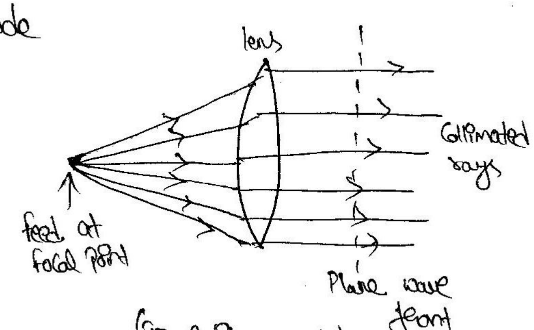
Unit - 6

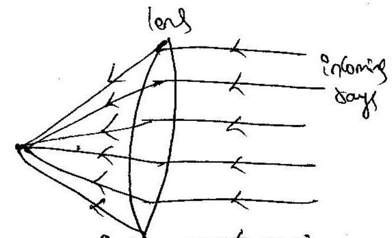
Lens Antennas :- & Antenna Measurements

In microwave frequencies in addition to horn Antenna, Paraboloid, we use " lens Antenna also" which is operated with frequencies ranges of 1000MHz - 10000 MHz from (1 GHz to 10 GHz)

The lens antennas can be used as transmitting & receiving mode

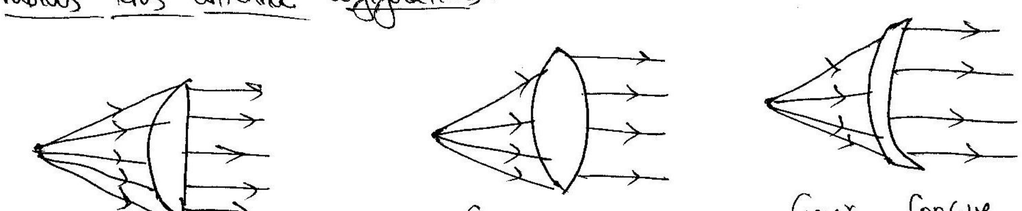


(Transmitting mode)



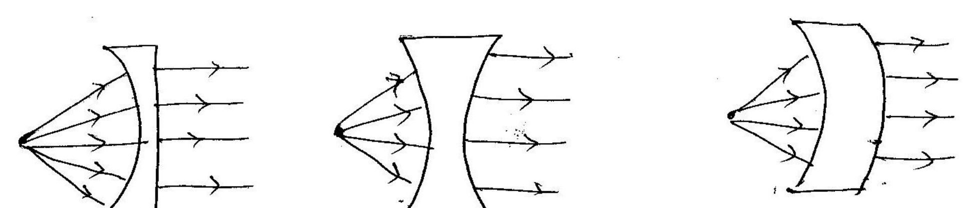
(Receiving mode)

Various lens antenna Configurations :-



Convex Plane   Convex - Convex   Convex - Concave

(a) Lens antenna with Refractive index $n > 1$ "



Concave Plane   Concave - Concave   Concave - Convex

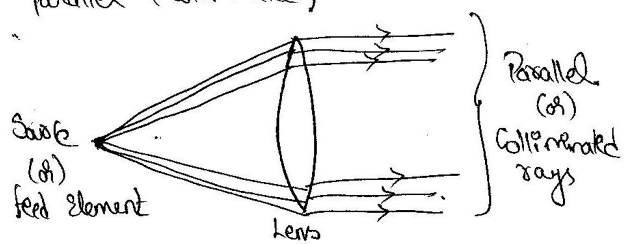
(b) Lens antenna with R.I "$n < 1$".

Principle of operation:-

→ This lens antenna gives the Principle of Equality Path length.

→ Same like Parabolic reflector the source is placed at a point called focal point. at a distance from the lens.

When the source radiates the waves and are incident on the lens and are refracted to another side and are Parallel (Collimnated) on the other side.



Transmitting lens Antenna.

Same Principle is applicable in case of receiving lens antenna ie Parallel rays are converted to spherical wavefronts to reach the point source.

The wave transmission from one medium to another medium depends on a factor called as Refractive index.

For lens medium it is greater than '1'

$$\mu = \frac{\text{velocity of wave in air}}{\text{velocity of wave in lens}} = \frac{c}{v} > 1$$

that means antenna obeys reciprocity theorem.

→ The purpose of lens is to get plane wavefront (Parallel rays) from the spherical wave. This lens antenna is used in conjuction with horn Antenna.

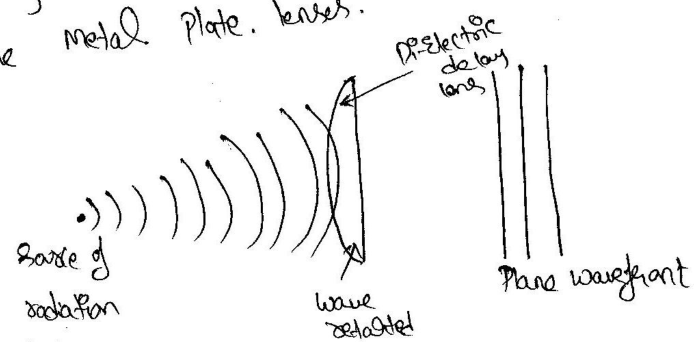
The Main application of lenses is to collimate incident divergent Energy to prevent it from spreading in undesired directions. These antenna are used to transform the divergent Energy in to the plane waves by properly choosing lens material and geometrical shape. These are used at very high frequencies as their dimension and weight become extremely large at lower frequencies.

The lense antenna classified as

1. Delay lens
2. Fast lens.

(1) Delay lens antenna in which the Electrical path length is increased by the lens medium and wave is retarded.

Eg:- Delay lens antenna & di-electric lenses and H-plane metal plate. lenses.



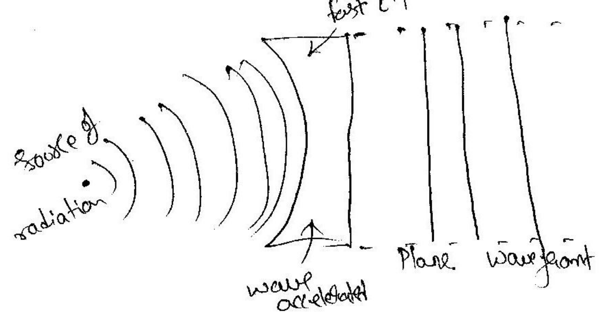
(a) Delay lens Antenna

The di-Electric lens antenna is further classified on basics of di-Electric used. Di-Electric lenses are constructed of either metallic (or) artificial di-Electric (or) non-metallic di-Electric such as polystyrene, lucite etc;

(2) Fast lens :-

It is the antenna in which Electrical Path length is decreased by the lens medium and the wave is accelerated.

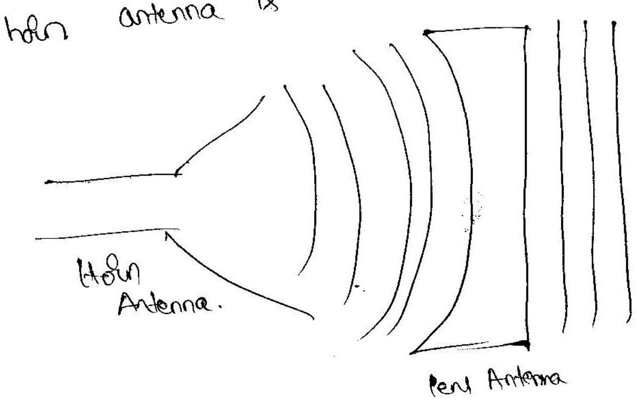
Fast lens is E-Plane metal Plate lenses.

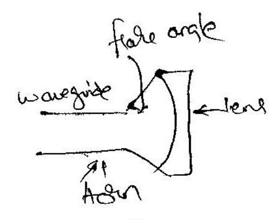


*) Feed Systems of Lens Antennas:-

Similar to Parabolic reflectors lens antenna are fed with horn Antenna. To have better Performance of lens antenna the aperture dimension of horn is selected Equal to focal length of lens antenna.

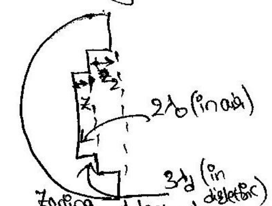
Bcoz feed becomes directive as compared to that in the reflector antennas For these Equal dimensions the flare angle of horn antenna is 53°.





(i) Zoning of lens:-

The weight of lens can be reduced by removing sections of lens, which is called Zoning of lens.



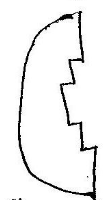
Zoning of lens antenna

Zoning

(a) Curved Surface Zoning

(b) Plane Surface Zoning

(a) Curved surface Zoning



(b) Plane surface zoning

In general zoning of lens is carried out in such a way that particular design frequency, the performance of lens antenna is not affected.

The zone step denoted by z. So in zoned lens antennas the thickness z of lens antenna is such that electrical length of thickness 'z' in di-electric is an integral length of 'λ' longer than in air.

z in di-electric may be $3\lambda_d$ and that in air is $2\lambda_0$ where λ_d and λ_0 are the wavelength in di-electric and air

For 1λ difference

$$\frac{z}{\lambda_d} - \frac{z}{\lambda_0} = 1$$

By R.I $n = \frac{\lambda_0}{\lambda_d}$

$$\Rightarrow \frac{z}{(\lambda_0/n)} - \frac{z}{\lambda_0} = 1 \Rightarrow \frac{(n-1)z}{\lambda_0} = 1$$

$$\Rightarrow \boxed{z = \frac{\lambda_0}{n-1}}$$ //.

*) Metal Plane Lens Antenna of E-Plane

To study the development of E-plane metal antenna, the knowledge of waveguide is required. Consider that TE_{10} mode wave is propagated through two parallel conducting plates of infinite extent.

The wave guide length denoted by 'λ_g'. Then the guide wavelength 'λ_0' through expression

$$\frac{1}{\lambda_g^2} = \left(\frac{1}{\lambda_0}\right)^2 - \left(\frac{1}{2a}\right)^2 \rightarrow (1)$$

where a = internal dimension of waveguide (or) spacing b/w two planes

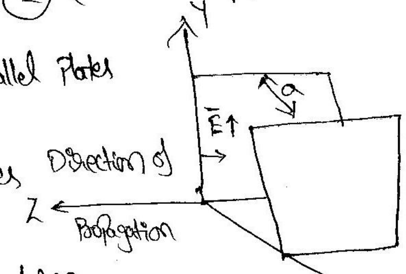
The phase velocity of wave in rectangular guide is always greater than velocity of wave in free space (i.e. $C = 3\times10^8$ m/sec)

$$V_p = \frac{C\lambda_g}{\lambda_0} \rightarrow (2) \quad \left(\because \lambda = \frac{C}{V}\right)$$

Consider wave propagation b/w two parallel plates separated. by

The electric field vector is parallel to plates

this indicates it is a part of rectangular waveguide with other dimension infinitely large.



Propagation through metal plate of E-plane

The effective R.I is given by

$$n = \frac{C}{V_p} = \frac{C}{\left(\frac{C\lambda_g}{\lambda_0}\right)} = \frac{\lambda_0}{\lambda_g} \rightarrow (3)$$

From Eq 1 multiplying B.S with. λ_0^2

$$\lambda_0^2\left(\frac{1}{\lambda_g}\right)^2 = \lambda_0^2\left(\frac{1}{\lambda_0}\right)^2 - \lambda_0^2\left(\frac{1}{2a}\right)^2$$

$$\frac{\lambda_0^2}{\lambda_g^2} = \frac{\lambda_0^2}{\lambda_0^2} - \left(\frac{\lambda_0}{2a}\right)^2$$

$$\frac{\lambda_0}{\lambda_g} = \sqrt{1-\left(\frac{\lambda_0}{2a}\right)^2} \rightarrow 4$$

But from Eq 3 the R.I ratio of free space wavelength to the wave guide wavelength is given by

$$n = \sqrt{1-\left(\frac{\lambda_0}{2a}\right)^2} \rightarrow 5$$

If the value n is practically always less than unit. The spacing b/w the plates for which the R.I becomes zero is called critical spacing b/w plates

$$0 = \sqrt{1-\left(\frac{\lambda_0}{2a}\right)^2}$$

S.O.B.S

$$0 = 1-\left(\frac{\lambda_0}{2a}\right)^2 \Rightarrow 1 = \left(\frac{\lambda_0}{2a}\right)^2$$

$$\Rightarrow (2a)^2 = \lambda_0^2$$

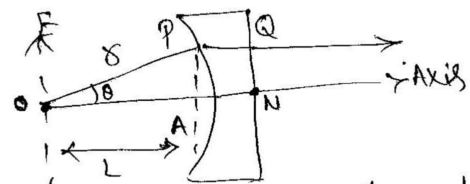
$$\Rightarrow \boxed{a = \frac{\lambda_0}{2}} \rightarrow 6$$

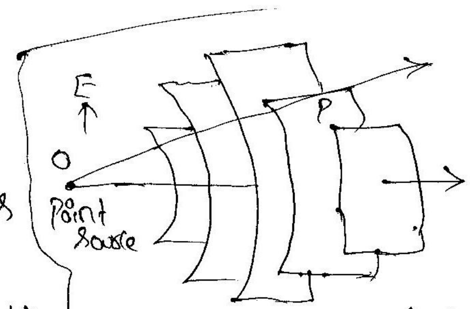
From this metal plate lens antenna can be developed using principles of waveguides in which many parallel plates with spacing used.

The main difference b/w ordinary lens & metal plate lens is in ordinary di-electric lens action depends on the retardation of wave. In metal plate lens action depends on acceleration of the wave in lens.

Simply :- Ordinary lens are slow down wavefront, while metal plate wavelens speed up wavefront.

Consider a plate which is on axis





Convergent E-plane metal plate lens.

According Fermat Principle, the shape of the plate is based on principle equality of electrical path length.

At Equality Electrical Path length

$$OPQ = OAN$$

If 'L' is focal length written as

$$\frac{L}{\lambda_0} = \frac{r}{\lambda_0} + \frac{L - r\cos\theta}{\lambda_g} \rightarrow (7)$$

λ_g - wavelength of lens
λ_0 = wavelength of free space.

Multiplying Eq (7) with λ_0 on B.S

$$L = r + \left(\frac{\lambda_0}{\lambda_g}\right)\left(L - r\cos\theta\right)$$

But the ratio of free space wavelength to wavelength in lens is effective R.I 'n'

$$L = r + n\left(L - r\cos\theta\right).$$

$$L = r + nL - nr\cos\theta$$

$$(L - nL) = r - nr\cos\theta$$

$$L(1-n) = r(1 - n\cos\theta)$$

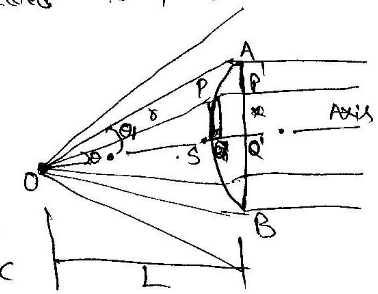
$$r = \frac{L(1-n)}{(1-n\cos\theta)} \quad //$$

[Or]

'PTO'

Non-metalic Di-Electric lens :-

Di-Electric lens is identical to optical lens Consider a non metalic di-electric and a feed element is placed infront of it so that the spherical wavefront are converted to plane wave and it depends on the geometry of the lens.



From fig the spherical wave incident on the lens and is denoted by the surface of the lens origin O to AB of the lens should have equal path length if we take isotropic lens.

let C is velocity of light in air and 'V' is velocity of light in medium for equal path length.

$$OP + PP' = OS + QQ'$$
$$OP + PP' = OS + SQ + QQ' \rightarrow (1)$$
$$\therefore \text{from fig} \quad PP' = QQ'$$

now Eq (1) is

$$OP + \cancel{QQ'} = OS + SQ + \cancel{QQ'}$$
$$OP = OS + SQ$$
$$r = L + x \rightarrow (2)$$

divide with 'c' on B.S

$$\frac{r}{c} = \frac{L}{c} + \frac{x}{V} \quad \left(\because \frac{x}{V} = V \text{ is} \rightarrow \text{velocity of light in medium}\right).$$

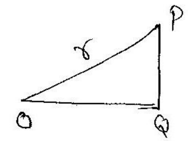
Multiple 'c' ob.s

$$\frac{r}{c} \times c = \frac{L}{c} \times c + \frac{x}{V}(c)$$
$$r = L + x\left(\frac{c}{V}\right) \quad \left(\because \text{R.I } n = \frac{c}{V}\right)$$
$$\boxed{r = L + n.x} \rightarrow (2)$$

Here x is given by

$$SQ = OQ - OS$$
$$x = OQ - L \rightarrow (3)$$

OQ is given by right angle Δle



$$\cos\theta = \frac{OQ}{OP}$$

$$OQ = OP\cos\theta$$

$$OQ = r\cdot\cos\theta$$

Eq (3)

$$x = r\cos\theta - L \rightarrow (4)$$

substitute x value in Eq (2)

$$r = L + (r\cos\theta - L).n$$

$$r = L + n\,r\cos\theta - nL$$

$$r - n r\cos\theta = L - nL$$

$$r(1 - n\cos\theta) = L(1-n)$$

$$\boxed{r = \frac{L(1-n)}{(1-n\cos\theta)}} \rightarrow (5)$$

//.

Here above expression gives geometry (or) contour of lens medium.

$$r = \frac{L(n-1)}{(n\cos\theta - 1)} \rightarrow (6)$$

The geometry depends on R.I 'n', distance from source to lens medium and also angle.

Let the angle 'θ' is small then $\boxed{r = L(n-1)}$

from Eq (4) cross multiply

$$r(n\cos\theta - 1) = L(n-1)$$

$$n\cos\theta - 1 = \frac{L(n-1)}{r}$$

$$n\cos\theta = \frac{L(n-1)}{r} + 1$$

L

if $r \rightarrow \infty$

$$n\cos\theta = 1$$

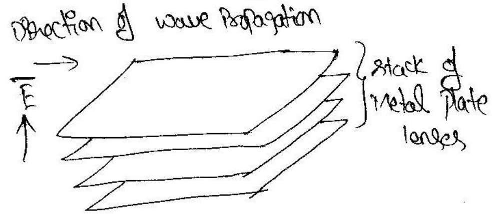
$$\cos\theta = \frac{1}{n}$$

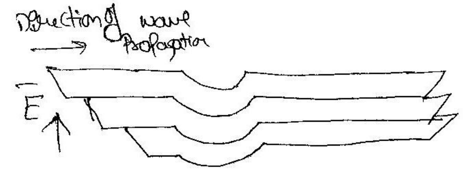
$$\theta = \cos^{-1}\left(\frac{1}{n}\right)$$

Means the angle with which the wave is on lens also depends on R.I //.

H-Plane Metal Plate lens Antenna:-

Can be achieved by arranging a stack of metal plates coinciding the orientation of H-plane (or) perpendicular to E-plane





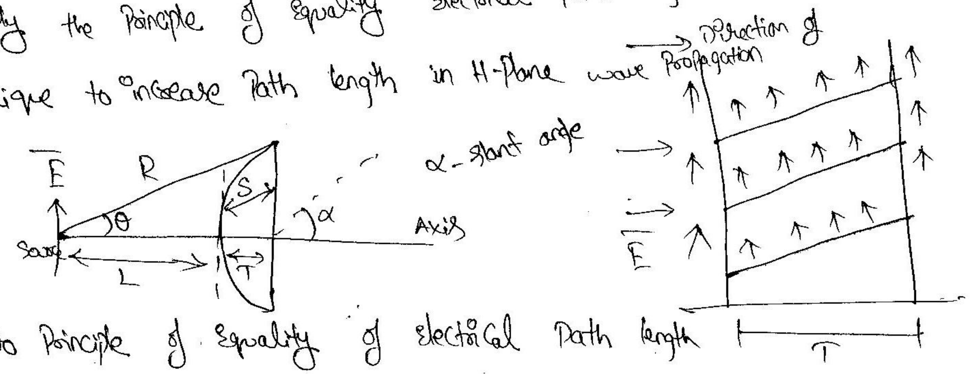
Deformed metal plate.

When the metal plates are arranged parallel to H-plane the velocity of the wave remain almost same but the path length decreases. now to increases the path length & to limit wave constraining there are two techniques.

1. To deform plates so that the path length is increased
2. In another the plates arranged in slanting mode.

Let us consider how to design a H-plane plate lens using the metal plate arranged in the slanting plate mode. for this we need to apply the principle of equality electrical path length.

Technique to increase path length in H-plane



Ac to principle of equality of electrical path length condition

$$R = L + \frac{R\cos\theta - L}{\cos\alpha}$$

let n be the R.I of slant plate lens medium, we can write

$$R = L + \frac{R\cos\theta - L}{\left(\frac{1}{n}\right)}$$

$$\therefore\ R = L + n R\cos\theta - nL$$

$$R - nR\cos\theta = L - nL$$

$$R(1 - n\cos\theta) = L(1-n)$$

$$R = \frac{(1-n)\cdot L}{n\cos\theta - 1}$$

Antenna Measurements:-

There are 2-types of antenna measurements there are

1) → Impeadence measurements
2) → Pattern measurements

* → under impeadence measurements we measure like i/p impeadence and mutual impeadence
* → under pattern measurement we measure like radiation pattern, directivity gain, Polarisation etc;

1) Impeadence Measurement :-

We measure the impeadence value at 2-freq (Low & high frequencies).

At Low freq to measure the impeadence we use wheat stone Bridge method. and at high freq we measure the impeadence by using slotted line method.

Impeadence at a pair of terminals is given by the ratio of volt and current $Z = \frac{V}{I}$

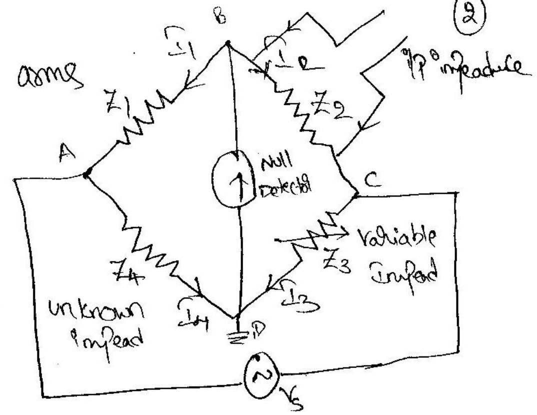
where Z - is a complex quantity and is given by $R+jX$

$$R+jX = \frac{V\angle\theta}{I\angle\theta}$$ $X \rightarrow$ is reactive component.

Bridge Method for impeadence measurements at Low freq's:-

At Lower freq's to measure the impeadence we use Wheat stone bridge method it contains 4-arms each is having an impeadance. This method is used to find unknown imped, Inductance, Capacitance.

②



Where Z_1 & Z_2 are called as ratio arms

$Z_3 \rightarrow$ variable impeadence

$Z_4 \rightarrow$ unknown "

We can change the variable impeadence till the galvonometer represents the null value (or) when the bridge is balanced then

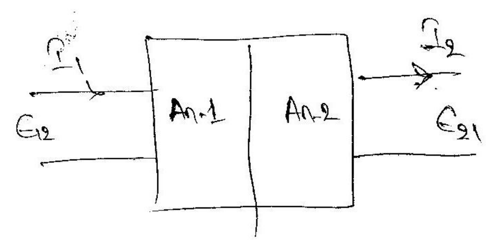
$$\frac{Z_1}{Z_2} = \frac{Z_4}{Z_3} \text{ (unknown)}$$

$$Z_4 = \frac{Z_1}{Z_2} \cdot Z_3 \Rightarrow \frac{Z_1\angle\theta_1}{Z_2\angle\theta_2} = \frac{Z_4\angle\theta_4}{Z_3\angle\theta_3}$$

$$\boxed{Z_1 Z_3 \angle\theta_1+\theta_3 = Z_2 Z_4 \angle\theta_4+\theta_2}$$

Reciprocity Theorem :-

It is defined as when an emf is applied at the terminals of antenna-1 and the current is measured at the terminals of antenna-2 (I_2) and same emf is replaced from i/p terminal to o/p terminal and the current is measured at the i/p terminal (antenna-1). If both currents are same then we say that antenna s/m is following reciprocity theorem.



$$\frac{E_{12}}{I_2} = \frac{E_{21}}{I_1}$$

Properties:-

1) Single antenna can be used as a transmitter and receiver.
2) The properties of both the antenna should be same i.e, field pattern of transmitter and receiver are equal.
3) Power flow at the terminals of antenna -1 (transmitter) is equal to the power flow at the terminals of antenna -2 (R_x)
4) Emf's (freq) at terminals of transmitter and receiver have same freq ranges.
5) The medium b/w the transmitter and receiver should be ideal & isotropic.

*) Slotted line Method for Measurement of i/p impedance of Antenna at High frequency:-

Slotted line is based on the characteristics of the travelling wave along the line. In other words parameters the i/p impedance can be determined such as voltage (or current) standing wave ratio (SWR) the spacing b/w voltage (or current) minimum and reference point at which impedance desired are known.

From the R.f transmission line fundamentals, the load impedance can be expressed in terms of reflection co-efficient 'K' as

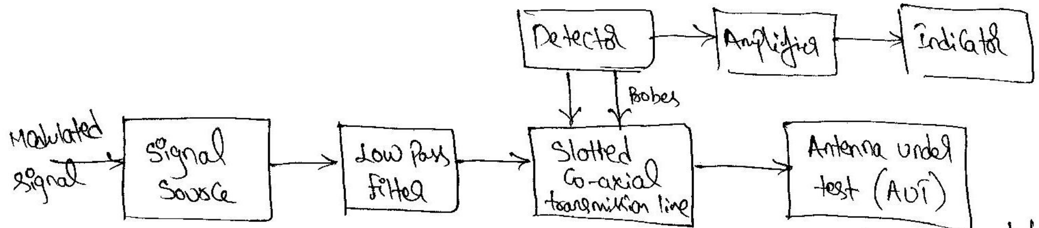
$$Z_L = Z_0\left[\frac{1+K}{1-K}\right]$$

But $K = \frac{S-1}{S+1}$, where $S = SWR = \left|\frac{V_{max}}{V_{min}}\right|$

Also $\theta = \pi - 2\beta d = \pi - 2\left(\frac{2\pi}{\lambda}\right)d = \pi\left(1-\frac{4d}{\lambda}\right)rad = 180\left(1-\frac{4d}{\lambda}\right)$ degrees.

d is spacing load and first voltage minimum. Thus using these equations we can calculate the load impedance, SWR.

The basic setup for measurement of SWR and i/p impedance is



A tx line used is slotted co-axial line which is terminated at load point in an antenna whose i/p impedance is to be measured. If the antenna impedance does not match with the characteristics impedance Z_0 of the transmission line, the reflection at load takes place. Thus incident and reflected wave produces standing wave along the transmission line feeding antenna.

A slotted line arrangement consists a co-axial cable with an axial slot along which probe and probe carriage moves. To the left end of the slotted line a signal source (either a transmitter (or) oscillator) is connected with to the right end antenna is connected. The probe is moved along the slot to obtain standing wave pattern resulting in variation in the o/p of crystal detector. While the probe is moved, two successive voltage max and min points are noted. The ratio of V_{max} and V_{min} gives value of SWR. Also the distance b/w load and first voltage minimum point gives parameters. Then we can calculate reflection co-efficient $K\angle\theta$ ''.

Then smith chart fundamental eq given by

$$Z_s = Z_0 \left[\frac{1 + K \cdot e^{-2\gamma l}}{1 - K \cdot e^{-2\gamma l}} \right] ''$$

*) Measurment of Directivity :-

Directivity is defined as the ratio of maximum value of radiation intensity to average value of radiation intensity

$$D = \frac{U_{max}(\theta,\phi)}{U_{avg}(\theta,\phi)}$$

WKT $U_{avg} = \frac{W_r}{\Omega} = \frac{W_r}{4\pi}$

$$\therefore D = \frac{U_{max}(\theta,\phi)}{\frac{W_r}{4\pi}} = \frac{4\pi\, U_{max}}{W_r}$$

WKT. $W_r = \int_0^{2\pi}\int_0^{\pi} U_{avg} \sin\theta \, . \, d\theta \, . \, d\phi$

$$D = \frac{4\pi\, U_{max}}{\int_0^{2\pi}\int_0^{\pi} U_{avg} \sin\theta . d\theta . d\phi} \rightarrow (1)$$

Directivity can be measured by knowning the Radiation intensity and it can be find by two methods

1. orrange slice method
2. Conical Method

1. Orrange slice method :-

In this method for a discrete value of 'π' radiation Pattern is measured w.r.t 'θ'.

→ Each Pattern is multiplied with a factor $\sin\theta$ and then integrated.

→ The integrated value for several Patterns are obtained according to eq (1).

* Conical Cut method (or) CM:-

In this method for discrete values of θ a set of patterns are measured (R.I) w.r.t ϕ.

→ Each pattern is multiplied by a factor $\sin\theta$ and is integrated, to get the directivity according to Eq (1).

* Measurement of Antenna Efficiency:-

Antenna efficiency depends on aperture area of the antenna, radiated power and loss in power.

→ Based on characteristics of antenna efficiencys 2-types

(a) Antenna Radiation Efficiency

(b) Antenna Apperture Efficiency.

(a) Antenna Radiation Efficiency:-

It is defined as the ratio of radiated power by total power

$$\eta_R = \frac{\text{Radiated Power}}{\text{total Power}} = \frac{W_r}{W_T} = \frac{W_r}{W_r+W_e} \rightarrow (1)$$

$$\eta_R = \frac{W_r}{W_r+W_e} = \frac{I_{rms}^2 R_r}{I_{rms}^2 R_r + I_{rms}^2 R_e}$$

$$\eta_R = \frac{R}{R_r+R_e} \rightarrow (2)$$

And also antenna efficiency is given by terms of gain's

$$G_p = \eta G_d$$

$$\eta = \frac{G_p}{G_d} \rightarrow (3) \qquad 0 \leq \eta \leq 1$$

from Eq (1)

$$\eta_R = \frac{W_r}{W_T} = \frac{W_T - W_L}{W_T} \quad \left(\because W_T = W_r + W_e\right) \quad W_r = W_T - W_L$$

$$\eta_R = \left(1 - \frac{\omega_L}{\omega_T}\right) \rightarrow (4)$$

from above Eq we know total Power by substracting total Power and received Power than we get loss in Power ~~from~~ received Power then we get loss in Power by substituting the value in Eq (4) we get radiation Efficiency.

(b) Antenna Apperture Efficiency :-

It is related to directive gain of an antenna and can be fixed by the concept of maximum apperture area and its optimum value and are related by

$$A_{emax} = \eta_A \cdot A$$

$$D = \frac{4\pi}{\lambda^2} A_{e max} = \frac{4\pi}{\lambda^2} \eta_A \cdot A$$

$$\eta_A = \frac{D\lambda^2}{4\pi A} \text{ //.}$$

*) Measurement of Gain (or) Directive gain *

Directive gain (or) gain of the antenna can be defined as ratio of radiation intensity of an antenna in a particular direction to the average radiation intensity of an antenna in that direction

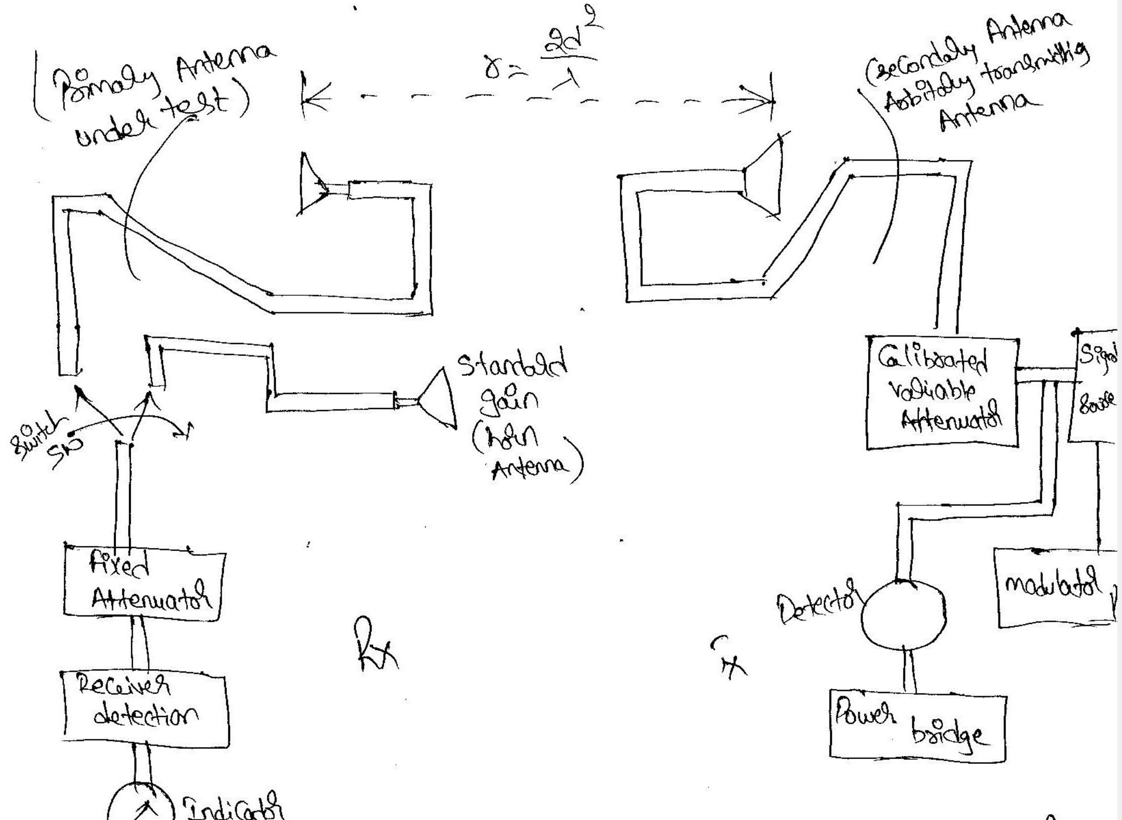
$$G_d = \frac{\text{R.I of an antenna in a particular direction}}{\text{Avg R.I of an antenna in that direction}} = \frac{U_{(\theta,\phi)}}{U_{avg}}$$

The gain of antenna can be measured by using 3-methods.

1. Direct Comparision method (or) Gain transfer.
2. Absolute gain method
3. Three antenna method.

(1) Direct Comparision method:-

In this method we are going to find the gain of the antenna by Comparing the signal strength of (transmitter (or) receiver with the standard gain antenna, mostly horn antenna Can be used as a standard gain antenna.



This method is used at high freq's and the Experimental setup Contains 2-Antenna's (Primary & secondary)

→ Primary Antenna has 2-Antennas (Standard gain antenna & Subject antenna) these are receivers

→ Secondary is a transmitter whose gain is unknown.

By using a switch we can change receiver position to either standard antenna (or) subject antenna.

To avoid the direct and indirect rays combination the antennas are separated by a distance of $r \geq \frac{2d^2}{\lambda}$

The primary antenna has 2-antennas standard and subject antenna they are seperated by proper distance to avoid the coupling b/w the antennas.

Attenuator is used to pass the signal of received strength power bridge is used to maintain stability in the power supply radiations.

→ Measurement :- 2-steps involved in measuring

1) The switch is connected to standard antenna and to the receiver. Antenna is adjusted in the direction of secondary antenna to have a max signal intensity. The transmitter is adjusted to radiate the signal by using attenuator & power bridge let the values be ω_1 & P_1

2) The switch is connected to subject antenna and to the receiver. The switch is connected to subject antenna and to the receiver signal intensity. The attenuator antenna is adjusted to get max signal intensity. The attenuator and power bridge are adjusted to receive the same readings at the receiver as we got in the previous case let the values be ω_2 & P_2.

Case(i) :- If $P_1 = P_2$ then the gains are equal and no corrections is required the power gain is given by

$$G_p = \frac{\omega_2}{\omega_1} \Rightarrow 10 \log G_p = 10 \log\left(\frac{\omega_2}{\omega_1}\right)$$

$$\Rightarrow \boxed{10 \log G_p = 10 \log(\omega_2) - 10 \log \omega_1}$$

Case(ii) If $P_1 \neq P_2$ then a correction pattern is multiplied with G_p that is given by correction of gain

$$G_c = G_p \cdot P \Rightarrow P = \frac{P_1}{P_2}$$

$$10 \log G_2 = 10 \log (G_p \cdot P)$$

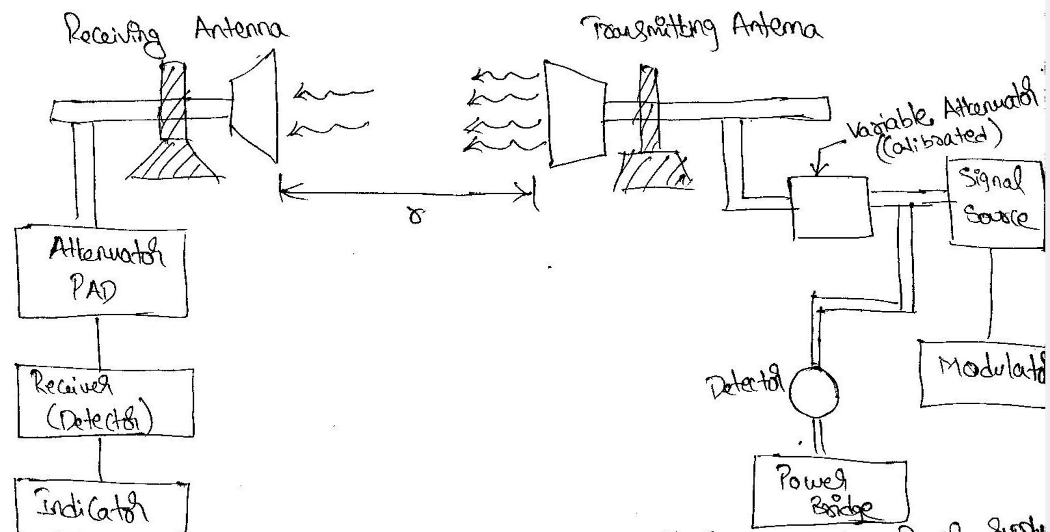
$$= 10 \log G_p + 10 \log P$$

$$\Rightarrow \boxed{10 \log G_c = 10 \log G_p + 10 \log \left(\frac{P_1}{P_2}\right)}$$

$$G_{(dB)} = G_{P(dB)} + P_{(dB)}$$

② Absolute Gain method :-

In this method we use two identical antennas to measure the gain and the experimental setup contains Primary antenna & secondary antenna which are seperated by a distance of $r \geq \frac{2d^2}{\lambda}$. Primary antenna is a receiver (R_x) & secondary antenna is transmitter (T_x)



Power bridge is used to maintain the stability in the Power supply radiations & the attenuator is used to allow the signals of required Power level & Receiver indicator display the signal intensity values.

Transmission formulae (or) friss formulae :-

It is defined as the ratio of radiated (received) Power to transmitted Power

$$\frac{W_r}{W_t} = \frac{A_{et}\, A_{er}}{\lambda^2 \cdot r^2}$$

Where A_{et} is Effective appeartule of transmitter
A_{er} - Effective " " receiver
r - distance b/w the antenna
λ - wavelength (or) height of antenna

$$\frac{W_r}{W_T} = \frac{G_t G_r}{\left(\frac{4\pi r}{\lambda}\right)^2}$$

∵ the antennas are identical so the effective appeartule are $A_{er} = A_{et}$

WKT relation b/w directivity & effective appeartule is

$$D = \frac{4\pi}{\lambda^2} A_e \rightarrow (1)$$

$$G_d = D$$

$$G_{d\,max} = \frac{4\pi}{\lambda} A_e \Rightarrow A_e = \frac{G_{d\,max}\,\lambda^2}{4\pi} \rightarrow (2)$$

In general $A_{et} = A_{er} = \frac{G_d \lambda^2}{4\pi}$

WK.T friss formulae in terms of

$$\frac{W_r}{W_T} = \frac{A_{er} \cdot A_{et}}{\lambda^2 r^2} = \frac{\left(\frac{G_d \lambda^2}{4\pi}\right)\left(\frac{G_d \lambda^2}{4\pi}\right)}{\lambda^2 \cdot r^2} = \frac{\left(\frac{G_d \lambda}{4\pi}\right)^2}{r^2}$$

Taking S.O.B.S $\sqrt{\frac{W_r}{W_t}} = \frac{G_d \lambda}{4\pi r} \Rightarrow \boxed{G_d = \frac{4\pi r}{\lambda}\sqrt{\frac{\omega_r}{\omega_t}}}$

Calculating values of r, λ, ω_r & ω_t we can measure gain in antenna.

(3) Three Antenna Method :- In this method we measure the unknown gain of 3-antenna and we require 3-set of measurement by considering a pair of antenna in each set

Step 1 :- Let antenna-1 be a transmitter and receiver and their power be transmitted power P_{t_1} & received power P_{r_1}.

Step-2 :- Let antenna -1, 3, be transmitter & receiver having transmitted power P_{t_2} & received power P_{r_2}.

Step 3:-

Let antenna - 2, 3 be another set having the transmitted & received Power P_{t_3} & P_{r_3}.

By friss formulae.

$$\frac{W_r}{W_t} = \frac{G_t G_r}{\left(\frac{4\pi r}{\lambda}\right)^2} \Rightarrow G_1 G_2 = \frac{W_{r_1}}{W_{t_2}} \times \left(\frac{4\pi r}{\lambda}\right)^2$$

$$10 \log (G_1 G_2) = 10 \log \left[\frac{W_r}{W_t} \times \left(\frac{4\pi r}{\lambda}\right)^2\right]$$

$$10 \log G_1 + 10 \log G_2 = 10 \log \left(\frac{W_r}{W_t}\right) + 10 \log \left(\frac{4\pi r}{\lambda}\right)^2 \rightarrow (1)$$

$$\frac{W_r}{W_t} = \frac{G_1 G_3}{\left(\frac{4\pi r}{\lambda}\right)^2} \Rightarrow G_1 G_3 = \frac{W_{r_1}}{W_{t_3}} \times \left(\frac{4\pi r}{\lambda}\right)^2$$

$$10 \log G_1 + 10 \log G_3 = 10 \log \left(\frac{W_r}{W_t}\right) + 10 \log \left(\frac{4\pi r}{\lambda}\right)^2 \rightarrow (2)$$

$$\frac{W_r}{W_t} = \frac{G_2 G_3}{\left(\frac{4\pi r}{\lambda}\right)^2} = G_2 G_3 = \frac{W_{r_2}}{W_{t_3}} \times \left(\frac{4\pi r}{\lambda}\right)^2$$

$$10 \log G_2 + 10 \log G_3 = 10 \log \left(\frac{W_r}{W_t}\right) + 10 \log \left(\frac{4\pi r}{\lambda}\right)^2 \rightarrow (3)$$

By knowing the values of r, λ, measuring the values of $\frac{W_{r_1}}{W_{t_1}}, \frac{W_{r_2}}{W_{t_2}}$ and $\frac{W_{r_2}}{W_{t_2}}$ we can calculate G_1, G_2, G_3 .//.

Derivation for transmission (or) friss formulae:-

This formulae is used to find the amount of Power loss that has been occured during the transmission of signal from transmitter to receiver.

Let A_{et} Effective apperture of the transmitter, A_{er} Effective apperture of receiver and D be distance b/w T_x & R_x. P_t be the transmitted Power, P_r be the radiated (or) received Power then Power received by receiving antenna per unit area is

$$P_R = \frac{P_T}{4\pi d^2} \; w/m^2 \rightarrow (1)$$

(13)

The Power gain density of the receiver depends on the gain of the transmitting antenna

$$P_D = G_T \times P_R$$

$$= G_T \times \left[\frac{P_T \, W/m^2}{4\pi d^2} \right] \rightarrow (2)$$

It also depends on effective aperture of receiver

$$P_D = G_T \times \frac{P_T}{4\pi d^2} \cdot A_{er} \rightarrow (3)$$

wkt the relation b/w gain & effective aperture of binody antenna is

$$G_T = \frac{4\pi}{\lambda^2} \cdot A_e \rightarrow (4)$$

Sub (4) in eq (3)

$$P_D = P_R = \frac{P_T}{\cancel{4\pi} d^2} A_{er} \cdot \frac{\cancel{4\pi}}{\lambda^2} A_{et}$$

$$P_D = \frac{P_T \cdot A_{er} A_{et}}{\lambda^2 \cdot d^2}$$

$$\boxed{\frac{P_R}{P_T} = \frac{A_{et} \cdot A_{er}}{\lambda^2 \cdot d^2}}$$

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$$G_T = \frac{4\pi}{\lambda^2} A_{et} = A_{et} = \frac{G_T \cdot \lambda^2}{4\pi}$$

$$G_R = \frac{4\pi}{\lambda^2} A_{er} = A_{er} = \frac{G_R \cdot \lambda^2}{4\pi}$$

$$\frac{P_R}{P_T} = \frac{\frac{G_T \lambda^2}{4\pi} \times \frac{G_r \cdot \lambda^2}{4\pi}}{\cancel{\lambda^2} d^2} = \frac{G_T \cdot G_r \cdot \lambda^2}{16\pi^2 \cdot d^2}$$

$$\frac{P_R}{P_T} = \frac{G_T \cdot G_r \cdot \lambda^2}{(4\pi \cdot d)^2} = \frac{G_T \cdot G_r}{\left(\frac{4\pi d}{\lambda}\right)^2} \text{ //.}$$