



I-1

## Neural Networks:-

Introduction:- By concept of biological neural networks may hold the key to success of solving intelligent tasks by machines. The new field is called Artificial Neural networks, which is more apt to describe it as parallel and distributed processing.

\* Brain :- Has A highly complex, non-linear & parallel complex Structural Constituents  $\rightarrow$  Neurons.

human brain Neurons are interconnected in a very complex way impossible to simulate.

$\therefore$  human brain got typically billions of nerve cells and trillions of interconnections.

\* The no of human brain are: Massively parallel no of neurons

- ✓ 10 - billions of neurons
- ✓ 60 - trillions of interconnections

$\rightarrow$  Artificial Neural Net :- (ANN) :-

NIAEFVN

Usefulness & Capabilities:-

i) It explores non-linearity in inter connections of nonlinear neurons.

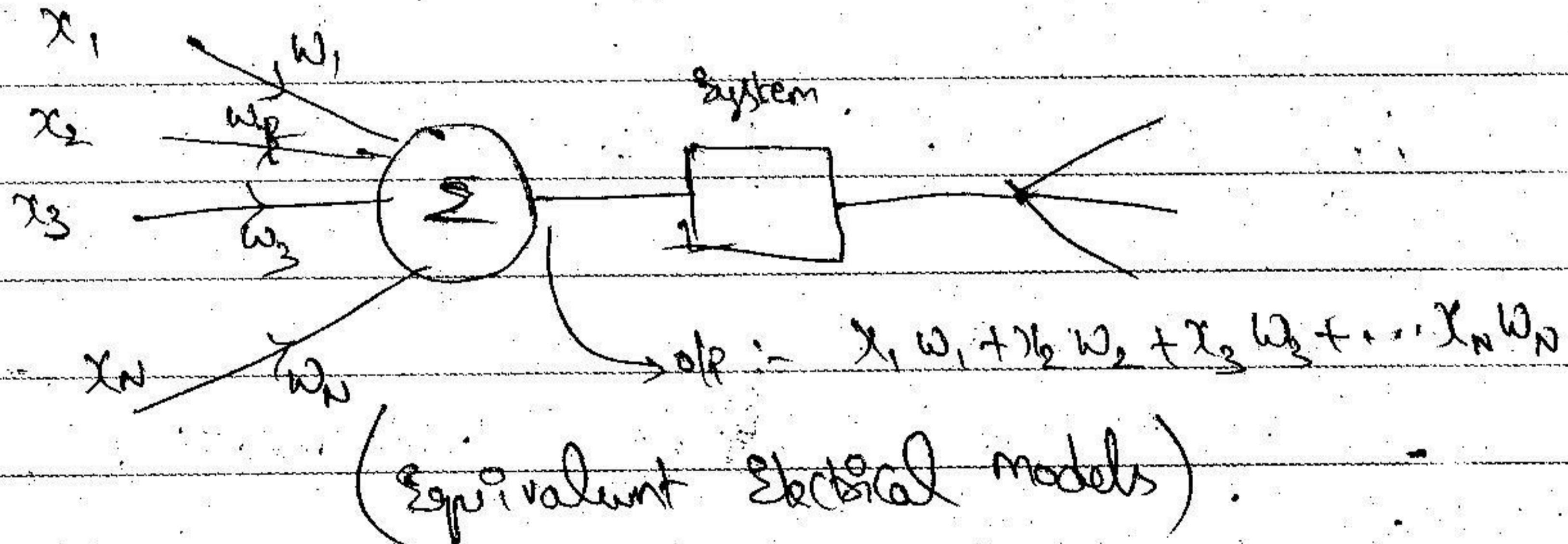
$\rightarrow$  Nonlinearity is distributed throughout.

\* (%) The NN are based on Parallel architecture of biological brains



- 2) IP & OP Mapping :- (e.g. Storing IP in a pattern. & op pattern is ~~can~~ can be different we have to do adjust)  
↳ Learning ability.
- 3) Adapting :- Can adapt the free parameters to changes in surrounding environment.
- 4) Credential Response :- Decision with a measure of confidence.
- 5) Fault tolerance :- (if connection is not working?)  
↳ Just degradation happens it is (Graceful degradation)
- 6) VLSI Implementability :- (based on concept of VLSI implementation to be designed neuron for computing).
- 7) Neuro-biological analogy :- All above should be considered.

Pyramidal Representation:



## Neural Net:-

(I-3) 1

- 1) Neural Net :- Is an artificial representation of the human brain that tries to simulate its learning process.
- An artificial neural net (ANN) is often called a Neural Net (NN).
- \* Traditionally (NN) is represented as network of biological neurons in nervous system that process & transmit information.
- ANN is interconnected group of artificial neurons that represents a mathematical model (or) computational model for information.
- \* Processing based on connectionist approach to computation.
- ANN is a net of simple processing elements (neurons) which can exhibit complex global behavior determined by connections b/w the processing elements & element parameters.  
↓  
(neurons)

## 2) Why Neural Net :-

- Conventional Computers are good for fast arithmetic & does what programmed does Program as to do.
- The conventional computers are not so good for interacting with noisy data (or) data from Environment, massive parallelism fault tolerance; and adapting to circumstances.
- \* → NN system help where we can not formulate an algorithm solution, as where we get lot of examples of behavior.
- NN follows different paradigm for computing.
- \* (i) von-Neuman machines based on processing/memory abstraction of human information processing
- \* (ii) The NN are based on parallel architecture of biological brains

→ NN are form of multiplexed coupled system, with (I-4)

- (i) Simple processing elements
- (ii) A high degree of interconnection
- (iii) Simple local message
- (iv) Adaptive interaction b/w elements.

3)

History :-

(i) McCulloch and Pitts (1943) :- are generally recognized as the

\* designer of first neural nw. which combine  
→ many signal processing unit together which could lead overall  
increase in computational power.

\* Suggested idea :- Neuron has a threshold level and once that level  
is reached neuron fires.

It is fundamental way which ANN's operate.

→ M&P nw had a fixed set of weights.

(ii) Hebb (1949) :- developed first learning rule: law of

\* 2-neurons are active at same time then the strength b/w  
them should be increased.

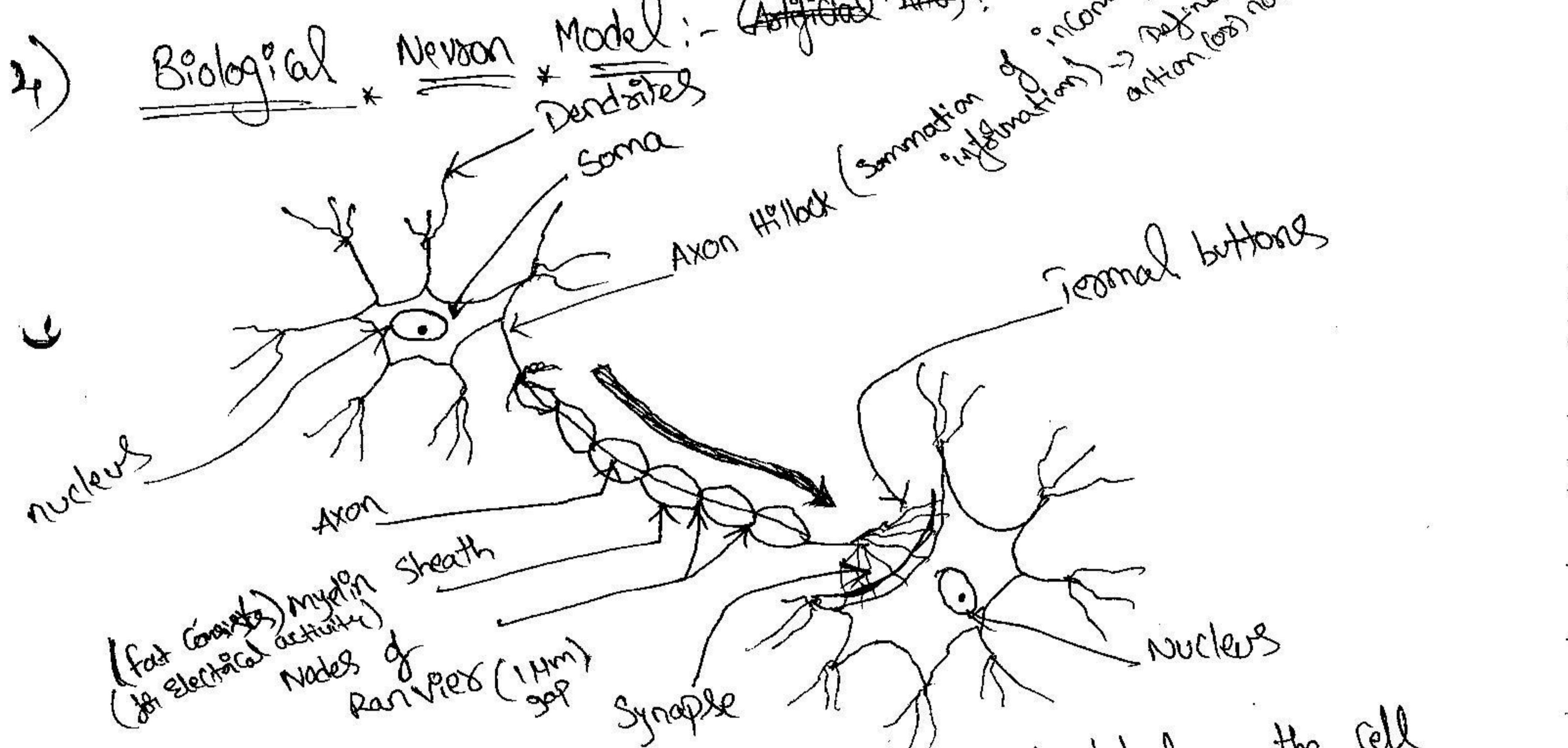
→ In 1950 & 1960's, many researchers (Brock, Minsky, Papert,  
& Rosenthal) worked on Perception.

→ The neural nw model could be forced to converge to  
collect weights that will solve the problem.

→ The weight adjustments (learning algorithm) used in

Perception was found more powerful than learning rule.  
used by Hebb.

- (iii) Minsky & Papert (1969) :- showed perception could not learn those functions which are not linearly separable. (3)  
 (iv) In (1985-1986) Parker & Lehn discovered a learning algorithm for multilayered now called back propagation that could solve problems that were not linearly separable. (2-5)



- (i) Dendrites :- are branching fibers that extended from the cell body (of) soma
- (ii) Soma (or) Cell body :- neuron contains the nucleus and other structures, support chemical processing & production of neurotransmitters.
- (iii) Axon :- is a singular fiber carries information away from the soma to the synaptic sites of other neurons, muscles, (or) glands.
- (iv) Axon Hillock :- is the site of summation for incoming information. At any moment, the collective influence of all neurons that conduct impulses to a given neuron will determine whether (or) not an action potential will be initiated at the axon hillock and propagated along the axon.

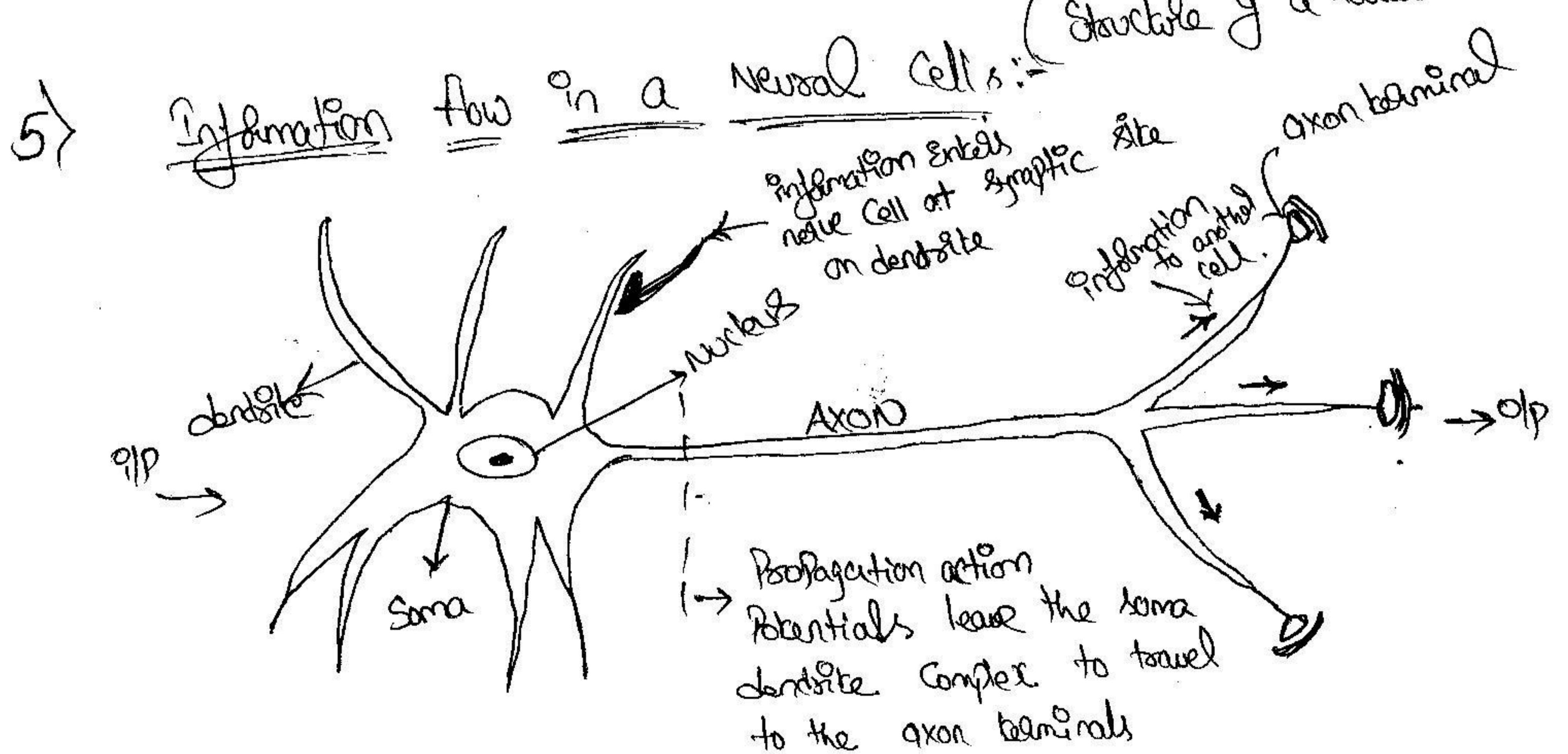
- (v) Myelin Sheath:- Consists of fat containing cells that insulate the axon from electrical activity.
- The insulation aids to increase rate of tx of signals
  - A gap exists b/w each myelin sheath cell along the axon.
  - When fat inhibits the propagation of electricity the signal jumps from one gap to the next.

(vi) Nodes of Ranvier:- Axon gaps about (1 μm) b/w myelin sheath cells along axon are since fat serves as a good insulator, the myelin sheath speeds the rate of tx of a electrical impulse along the axon.

(v) Synapse:- Is the point connection b/w two neurons (or) a neuron and muscle (or) a gland.

→ Electro chemical communication b/w neurons takes place at these junctions

(vi) Terminal Buttons:- A small knobs at the end of an axon that release chemicals called neurotransmitters.



(5)

- Dendrites receive action from other neurons
- Soma Process the incoming activation & converts them into (1-7)
- Cell activation
- Axons acts as transmission lines to send activation to other neurons
- Synapses the junctions allow signal to flow the axon and dendrites.
- The process of transmission is by diffusion of chemical called neurotransmitter

## 6) Applications of Neural Net:-

- (i) Character Recognition:- This has become very important as handled devices like the Palm Pilot are becoming increasingly popular.  
Neural net can be used to recognize handwritten characters.
- (ii) Image Comprehension:- NN can receive and process vast amount of information at once making them useful in image comprehension
- (iii) Stock market Prediction:- Many factors weigh in whether a given stock will go up on any given day since neural net can examine a lot of information quickly and sort it all out, they used to predict stock price.
- (iv) Traveling Salesman Problem:- This NN can solve the traveling salesman problem but only to a certain degree of approximation
- (v) Medicine, Electronic Nose, Safety, Loan applications:- Some applications that their proof of concept stage with acceptance of NN that will decide whether grant loan or not.
- (vi) Miscellaneous Applications:- Some interesting (albeit at times a little absurd) applications of NN.

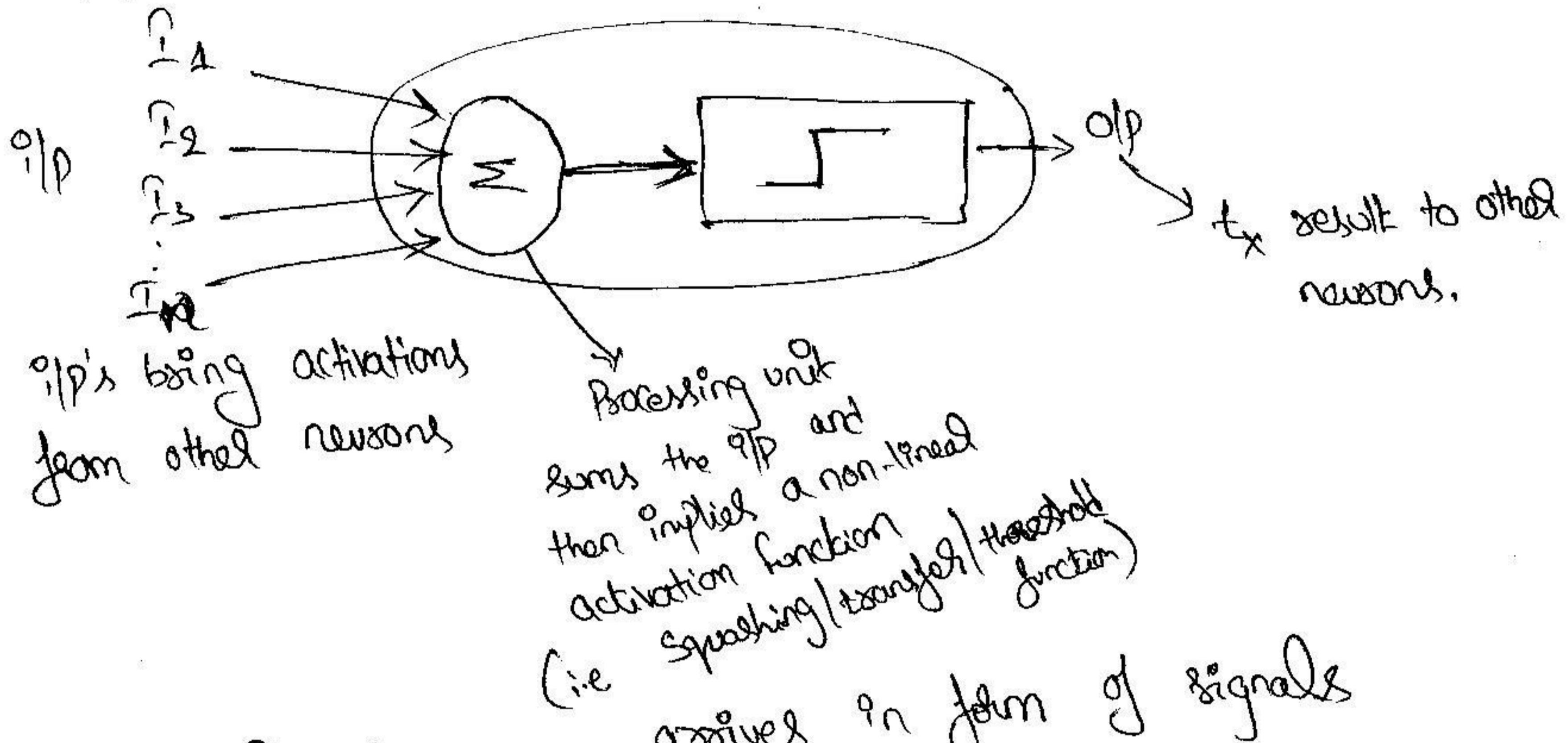
## 7) Artificial Neuron Model:-

(I-8)

6

### (i) The McCulloch-Pitts Neuron:-

- \* This is simplified model of real neuron, known as threshold neuron.



- The  $O_{IP}$  to neuron arrives in form of signals
- The signal builds up in the cell
- The signal discharges (cell fires) through the  $O_{IP}$
- Finally the cell starts building up signal again.

## 8) Notations:-

- i) Scalar:- The no. of  $x_i$  can be added up to give scalar no.  
 $S = x_1 + x_2 + x_3 + \dots + x_n = \sum_{i=1}^n x_i$

- ii) vector:- An ordered set of related no. Row vectors ( $1 \times n$ )  
 $x = (x_1, x_2, \dots, x_n), y = (y_1, y_2, y_3, \dots, y_n)$

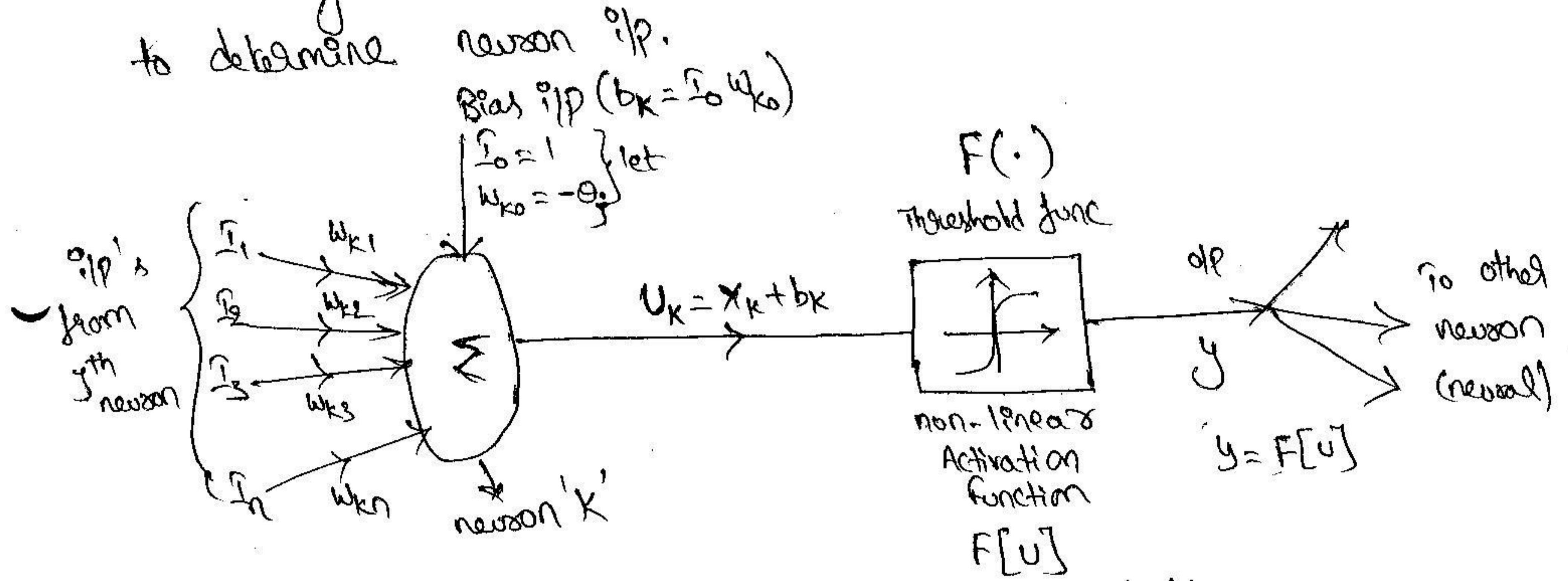
- iii) Add:- Two vectors of same length added give another vector  
 $\tilde{x} = x + y = (x_1 + y_1, x_2 + y_2, x_3 + y_3, \dots, x_n + y_n)$

- iv) Multiply:- Two vectors of same length multiplied give scalar  
 $P = x \cdot y = x_1 y_1 + x_2 y_2 + x_3 y_3 + \dots + x_n y_n = \sum_{i=1}^n x_i y_i$

7) Artificial Neuron & its operation:-

(7, 9) 4

- Similar to biological neuron.
  - Artificial neuron receives many ip's representing the ip of other neurons
  - Each ip is multiplied by a corresponding weight ; Analogue to synaptic strength.
  - All weights are summed and passed through a Activation function to determine neuron ip.



where  $\theta_j$  = j<sup>th</sup> neuron activation threshold

where  $\theta_j = f^{-1}$  neuron activation function  
 If  $[I_1, I_2, I_3, \dots, I_n]$  are inputs from  $j^{\text{th}}$  neuron which having  
 $w_{j1}, w_{j2}, \dots, w_{jn}$  and so on.

Synaptic weights of  $k^{th}$  neurons  $\{w_{k1}, w_{k2}, w_{k3}, \dots, w_{kn}\}$  represent input to  $k^{th}$  neuron.

Synaptic weights  $(w_1, w_2, \dots, w_n)$  as input for  $k^{th}$  neuron.

bias input ( $b_K = g_0 \cdot w_{K0}$ ) as input to  $a_K$ , then

where let  $(i_0 = 1 ; \text{ & } w_{k_0} = -\theta_j)$  then.

The off of  $k^{\text{th}}$  neuron is

$$x_k = \sum w_{k1} + w_{k2} + \dots + w_{kn} \rightarrow a$$

$$\Rightarrow x_k = \sum_{j=1}^n c_j \cdot w_{kj} \rightarrow \theta (\because j = 1, 2, \dots, n)$$

now complete dp of  $\hat{y}_k$  is  $\sum_{j=1}^J y_j$  with summation of bias  $q_P(b_k)$  is  $\sum_{j=1}^J b_j$

$$U_K = x_K + b_K \rightarrow \mathbb{C}$$

$$\Rightarrow U_k = \sum_{j=1}^n i_j w_{kj} + b_k \rightarrow \textcircled{d} \quad (\hat{i}-10)$$

now let us consider from bias o/p  $b_k$  & that

$$b_k = i_0 \cdot w_{k0} \quad \text{with}$$

case(i)  $w_{k0} = -\theta_j$  & ( $i_0 = 1$  &  $i_0 = 0$ ) if then.

Eq \textcircled{d} is

$$U_k = \sum_{j=0}^n i_j w_{kj} - \theta_j \rightarrow \textcircled{e} \quad (\because \text{if } b_k = -\theta_j)$$

$$U_k = \sum_{j=0}^n i_j w_{kj} \rightarrow \textcircled{f} \quad (\because \text{if } b_k = 0)$$

$$\therefore \boxed{U_k = \sum_{j=0}^n i_j w_{kj}} \rightarrow \textcircled{g}$$

$$\boxed{U_k = i \cdot w} \rightarrow \textcircled{h}$$

The vector form representation is

to determine the complete o/p of neuron w/o 'y' from

concept of McCulloch & Pitts model:-

McCulloch - Pitts Model:- (MP)

McCulloch - Pitts model has threshold level and once reached

defines neurons has fixed.

that level is fixed.

→ whole MP model has fixed set of weights.

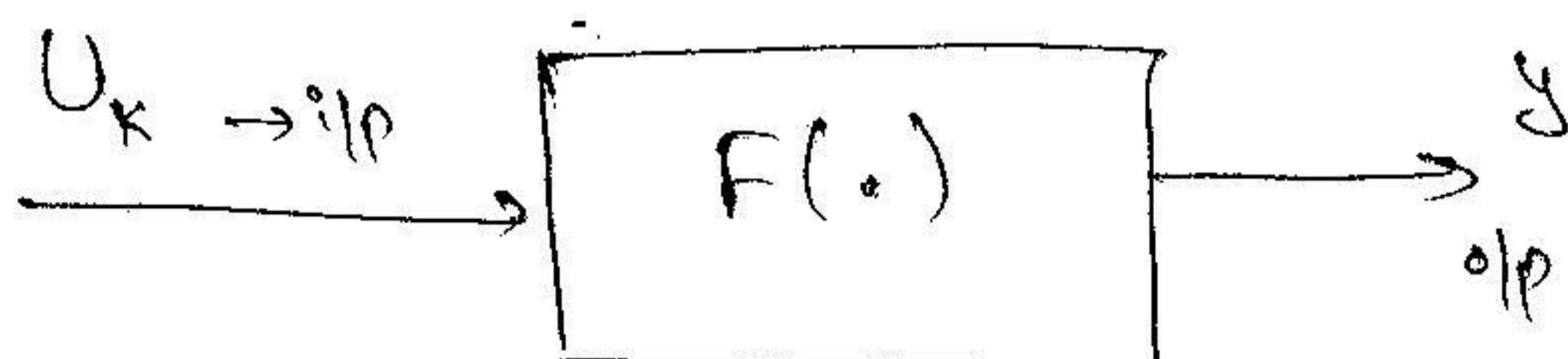
→ to define threshold for neuron the o/p of  $k^{th}$  neuron as

→ to define threshold for neuron the o/p of  $k^{th}$  neuron as

to be applied for functional representation. In order to

defined linear o/p to non linear threshold level representation.

Activation function.



The functional Representation is

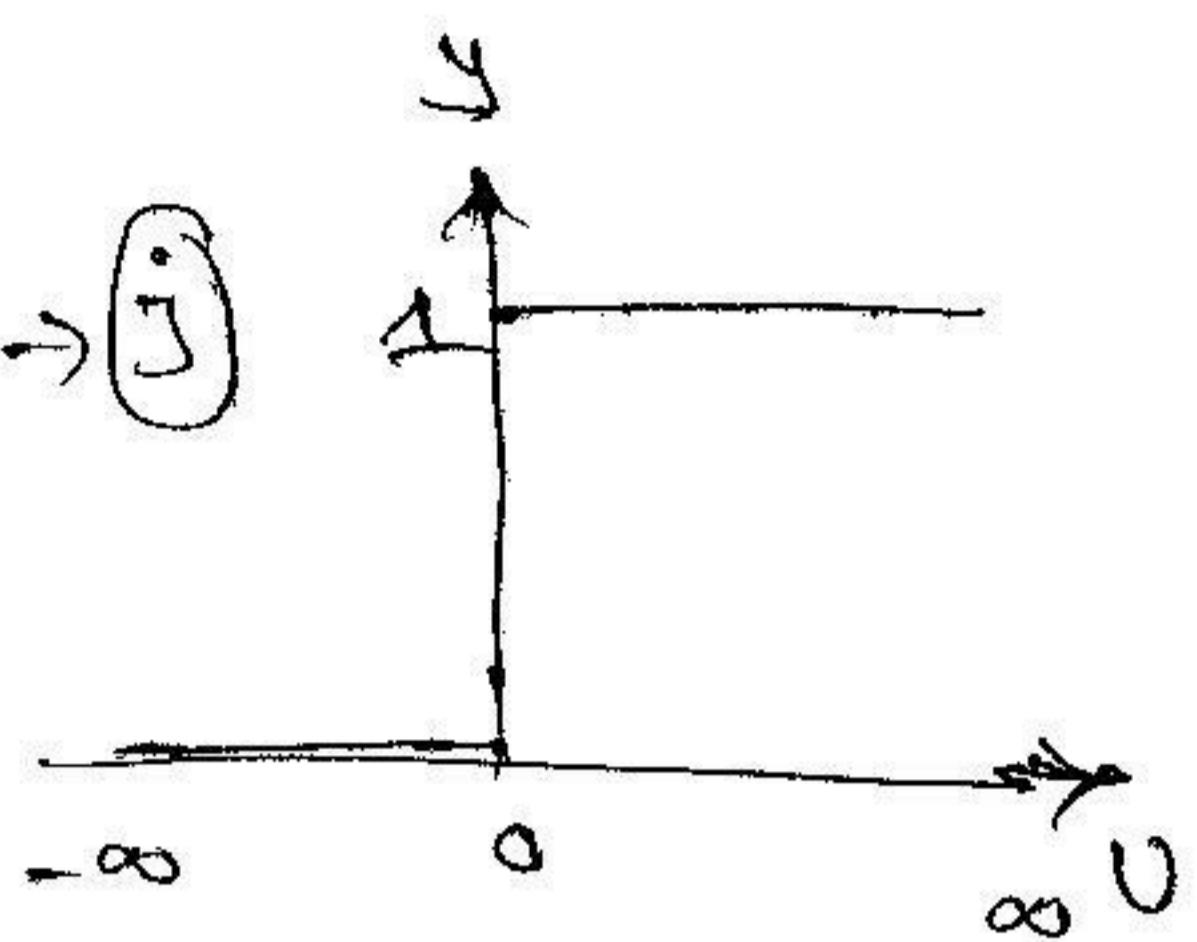
(I-11) ⑨

$$y = F(u) \rightarrow i$$

where an o/p  $\in$  o/p mapping from  $u$  to  $y$ .

(i) threshold function:-

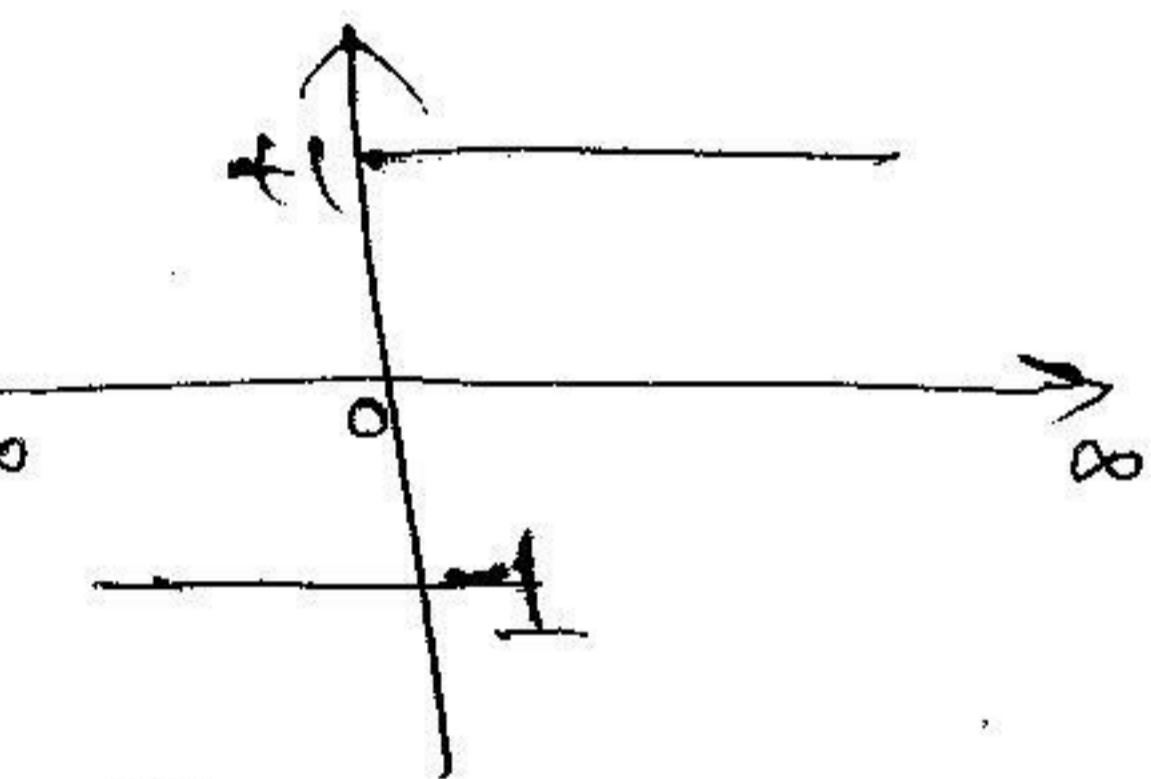
$$\begin{cases} \text{if } u_k \geq 0 \text{ then } y = 1 \text{ (+ve)} \\ \text{if } u_k < 0 \text{ then } y = 0 \text{ (-ve) others} \end{cases} \rightarrow j$$



(ii) Sigmoid function (or) (S-shape function):-

to define o/p is at  $+1$  (or)  $-1$

$$\begin{cases} \text{if } \text{sgn}(u_k) \geq 0 \text{ then } y = 1 \\ \text{if } \text{sgn}(u_k) < 0 \text{ then } y = -1 \end{cases} \rightarrow k$$



$\therefore$  the total o/p ~~if~~ 'y' is represented

$$y = \text{sgn} \left( \sum_{k=1}^n i_j w_{kj} - \theta \right) \rightarrow l$$

here  $\theta \rightarrow$  the neuron activation threshold of  $k^{th}$  neuron.

$$\begin{cases} \text{if } \sum_{k=1}^n i_j w_{kj} \geq 0 \text{ then o/p } y = 1 \\ \text{if } \sum_{k=1}^n i_j w_{kj} < 0 \text{ then o/p } y = 0 \end{cases} \rightarrow m$$

$$\sum_{k=1}^n i_j w_{kj} < 0 \text{ then o/p } y = 0$$

to identify the o/p  $y = +1$  (or)  $-1$  then, from above  $m$  eq  $\&$   $l$

~~eq~~ and comparing with Eq  $k$ . we define

the o/p of neuron is ' $+1$  (or)  $-1$ '

Note :-

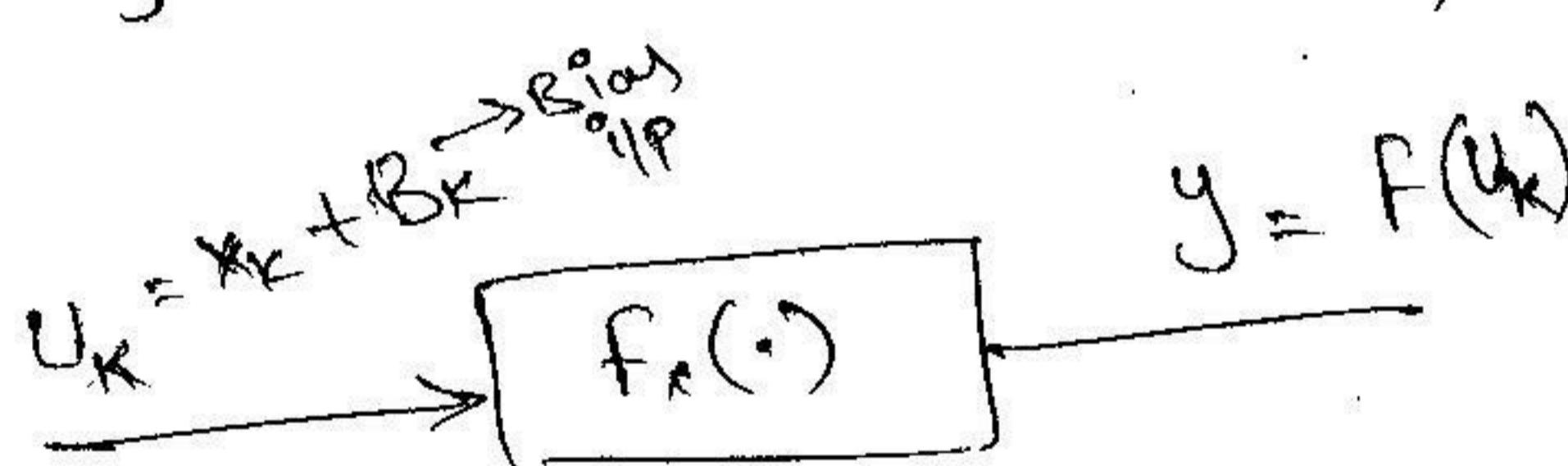
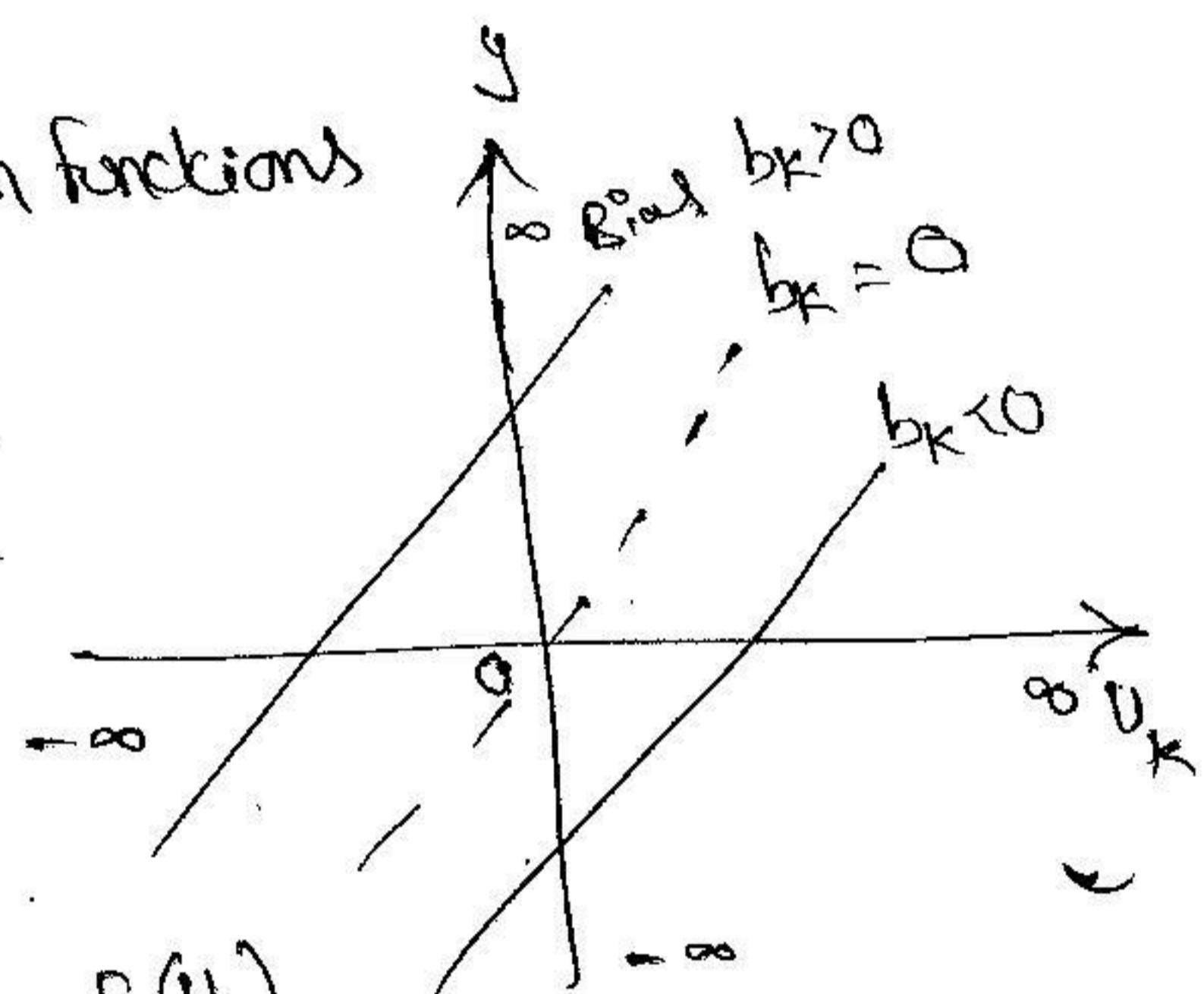
- As ip applied from  $j^{th}$  neuron are summed with weighted ip and bias of and given an op which is said to be function of of  $k^{th}$  neuron represented 'linear function'
- the op of summation of ip & synaptic weights are taken as ip to Activation function.
- In order to fitted the non-linear function of total. op response of Artificial neuron n/w. either (+ve or -ve).

### 2) Activation function:- (F)

Perform a mathematical operations on the signal of. the

most common activation function are

- threshold function
- linear function
- piecewise Linear function.
- Sigmoidal (S-shape) function
- Tangent hyperbolic function.



$$x_k = \sum_{j=1}^n x_j w_{kj} - \theta_j$$

$\theta_j$  → Threshold interval of  $j^{th}$  neuron.

- The Sigmoid & hyperbolic-tangent functions perform well for prediction and process forecasting types of problems
- Radial bias function proves more effective for n/w & highly recommended
- Junction for those problems involving fault diagnosis.
- Hard limited for classification problems.

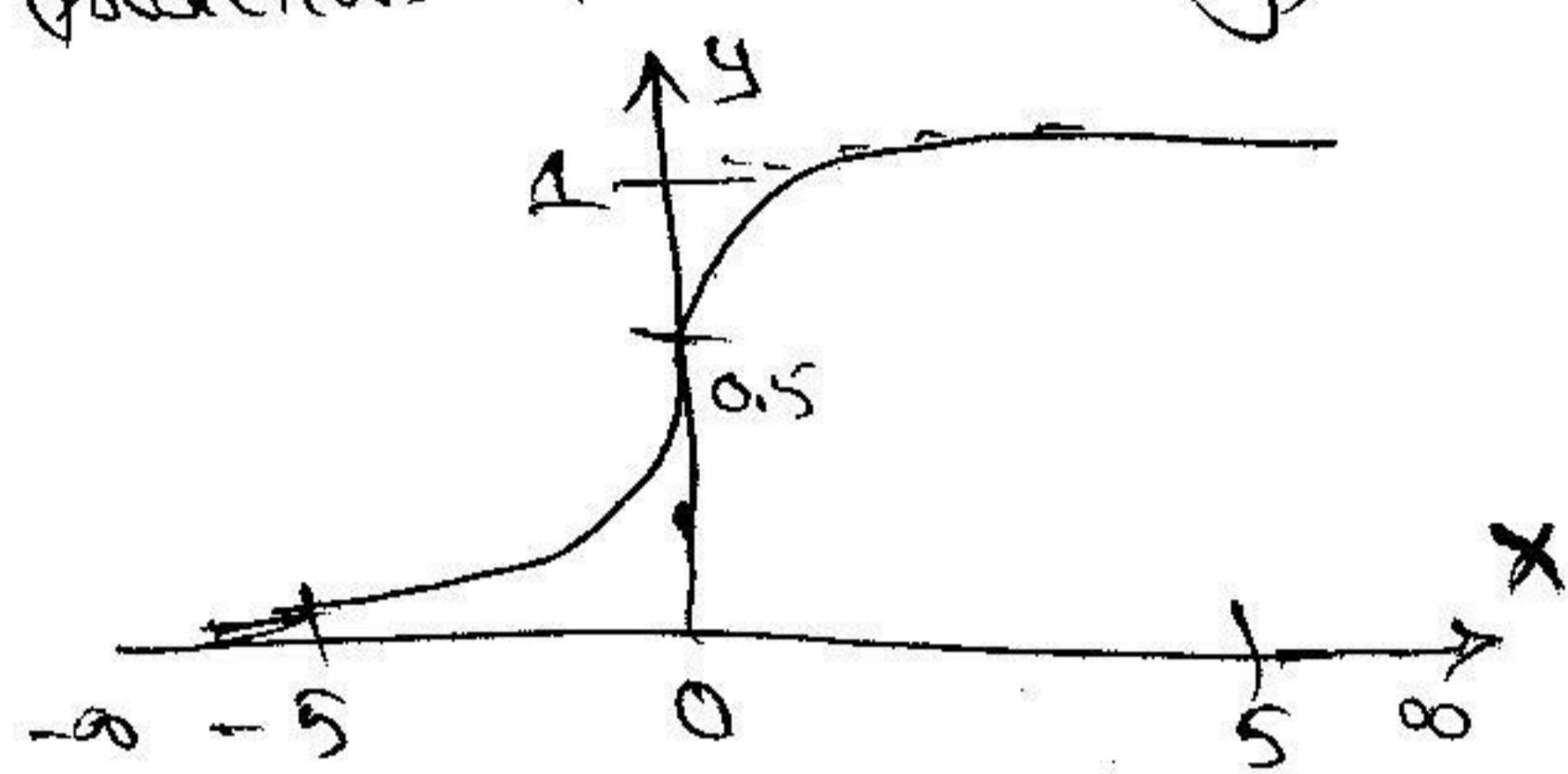
## Types of Activation functions:-

→ behavioral of artificial neuron depends on both synaptic weight and activation function can be represented ;  $\log, e^x, x^3 \dots$  etc., mathematically.

→ Sigmoid functions are commonly used activation functions in multilayered feed forward neural net.

(i) Sigmoid function (unipolar sigmoid) :- (used in Prediction & Pattern Recognition).

$$y(x) = f(x) = \frac{1}{1+e^{-x}} \rightarrow (a)$$



range is  $0 < y < 1$

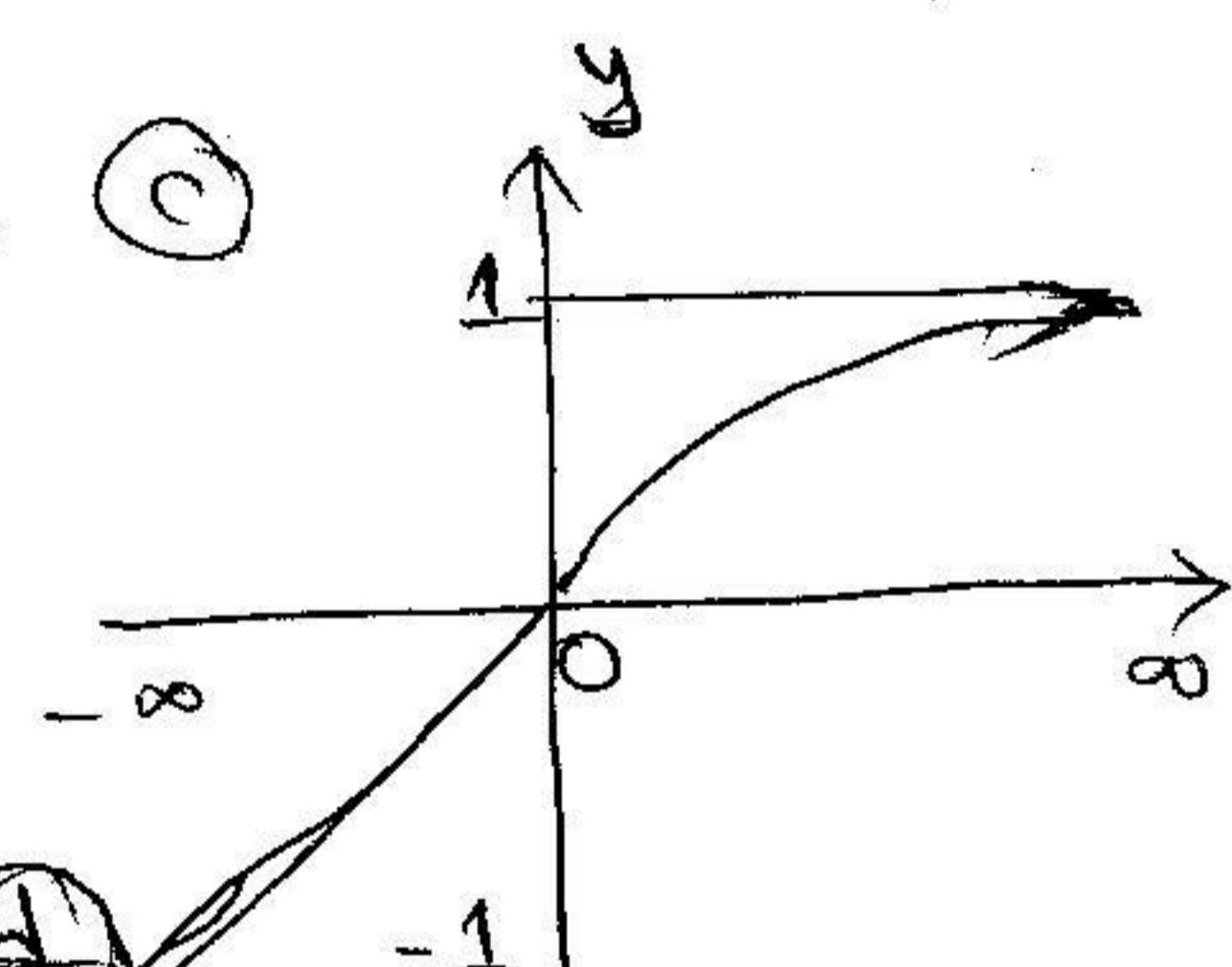
The derivative of function is

$$y'(x) = f'(x) = f(x)(1-f(x)) \rightarrow (b)$$

$$\because x = \sum_{j=1}^n x_j w_{nj} - \theta_j$$

(ii) Hyperbolic tangent (bipolar sigmoid) function:- (Prediction & Pattern Recognition)

$$g(x) = f(x) = \tanh(x) = \frac{e^x - e^{-x}}{e^x + e^{-x}} \rightarrow (c)$$



range of signal is  $-1 < y < 1$

derivative of

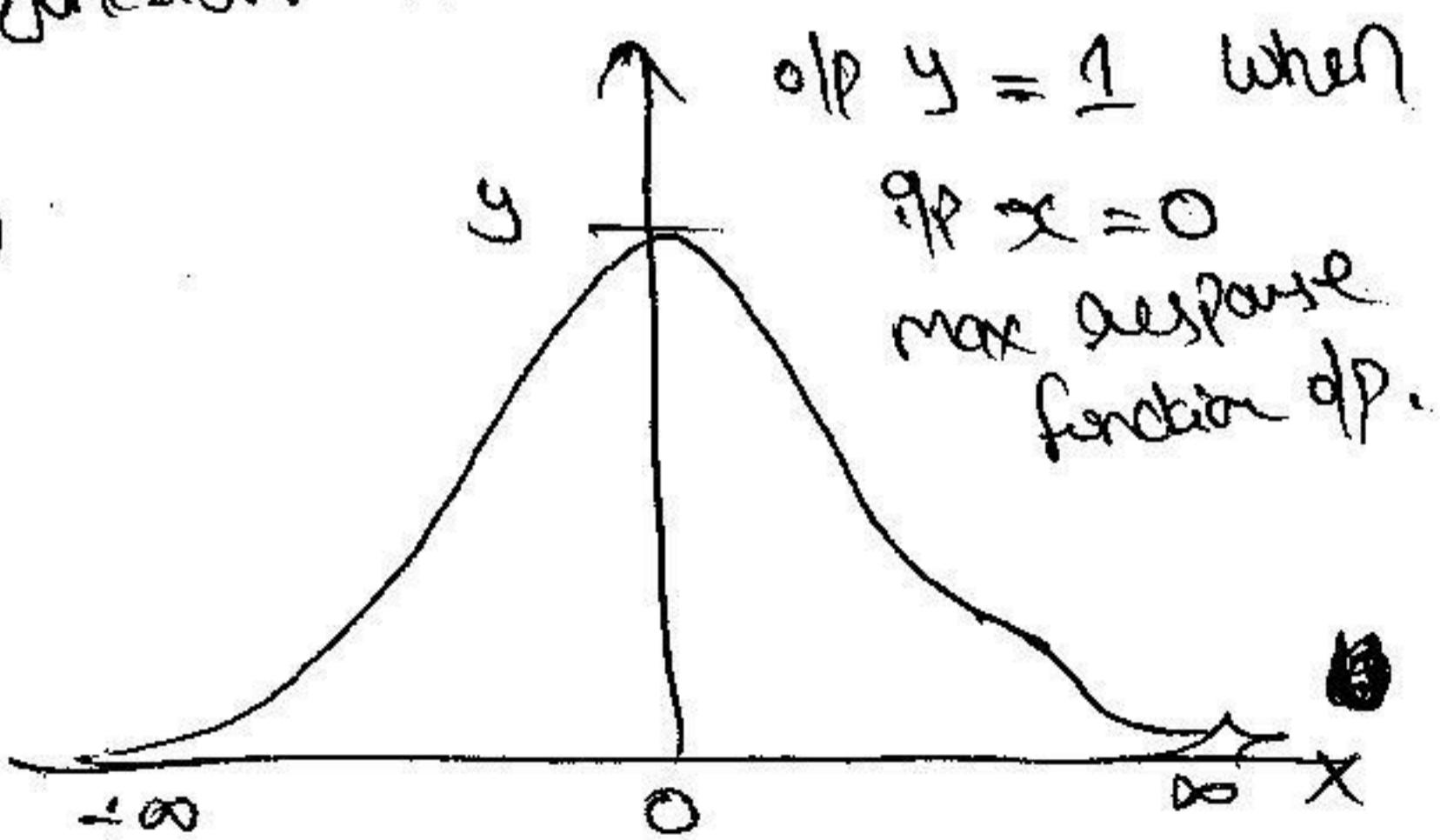
$$y'(x) = f'(x) = 1 - [f(x)]^2 \rightarrow (d)$$

(iii) Radial basis function :-(used for fault diagnosis problems)  
The Gaussian function is mostly commonly used in radially symmetric function, the characteristic function is in mathematic description.

$$y(x) = f(x) = e^{(-x^2/2)} \rightarrow (e)$$

derivative

$$y'(x) = F'(x) = -x \cdot e^{-x^2/2} \rightarrow (f)$$



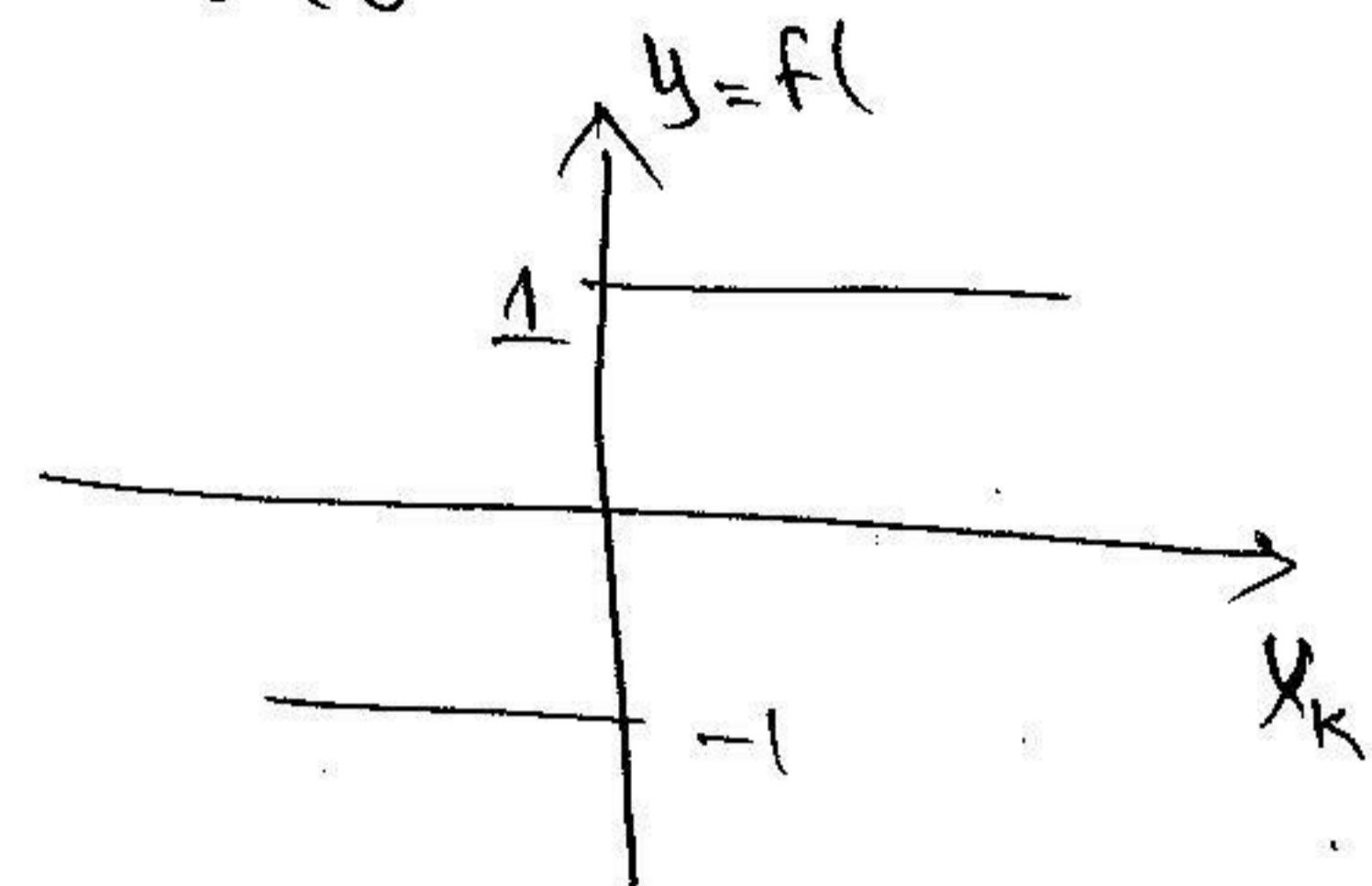
(QV) Hard Limited:- (Classification of Problems):- (I-14) 12

It is mostly used in classification of problems

$$F(x_k) = \text{sgn}(x_k) = \begin{cases} +1 & v \geq 0 \\ -1 & v < 0 \end{cases}$$

This function is not differentiable

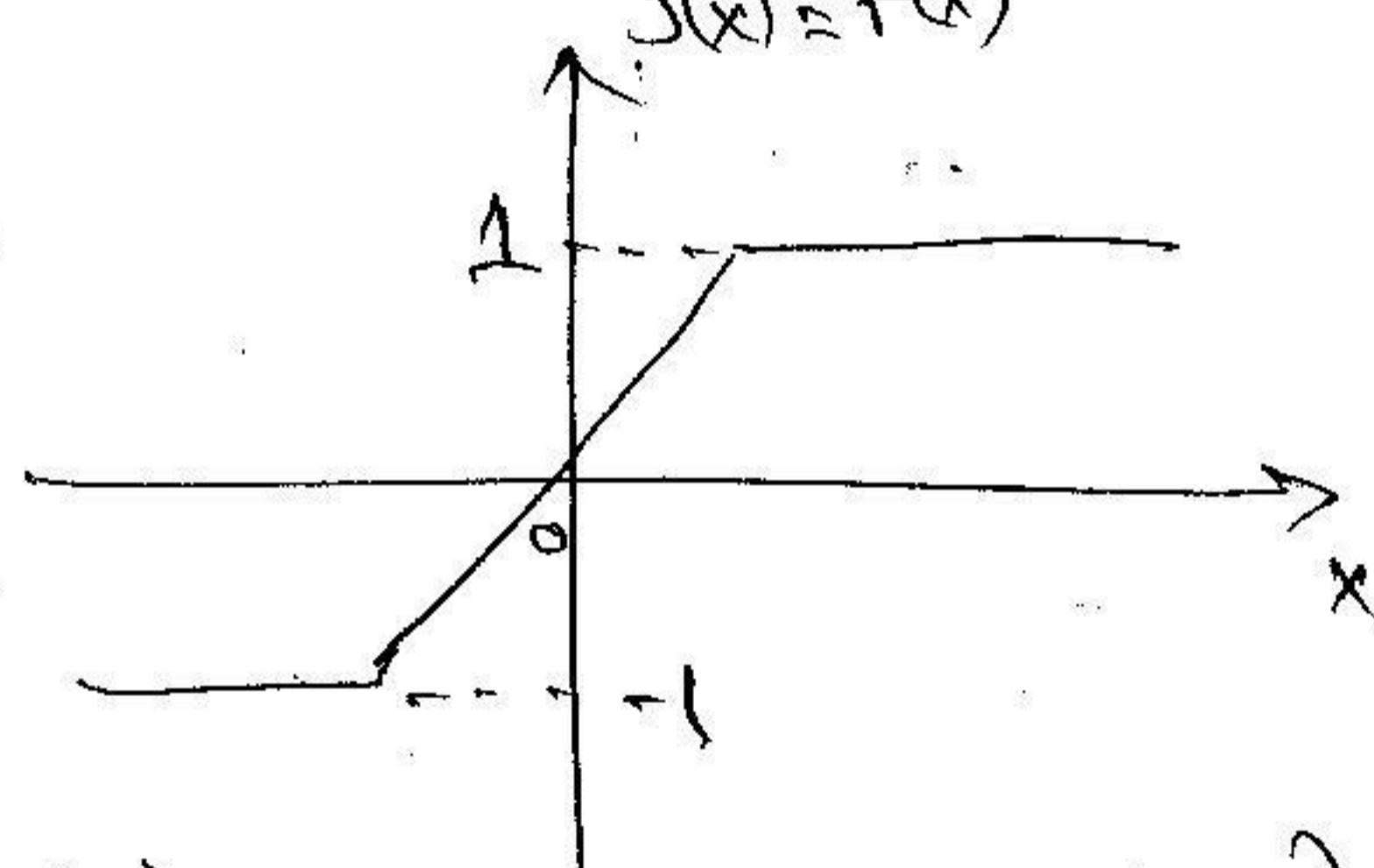
$\therefore$  It cannot be used for continuation type of applications.



(V) Piecewise linear function:- (Called Saturating linear func)

This activation function is called saturating linear function and can have either a binary (or) bipolar range for saturation limits of dp.

$\rightarrow$  The mathematical model is



$$y(x) = F(x) = \begin{cases} -1 & \text{if } \text{sgn}(x) < 0 \rightarrow (\text{-ve weighted}) \\ \text{sgn}(x) & \text{if } -1 \geq x \geq 1 \rightarrow (\text{proportional value to -1 to 1}) \\ 1 & \text{if } \text{sgn}(x) \geq 0 \quad (\text{+ve weighted sum}) \end{cases}$$

## Sigmoidal function (S-shape function) :-

The non-linear curved S-shape function is called the sigmoid function.

→ It is most common type of activation used to construct neural net.

→ It is mathematically well behaved differentiable and strictly increasing function.

$$y = f(x) = \frac{1}{1 + e^{-\alpha x}} ; 0 \leq f(x) \leq 1$$

$$y = f(x) = \frac{1}{1 + e^{(-\alpha x)}} ; 0 \leq f(x) \leq 1$$

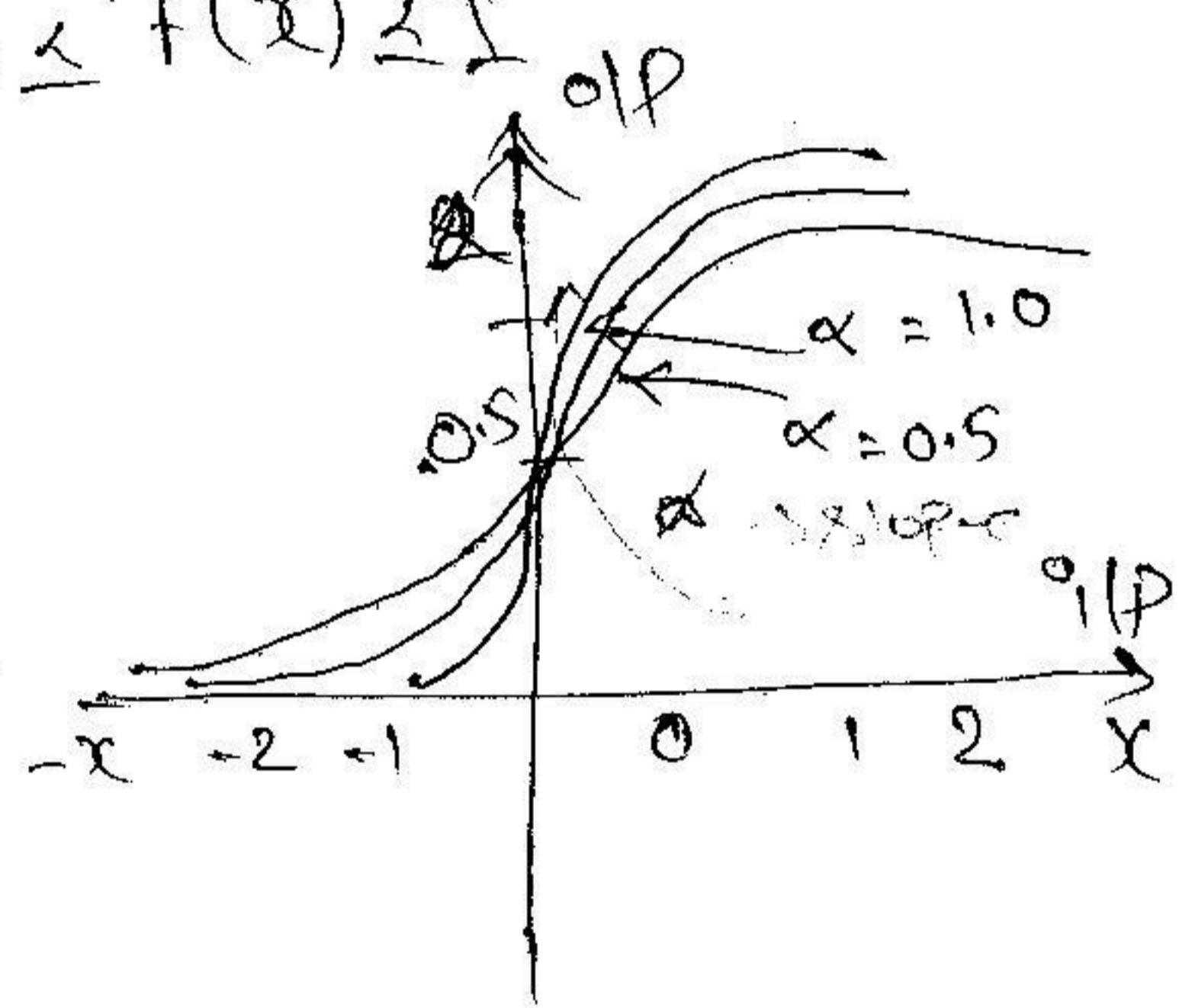
This explained as

$$\begin{aligned} y &= 0 && \text{for large -ve } ^{\circ}\text{IP values} \\ y &= 1 && \text{for large +ve value with.} \end{aligned}$$

$\alpha \rightarrow$  Slope Parameter.

$\lambda \rightarrow$  also be represented as Parameter.

By varying  $\alpha$ -different shapes of function can be obtained which adjusts the abruptness of the function as it changes b/w the two asymptotic values.



q) Neural nw viewed as Directed graphs:-

Signal flow graph without ~~rules~~ a well-defined set of rules were originally developed for linear nw. The presence of non-linearity in model of a neuron limits the scope of their application to neural nw.

→ A signal flowgraph is a nw of directed links (branches) that are interconnected at certain points called nodes. A typical node  $j$  has associated node signal  $x_j$ . A typical direct link originated at node  $j$  and terminates on node  $k$ ; it has associated transfer function (or) transmission that specifies manner of signal.

→ The flow of signals in various parts of the graph is dictated by three rules.

Rule 1:- A signal flows along a link only in direction defined by arrow on the link.

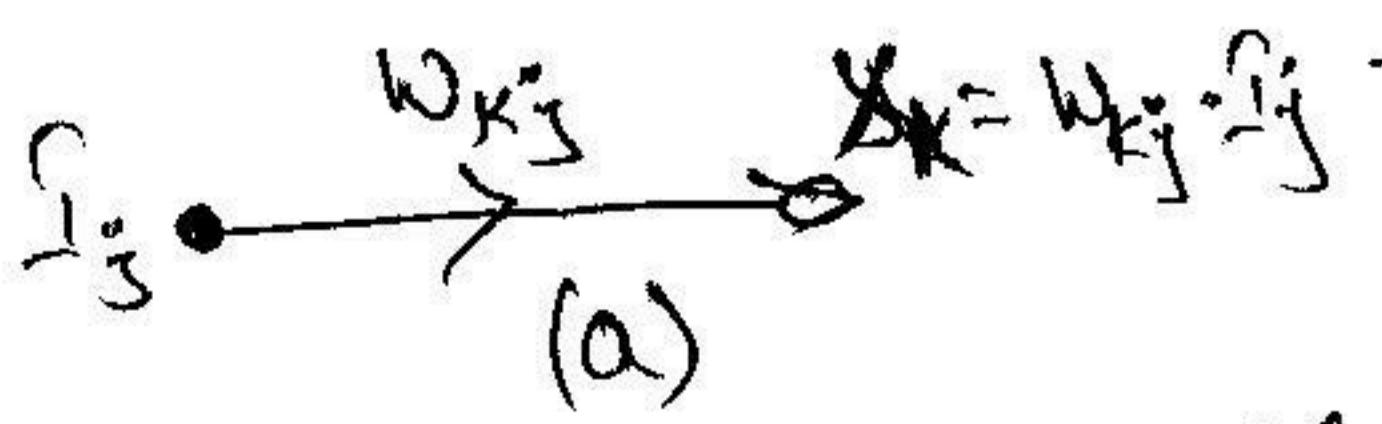
2-different link types → Synaptic link ✓ →  $w_{kj} \cdot x_j$  op

ii) Synaptic link :- behavior is governed by linear op - op.

relation (ie  $y_k$  multiplied with  $w_{kj}$  gives  $y_k$  op).

ii) Activation link :- behavior is governed in general by a non-linear op relation

(i.e.  $y = f(\cdot)$  is non linear activation function).



$$x_j \xrightarrow{w_{kj}} x_k = w_{kj} \cdot x_j$$

(b)

$$y_i \quad y_j \quad y_k = y_i + y_j$$

(c)

$$x_j \xrightarrow{f(\cdot)} y_k = f(x_j)$$

(d)

Rule - 2:-

A node signal equals the algebraic sum of all signals entering Pertinent node via incoming links (fig.-c)

Rule - 3:-

The signal at a node is transmitted to each outgoing link originating from that node with transmission being entirely independent of transfer functions of outgoing links. (fig-d)  
is Case of synaptic divergence (or) fan-out.

→ A neural  $\text{N}_w$  is a directed graph consisting of nodes with interconnecting synaptic and activation link are characterized by four properties:

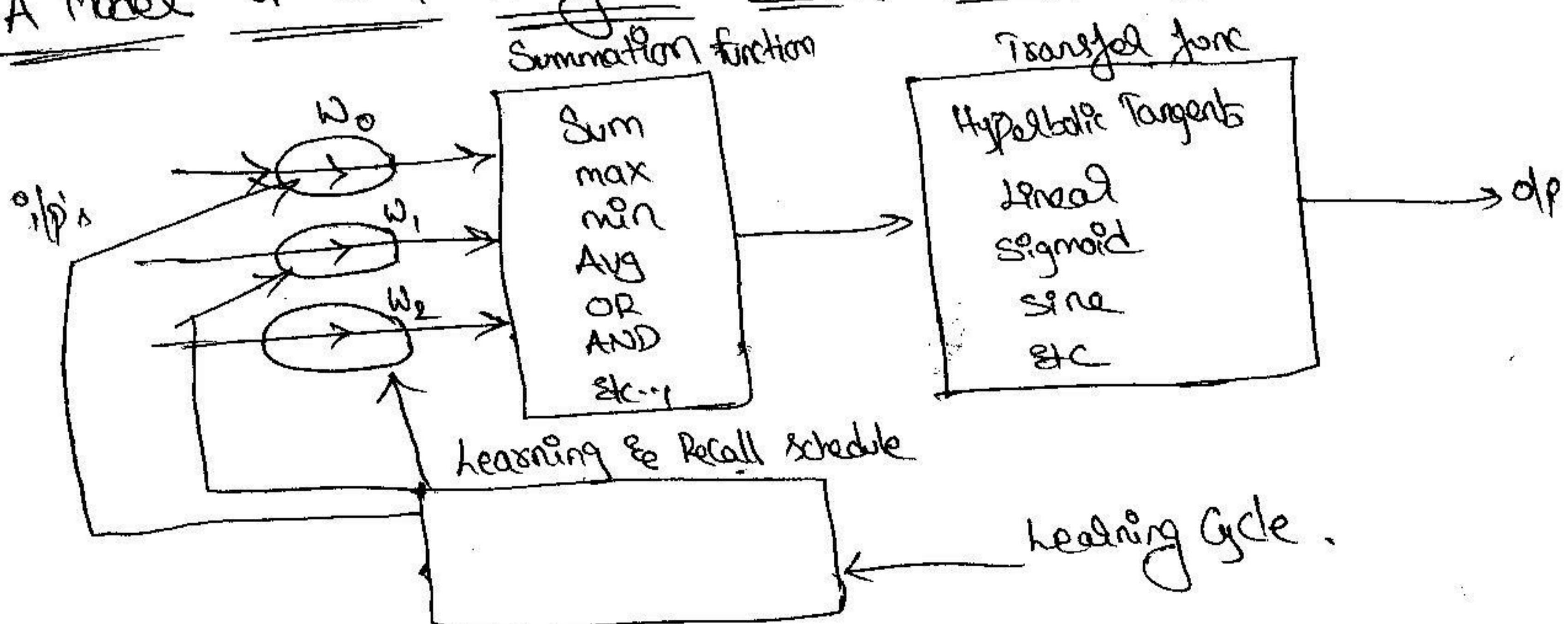
- i) Each neuron is represented by set of linear synaptic links and externally applied bias and possibly non-linear activation link. The bias is represented by synaptic links connected to off fixed at +1.
- ii) The synaptic link of neuron weight their respective off signal.
- iii) The weighted sum of off signals defines the induced total field of neuron in question.
- iv) The activation link squash the induced total field of neuron to produce op.

### 3 - Graphical Representation of neuron:-

- ✓ i) Block diagram providing functional description of  $\text{N}_w$
- ✓ ii) Architectural graph, describing the  $\text{N}_w$  layout.
- ✓ iii) Signal flow graph; providing a complete description of signal flow in the  $\text{N}_w$ .

10) History:-

<u>Author</u>	<u>Year</u>	<u>Idea</u>	<u>Implementation.</u>
1) McCulloch - Pitts	1943	neuron has threshold level once reached neuron fires	1 <sup>st</sup> Artificial neuron model.
2) Hebb	1949	If 2-neurons are active same time then strength b/w them should be increased	1 <sup>st</sup> learning rule
3) Rosenblatt	1950 & 1960	Converge to Correct weights & solve Problems	Introduced Perceptron (learning algorithm)
4) Minsky & Papert	1969	Perceptron could not learn function which are not linear	-
5) Parker & LeCun	1985-1986	Learning algorithm for multilayer called Back Propagation	

11) A Model of a Processing Element (Electronic Implementation):-

## Why to use Linear Model and Non-Linear Model in neural n/w:-

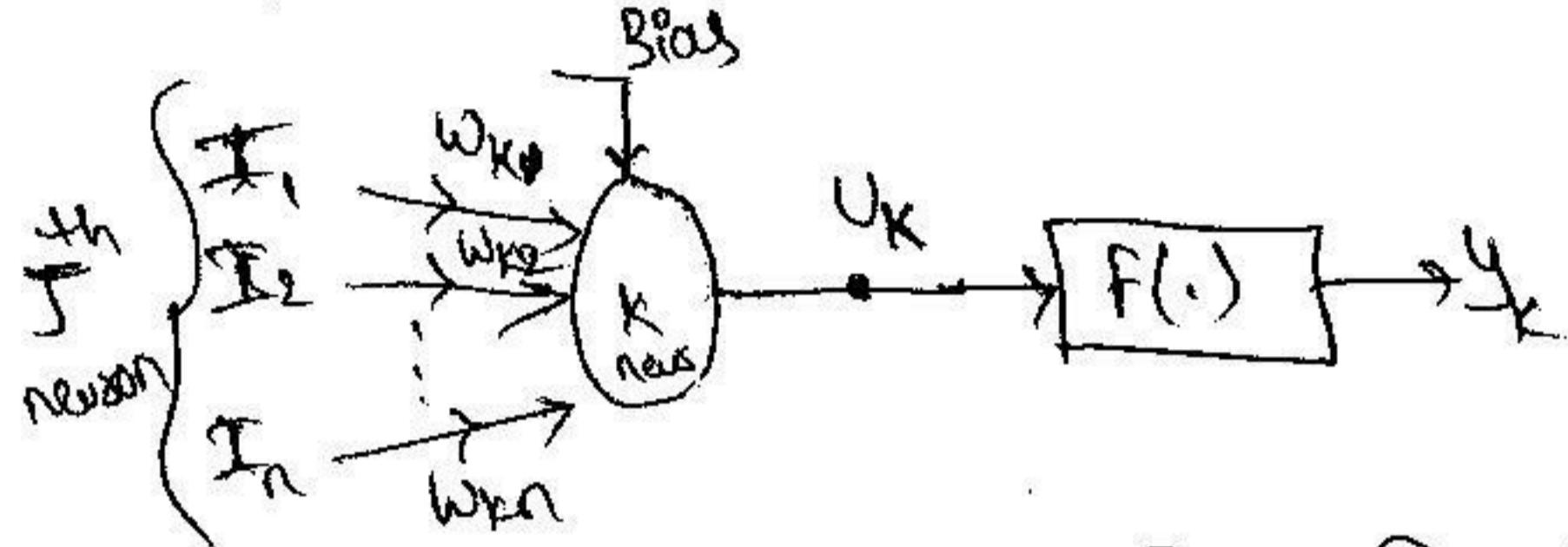
for the concept of single layer (or) Multilayer neural n/w to identify the active neurons in the network based on Activation functional problem it is identified.

It is used in data fit model.

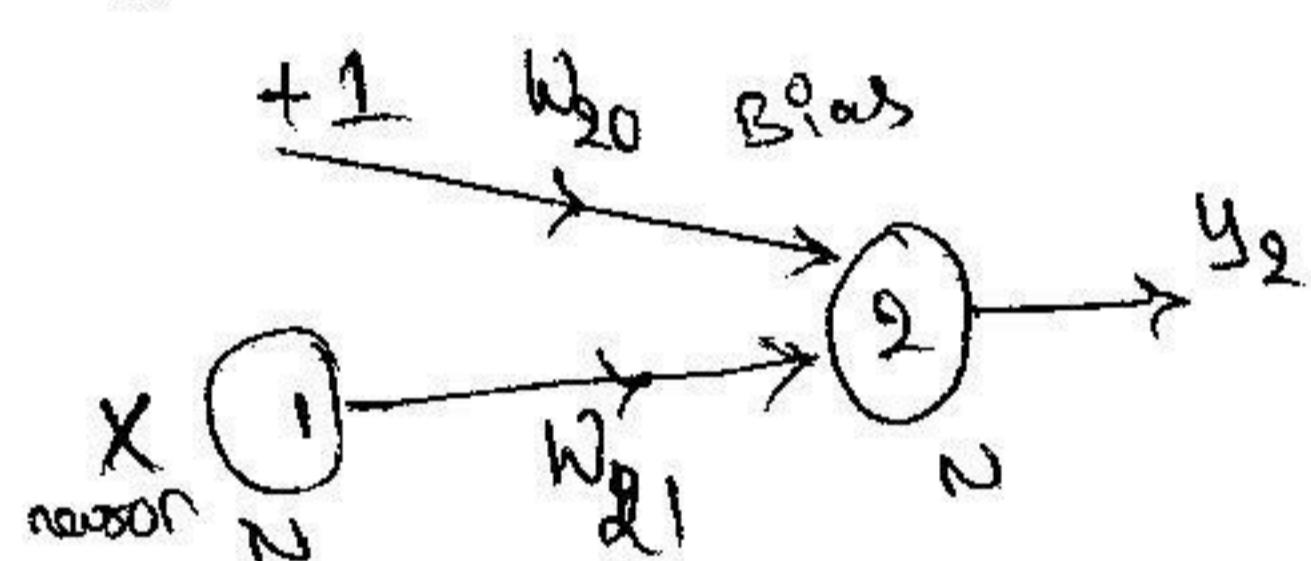
Linear model :-  $(y_k = u_k)$

$$u_k = \sum_{j=1}^n i_j \cdot w_{kj}$$

$$u_k = i_1 w_{k1} + i_2 w_{k2} + i_3 w_{k3} + \dots + i_j w_{kj}$$



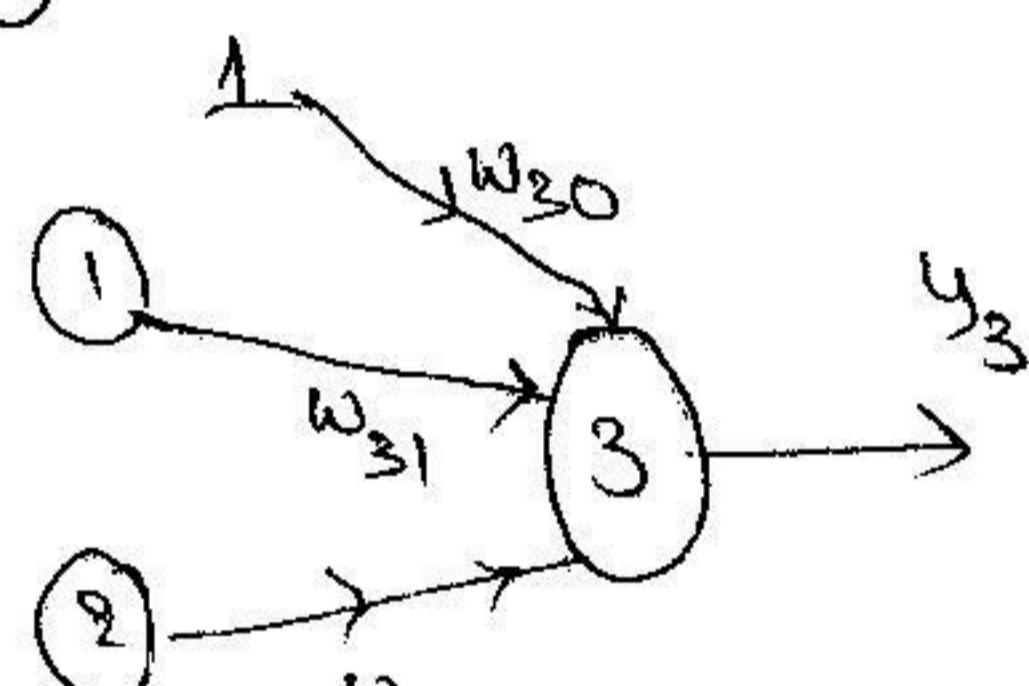
If 2-nodes of ① & ② are considered.



$$y_2 = w_{21}x + w_{20}$$

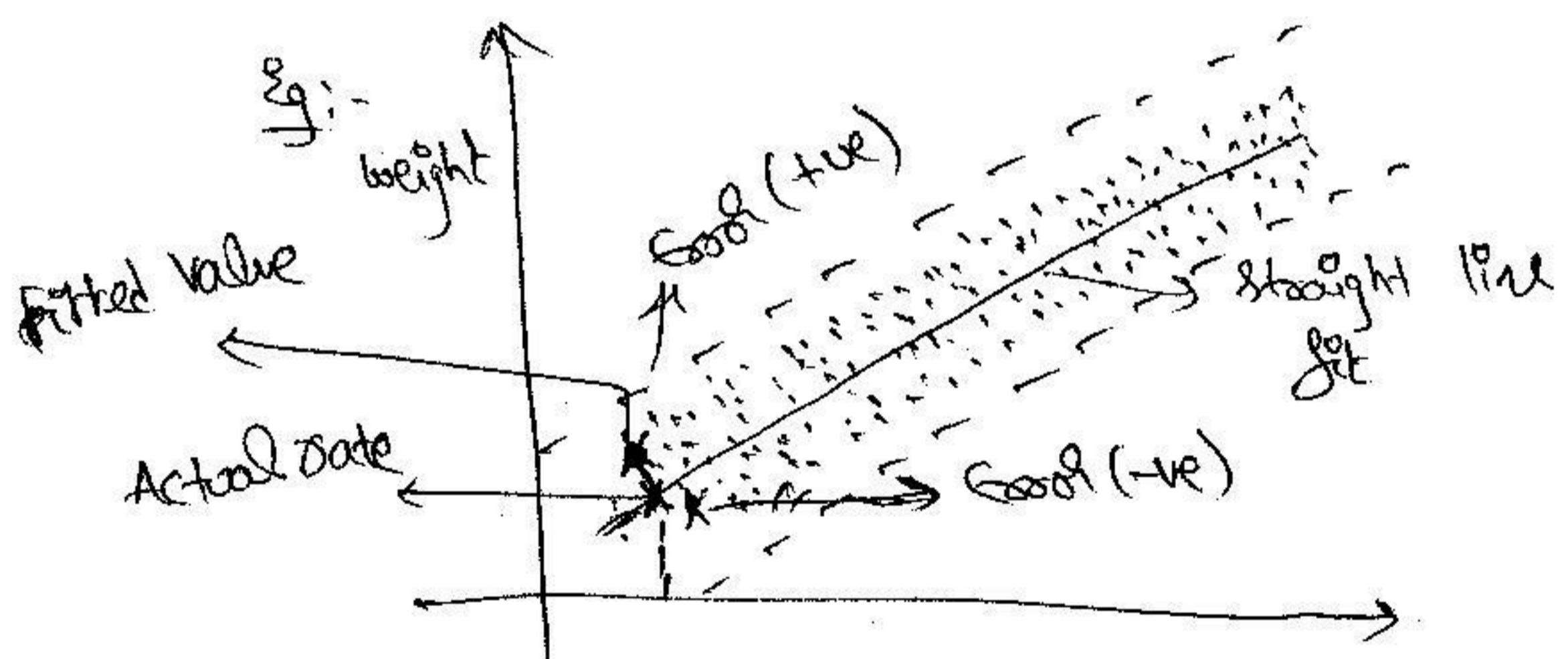
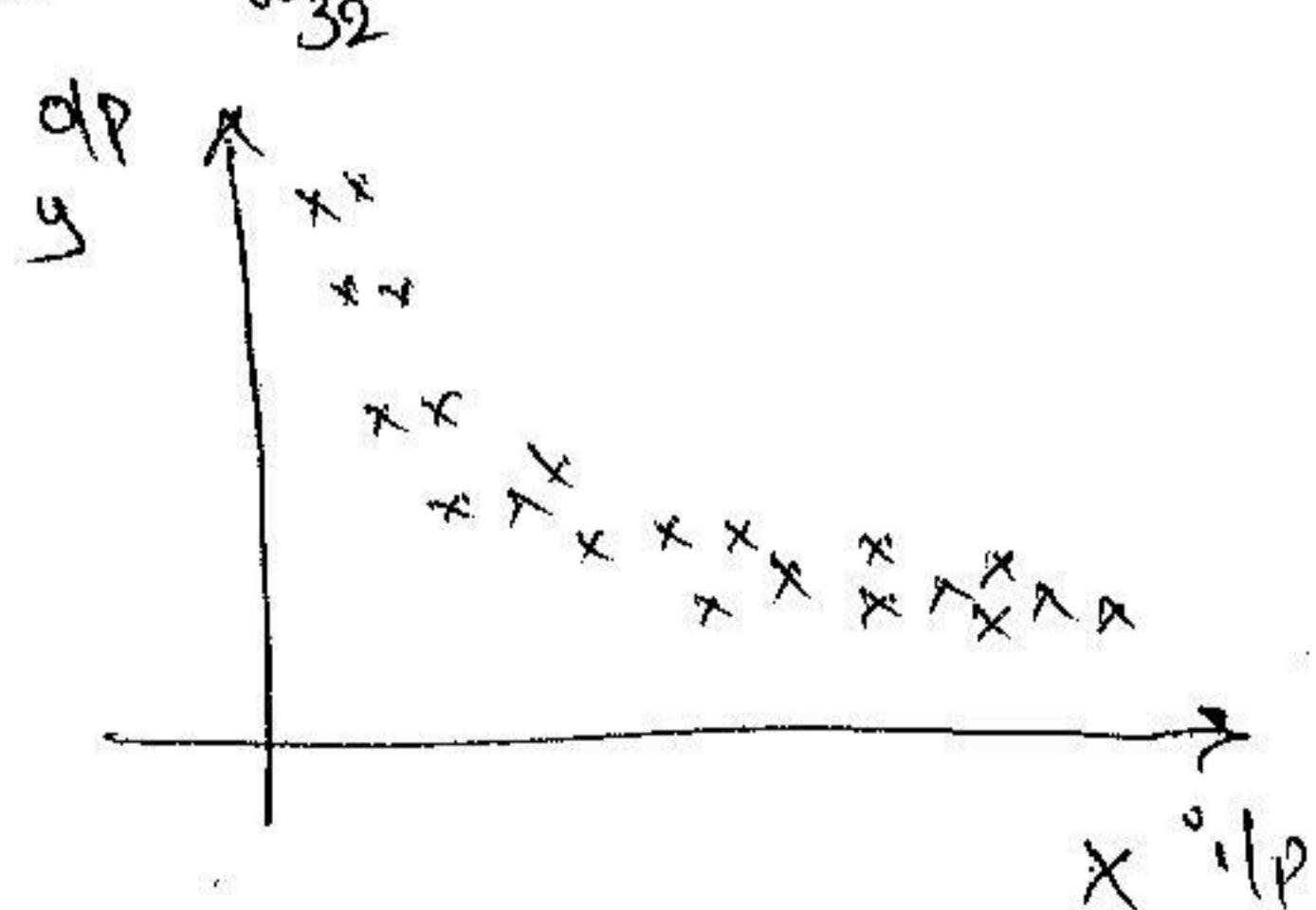
slope      ↓       $\downarrow$   
 $y = mx + c$        $\hookrightarrow$  intercept  
 slope / Gradient (direction)  
 bias is intercept

If 3-nodes  $N_1, N_2, N_3$  are considered



$$y_3 = w_{30} + w_{31} \cdot x_1 + w_{32} \cdot x_2$$

$y_3$  is dependent upon 2 ip  $x_1$  &  $x_2$



Note:-

- The straight line fit is possible in less no. of ip's.
- It is not approximately in more no. of ip's.
- The " " " is not Appropriate with (non-linear model).
- So we come up with (Binary/Step/Unipolar/Threshold) hard limit / Sigmoid functions

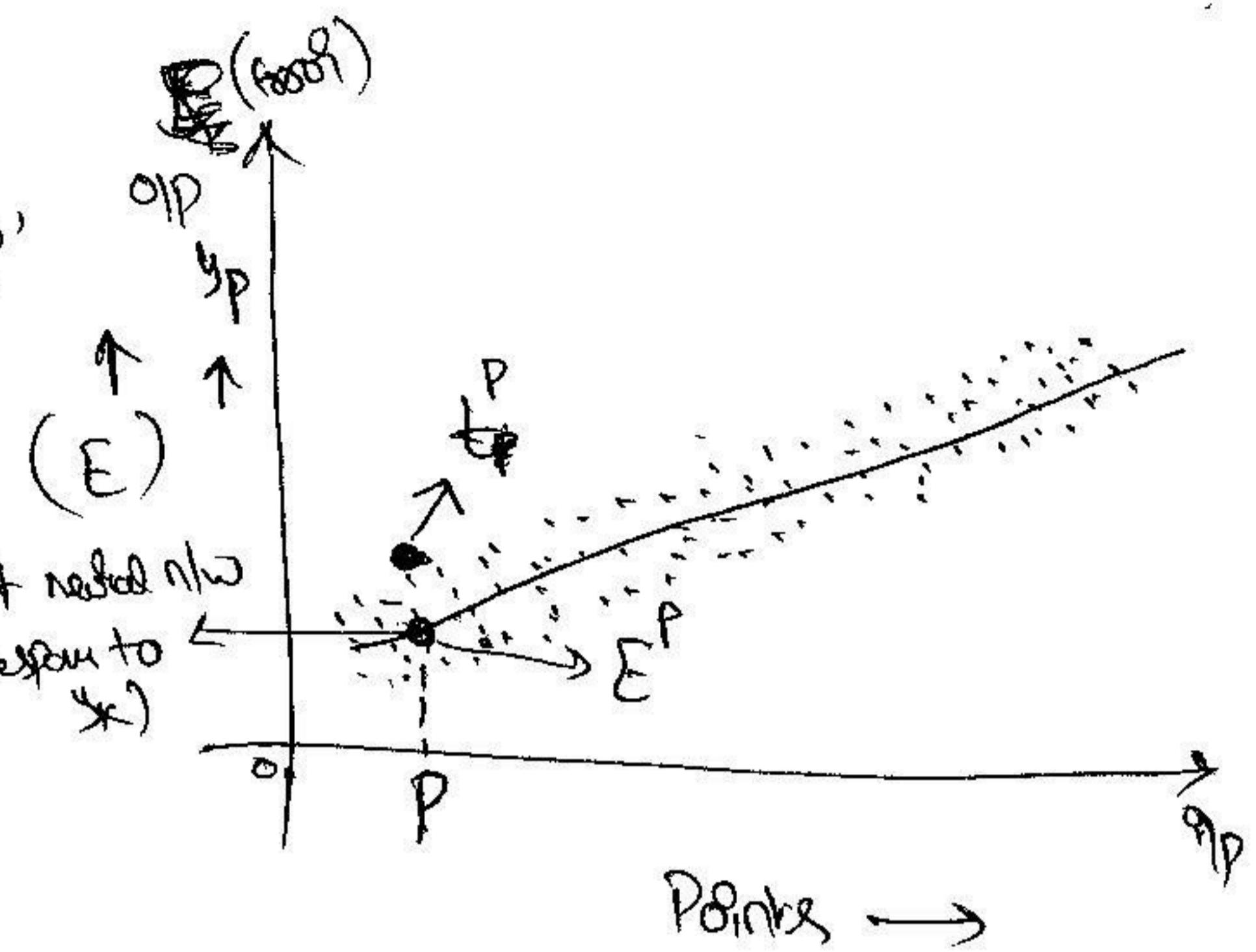
(i) Represent on Good :- (E)

let  $E^P \rightarrow$  Error for Particular Point 'P'

$E^P \rightarrow$  target dp for Point 'P'

$y^P \rightarrow$  fitted dp (o) now Repres. Curve fit need new job (Response to \*)

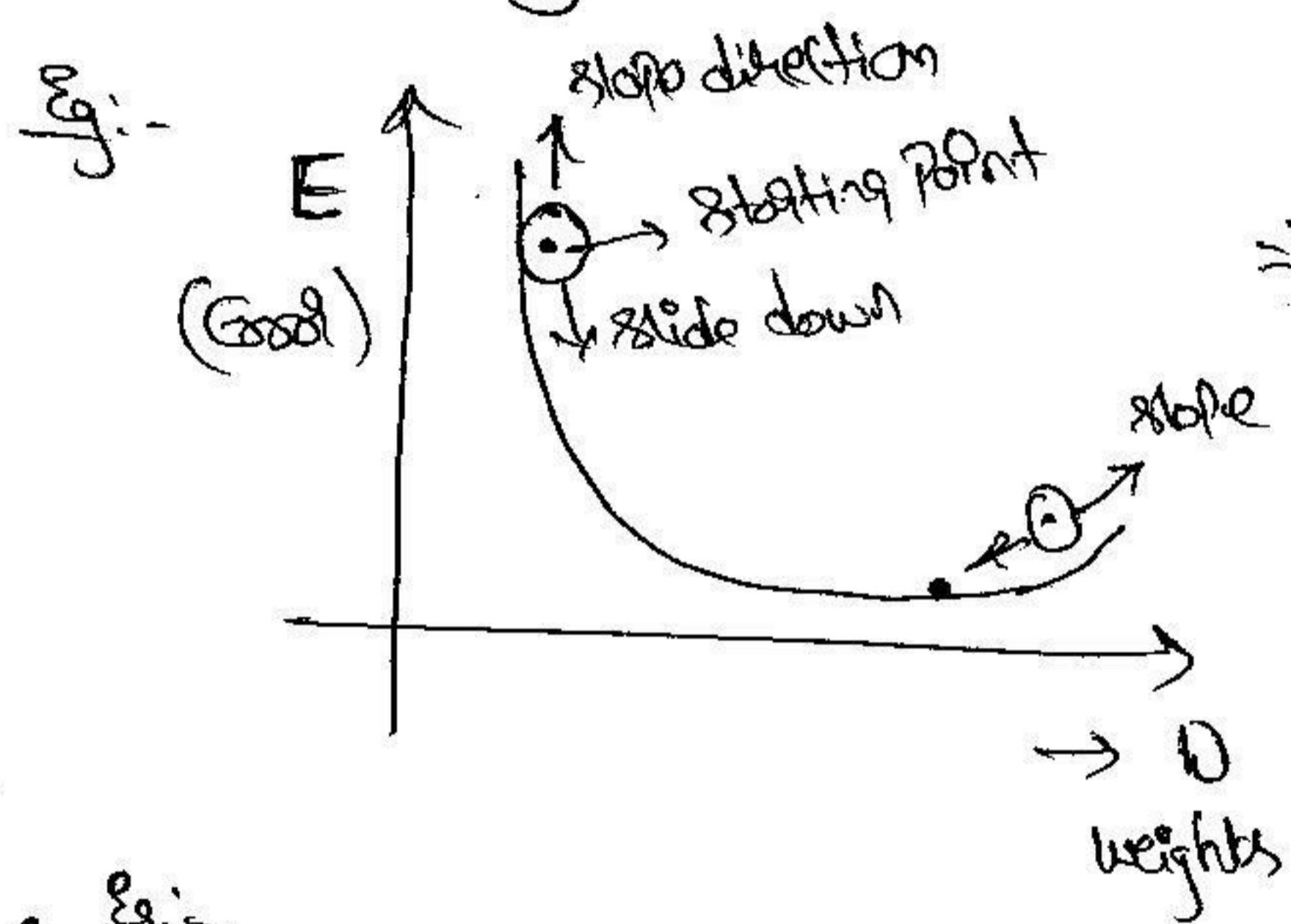
$$\therefore E^P = (t^P - y^P)^2 \rightarrow ①$$



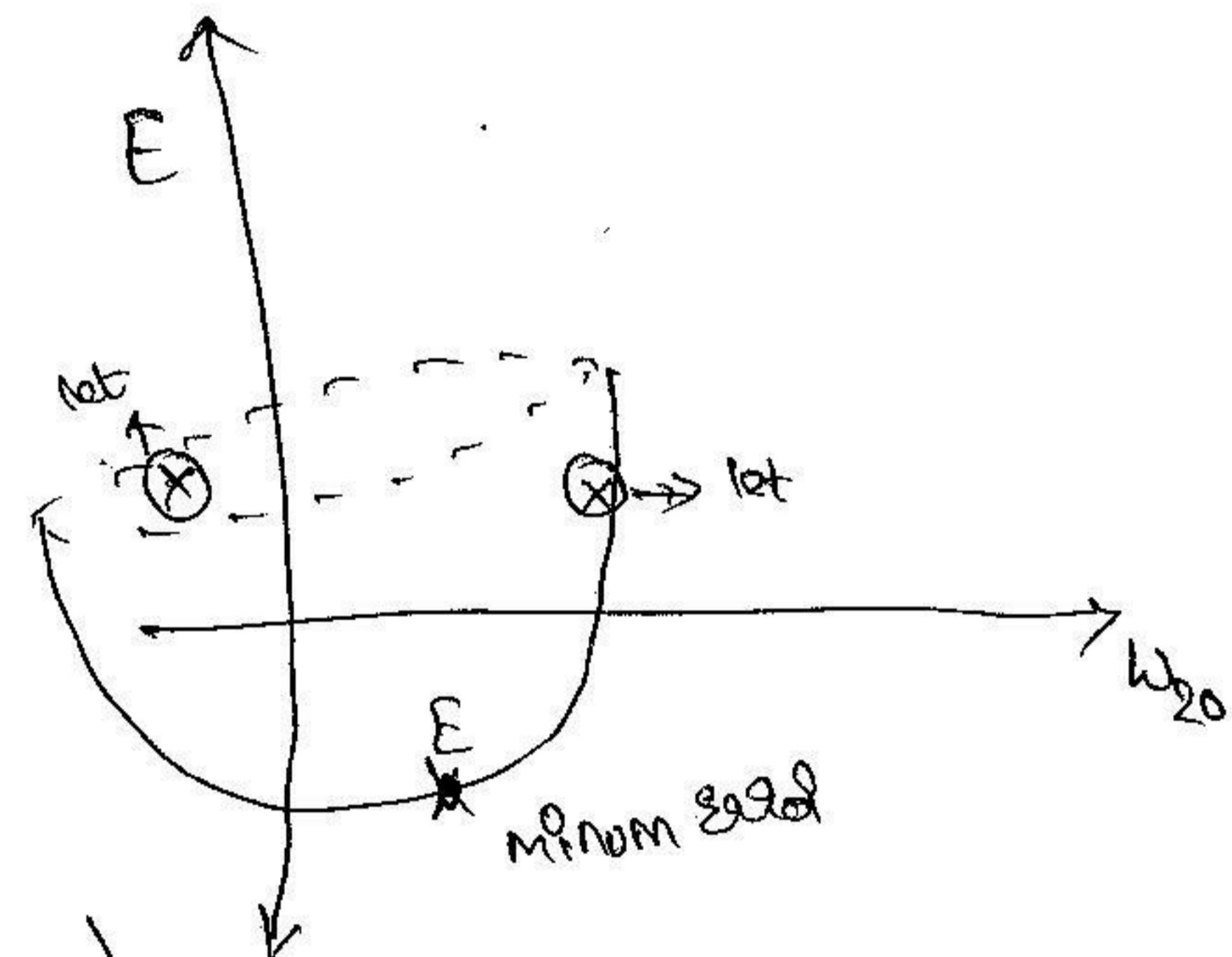
∴ Total error of all set up points are given as

$$E = \sum_P E^P = \sum_P (t^P - y^P)^2 \rightarrow ②$$

The main object is to find a minimum error in slope plane.



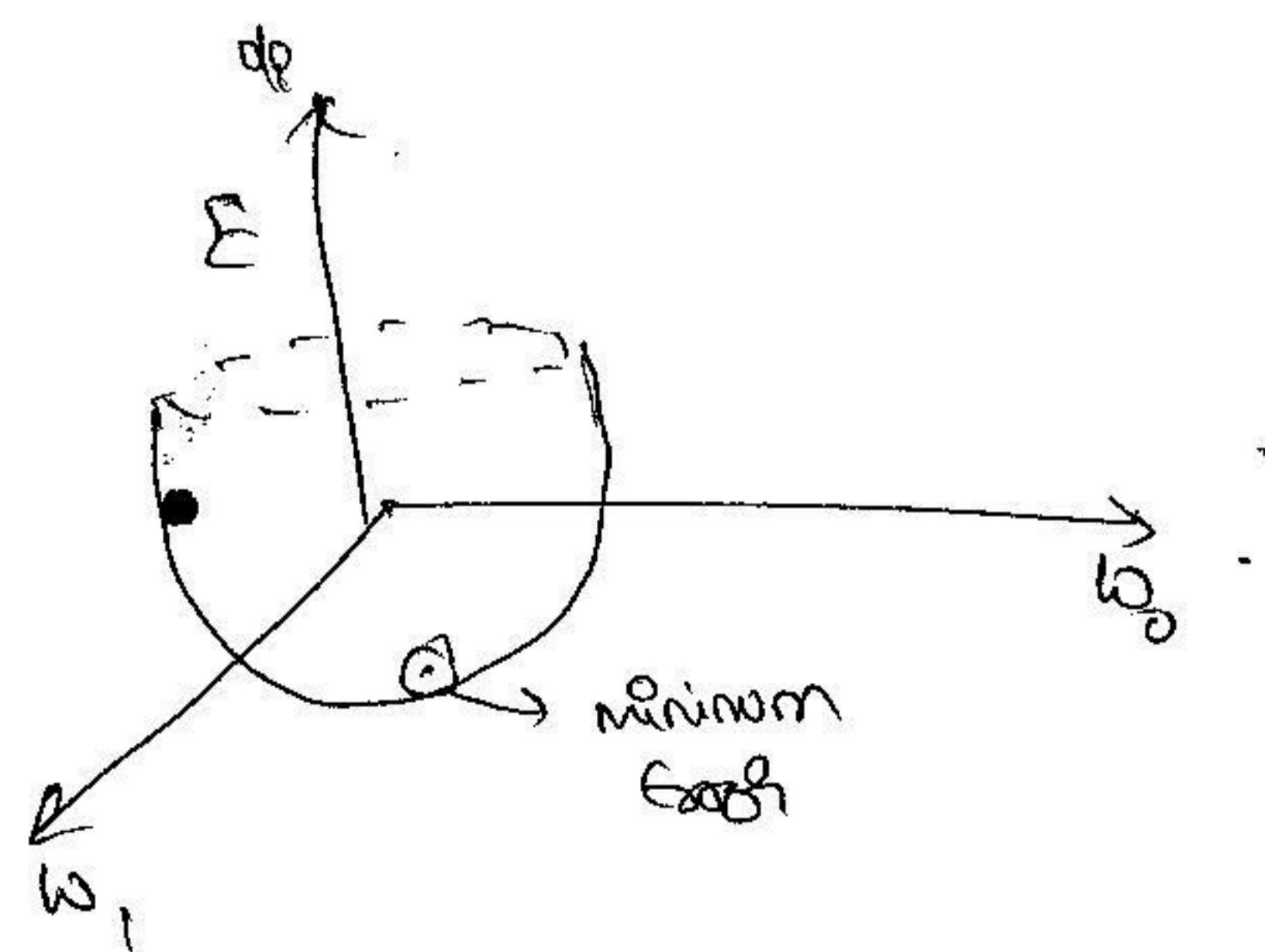
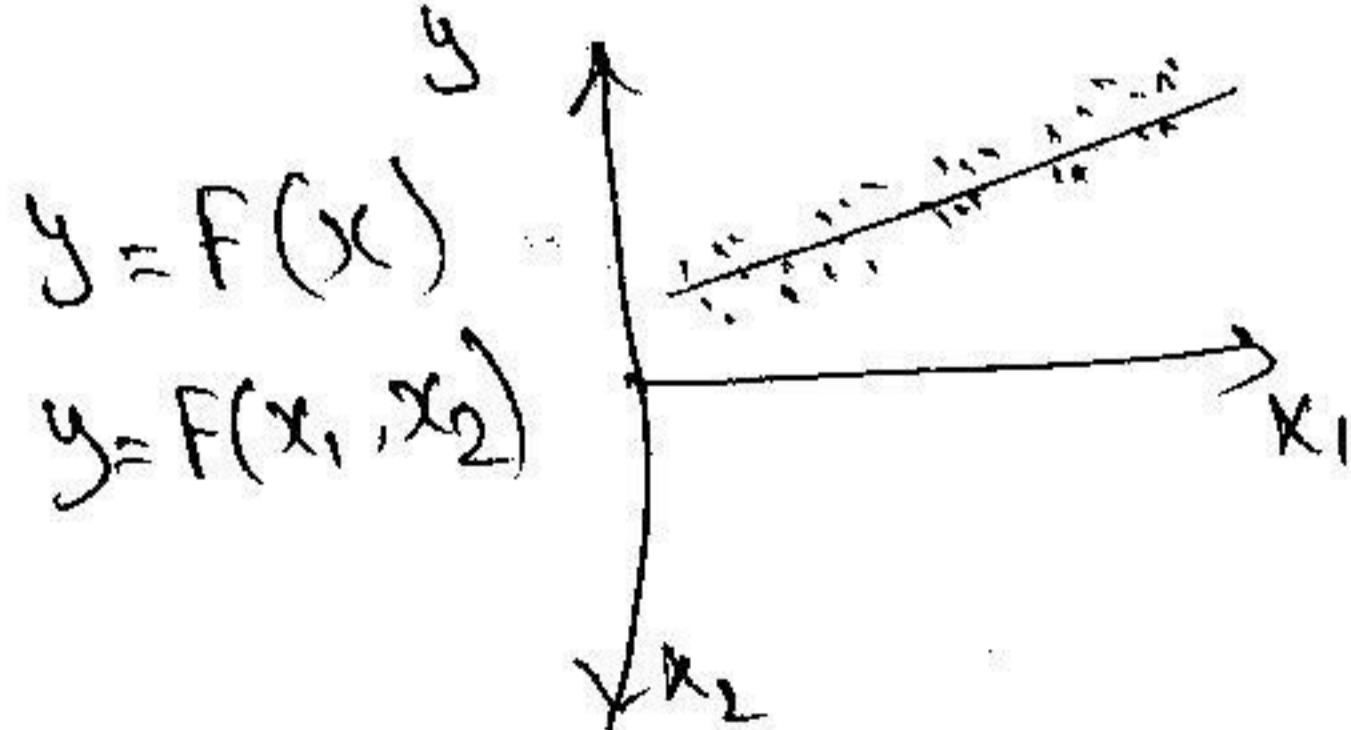
$$\Rightarrow \frac{\partial E}{\partial w}$$



(Ball in net rolled across slope direction)  $w_{21}$

→ This technique is known as steepest descent technique.

\*> Gradient Descent Algorithm :-



Gradient Descent algorithm:-  $G_i = \frac{\partial E}{\partial w_i}$

(i-21)

Let  $P \rightarrow \text{no. of points/observations or no. of points}$

$E \rightarrow \text{Combined Error}$

w.r.t

$$E = \sum_P E^P \rightarrow ①$$

$O \rightarrow \infty$  of different outputs

$$E^P = \frac{1}{2} \sum_0 (t_0^P - y_0^P) \rightarrow ②$$

$t_0^P \rightarrow \text{target op}$  } @ 1-op  
 $y_0^P \rightarrow \text{op of system}$  } unit  
 Actually

for several op

$$y_1 = f_1(x_1, x_2, \dots, x_n)$$

$$y_2 = f_2(x_1, x_2, \dots, x_n)$$

$$\vdots$$

$$y_m = f_m(x_1, x_2, \dots, x_n)$$

$\therefore$  the gradient descent is related as  $G_i = \frac{\partial E}{\partial w_{ij}} = \frac{\partial}{\partial w_{ij}} \sum_P E^P \rightarrow ③$

$$G_i = \sum_P \frac{\partial E^P}{\partial w_{ij}} \rightarrow ④$$

Applying chain rule of differentiation.

$$\frac{\partial E}{\partial w_{ij}} = \frac{\partial E^P}{\partial y_0} \cdot \frac{\partial y_0}{\partial w_{ij}} \rightarrow ⑤$$

From eq ② differentiating with  $\partial w_{0j}$

$$\therefore \frac{\partial E}{\partial w_{ij}} = -(t_0^P - y_0^P) \rightarrow ⑥$$

$$\text{w.r.t } y_0 = \sum_j w_{0j} \cdot x_j \rightarrow ⑦$$

From eq ⑦ differentiating eq w.r.t  $\partial w_{0j}$

$$\frac{\partial y_0}{\partial w_{0j}} = \frac{\partial}{\partial w_{0j}} \sum_j w_{0j} \cdot x_j$$

$$\Rightarrow \frac{\partial y_0}{\partial w_{0j}} = x_j \rightarrow ⑧$$

(I-22)

$\therefore$  now Eq (5) is written as

$$\frac{\partial E}{\partial w_{0i}} = - (t_0^* - y_0) \cdot x_i \rightarrow 9 \rightarrow (\text{for -ve direction})$$

$\therefore$  in correction representation is

$$\boxed{\Delta w_{0i} = \eta (t_0 - y_0) \cdot x_i} \rightarrow \text{correction.} \rightarrow 10$$

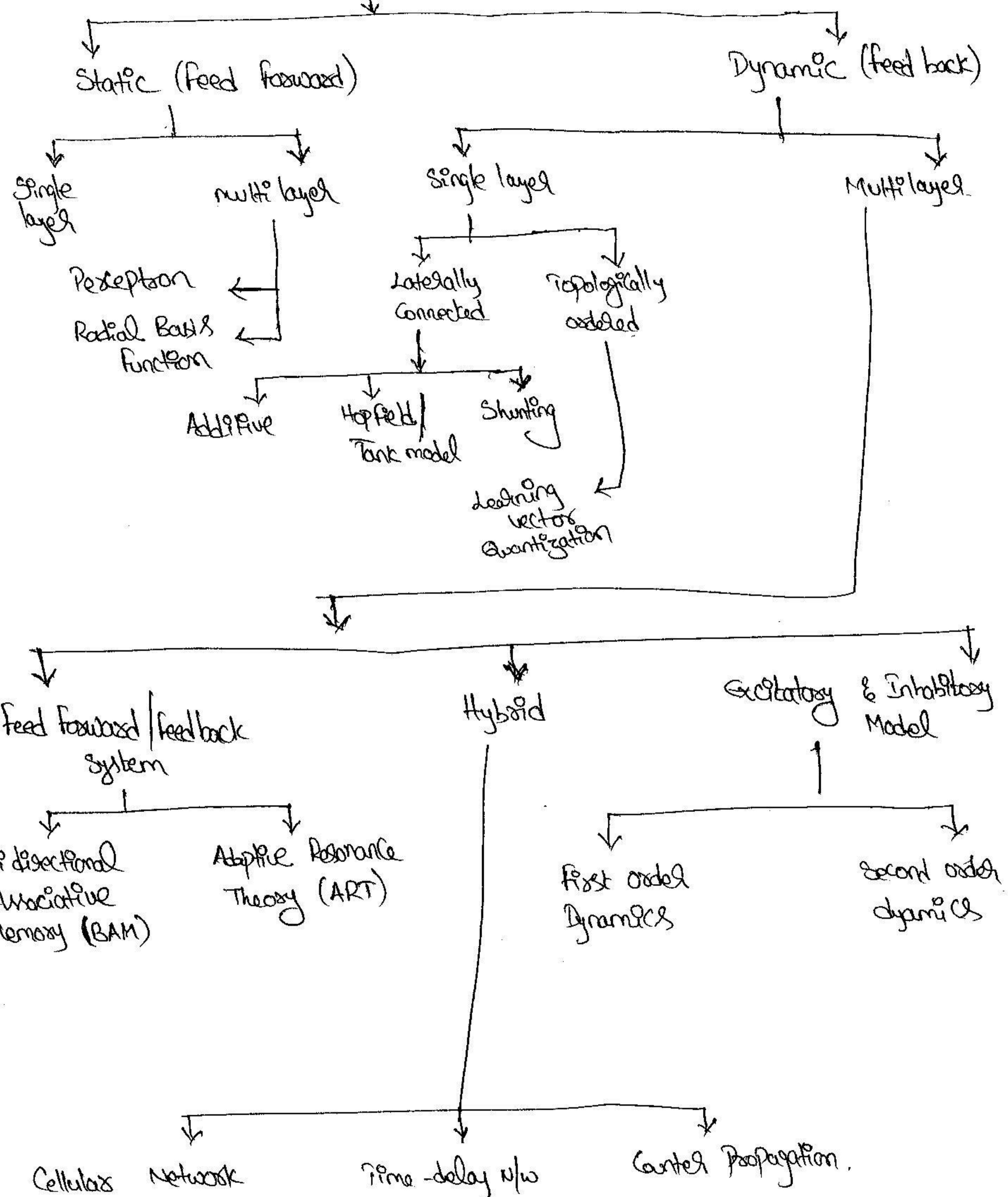
$\uparrow$   $\rightarrow$  rate at which it is falling is controlled by  $\eta$

$\downarrow$   $\rightarrow$  learning rate

$$\therefore \boxed{w_{0i} = w_{0i}^{(\text{old})} + \Delta w_{0i}^{(\text{new})}} \rightarrow 11$$

$$f(v_k) = \frac{1}{1 + e^{-(av_k)}}$$

12) Classification of Artificial Neural network :- (I-23) 17



### 13) Neuron Modeling for Artificial Neural Systems:

Mcculloch-Pitts model of neuron is characterized by its formalism and its elegant, precise mathematical definition.

- Model makes several drastic simplifications
- It allows 0,1 states only. Operates under a discrete time assumption. in brief.
- Weights and neurons threshold are fixed in model and no interaction among nw neurons take place except signal flow.

$$\text{the op } x_1, x_2 \dots x_n \rightarrow x'_p \\ w_1, w_2 \dots w_n \rightarrow w_p$$

Symbolic Representation of op is

$$O_p = f(w^t \cdot x) \text{ (or)} \rightarrow \textcircled{a} \text{ multiplicative weights}$$

$$O_p = f\left(\sum_{i=1}^n w_i x_i\right) \rightarrow \textcircled{b}$$

where weighted vector defined as  $w = [w_1 \ w_2 \ \dots \ w_n]^t$

and  $x$  is op vector

$$x = [x_1 \ x_2 \ x_3 \ \dots \ x_n]^t$$

( $t \rightarrow$  denotes transposition)

where function

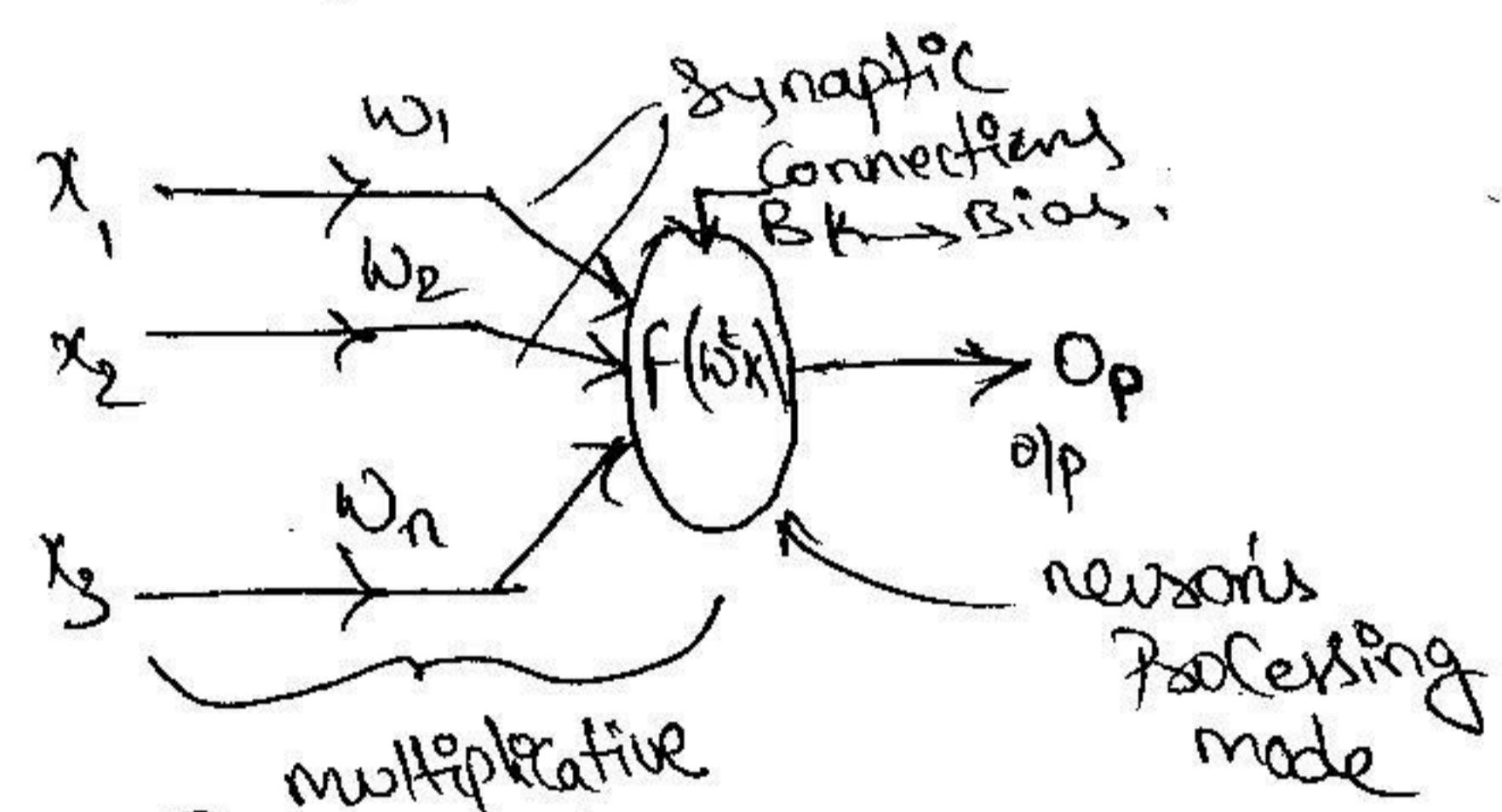
$f(w^t x)$  → referred as Activation function.

Q-Models Mainly introduced:-

(i) neural nw are meant to be artificial neural nw consisting

of neuron models.

(ii) neurons are meant to be artificial neuron models.



The typical activation used are

$$f(O_p) = \frac{2}{1 + e^{-\alpha O_p}} - 1 \rightarrow ①$$

and

$$f(O_p) = \text{sgn}(O_p) = \begin{cases} +1, & O_p > 0 \\ -1, & O_p < 0 \end{cases} \rightarrow ②$$

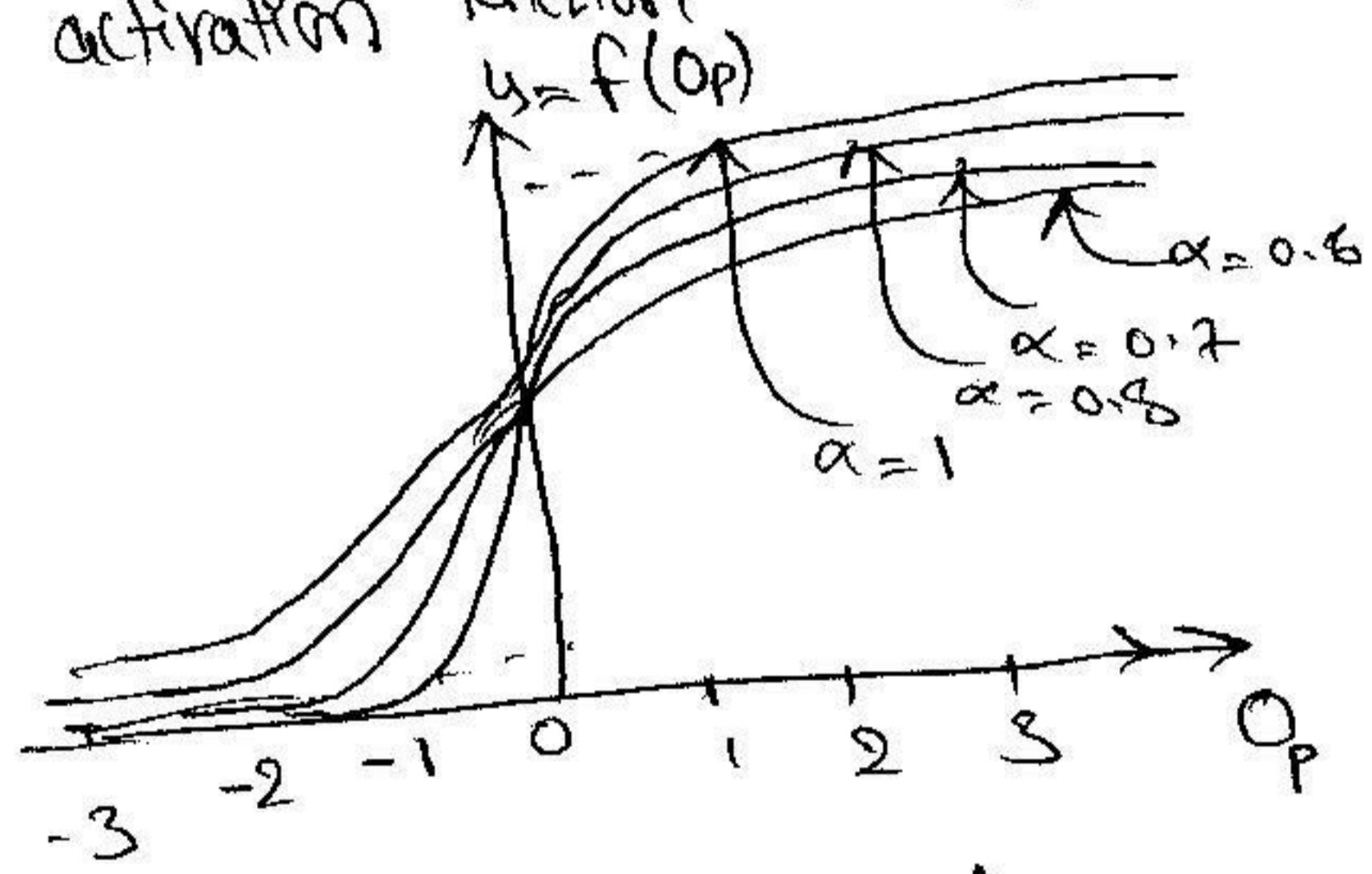
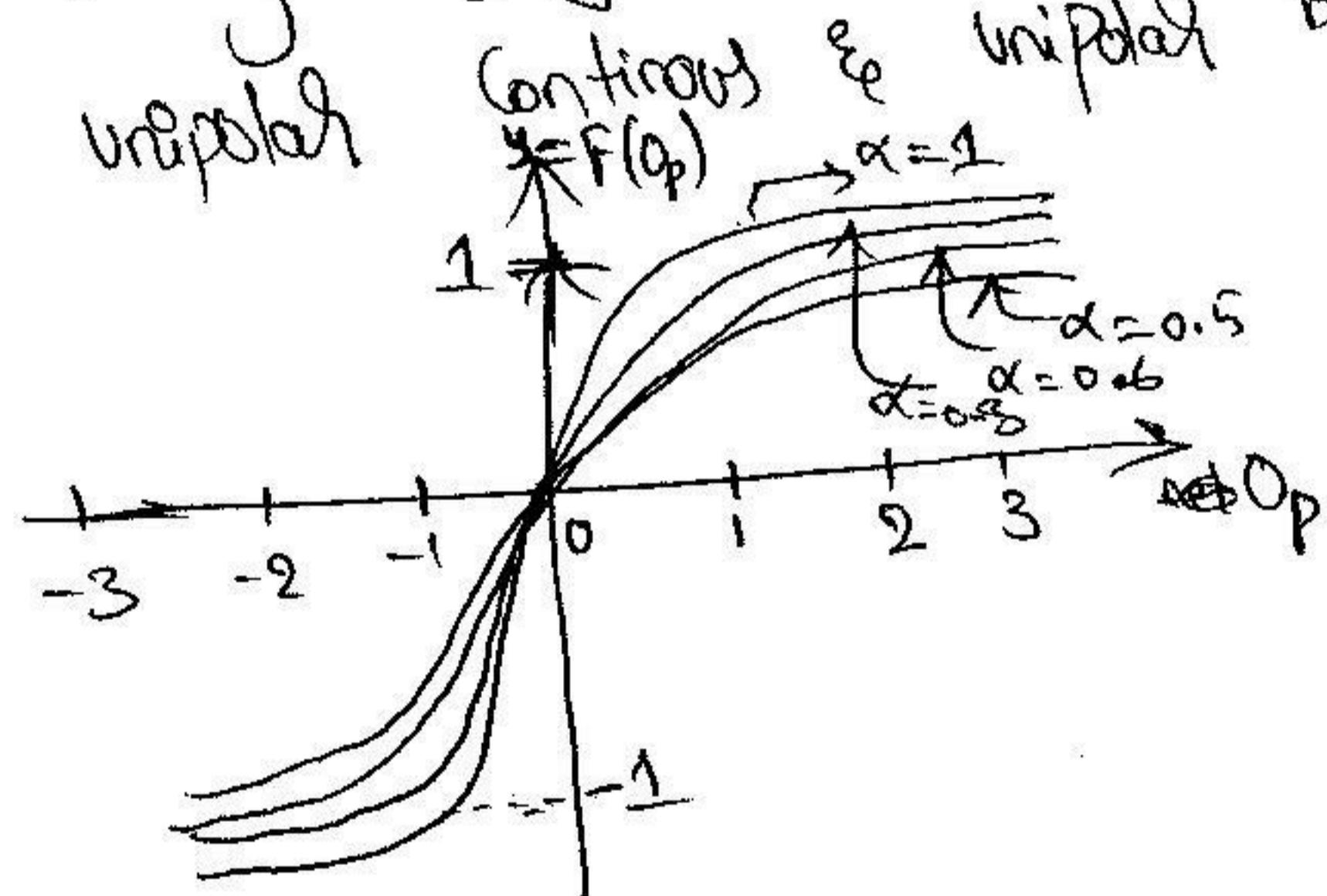
where  $\alpha > 0$  in eq ① is proportional to neuron gain determined the steepness of continuous function  $f(O_p)$  near  $O_p = 0$ . If  $\alpha \rightarrow \infty$  (infinite) then limit continuous function becomes the sign ( $O_p$ ) function defined in Activation are called bipolar functions.

(as) bipolar binary function

bipolar used to point out both (+ve & -ve) response of

neuron.

→ By shifting and scaling the bipolar activation function defined by unipolar



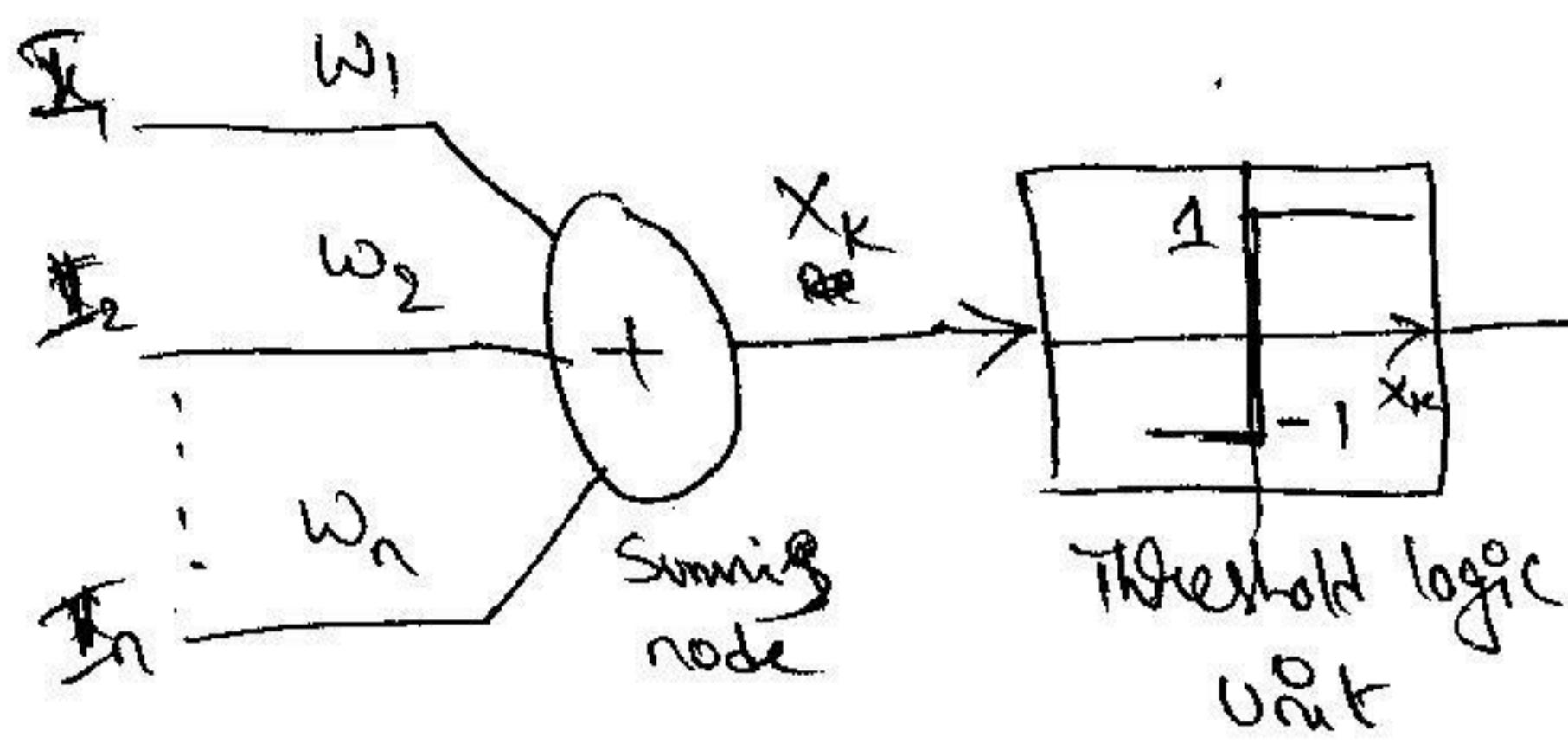
unipolar continuous.

Bipolar Continuous

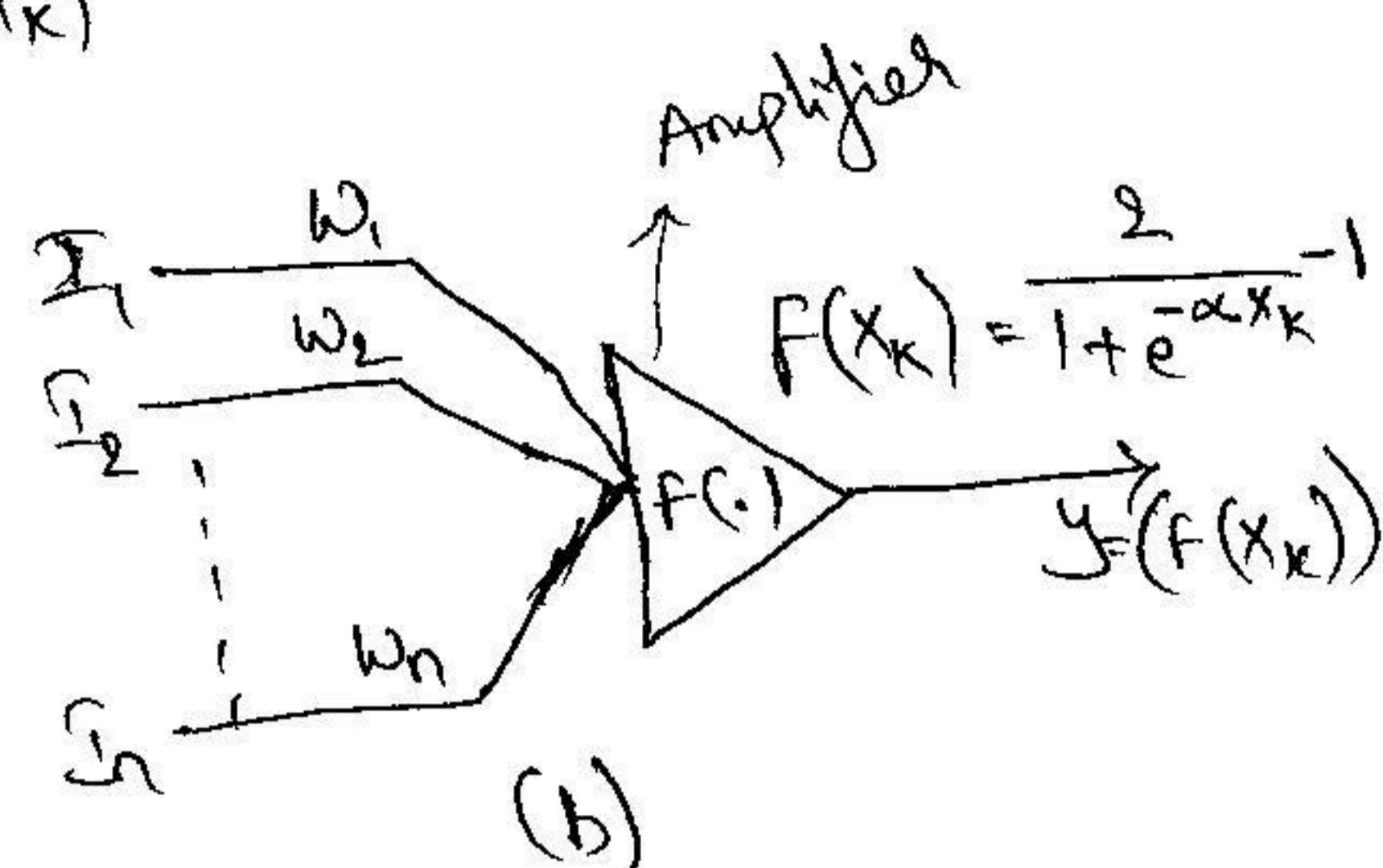
respectively:-  $f(O_p) = \frac{1}{1 + e^{-\alpha O_p}}$  → ③

$$f(O_p) = \begin{cases} 1, & O_p \geq 0 \\ 0, & O_p < 0 \end{cases} \rightarrow ④$$

eq ① & ④ are Sigmoid function characteristics. as compared to hard limiting function given in eq ② & ③.



(a) hard limiting neuron  
(binary Perceptron)



(b) soft limiting neuron  
(continuous Perceptron)

→ In case of Continuous activation function, above figure-(a) is model used and shown in (b). The neuron is depicted at a summing high gain saturating amplifier which amplifies its off signal ' $w^t$ '.  
These two model are called discrete (binary) & continuous Perceptron.

→ The discrete Perceptron introduced by Rosenblatt was first learning machine.

If neuron of the discrete (binary) or continuous given a layer of  $m$ -neuron their off's  $O_1, O_2, \dots, O_m$  are arranged in layer.

$$O_p = \{O_1, O_2, \dots, O_m\}$$

where  $O_p$  is off signal of  $i^{th}$  neuron. The domain of vector  $O'$  in  $m$ -dimensional space as follows  $i=1, 2, \dots, m$

$$(-1, 1)^m = \{O \in R^m ; O_i \in (-1, 1)\}$$

$$(0, 1)^m = \{O \in R^m ; O_i \in (0, 1)\}.$$

## (Lecture - 4)

### ① Models of ANN:-

#### ① Feed forward w/w:-

let  $\underline{I} = [I_1, I_2, \dots, I_n]^t \rightarrow \textcircled{a}$ ;  $W_{kj} = \{w_{k1}, w_{k2}, \dots, w_{km}\}^t$

$$Y = [y_1, y_2, \dots, y_n]^t \rightarrow \textcircled{b}$$

as w.l.o.g for 1-neuron  $y_j = f(V_k) = f[W_k^t \cdot \underline{I}] \rightarrow \textcircled{c}$  for  $j=1 \rightarrow n$ .

let us consider a non-linear matrix operation denoted by ' $\Pi$ '.

$$\therefore y = \Pi[W, \underline{I}] \rightarrow \textcircled{d}$$

$$\therefore W = \begin{bmatrix} w_{11} & w_{12} & w_{13} & \dots & w_{1n} \\ w_{21} & w_{22} & w_{23} & \dots & w_{2n} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ w_{m1} & w_{m2} & w_{m3} & \dots & w_{mn} \end{bmatrix} \quad \begin{array}{l} s=1 \rightarrow n \\ k=1 \rightarrow m \\ \rightarrow \textcircled{e} \end{array}$$

and  $\therefore y = \Pi(\cdot) = \begin{bmatrix} \Pi(\cdot) & 0 & \dots & 0 \\ 0 & \Pi(\cdot) & \dots & 0 \\ 0 & 0 & \Pi(\cdot) & \dots & 0 \\ \vdots & \dots & \dots & \dots & \Pi(\cdot) \end{bmatrix} \rightarrow \textcircled{f}$

$\therefore \underline{I} \& Y$  are o/p & o/p patterns involves in time delay for activation

$$\therefore \boxed{y(t) = \Pi[W, \underline{I}(t)]} \rightarrow \textcircled{g}$$

e.g. multilayer ANN:-

$$\text{W.R.T } f(\text{net}) := \text{Sgn}(\text{net}) = \begin{cases} 1 & \text{net} \geq 0 \rightarrow \textcircled{a} \\ 0 & \text{net} < 0 \end{cases}$$

$\therefore$  The response should satisfy

$$O_5 = \text{Sgn}(O_1 + O_2 + O_3 + O_4 - b_K) \rightarrow \textcircled{b}$$

where response should be  $O_1 + O_2 + O_3 + O_4 + O_5 = 1$

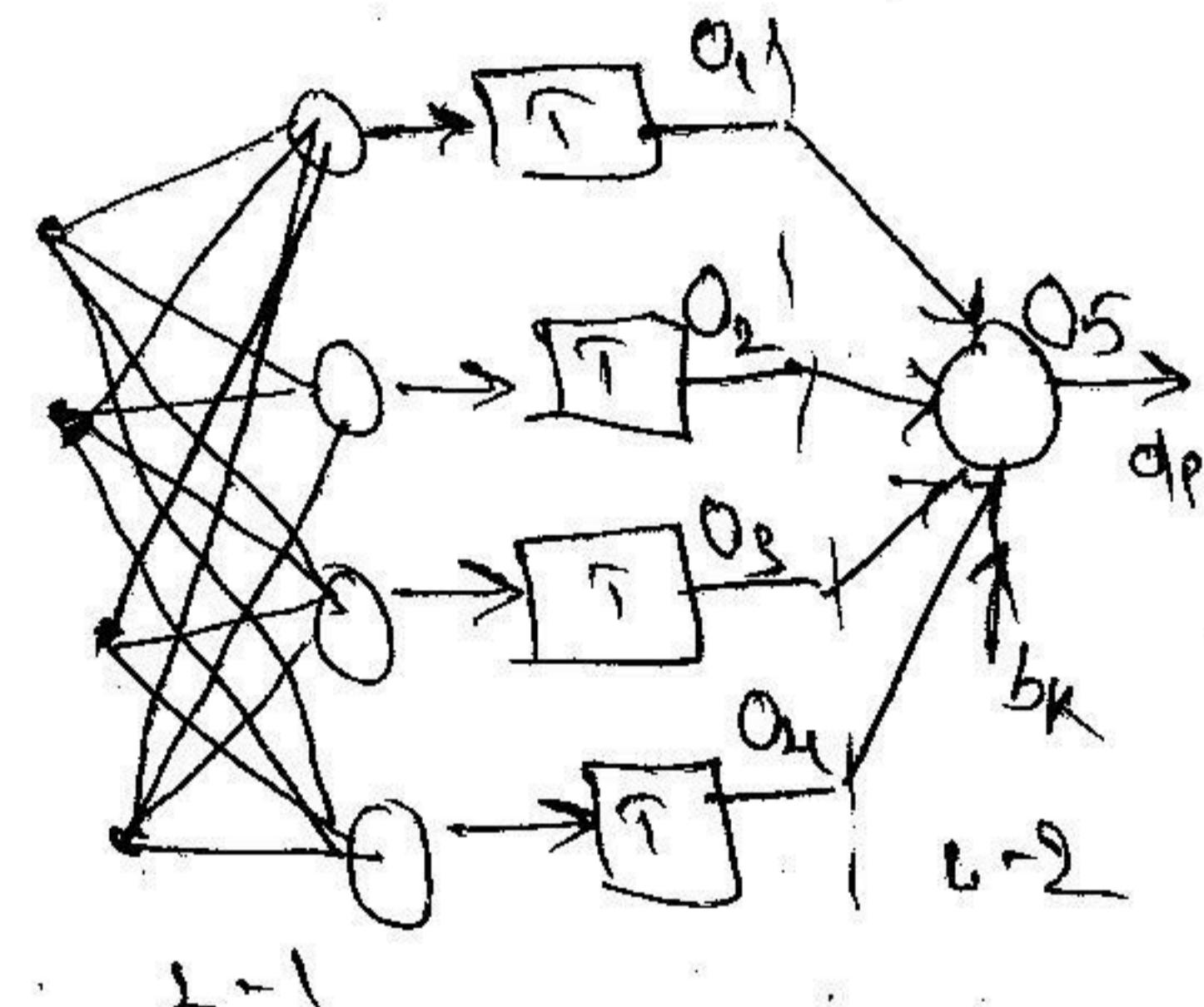
(Competitive learning)

Expected

$y = f(X_k) = \dots \rightarrow \textcircled{c}$  leading toward the  $k^{th}$  neuron

whole weights  $w_j$  contain weights leading toward the o/p node is

$$W_k = \{w_1, w_2, \dots, w_m\}^t \rightarrow \textcircled{c}$$



Q: In general Activation function  $\phi$  is given as  

$$y = \text{sgn}(v_k) = \frac{1}{1+e^{-av_k}} \rightarrow \text{C} \left( \phi = \frac{2}{1+e^{-av_k}} - 1 \right).$$
 typical Activation function

$\therefore$  In multilayer the bipolar activation function obtain for feed forward is

$$\phi = \begin{cases} \frac{2}{1+e^{-(I_1-\theta_1)\lambda}} - 1 \\ \frac{2}{1+e^{(I_2-\theta_2)\lambda}} - 1 \\ \frac{2}{1+e^{(I_3-\theta_3)\lambda}} - 1 \\ \frac{2}{1+e^{(I_4-\theta_4)\lambda}} - 1 \end{cases} \rightarrow 1^{\text{st}} \text{ layer bipolar AG}$$

$\therefore$  for second layer  $O_B = \frac{2}{(3.5 - O_1 - O_2 - O_3 - O_4) \cdot \lambda} - 1$

### ② feed backward:-

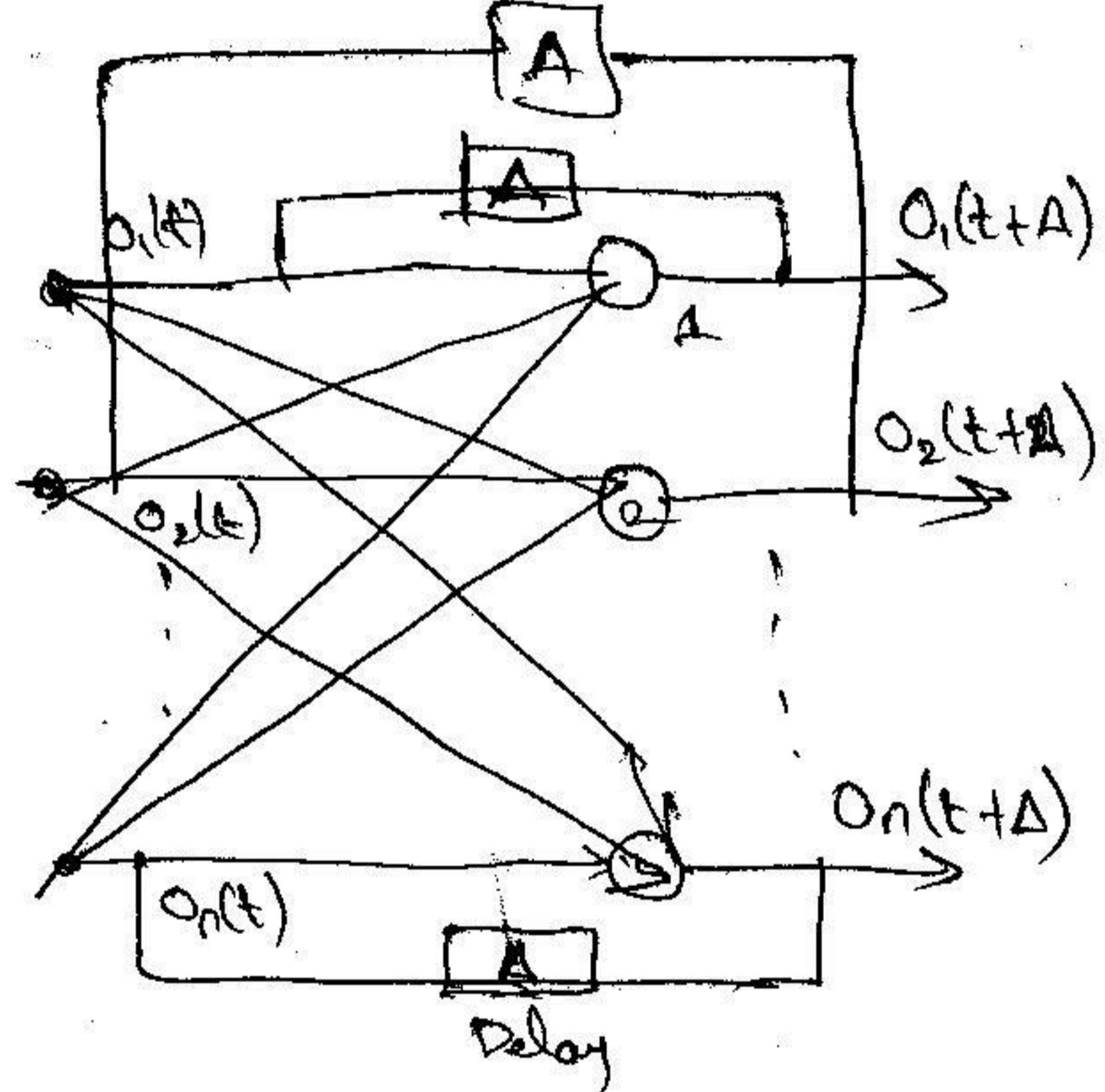
The control of  $\phi$  or through  $\phi$  between control is especially meaningful if present  $\phi$  say  $O(t)$  say  $O(t)$  control  $\phi$  as  $O(t+\Delta)$   
 $t \rightarrow$  time  $\Delta$  day

If we have discrete variable performance then  $\Delta, 2\Delta, 3\Delta, \dots$  called clock time

for discrete ANNs converted

$$O^{k+1} = R [w_{0k}^k] \text{ for } k=1, 2, \dots$$

e.g.:  $O^1 = R [w^0 x^0]$   
 $O^2 = R [w R [w^0 x^0]] \dots \dots O^{k+1} = R [w R [R [w^0 x^0]]]$



## Models of Artificial neural networks:-

(i-27) ~~Q1~~

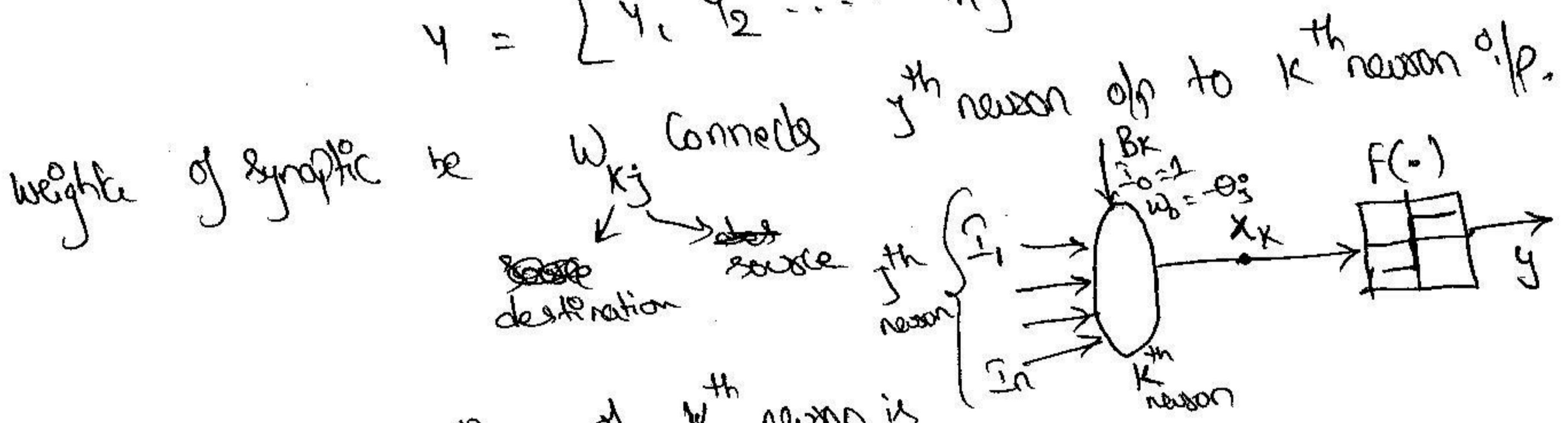
→ An introductory to artificial network is a physical cellular networks that are able to acquire, store and utilize knowledge has been related to network Capabilities and Performance.

### (i) feed forward network:-

Let us consider an elementary feed forward architecture of  $m$ -neurons are receiving  $n$ -inputs. Its op & ip vectors are respectively be

$$\underline{i} = [i_1, i_2, \dots, i_n]^t \rightarrow \textcircled{a}$$

$$\underline{y} = [y_1, y_2, \dots, y_m]^t \rightarrow \textcircled{b}$$



we thus write activation of  $k^{\text{th}}$  neuron is

$$x_k = \sum_{j=1}^n i_j \cdot w_{kj} + b_k \quad \text{for } j=1, 2, \dots, n. \quad (\because b_k = \theta_j - \theta_j) \quad (\because b_k = 0)$$

The non-linear transformation involving the activation function  $y = f(x_k)$  for  $j=1, 2, \dots, n$  completes processing of  $i$ . the transformation performed by  $m$  neurons. an nw is strongly non-linear mapping

expressed as

$$y = f(x_k) = f(w_k^t \cdot i) \quad \text{for } j=1, 2, \dots, n \rightarrow \textcircled{d}$$

whole weights of node is

$$w_k = [w_{k1}, w_{k2}, \dots, w_{km}]^t \rightarrow \textcircled{e}$$

By introducing a non-linear matrix operation ' $\Gamma$ ', the mapping of  $\text{Q}_P$  space  $\mathbf{i}$  to  $\text{Q}_P$  space  $\mathbf{y}$  implemented by  $\eta/\nu$  expressed.

$$\mathbf{y} = \Gamma[\mathbf{w}, \mathbf{i}] \rightarrow \textcircled{F}$$

where  $\mathbf{w}$  is weight matrix also called Connection matrix

$$\mathbf{w} = \begin{bmatrix} w_{11} & w_{12} & w_{13} & \dots & w_{1n} \\ w_{21} & w_{22} & w_{23} & \dots & w_{2n} \\ w_{31} & w_{32} & w_{33} & \dots & w_{3n} \\ \vdots & & & & \\ w_{m1} & w_{m2} & w_{m3} & \dots & w_{mn} \end{bmatrix} \rightarrow \textcircled{g}$$

$j = 1 \rightarrow n$   
 ~~$k = 1 \rightarrow m$~~

and

$$\mathbf{y} = \Gamma(\cdot) = \begin{bmatrix} f(\cdot) & 0 & \dots & 0 \\ 0 & f(\cdot) & \dots & 0 \\ \vdots & & & \\ 0 & 0 & \dots & f(\cdot) \end{bmatrix} \rightarrow \textcircled{h}$$

The matrix of function operator ' $\Gamma$ ' operates Component wise on activation values  $x_k$  of each neuron.

The  $\text{Q}_P$  &  $\text{Q}_P$  vectors  $\mathbf{i}$  &  $\mathbf{y}$  called as  $\text{Q}_P$  &  $\text{Q}_P$  Pattern

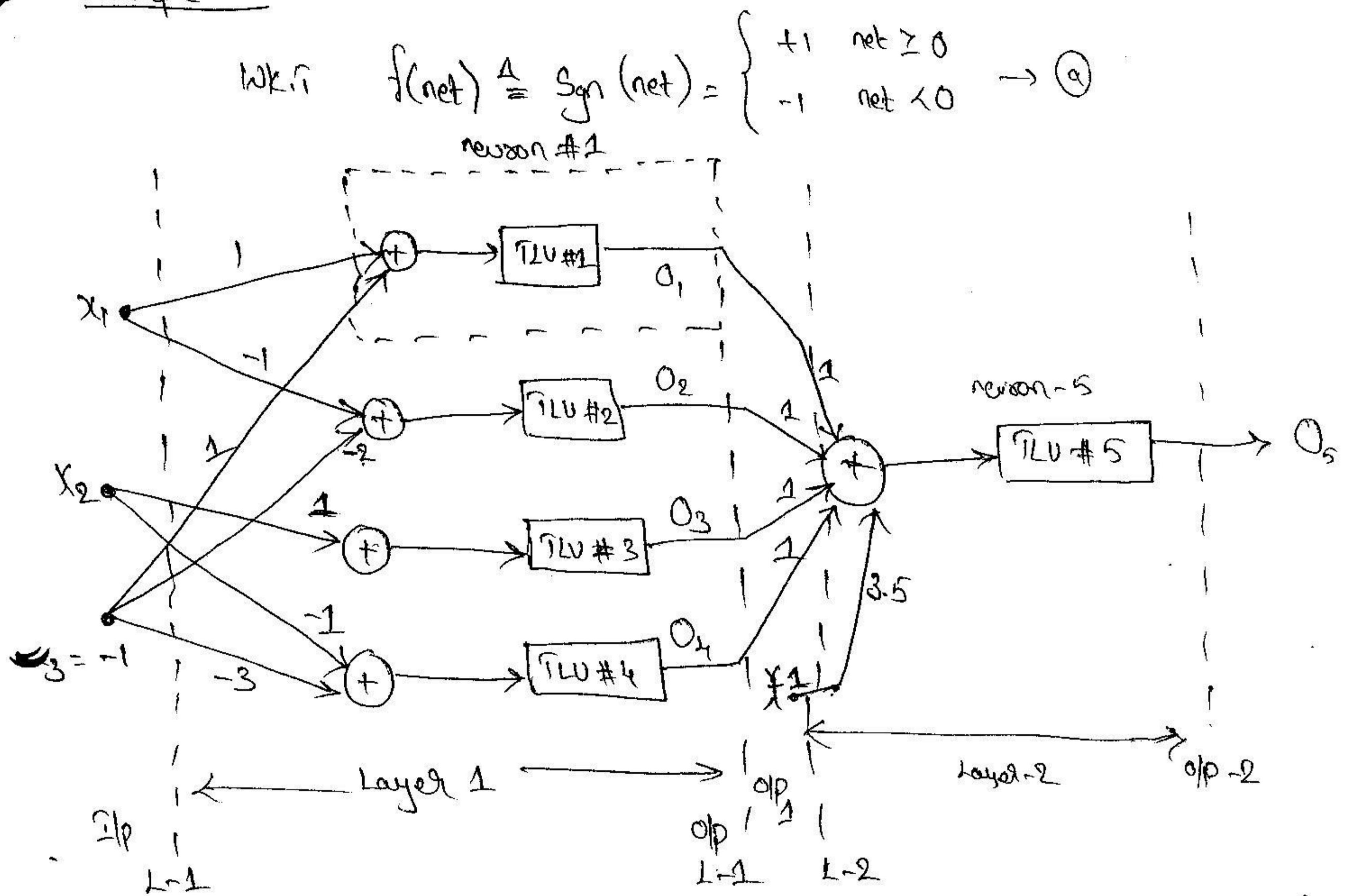
$\therefore$  It involves time delay b/w  $\mathbf{i}$  &  $\mathbf{y}$  so we can

write Eq.  $\textcircled{F}$  as

$$\boxed{\mathbf{y}(t) = \Gamma[\mathbf{w}, \mathbf{i}(t)] \rightarrow \textcircled{i}}$$

- The generic feedforward is characterized by lack of feedback.
- This type of connection  $\eta/\nu$  can be connected in cascade to create a multilayered  $\eta/\nu$ .

Example 1:-



$$\therefore \text{The response of } O_5 = \text{Sgn}(O_1 + O_2 + O_3 + O_4 - 3.5)$$

the  $O_5$  neuron respond +1 if  $O_1 = O_2 = O_3 = O_4 = 1$

$\therefore$  The bipolar activation function, obtain for first layer.

$$O_P = \begin{bmatrix} \frac{2}{1+e^{(1-x_1)\lambda}} - 1 \\ \frac{2}{1+e^{(x_2-2)\lambda}} - 1 \\ \frac{2}{1+e^{(-x_2)\lambda}} - 1 \\ \frac{2}{1+e^{(x_2-3)\lambda}} - 1 \end{bmatrix}$$

$\therefore$  for second layer,

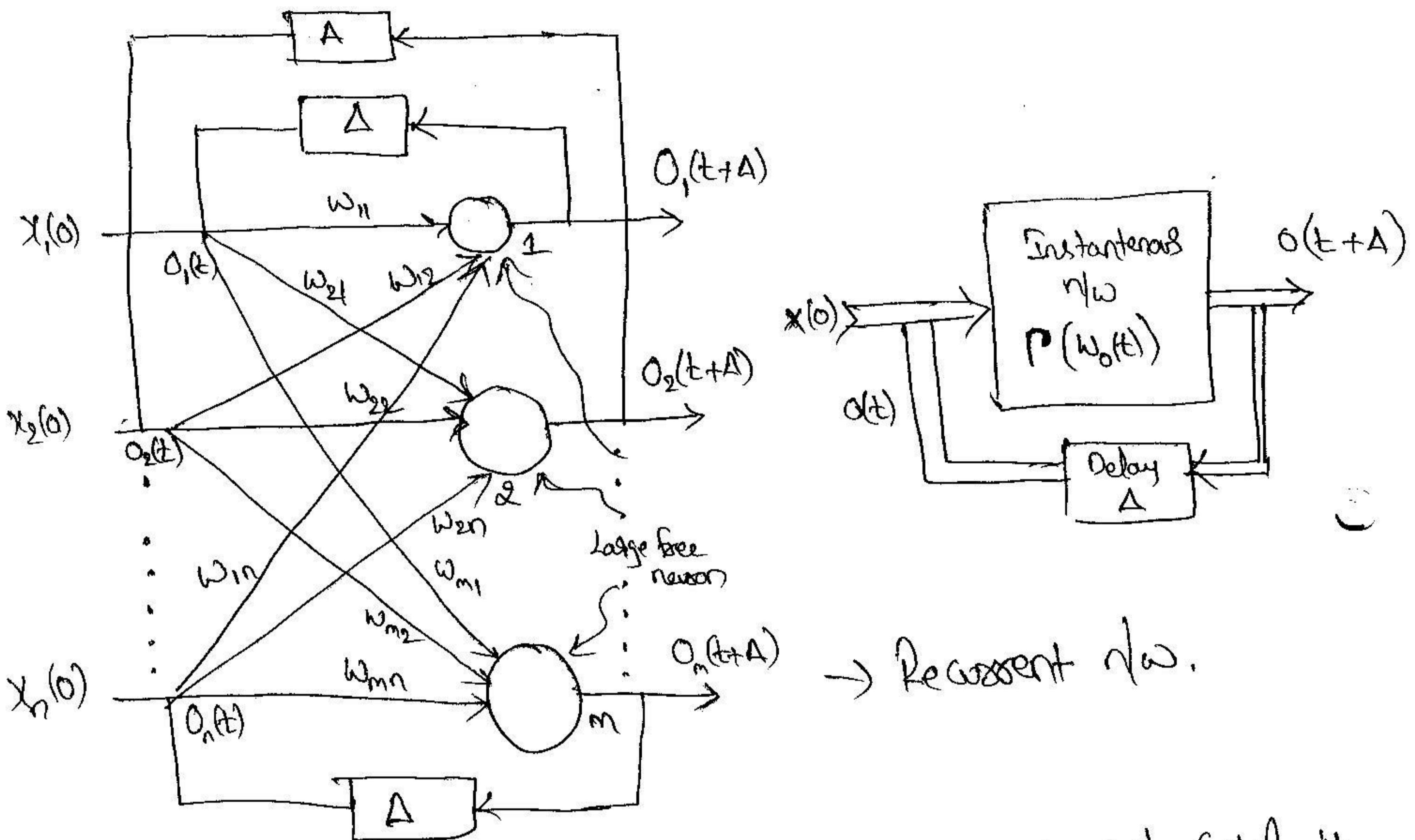
$$O_5 = \frac{2}{1+e^{(3.5-O_1-O_2-O_3-O_4)\lambda}} - 1$$

(ii)

## Feedback Network:-

Can be obtained from the feed forward n/w neuron dp to their IP's.

(I-30)



The control of dp  $O_i$  through especially meaningful if present the time  $\Delta$  (delay) elapsed b/w elements in feedback loop.

feed forward n/w mapping of  $O(t)$  into  $O(t+\Delta)$  is

$$O(t+\Delta) = \Gamma [w_o(t)]$$

There are two main categories of single layer feedback n/w. If we consider time as discrete variable then n/w performance at discrete time instances  $\Delta, 2\Delta, 3\Delta, \dots$  the system is called discrete time.

for discrete time artificial neural system, we have connected to

$$O^{k+1} = \Gamma [w_o^k], \text{ for } k = 1, 2, \dots \rightarrow \textcircled{a}$$

$k \rightarrow$  instant number.

$\therefore$  response at  $k+1$  instant depends on entire history of the starting at  $k=0$ .

(9-31) ~~Q~~

$$O^1 = \Gamma [w x^0]$$

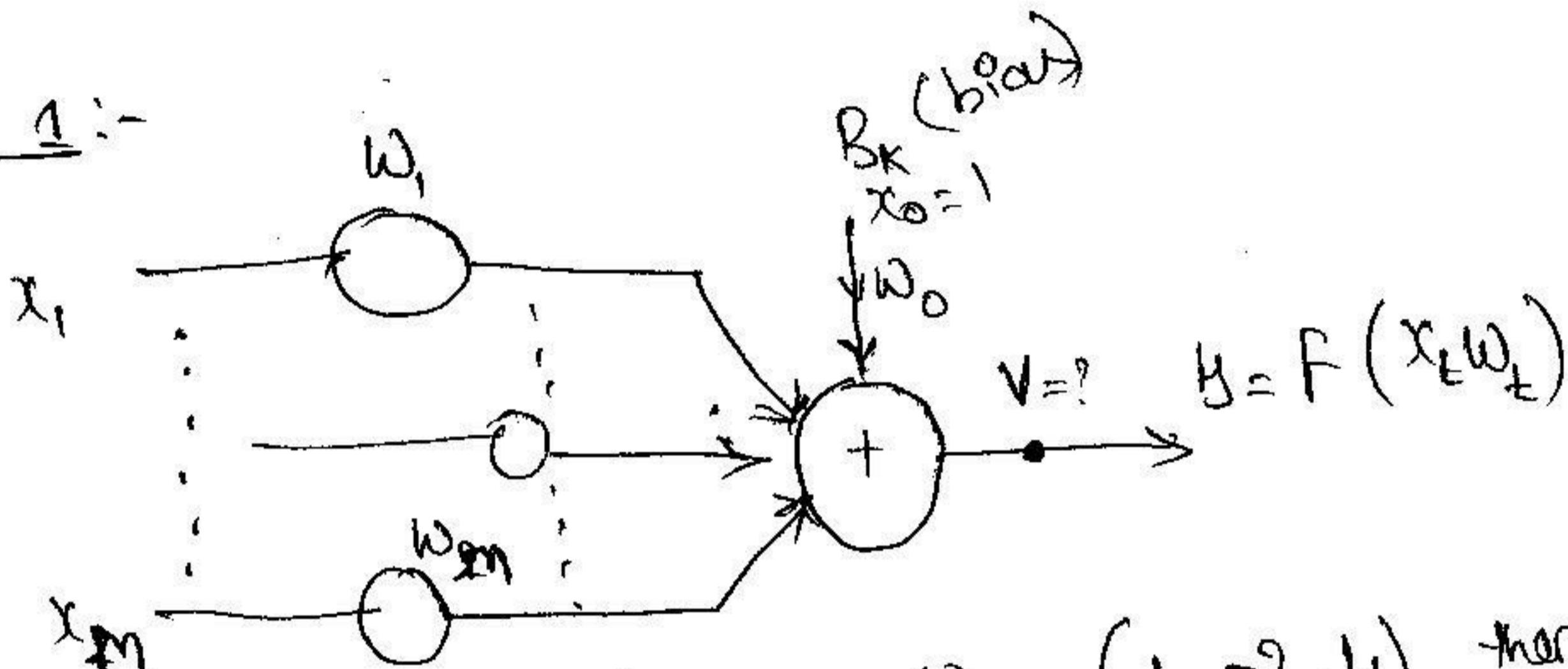
$$O^2 = \Gamma [w \Gamma [w x^0]] .$$

$$\vdots$$

$$O^{k+1} = \Gamma [w \Gamma [w \Gamma [w x^0]] \dots]$$

A system with discrete time inputs and discrete data representation is called an automaton.

Example 1 :-



Let  $x = (0, 1, 1)$ ,  $w = (1, -2, 4)$  then

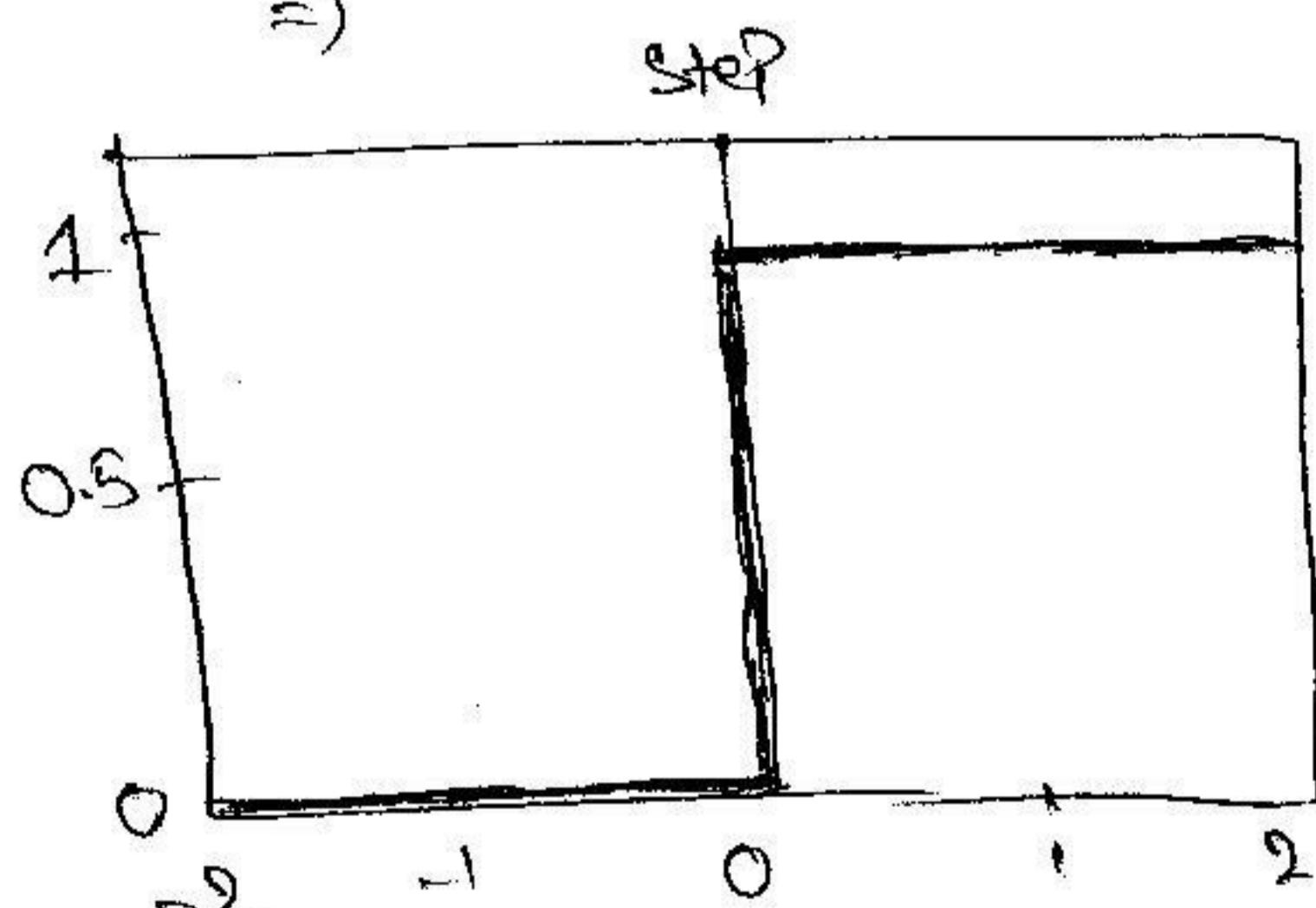
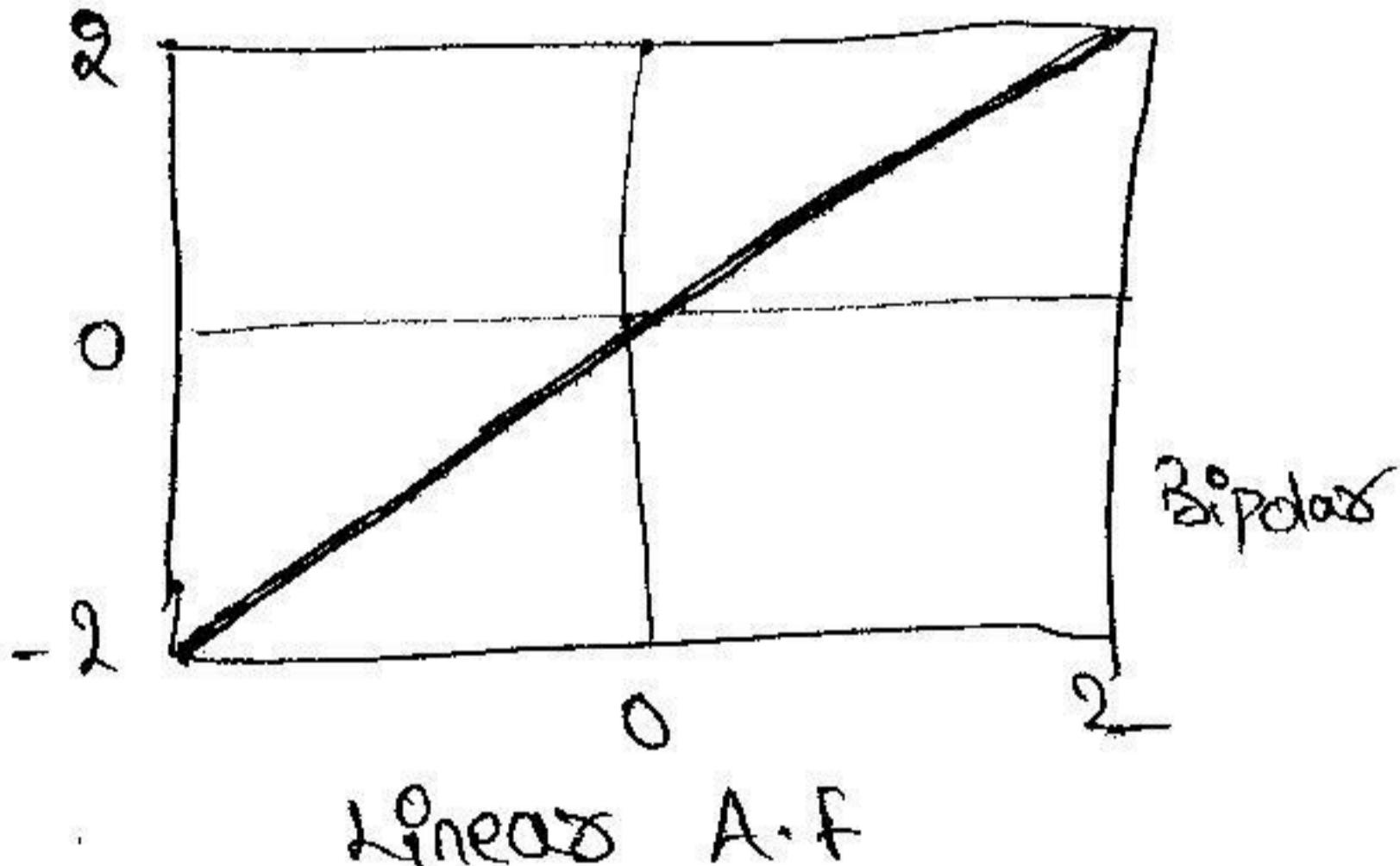
$$v = w_1 x_1 + w_2 x_2 + w_3 x_3 + \dots + w_m x_m$$

$$v = 1 \cdot 0 + (-2) \cdot 1 + (4) \cdot 1 = 2$$

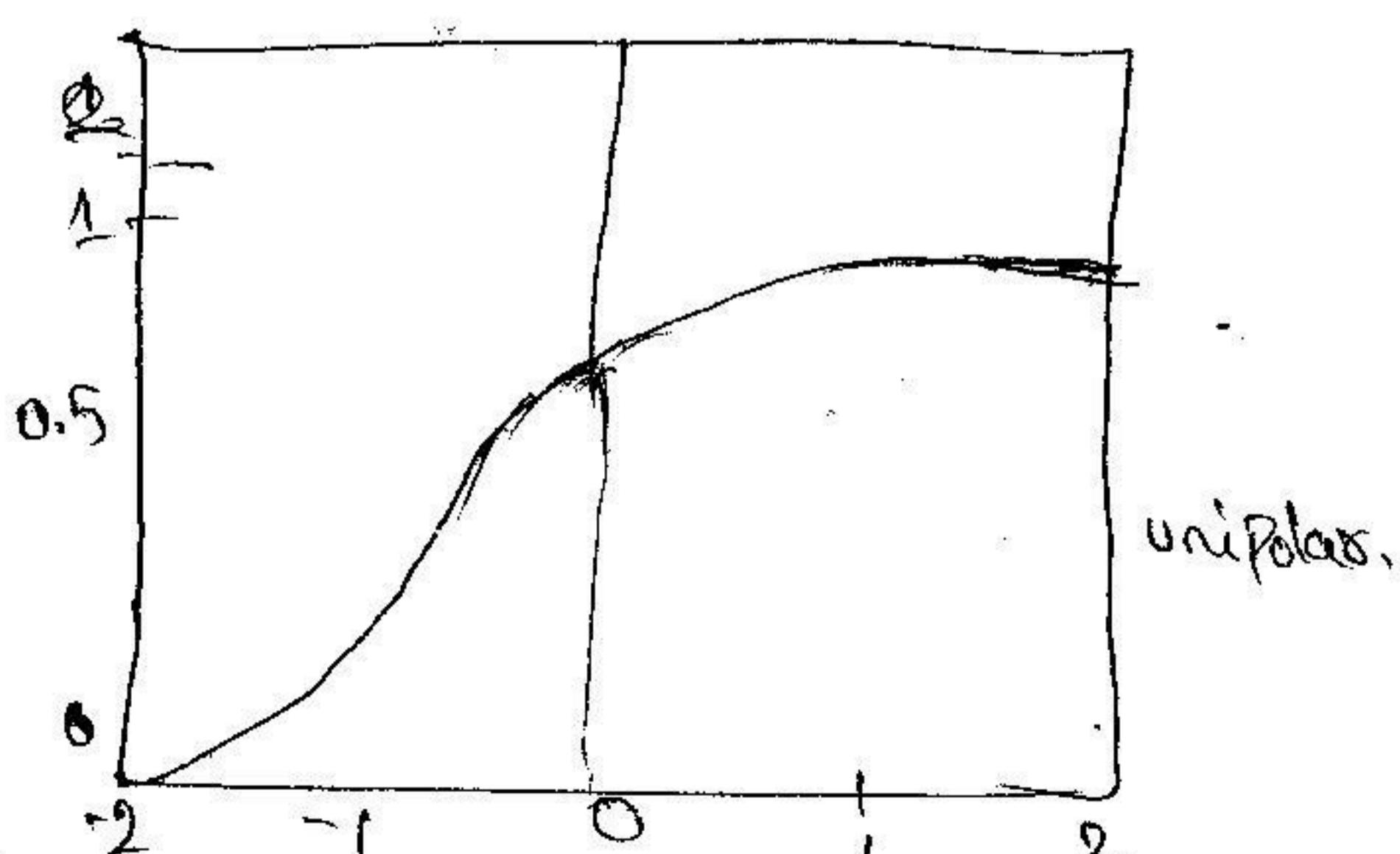
$$v = 2 \rightarrow (\text{Positive})$$

Activation function:-

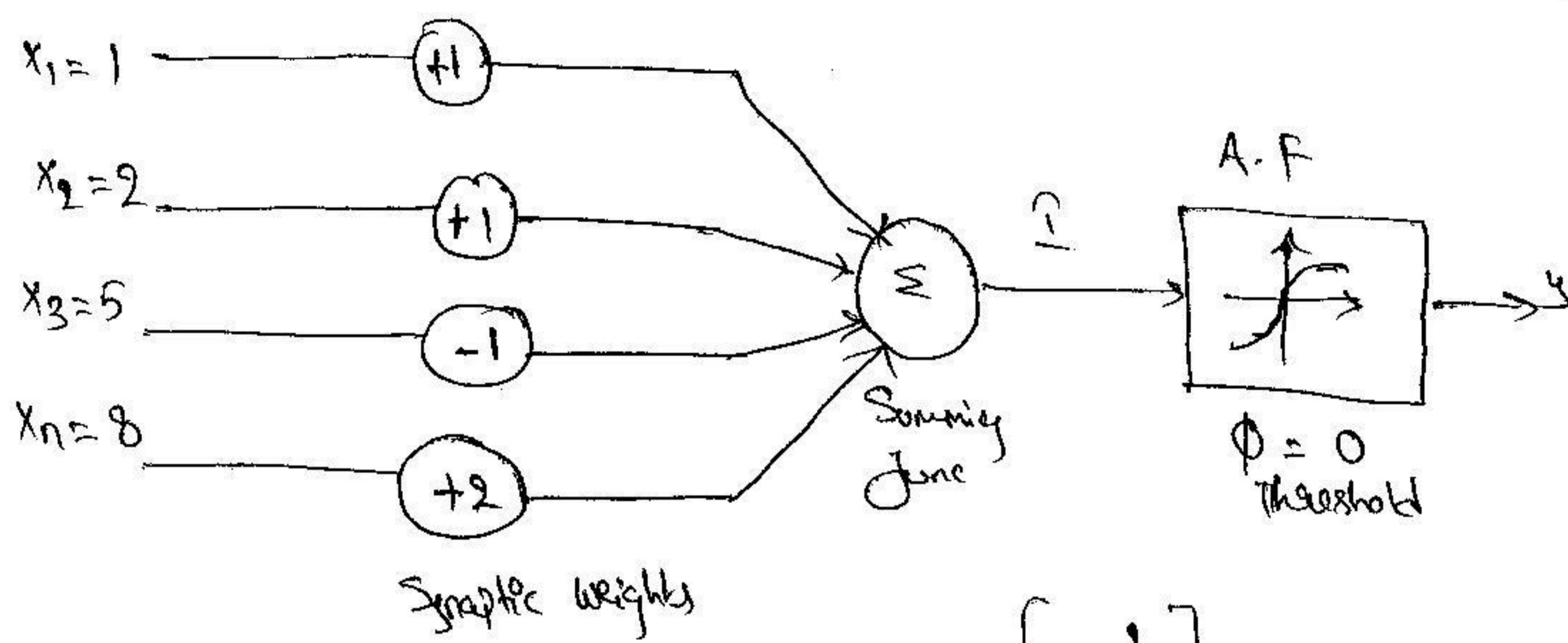
$$y = f(v) \Rightarrow v = a + b x$$



Sigmoid function  $y = \text{Sgn}(v)$



Ex-2



$$\Sigma = \mathbf{x}^T \cdot \mathbf{w} = [1 \ 2 \ 5 \ 8] \begin{bmatrix} 1 \\ 1 \\ -1 \\ 2 \end{bmatrix} = [1 \times 1 + 2 \times 1 + 5 \times (-1) + 8 \times 2] = [14]$$

With a binary A-F the op is '1'.

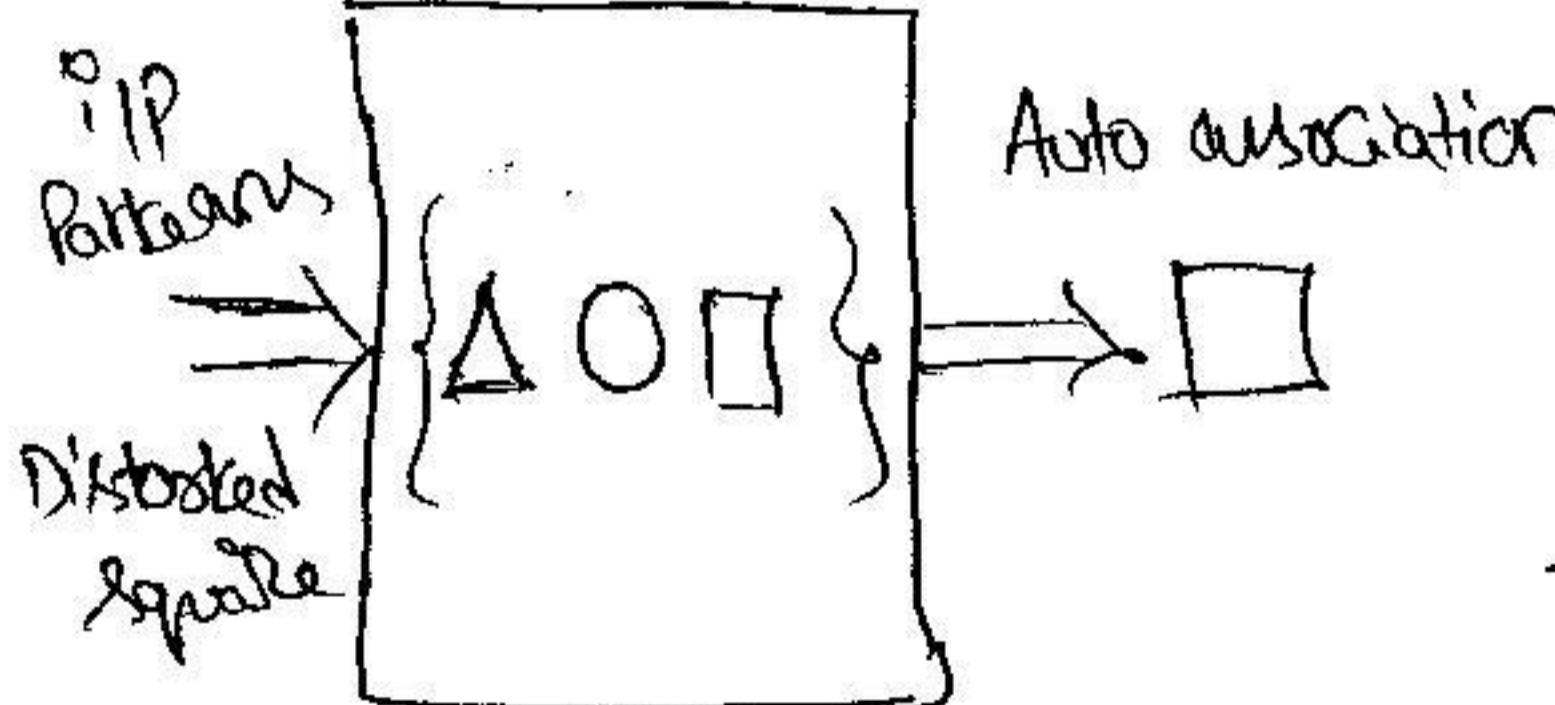
#### Neural Processing:-

The process of computation of  $O_p$  for a given  $X$  performed by the nw is known as Recall.

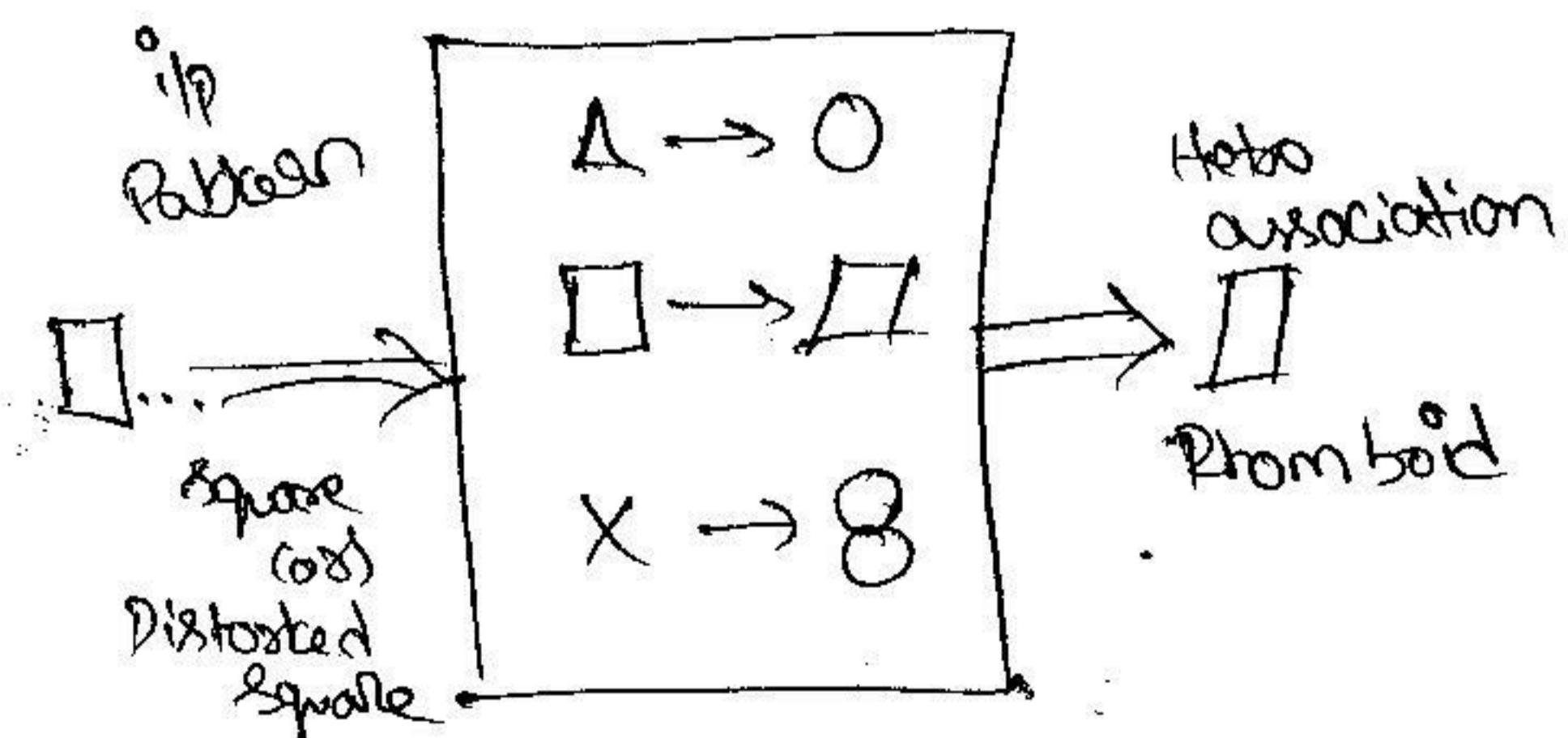
Recall is a proper processing phase for neural nw. and its objective is to retrieve information.

Recall corresponds to decoding of stored content which may have been encoded in a nw previously.

Assume that a set of patterns can be stored in nw. If network is presented with a pattern from the set it may associate the input with the stored pattern. This process is called autoassociation.



(a) Auto association.



(b) Hetero association.

→ In hetero association processing, the association b/w pair of patterns are stored.

→ A square o/p pattern generated at q/p resides in stored at o/p.  
It can be inferred at that the rhomboid and square constitute one pair of stored patterns.

→ Classification can be understood as a special case of hetero association.  
where association is now b/w (q/p & o/p pattern) and second member of hetero associative pair which is supposed to indicate the q/p's class members.

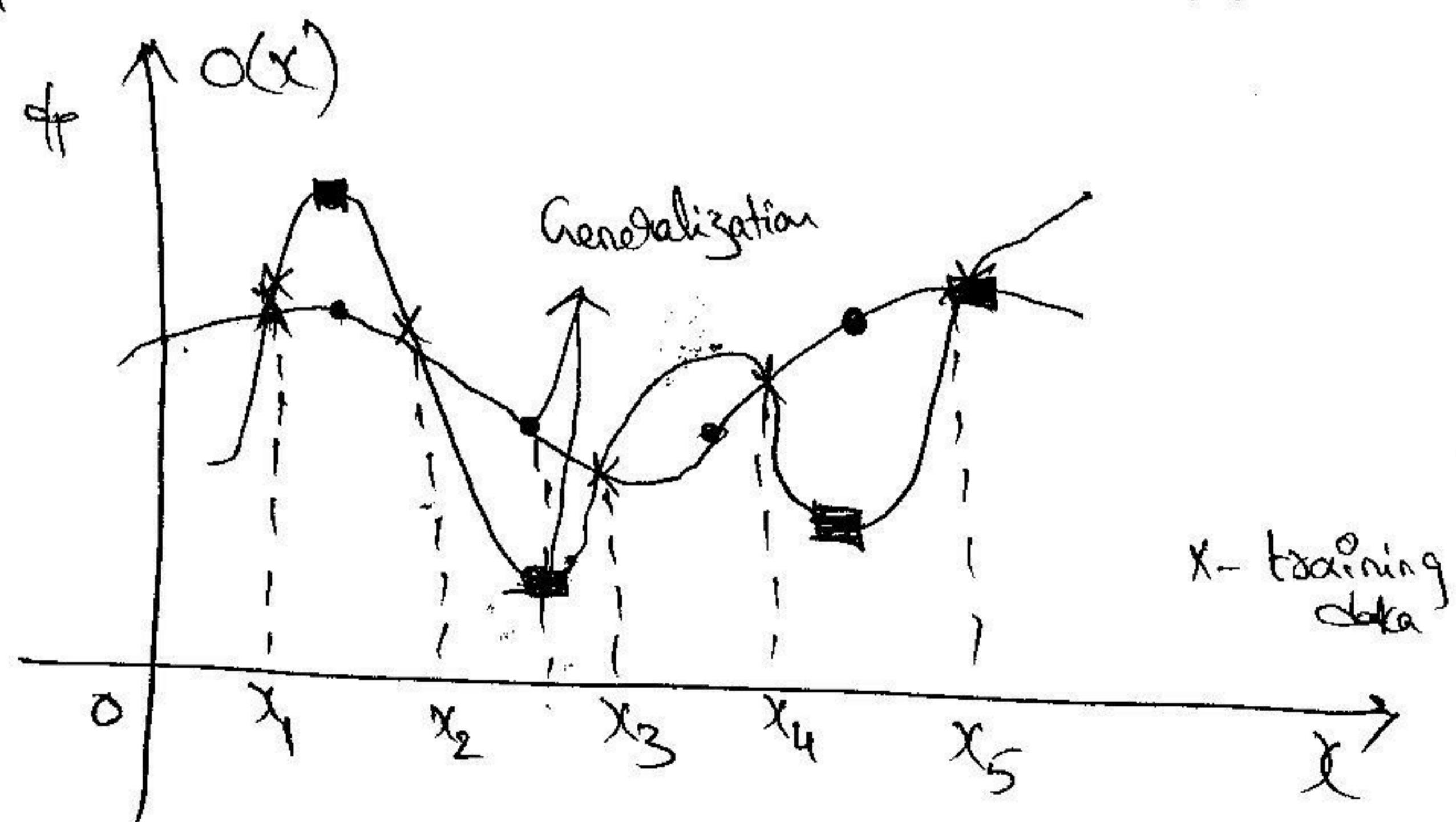
→ If the desired response is class member but the o/p pattern does not exactly corresponds to any pattern in set, the processing is called recognition.

→ When class membership for one of pattern in the set is recalled, recognition becomes identical to classification.

→ The distinct strength of refal n/w is their ability to generalize i.e.,  
→ The n/w is said to be generalized well when it sensibly interpolates o/p pattern that are new to n/w.

→ Assume that n/w has been trained using data o/p  $x_1, x_2 \dots x_5$  and o/p  $O(x)$  then.

- → Good generalization
- → Bad "



## Training methods of ANN:-

Consists 2-Parts. (1 is Activation State  
2 is dynamic synaptic weights)

Term: Short term ; long term

- ① Short term  $\rightarrow$  neural n/w is modeled by Activation State of n/w
- ② long term  $\rightarrow$  " " " Encoded by information in synaptic weights due to learning.

Def. of learning:-

is a process of neural n/w are adapted through a process of simulation by environment in which n/w is embedded.

(i) Supervised learning: - Requires pairing of each op vector with target vector  $(t)^t$

representing desired op  $y_k$ . all these are called training pair.

$\rightarrow$  usually n/w is trained over number of training pairs.

$\rightarrow$  An op vector is applied, the op vector of network is calculated and compared to corresponding target vector and difference (err) is fed back through the algorithm and tends to minimize err. n/w & weight are changed according to algorithm and tends to minimize err.

Supervised learning: - An external signal known as teacher controls learning and incorporate information.

(ii) Unsupervised learning: - no external signal (teacher) is used in learning and incorporate information.

$\rightarrow$  unsupervised learning is more plausible model of training in biological system because the neural n/w relies upon both internal and local information.

$\rightarrow$  unsupervised learning is more plausible model of training in biological system if requires no target vector for the op. and hence no confusion to predetermined ideal responses. The training set consists of solely of op vectors.

$\rightarrow$  The training algorithm modifies n/w weights to produce op vector to produce in same pattern.

## Types of basic learning mechanism:- (EMHC B)

- (i) Error Collection mechanism
- (ii) Memory Based learning "
- (iii) Hebbian learning
- (iv) Competitive learning
- (v) Boltzmann learning.

### (a) Error collection learning:-

(Recognition)

A n/w is trained so that application of set of ip produces desired op's.

Each such ip op is referred to vector.

Training is accomplished by sequentially applying ip vectors while adjusting

n/w weights according to predetermined procedure.

During training the n/w weight gradually converge to values such that

each ip vector produces the desired op vector.

Let us consider:- a time index , then the error at  $n^{th}$  instant case obtained as

function of actual op of op layer be  $o_k(n)$  for  $k^{th}$  neuron and desired

target response (or) target op of n/w be  $d_k(n)$  for  $k^{th}$  node.

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$\therefore$  op layer send signal denoted by  $e_k(n)$ .

$$e_k(n) = d_k(n) - o_k(n) \rightarrow (a)$$

The error signal  $e_k(n)$  is used to adjust the synaptic weight of neuron  $k$

The Cost function (or) Performance index  $E(n)$  is given as

$$E(n) = \frac{1}{2} \cdot e_k^2(n). \rightarrow (b)$$

$\rightarrow$  This kind of learning based on many learning rules which are

desired in order to update the weights of n/w. like

(i) Reception learning rule

(ii) Delta learning rule

(iii) Window Hoff learning rule.

(i) Perception learning rule :- (Rule is applicable only for binary neuron response) (1-36)  
 Learning Signal known as error signal defined as difference b/w the desired and actual neuron response

$$e_i = d_i - O_i \rightarrow \textcircled{a}$$

where  $O_i = \text{Sgn}(w_i \cdot x)$  and  $d_i \rightarrow \text{desired response}$

weight adjustments in this method

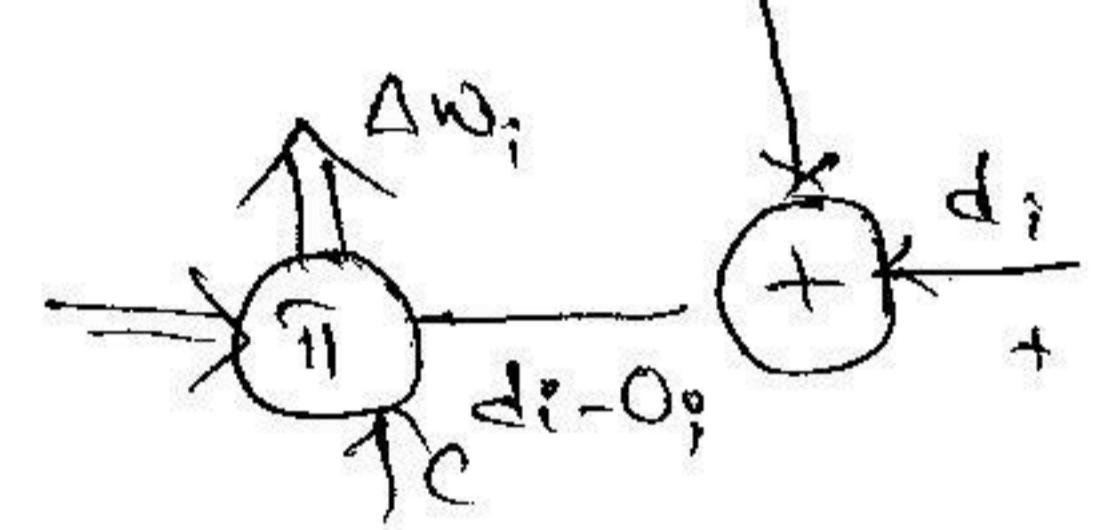
$\Delta w_i$  and  $\Delta w_{ij}$  are defined as



$$\Delta w_i = c [d_i - \text{Sgn}(w_i \cdot x)] x \rightarrow \textcircled{b}$$

$$\Delta w_{ij} = c [d_i - \text{Sgn}(w_i \cdot x)] x_j \rightarrow \textcircled{c}$$

for  $j = 1 \rightarrow n$ .



the relationship  $\textcircled{b} \Leftrightarrow \textcircled{c}$  express the rule for bipolar binary case.  
 under this rule weights are adjusted if and only if incorrect.

Since the desired response is either 1 (or) -1 the weight adjustment reduces

$$\text{to } \Delta w_i = \pm 2c x$$

$c \rightarrow \text{learning constant}$

(ii) Delta learning rule :-

→ introduced for training a neural nw.

→ This learning rule is applicable only for continuous activation function.

→ This rule can be derived from condition of least square error Ls

'd<sub>i</sub>' and 'O<sub>i</sub>'

→ The squared error may be defined as

$$E = \frac{1}{2} (d_i - O_i)^2 = \frac{1}{2} [d_i - f(w_i \cdot x)]^2 \rightarrow \textcircled{a}$$

→ The error gradient vector with respect to  $w_i$  may be written as

$$\nabla E = -(d_i - O_i) f'(w_i \cdot x) x \rightarrow \textcircled{b}$$

The elements of gradient vector are  $\frac{\partial E}{\partial w_{ij}} = -(d_i - O_i) f'(w_i \cdot x) x_j$  for  $j = 1 \rightarrow n$ .  $\hookrightarrow \textcircled{c}$

In order to minimize the cost function  $E$ , it requires weight changes to be in 've gradient direction.

$$\therefore \Delta w_i = -\eta \nabla F \rightarrow (d)$$

where  $\eta$  is a positive constant

from eq (b) & (d) we write obtain

$$\Delta w_i^T = \eta (d_i - o_i) f'(net_i) x \rightarrow (e)$$

$\Delta w_i^T = \eta (d_i - o_i) f(w_i x) \cdot x$  and single weight adjustment becomes

since  $net_i = w_i x$  and  $\Delta w_i = \eta (d_i - o_i) f'(net_i) x_j$  for  $j=1, 2, \dots, n$

$$\Delta w_{ij} = \eta (d_i - o_i) f'(net_i) x_j$$

for this we, the weights can be initialized to any value.

(iii) Window-Hoff learning rule:  
this rule is independent of activation function of neurons. It minimizes the squared error b/w the desired  $d_i$  and the neurons activation

$$value \quad net_i = w_i x$$

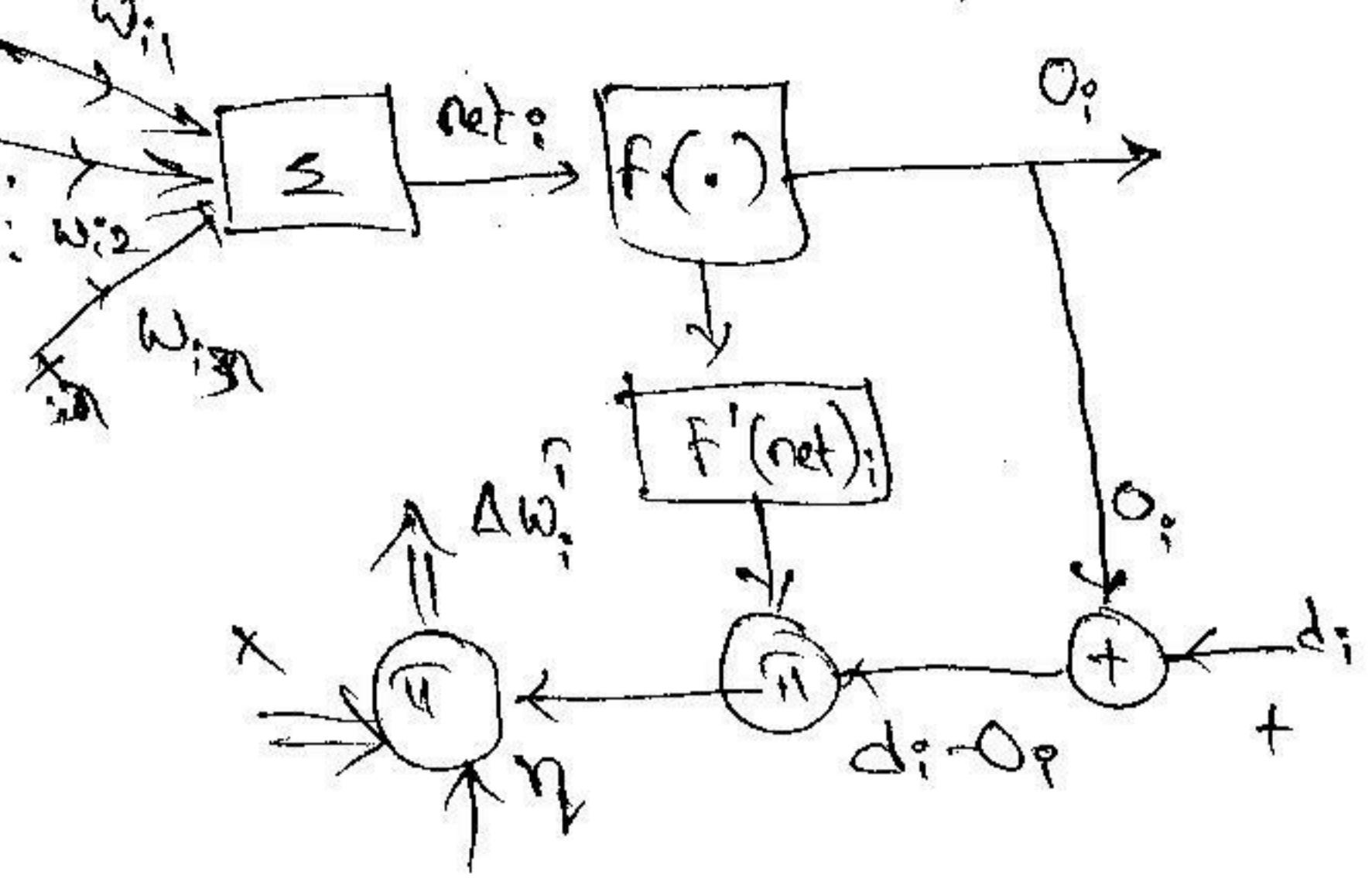
$$\therefore error \quad e_i = d_i - o_i = d_i - w_i x \rightarrow (a)$$

$$using \quad learning \quad rule \quad change \quad in \quad weight \quad vector \quad \Delta w_i = \eta (d_i - w_i x) x \rightarrow (b)$$

single weight adjustment written as  $\Delta w_i = \eta (d_i - w_i x)$  for  $i=1 \rightarrow n \rightarrow (c)$

this rule is special case of delta learning rule i.e.  $f(net) = net$  and

$f'(net) = 1$  this rule is also called as least mean square (LMS) learning rule.



$L \rightarrow L$

(1)

(b) Memory based learning:-

Let consider 1-pattern of  $\bar{x}$  the  
 $\bar{x}$  is vector and of o/p of  $\bar{x}$   
is  $d_i \rightarrow$  vector.

$$\{\bar{x}_i, d_i\}_{i=1}^n \rightarrow 1\text{-pattern}$$

$$\{\bar{x}_i\}_{i=1}^n =$$

det an o/p be  $\bar{x}_{test}$  and what is o/p of system. out of these  
vectors  $\bar{x}_{test}$  searches for nearest vector among  $m$  vector o/p.  
if 1 of is closest among 'm'

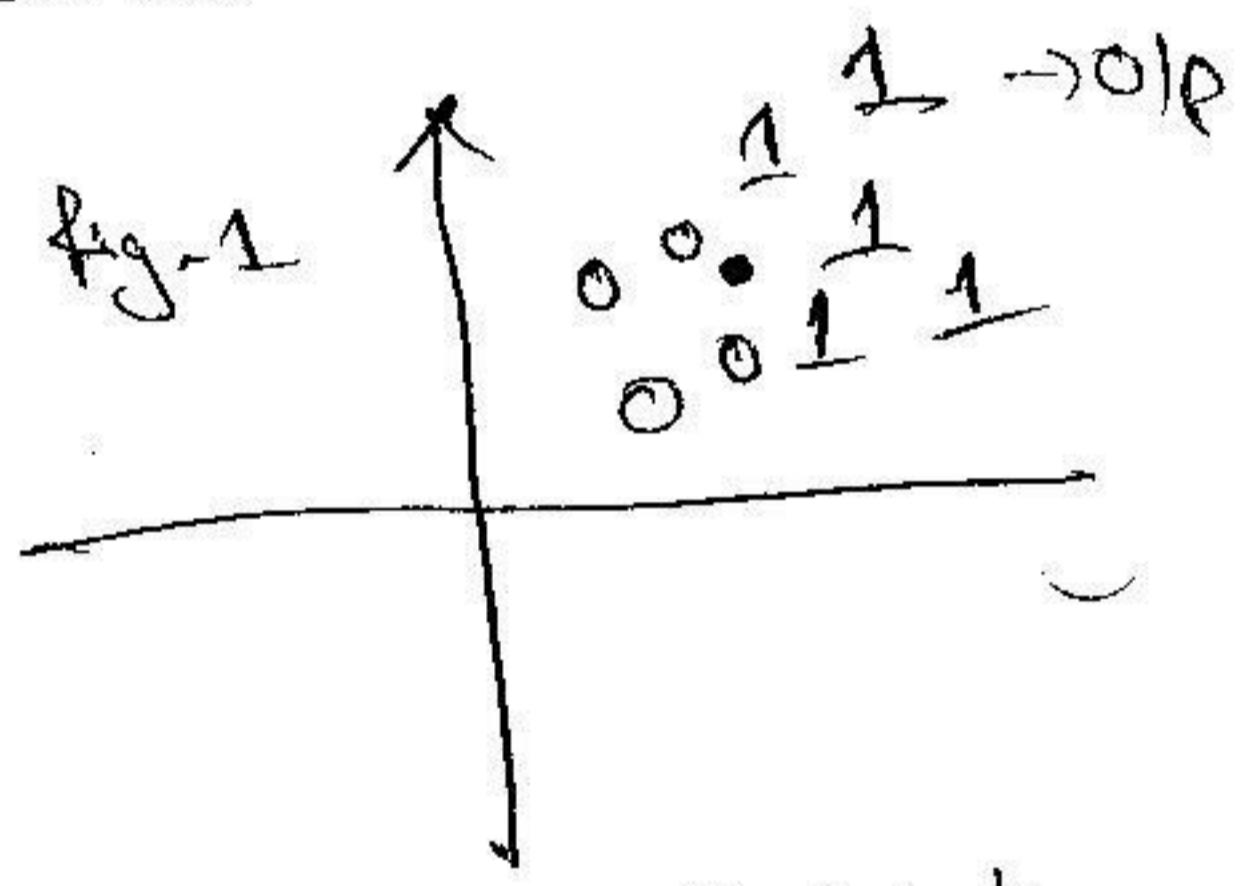
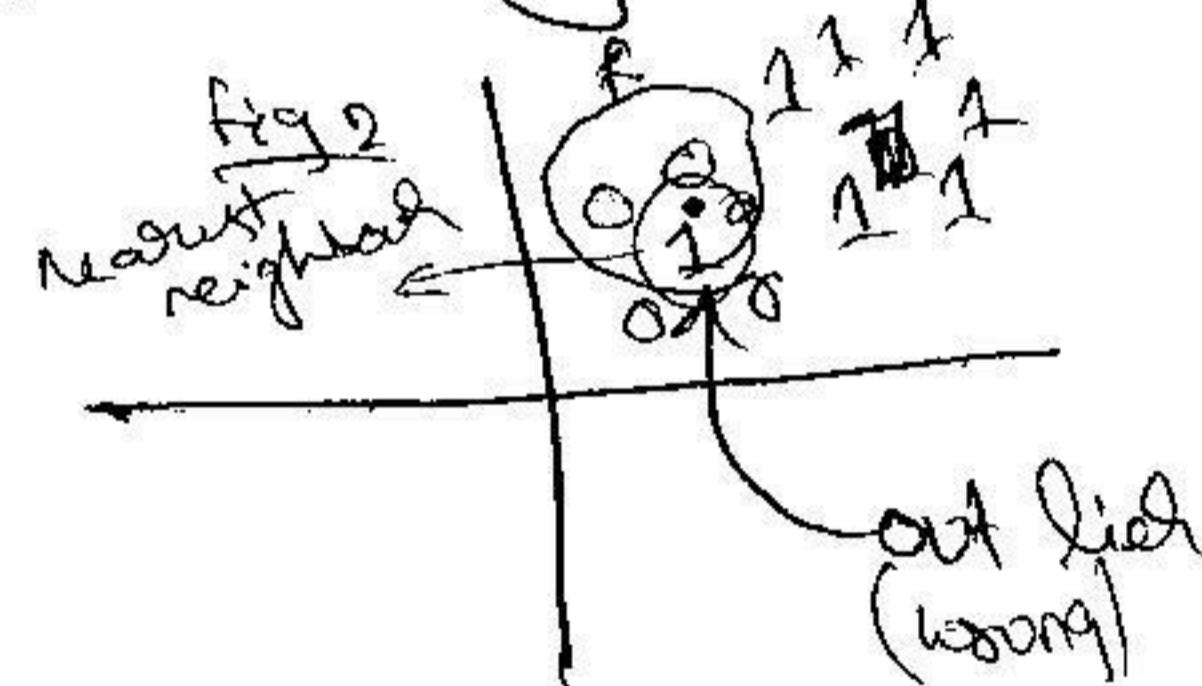
$$x'_n \in \{\bar{x}_1, \bar{x}_2, \bar{x}_3, \dots, \bar{x}_m\}$$

nearest neighbor of  $\bar{x}_{test}$  if minimum of euclidean distance  $d_{eu}$

$d(\bar{x}_i \& \bar{x}_{test})$  is searched by varying  $i = 1 \rightarrow N$

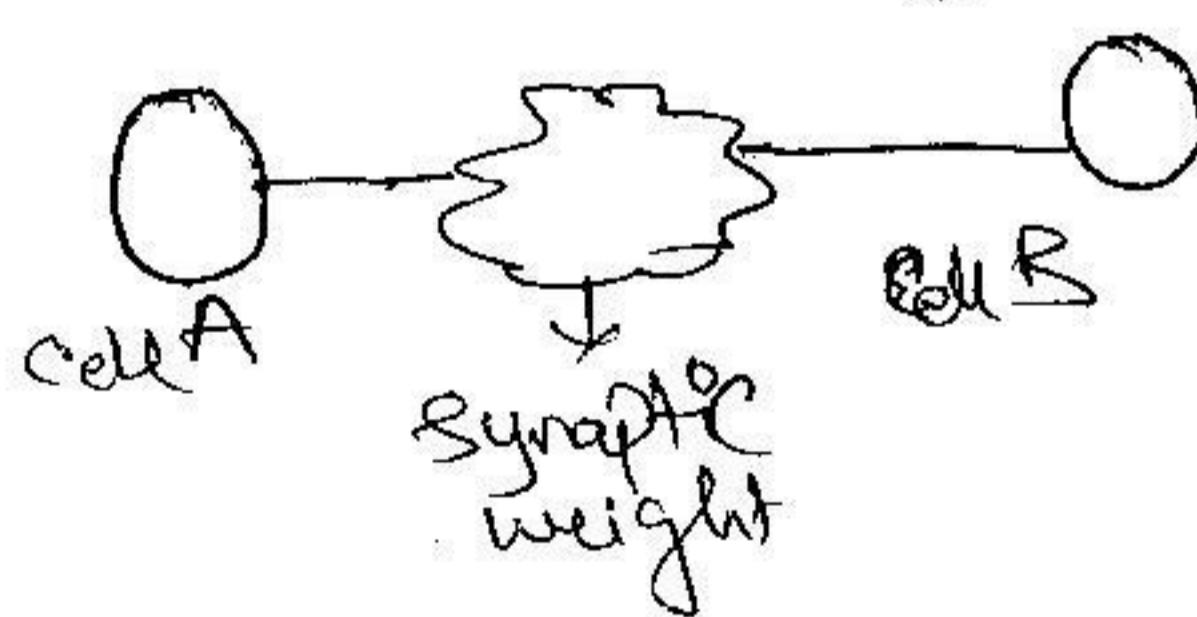
$$\min d(\bar{x}_i, \bar{x}_{test}) = d(x'_n, \bar{x}_{test})$$

let us consider patterns are of form  
( $K$ -nearest neighbor classification)



A's efficiency in firing B metabolic changes happen so that efficiency of A firing B increased.

(c) Hebbian learning:-



Hebbian Synapses  $\rightarrow$  3-character

a) very much time dependent (Synchronous must be)

b) local

c) strongly interactive

If two neurons on either side of synapse (connection) are activated simultaneously (ie synchronously) then the strength of that synapse is selectively increased (1-39)

If two neurons on either side of a synapse are activated asynchronously, then that synapse selectively weakened (or) eliminated.

→ Hebbian synapse defined as synapse that uses a time dependent, highly local, and strongly interactive mechanism to increase synaptic efficiency as a function of correlation b/w presynaptic & post synaptic activity.

Key Properties:-

- (i) time dependent mechanism:- Refers modification in a hebbian synapse depend on exact time of occurrence of presynaptic & post signals.
- (ii) local mechanism:- A synapse is transmission site where information-bearing signals are in spatiotemporal contiguity.
- (iii) interactive mechanism:- change of hebbian synapse depends on signals on b.s of synapse. ie hebbian learning depends on a true interaction b/w presynaptic & post synaptic signals in the sense that we can not make a prediction from either one of these two activities by itself.
- (iv) conjunctive (or) correlational mechanism:- According to one of hebbian interpretation the co-occurrence of pre & post synaptic signals is sufficient to produce synaptic modification. for another interpretation of hebbian learning one may think of interactive mechanism characterizing a hebbian synapse an statistical term.

(v) Mathematical relation:- Consider a synaptic weight  $w_{kj}$  of neuron k with pre & post synaptic signals by  $x_j$  and  $o_k$  respectively.

∴ the change of weight is  $\Delta w_{kj}(n) = f(o_k(n), x_j(n)) \rightarrow (a)$

↓  
Post synaptic      ↓  
                        Pre synaptic

Simple form of Hebbian is given as

$$\Delta w_{kj}(n) = \eta \cdot o_k(n) \cdot x_j(n) = \sum_{j=1}^n f(w_j \cdot x_j(n)) \quad \text{for } j=1 \rightarrow n. \rightarrow (b)$$

where  $\eta \rightarrow$  learning rate & positive constant.  
 { the ly correlated leads synaptic strengthening.  
 -vely " (as) on correlated synaptic weakening } (binary Activation)

(iii) Classification of synaptic modification:-

- (i) Hebbian :- synapse increases its strength with the collation
- (ii) Anti hebbian :- " " " but " -ve "
- (iii) non - "  :- which doesn't involve hebbian mechanism of either kind.

(iv) Covariance hypothesis :- In this hypothesis Pre- & Post Synaptic signals are replaced by deviation of Presynaptic & Post synaptic signals from their respective average values over a certain time interval.

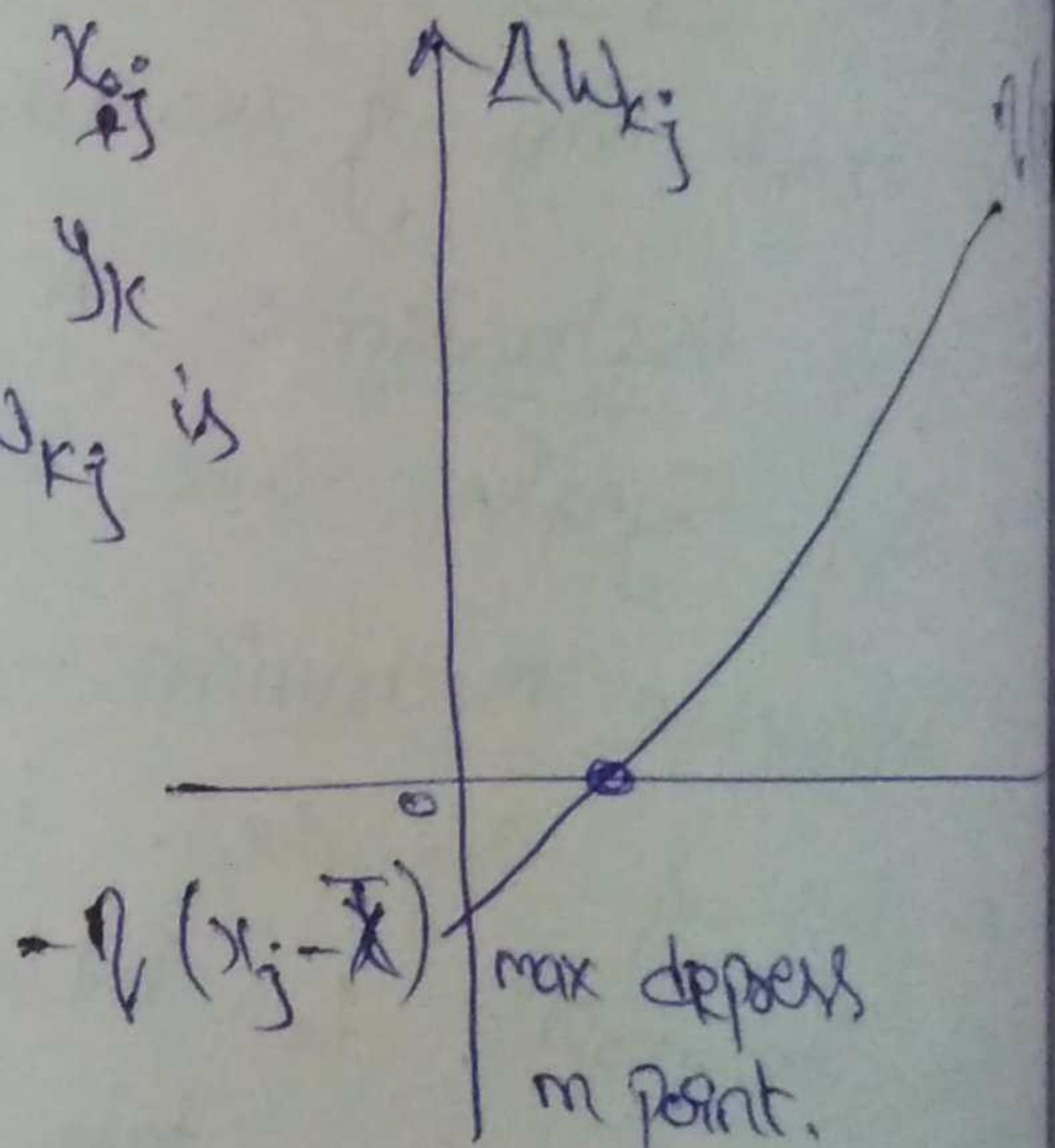
Let  $\bar{x} =$  time averaged value of  $x_j$

$\bar{y} =$  " " " " "  $y_k$

The adjustment applied to synaptic weight  $w_{kj}$  is

$$\Delta w_{kj} = \eta (x_j - \bar{x})(y_k - \bar{y})$$

$\therefore$  the hebbian rule represents poorly feedforward, unsupervised learning.



(i)  $w_{kj} \uparrow$  if  $x_j > \bar{x}$  &  $y_k > \bar{y}$

(ii)  $w_{kj} \downarrow$  if either  $\rightarrow$  (a)  $x_j > \bar{x}$  &  $y_k < \bar{y}$   
                                  (b)  $x_j < \bar{x}$  &  $y_k > \bar{y}$

(iii)  $w_{kj} = 0$  no updating (if it's within the average)  $\rightarrow$  no change in weights.

## Competitive learning :-

(2-4)

In competitive learning only a single o/p neuron is active at any one time.

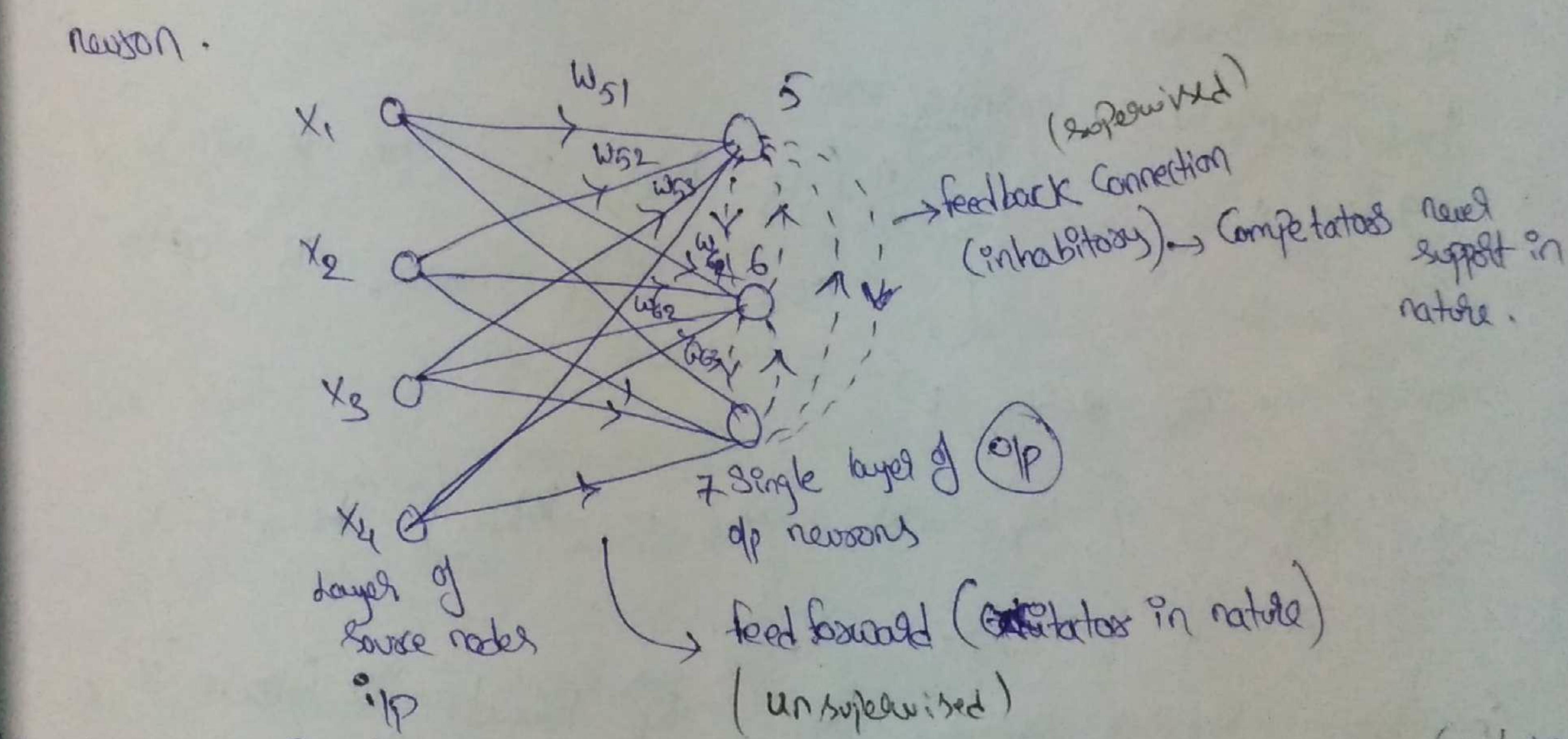
→ A set of neurons that are all the same except for some randomly distributed synaptic weights, which therefore respond differently to given set of ip patterns

→ A limit imposed on strength of each neuron

→ A mechanism that permits 2-neurons to compete for the right

to respond to a given subset of o/p such that only one o/p neuron (or) only one neuron per group is active (ie on) at a time.

→ The neuron that wins the competition is called a winner take all neuron.

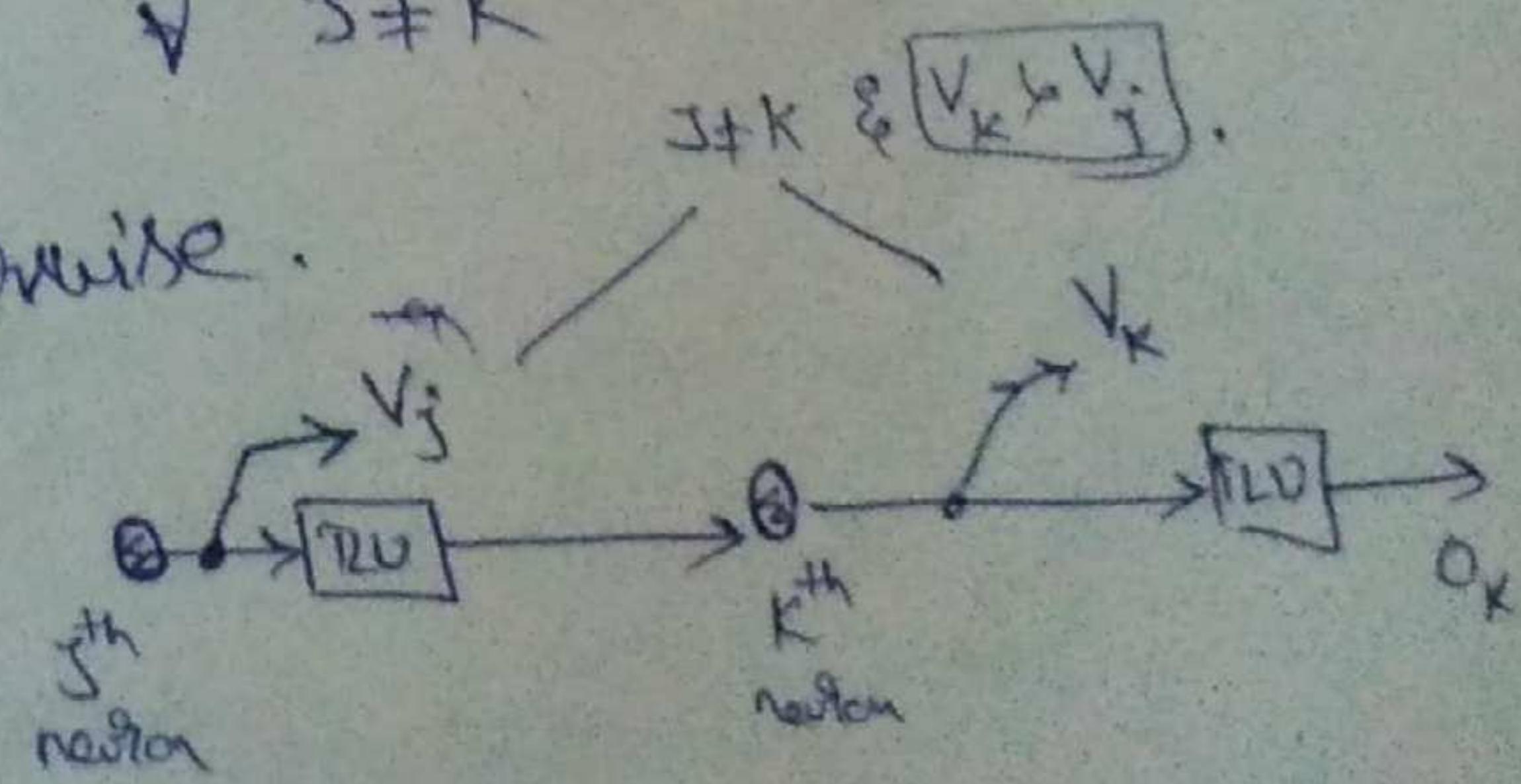


### (i) Mathematical Relation:-

for a neuron 'k' to be winning neuron it's induced local field \$V\_k\$ for a specified ip pattern 'x' must be the largest among all neurons in the

→ the ip signal \$J\_k\$ of winning neuron 'k' is set equal to one and that the ip signal of all the neurons that lose the competition are set equal to zero

$$J_k = \begin{cases} 1 & \text{if } V_k > V_j \quad \forall j \neq k \\ 0 & \text{otherwise.} \end{cases}$$



where local field  $v_k$  represents combined action of all forward and feedback o/p to neuron 'k'.  
 Let  $w_{kj}$  denote synaptic weight connecting o/p node  $j \rightarrow k$  neuron.  
 If each neuron is allotted a fixed amount of synaptic weight (i.e. all synaptic weights are positive) which is distributed among all o/p nodes i.e.

$$\sum_j w_{kj} = 1 \text{ for all } k.$$

→ The neuron then learn by shifting weights from inactive to

input nodes. to a particular o/p pattern, no learning

→ If neuron does not respond place in neuron

learn competitive learning rule :-

$$\Delta w_{kj} = \begin{cases} \eta(x_j - w_{kj}) & \text{if neuron } k \text{ wins} \\ 0 & \text{if neuron } k \text{ loses} \end{cases}$$

moving synaptic vector 'k'.

$$\vec{w}_k = [w_{k1}, w_{k2}, w_{k3}, \dots, w_{km}] \quad k \rightarrow m-1$$

$$\vec{x} = [x_1, x_2, \dots, x_m]$$

Competitive learning defines moving  $\vec{w}_k$  towards o/p pattern  $\vec{x}$ ,

(ii) Winner Take All learning rule :-

It is example of competitive learning and is used for unsupervised training (feed forward) network.

→ The winner take all learning is used for learning statistical properties of o/p's

→ The learning is based on premise that one of neuron say the  $m^{th}$ , has the maximum response due to o/p  $\vec{x}$ .

This neuron is declared the winner. As a result of winning

weight vectors  $W_m$ .

(I-43)

$$W_m = [w_{m1}, w_{m2}, \dots, w_{mn}]$$

The weights adjusted given as unsupervised learning step.

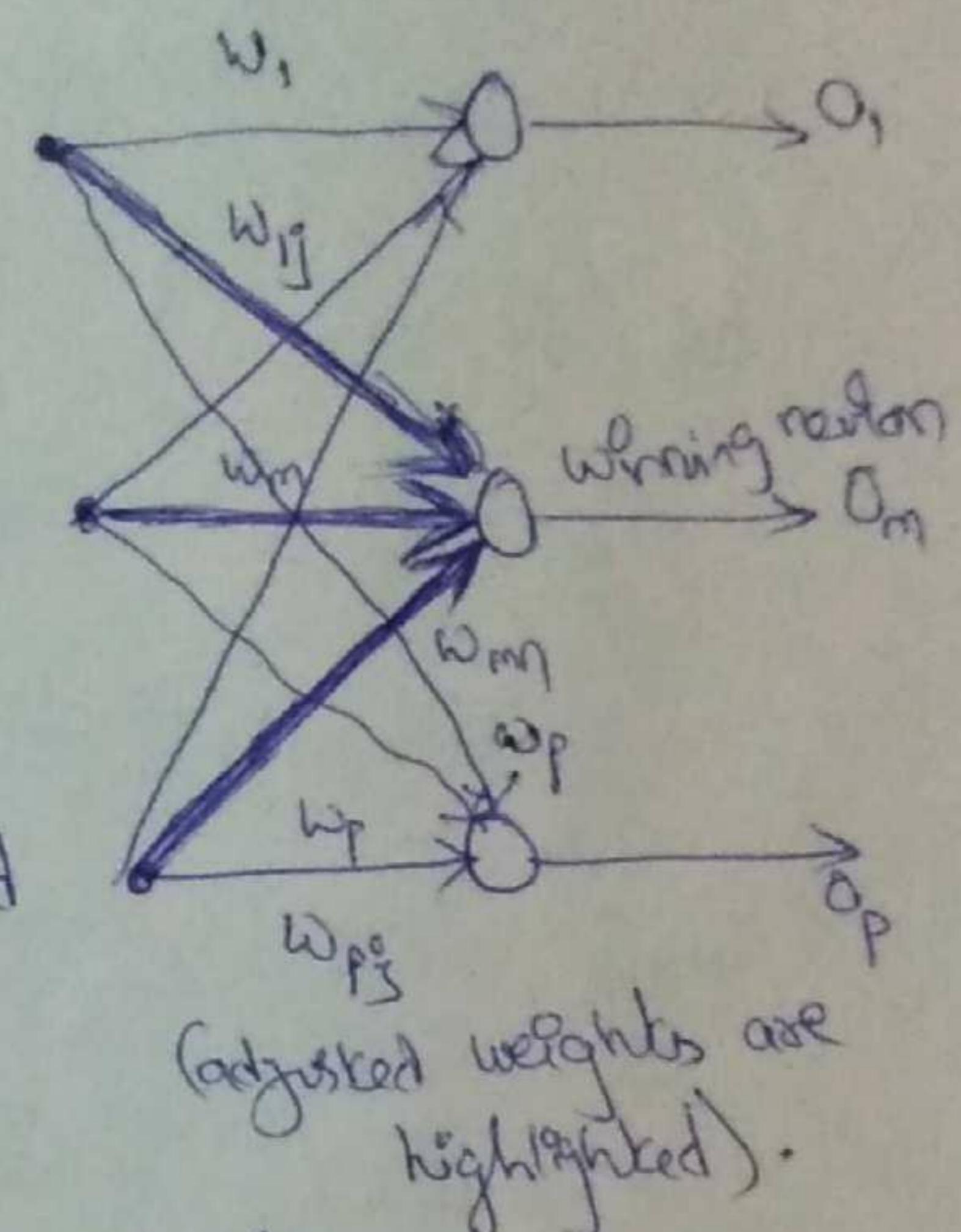
Computed as  $\Delta W_m^T = \gamma (x - W_m^T) \rightarrow (b)$

for individual weight adjustment  $\Delta w_{mj} = \gamma (x_j - w_{mj}) \text{ for } j=1 \rightarrow n \rightarrow (c)$

$\gamma \rightarrow$  small learning constant.

winner selection is based on criterion of maximum activation among all p-neurons participating in a competition.

$$w_m x = \max(w_i x) \quad i=1, 2, \dots, p.$$



### (e) Boltzmann learning:-

It is a statistical learning algorithm derived from ideas rooted in statistical machines.

In Boltzmann machine the neurons form a recurrent structure and they operate in a binary manner, denoted as +1/-1

↑ ↓  
ON OFF

→ In Boltzmann machine no hidden layer is present.

→ This machine is characterized by an energy function E.

$$E = -\frac{1}{2} \sum_j \sum_{k \neq j} w_{kj} x_k x_j \rightarrow (a)$$

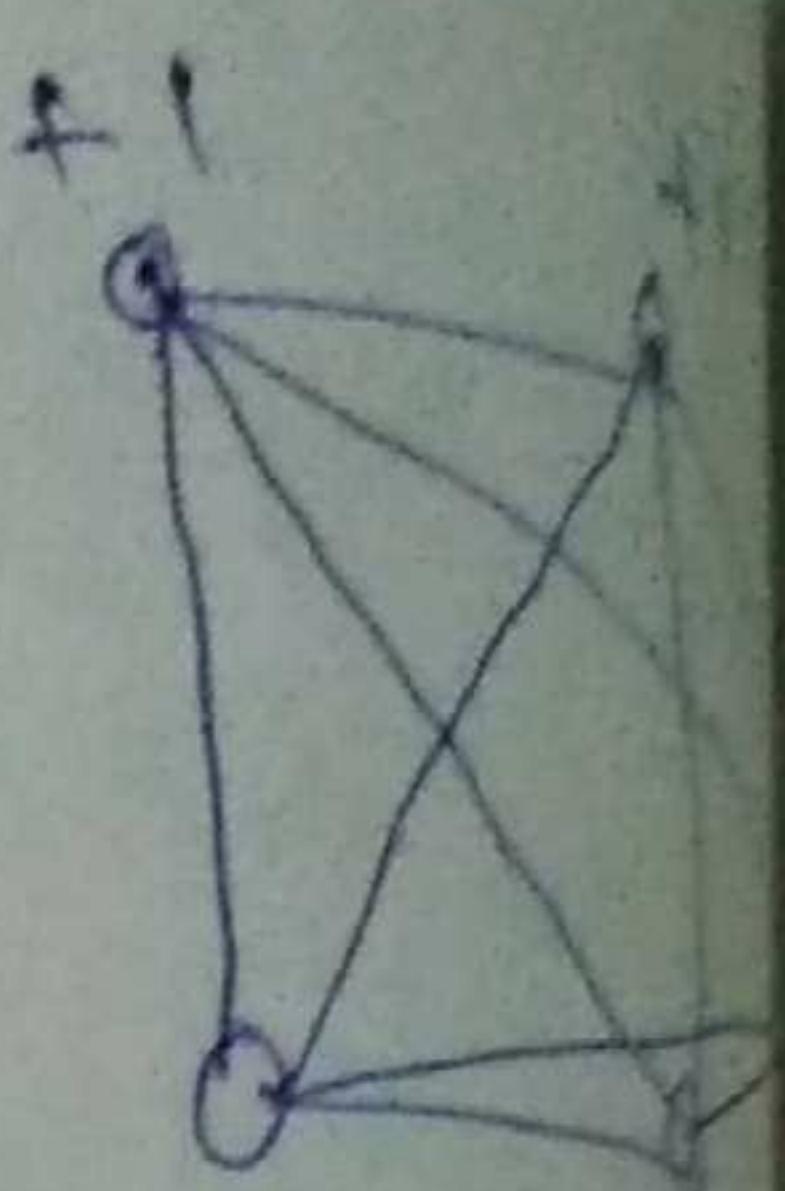
$x_j$  is state of neuron j and  $w_{kj}$  is synaptic weight connecting neuron j to k. The fact that  $j \neq k$  means only one of neurons in machine has self feedback.

→ Boltzmann learning defined particular state of neuron we will have energy function.

If : Pickup Particular neuron 'k' change state from  $(-1) \rightarrow (+1)$  it is said to be recomplete

$\therefore$  The Probability of neuron flip 'k' is  $x_k$

$$P(x_k \rightarrow -x_k) = \frac{1}{1 + e^{(-\Delta E_k/T)}}$$



$\Delta E_k \rightarrow$  Change of Energy resulting from such a flip  $(-1) \rightarrow (+1)$  (or)  $(+1) \rightarrow (-1)$

$\gamma \rightarrow$  Pseudo temperature.

The neurons of Boltzmann machine Partition in two ways

- 1) visible  $\rightarrow$  Provide interface b/w network (Available at op)
- 2) hidden  $\rightarrow$  It operates freely (in variable at op).

There are 2-modes of operation.

$\rightarrow$  clamped condition in which visible neurons are all clamped specific state determined by environment. (either  $+1/-1$ )

$\rightarrow$  free running condition in which all neurons (visible & hidden) allowed to operate freely.

Let  $\rho_{kj}^+$  = Correlation between neuron  $k$  and neuron  $j$  in clamp condition  
are binding qualities of  $(+1 \text{ (or)} -1)$

$\rho_{kj}^-$  = Correlation b/w neurons in free running condition.  
 $\Delta w_{kj} = \eta (\rho_{kj}^+ - \rho_{kj}^-) \beta_{jk}$