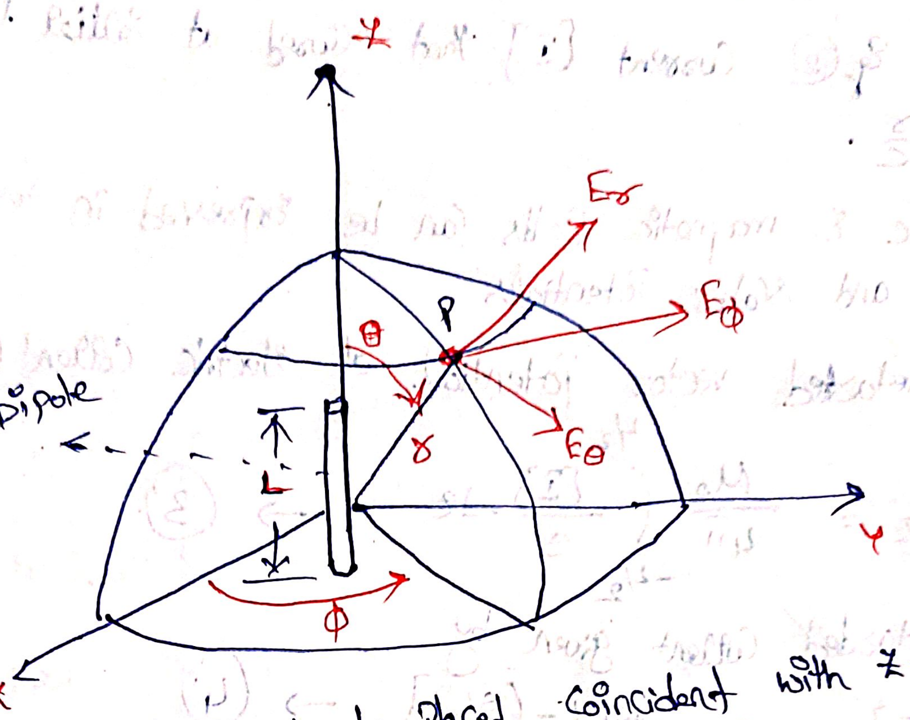
*) Fields of Short Dipole :-

Let consider to find field everywhere around a short dipole.



→ Dipole of length L be placed coincident with Z-axis

→ Electric field components E_r, E_θ, E_ϕ.

→ The current flowing in dipole the effect of current is not felt instantaneously at point-P, but only after an interval equal to the time required for the disturbance to propagate over the distance 'r'.

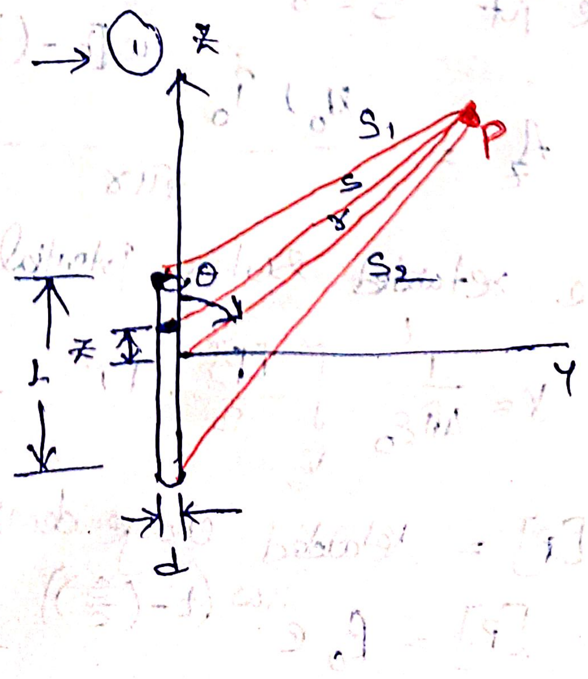
The current I is given as

$$I = I_0 \cdot e^{j\omega t} \rightarrow (1)$$

By Lorentz
The propagation (or retardation) time done is

$$[I] = I_0 \cdot e^{j\omega \{t - (\frac{r}{c})\}} \rightarrow (2)$$

[I] = retarted current



The retardation time $\frac{r}{c}$ results in phase retardation.

$$\frac{\omega r}{c} = \frac{2\pi f r}{c} \text{ radians} = 360^\circ f r/c = 360^\circ t/T$$

w.k.t $T = \frac{1}{f}$

from Eq ② Current $[I]$ that occured at earlier time $t - \frac{r}{c}$.

→ Electric & magnetic fields can be expressed in terms vector and scalar potentials

→ The retarded vector potential of electric current (A_z) is

$$A_z = \frac{\mu_0}{4\pi} \int_{-L/2}^{L/2} \frac{[I]}{s} \cdot dz \rightarrow ③$$

as retarded current given by

$$[I] = I_0 e^{j\omega(t - (s/c))} \rightarrow ④$$

z = distance to a point on the conductor
I_0 = Peak value in time of current
μ_0 = Permeability of free space $= 4\pi \times 10^{-7} Hm^{-1}$

if distance from dipole is large compared to length $(r >> L)$ and wavelength is large compared to length $(\lambda >> L)$

we put $s = r$

$$A_z = \frac{\mu_0 L I_0 e^{j\omega[t - (\frac{r}{c})]}}{4\pi r} \rightarrow ⑤$$

The retarded scalar potential V of charge distribution is:

$$V = \frac{1}{4\pi\varepsilon_0} \int_V \frac{[\rho]}{s} \cdot d\tau \rightarrow ⑥$$

$[\rho]$ = Retarded charge density

$$[\rho] = \rho_0 e^{j\omega(t - (\frac{s}{c}))}$$

$d\tau$ = infinitesimal volume element

ε_0 = Permitivity (or) dielectric constant of free space

$= 8.85 \times 10^{-12}\ F m^{-1}$

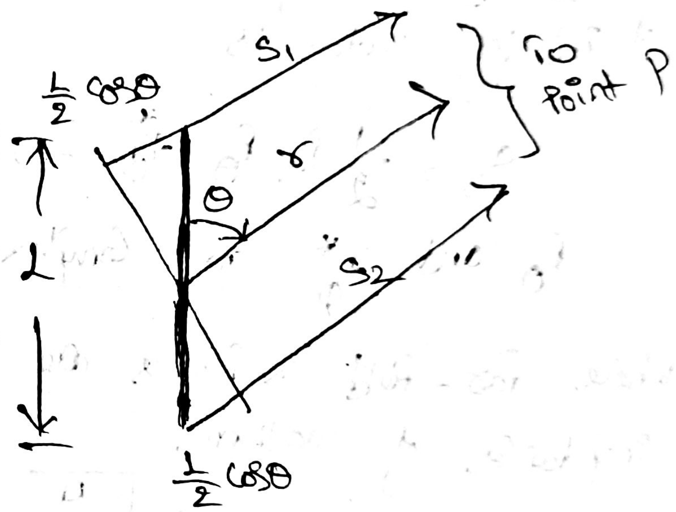
$$V = \frac{1}{4\pi\varepsilon_0}\left\{\frac{[q]}{S_1} - \frac{[q]}{S_2}\right\} \rightarrow (7)$$

where

$$[q] = \int [I] \cdot dt = I_0 \int e^{j\omega(t-(\frac{S}{c}))} \cdot dt = \frac{[I]}{j\omega} \rightarrow (8)$$

substituting (8) to (7)

$$V = \frac{I_0}{4\pi\varepsilon_0 j\omega}\left[\frac{e^{j\omega[t-(\frac{S_1}{c})]}}{S_1} - \frac{e^{j\omega[t-(\frac{S_2}{c})]}}{S_2}\right] \rightarrow (9)$$



when $r \gg L$, the line connecting the ends of dipole of Point P

$$S_1 = r - \frac{L}{2}\cos\theta \rightarrow (10)$$

$$S_2 = r + \frac{L}{2}\cos\theta \rightarrow (11)$$

substitute (10) & (11) into (9)

Electric fields of short dipole

$$E_r = \frac{I_0 L \cos\theta\, e^{j\omega[t-(\frac{r}{c})]}}{2\pi\varepsilon_0}\left(\frac{1}{cr^2} + \frac{1}{j\omega r^3}\right) \rightarrow (12)$$

$$E_\theta = \frac{I_0 L \sin\theta \cdot e^{j\omega(t-(\frac{r}{c}))}}{4\pi\varepsilon_0}\left(\frac{j\omega}{c^2 r} + \frac{1}{cr^2} + \frac{1}{j\omega r^3}\right) \rightarrow (13)$$

from Eq (12) & (13) the relation used as

$\mu_0 \varepsilon_0 = \frac{1}{c^2}$ where c = velocity of light

*) Radiation Resistance of short Dipole :-

→ The Pointing vector of far field is integrated over large sphere to obtaine total Power radiated

The avg Power vector

$$S = \frac{1}{2} Re\,(E \times H^*) \rightarrow (1)$$

where Power is $= I^2 R$

I is rms current on dipole

R is resistance

The far field Component are E_θ & H_ϕ so radial Component of pointing vector is

$$S_r = \frac{1}{2} Re\, E_\theta \cdot H_\phi^* \rightarrow (2)$$

E_θ and H_ϕ^* are Complex

where far-field Components are related by intrinsic impedance of medium.

$$E_\theta = H_\phi Z = H_\phi \sqrt{\frac{\mu}{\varepsilon}} \rightarrow (3)$$

$$\therefore S_r = \frac{1}{2} Re\, Z\, H_\phi \cdot H_\phi^* = \frac{1}{2} |H_\phi|^2 Re Z = \frac{1}{2} |H_\phi|^2 \sqrt{\frac{\mu}{\varepsilon}} \rightarrow (4)$$

The total Power P radiated is then

$$P = \iint S_r \cdot dS$$

$$= \frac{1}{2}\sqrt{\frac{\mu}{\varepsilon}} \int_0^{2\pi}\int_0^{\pi} |H_\phi|^2 r^2 \sin\theta \cdot d\theta \cdot d\phi \rightarrow (5)$$

$|H_\phi|$ is absolute value of magnetic field.

$$|H_\phi| = \frac{\omega I_0 L \sin\theta}{4\pi c r} \rightarrow (6)$$

Substituting eq (5)

$$P = \frac{1}{32}\sqrt{\frac{\mu}{\varepsilon}}\frac{\beta^2 I_0^2 L^2}{\pi^2}\int_0^{2\pi}\int_0^{\pi}\sin^3\theta \cdot d\theta \cdot d\phi \rightarrow (7)$$

The double integral equals to $\frac{8\pi}{3}$. and (7) becomes

$$P = \sqrt{\frac{\mu}{\varepsilon}}\frac{\beta^2 \cdot I_0^2 L^2}{12\pi} \rightarrow (8)$$

∴ P must be equal to square of rms current I flowing on dipole time a resistance R_r.

$$\sqrt{\frac{\mu}{\varepsilon}} \cdot \frac{\beta^2 I_0^2 L^2}{12\pi} = \left(\frac{I_0}{\sqrt{2}}\right)^2 R_r \rightarrow (9)$$

on solving R_r

$$R_r = \sqrt{\frac{\mu}{\varepsilon}}\frac{\beta^2 L^2}{6\pi} \rightarrow (10)$$

In air (or) vaccum $\sqrt{\frac{\mu}{\varepsilon}} = \sqrt{\frac{\mu_0}{\varepsilon_0}} = 377 = 120\pi\,\Omega$

$$\boxed{\text{Dipole with uniform current} \quad R_r = 80\pi^2\left(\frac{L}{\pi}\right)^2 = 80\pi^2 L_\lambda^2 = 790 L_\lambda^2\ (\Omega)}$$