) Antenna Efficiency.

(i) Radiation Pattern :- (or) Antenna Pattern is defined as a "mathematical function" & a graphical representation of the radiation properties of the antenna as a function of space Co-ordinates.

*) → Radiation Properties includes Power flux density, radiation intensity, field strength, directivity phase (or) Polarization..

*) → The Radiation Property of most Concern in 2-dimensional & 3-dimensional spatial distribution of radiated Energy.

*) → A trace of received Power at a Constant radius is called the Power Pattern. (r)

*) → A graph of spatial variation of the electric field along a Constant radius is called an amplitude field Pattern (E (volt/meter)).

In Practice, the 3-dimensional is measured and recorded in a series of 2-dimensional Pattern. where the Co-ordinates system usually used for the same is Spherical Co-ordinates (r, θ, ϕ).

The antenna is assumed to be located at origin of spherical Co-ordinate system and field strength is specified at points on the spherical surface of radius (r).

The shape of radiation Pattern does not depend on radius 'r'.

where $r \gg \lambda$

The distribution of Em wave Energy $E = \frac{V}{d}$ (V/m)

The direction of field strength (E) for radiation field is always tangential to spherical surface of imaginary sphere of radius (r).

For vertical dipole Electric field strength 'E' is in direction 'θ' for the horizontal loop in direction of 'ϕ'.

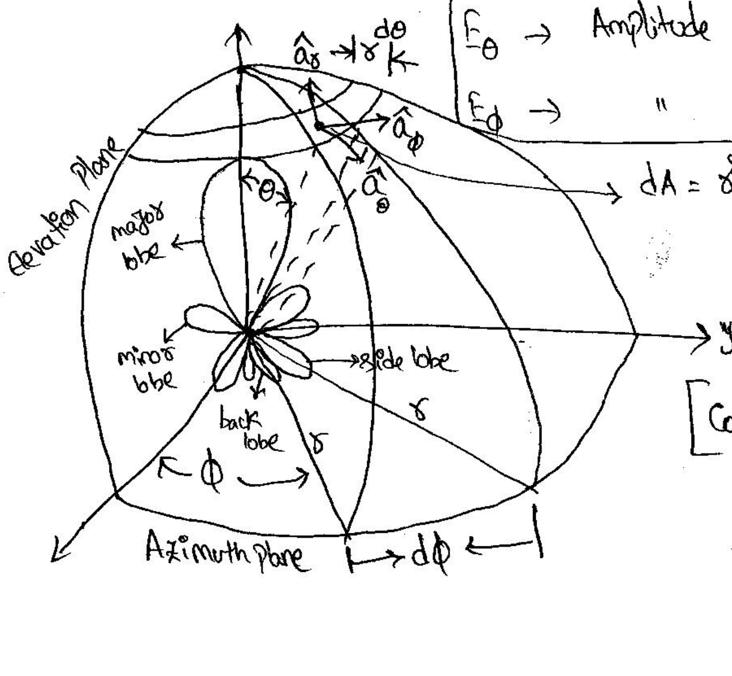
Where the radiation of field strength may have Components E_θ and E_ϕ

where $E = \sqrt{E_\theta^2 + E_\phi^2}$

$E \rightarrow$ Total Electric field strength

$E_\theta \rightarrow$ Amplitude of θ Component

$E_\phi \rightarrow$ " " ϕ "



[Co-ordinate System for Antenna Analysis.]

The Entire radiation ~~is~~ divided in to 3 - regions

Radiation Pattern

ⓐ main lobe ⓑ side lobe ⓒ Back lobe.

ⓐ Main lobe :- The region where we get maximum Energy for an antenna there may be one (or) more main lobes

ⓑ Side lobe :- The region where we get minimum & less Energy and are adjusted to the main lobes.

ⓒ Back lobes :- These are same as side lobes but these regions are adjusted to main lobes.

ii) Radiation intensity (or) Power radiation Pattern :-

→ Radiation intensity is defined as the ratio of differential Power to unit solid angle.

It is represented by 'U'

$$U = \frac{\text{diff Power}}{\text{unit solid angle}}$$

The radiation intensity of an antenna is defined as

$$U(\theta, \phi) = r^2 P_d(\theta, \phi)$$

Then the total Power radiated can be expressed in terms of the radiation intensity as

$$P_{rad} = \oint_S P_d(\theta, \phi) \, ds$$

$$= \oint_S P_d(\theta, \phi) \left[r^2 \sin\theta \cdot d\theta \cdot d\phi\right]$$

$$= \int_{\phi=0}^{2\pi} \int_{\theta=0}^{\pi} \left[P_d(\theta, \phi) r^2\right] \sin\theta \cdot d\theta \cdot d\phi$$

let $d\Omega = \sin\theta \cdot d\theta \cdot d\phi$ be the differential solid angle in steradian

$$P_{rad} = \int_{\phi=0}^{2\pi} \int_{\theta=0}^{\pi} U(\theta,\phi) \cdot d\Omega$$

Thus the radiation intensity $U(\theta,\phi)$ is expressed in watts/sr (watts/steradian)
it is defined as avg Power Per unit solid angle.
The avg of value of radiation intensity is

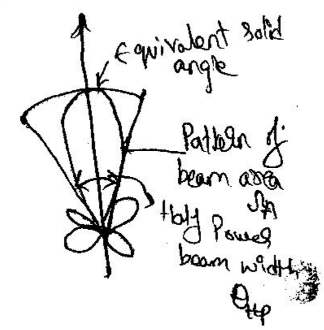
$$U_{avg} = \frac{P_{rad}}{4\pi}$$

Using Radiation intensity $U(\theta,\phi)$ we can also calculate normalized Power Pattern as ratio of radiation intensity $U(\theta,\phi)$ it is maximum value $U(\theta,\phi)_{max}$

$$P_{dn}(\theta,\phi) = \frac{U(\theta,\phi)}{U(\theta,\phi)_{max}}$$ //.

*) Beam solid Angle (or) Beam Area (Ω_A)
Generally Antenna Pattern of Beam Area (or) solid angle expressed in steradian. It is defined as integral of normalized Power over a sphere.

$$\Omega_A = \int_{\phi=0}^{2\pi} \int_{\theta=0}^{\pi} P_{dn}(\theta,\phi) \sin\theta \, d\theta \cdot d\phi$$



but $d\Omega = \sin\theta \cdot d\theta \cdot d\phi$

$$\boxed{\therefore \Omega_A = \int_{\phi=0}^{2\pi} \int_{\theta=0}^{\pi} P_{dn}(\theta,\phi) \cdot d\Omega \text{ steradian}}$$

many times Beam area is described as angles subtended by half Power Points of main lobe.

$$\boxed{\text{Beam area} = \Omega_A = \theta_{HP}\,\phi_{HP} \text{ steradian}}$$

iii) **Gain** :- It is defined as the ratio of maximum Radiation intensity from subject & test antenna. to the average radiation intensity of reference antenna.

$$\text{Gain}(G) = \frac{\text{Max R.I of test antenna}}{\text{Avg R.I of reference Antenna}}$$

$$\boxed{G = \frac{U_t}{U_R}}$$

iv) **Directive Gain** (G_d) :-

It is defined as a ratio of radiation intensity of an antenna in a direction to the avg radiation intensity of standard Antenna in that direction.

$$G_d = \frac{\text{R.I of an Antenna in a direction}}{\text{Avg R.I of standard antenna in that direction}}$$

$$G_d = \frac{U(\theta,\phi)}{U_{avg}(\theta,\phi)} = \frac{U(\theta,\phi)}{\omega_r/4\pi}$$

$$G_d = \frac{4\pi U(\theta,\phi)}{\omega_r}$$

$$G_d\,(db) = 10 \log G_d$$

$$\boxed{G_d\,(db) = 10 \log G_d}$$

It depends on radiation intensity (RI) radiated Power but does not depends on i/p Power.

v) Power gain:- (G_p)

Is a ratio of two powers densities of an antenna in a direction to avg power density of standard antenna in that direction

$$G_p = \frac{\text{Power density of an Antenna in direction}}{\text{Avg Power density of Standard Antenna}}$$

$G_p = \eta_r G_d$ $\quad$ ($\because \eta$ = efficiency for Antenna)

$G_p = G_d$

$$G_p = \frac{\phi(\theta, \phi)}{\frac{W_T}{4\pi}} \quad (\because W_T = W_r + W_l = \text{Total i/p Power})$$

$$\boxed{G_p = \frac{4\pi\, \phi(\theta, \phi)}{W_T}}$$

In terms of Power i/p (or) Power gain in 'dB'

$$\boxed{G_p = 10 \log_{10}(G_p)}$$

vi) Directivity (D) :-

The maximum directive gain is called Directivity (D) of an antenna.

Defined as ratio of Maximum radiation intensity to its average radiation intensity i.e.,

$$\text{Directivity (D)} = \frac{\text{max Radiation intensity to test antenna}}{\text{Avg Radiation intensity of test Antenna.}}$$

$$D = \frac{U\phi(\theta,\phi)_{max}}{\phi_{av}} \text{ both of test antenna}$$

Directivity of antenna for isotropic Antenna :-

$$\text{Directivity (D)} = \frac{\text{max Radiation intensity of subject (or) test Antenna}}{\text{Radiation intensity of isotropic Antenna}}$$

$$D = \frac{U\phi(\theta,\phi)_{max} \text{ (test antenna)}}{\phi_0 \text{ (isotropic Antenna)}}$$

vii) Radiation Efficiency (or) Antenna Efficiency :- (η)

The efficiency of an Antenna is defined as the ratio of Power radiated to the total i/p power supplied to the antenna and denoted as η (or) K.

$$\text{Antenna Efficiency} = \frac{\text{Power Radiated}}{\text{Total i/p Power}}$$

$$\eta = \frac{W_r}{W_T} = \frac{W_r}{W_r + W_l}$$

$$= \frac{W_r}{W_T} \times \frac{4\pi\,U(\theta,\phi)}{4\pi\,U(\theta,\phi)}$$

$$= \frac{4\pi U(\theta,\phi)}{W_T} \cdot \frac{W_r}{4\pi U(\theta,\phi)}$$

$$\eta = G_p \cdot \frac{1}{G_d} = \frac{G_p}{G_d} = \frac{W_r}{W_r + W_t}$$

W_r = Power radiated

W_t = ohmic losses

If current flowing in antenna is I then

$R_r \to$ Radiation resistance

$$\eta = \frac{I^2 R_r}{I^2(R_r + R_l)}$$

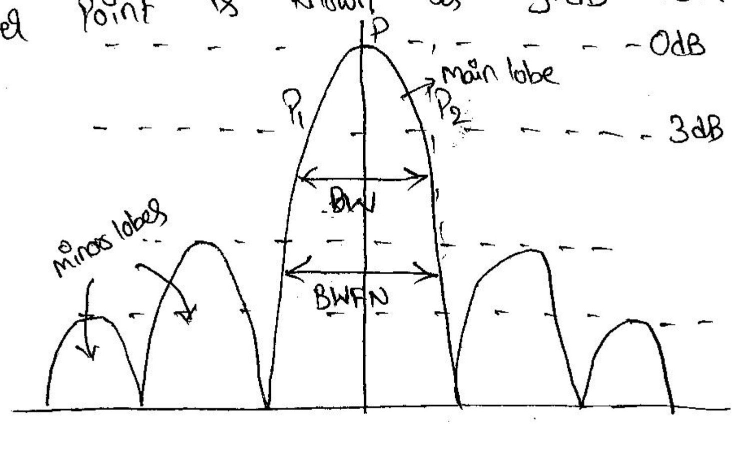
$$\boxed{\eta\,\% = \frac{R_r}{R_r + R_l} \times 100}$$

$R_l \to$ ohmic loss resistance of antenna conductor

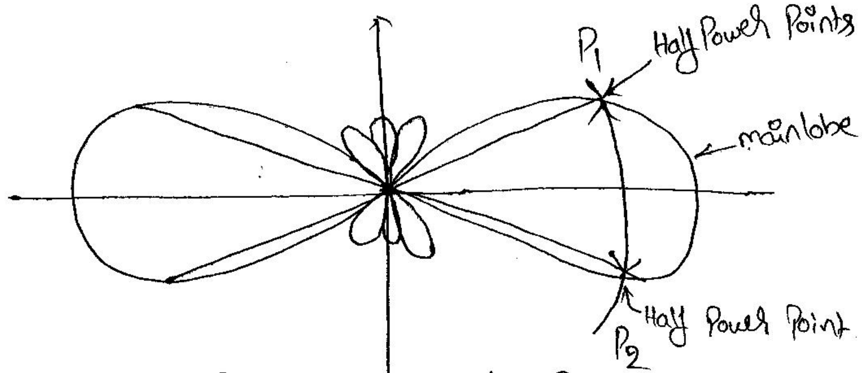
viii) Antenna Beam width :-

Antenna Beam width is a measure of directivity of the antenna. The Antenna beam width is an angular width in degrees. It is measured on a radiation Pattern major lobe.

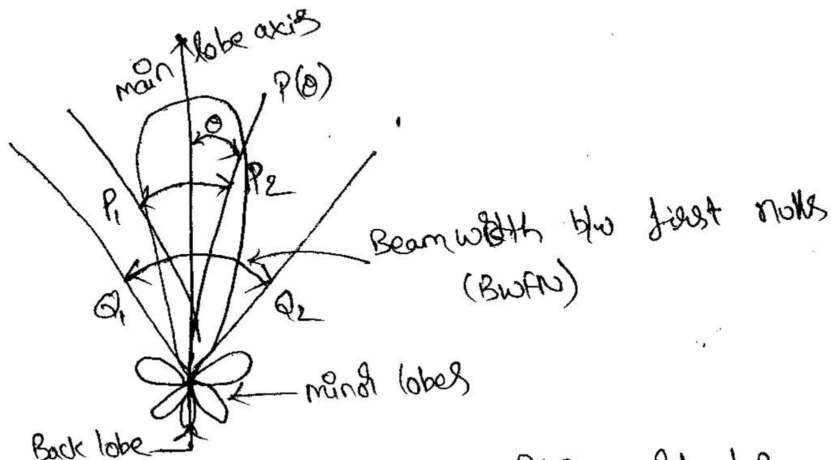
The antenna beamwidth is defined as the angular width in degree b/w the two points in a major lobe of a radiation Pattern where the radiated power decreases to half of the maximum value. This is called Beam width. It's half Power (or) half Power Point is known as 3-dB beam width



Antenna Pattern in rectangular Co-ordinates (or) Logarithmic Scale



Beam width angle $P_1 P_2$.



Antenna Power Pattern in Polar-Co-ordinates.

The beam width is also called as Half Power Beam Width (HPBW) because it is measured b/w two points on major lobe where power is half of its maximum power. The two points P_1 & P_2 the angular width between points P_1 & P_2 is nothing but antenna beamwidth (or) Half Power beamwidth (HPBW).

It is clear power is maximum at point 'P' but at points P_1 & P_2 the power is 3-dB down the maximum power.

Many times Radiation Pattern of antenna is described as in term of angular beamwidth b/w first nulls (or) first side lobes. Then such an angular beamwidth is called Beamwidth b/w first nulls (BWFN).

The directivity (D) of antenna is related with beam solid angle (Ω_A) (or) beam area 'B' through expression

$$\boxed{D = \frac{4\pi}{\Omega_A} = \frac{4\pi}{B}}$$

B = Beam area.

B = (HPBW) in horizontal plane X (HPBW) in vertical plane

= (HPBW) in E-plane X (HPBW) in H-plane

(or)

$$\boxed{B = \theta_E \times \theta_H}$$

$$\therefore \boxed{D = \frac{4\pi}{\theta_E \theta_H} \dots \text{ if } \theta_E \text{ and } \theta_H \text{ are in radians}}$$

Then

$$\boxed{D = \frac{4\pi (1 \text{ radi})^2}{\theta_E^\circ \theta_H^\circ} = \frac{4\pi (57.3)^2}{\theta_E^\circ \theta_H^\circ} = \frac{41257}{\theta_E^\circ \theta_H^\circ}}$$

(ix) Antenna Beam Efficiency :- (BE)

Antenna beam efficiency is the parameter that is frequency used to judge the quality of transmitting and receiving antennas For the antenna with major lobe coincident with z-axis the beam efficiency is given as

$$BE = \frac{\text{Power transmitted (or) received with in cone angle } \theta_1}{\text{Power transmitted (or) received) by antenna}}$$

$\theta_1 \rightarrow$ half angle of the cone within percentage of total power is.

Mathematically Beam efficiency (BE) = $$\frac{\int_{\phi=0}^{2\pi} \int_{\theta=0}^{\pi} U(\theta,\phi) \cdot \sin\theta \cdot d\theta \cdot d\phi}{\int_{\phi=0}^{2\pi} \int_{\theta=0}^{\pi} U(\theta,\phi) \cdot \sin\theta \cdot d\theta \cdot d\phi}$$

If θ_1 is choosen as angle where first nulls (or) minimum occurs the beam efficiency expressed as in terms (Ω_m) and beam area (Ω_A)

Beam Efficiency can be defined as ratio of main beam area to total beam area denoted as (ε_M)

$$BE \text{ (or) } \varepsilon_M = \frac{\Omega_M}{\Omega_A} = \frac{\text{Main beam area}}{\text{Total beam area}}$$

Total beam area consists of main beam area (Ω_M) and minor lobe area (or) solid angle Ω_m i.e

$$\boxed{\Omega_A = \Omega_M + \Omega_m}$$

Total beam area = Main beam area + Minor lobe area.

$$1 = \frac{\Omega_M}{\Omega_A} + \frac{\Omega_m}{\Omega_A}$$

$$\boxed{1 = \varepsilon_M + \varepsilon_m}$$

where

$$\varepsilon_m = \frac{\Omega_m}{\Omega_A} = \text{stray factor} = \frac{\text{Minor lobe area}}{\text{Total beam area}}$$

$$\varepsilon_M = \frac{\Omega_M}{\Omega_A} = \text{Beam Efficiency.}$$

(X) <u>Front to Back Ratio (FBR)</u>:-

Is the ratio of Power radiated in desired direction to the Power radiated in the opposite direction.

$$FBR = \frac{\text{Power radiated in desired direction}}{\text{Power radiated in opposite direction.}}$$

ON the frequency of operation. FBR is depended, so when frequency of an antenna changes. the FBR also changes. Similarly the FBR depends on the spacing b/w the antenna Elements. If spacing b/w antenna Elements increases the FBR decreases. The FBR also depends on Electrical length & Parasitic Elements of the antenna.

The FBR can be raised by diverting the gain of backward direction response of antenna to the front (or) forward (or) desired direction by adjusting the length of Parasitic elements. the method of adjusting the Electrical length of the Parasitic Element is called tuning."

(XI) Antenna Temperature (T_A):-

Antenna Radiation Pattern is effected by temperature with its radiation resistance and it does not depends on location (or) direction of antenna. In general temperature and Power are related as

$$P \propto T_A$$

$$P = K_A T_A \rightarrow ①$$

$P \rightarrow$ Noise Power per unit BW W/Hz

$K \rightarrow$ Boltzmann's Constant $= 1.38 \times 10^{-23}$ J/K

$T_A \rightarrow$ Absolute temp of Antenna in K.

Let an antenna placed at Environmental temperature and the radiation pattern depends on physical size of antenna due to Environmental temperature the noise Power is

$$P = S \cdot A_e \cdot B \rightarrow ②$$

P → Noise Power ; B → Bandwidth

S → Power density (or) Power spectral density

A_e → Effective aperature of antenna

from eq ① & ② The flux density of antenna is

$$K T_A \cancel{B} = S \cdot A_e \cancel{B}$$

$$S = \frac{K \cdot T_A}{A_e} \quad W/m^2\,Hz \rightarrow ③$$

∴ Temperature of Antenna is written as

from above eq $\boxed{T_A = \frac{S \cdot A_e}{K} \ {}^{\circ}k}$

If the source is small compared to the beam solid angle 'Ω_A'

then the source temperature can be given.

$$\boxed{T_s = \frac{\Omega_A}{\Omega_s} T_A \ {}^{\circ}k}$$

Ω_A = Beam solid angle in steradian

Ω_s = Source solid angle in steradian.

(xii) Effective Aperture (or) Effective Area (A_e):-

A transmitting antenna transmits electromagnetic waves and receiving antenna receives a fraction of the same.

On consideration an antenna to have effective area (or) aperture over which it extracts electromagnetic energy from travelling electromagnetic waves.

The ratio of Power received at the antenna load terminal to the Poynting vector of average Power density.

$$\boxed{A_e = \frac{P_{received}}{\bar{P}_{avg}} \; m^2} \rightarrow (1)$$

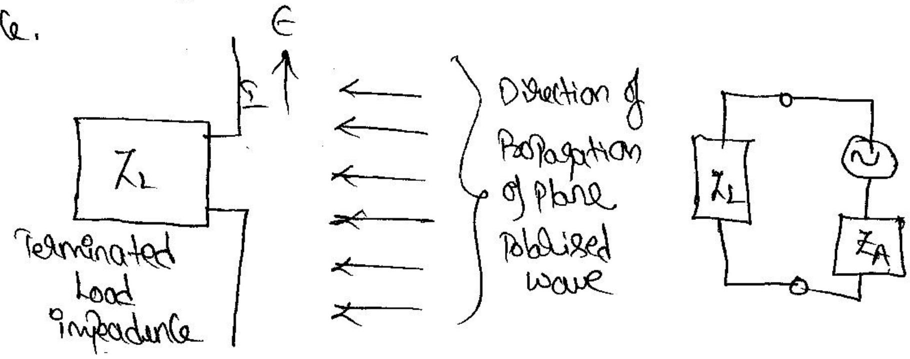
Power received is denoted as (P_R) measured in watts
Power density measured in watts/m^2.

$$(or) \quad A_e = \frac{W}{P} \equiv A \rightarrow (2)$$

$$W = PA$$

W = Power received in watts
P = Poynting avg Power density
A = Effective (or) Capture area (or) Effective aperture in m^2.

Let a receiving antenna be placed in the field of plane Polarised travelling waves in having an effective area 'A' and the receiving antenna (dipole) is terminated at load impedance.



I → terminal current, then received Power

$$W = I^2_{rms} R_L \rightarrow (3)$$

R_L → load resistance in 'Ω'.
I_{rms} → Terminal rms current

$$A = \frac{W}{P} = \frac{I_{rms}^2 R_L}{P} \rightarrow (4)$$

Since the antenna extracts energy from incident electromagnetic waves, delivers the same to terminated load impedance Z_L and power flowing per square meter (or) Poynting vector P W/m². The entire system replaced by equivalent circuit

V = Equivalent Thevenins voltage

Z_A = " " impedance

$$I_{rms} = \frac{\text{Equivalent voltage}}{\text{Equivalent impedance}} = \frac{V}{Z_L + Z_A} \text{ Amp} \rightarrow (5)$$

where $Z_A = R_A + jX_A$ (Complex antenna impedance)

$Z_L = R_L + jX_L$

now:-

$$I_{rms} = \frac{V}{Z_A + Z_L} = \frac{V}{Z_T} \rightarrow (6)$$

$$I_{rms} = \frac{V}{(R_A + jX_A) + (R_L + jX_L)}$$

$$|I_{rms}| = \frac{|V|}{\sqrt{(R_A + jX_A)^2 + (R_L + jX_L)^2}}$$

Squaring on Both sides

$$|I_{rms}|^2 = \frac{V^2}{(R_A + R_L)^2 + (X_A + X_L)^2} \rightarrow (7)$$

$$\therefore \quad W = I_{rms}^2 R_L = \frac{V^2 R_L}{(R_A + R_L)^2 + (X_A + X_L)^2} \rightarrow (8)$$

$$A_e = \frac{W}{P}$$

$$A_e = \frac{V^2 R_L}{P(R_A + R_L) + P(X_A + X_L)^2} \rightarrow (9)$$

A/c to maximum Power transfer theorem,

$$X_L = -X_A$$

$$R_L = R_A = (R_r + R_l)$$

$$R_L = R_r \quad \text{if } R_l = 0$$

from Eq (9)

$$A_e = \frac{V^2 R_L}{P[(2R_L)^2 + 0]}$$

$$A_e = \frac{V^2 \cancel{R_L}}{4 P R_L^{\cancel{2}}} = \frac{V^2}{4 P R_L}$$

$$\therefore \boxed{A_e = \frac{V^2}{4 P R_L}}$$

(xiii) <u>Effective Length</u> :-

It represents effectiveness of antenna as radiator (or) Electromagnetic wave energy.

It is defined as ratio of incident (or) open circuit voltage to incident (or) field strength.

$$l_{eff} = \frac{\text{Open Ckt voltage}}{\text{incident field strength}}$$

$$l_{eff\,(received)} = \frac{V}{E} \rightarrow (1)$$

we know that

$$A_e = \frac{V^2 R_L}{P[(R_A + R_L)^2 + (X_A + X_L)^2]}$$

$$V^2 = \frac{A_e P\left[(R_A+R_L)^2 + (X_A+X_L)^2\right]}{R_L}$$

$$V = \sqrt{\frac{A_e P\left[(R_A+R_L)^2 + (X_A+X_L)^2\right]}{R_L}} \rightarrow (2)$$

$$P = \frac{E^2}{Z}$$

$$V = \sqrt{\frac{A_e E^2\left[[R_A+R_L]^2 + [X_A+X_L]^2\right]}{Z R_L}}$$

$$l_e = \frac{V}{E} = \sqrt{\frac{A_e (R_A+R_L)^2 + (X_A+X_L)^2}{Z R_L}}$$

By maximum Power theorem

$$X_A = -X_L \;;\; R_A = R_L$$

$$l_{eff} = \frac{V}{E} = \sqrt{\frac{A_e (2R_L)^2}{Z R_L}}$$

$$l_{eff} = \sqrt{\frac{4 A_e R_L}{Z}}$$

$$l_{eff} = 2\sqrt{\frac{A_e R_L}{Z}}$$

$$l_{eff} = 2\sqrt{\frac{A_e (R_r + R_L)}{Z}}$$

$$\boxed{\therefore\; l_{eff} = 2\sqrt{\frac{A_e R_r}{Z}}}$$

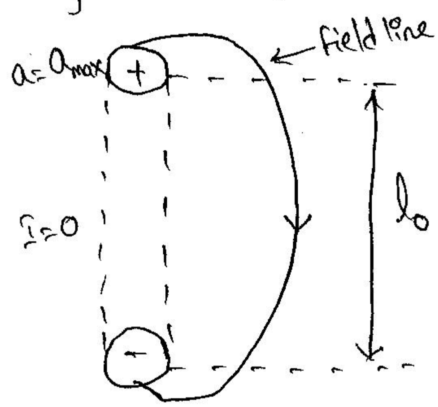
*) Fields from Oscillating Dipole :-

In this we discuss the radiation mechanism of an oscillating charge which moves back and forth in simple harmonic motion along a dipole.

Let us consider a dipole Antenna with two equal and opposite charges oscillating up and down with simple harmonic motion.

Consider l_0 be the maximum separation b/w two equal and opposite charges while l be an instataneous separation b/w charges.

Let us analyze the Electric field by taking into a account a single Electric field line.

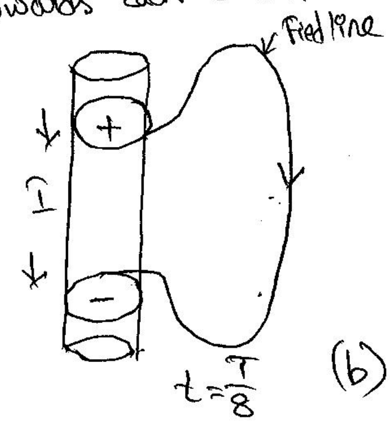


$t = 0$ (a)

*) The equal and opposite charges are at maximum separation of 'l_0', $t = 0$.

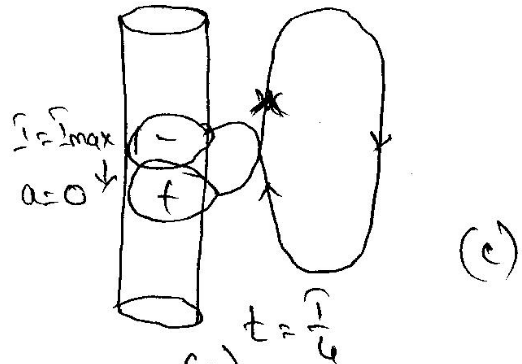
*) Here the acceleration of charges is maximum i.e a_{max} as reverse direction current ($I = 0$).

Let 'T' be the period of oscillation. Then after period $t = \frac{T}{8}$ the charges move towards each other.



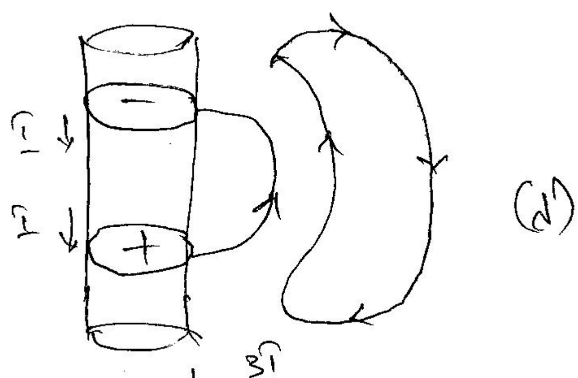
$t = \frac{T}{8}$ (b)

After a time period $t = \frac{T}{4}$ the charges reach at Mid Point of a axis of the dipole. At this instant the acceleration of charges becomes zero ($a = 0$) with current of maximum value ($I = I_{max}$)

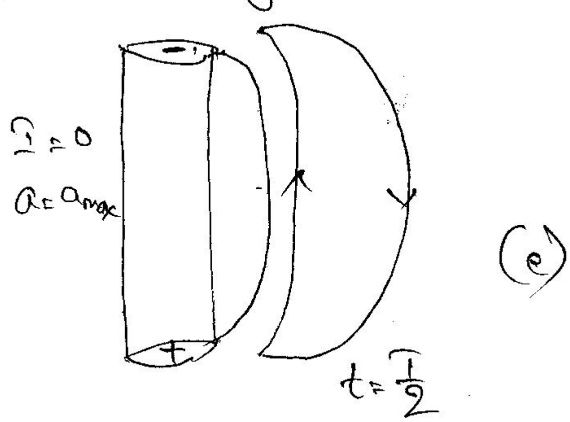


(c)

After a time period $t = \frac{3T}{8}$ a field line is completely detached and released from dipole. After detachment of field line new field line starts developing.



After $t = \frac{T}{2}$ instant the charges reach to the exactly opposite ends of dipole as shown below. Again the charges gain maximum acceleration ($a = a_{max}$) with current being zero ($I = 0$)



*) Relation Between Max. Aperture and Gain (or) Directivity:-

The radiation Pattern is same for transmitting and receiving antennas.

By virtue of Reciprocity theorem and hence the idea of directivity which its self is related with shape of radiation Pattern is extended for receiving antennas also.

In Practice that the directivity of receiving antennas are directly Proportional to maximum effective apertures.

Let there be two Antenna A and B whose directivities and maximum effective apertures are denoted by D_a, D_b, and $(A'_{ea})_{max}$ and $(A'_{eb})_{max}$

$$D_a \propto (A_{ea})_{max}$$

$$\therefore \quad D_b \propto (A_{eb})_{max}$$

$$\therefore \quad \frac{D_a}{D_b} = \frac{(A_{ea})_{max}}{(A_{eb})_{max}}$$

For antenna with loss, gain will be less than directivity by a factor which corresponds to efficiency. The directivity and gain given as

$$G_o = kD$$

$G_o \to$ Gain of transmitting (or) receiving antenna

$k \to$ Efficiency factor

$D \to$ Directivity.

If now the losses of efficiency factor k and mismatch are included, the k can be replaced by effectiveness ratio α i.e

$$G_o \propto D$$

Let us now assume that G_{oa}, α_a, D_a being the gain, Effectiveness ratio and directivity of antenna A and G_{ob}, α_b, D_b the Corresponding quantities for antenna B, then

$$G_{oa} = \alpha_a D_a \quad \ldots\ldots \text{ for Antenna A}$$
$$G_{ob} = \alpha_b D_b \quad \ldots\ldots \text{ for '' B}$$

$$\frac{G_{oa}}{G_{ob}} = \frac{\alpha_a D_a}{\alpha_b D_b} = \frac{\alpha_a (A_{ea})_{max}}{\alpha_b (A_{eb})_{max}} \rightarrow (3)$$

from Def.
$$\alpha_a = \frac{A_{ea}}{(A_{ea})_{max}} \rightarrow (4)$$

$$(A_{ea}) = \alpha_a (A_{ea})_{max}$$
$$(A_{eb}) = \alpha_b (A_{eb})_{max} \rightarrow (5)$$

from eq (3)

$$\frac{G_{oa}}{G_{ob}} = \frac{A_{ea}}{A_{eb}}$$

when A_{ea} and A_{eb} are the Effectiveness apertures A & B

let us assume that Antenna A is an isotropic antenna then its directivity $D_a = 1$

now from eq (1).

$$\frac{D_a}{D_b} = \frac{1}{D_b} = \frac{(A_{ea})_{max}}{(A_{eb})_{max}}$$

(or)

$$\boxed{\left(A_{ea}\right)_{max} = \frac{\left(A_{eb}\right)_{max}}{D_b}}$$

This above Eq suggests that if the maximum effective aperture and directivity of antenna B (or) Any Antenna are know the ratio of two will give the maximum effective aperture of an isotropic antenna.

The directivity of any antenna can be calculated by

$$\boxed{D_b = \frac{\left(A_{eb}\right)_{max}}{\left(A_{ea}\right)_{max}}}$$

Reciprocity theorem:-

Statement:- If an emf is applied to the terminals of an antenna 1 and the current measured at the terminals of another antenna 2, then an equal current both in amplitude and phase will be obtained at the terminals of antenna no 1 if the same emf is applied to the terminals of antenna 2.

(or)

If a current I_1 at the terminals of Antenna 1 induces an emf E_{21} at open terminals of antenna 2 and current I_2 at the terminals of Antenna 2 induces emf of E_{12} at the open ~~circuit~~ terminals of Antenna 1 then $E_{12} = E_{21}$

Assumptions:-

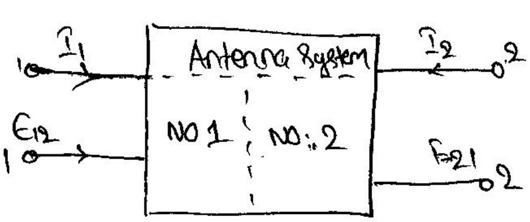
(i) Emf are in same frequency

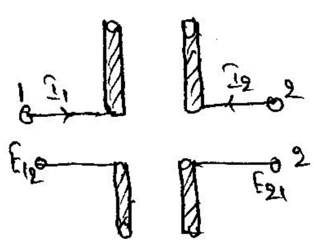
(ii) Medium b/w the two Antenna are linear, Passive and isotropic.

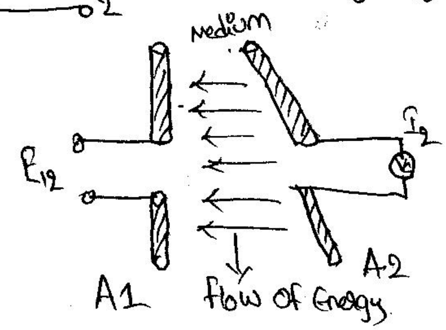
(iii) Generator producing emf and Ammeter for measuring the current have zero impedance.

Explanation:-

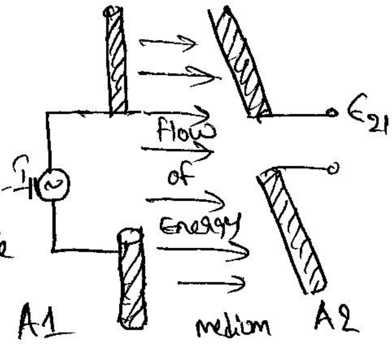
A transmitter of frequency 'f' and zero impedance be connected to the terminals of Antenna 2 which is generating a current I_2 and including an EMF E_{12} at the open terminals of Antenna 1.







(2) Now the same transmitter is transferred to Antenna 1 which is generating a current I_1 and inducing a voltage E_{21} at the open terminals of Antenna 2.

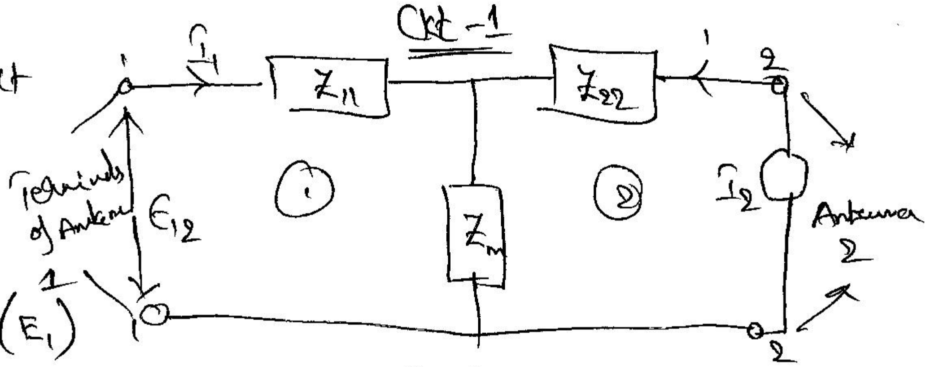


Thus A/c statement of reciprocity theorem

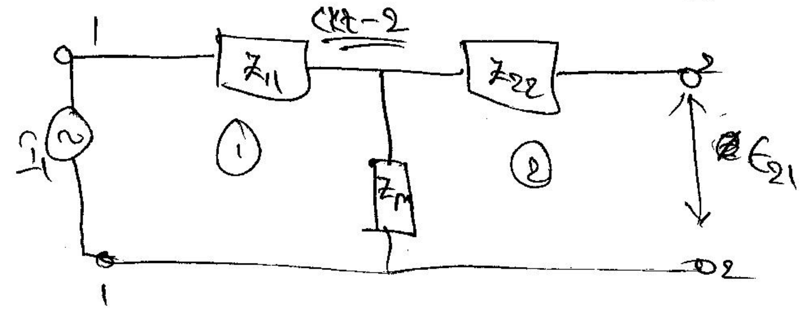
$$I_1 = I_2 \text{ Provided } E_{12} = E_{21}$$

∵ The ratio of emf to current is an impedance therefore the ratio $\frac{E_{12}}{I_2}$ is given as transfer impedance Z_{12} as in case I, and so also the ratio $\frac{E_{21}}{I_1}$ as transfer impedance Z_{21} as in II case.

The Equivallent Ckt



The ratio of voltage (E_1) of 1 ckt to current I_2 in second ckt is defined as transfer impedance



Z_T (or) Z_{12} i.e

$$Z_T = Z_{12} = \frac{E_1}{I_2} \rightarrow (1)$$

Thus from the reciprocity it follows that the two ratios i.e two impedance are equal i.e

$$Z_{12} = Z_{21} \rightarrow (2)$$

where the mutual impedance (Z_m) b/w two antennas is

$$Z_m = Z_{12} = Z_{21} = \frac{E_{12}}{I_2} = \frac{E_{21}}{I_1} \rightarrow (3)$$

$$\boxed{\frac{E_{12}}{I_2} = \frac{E_{21}}{I_1}} \rightarrow (4)$$

<u>Proof</u>:- To prove Reciprocity theorem for antennas, the space b/w the Antenna 1 & Antenna 2 are replaced by a n/w of linear, passive and bilateral impedance.

Z_{11}, Z_{22} = Self impedance Antenna 1 & Antenna 2

Z_m = mutual impedance b/w 2 Antennas

Now applying Kirchoffs mesh law

from <u>ckt-1</u>:-fig from loop '2'

$$(Z_{22}+Z_m)I_2 - Z_m I_1 = 0 \quad (\because \text{no voltage in loop 2})$$

$$I_2 = I_1 \frac{Z_m}{(Z_{22}+Z_m)} \rightarrow (5)$$

for <u>loop</u> ~~loop~~ <u>1</u>:-

$$(Z_{11}+Z_m)I_1 - Z_m I_2 = E_{12}$$

$$(Z_{11}+Z_m)I_1 - Z_m\left[I_1\left(\frac{Z_m}{Z_{22}+Z_m}\right)\right] = E_{12} \rightarrow (6)$$

$$\Rightarrow I_1\left[\frac{(Z_{11}+Z_m)(Z_{22}+Z_m) - Z_m^2}{Z_{22}+Z_m}\right] = E_{12}$$

$$\Rightarrow I_1\left[\frac{Z_{22}Z_{11} + Z_m Z_{11} + Z_{22}Z_m + Z_m^2 - Z_m^2}{(Z_{22}+Z_m)}\right] = E_{12}$$

$$(\text{or}) \quad I_1 = \frac{E_{12}(Z_{22}+Z_m)}{Z_{11}Z_{22} + Z_m(Z_{11}+Z_{22})} \rightarrow (7)$$

Substituting Eq 7 in Eq 6

$$i_2 = \frac{E_{12}(Z_{22}+Z_m)\cdot Z_m}{[Z_{11}\times Z_{22}+Z_m(Z_{11}+Z_{22})]\cdot(Z_{22}+Z_m)}$$

$$i_2 = \frac{E_{12}\cdot Z_m}{Z_{11}\times Z_{22}+Z_m(Z_{11}+Z_{22})} \rightarrow 8$$

Simillarly I_1 in Ckt-2 given by

$$I_1 = \frac{E_{21}\cdot Z_m}{Z_{22}\cdot Z_{11}+Z_m(Z_{22}+Z_1)} \rightarrow 9$$

Clearly Eq 8 & 9 are same exect value of emf

Ac to theorem statement

$$E_{12} = E_{21} \quad \text{if } I_1 = I_2$$

$\therefore$ Applying Condition:- $I_1 = I_2$

$$\frac{E_{12} Z_m}{[Z_{11}\cdot Z_{22}+Z_m](Z_{11}+Z_{12})} = \frac{E_{21}\cdot Z_m}{[Z_{11}\cdot Z_{22}+Z_m](Z_{11}+Z_{22})}$$

$$\boxed{\therefore\ E_{12} = E_{21}}$$

Hence Proved