(61) unit - II 18/12/08

Circular wave guides:-

* Circular wave guides tends to twist the waves as these waves Propagates through a wave guide mainly circular wave guides are used with rotating antennas as in radars.

A circular wave guide is a cylindrical hallow metalic tube with uniform circular crossection of a finite radius.

* A circular wg is nothing but a tubular, circular conductor.

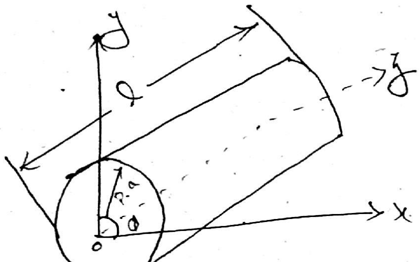


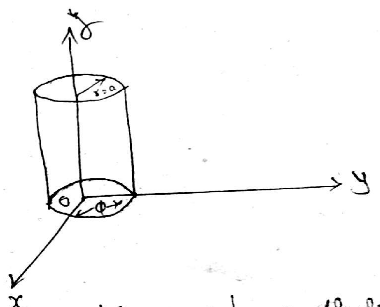
Figure shows a circular wg of inner radius here radius ρ = a and length 'l'.

here the angle φ varies from 0 to 2π and ρ is varies from 0 to a and l varies along z-axis

the helmoltz wave eq for TE & TM wave travelling in z-direction in a circular wg are given by (62)

$\nabla^2 H_z = 0$ & $\nabla^2 E_z = 0$

Solutions of wave Eq. in a circular wg.



Same as rectangular wg only a sinusoidal steady state (or) freq domain solution will be given for circular wg.

Here fig shows a cylindrical co-ordinate system. the helmoltz Eq in cylindrical co-ordinates are given by.

$$\frac{1}{r}\cdot\frac{\partial}{\partial r}\left(r\cdot\frac{\partial\psi}{\partial r}\right)+\frac{1}{r^2}\cdot\frac{\partial^2\psi}{\partial\phi^2}+\frac{\partial^2\psi}{\partial z^2}=\gamma^2\psi \rightarrow (1)$$

By using separation of variables method the solution is assumed in the form of

$$\psi = R_{(r)}\,\Phi_{(\phi)}\cdot Z_{(z)} \rightarrow (2)$$

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$R_{(r)}$ is a func of 'r' only

$\phi_{(\phi)}$ is a fun of ϕ only

$Z_{(z)}$ is " " " z "

By substituting Eq (2) in Eq (1) we get

$$\Rightarrow \frac{1}{r}\cdot\frac{\partial}{\partial r}\left(r\cdot\frac{\partial(R_{(r)}\cdot\phi_{(\phi)}\,Z_{(z)})}{\partial r}\right)+\frac{1}{r^2}\cdot\frac{\partial^2(R_{(r)}\cdot\phi_{(\phi)}\cdot Z)}{\partial(R\phi Z)^2}+\frac{\partial^2(R\phi Z)}{\partial z^2}=\gamma^2(R\phi Z)$$

$$\Rightarrow \frac{1}{r}\cdot\frac{d}{dr}\left(r\cdot\frac{d(R\phi Z)}{dr}\right)+\frac{1}{r^2}\cdot\frac{d^2(R\phi Z)}{d(R\phi Z)^2}+\frac{d^2(R\phi Z)}{d(z)^2}=\gamma^2(R\phi Z)$$

$$\frac{1}{r}\cdot(\phi Z)\cdot\frac{dR}{dr}+\frac{1}{r^2}\cdot RZ\cdot\frac{d^2\phi}{d\phi^2}+R\phi\cdot\frac{dz}{dz^2}=\gamma^2(R\phi Z)$$

Dividing trough out by $R\phi Z$.

$$\frac{1}{Rr}\frac{d}{dr}\left(r\frac{dR}{dr}\right)+\frac{1}{\phi r^2}\cdot\frac{d^2\phi}{d\phi^2}+\frac{1}{Z}\frac{d^2z}{dz^2}=\gamma^2 \rightarrow (3)$$

$\because$ The sum of three independent variable is a Constant i.e each term must be Equal to a Constant.

hence the third term may be set Equal to a Constant γ_g^2

(4)

i.e $$\frac{d^2 z}{dz^2}=\gamma_g^2 z. \rightarrow (4)$$

The solution of this Eqn is given by $z=A\cdot e^{-\gamma_g z}+B\cdot e^{\gamma_g z}\rightarrow(5)$ where γ_g is propagation Constant of the wave in a 'wg'.

By substituting (4) in (3)

$$\frac{1}{Rr}\frac{d}{dr}\left(r\frac{dR}{dr}\right)+\frac{1}{\phi r^2}\frac{d^2\phi}{d\phi^2}+\frac{1}{\cancel{z}}\cdot\gamma_g^2\cancel{z}=\gamma^2$$

$$\frac{1}{Rr}\frac{d}{dr}\left(r\cdot\frac{dR}{dr}\right)+\frac{1}{\phi r^2}\frac{d^2\phi}{d\phi^2}+\gamma_g^2-\gamma^2=0$$

$$\frac{1}{Rr}\cdot\frac{d}{dr}\left(r\cdot\frac{dR}{dr}\right)+\frac{1}{\phi r^2}\cdot\frac{d^2\phi}{d\phi^2}-(\gamma_g^2-\gamma^2)=0$$

multiplying above Eq by r^2 on Bs.

$$\frac{r}{R}\cdot\frac{d}{dr}\left(r\cdot\frac{dR}{dr}\right)+\frac{1}{\phi}\cdot\frac{d^2\phi}{d\phi^2}-(r^2\gamma_g^2-r^4)=0\rightarrow(6)$$

The second term is a func of 'ϕ' only.

hence Equating the second term to a Constant '$-n^2$'

$$\frac{d^2\phi}{d\phi^2}=-n^2\phi\rightarrow(7)$$

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The Solution of this Eq. can be given as,

$$\phi = A_n \sin(n\phi) + B_n \cos(n\phi) \rightarrow (8).$$

By substituting Eq (7) in (6)

$$\frac{\gamma}{R}\frac{d}{d\gamma}\left(\gamma.\frac{dR}{d\gamma}\right) + \cancel{\phi} - n^2\phi - (\gamma^2\gamma_g^2 - \gamma^4) = 0 \rightarrow (8)$$

by multiplying this Eq by 'R'

$$\gamma.\frac{d}{d\gamma}\left(R.\gamma.\frac{dR}{d\gamma}\right) - \frac{1}{\cancel{\phi}}n^2R\cancel{\phi} - R(\gamma^2\gamma_g^2 - \gamma^4) = 0$$

$$\gamma.\frac{d}{d\gamma}\left(R.\gamma.\frac{dR}{d\gamma}\right) - R\left(n^2\cancel{\phi} + (\gamma^2\gamma_g^2 - \gamma^4)\right) = 0$$

in place of $\gamma_g^2 - \gamma^2 = Kc^2$

$$\gamma\frac{d}{d\gamma}\left(2\gamma.\frac{dR}{d\gamma}\right) - R\left(n^2 + Kc^2.\gamma^2\right) = 0 \rightarrow (9)$$

~~take R~~

This $\gamma\frac{d}{d\gamma}$ is Bessel ~~sphere~~ of order 'n'

$$K_c^2 = \gamma_g^2 - \gamma^2$$

$$K_c^2 + \gamma^2 = \gamma_g^2$$

This Eq is called characteristic Eq of Bessel fun.

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∴ for lossless wg the characteristic Eq reduces to

$$\beta_g = \pm\sqrt{\omega^2\mu\varepsilon - K_c^2}$$

$$K_c^2 + \gamma^2 = (j\beta_g)^2$$

$$K_c^2 - \omega^2\mu\varepsilon = -\beta_g^2$$

$$\beta_g^2 = \omega^2\mu\varepsilon - K_c^2$$

$$\beta_g = \pm\sqrt{\omega^2\mu\varepsilon - K_c^2}.$$

The Solution of Bessels Eq are

$$R = C_n J_n(K_c\gamma) + D_n N_n(K_c\gamma) \rightarrow (12)$$

where $J_n(K_c\gamma)$ is the 'n' th order Bessel func of the first kind representing a standing wave of $\cos(K_c\gamma)$ for $\gamma < a$.

$N_n(K_c\gamma)$ is the n^{th} order Bessel func of the second kind representing a standing wave of sine of $K_c\gamma$ for all 'γ' greater a $(\gamma > a)$

∴ The total solution of helmoltz solution in Cylindrical Co-ordinates are given by

$$\psi = \left[C_n J_n(K_c\gamma) + D_n N_n(K_c\gamma)\right]\left(A_n\sin(n\phi) + B_n\cos(n\phi)\right).e^{\pm j\beta_g z} \rightarrow (13)$$

Propagation of TE waves in Cylindrical Co-ordinates:

(6x) ~~for~~ TE ~~waves~~ in Cylindrical ~~Co-ordinates~~

∴ for TE wave to propagate

$E_z = 0$; $H_z \neq 0$

From Maxwell Eq $\nabla^2 H_z = -\omega^2 \mu \varepsilon H_z \rightarrow (1)$

Expanding $\nabla^2 H_z$ in Cylindrical Co-ordinates.

$$\frac{\partial^2 H_z}{\partial \rho^2} + \frac{1}{\rho}\cdot\frac{\partial H_z}{\partial \rho} + \frac{1}{\rho^2}\cdot\frac{\partial^2 H_z}{\partial \phi^2} + \frac{\partial^2 H_z}{\partial z^2} = -\omega^2 \mu \varepsilon H_z \rightarrow (2)$$

But we know $\frac{\partial^2}{\partial z^2} = \gamma^2$

$$\frac{\partial^2 H_z}{\partial \rho^2} + \frac{1}{\rho}\cdot\frac{\partial H_z}{\partial \rho} + \frac{1}{\rho^2}\cdot\frac{\partial^2 H_z}{\partial \phi^2} + \gamma^2 H_z = -\omega^2 \mu \varepsilon H_z$$

$$\frac{\partial^2 H_z}{\partial \rho^2} + \frac{1}{\rho}\cdot\frac{\partial H_z}{\partial \rho} + \frac{1}{\rho^2}\cdot\frac{\partial^2 H_z}{\partial \phi^2} + \gamma^2 H_z + \omega^2 \mu \varepsilon H_z = 0$$

$$\frac{\partial^2 H_z}{\partial \rho^2} + \frac{1}{\rho}\cdot\frac{\partial H_z}{\partial \rho} + \frac{1}{\rho^2}\cdot\frac{\partial^2 H_z}{\partial \phi^2} + H_z(\gamma^2 + \omega^2 \mu \varepsilon) = 0$$

But we know $\gamma^2 + \omega^2 \mu \varepsilon = h^2$

$$\frac{\partial^2 H_z}{\partial \rho^2} + \frac{1}{\rho}\cdot\frac{\partial H_z}{\partial \rho} + \frac{1}{\rho^2}\cdot\frac{\partial^2 H_z}{\partial \phi^2} + H_z h^2 = 0 \rightarrow (3)$$

This partial differential eq can be solved by using separation of variables method.

∴ The solution is assumed in the form of

$H_z = PQ \rightarrow (4)$

where 'P' is a func of 'ρ' only and 'Q' is func of ϕ only

Now By substituting eq (4) in eq (3)

$$\frac{\partial^2 PQ}{\partial \rho^2} + \frac{1}{\rho}\cdot\frac{\partial PQ}{\partial \rho} + \frac{1}{\rho^2}\cdot\frac{\partial^2 PQ}{\partial \phi^2} + PQh^2 = 0.$$

$$Q\frac{\partial^2 P}{\partial \rho^2} + \frac{Q}{\rho} + \frac{P}{\partial \phi^2}\frac{1}{\rho} + PQh^2 = 0.$$

now we multiplying this eq by $\frac{\rho^2}{PQ}$

$$\frac{\rho^2}{P}\cdot\frac{\partial^2 P}{\partial \rho^2} + \frac{\rho}{P}\cdot\frac{dP}{d\rho} + \frac{1}{Q}\cdot\frac{d^2 Q}{d\phi^2} + h^2 \rho^2 = 0 \rightarrow (5)$$

first, IInd, IIIrd term of eq (5) are func of 'ρ' only and third term is func of ϕ only

hence $\frac{1}{Q}\cdot\frac{\partial^2 Q}{d\phi^2} = -n^2$

where n^2 is constant.

Then eq (5) become.

$$\frac{\rho^2}{P}\cdot\frac{d^2 P}{d\rho^2} + \frac{\rho}{P}\cdot\frac{dP}{d\rho} - n^2 + h^2 \rho^2 = 0.$$

now this Eq is multiplying by 'P'.

(69)

$$\rho^2 \cdot \frac{d^2P}{d\rho^2} + \rho\frac{dP}{d\rho} + (\rho^2h^2 - n^2)P = 0 \rightarrow (6)$$

This is similar to the Bessel Eq of the form

$$x^2 \cdot \frac{d^2y}{dx^2} + x \cdot \frac{dy}{dx} + (x^2 - n^2)y = 0$$

for this Bessel Eq solution is ~~P = Cn~~

$y = C_n J_n(x)$ where J_n is a n^{th} order bessel func of the first kind and C_n is constant.

now in order to ~~P~~ bring Eq (6) in standard Bessel Eq we write

$$(\rho h)^2 \cdot \frac{d^2P}{d\rho^2} + \rho\frac{dP}{d\rho} + (\rho^2h^2 - n^2)P = 0.$$

∴ The solution of this Eq is given by.

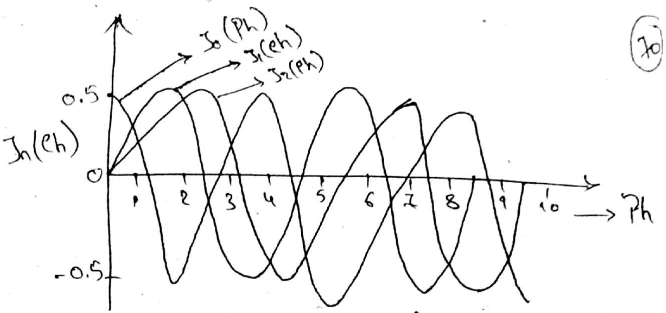
$$P = C_n J_n(\rho h) \rightarrow (7) \text{ and.}$$

$$\frac{1}{Q} \cdot \frac{d^2Q}{d\phi^2} = -n^2 \rightarrow (8)$$

∴ The solution (8) general solution is given by

$$Q = A_n \sin(n\phi) + B_n \cos(n\phi)$$

(70)



Graph shows n^{th} order Bessel func ie $J_n(\rho h)$ of the first kind for some integer values of 'n'.

∴ The complete solution becomes as for Eq (4)

$$P = C_n J_n(\rho h)$$

$$H_z = [C_n J_n(\rho h)][A_n \sin n\phi + B_n \cos n\phi].$$

$$x \sin\phi + y\cos\phi$$

$$= C_n J_n(\rho h)\sqrt{A_n^2 + B_n^2}\cos\left(n\phi + \tan^{-1}\left(\frac{A_n}{B_n}\right)\right);$$

$$= C_n J_n(\rho h)\, C_n' \cos n\phi.$$

∴ $C_n' = \sqrt{A_n^2 + B_n^2}$

$$n\phi = \tan^{-1}\left(n\phi + \frac{A_n}{B_n}\right)$$

⇒ $= C_0 J_n(\rho h) \cos n\phi$

where $C_0 = C_n \cdot C_n'$

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If we consider a sinusoidal variation (71)

then $H_z = C_0 J_n(\rho h) \cos n\phi \cdot e^{-\gamma z} \rightarrow (9)$

of Boundary Conditions

now by applying Boundary Conditions.

we know that all along the surface of circular waveguide at $\rho = a$

i.e $E_\phi = 0$ $\forall$ value of ϕ varying b/w (0 to 2π)

i.e $\frac{\partial H_z}{\partial \rho}_{(\rho=a)} = 0$.

$\Rightarrow J_n'(ah) = 0$

$\Rightarrow$ hear the prime indicated differenciation w.r.t to (ah)

The m^{th} root of this Eq is denoted by P'_{nm} which are the Eigen values. are given by

$\therefore P'_{nm} = (ah) \rightarrow (10)$

Eq (9) now reduces to $H_z = C_0 J_n'(ah) \cos n\phi \cdot e^{-\gamma z}$

$h = P'_{nm}/a$

This Eq represents all possible solutions of H_z for TE NM waves in circular wg

the values of 'h' is given from Eq (10)

i.e $h = (P'_{nm}/a)$ (72)

The various field components $E_\phi, E_\rho, H_z, H_\phi, H_\rho$ can be obtained by using cylindrical co-ordinates by using Maxwells curl Eq.

$\therefore$ The field components are given by

$E_\rho = \frac{-j\omega\mu}{h^2} \cdot \frac{1}{\rho} \cdot \frac{\partial H_z}{\partial \phi} \rightarrow (11)$

$E_\phi = \frac{j\omega\mu}{h^2} \cdot \frac{\partial H_z}{\partial \rho}$

$E_z = 0$

$H_\rho = \frac{-\gamma}{h^2} \cdot \frac{\partial H_z}{\partial \rho}$

$H_\phi = \frac{-\gamma}{h^2} \cdot \frac{1}{\rho} \cdot \frac{\partial H_z}{\partial \phi}$.

$H_z = C_0 J_n'(ah) \cdot \cos n\phi \cdot e^{-\gamma z}$.

Where $h^2 = \gamma^2 + \omega^2 \mu \varepsilon$

By substituting Eq for H_z in above Eq with $h = (P'_{nm}/a)$ the complete field Eq for TE NM waves in circular wg can be given as

$(P'_{nm}/a)^2 = \gamma^2 + \omega^2 \mu \varepsilon$

$E_\rho = C_{0\rho} J_n\left(\frac{P'_{nm}}{a}\rho\right) \sin n\phi \cdot e^{-\gamma z}$

$E_\phi = C_{0\phi} \cdot J_n'\left(\frac{P'_{nm}}{a}\rho\right) \cos n\phi \cdot e^{-\gamma z}$

$E_z = 0$

$H_\rho = \frac{-C_{0\phi}}{Z_z} \cdot J_n'\left(\frac{P'_{nm}}{a}\rho\right) \cos n\phi \cdot e^{-\gamma z}$

$H_\phi = \frac{C_{0\rho}}{Z_z} J_n\left(\frac{P'_{nm}}{a}\rho\right) \cdot \sin n\phi \cdot e^{-\gamma z}$

(73) $H_z = C_0 \cdot J_n\left(a \cdot \frac{P'_{nm}}{a}\right) \cdot \cos n\phi \cdot e^{-\gamma z}$

$= C_0 J_n(P'_{nm}) \cdot \cos n\phi \cdot e^{-\gamma z}$

Propagation of TM waves in Circular Wg:-

For TM wave to propagate in circular 'Wg': $H_z = 0$; & $E_z \neq 0$

from maxwells Eq. $\nabla^2 E_z = -\omega^2 \mu \varepsilon E_z$

The solution of this Eq (simillar to TE waves) is given by.

$E_z = C_0 \cdot J_n(\rho h) \cdot \cos n\phi \cdot e^{-\gamma z}$

By applying Boundary Conditions to above Eq

ie $E_z = 0$, at $\rho = a$

we get

$J_n(ah) = 0$

the values of these roots ie $J_n(ah) = 0$ are called Eigen values and are denoted by 'P_{nm}'.

where $P_{nm} = ah$

The various field components are obtained by using max wells curl Eq.

30/12/08

* Radiation losses:-

1) TEM transmission.

2) uniform di-electric in the neighbour hood of the strip equal in magnitude to an effective value

3) neglect of radiation from the transfers electric field component parallel to strip.

4) substrate thickness must be less than free space wave length.

Ewing's results shows that the ratio of radiated power to total decipated power. for an open circuit micro strip line is given as

$$\frac{P_{rad}}{P_t} = 240\pi^2\left(\frac{h}{\lambda_0}\right)^2 \frac{F(\varepsilon_{re})}{Z_0} \quad \text{--- (1)}$$

where $F(\varepsilon_{re})$ is radiation factor written by

$$F(\varepsilon_{re}) = \frac{\varepsilon_{re}+1}{\varepsilon_{re}} - \frac{\varepsilon_{re}-1}{2\sqrt{\varepsilon_{re}}\,\varepsilon_{re}} \ln\left(\frac{\sqrt{\varepsilon_{re}}+1}{\sqrt{\varepsilon_{re}}-1}\right)$$

Where ε_{re} is the effective di-electric constant and λ_0, $\lambda_0 = \frac{c}{f}$ is free space wave length.

the radiation factor decreases with ↑ di-electric the substrate constant.

thats why. alternatively eq(1) can be expressed as

$$\frac{P_{rad}}{P_t} = \frac{R_r}{Z_0} \rightarrow (3)$$

where R_r is radiation resistance of an open circuited micro strip and is given by

$$R_r = 240\pi^2 (h/\lambda_0)^2 F(\varepsilon_{re})$$

The ratio of radiation resistance (R_r) to the real part of characteristic impeadence Z_0 of the micro strip line is equal to a small factor of power radiated from a single open circuit dis continuity.

Quality factor (Q) of micro strip line.

Many of the micro wave integrated circuits requires very high quality factor. and also very high quality resonant circuits.

But it is limited by the radiation losses of the substrates and the thrown radiation losses. of di-electric constants.

W.k.t but uniform current distribution in the micro strip line the dhming atoriration constant of wire is given as.

$$\alpha_c = \frac{8.686 R_s}{Z_0 \omega}$$

W.k.T the characteristic impeadence of wire micro strip line is

$$Z_0 = \frac{h}{\omega}\sqrt{\frac{\mu}{\varepsilon}} = \frac{377}{\sqrt{\varepsilon_r}} \frac{h}{\omega} (\Omega)$$

The wave length of micro strip line is given as

$$\lambda_g = \frac{30}{f\sqrt{\varepsilon_r}} \text{ cm} \rightarrow (1)$$

where 'F' is freq. in GHz

∴ Q_c is related to the conductor attenuation constant for $Q_c = \frac{27.3}{\alpha_c}$

Where Q_c is quality factor of (Ω) attenuation constant.

where α_c is the (Ω) for conductor attenuation constant in dB/λ_g

∴ Q_c of a wide (Ω) strip line is expressed as $Q_c = 39.5 (h/R_s) f \rightarrow (3)$

$Q_c = 39.5\left(\frac{h}{2}\right)\Gamma \rightarrow (3)$

where h is measured in 'Cm'

R_s is expressed as $\sqrt{\frac{\pi\mu f}{\sigma}}$ $\mu = 4\pi \times 10^{-7} H/m$

$$R_s = 2\pi\sqrt{\frac{f}{\sigma}} \ \Omega/sq$$

finally the quality factor (Q_c) of a wire micro strip line is $Q_c = 0.63h\sqrt{\sigma f_{GHz}}$

where 'σ' is conductivity of di-electric substrate board. in $\mho/cm$

for a copper strip conductivity

$$\sigma = 5.8 \times 10^{7} \mho/cm$$

and 'Q_c' becomes $Q_{cu} = 4.797\, h\sqrt{f_{GHz}}$

similarly a quality factor is related to the di-electric attenuation constant.

$$Q_d = \frac{27.3}{\alpha_d}$$

we know $\alpha_d = 27.3\left(\frac{q\epsilon_r}{\epsilon_{re}}\right)\frac{\tan\theta}{\lambda g}$. $dB/\lambda g$

$$Q_d = \frac{\epsilon_{re}}{q\epsilon_r} \cdot \frac{\lambda g}{\tan\theta}$$

$$Q_d \simeq \lambda g/\tan\theta$$

hence the quality factor for the di-electric attenuation constant of a micro strip line is approximatly equal to the reciprocal of the di-electric loss tangent. $(\tan\theta)$.

$$\tan\theta = \frac{\sigma}{\omega\epsilon}$$

$$Q_d = \frac{\omega\epsilon\lambda g}{\sigma}$$

31/12/08

Cavity Resonator:-

Cavity Resonator is a peace of 'wg' closed at both ends by conducting (or) metalic planes. (or)

Cavity Resonator is a metalic enclosed that confines electromagnetic energy.

(or)

Closing of co-axial lines (or) 'wg' & may be considered as forms of a more general type of resonator. Called the cavity resonator which may consiste of a cavity completely enclosed by conducting walls

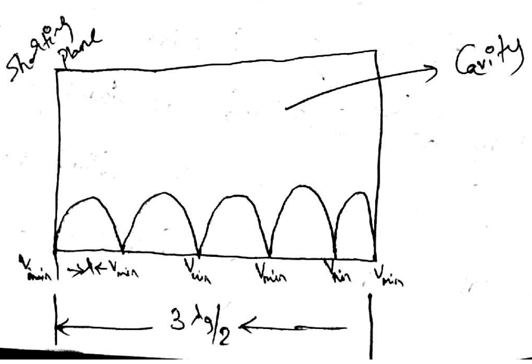
The electrical and magnetic energies are stored inside the cavity determines its ~~inductance~~ inductance and capacitance.

→ Cavity resonators are formed by shorting two ends of a section of a 'wg'.

→ When one end is terminated of the 'wg' in a shorting plate there will be reflections and hence standing waves occurs. Standing waves and oscillations will take place in resonator exits

→ When another end is terminated by shorting plate is kept at a distance of a multiple of $\lambda_g/2$ then hallow space is formed and this space can support a signal which bounces back and forth. b/w the 2 - shorting plates.

→ This results in resonance and hence the hallow space is called cavity. and resonator is called cavity resonator.



Rectangular cavity resonator, circular cavity resonator and re-entrant cavity resonator are commonly used in many μwave applications.

Micro wave cavity resonators are tuneable circuits used in micro wave oscillators, ~~appli~~ amplifiers, wave meters, and filters.

→ The micro wave cavity resonator is related to tuned circuit at low freq having a resonant frequency.

$$f_r = \frac{1}{2\pi\sqrt{LC}}$$

→ Cavity resonator can resonate at only one particular frequency. Just as a parallel resonant circuit.

→ Propagation in cavity resonator may take place in various directions.

i.e more than one direction and in various modes. In general cavity resonators are capable of possibility of large number of possible modes of resonance.

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(Properties of Cavity resonator :-

1) The Cavity resonator Should resonate at only one particular frequency.

2) With in the Cavity various TE & TM modes Exits.

3) The Energy dissipated by the finite Conductivity of the Cavity walls determines its Equalent resistance.

4) When frequency of i/p Signal is Equal to a resonant frequency a maximum amplitude of the Standing waves occurs. and the Peack energies stored in the Electric and magnetic fields are Equal.

5) The mode with lowest resonant freq is Called the dominant mode.

6) They have high quality factor (Q)

7) They have high Shunt impeadence

8) Theoritically infinite modes occurs in a resonant circuits.

9) Coupling is zero when the Plane of loop is rotated so that it is Parallel to magnetic flux.

2/1/09

Rectangular Cavity resonator :-

" " " is a rectangular 'wg' with 2. Sides closed by Conducting walls are metalic walls.

The wave Eq for Electro magnetic fields in side the Cavity. In side Cavity resonator Should satisfy maxwells Eq and it is subject to the boundary Conditions that the Electric field tangential to and magnetic field ~~Para~~ normal to metal walls must dis appears.

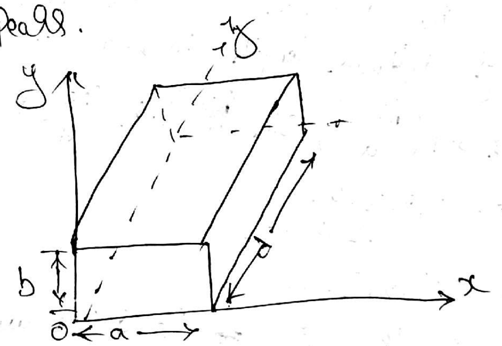


figure shows. the Co-ordinates of a rectangular Cavity resonator.

The wave Eq in the rectangular Cavity resonator Should Satisfies boundary Conditions of zero tangential electric field of four of the walls.

It is necessary to choose the harmonic func of z-direction to satisfy the boundary condition.

Can be given as.

$$H_z = H_{0z} \cos\left(\frac{m\pi x}{a}\right) \cdot \cos\left(\frac{n\pi y}{b}\right) \cdot \sin\left(\frac{p\pi z}{d}\right)$$

where $m = 0, 1, 2, 3, \ldots$, represents the number of half wave lengths in the x-direction, and $n = 0, 1, 2, 3, \ldots$ represents the number of half wave lengths in y-direction and $p = 1, 2, 3, \ldots$ represents the number of half wave length in z-direction and

$$E_z = E_{0z} \sin\left(\frac{m\pi x}{a}\right) \cdot \sin\left(\frac{n\pi y}{b}\right) \cdot \cos\left(\frac{p\pi z}{d}\right)$$

here $m = 1, 2, 3, 4, \ldots$

$n = 1, 2, 3, 4, \ldots$

and $p = 0, 1, 2, 3, \ldots$

The separation Eq for TE & TM waves is given by

$$K^2 = \left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2 + \left(\frac{p\pi}{d}\right)^2.$$

for a loss less di-electric

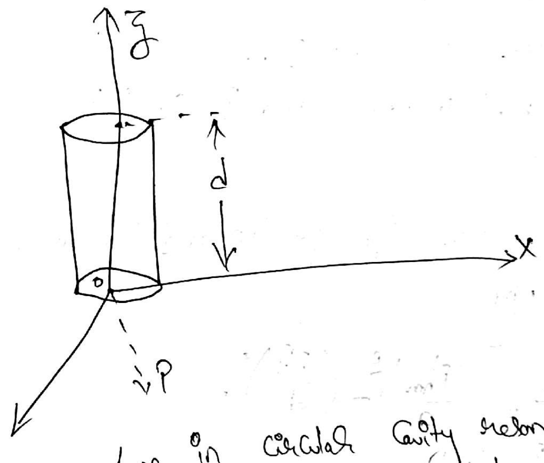
$$K^2 = \omega^2 \mu \varepsilon$$

∴ The resonant freq is expressed as.

$$f_r = \frac{1}{2\sqrt{\mu\varepsilon}} \sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2 + \left(\frac{p}{d}\right)^2}$$

Circular Cavity ~~resolution~~: resonator

" " is a circular 'wg' with 2-sides closed by metalic walls (or) conducting walls.



The wave func in circular cavity resonator should satisfy maxwells Eq and subject to the same boundary conditions. dis-

hence it is necessary to use the harmonic func in z-direction to satisfy the boundary

Condition there harmonic func can be given as.

$$H_z = H_{oz}\, J_n\left(\frac{P'_{nm}\rho}{a}\right) \cos n\phi \sin\left(\frac{P\pi z}{d}\right).$$

where $n = 0, 1, 2, 3 \ldots$
and $m = 1, 2, 3, 4 \ldots$
$P = 1, 2, 3, 4 \ldots$

where J_n is the Bessel func of the first guide where H_{oz} is amplitude of magnetic field and

$$E_z = E_{oz} \cdot J_n\left(\frac{P_{nm}\rho}{a}\right)$$

where $n = 1, 2, 3 \ldots$
$m = 1, 2, 3, 4 \ldots$ and $P = 0, 1, 2, 3$

P_{oz} is

the seperation Eq for TE and TM waves are given by

$$k^2 = \left(\frac{P'_{nm}}{a}\right)^2 + \left(\frac{P\pi}{d}\right)^2.$$

and ~~k^2 TM wave~~

$$k^2 = \left(\frac{P_{nm}}{a}\right)^2 + \left(\frac{P\pi}{d}\right)^2 \text{ TM waves}$$

we know $k^2 = \omega^2 \mu \varepsilon$

hence the resonance freq given by.

$$f_r = \frac{1}{2\pi\sqrt{\mu\varepsilon}} \sqrt{\left(\frac{P'_{nm}}{a}\right)^2 + \left(\frac{P\pi}{d}\right)^2}$$

for TM wave $f_r = \dfrac{1}{2\pi\sqrt{\mu\varepsilon}} \sqrt{\left(\frac{P_{nm}}{a}\right)^2 + \left(\frac{P\pi}{d}\right)^2}$

Expressions for various field components in a rectangular cavity resonator:-

For TE Waves:-

$$H_x = H_{ox} \cdot \cos\left(\frac{n\pi}{x}\right) \cdot \cos\left(\frac{m\pi}{x}\right) \cdot \sin\left(\frac{P\pi}{d}\right).$$

$$H_y = \frac{1}{k_c^2} \cdot \frac{\partial^2 H_z}{\partial y \partial z}$$

now differ H_z w.r.t. y, z

$$H_y = \frac{1}{k_c^2} \cdot \frac{\partial^2}{\partial y \partial z}\left(H_{oz} \cos\left(\frac{n\pi}{x}\right) \cdot \cos\left(\frac{m\pi}{x}\right) \sin\left(\frac{P\pi}{d}\right)\right)$$

$$H_y = \frac{H_{oz}}{k_c^2} \sin\frac{P\pi}{d}\left(\frac{-n\pi}{x}\right)\left(-\sin\left(\frac{m\pi}{x}\right)\right)\left(\frac{-m\pi}{x}\right). \left(-\sin\left(\frac{m\pi}{x}\right)\right)$$

$$H_y = -\left(\frac{H_{ox}}{k_c^2}\right).$$

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$$H_x = \frac{1}{k_x^2} \cdot \frac{\partial H_z}{\partial x\, \partial y}$$

$$H_x = \frac{1}{k_x^2} \frac{\partial}{\partial x\, \partial y}\left(H_{oz} \cos\left(\frac{n\pi}{a}\right) \cos\left(\frac{m\pi}{a}\right) \sin\left(\frac{p\pi}{d}\right)\right)$$

$$= \frac{H_{oz}}{k_x^2}\left(-\sin\left(\frac{n\pi}{a}\right)\cdot\left(\frac{n\pi}{a}\right)\right)\left(-\sin\left(\frac{m\pi}{a}\right)\cdot\frac{m\pi}{a}\right) \cos\left(\frac{p\pi}{d}\right)\cdot\left(\frac{p\pi}{d}\right)$$

$$\Rightarrow -\frac{H_{oz}}{k_x^2} \sin\left(\frac{n\pi}{a}\right)\cdot\left(\frac{m\pi}{a}\right)\left(-\sin\left(\frac{m\pi}{\cancel{a}}\right)\cdot\frac{m\pi}{\cancel{a}}\right) \cos\left(\frac{p\pi}{d}\right)\cdot\left(\frac{p\pi}{d}\right).$$

$$\Rightarrow \frac{-H_{oz}}{k_x^2}\left(\sin\left(\frac{m\pi z}{a}\right)\left(\frac{m\pi}{a}\right) \cdot \cos\left(\frac{n\pi y}{b}\right)\left(\frac{n\pi}{b}\right).\right.$$
$$\cos\left(\frac{p\pi z}{d}\right)\left(\frac{p\pi}{d}\right)\left(\frac{-H_{oz}}{k_c^2}\right)\left(\frac{mp\pi^2}{ad}\right)$$
$$\sin\left(\frac{m\pi z}{a}\right) \cdot \cos\left(\frac{n\pi x}{a}\right) \cdot \cos\left(\frac{n\pi y}{b}\right)$$
$$\cos\left(\frac{p\pi z}{d}\right).$$

we know $E_x = 0$; $E_y = \frac{J\omega\mu}{k_c^2} \cdot \frac{\partial H_z}{\partial x}$..

now diffrentate

$$E_x = \frac{-J\omega\mu}{k_c^2} \cdot \frac{\partial H_z}{\partial y},$$

$$E_x = \frac{-J\omega\mu}{k_c^2} \frac{\partial}{\partial y}\left(H_{oz} \cos\left(\frac{n\pi}{a}\right) \cos\left(\frac{m\pi}{b}\right) \cdot \sin\left(\frac{p\pi}{d}\right)\right)$$

$$E_x = \frac{-J\omega\mu}{k_c^2} \cdot H_{oz} \cos\left(\frac{n\pi x}{a}\right) \cdot \sin\left(\frac{m\pi x}{b}\right) \cdot \sin\left(\frac{p\pi x}{d}\right)$$

for TM wave. $H_z = 0$; $E_z \neq 0$ but

$$E_z = E_{oz} \sin\left(\frac{n\pi x}{a}\right) \sin\left(\frac{m\pi x}{b}\right) \cdot \cos\left(\frac{p\pi}{d} z\right)$$

$$E_y = \frac{1}{k_c^2} \frac{\partial^2 H_z}{\partial y\, \partial y}$$

$$E_y = \frac{1}{k_c^2} \cdot \frac{\partial^2}{\partial y\, \partial y}\left(E_{oz} \sin\left(\frac{n\pi x}{a}\right) \cdot \sin\left(\frac{m\pi x}{b}\right) \cdot \cos\left(\frac{p\pi}{d} x\right)\right)$$

$$E_y = -\frac{E_{oz}}{k_c^2} \cos\left(\frac{n\pi x}{a}\right) \cdot \cos\left(\frac{m\pi x}{b}\right) \cdot \sin\left(\frac{p\pi z}{d}\right)$$

$$E_x = \frac{1}{k_c^2} \cdot \frac{\partial^2 E_z}{\partial x\, \partial y}.$$

$$E_x = \frac{1}{k_c^2} \frac{\partial^2}{\partial x\, \partial y}\left(E_{oz} \sin\left(\frac{n\pi x}{a}\right) \sin\left(\frac{m\pi x}{b}\right) \cdot \cos\left(\frac{p\pi z}{d}\right)\right)$$

$$= \frac{-E_{oz}}{k_c^2}\left(\cos\left(\frac{n\pi x}{a}\right) \cdot \cos\left(\frac{m\pi x}{b}\right)\left(-\sin\left(\frac{p\pi z}{d}\right)\right)\right).$$

and $H_y = 0$;

$$H_x = \frac{J\omega\varepsilon}{k_c^2} \cdot \frac{\partial E_z}{\partial y} =$$

$$H_y = \frac{-J\omega\varepsilon}{k_c^2} \cdot \frac{\partial E_z}{\partial x}$$

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$$H_y = \frac{-j\omega\varepsilon}{k_c^2} \cdot \frac{\partial}{\partial x}\left(E_{oz} \sin\left(\frac{n\pi x}{a}\right) \cdot \sin\left(\frac{m\pi x}{b}\right) \cdot \cos\left(\frac{p\pi z}{d}\right)\right)$$

$$= \cancel{\frac{-j\omega\varepsilon}{k_c^2} \cdot \frac{\partial}{\partial x}}\ \cancel{\varepsilon}$$

$$= \frac{E_{oz}}{k_c^2} \sin\left(\frac{n\pi x}{a}\right) \cos\left(\frac{m\pi x}{b}\right) \cdot \sin\left(\frac{p\pi z}{d}\right)$$

where k_c is cut off wave number and it is given by $k_c^2 = \left(\frac{n\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2$.

Expressions for various field components in a circular cavity resonator:-

For TE waves:-

$$H_z = H_{oz} J_n\left(\frac{P'_{nm}}{a}\rho\right) \cos n\phi \sin\left(\frac{p\pi z}{d}\right)$$

$$H_\phi = -H_{oz}\left(\frac{p\pi}{d}\right)\left(\frac{n}{\rho}\right)\left(\frac{a}{P'_{nm}}\right)^2 J_n\left(\frac{P'_{nm}}{a}\rho\right) \sin\phi \cos\left(\frac{p\pi x}{d}\right).$$

$$H_\rho = H_{oz}\left(\frac{p\pi}{d}\right)\left(\frac{a}{P'_{nm}}\right) J'_n\left(\frac{P'_{nm}\rho}{a}\right) \cos n\phi \cdot \cos\left(\frac{p\pi z}{d}\right)$$

$$E_z = 0$$

$$E_\phi = j\omega\mu H_{oz}\left(\frac{a}{P'_{nm}}\right) J'_n\left(\frac{P'_{nm}}{a}\rho\right) \cos n\phi \sin\left(\frac{p\pi z}{d}\right)$$

$$E_\rho = j\omega\mu H_{oz}\left(\frac{n}{\rho}\right)\left(\frac{a}{P_{nm}}\right)^2 J_n\left(\frac{P_{nm}}{a}\rho\right) \sin n\phi \sin\left(\frac{p\pi z}{d}\right)$$

For TM waves:-

$$E_z = E_{oz} J_n\left(\frac{P_{nm}}{a}\right) \cos\left(\frac{n\pi}{a}\right) \sin\left(\frac{m\pi}{a}\right).$$

$$E_\phi = E_{oz}\left(\frac{p\pi}{d}\right)\left(\frac{n}{\rho}\right)\left(\frac{a}{P_{nm}}\right)^2 J_n\left(\frac{P_{nm}}{a}\rho\right) \sin n\phi \sin\left(\frac{p\pi z}{d}\right).$$

$$E_\rho = -E_{oz}\left(\frac{p\pi}{d}\right) \cdot \left(\frac{a}{P_{nm}}\right) J'_n\left(\frac{P_{nm}}{a}\rho\right) \cdot \cos n\phi \sin\left(\frac{p\pi z}{d}\right)$$

$$H_z = 0$$

$$H_\phi = -j\omega\varepsilon E_{oz}\left(\frac{a}{P_{nm}}\right) J_n\left(\frac{P_{nm}\rho}{a}\right) \cos n\phi \cos\left(\frac{p\pi z}{d}\right)$$

$$H_\rho = j\omega\varepsilon E_{oz}\left(n/\rho\right)\left(a/P_{nm}\right)^2 J_n\left(\frac{P_{nm}}{a}\rho\right) \sin n\phi \cos\left(\frac{p\pi z}{d}\right).$$

Expression for Resonant frequency in a rectangular cavity resonator:-

We know that A rectangular 'wg' k_c^2

$$h^2 = \gamma^2 + \omega^2\mu\varepsilon = A^2 + B^2 = \left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2$$

$$\gamma^2 + \omega^2\mu\varepsilon = \left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2$$

$$\omega^2\mu\varepsilon = \left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2 - \gamma^2.$$

We know that for wave propagation.

$$\gamma = j\beta$$

$$\therefore \gamma^2 = -\beta^2$$

If a wave has to exists in a cavity resonator there must be a ~~fate~~ phase change corresponding to a given ~~waveleng~~ guide wave length.

$$\beta = 2\pi/\lambda_g$$

The condition for the resonator to resonate is

$$\beta = P\pi/d$$

where P = constant its value starts from 1 .... ∞ that indicates half wave variation of either electric or magnetic fields along the z-direction.

D is the length of resonator.

$$F = F_0, \quad \omega = \omega_0 = 2\pi f_0 \quad \text{and} \quad \beta = \left(\frac{P\pi}{d}\right).$$

By substituting the value of 'β' ~~on~~ where $\omega = \omega_0$ in Eq (1)

$$\omega_0^2 \mu \varepsilon = \left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2 + \left(\frac{P\pi}{d}\right)^2$$

$$\omega_0^2 = \frac{1}{\mu\varepsilon}\left[\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2 + \left(\frac{P\pi}{d}\right)^2\right]$$

$$(2\pi f_0)^2 = \frac{1}{\mu\varepsilon}\left[\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2 + \left(\frac{P\pi}{d}\right)^2\right].$$

$$f_0^2 = \frac{1}{(2\pi)^2\mu\varepsilon}\left[\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2 + \left(\frac{P\pi}{d}\right)^2\right]$$

$$f_0 = \frac{1}{2\pi\sqrt{\mu\varepsilon}}\sqrt{\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2 + \left(\frac{P\pi}{d}\right)^2}.$$

But we know $\frac{1}{\sqrt{\mu\varepsilon}} = C$

$$f_0 = \frac{C}{2\pi}\sqrt{\pi^2\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2 + \left(\frac{P}{d}\right)^2}$$

$$= \frac{C}{2}\sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2 + \left(\frac{P}{d}\right)^2}$$

For both TE & TM waves the resonant freq is same in a rectangular cavity resonator.

General mode of propagation in a rectangular cavity resonator is :-

TE_{mnp} , TM_{mnp} ; TE_{mnp} TM_{mnp}

Expression for ~~the~~ resonant frequency in a circular cavity resonator:-

Circular end plates are used to short the circular both ends.

a = radius of the 'wg'

d = length (or) height of ~~the~~ circular wg.

We know condition for resonance is $\beta = \left(\frac{P\pi}{d}\right)$.

For circular ω_g we know that

$$h^2 = \gamma^2 + \omega^2 \mu \varepsilon$$

and also w.k.t :-

$$h = \frac{P'_{nm}}{a} \text{ for TE waves}$$

$$\therefore h^2 = \left(\frac{P'_{nm}}{a}\right)^2$$

$$\omega^2 \mu \varepsilon = h^2 - \gamma^2$$

$$\omega^2 \mu \varepsilon = \left(\frac{P'_{nm}}{a}\right)^2 - \gamma^2$$

For wave propagation $\gamma = j\beta$ and for resonance

$\beta = \frac{P\pi}{d}$ and $\omega = \omega_0$

$\therefore$ Eq (1) becomes $\gamma^2 = j^2\beta^2 = -\beta^2$

$$\omega_0^2 \mu \varepsilon = \left(\frac{P'_{nm}}{a}\right)^2 + \left(\frac{P\pi}{d}\right)^2$$

$$(2\pi f_0)^2 \mu \varepsilon = \frac{1}{\mu \varepsilon}\left[\left(\frac{P'_{nm}}{a}\right)^2 + \left(\frac{P\pi}{d}\right)^2\right]$$

$$f_0 = \frac{1}{2\pi\sqrt{\mu\varepsilon}}\sqrt{\left(\frac{P'_{nm}}{a}\right)^2 + \left(\frac{P\pi}{d}\right)^2}$$

$$f_0 = \frac{c}{2\pi}\sqrt{\left(\frac{P'_{nm}}{a}\right)^2 + \left(\frac{P\pi}{d}\right)^2}$$

For TM waves $f_0 = \frac{c}{2\pi}\sqrt{\left(\frac{P_{nm}}{a}\right)^2 + \left(\frac{P\pi}{d}\right)^2}$

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Dominant modes :-

We know that for rectangular cavity resonator the resonant freq is

$$f_r = \frac{c}{2\pi}\sqrt{\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2 + \left(\frac{P\pi}{d}\right)^2}$$

For $a > b < d$ the dominant mode is TE_{101} mode

For circular cavity resonator the resonant freq is

$$f_r = \frac{1}{2\pi\sqrt{\mu\varepsilon}}\sqrt{\left(\frac{P'_{nm}}{a}\right)^2 + \left(\frac{P\pi}{d}\right)^2}$$

For TM waves

$$f_r = \frac{1}{2\pi\sqrt{\mu\varepsilon}}\sqrt{\left(\frac{P_{nm}}{a}\right)^2 + \left(\frac{P\pi}{d}\right)^2}$$

When $d > 2a$ then TE_{111} is the dominant mode.

When $2a > d$ then TM_{110} is the dominant mode

Q-factor (&) Coupling Co-efficients of cavity resonator :-

The quality factor 'Q' is a measure of the frequency selectivity of a resonant (or) anti resonant circuit. and it is defined as

$$Q = 2\pi \times \frac{\text{maximum energy stored}}{\text{Energy dissipated per cycle}}$$

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$$Q = \frac{\omega W}{P} \quad \text{— (1)}$$

where W is maximum Energy stored 'P' is the avg Power loss. at resonant freq the Electric and magnetic fields are Equal. When the electric energy is maximum the magnetic Energy is zero. and vise versa.

The total Energy stored in resonator is obtained by integrating the Energy density over the value of the resonator.

$\omega = 2\pi f_r$

~~Q = 2π~~

$$Q = \frac{2\pi f_r W}{P}$$

$$W_e = \int_v \frac{\epsilon}{2}|E|^2 dv = \omega_m = \int_v \frac{\mu}{2}|H|^2 dv = W \rightarrow (2)$$

where $|E|$ & $|H|$ are peak value of Electric and magnetic field intensities the avg Power loss in resonator is Evaluated & obtained by integrating Power density over the surface of resonator.

$$\text{here } P = \frac{R_s}{2}\int_s |H_t|^2 da \rightarrow (3)$$

where H_t is the Peak value of the tangential magnetic field and R_s is surface resistance of the resonator.

By substituting eq (2) & (3) in (1)

$$Q = \frac{2\pi f_r \int_v \frac{\epsilon}{2}|E|^2 dv}{\frac{R_s}{2}\int_s |H_t|^2 . da} = \frac{\omega\mu \int_v |H|^2 . dv}{R_s \int_s |H_t|^2 . da} \rightarrow (4)$$

∵ the Peak value of the magnetic intensity is related to its tangential and normal components by

$$|H|^2 = |H_t|^2 + |H_n|^2.$$

where H_n is the Peak value of the normal magnetic intensity.

the value of $|H_t|^2$ at the resonator is approximatly twise the value of $|H_n|^2$.

∴ the quality factor (Q)

the Eq (4) can be expressed as

$$Q = \frac{\omega\mu\,(\text{value})}{2R_s\,(\text{surface})} \rightarrow (5)$$

An unloaded resonator can be represented by either a series or a parallel resonant circuit.

The resonant freq and the unloaded quality factor 'Q_0' of a cavity resonator given by

$$f_0 = \frac{1}{2\pi\sqrt{LC}} \rightarrow (6)$$

$$Q_0 = \frac{\omega_0 L}{R} \rightarrow (7)$$

The loaded quality factor (Q) of system is given by

$$Q_l = \frac{\omega_0 L}{R + N^2 Z_g}$$

The coupling co-efficient of system is defined as $k = \frac{N^2 Z_g}{R}$

where N is transformer turns ratio and Z_g is internal impedance and 'R' is series resistance

hence the loaded quality factor would becomes

$$Q_l = \frac{\omega_0 L}{R(1+k)} \rightarrow (8)$$

$\therefore$ Q_l becomes $= \frac{Q_0}{1+k} \rightarrow (9)$

by re-arrangement of eq (8) and (9) we get

$$\cancel{Q_l} = \frac{\cancel{\omega_0 L}}{R}$$

$$\frac{1}{Q_l} = \frac{1}{Q_0} + \frac{1}{Q_{ext}}$$

where $Q_{ext} = \frac{Q_0}{k} = \frac{\omega_0 L}{Rk}$

Q_{ext} = External quality factor.

There are 3-types of quality factor:-

1) Critical coupling :- If the resonator is matched to a generator then $k = 1$.

* The loaded Q_l is given by

$$Q_l = \frac{1}{2} Q_{ext} = \frac{1}{2} Q_0$$

hence $Q_{ext} = 2Q_0$

2) Over coupling :- If 'k' is greater than one at a voltage maximum the cavity terminals are in the o/p line at resonance hence the normalised impedance at voltage maximum is standing wave ratio.

i.e 'ρ'

$\therefore k = \rho$

hence ~~loaded~~ $Q_L = \dfrac{Q_0}{1+\rho}$ and $Q_{ext} = \dfrac{Q_0}{\rho}$

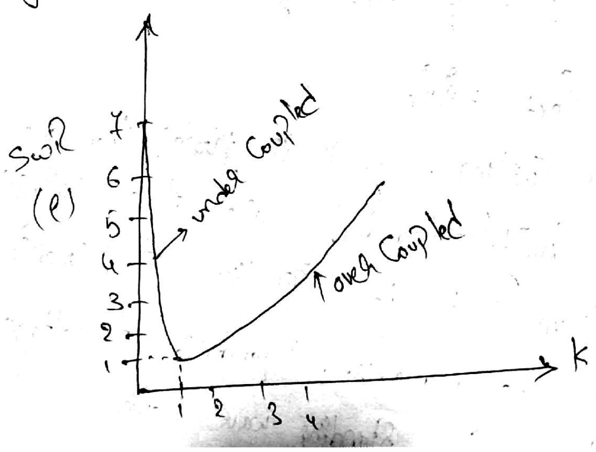
3) Under Coupling :- If 'k' is less than ~~0~~ 1 are at voltage minimum and the i/p terminal is equal to reciprocal of standing wave ratio.

$k = \dfrac{1}{\rho}$.

$Q_L = \dfrac{Q_0 \rho}{1+\rho}$

$Q_{ext} = $ '$Q_0 . \rho$'

The relationship of Coupling Co-efficients 'k' and stand wave ratio is shown in below figure.



5/01/09

1) A rectangular Cavity resonator of width 'a', height 'b', and length 'd' is to resonance with V_{t} : 101V show that the freq of resonance $f_r = \dfrac{c}{2d}\sqrt{1+\dfrac{d^2}{a^2}}$ if $f_r = 10\,GHz$ a = 2cm b = 1cm.. find 'd'.

2) A circular Cavity resonator of radius 'a' and height 'd' is to resonance with TE_{111} mode show that the freq of resonance $f_r = \dfrac{c}{2d}\sqrt{\left(1+\dfrac{[illegible]}{a}\right)^2}$ assume that oscillatory func ~~[illegible]~~

$x'_{11} = 3.832$