DIGITAL SIGNAL PROCESSING

HAND NOTES

BY

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Digital Signal Processing :-

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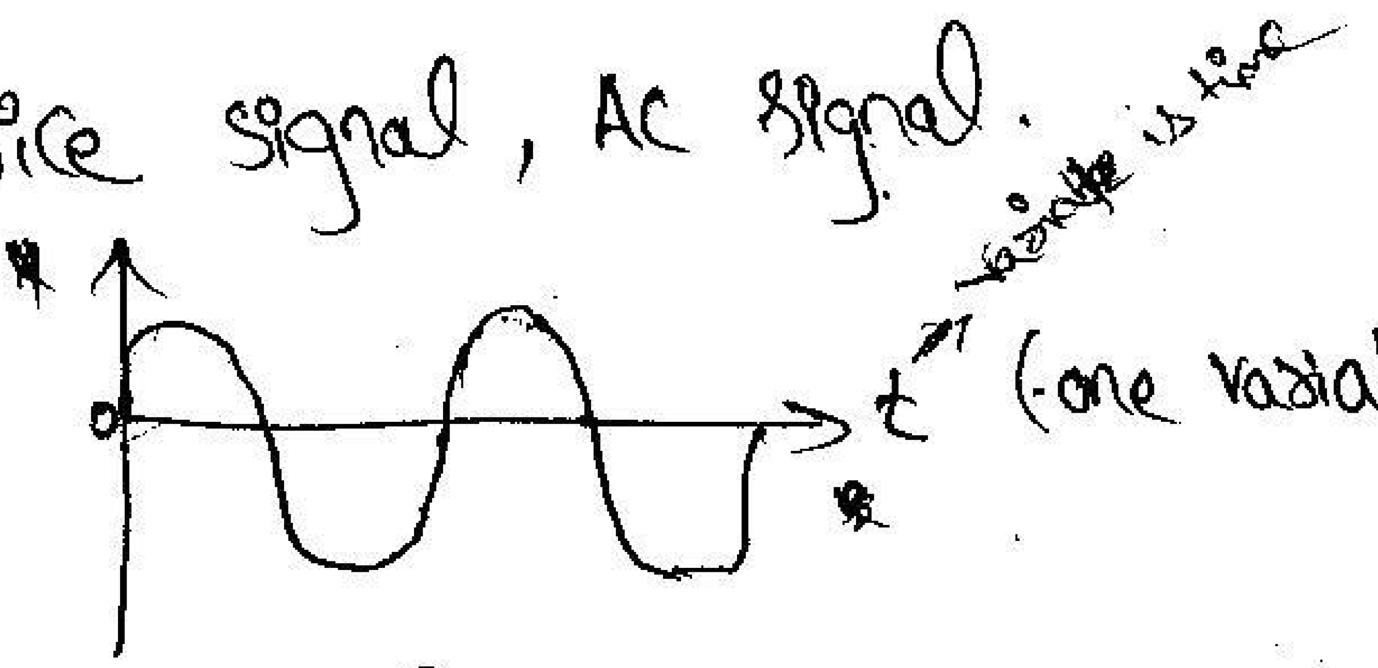
UNIT - 1

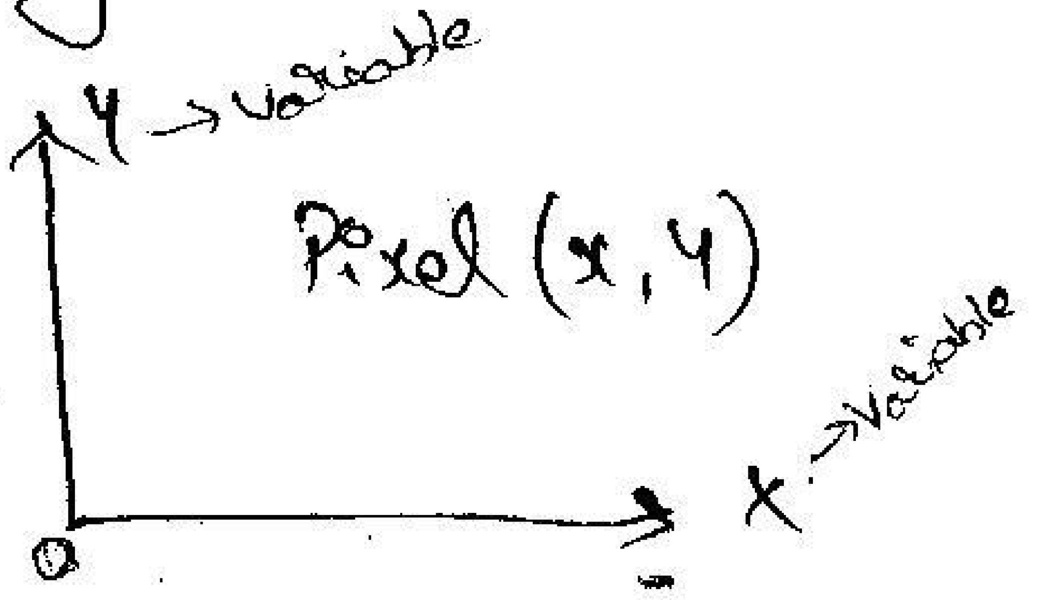
INTRODUCTION TO Digital Signal Processing :-

*) **Signal :-** The Physical quality which varies with time space (or)
The More independent variables $X(t) = F(x_1, x_2, x_3, x_4 \ldots\ldots)$
x_i = time, Space, temperature etc., ----> (independent variables)
Eg :- Audio, video, ECG (Electro Chordio Groom), AC Power supply signal.

*) **One - Dimensional signal :-**
If signal depends on one - variable is called one dimensional signal.
Eg :- voice signal, AC signal.



*) **Two - Dimensional Signal :-**
If the signal depends on two variable is called two dimensional signal.
Eg :- Pixel has two values which consists of X-dimension & Y-dimension
Eg :- Picture, video signal



*) Multi - Dimensional Signal :-

If the signal depends 1 (or) more than 2-variables is called Multi dimensional signal.

Eg :- Speed of the winds.

*) Classification of Signals :-

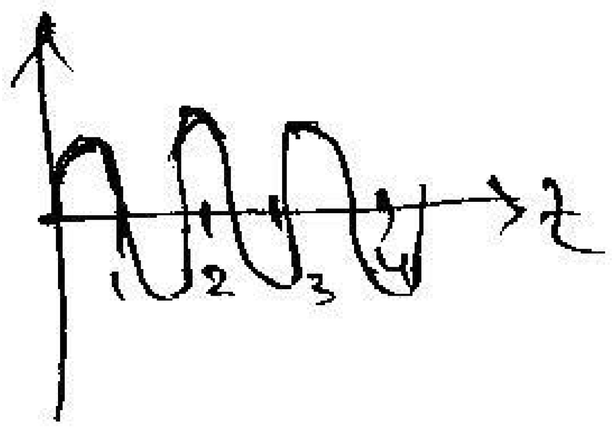
(a) Continous time signals
(b) Discrete - time signals
(c) Digital signals.

(a) Continous time signals :- (or) Analog signal → denoted by $x(t)$

is defined for every instant of time

(or)

Defined over a continous range of time.



Eg :- A mathematical functions is

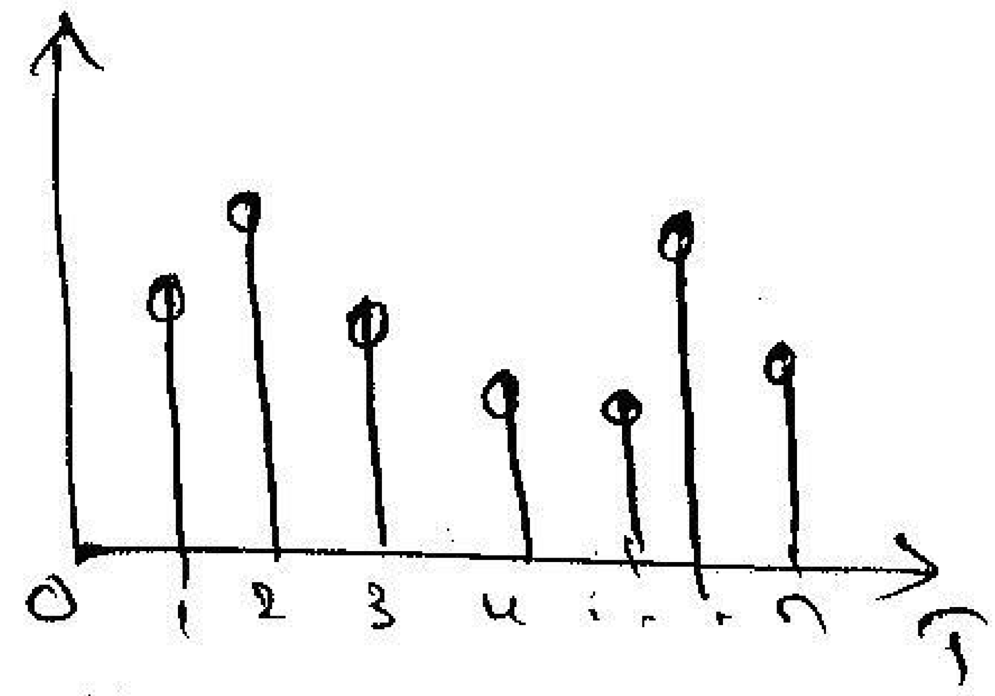
$A \sin(\omega t)$ and $a + bt$

(b) Discrete - time signals :- denoted by $x(n)$

The discrete time signals are defined at discrete instants of time.

Eg :- Sampling Period :

$$x(nT) = x(t)\Big|_{t=nT}$$



T = Sampling interval

n = integer ranging from $(-\infty$ to $\infty)$. Called time index.

For Convenience we write $x(nT) = x(n) = 0, \pm 1, \pm 2 \ldots\ldots$

Thus represented as $x(-2), x(-1), x(0), x(1), x(2) \ldots$

(c) Digital signal:-

Is nothing but the distance time signal which takes the infinite values.

The digital signal is binary signal which takes as whose values Equal to '1' (or) '0'

i.e $x(n) = '0'$ (or) $'1'$ $(-\infty$ to $\infty)$

for $n = -2, -1, 0, 1, 2 \ldots$

Digital Signals that are discrete in time and quantized in amplitude are digital signals.

Problem 1:- Sketch the Continous time signal $x(t) = 2e^{-2t}$ for an interval $0 \leq t \leq 2$. Sample the Continous-time signal with a sampling Period $T = 0.2$ sec and sketch the discrete time signal.

Solution:- Given $x(t) = 2e^{-2t}$ at interval $0 \leq t \leq 2$

at $t = 0$, $x(0) = 2.e^{-2(0)} = 2.e^{0} = 2$

$t = 0.2$, $x(0.2) = 2.e^{-2(0.2)} = 2.e^{-0.4} = 1.34$

"Continous signal" $t = 0.4$, $x(0.4) = 2.e^{-2(0.4)} = 2.e^{-0.8} = 0.89$

$t = 0.6$, $x(0.6) = 2.e^{-2(0.6)} = 2.e^{-(1.2)} = 0.60$

$t = 0.8$, $x(0.8) = 2.e^{-2(0.8)} = 2.e^{-1.} = 0.40$

$t = 1$, $x(1) = 2.e^{-2(1)} = 0.270$

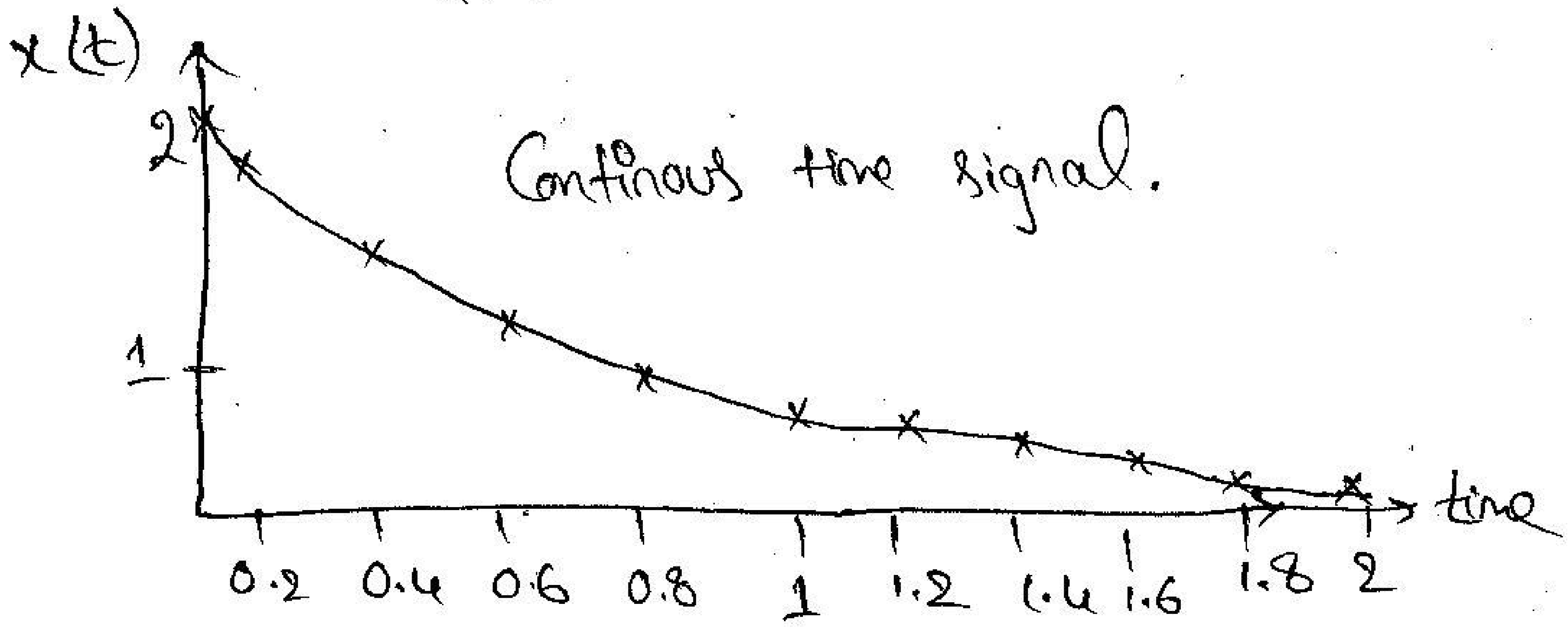
$t = 1.2$, $x(1.2) = 2.e^{-2(1.2)} = 0.1814$

$t = 1.4, x(1.4) = 2 \cdot e^{-2(1.4)} = 0.1216$

$t = 1.6, x(1.6) = 2 \cdot e^{-2(1.6)} = 0.081$

$t = 1.8, x(1.8) = 2 \cdot e^{-2(1.8)} = 0.054$

$t = 2, x(2) = 2 \cdot e^{-2(2)} = 0.0366$



Given sampling time Period $T = 0.2$

for Discrete time signal. we know.

$$x(nT) = x(t)\Big|_{t=nT} \quad \text{where } n = 0, 1, 2, 3 \ldots \infty$$

$$= x(t)\Big|_{t = 0.2 \times n} \quad (\because T = 0.2)$$

$$= x(0.2 \times n)$$

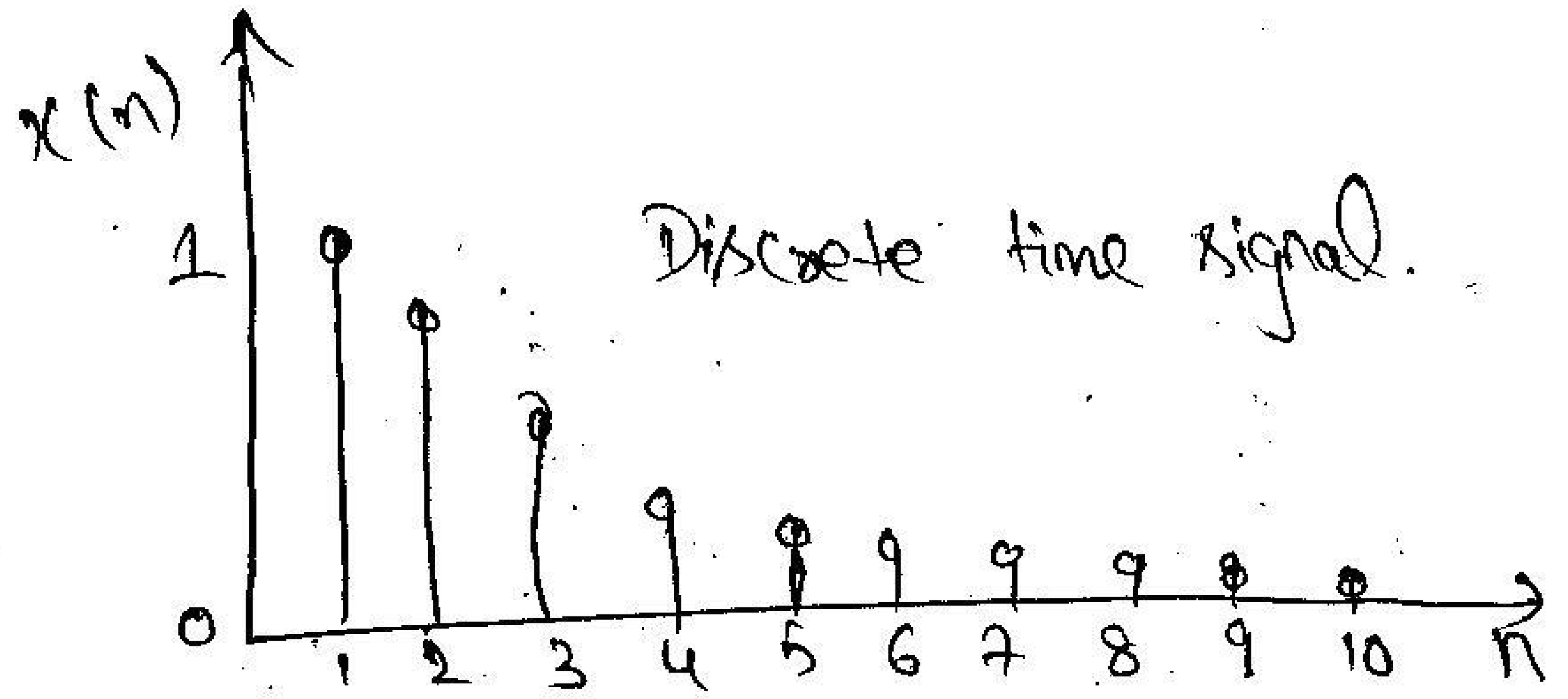
$$\Rightarrow x(n) = 2 \cdot e^{-2(0.2n)} \quad \text{where "} x(t) = 2 \cdot e^{-2t} \text{ given"}$$

$$x(n) = 2 \cdot e^{-0.4n}$$

$x(0) = 2 \cdot e^{-0.4 \times 0} = 2$	$x(6) = 0.1814$
	$x(7) = 0.1216$
$x(1) = 1.3406$	$x(8) = 0.0815$
$x(2) = 0.8987$	$x(9) = 0.0546$
$x(3) = 0.6024$	$x(10) = 0.0366$
$x(4) = 0.4038$	
$x(5) = 0.2707$	

The sequence $x(n)$ can be written as

$$x(n) = \{2, 1.34, 0.89, 0.602, 0.403, 0.2707, 0.1814, 0.1216, 0.0815, 0.0546, 0.0366\}$$



Problem 2:- Sketch the signal $x(t) = \sin 7t + \sin 10t$ for an interval $0 \leq t \leq 2$ sample the signal with a sampling Period $T = 0.2$ sec and sketch the discrete time signal.

Given:- $x(t) = \sin 7t + \sin 10t$; $T = 0.2$ sec

for interval of $0 \leq t \leq 2$

Continous time signal:- w.k.T $x(t) = \sin 7t + \sin 10t$

$t = 0$; $x(0) = \sin 7(0) + \sin 10(0) = 0$

$t = 0.2$; $x(0.2) = \sin 7(0.2) + \sin 10(0.2) = 1.89$

$t = 0.4$; $x(0.4) = \sin 7(0.4) + \sin 10(0.4) = -0.4218$

$t = 0.6$, $x(0.6) = \sin 7(0.6) + \sin 10(0.6) = -1.1510$

$t = 0.8$; $x(0.8) = \sin 7(0.8) + \sin 10(0.8) = 0.3581$

$t = 1$, $x(1) = \sin 7(1) + \sin 10(1) = 0.1130$

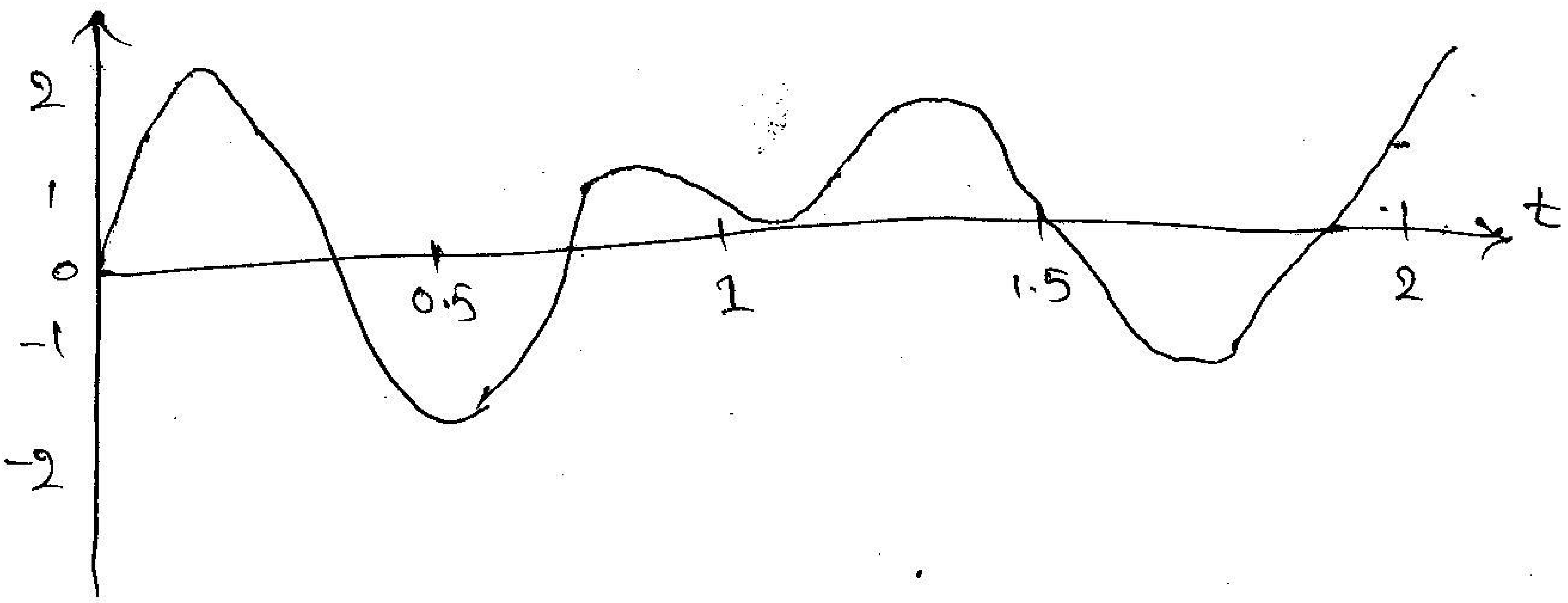
$t = 1.2$; $x(1.2) = \sin 7(1.2) + \sin 10(1.2) = 0.3180$

$t = 1.4$; $x(1.4) = \sin 7(1.4) + \sin 10(1.4) = 0.6241$

$t = 1.6$; $x(1.6) = \sin 7(1.6) + \sin 10(1.6) = -1.2671$

$t = 1.8$; $x(1.8) = \sin 7(1.8) + \sin 10(1.8) = -0.7174$

$t = 2$; $x(2) = \sin 7(2) + \sin 10(2) = 1.9036$



Discrete time signal:-

Given $T = 0.2$ interval

w.k.T $\quad$ ~~x(t)~~ $= x(nT) = x(t)\Big|_{t=nT}$

$\because T = 0.2$

$x(nT) = x(t)\Big|_{t = n\times 0.2}$

$x(nT) = x(0.2n)$ where $n = 0,1,2,3\ldots$

at $n=0$; $x(0) = \sin 7(0.2\times 0) + \sin 10(0.2\times 0) = 0$

at $n=1$; $x(1) = \sin 7(0.2\times 1) + \sin 10(0.2\times 1) = 0.059$

at $n=2$; $x(2) = \sin 7(0.2\times 2) + \sin 10(0.2\times 2) = 0.118$

$n=3$; $x(3) = \sin 7(0.2\times 3) + \sin 10(0.2\times 3) = 0.17$

$n=4$; $x(4) = \sin 7(0.2\times 4) + \sin 10(0.2\times 4) = 0.23$

$n=5$; $x(5) = \sin 7(0.2\times 5) + \sin 10(0.2\times 5) = 0.29$

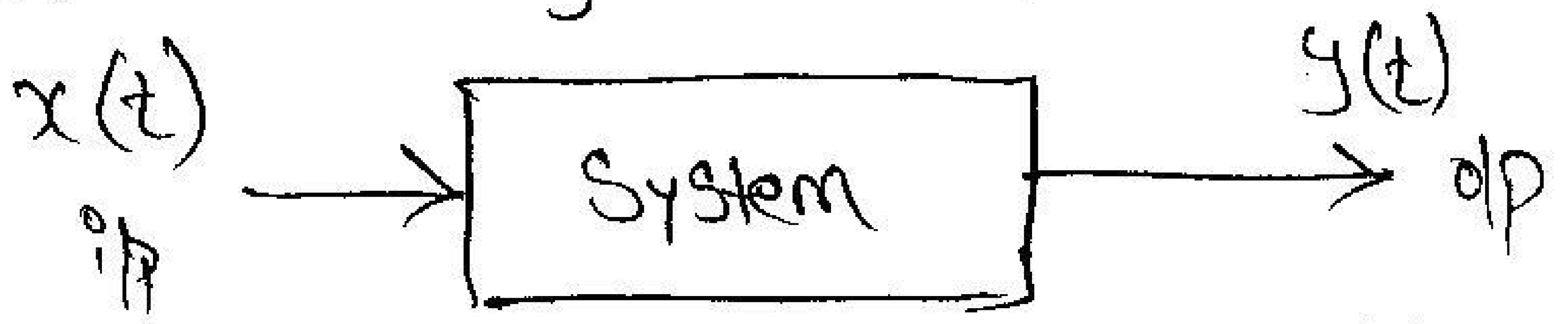
$n=6$; $x(6) = \sin 7(0.2\times 6) + \sin 10(0.2\times 6) =$

$\vdots$

$n=10$; $x(10) = \sin 7(0.2\times 10) + \sin 10(0.2\times 10) =$

3. Problem:- Sketch the signal $x(t) = e^{-t^2/2}$ for $-1 \le t \le 1$. Sample the signal with a sampling period $T = 0.1$ sec and sketch the discrete time signal.

*) **System:-** A system is an inter connection of components. It is a physical device that performs an operation on an i/p signal and produces another signal as o/p.



$y(t)$ = operation on $x(t)$

Mathematically $y(t) = T[x(t)]$

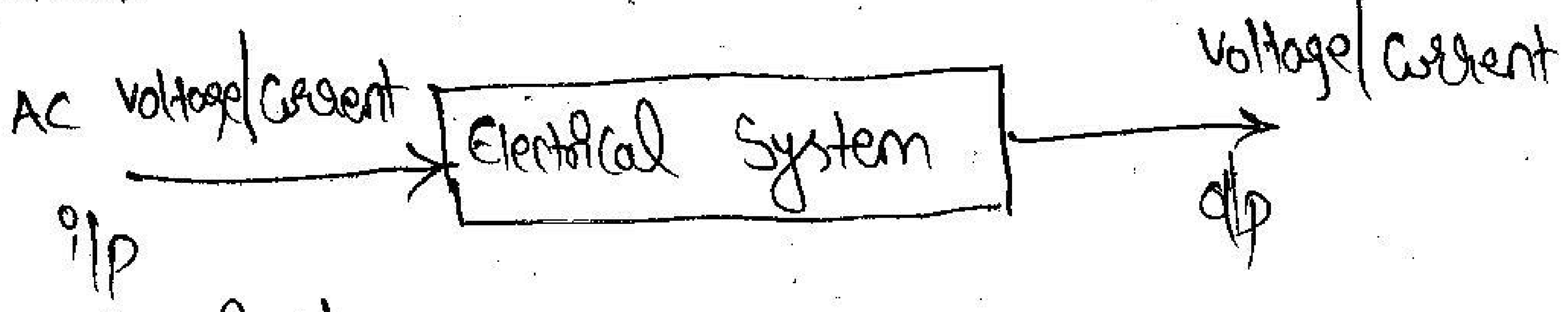
Represents $x(t)$ is transformed to $y(t)$

(or)

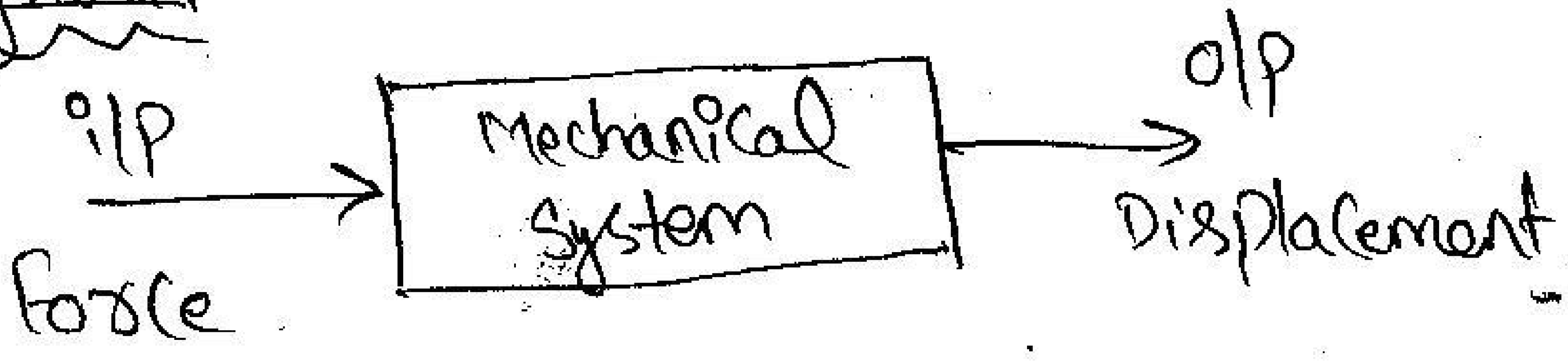
$y(t)$ is the transformed form of $x(t)$

Eg:- ① **An Electrical System:-**

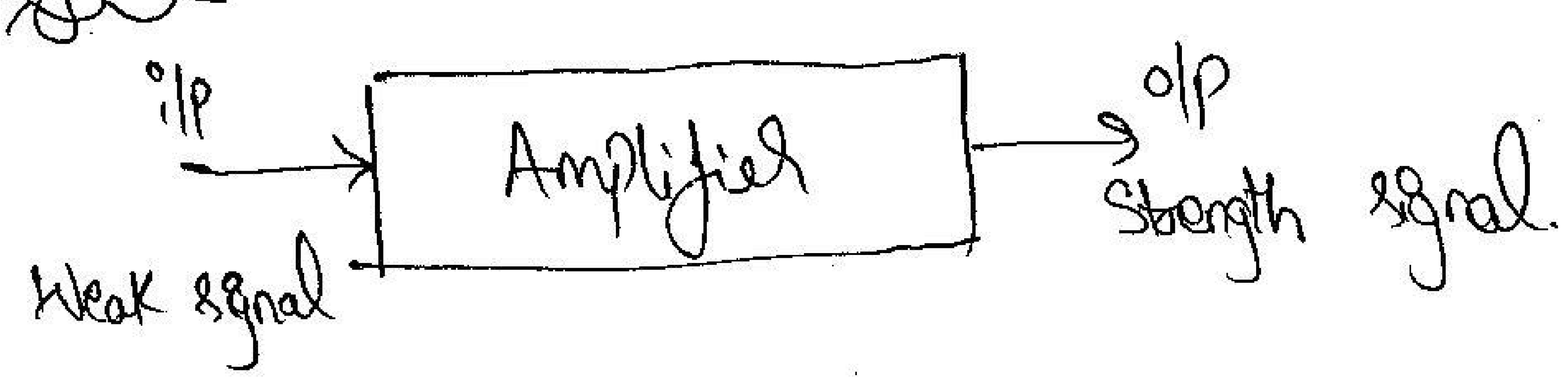
An electrical ckt is a system with i/p's equal to driving voltage/currents and with o/p equal to voltage/currents at various points in circuit



② **Mechanical System:-**



③ **Amplifier System:-**



*) Classification of System :-

(i) Continuous - time system
(ii) Discrete - time System.

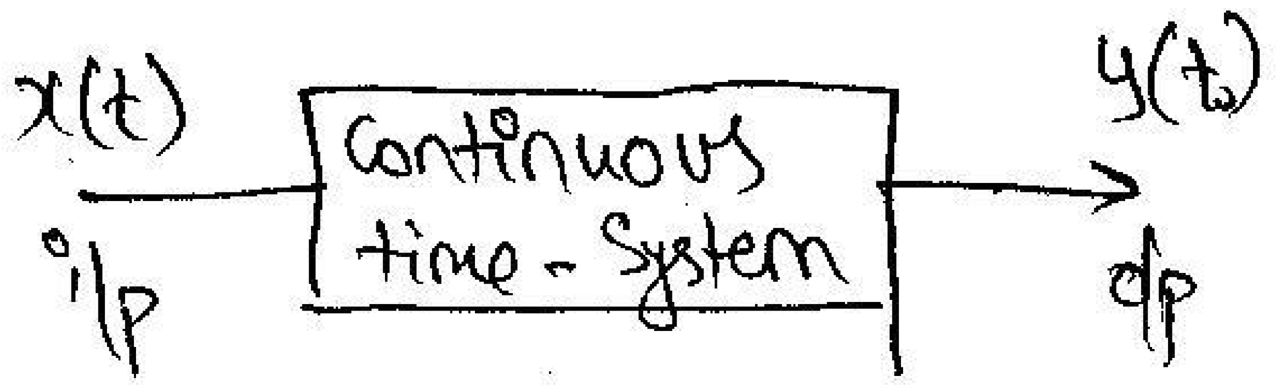
(i) Continuous -time system :-

It is one which operates on continuous time signal and Produces a Continuous - time o/p signal.

if i/p & o/p $x(t)$ & $y(t)$ are Continous then

$x(t)$ is transformed to $y(t)$

$$\therefore y(t) = T[x(t)]$$

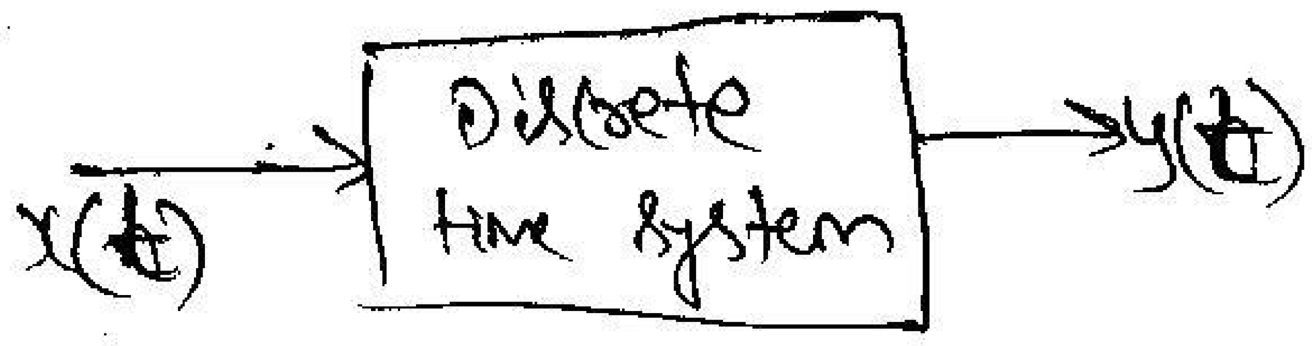


(ii) Discrete - time System :-

is one which operates on discrete time signal and Produces a discrete - time o/p signal.

if i/p $x(n)$ and o/p $y(n)$ then

$$y(n) = T[x(n)]$$



*) Behaviours of a System :-

This is from finding mathematical model of system. Generally mathematical model Consists of Collection of Equations describing the relationship b/w i/p & o/p signals of system.

There are 2-types of basic's of mathematical models

(a) I/p / o/p Representation describing the relationship b/w the i/p and o/p signals of a system.

(b) The state or internal model describing the relationship among the i/p, state and o/p signals of a system.

The i/p / o/p representation of system can be divided into 4-types:

1. The i/p / o/p differential equation (for continuous-time system) (or) difference equation (for descrete-time systems).
2. The convolution model.
3. The Fourier transform representation
4. The system function representation.

*) **Signal Processing:-**

* A System is defined as a physical device that performs an operation on a signal.

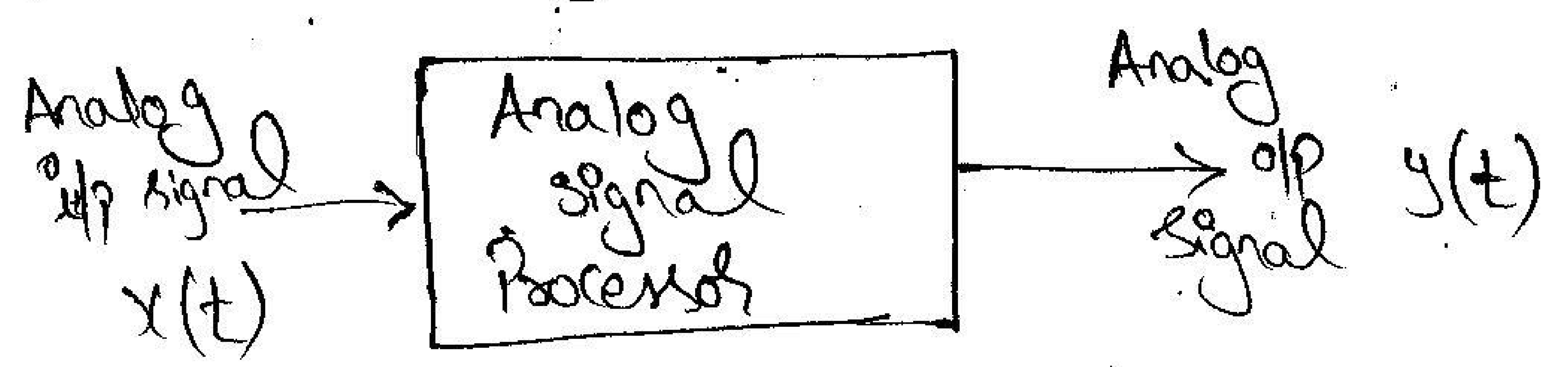
* Signal processing is any operation that changes the characteristics of a signal.

*) These characteristics include the amplitude, shape, phase, and frequency content of the signal.

→ 2 types of signal processings:-

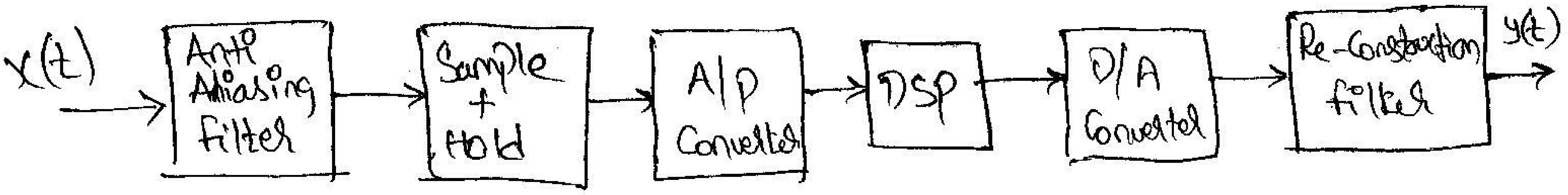
(a) Analog signal processing
(b) Digital signal processing.

(a) **Analog signal Processing:-** The system that process the analog signal is known as analog signal processing system.



(b) Digital Signal Processing System:-

The system that Process the digital signal is known as digital signal Processing system.



*) The i/p signal is applied to an antialiasing filter, which is a low pass filter used to removes high frequencys noise and to band limit the signal.

*) The sample and hold device provides the i/p to the ADC and will be required if the i/p signal must remain relatively constant during the conversion of Analog signal to digital format.

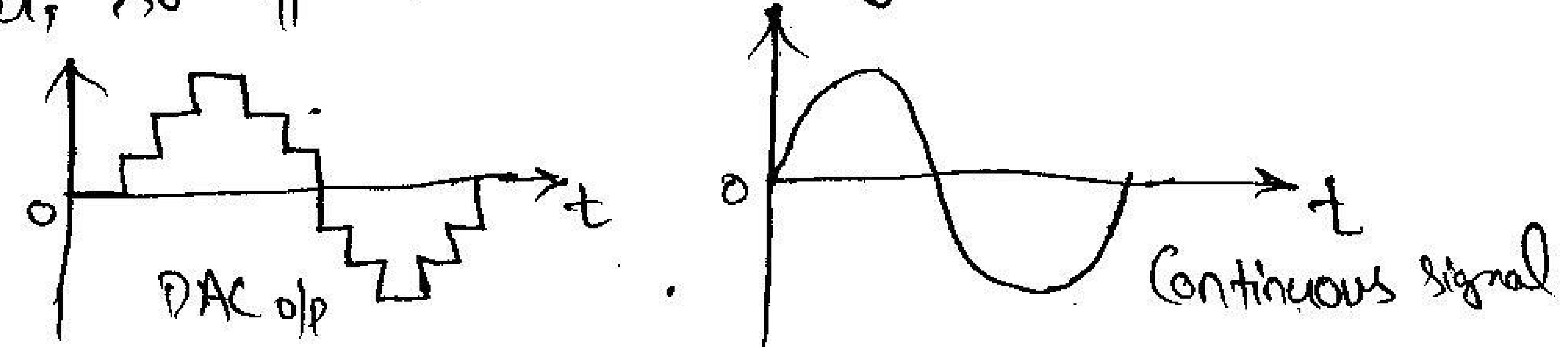
*) The o/p of sample and hold serves i/p to A/D converter the o/p of ADC is an N-bit binary no:- depending on the value of analog signal at its i/p. The ADC i/p signal is limited range 0 to +10V if unipolar, and bipolar means -5V to 5V.

*) After converted digital form, the signal can be processed using digital techniques.

*) The DSP processor may be large programmable digital computer/µP's. to perform desired operation.

*) The Digital signal is applied to DAC. The o/p of DAC is continous but not smooth, which contains unwanted high freq components.

* To eliminate high freq components, the o/p of DAC is applied to a reconstruction filter, so o/p is continous signal.



$x(t)$ → Anti Aliasing filter → Sample + Hold → A/D converter → DSP → D/A converter → Reconstruction filter →

Advantages & Limitations of Digital signal processing

Advantages:-

Digital signal processing posesses several advantages over analog signal processing.

1. **Greater accuracy :-** The tolerance of the circuit compo used to design the analog filters affects the accuracy, whereas DSP provides superior control of accuracy.
2. **Cheaper :-** In many applications, digital realization is comparatively cheaper than its analog counter part.
3. **Ease of Data storage :-** Digital signals can be easily stored on magnetic media without loss of fidelity and can be processed off-line in a remote laboratory
4. **Implementations of sophisticated algorithms :-** The DSP allows to implement sophisticated algorithms when compared to its analog counterpart
5. **Flexibility in configuration :-** A DSP system can be easily reconfigured by changing the program. Reconfi of analog system involves redesign of system hardwa

6. ~~Application~~

6. **Applicability of VLF signals :-** The very low frequenc signals such as those occuring in seismic applic can be easily processed using a digital signal proces when compared to an analog processing system.
7. **Time sharing :-** DSP allows the sharing of a given processor among a number of signals by time sharing thus reducing the cost of processing a signa

Limitations :-

1. System com
digital proces
Such as A/D
2. Bandwidth li
Ba
information los
the bandwidth.
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and digital
3. power consu
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containing
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Applications of

1. Tele commu
Telephone diall
multiplexing,
Phone, FAX.
2. consumer e
synthesizer,
sound record
3. Instrumenta
filter, PLL,
control, Pr

→ D/A converter → Reconsturction filter

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Limitations :-

System complexity :- system complexity increases in the digital processing of an analog Signal because of devices Such as A/D & D/A converters and their associated filters

Bandwidth limited by sampling rate :-

Band limited signals can be sampled without information loss if the sampling rate is more than twice the bandwidth. Therefore, signals having extremely wide bandwidths require fast sampling rate A/D converters and fast digital signal processors. But there is a practical limitation in the speed of operations of A/D converters and digital signal processors.

power consumption :- A variety of analog processing algorithms can be implemented using passive circuit elements like inductors, capacitors and resistors that do not need much power. Whereas a DSP chip containing over 4 lakh transistors ~~disappears~~ dissipates more power.

Applications of DSP :-

Tele communication :- Echo cancellation in telephone n/w, Telephone dialling application, modems, Line repeaters, channel multiplexing, Data encryption, Video conferencing, cellular phone, FAX.

consumer electronics :- Digital Audio/TV, electronic music synthesizer, educational toys, FM stereo applications, sound recording applications.

Instrumentation and control : spectrum analysis, Digital filter, PLL, function generator, servo control, Robot control, process control.

4. Image processing :- Image compression, image enhancem[ent], image analysis and recognition.

5. Medicine : Medical diagnostic instrumentation such as computerized Tomography (CT), X-ray scanning, Magne[tic] resonance imaging, spectrum analysis of ECG, EEG signals to detect the various disorders in heart and brain, patient monitoring.

6. Speech processing :- speech analysis methods are used in automatic speech recognition, speaker verification and speaker identification. speech synthesis techniques include conversion of written text into speech.

7. Seismology :- DSP techniques are employed in the geoph[ysical] exploration for oil & gas, detection of underground nucle[ar] explosion & earthquake monitoring.

8. Military :- Radar signal processing, sonar signal proces[sing], Navigation, secure communications.

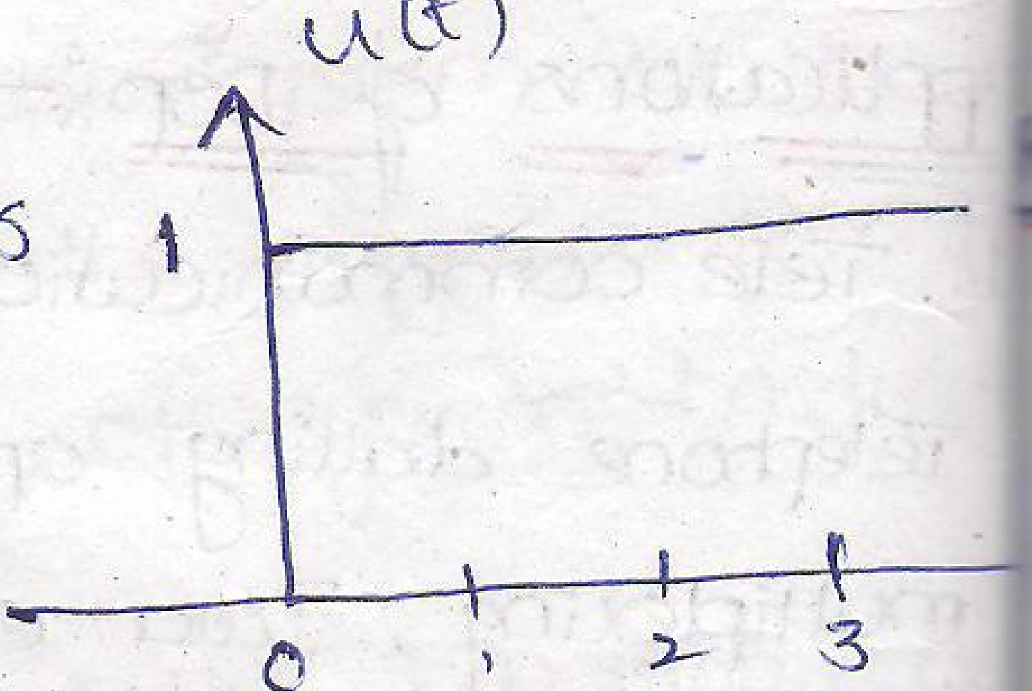
continuous time signals :-

1. unit step function.

unit step function is defined as

$u(t) = 1$ for $t \geq 0$

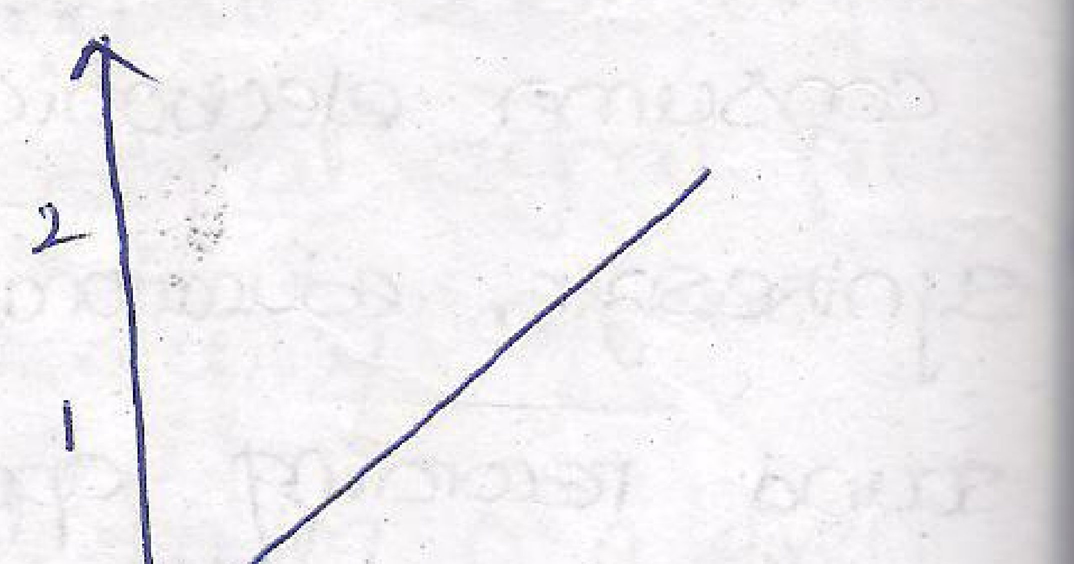
$= 0$ for $t < 0$



2. Unit Ramp Function :-

$r(t) = t$ for $t \geq 0$

$= 0$ for $t < 0$



(or)

$r(t) = t\,u(t)$

$r(t) = \int_{-\infty}^{t} u(\tau)\,d\tau = \int_{0}^{t} d\tau = t \quad (t>0)$

Impulse function

$\int_{-\infty}^{\infty} \delta(t)$

and $\delta(t)$ =

The 1st conditi[on]

the impulse is

States that $\delta($

values of t.

amplitude ever

Sinusoidal signal

$x(t) =$

where A = a

Ω = f

θ = ph

Real exponential

$x(t) =$

Representation of

1. Graphical

2. Functional

3. Tabular

4. Sequence

Graphical repres[entation]

Let us cons[ider]

$x(0) = 2$, $x(1) = 2$

This can

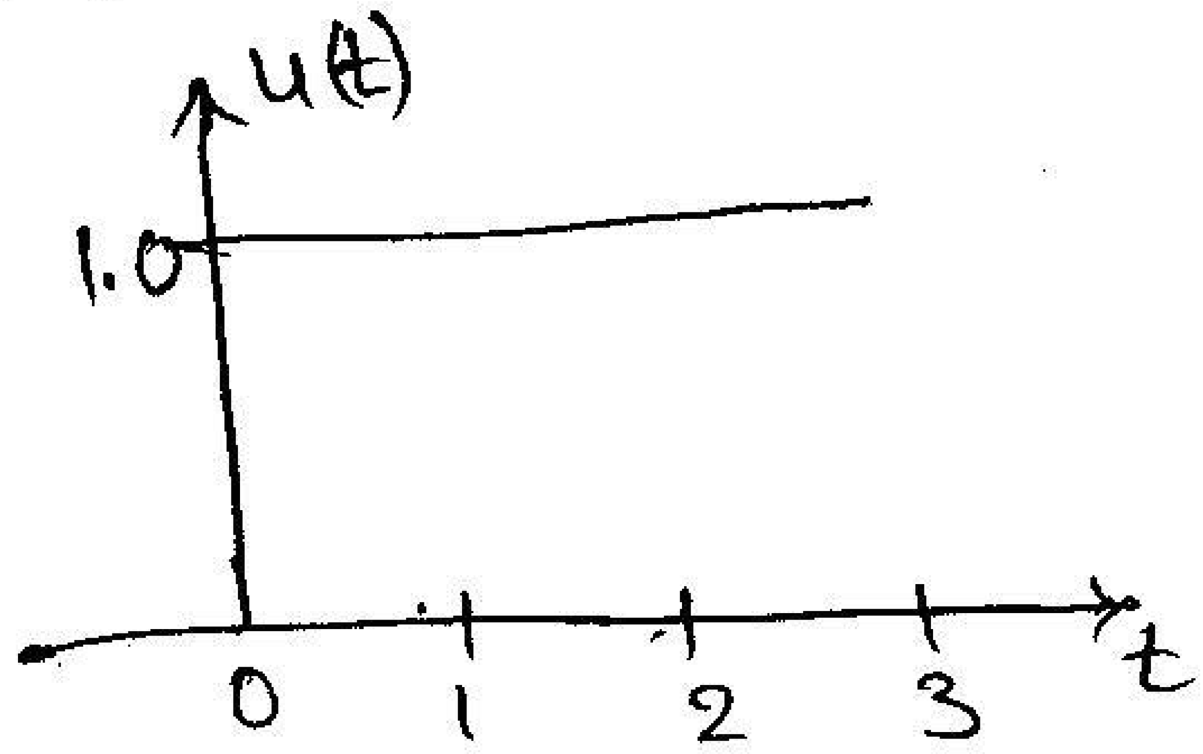
Elementary Continuous time signals:-

1. unit step function:- Amplitude of u(t) is Equal to one.

$$u(t) = 1 \text{ for } t \geq 0$$
$$= 0 \text{ for } t < 0$$

*) If the argument 't' in brackets is less than zero (0), the unit step function is zero

*) If the argument 't' inside " " greater " " (or) Equal to zero the unit step function is unity.



2. unit Ramp function:-

is defined as

$$r(t) = t \text{ for } t \geq 0$$
$$= 0 \text{ for } t < 0$$

(or)

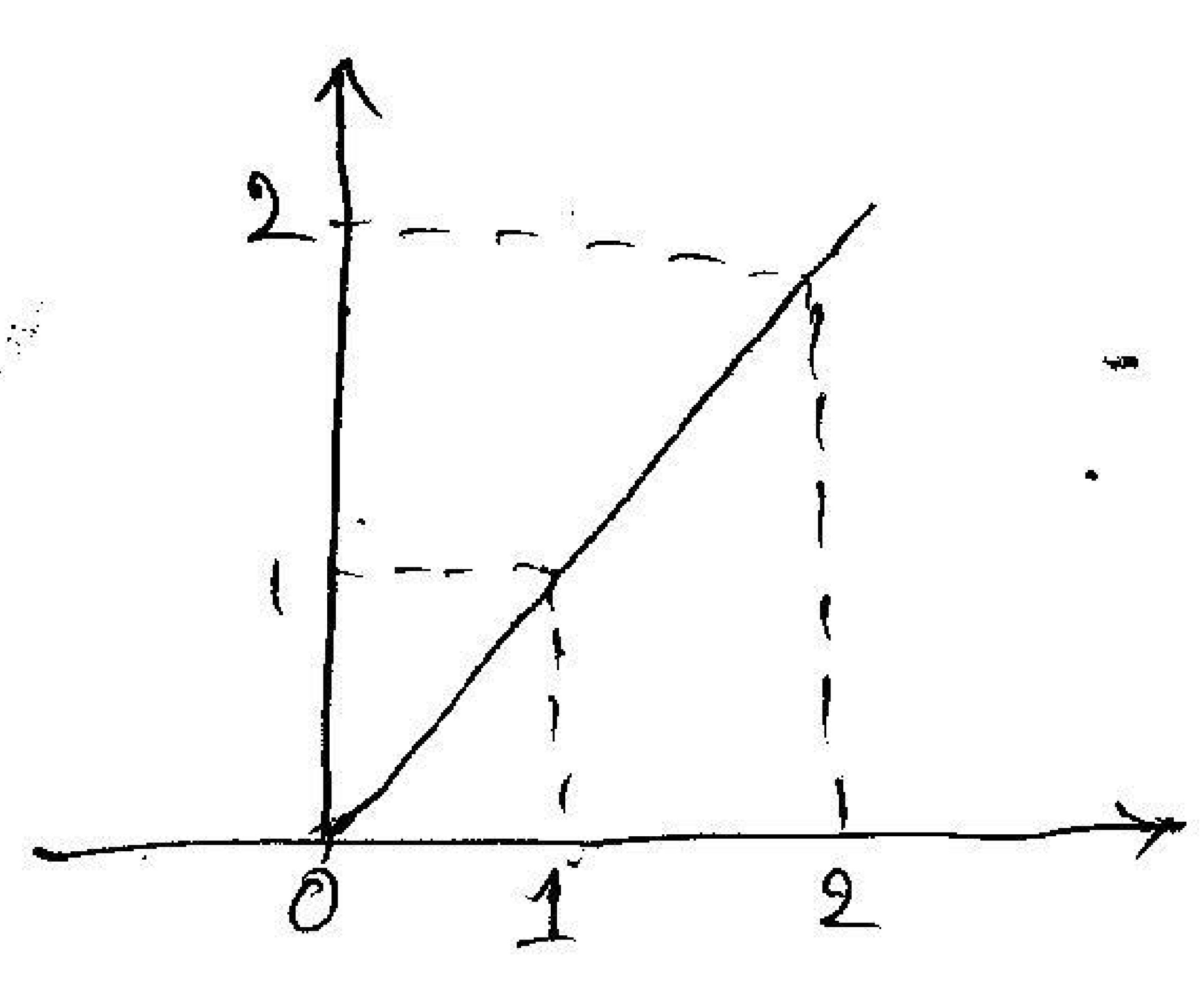
$$\therefore r(t) = t\, u(t)$$

The ramp function can be obtained by applying unit step function to an integrator.

$$r(t) = \int_{-\infty}^{t} u(\tau)\cdot d\tau = \int_{0}^{t} d\tau = t \text{ (in the interval } t > 0)$$

The unit step function can be obtained by differentiating unit ramp

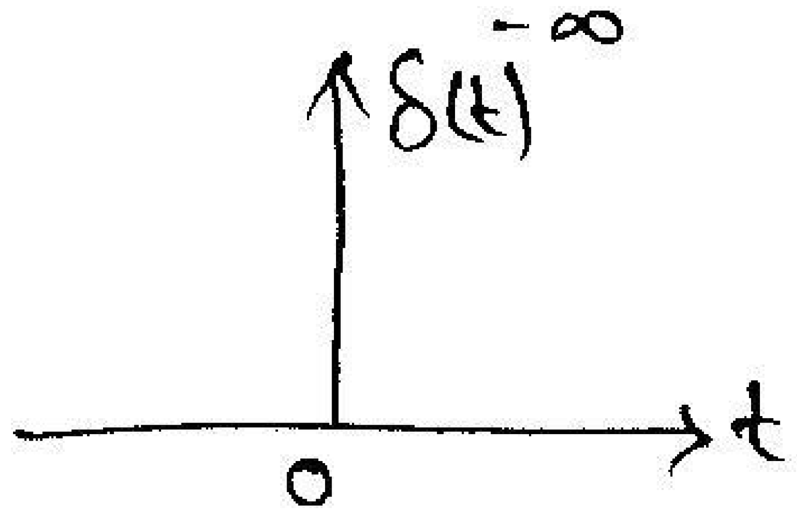
Thus $u(t) = \dfrac{d\,r(t)}{dt}$



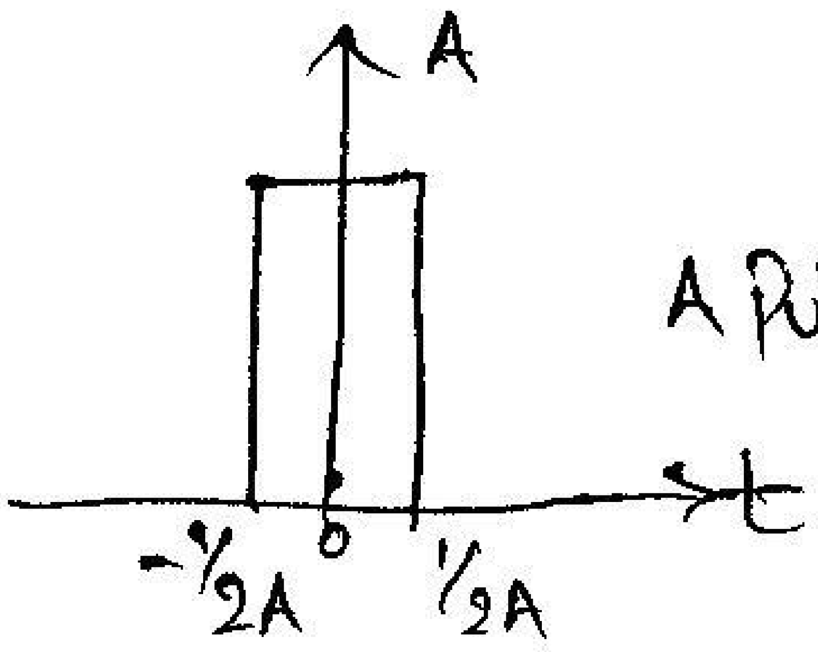
3. Impulse Function:- $\delta(t)$

The unit impulse function $\delta(t)$ is defined as

$$\int_{-\infty}^{\infty} \delta(t).dt = 1 \quad \& \quad \delta(t) = 0 \text{ for } t \neq 0$$



An impulse



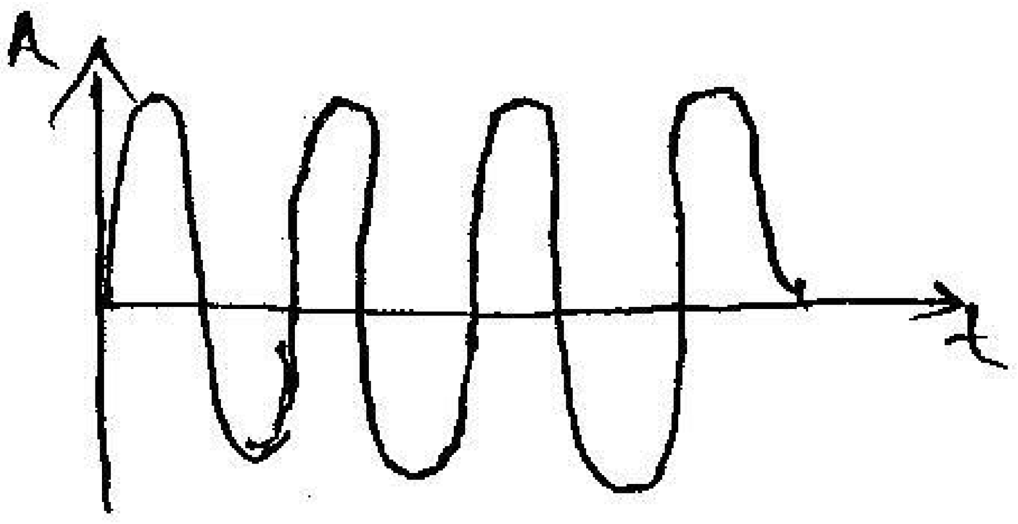
A Pulse

*) First Condition states that the area under the impulse is '1'

*) Second " " " $\delta(t)$ is zero for all non zero value of 't'.

*) In impulse function has zero amplitude every where except at $t = 0$.

4. Sinusoidal signal:-



A Continuous time sinusoidal signal

$$x(t) = A \sin(\Omega t + \theta)$$

A = Amplitude ; Ω = frequency in radius per second

θ = Phase angle in radians.

5. Real exponential signal:-

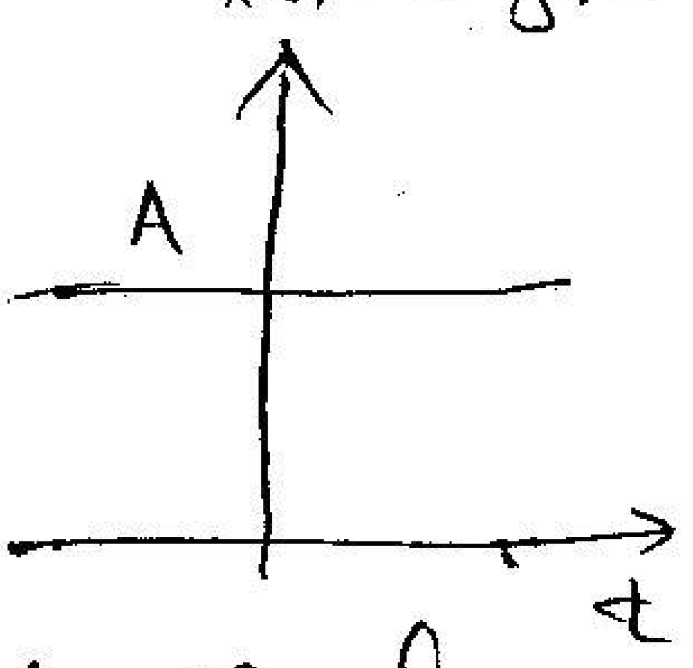
A real exponential signal is defined as

$$x(t) = A.e^{at}$$

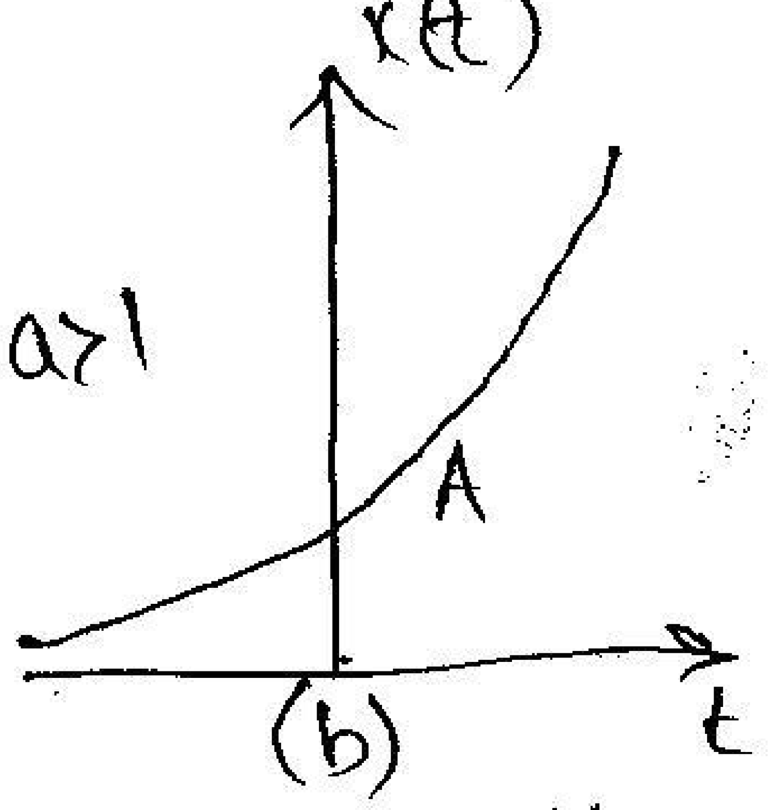
'A' and 'a' both are real. Depending on value of 'a' we get different signals.

If 'a' is Positive the signal $x(t)$ is growing exponentially.

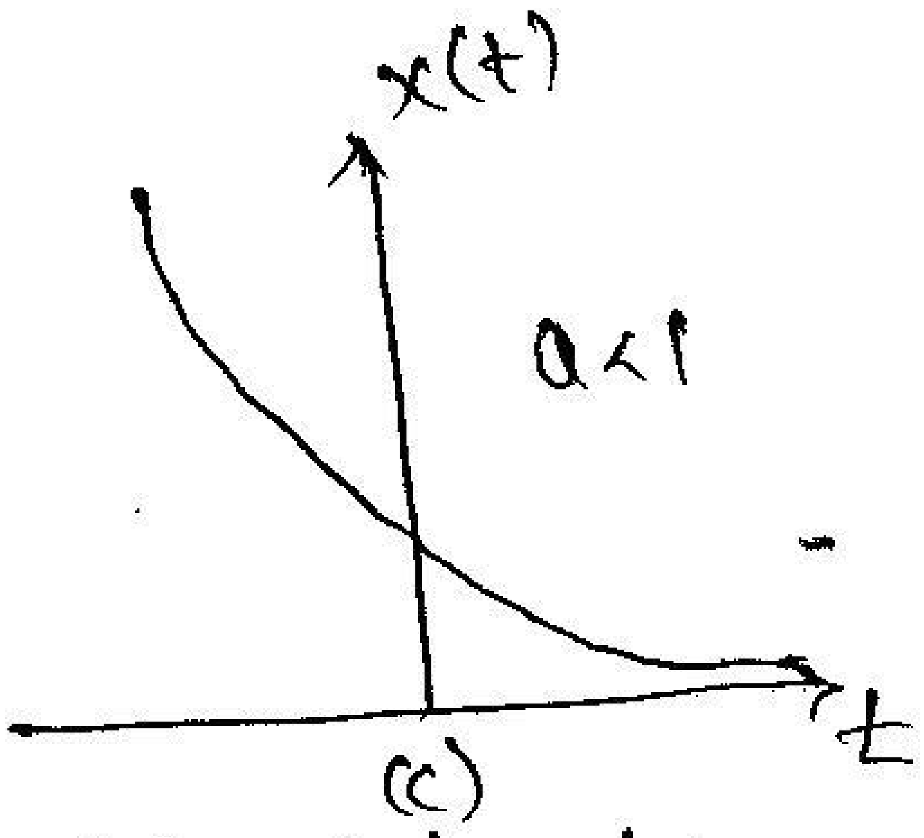
$x(t) = e^{at}$ for $a = 0$



dc - signal



(b)
Exponentially growing signal



(c)
exponentially decaying signal.

*) Representation of Discrete-time Signals:-

11

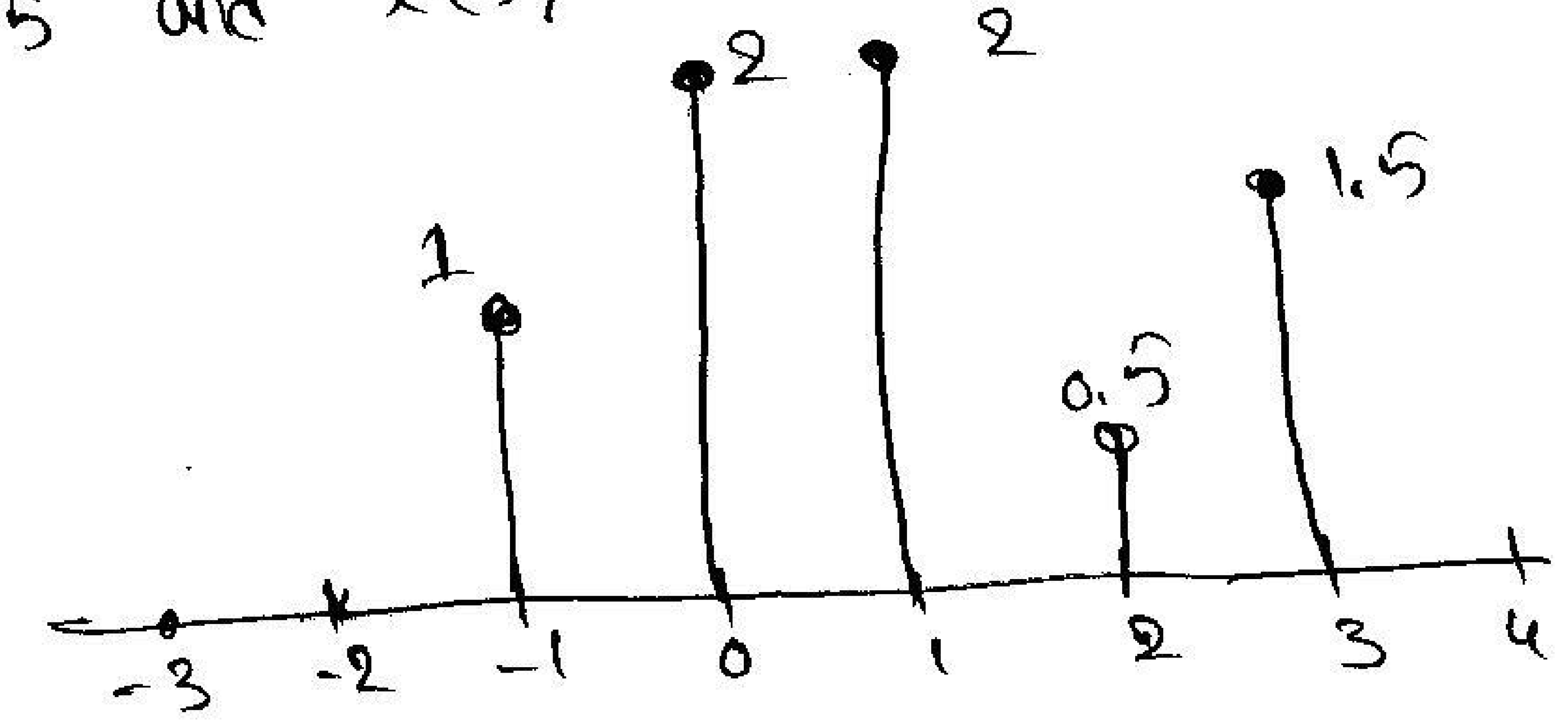
There are 4-types

1) Graphical representation
2) functional "
3) Tabular "
4) sequence "

1. Graphical Representation:-

Eg:- let us consider $x(n)$ with $x(-1)=1$; $x(0)=2$; $x(1)=2$; $x(2)=0.5$ and $x(3)=1.5$

Sol:-



2. Functional Representation:-

The discrete time signal is represented functionally as

$$x(n) = \begin{cases} 1 & \text{for } n=-1 \\ 2 & \text{for } n=0,1 \\ 0.5 & \text{for } n=2 \\ 1.5 & \text{for } n=3 \\ 0 & \text{other wise} \end{cases}$$

3. Tabular representation:-

n	-1	0	1	2	3
x(n)	1	2	2	0.5	1.5

4. Sequence Representation:-

A finite duration sequence with time origin ($n=0$) indicated by symbol '↑' is

$$x(n) = \{1, \underset{\uparrow}{2}, 2, 0.5, 1.5\}$$

A finite duration sequence can be represented as

$$x(n) = \{\ldots 0, 2, \underset{\uparrow}{1}, -1, 3, 2, \ldots\}$$

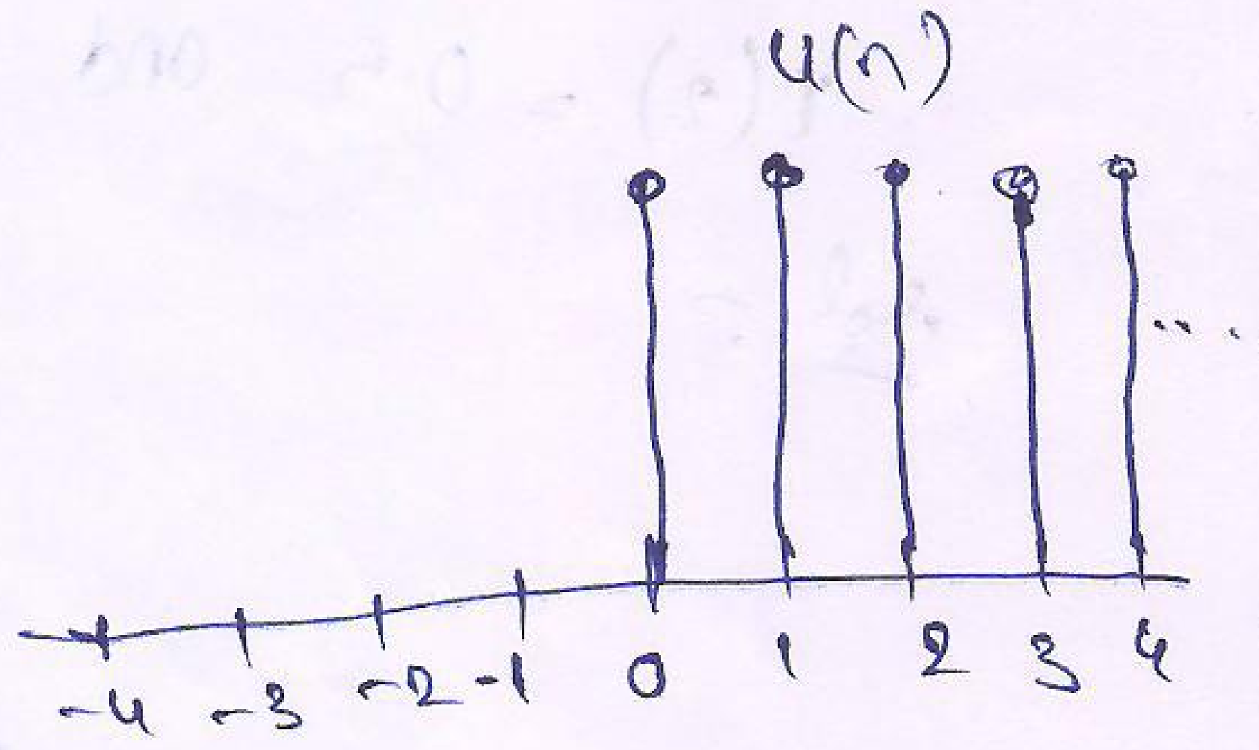
A finite duration sequence that satisfies the condition $x(n) = 0$ for $n < 0$ can be represented as

$$x(n) = \{2, 4, 6, 8, -3\}.$$

12) *) **Elementary Discrete-time Signals:-**

1. **Unit step sequence:-**

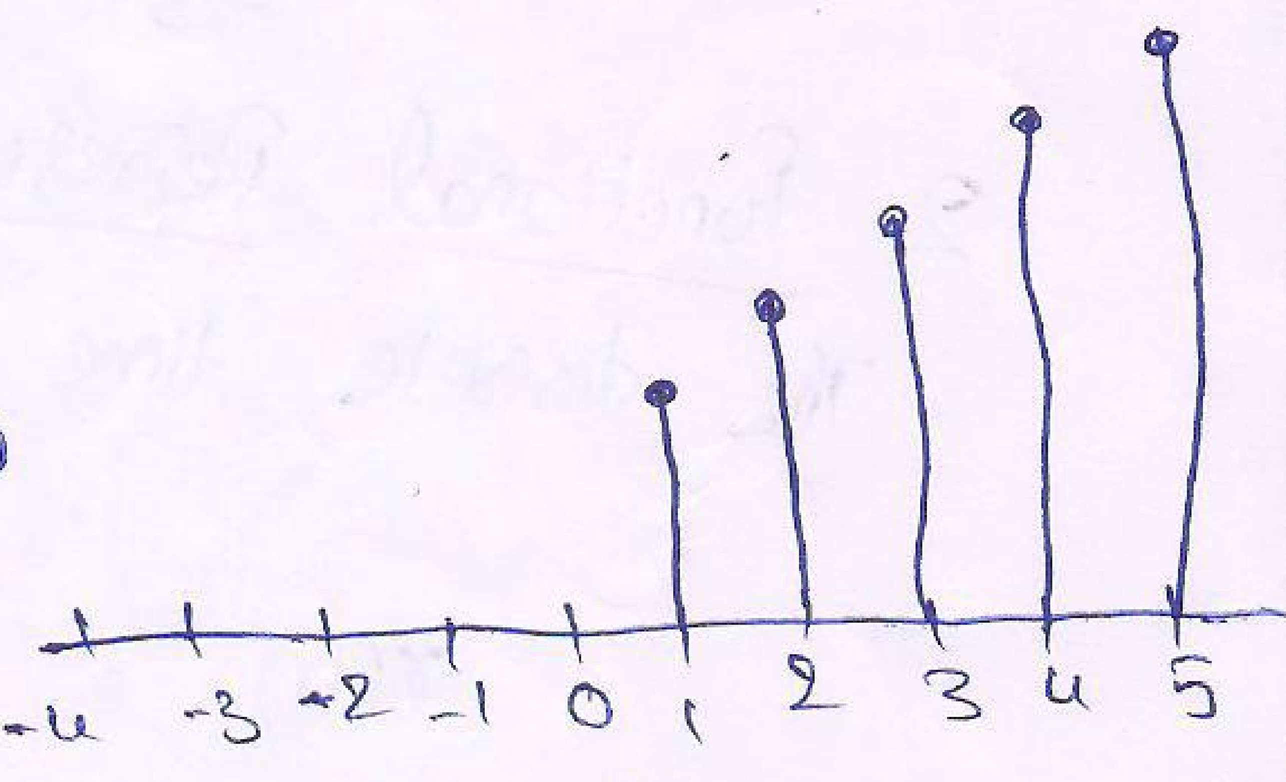
$$u(n) = 1 \text{ for } n \geq 0$$
$$= 0 \text{ for } n < 0$$



2. **Unit ramp sequence:-**

defined

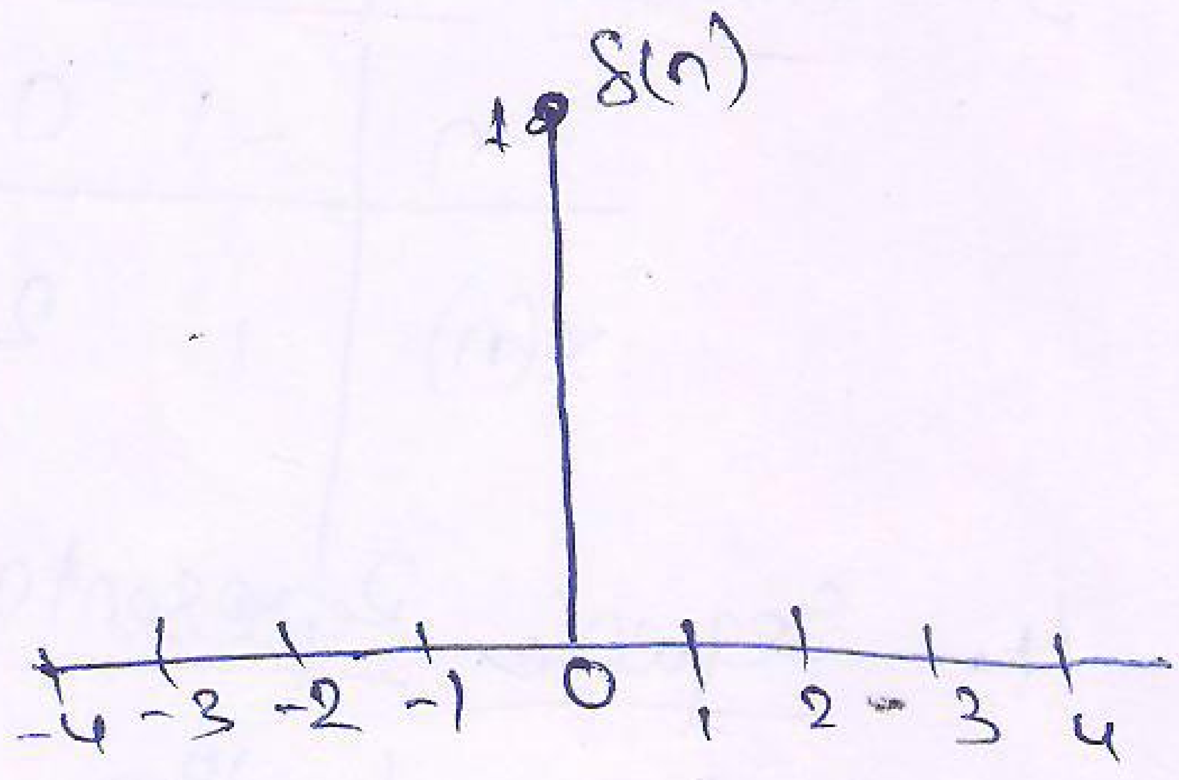
$$r(n) = n \text{ for } n \geq 0$$
$$= 0 \text{ for } n < 0$$



unit ramp Sequence

3. **Unit sample sequence (unit impulse sequence):-**

$$\delta(n) = 1 \text{ for } n = 0$$
$$= 0 \text{ for } n \neq 0$$



unit impulse function has following conditions

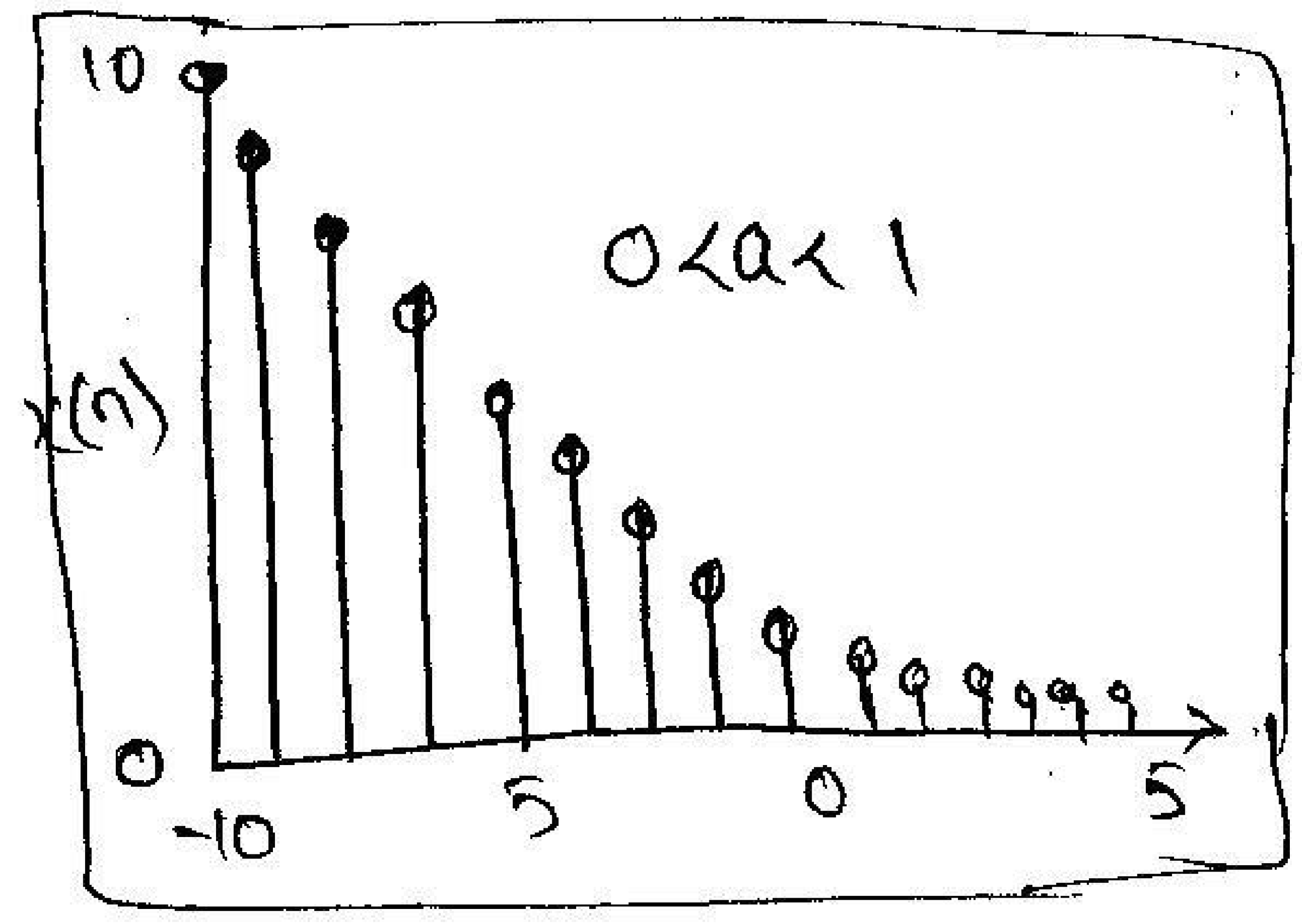
$$\delta(n) = u(n) - u(n-1)$$

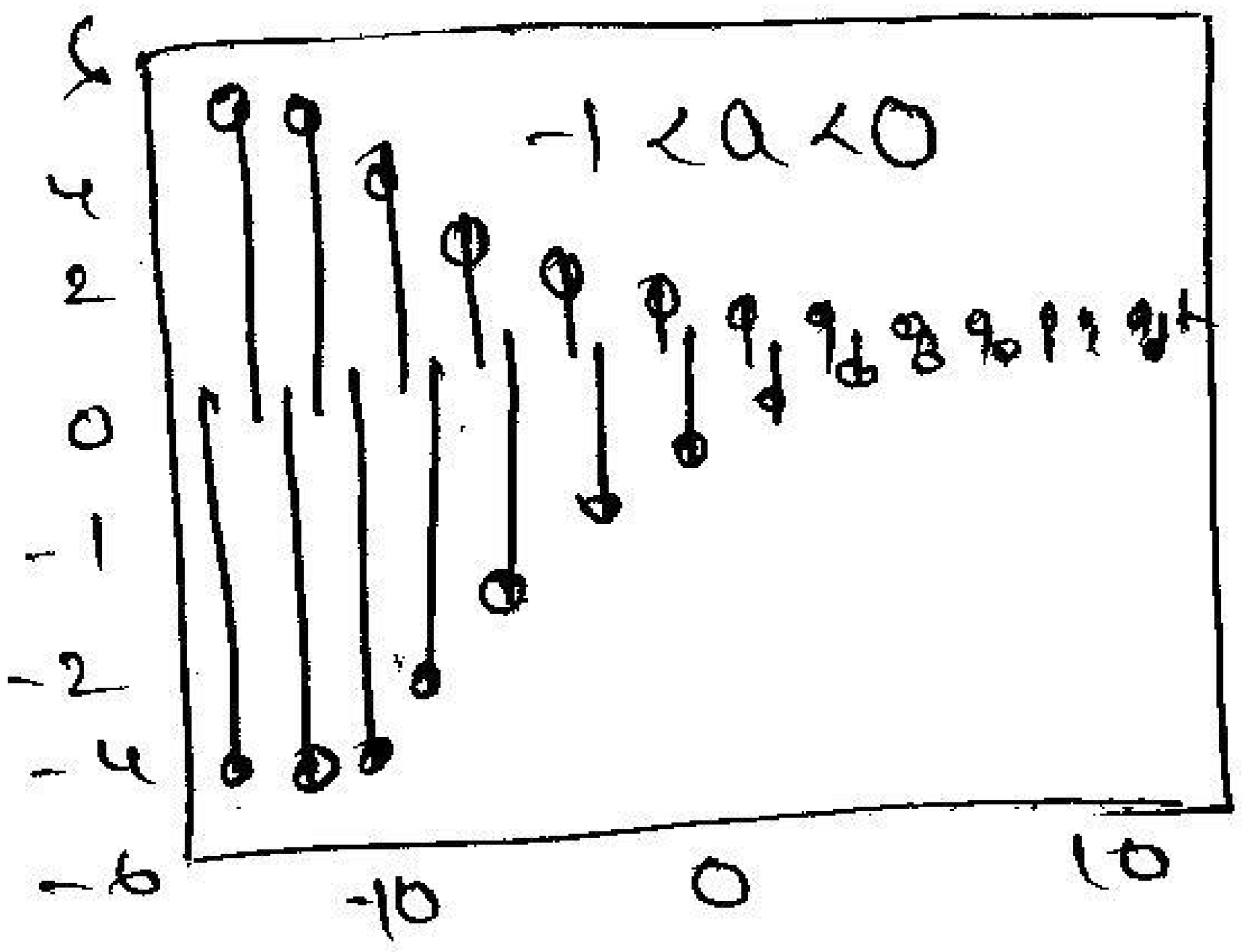
$$u(n) = \sum_{k=-\infty}^{n} \delta(k)$$

$$\sum_{n=-\infty}^{\infty} x(n) \cdot \delta(n-n_0) = x(n_0).$$

4. Exponential Sequence:-

is a sequence of form $x(n) = a^n$ for all 'n'.





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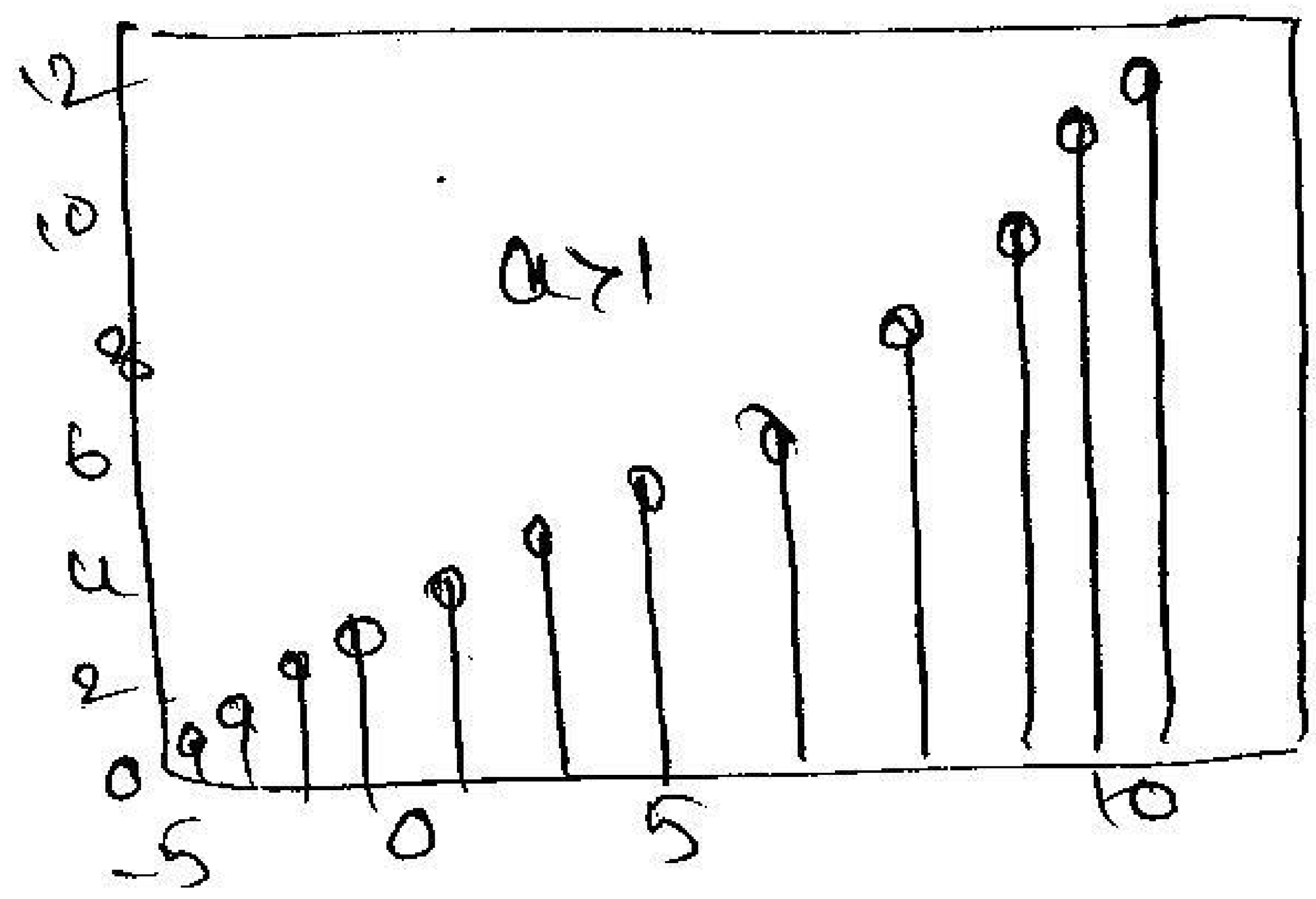
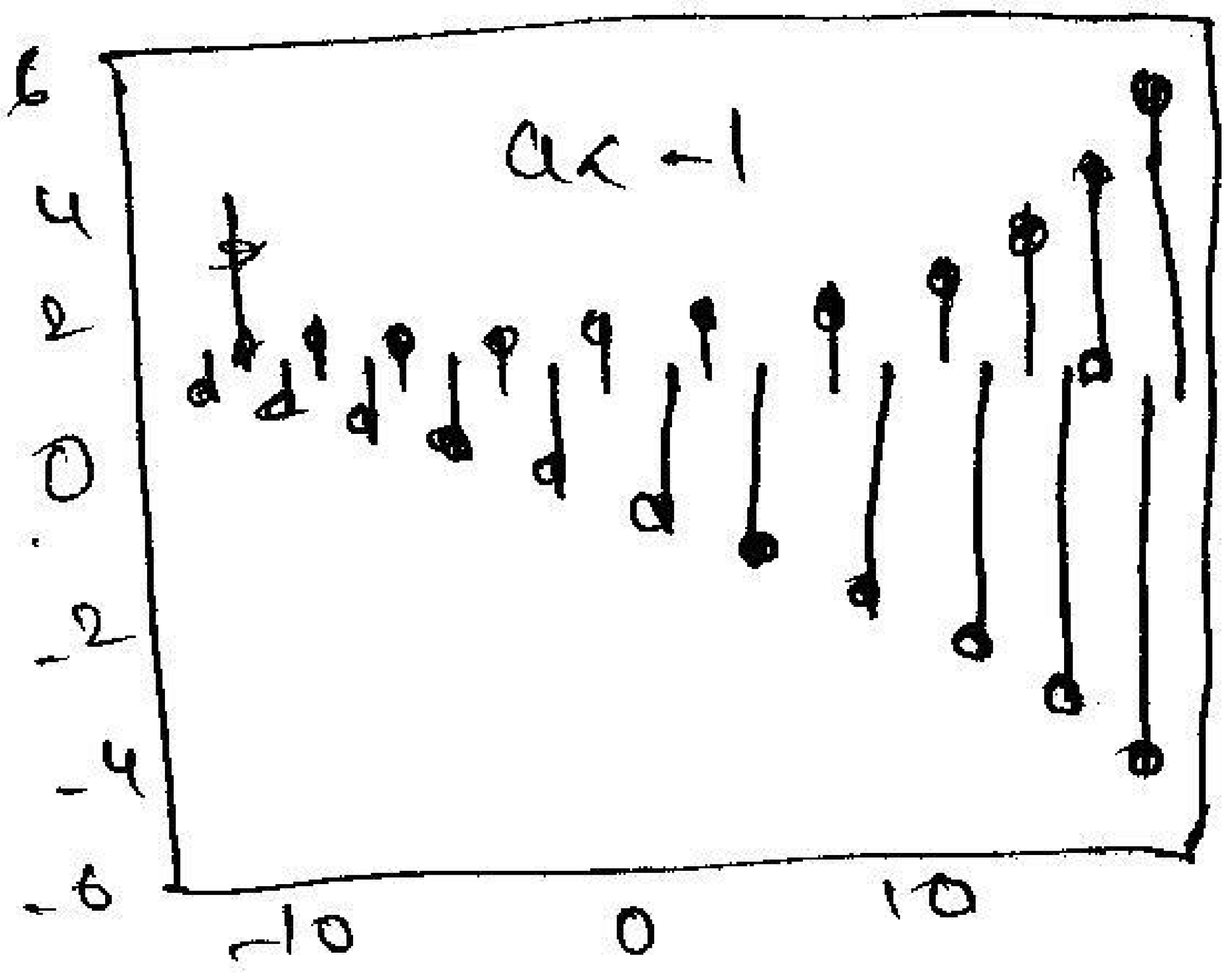




5. Sinusoidal Signal:-

discrete sinusoidal given as

$$x(n) = A \cos(\omega_0 n + \phi)$$

ω_0 = frequency ; ϕ = Phase ;

By using Euler's identity

$$A \cos(\omega_0 n + \phi) = \frac{A}{2} e^{j\phi} \cdot e^{j\omega_0 n} + \frac{A}{2} \cdot e^{-j\phi} \cdot e^{-j\omega_0 n}$$

since $|e^{j\omega_0 n}|^2 = 1$.

(6) Complex Exponential Signal:-

given by:-

$$x(n) = a^n \cdot e^{j(\omega_0 n + \phi)}$$

$$= a^n \cdot \cos(\omega_0 n + \phi) + j \cdot a^n \sin(\omega_0 n + \phi)$$

for $|a| = 1$, the real and imaginary parts of complex exponential sequence are of sinusoidal.

$|a| < 1$ the amplitude of sinusoidal decays exponentially

$|a| > 1$ " " " " increases " .

13) *) Classification of Discrete Time Signals:-

1. Energy signals and Power signals:-

For discrete-time signal $x(n)$ the Energy 'E' is defined as

$$E = \sum_{n=-\infty}^{\infty} |x(n)|^2$$

The average Power of discrete signal $x(n)$ is defined as

$$P = \lim_{N \to \infty} \frac{1}{2N+1} \sum_{n=-N}^{N} |x(n)|^2$$

*) A signal is Energy signal if and only if total Energy of the signal is finite.

*) the signal is said to be Power signal if average Power of signal is finite. $(E = \infty)$

*) A signal which do not satisfy above Properties are neither Energy nor Power signals.

Eg:- Determine the values of Power and Energy of the following signals.

(i) $x(n) = \left(\frac{1}{3}\right)^n u(n)$; (ii) $x(n) = e^{j\left(\frac{\pi}{2}n + \frac{\pi}{4}\right)}$

(iii) $x(n) = \sin\left(\frac{\pi}{4}n\right)$ (iv) $x(n) = e^{2n} u(n)$.

(i) $x(n) = \left(\frac{1}{3}\right)^n u(n)$

$$\sum_{n=0}^{\infty} a^n = \frac{1}{1-a} \text{ for } a<1$$

The Energy signal

unit step sequence in discrete

$$E = \sum_{n=-\infty}^{\infty} |x(n)|^2$$

$$\left(\because \begin{array}{l} u(n) = 1 \text{ for } n \geq 0 \\ u(n) = 0 \text{ for } n < 0 \end{array}\right)$$

$$= \sum_{n=0}^{\infty} \left[\left(\frac{1}{3}\right)^n\right]^2$$

$$\left[\because 1 + a + a^2 + a^3 + \dots \infty = \frac{1}{1-a}\right]$$

$$= \sum_{n=0}^{\infty} \left(\frac{1}{9}\right)^n$$

$$= \frac{1}{1-\frac{1}{9}} = \frac{9}{8}$$

$$\sum_{n=0}^{N} a^n = \frac{1-a^{N+1}}{1-a} \quad \forall\, a \neq 1$$

The Power signal $\lim_{N\to\infty} \frac{1}{2N+1} \sum_{n=-N}^{N} |x(n)|^2$

$$\text{no:- } \lim_{N\to\infty} \frac{1}{2N+1} \sum_{n=0}^{N} \left(\frac{1}{9}\right)^n \qquad \left(\frac{1-a^{N+1}}{1-a}\right)$$

$$= \lim_{N\to\infty} \frac{1}{2N+1} \left[\frac{1-\left(\frac{1}{9}\right)^{N+1}}{1-\left(\frac{1}{9}\right)}\right]$$

$$= 0$$

The Energy is finite and Power is zero
∴ signal is Energy signal.

(ii) $x(n) = e^{j\left(\frac{\pi}{2}n + \frac{\pi}{4}\right)}$ (Q1)

for Energy signal

$$E = \sum_{n=-\infty}^{\infty} |x(n)|^2$$

$$= \sum_{n=-\infty}^{\infty} \left| e^{j\left(\frac{\pi}{2}n + \frac{\pi}{4}\right)} \right|^2 \qquad \boxed{\because \left| e^{j(\omega+\theta)} \right| = 1}$$

$$= \sum_{n=-\infty}^{\infty} 1 = \infty$$

Power signal:-

$$P = \lim_{N\to\infty} \frac{1}{2N+1} \sum_{n=-N}^{N} |x(n)|^2 \qquad \boxed{\because \sum_{n=-N}^{N} 1 = 2N+1}$$

$$= \lim_{N\to\infty} \frac{1}{2N+1} \sum_{n=-N}^{N} \left| e^{j\left(\frac{\pi}{2}n + \frac{\pi}{4}\right)} \right|^2$$

$$= \lim_{N\to\infty} \frac{1}{2N+1} \sum_{n=-N}^{N} 1$$

$$= \lim_{N\to\infty} \frac{1}{2N+1} (2N+1) = 1$$

$\therefore$ The signal is power signal.

(iii) $x(n) = \sin\left(\frac{\pi}{4}n\right)$

$$E = \sum_{n=-\infty}^{\infty} \left| \sin^2\left(\frac{\pi}{4}n\right) \right| = \sum_{n=-\infty}^{\infty} \left[\frac{1 - \cos\left(\frac{\pi}{2}n\right)}{2} \right] = \infty \text{ infinite}$$

$$P = \lim_{N\to\infty} \frac{1}{2N+1} \sum_{n=-N}^{N} \left| \sin^2\left(\frac{\pi}{4}n\right) \right|$$

$$= \lim_{N\to\infty} \frac{1}{2N+1} \sum_{n=-N}^{N} \left[\frac{1 - \cos\left(\frac{\pi}{2}n\right)}{2} \right]$$

$$= \frac{1}{2} \lim_{N\to\infty} \frac{1}{2N+1} \sum_{n=-N}^{N} 1$$

$P = \frac{1}{2}$ .. finite.

(iv) $x(n) = e^{2n} u(n)$

$$E = \sum_{n=-\infty}^{\infty} |x(n)|^2 = \sum_{n=-\infty}^{\infty} e^{4n} \quad \left(\because \begin{array}{l} u(n) = 1 \text{ for } n \geq 0 \\ u(n) = 0 \text{ for } n < 0 \end{array} \right)$$

$$= 1 + e^4 + e^8 + \cdots \infty \quad \left(\because 1 + e^4 + e^8 + e^{16} + \cdots = \infty \right)$$

$$= \infty$$

$$P = \lim_{N \to \infty} \frac{1}{2N+1} \sum_{n=-N}^{N} |x(n)|^2$$

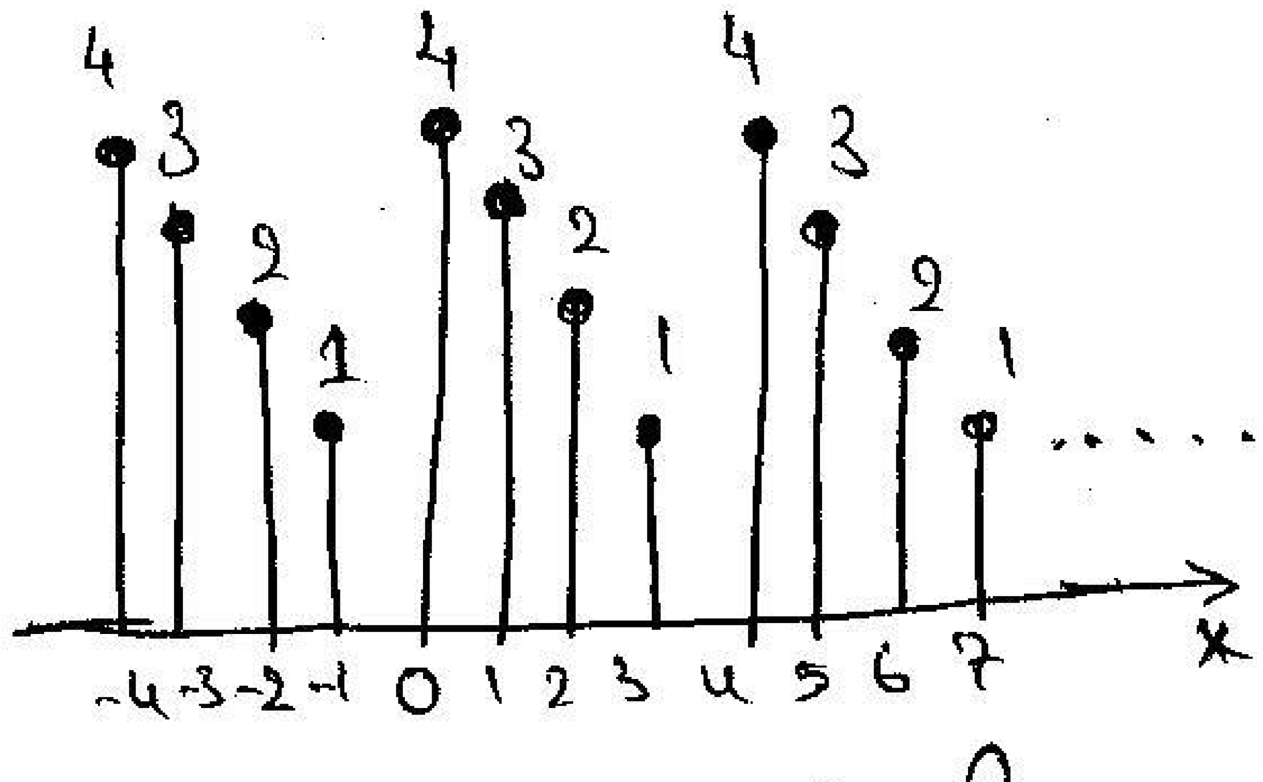
$$= \lim_{N \to \infty} \frac{1}{2N+1} \sum_{n=-N}^{N} e^{4n}$$

$$= \lim_{N \to \infty} \frac{1}{2N+1} \left[\frac{e^{4(N+1)} - 1}{e^4 - 1} \right] = \infty$$

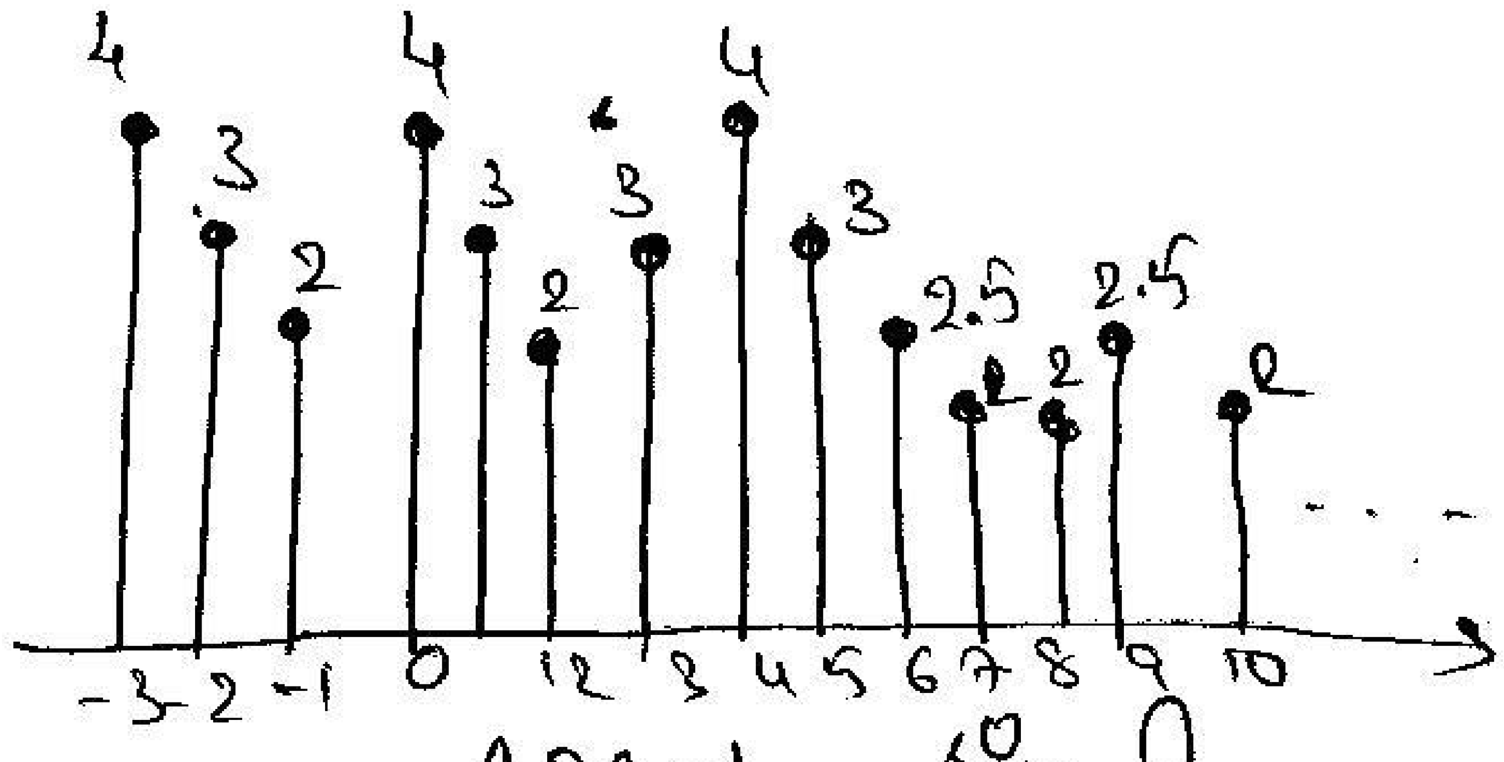
$\therefore$ The signal Power & Energy are infinite.

2. Periodic & aperiodic signals:-

A discrete signal $x(n)$ is said to be periodic with period N if $x(N+n) = x(n)$ for all n. $\rightarrow$ (a)



Periodic Signal.



aperiodic Signal

All continuous time sinusoidal signal are periodic but all the discrete time sinusoidal sequences are not periodic

$$x(n) = A \sin(\omega_0 n + \theta). \rightarrow (b)$$

A discrete time sequence is periodic if it satisfies the condition

$$x(N+n) = x(n) \quad \rightarrow (c)$$

$$x(n+N) = A \sin(\omega_0(n+N)+\theta)$$
$$= A \sin(\omega_0 n + \omega_0 N + \theta) \rightarrow (d)$$

where A = Amplitude ; ω_0 and θ are frequency & Phase shift above Eq (d) satisfies if and only if $\omega_0 N$ is the integer multiple of 2π.

$$\omega_0 N = 2\pi m$$

$$\boxed{\frac{\omega_0}{2\pi} = \frac{m}{N}}$$ rational no:-

(or

$$\boxed{N = 2\pi\left(\frac{m}{\omega_0}\right)}$$

Eg:- Determine whether (or) not each of following signals is periodic. if a signal is periodic. specify fundamental.

(i) $x(n) = \cos\left(\frac{2\pi}{3}\right).n$

$$x(n) = \cos\left(\frac{2\pi}{3}\right).n$$

$$\omega_0 = \frac{2\pi}{3}$$

Compare $\omega_0 = \frac{2\pi m}{N}$

$$\frac{\cancel{2\pi}}{3} = \frac{\cancel{2\pi} m}{N} \Rightarrow \frac{m}{N} = \frac{1}{3} \text{ rational no:-}$$

$$N = \frac{3m}{1} \Rightarrow 3m$$

for smallest value of 'm', so that N-becomes integer 'm=1'

$$\therefore N = 3$$

3. Causal & Non-Causal Signal:-

A signal. $x(n)$ is said to be Causal if its value is zero for $n<0$. otherwise signal is non-causal..

Eg:- $x_1(n) = a^n u(n)$

$x_2(n) = \{\underset{\uparrow}{1}, 2, -3, -1, 2\}$

Eg:- non Causal:- $x_1(n) = a^n u(-n+1)$

$x_2(n) = \{1, -2, \underset{\uparrow}{1}, 4, 3\}$.

4. Symmetric (Even) and Antisymmetric (odd signals):-

(*) A discrete time signal $x(n)$ is said to be a symmetric if it satisfies the condition

$$x(-n) = x(n) \quad \forall n$$

Eg:- $x(n) = \cos \omega n$

(*) The signal is said to be an odd signal if satisfies the condition

$$x(-n) = -x(n) \quad \forall n$$

Eg:- $x(-n) = A \sin \omega n$

(*) A signal $x(n)$ can be expressed as the sum of even and odd components. i.e

$$x(n) = x_e(n) + x_o(n) \longrightarrow (1)$$

$x_e(n)$ = Even component ; $x_o(n)$ = odd component

Replace n by '$-n$' in eq (1)

$$\therefore x(-n) = x_e(-n) + x_o(-n)$$

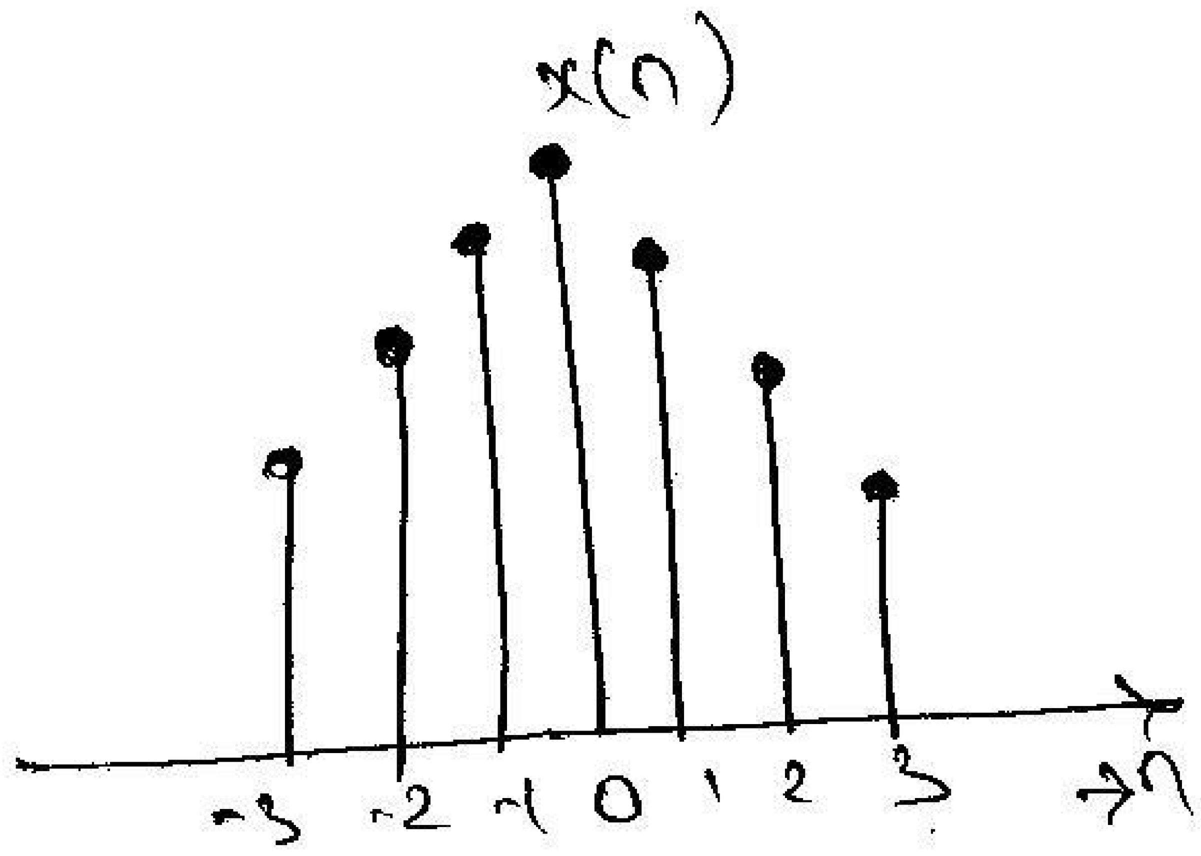
$$= x_e(n) - x_o(n) \rightarrow (2)$$

Adding Eq ① & ②

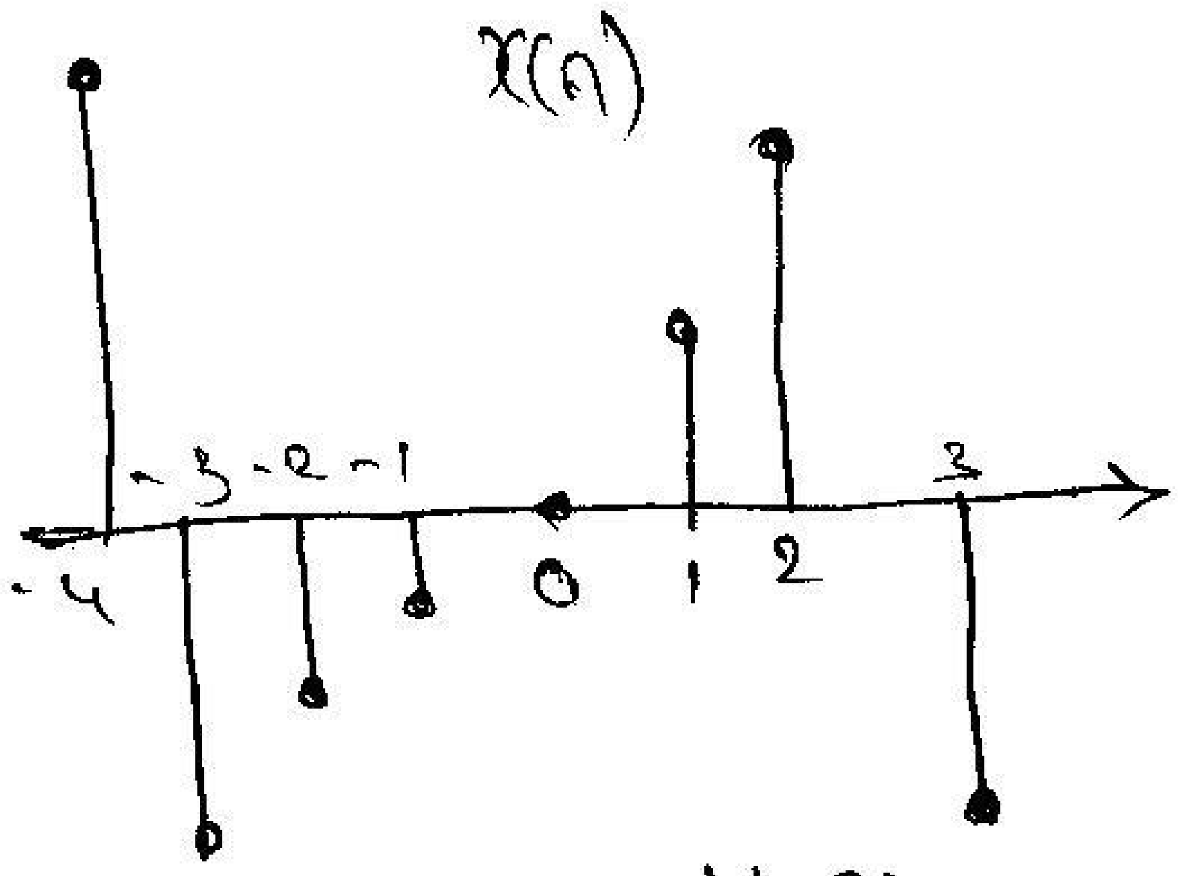
$$2x_e(n) = x(n) + x(-n)$$

$$\Rightarrow x_e(n) = \frac{1}{2}\left[x(n) + x(-n)\right]$$

similly:- $$x_o(n) = \frac{1}{2}\left[x(n) - x(-n)\right]$$



Even
Symmetric



odd signals
Antisymetric

14) *) Operation on Signals:-

The mathematical transform from one signal to another is given

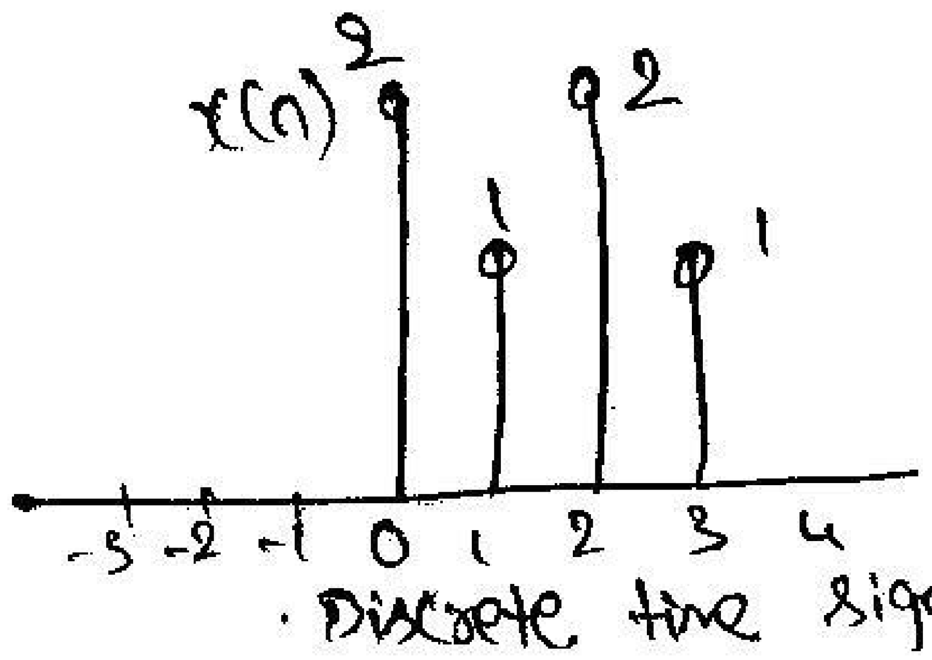
$$y(n) = T\left[x(n)\right]$$

① Shifting:- The shift operation takes the i/p sequence and shifts the values by an integer increment of the independent variable. The shifting may Delay (or) Advance the sequence in time.

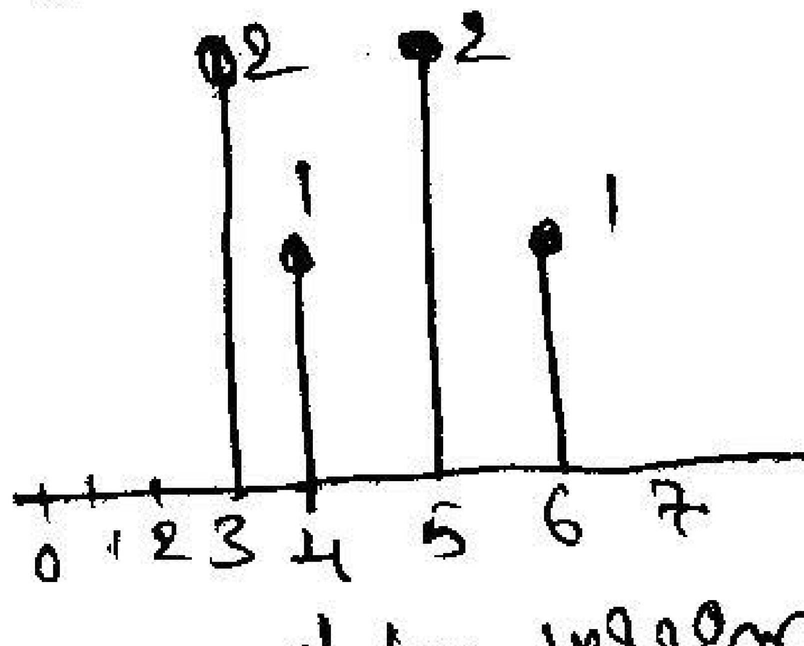
$$y(n) = x(n-k)$$

$y(n)$ → o/p, $x(n)$ → i/p

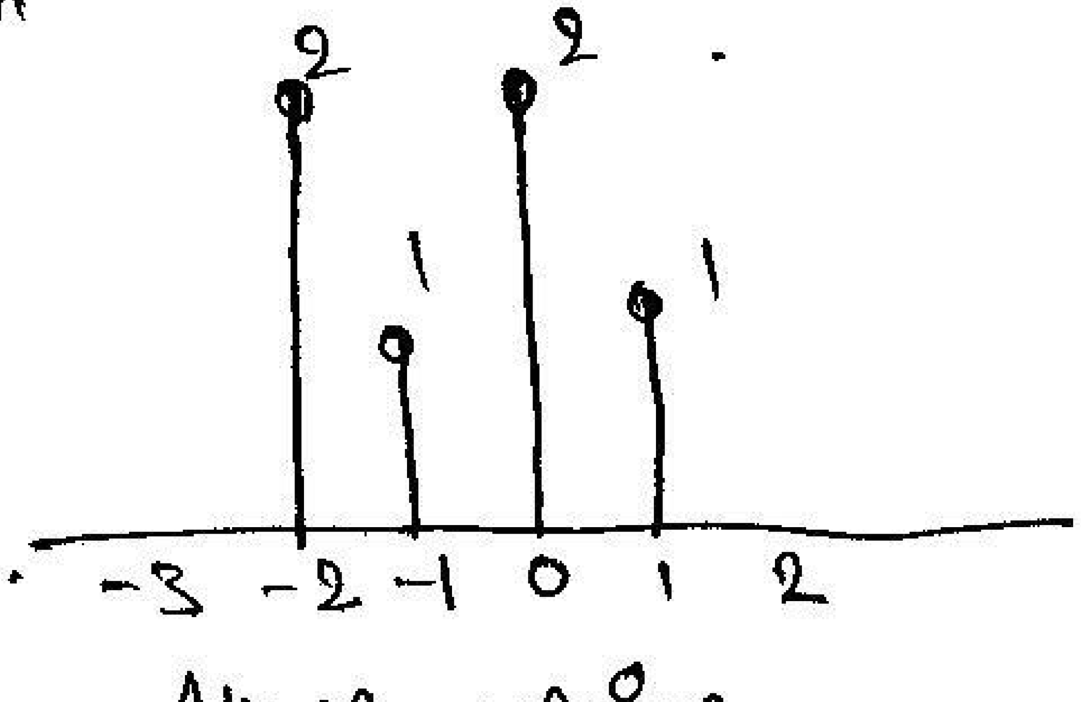
If k is +ve the shifting delays the sequence
k is -ve the " advances " "



. Discrete time signal

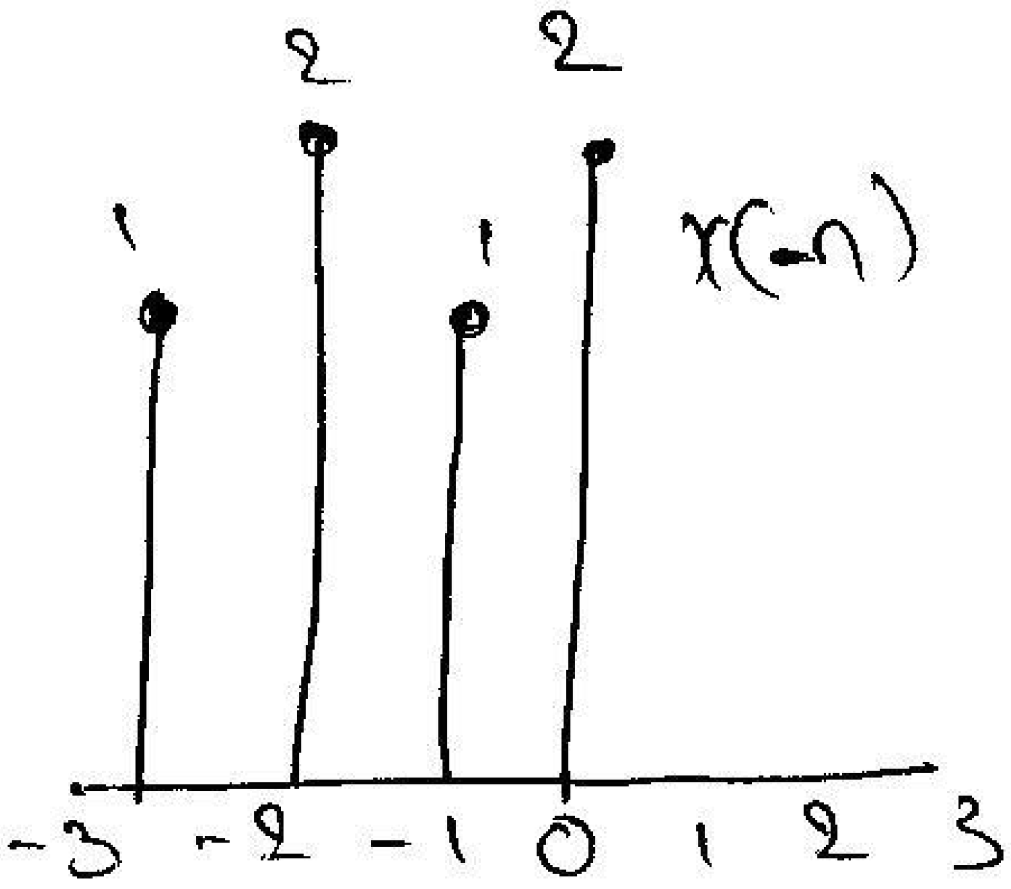


delay version.

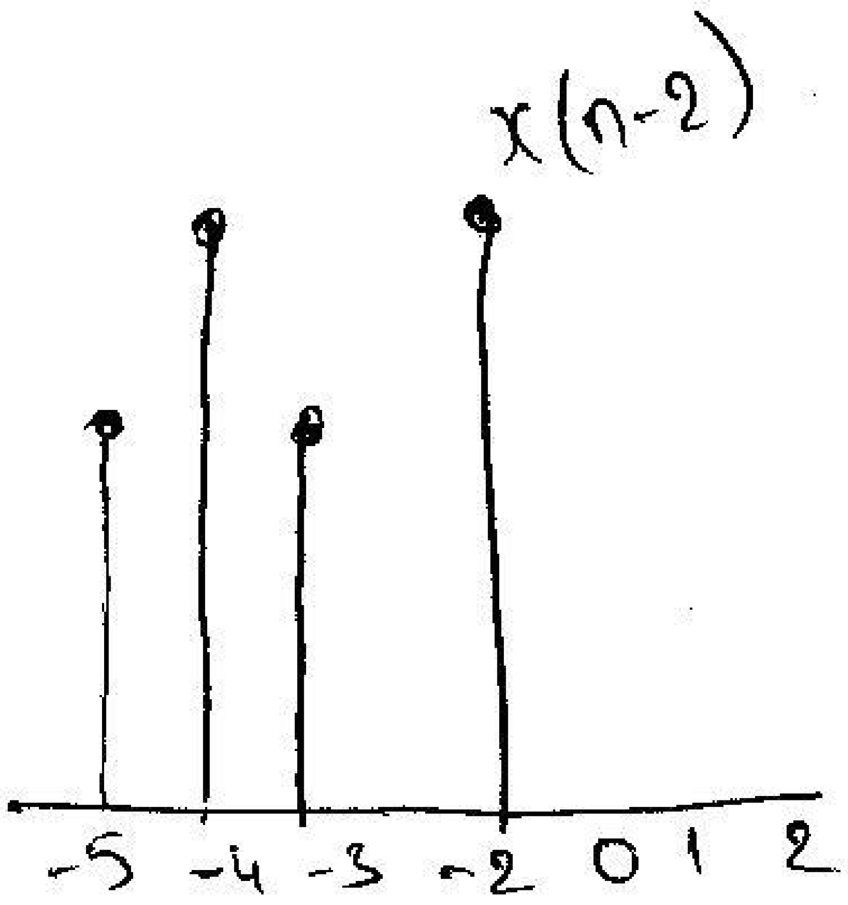


Advance version.

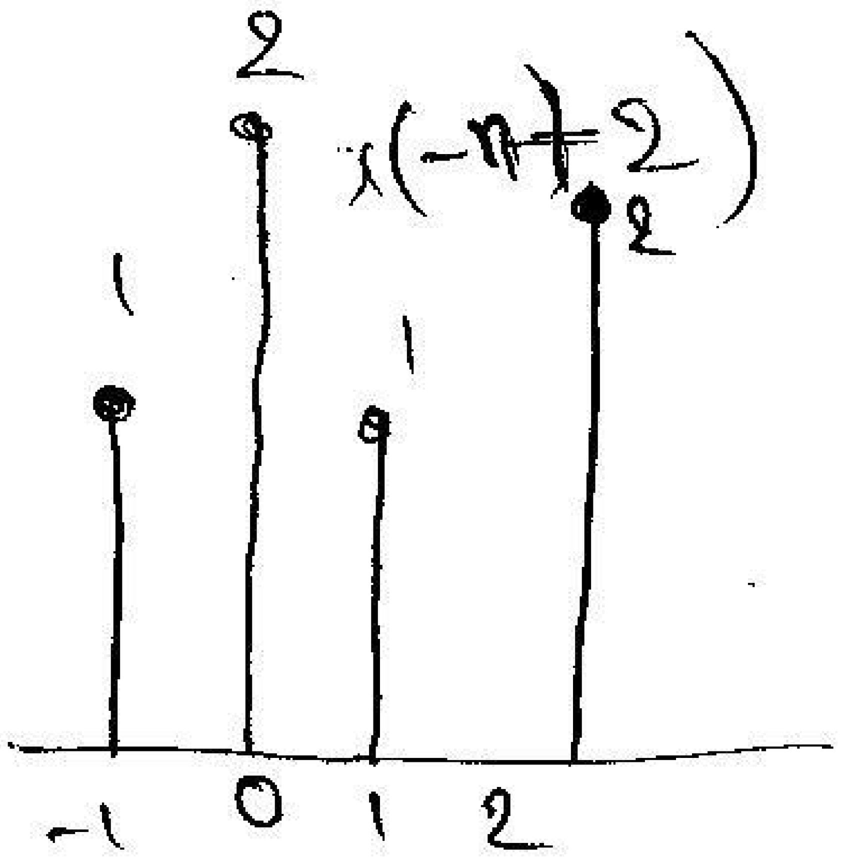
(2) Time Reversal :- denoted as $x(-n)$



Shifted version



Advance



delay.

(3) Time Scaling :-

This is obtained by replacing n by λn in sequence $x(n)$

$$\therefore \quad y(n) = x(\lambda n)$$

Eg:-

let $x(n)$ is a sequence. if $\lambda = 2$ then

$$y(n) = x(2n)$$

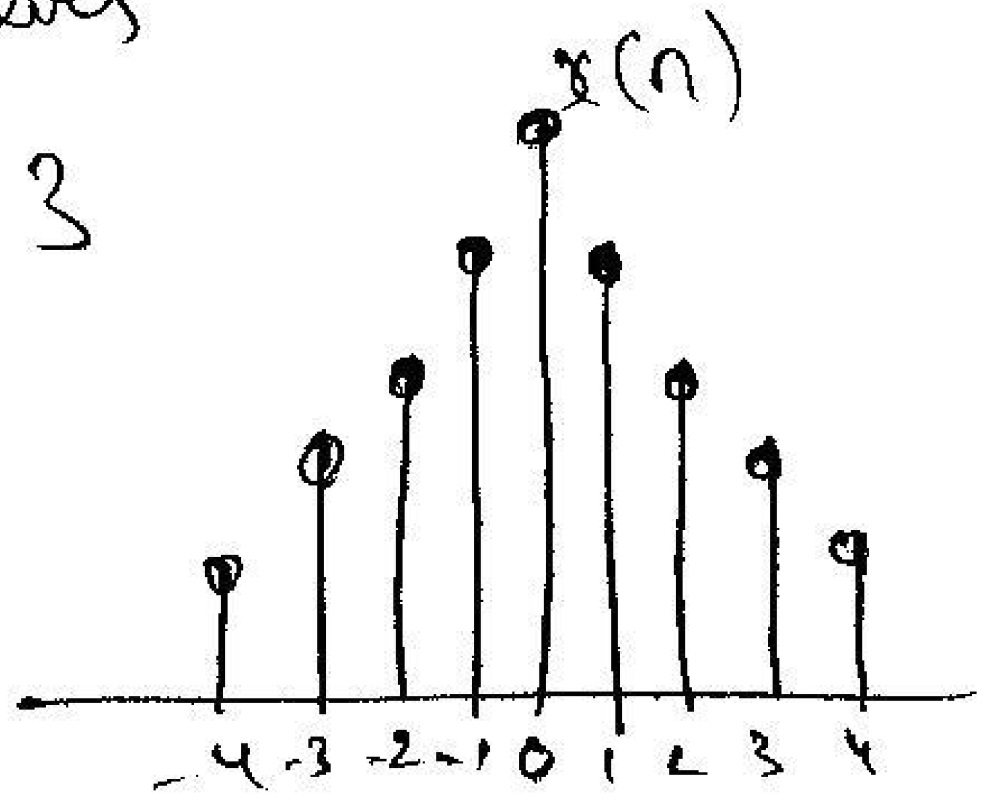
the graph is ploted by using different values

for:- $n = -1$; $y(-1) = x(-2) = 3$

silly $y(0) = x(0) = 5$

$y(1) = x(2) = 3$

$y(2) = x(4) = 1$



(4) Scalar Multiplication :-

The signal $x(n)$ is multiplied by a scale factor a

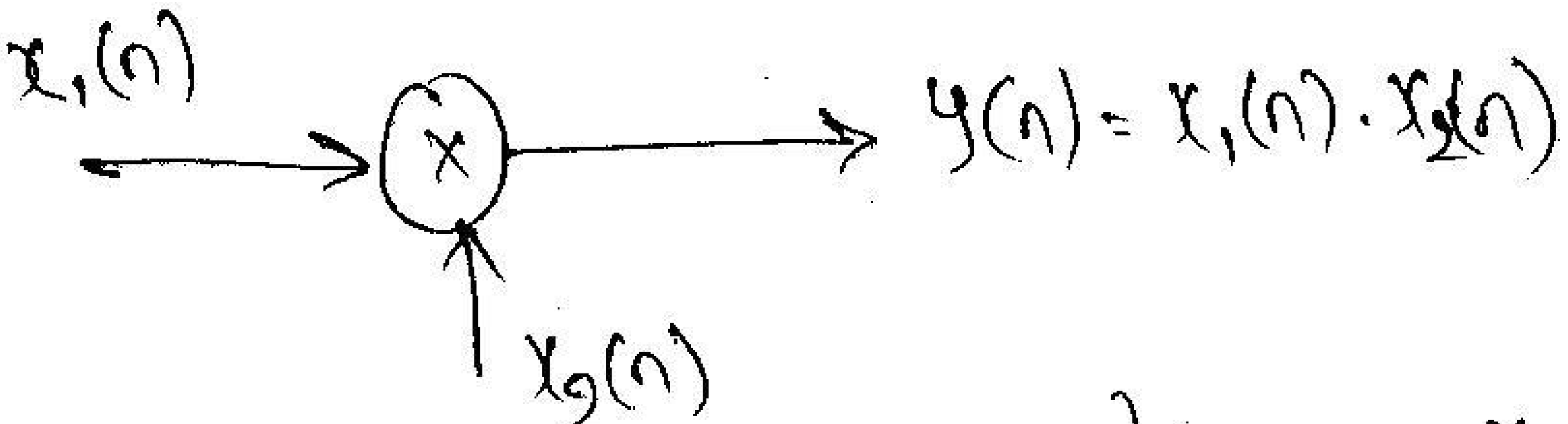
$$x(n) \xrightarrow{\quad a \quad} y(n) = a \cdot x(n)$$

Eg:- $x(n) = \{1, 2, 1, -1\}$; and $a = 2$;

$y(n) = a\,x(n) = \{2, 4, 2, -2\}$.

5) Signal Multiplier:-

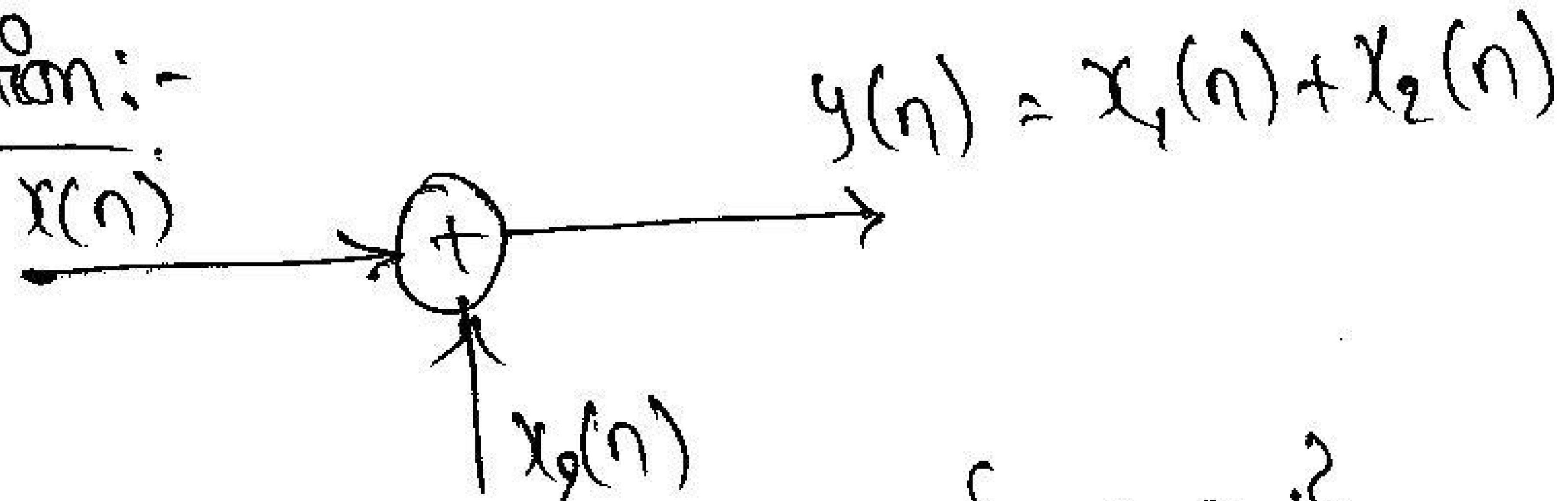
Multiplication of 2-signal sequences to form another seq.



eg:- $x_1(n) = \{-1, 2, -3, 2\}$ and $x_2 = \{1, -1, -2, 1\}$

then $x_1(n) \cdot x_2(n) = \{(-1 \times 1), (2 \times -1), (-3 \times -2), (-2 \times 1)\}$
$= \{-1, -2, 6, -2\}$.

6) Addition operation:-



$x_1(n) = \{1, 2, 3, 4\}$; $x_2(n) = \{4, 3, 2, 1\}$
$x_1(n) + x_2(n) = \{5, 5, 5, 5\}$.

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*) Classifications of Discrete time Systems:-

1. Static & dynamic systems
2. Causal and Non-Causal systems
3. Linear and Non-linear systems
4. Time variant and Time invariant systems
5. FIR & IIR system
6. Stable and unstable systems.

1. Static & Dynamic Systems :-

A discrete time system is called static (or) memoryless if its o/p at any instant depends on the i/p samples at the same time, but not on past (or) future sample of i/p. otherwise the system is dynamic.

$$y(n) = a\,x(n)$$
$$y(n) = a\,x^2(n)$$
} static

$$y(n) = x(n-1) + x(n-2)$$
$$y(n) = x(n+1) + x(n)$$
} dynamic.

2. Causal & Non-Causal Systems :-

A system is said to be causal if the o/p of system at any time n depends only at present & past i/p's but doesnot depends on future i/p's.

$$y(n) = F[x(n), x(n-1), x(n-2) \ldots].$$

If the o/p of a system depends on future i/p, the system is said to be non-causal (or) anticipatory

$y(n) = x(n) + x(n-1)$ causal system

$y(n) = x(2n)$ non-causal system.

Eg:- $y(n) = x(n) + \dfrac{1}{x(n-1)}$

for $n = -1$ $\quad y(-1) = x(-1) + \dfrac{1}{x(-2)}$

$n = 0$ $\quad y(0) = x(0) + \dfrac{1}{x(-1)}$

$n = 1$ $\quad y(1) = x(1) + \dfrac{1}{x(0)}$

for all values of 'n' the o/p depends on present & past i/p.

$\therefore$ System is Causal.

Eg:- (ii) $y(n) = x(n^2)$

$n = -1$; $y(-1) = x(1)$

$n = 0$; $y(0) = x(0)$

$n = 1$; $y(1) = x(1)$

$\forall$ value of n, (except for $n=0$ & $n=1$), the system depends on future i/p. so the system is non causal.

3. <u>Linear & Non-linear systems</u>:-

A system satisfies the superposition principle is said to be linear system.

Super position ~~theorem~~ states that the response of the system to a weighted sum of signals should be equal to the corresponding weighted sum of the o/p's of sm to each of i/p signals.

A system is linear if and only if

$$T\left[a_1 x_1(n) + a_2 x_2(n)\right] = a_1 T\left[x_1(n)\right] + a_2 T\left[x_2(n)\right]$$

~~y(n) = ax(n)~~

Eg:- $y(n) = n\,x(n)$

$$y_1(n) = T\{x_1(n)\} = n.x_1(n)$$

$$y_2(n) = T[x_2(n)] = n.x_2(n)$$

$$a_1 T[x_1(n)] + a_2 T[x_2(n)] = a_1 n.x_1(n) + a_2 n\,x_2(n)$$

∴ o/p due to weighted sum of i/p is

$$y_3(n) = T\{a_1 x(n) + a_2 x(n)\} = a_1 n\,x_1(n) + a_2 n\,x_2(n)$$

(Equal).

∴ Superposition principle is satisfied.

4. Time Variant & Time invariant system :-

A system is said to be time variant if the characteristics of the system do not change with time.

$$y(n,k) = T\{x(n-k)\}.$$

Delay the o/p sequence by k-samples, denote it as $y(n-k)$ if

$$y(n,k) = y(n-k)$$

∀ Possible value of k, the system is time invariant on the other hand. $y(n,k) \neq y(n-k)$.

Eg:- $y(n) = x(n) + x(n-1)$

Given $y(n) = T[x(n)] = x(n) + x(n-1)$

if the i/p is delayed by k-units in time, we have

$$y(n,k) = T[x(n-k)] = x(n-k) + x(n-k-1)$$