

Neural Networks & fuzzy logic

1(a)

Ans.

Applications of Neural Networks :-

(2 Marks)

- (i) character Recognition :-
 - (ii) Image compression :-
 - (iii) Stock market prediction :-
 - (iv) Travelling Salesman Problem :-
 - (v) Medicine, electronic nose, security and loan applications :-
 - (vi) Miscellaneous applications :-
- } 1 mark } 1 m

1(b)

Ans.

Training methods of Artificial neural network :-

(2 Marks)

- 2 - types
 - i) Supervised learning definition
 - ii) Unsupervised learning definition.
- } 1m } 1m

→ training methods of ANN consists of two Part

(a) Activation state (b) dynamic Synaptic weights.

which are further represented as short term and long term.

short term :- neural nw is modeled by activation state of nw.

long term :- neural nw is encoded by information in synaptic weights due to learning.

1(c):-

Delta learning Rule :-

(2 Marks)

Ans Delta learning rule introduced for training neural network.

which is applicable only for continuous activation function.
→ This rule can be derived from condition of least square error between desired d_i and actual \hat{o}_i

→ the squared error defined as

$$E = \frac{1}{2} (d_i - \hat{o}_i)^2 = \frac{1}{2} [d_i - f(w_0 x)]^2 \rightarrow (a)$$

the error gradient vector w.r.t w_0 written as

$$\nabla E = -(d_i - \hat{o}_i) f'(w_0 x) x \rightarrow (b)$$

∴ Elements of gradient vector are $\frac{\partial E}{\partial w_{0j}} = -(d_i - \hat{o}_i) f'(w_0 x_j) x_j$ for $j=1 \rightarrow n$.

∴ The weight adjustment given by

$$\Delta w_{0j} = l (d_i - \hat{o}_i) f'(w_0 x_j) x_j \text{ for } i=1 \rightarrow n.$$

1(d):-

Perception:-

(2 Marks)

Ans → the perception is a model of artificial neuron, which is a single layer ~~retror~~ neural network whose weights and bias could be trained to produce a correct target (or) desired response.

→ Perception is a program that learn concepts i.e it can learn to respond with (true (1)) and (false (0)).

→ The training technique used is called perception learning rule.

(e) Pattern representations:-

Classification Problem

{ Spatial Pattern }
Temporal Pattern.

1m

Example:- of Spatial Pattern:-

Pictures, Video Images, Fingerprint, weather map.

Ex:- of Temporal Patterns:-

Speech signals, signal vs time recorded by sensors, etc.

1(e):- Ane.

Recurrent Auto associate Memory:- (2 Marks)

→ Associative memory can be implemented using networks with or without feedback but in recurrent autoassociate the simplest kind of feedback that the output of a network is used repetitively as a new IP until the pocket converges.

→ The function of an associative memory is to recognize previously learned input vectors; even in the case where some noise has been added. in finding cluster centroids in input space.

1(f):-

Ane

Spatio-Temporal Patterns:-

(2 Marks)

A spatio temporal neural network differs from other neural networks in two ways

(i) Propagation delay along links and sequence play an important role in the computation carried out by the network.

(ii) The representational state of network depends not only on which nodes are firing, but also on relative firing times of nodes.

→ The representational significance of node varies with time and depends on firing state of other nodes. The use of

Recurrence and multiple links with variable Propagation delay provides rich mechanism for integration, Context sensitivity, feature extraction and pattern recognition. Recurrent links enable node to integrate and differentiate ifp. detect the onset of features and measure their duration. At the same time, multiple links with variable propagation delays between node serves as a short term memory and allow the network to maintain context over a window of time.

1(g) i) Classical sets operations:-

$$\text{Union} \rightarrow A \cup B = \{x | x \in A \text{ or } x \in B\}$$

$$\text{Intersection} \rightarrow A \cap B = \{x | x \in A \text{ and } x \in B\}$$

$$\text{Complement} \rightarrow \bar{A} = \{x | x \notin A, x \in X\}$$

$$\text{Difference} \rightarrow A \setminus B = \{x | x \in A \text{ and } x \notin B\}$$

2 marks

1 mark

with Venn diagrams

ii) Fuzzy sets operations:-

$$\text{union} \rightarrow \mu_{A \cup B}(x) = \mu_A(x) \vee \mu_B(x)$$

$$\text{Intersection} \rightarrow \mu_{A \cap B}(x) = \mu_A(x) \wedge \mu_B(x)$$

$$\text{Complement} \rightarrow \mu_{\bar{A}}(x) = 1 - \mu_A(x).$$

1 mark

with Venn diagram

1(h)
Ans.

Fuzz logic :-

2 marks

Definitions ; logical connectives.

↳ (i) Negation } notation.

↳ (ii) Disjunction

↳ (iii) Conjunction

↳ (iv) Implication

1 (i)

fuzzification :-

→ 2marks

Definition and explanation →

1 mark

membership function representing with example → 1m

1 (ii) :-

Problem:-

2 marks

Given $x = (a, b, c, d)$

$y = (1, 2, 3, 4)$

$$\tilde{A} = \{(a, 0), (b, 0), (c, 0.6), (d, 1)\} \quad ; \quad \tilde{B} = \{(1, 0.2), (2, 1), (3, 0.8), (4, 0)\}$$

$$\tilde{C} = \{(1, 0), (2, 0.4), (3, 1), (4, 0.8)\}$$

$$(i) \text{ If } x \text{ is } \tilde{A} \text{ THEN } y \text{ is } \tilde{B} = (\tilde{A} \times \tilde{B}) \cup (\tilde{A}^c \times y)$$

By using Zadeh's notation.

$$\tilde{A} = \left\{ \frac{0}{a} + \frac{0}{b} + \frac{0.6}{c} + \frac{1}{d} \right\} = \left\{ \frac{1}{a} + \frac{1}{b} + \frac{1}{c} + \frac{1}{d} \right\}$$

$$\tilde{B} = \left\{ \frac{0.2}{1} + \frac{1}{2} + \frac{0.8}{3} + \frac{0}{4} \right\} = \left\{ \frac{1}{1} + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} \right\}$$

$$\tilde{C} = \left\{ \frac{0}{1} + \frac{0.4}{2} + \frac{1}{3} + \frac{0.8}{4} \right\} = \left\{ \frac{1}{1} + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} \right\}$$

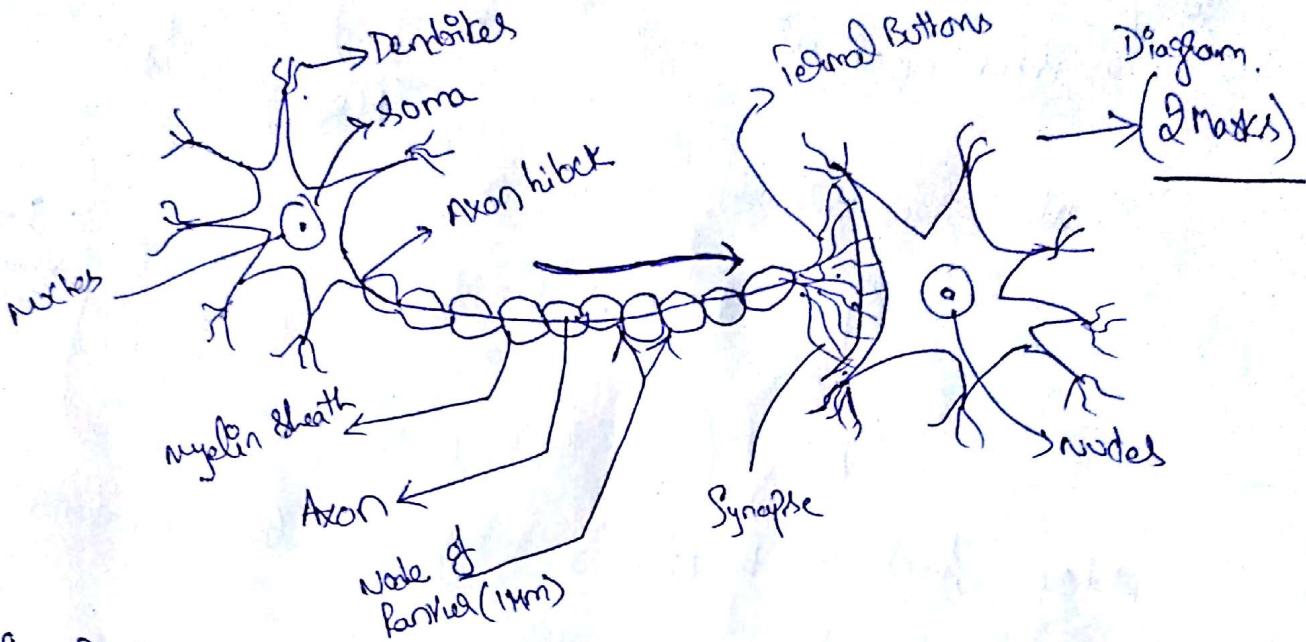
$$(\tilde{A} \times \tilde{B}) \cup (\tilde{A}^c \times y) = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \end{bmatrix} \quad " \quad \text{1m}$$

(ii) If x is \tilde{A} THEN y is \tilde{B} ELSE y is \tilde{C}

$$R = (\tilde{A} \times \tilde{B}) \cup (\tilde{A}^c \times \tilde{C})$$

$$\therefore (\tilde{A} \times \tilde{B}) \cup (\tilde{A}^c \times \tilde{C}) = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \end{bmatrix} \quad \text{1m}$$

Structure of Biological neuron :- (5 marks)



- (i) Dendrites :-
- (ii) Soma (or) Cellbody :-
- (iii) Axon :-
- (iv) Axon hillock :-
- (v) Myelin Sheath :-
- (vi) Node of Ranvier :-
- (vii) Synapse :-
- (viii) Terminal Buttons :-

→ (2 marks)

→ Information flow in neural cells :- (1 mark):

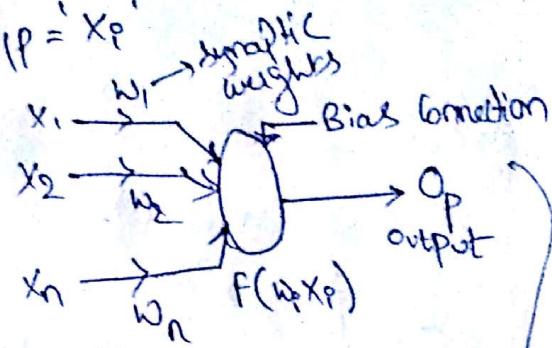
Neuron modeling for artificial neural systems:- (5 marks)

Mcculloch Pitts model of neuron is characterized by its formalism and the elegant Bochner mathematical model.

→ This model allows 0,1 states only operated under a discrete time assumptions. in larger networks.

→ Weights and neuron threshold fixed in model and weight interaction among network neurons take place except signal.

Let $x_1, x_2, x_3, \dots, x_n$ be $\vec{x}^T = [x_1, x_2, \dots, x_n]^T$
 $w^t = [w_1, w_2, \dots, w_n]^T$ be
 weighted synapses



1 mark

∴ Symbolic representation of O_p is

$$O_p = f(w^t \cdot \vec{x}) \rightarrow (a)$$

$$O_p = f\left(\sum_{i=1}^n w_i x_i\right) \rightarrow (b)$$

where function $f(w^t \cdot \vec{x})$ → referred as Activation function.

S-models:-

(i) neural network are meant to be artificial neural networks consisting of neuron models

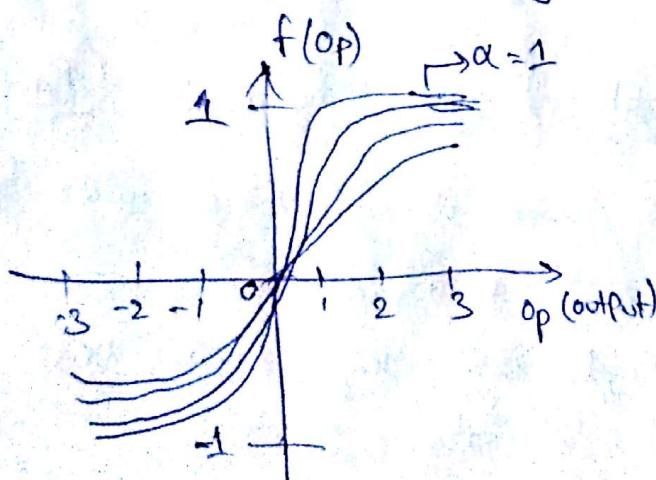
(ii) neurons are meant to be artificial neuron models.

→ typical activation function used is

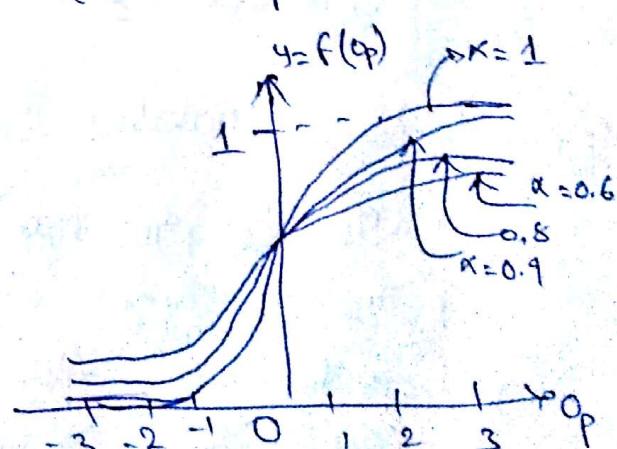
1 mark

$$f(O_p) = \frac{2}{1+e^{-k \cdot O_p}} - 1 \rightarrow (c)$$

$$f(O_p) = \text{Sgn}(O_p) = \begin{cases} +1 & O_p > 0 \\ -1 & O_p \leq 0 \end{cases} \rightarrow (d)$$



Bipolar Continuous



Unipolar Continuous.

Activation function :-

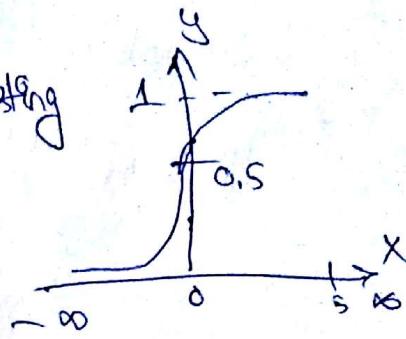
(3 marks)

5

(i) Sigmoid function :- (unipolar sigmoid)

used in Prediction & Process forecasting

$$y(x) = f(x) = \frac{1}{1 + e^{-x}}$$



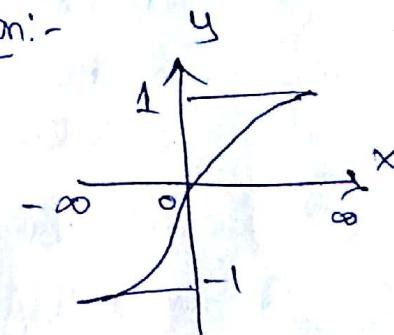
range is $0 < y < 1$

The derivative function is $y'(x) = f'(x) = f(x) \cdot (1 - f(x))$.

(ii) Hypotholic tangent (bipolar sigmoid) function:-

used in Prediction & Process forecasting

$$y(x) = f(x) = \tanh(x) = \frac{e^x - e^{-x}}{e^x + e^{-x}}$$



range is $-1 < y < 1$

derivative $y'(x) = f'(x) = [1 - f(x)]^2$

(iii) Radial basis function:-

→ used for fault diagnosis problems.

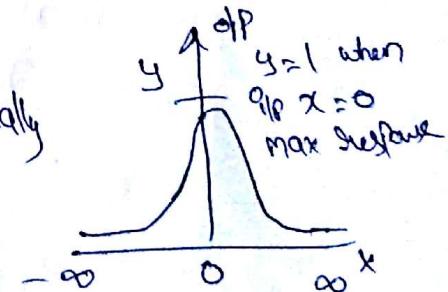
→ Gaussian function is mostly common used in radially symmetric function

$$(-x^2/2)$$

$$y(x) = f(x) = e^{-x^2/2}$$

range $0 < y < 1$

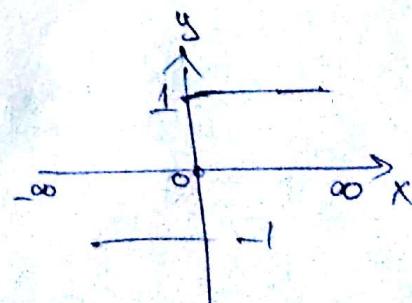
$$\text{derivative } y'(x) = f'(x) = -x \cdot e^{-x^2/2}$$



(iv) Hard limited:-

used in classification of patterns

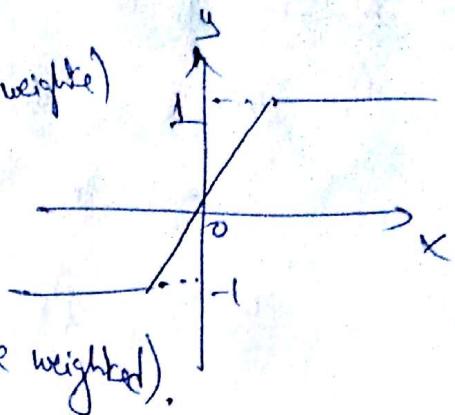
$$f(x) = \text{Sgn}(x) = \begin{cases} 1 & x \geq 0 \\ -1 & x < 0 \end{cases}$$



(v) Piecewise Linear Function:-

→ Saturating Piece. function. \rightarrow (-ve weighted)

$$y(x) = f(x) = \begin{cases} -1 & \text{if } \text{sgn}(x) < 0 \\ \text{sgn}(x) & \text{if } -1 \geq x \geq 1 \\ 1 & \text{if } \text{sgn}(x) > 0 \end{cases}$$



(+ve weighted).

3(a)

Learning mechanisms:- (Any two types)

(5 marks)

- (i) Error correction mechanism $\xrightarrow{\text{Perception}} \xrightarrow{\Delta \text{ window Hoff.}}$
- (ii) memory Based learning mechanism.
- (iii) Hebbian learning mechanism.
- (iv) Competitive learning mechanism.
- (v) Boltzmann learning mechanism.

(i) Error correction mechanism:- (used in Recognition of etc.).

Consider a time order, then the error at n^{th} instant can be obtained as function of actual o/p. be $O_k(n)$ and desired target response be $d_k(n)$ for k^{th} neuron.

\therefore o/p layer error signal is

$$e_k(n) = d_k(n) - O_k(n) \rightarrow (a)$$

cost function (or) performance index $F(n)$ is

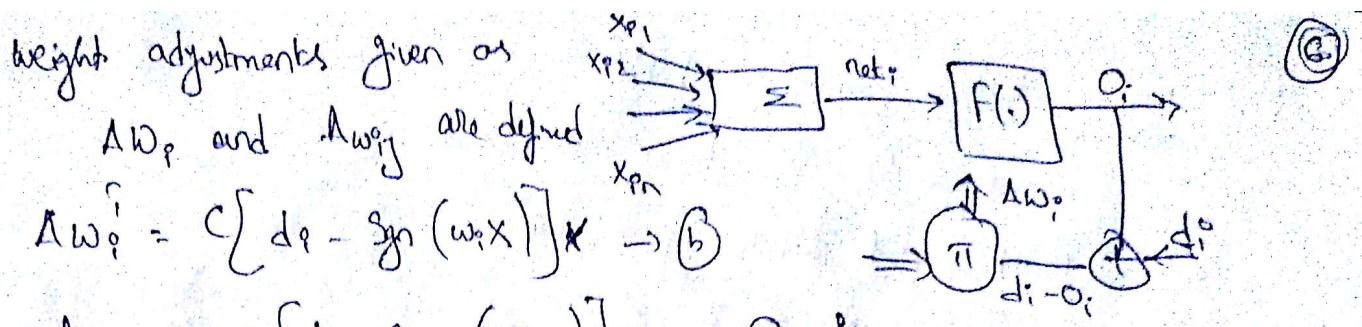
$$F(n) = \frac{1}{2} \cdot e_k^2(n) \rightarrow (b)$$

(a) Perception learning rule:- (Applicable for binary response).

Let 'i' be neuron then error learning is

$$e_i = d_i - O_i \rightarrow (c)$$

$$O_i = \text{sgn}(w_i x_i)$$



$$\Delta w_i = C [d_i - \text{sgn}(w_i \cdot x)] x_i \rightarrow (b)$$

$$\Delta w_{ij} = C [d_i - \text{sgn}(w_i \cdot x)] x_j \rightarrow (c) \text{ for } j=1 \rightarrow n.$$

The desired response is either 1 (or) -1

C = learning Constant.

(b) Delta learning rule:- (used for continuous activation function)

The squared error defined as

$$E = \frac{1}{2} (d_i - O_i)^2 = \frac{1}{2} [d_i - f(w \cdot x)]^2 \rightarrow (a)$$

Error gradient

$$\nabla E = -(d_i - O_i) f'(w \cdot x) x \rightarrow (b)$$

$$\text{gradient vector } \frac{\partial E}{\partial w_{ij}} = -(d_i - O_i) f'(w \cdot x) x_j \text{ for } j=1 \rightarrow n. \rightarrow (c)$$

∴ The weight adjustment

$$\Delta w_{ij} = l (d_i - O_i) f(\text{net}_i) x_j \text{ for } j=1 \rightarrow n.$$

(c) Widrow-Hoff learning rule:-

Used for independent activation function.

Error represent for ith neuron be

$$e_i = d_i - O_i = d_i - w_i \cdot x \rightarrow (a)$$

using learning rule change in weight vector.

$$\Delta w_i = l (d_i - w_i \cdot x) x \rightarrow (b)$$

Weight adjustment

$$\Delta w_{ij} = l (d_i - w_i \cdot x) x_j \text{ for } j=1 \rightarrow n. \rightarrow (c)$$

(ii) memory based learning mechanism:

used in identifying the error.

Let us consider 1-Pattern of x then.

\vec{x} is vector and dp of x is

$dp \rightarrow$ vector.

$$\{\vec{x}_i, dp_i\}_{i=1}^N \rightarrow \text{for 1-Patter.}$$

Let an o.p be \vec{x}_{test} and dp of system be vector as X_{test} .

for nearest match among m vector o.p.

\therefore Closest among 'm' of 1-pattern is \vec{x}_n .

$$\vec{x}_n \in \{\vec{x}_1, \vec{x}_2, \vec{x}_3, \dots, \vec{x}_n\}$$

\downarrow
nearest neighbor.

i. evident distance minimum is related as

$d(\vec{x}_i, \vec{x}_{test})$ is searched by varying $i = 1 \rightarrow N$

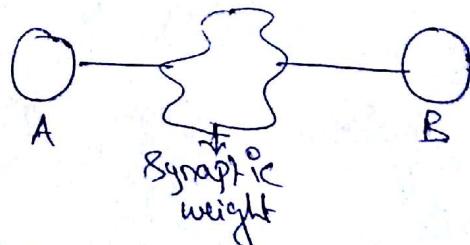
$$\min d(\vec{x}_i, \vec{x}_{test}) = d(\vec{x}_n, \vec{x}_{test}).$$

(iii) Hebbian learning mechanism:

node A is ~~more~~ efficiently in firing

B metabolic changes between

two nodes increases.



→ Positive Correlated leads synaptic strengthening.

→ negative Correlated leads synaptic weakening.

Mathematical relations: - in weight adjustments

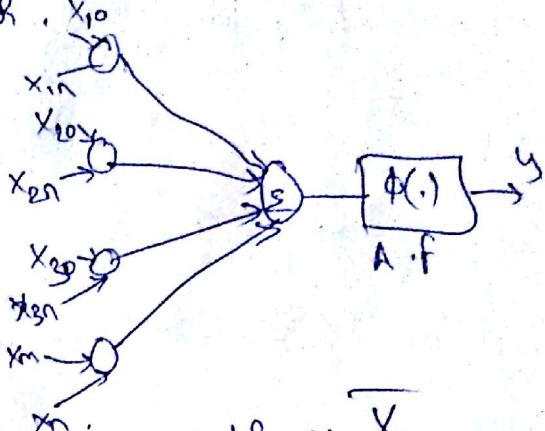
$$\Delta W_{kj}(n) = f(y_k(n)) \cdot x_j(n)$$

\downarrow
Post Synaptic

→ Pre Synaptic signal.

$$\therefore \Delta W_{kj}(n) = f(y_k(n)) \cdot x_j(n)$$

$n \rightarrow$ learning Constant.



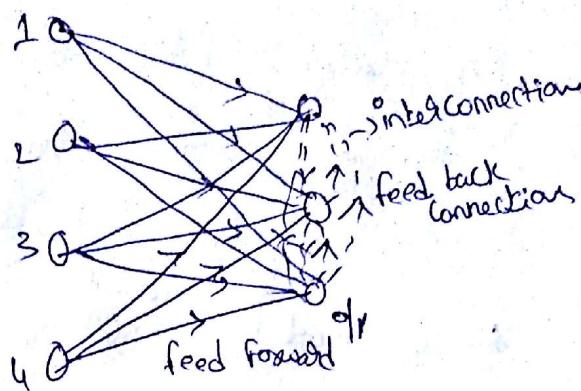
(iv) Competitive learning:-

In this learning mechanism only a single dp neuron is active at any one time.

$$q_k = \begin{cases} 1 & \text{if } v_k > v_j \text{ for } j \neq k \\ 0 & \text{otherwise.} \end{cases}$$

where the sum of total of synaptic weights = 1

$$\sum_j w_{kj} = 1 \text{ for all } k^{\text{th}} \text{ neuron.}$$



(v) Boltzmann learning mechanism:-

In Boltzmann machine no hidden layer is present and it is characterized by energy function E.

$$E = -\frac{1}{2} \sum_j \sum_k w_{kj} x_k x_j \rightarrow @ \text{ for all } j \neq k$$

If particular neuron k pick up then k neuron state changes

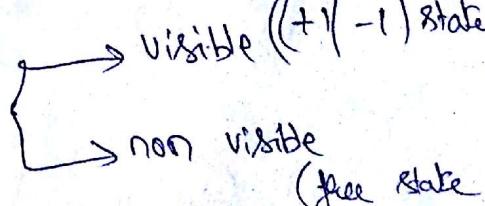
from (-1) to $(+1)$ state.

at neuron k flip state is

$$P(x_k \rightarrow -x_k) = \frac{1}{1 + e^{(-\Delta E_k / T)}}.$$

$\Delta E_k \rightarrow$ change of energy resulting from such a flip (constant).
 $T \rightarrow$ pseudo temperature.

Boltzmann learning categorized in 2 states,



The weight adjustment b/w free running condition of neurons

$$\Delta w_{kj} = \eta (e_{kj}^+ - e_{kj}^-) \delta f k l.$$

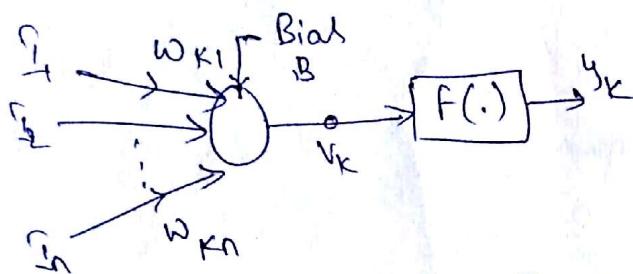
3(b):- Use of linear model and non linear model in ANN:-

5 marks

for concept of Single layer (or) multilayer neural network

to identify the active neuron in network based on Activation function process it is identified.

Linear model :- used in data fit model

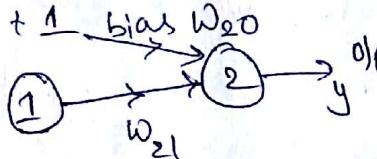


$$v_k = \sum_{j=1}^n i_j w_{kj}$$

$$v_k = i_1 w_{k1} + i_2 w_{k2} + \dots + i_n w_{kn}$$

1 mark

If 2 nodes (1) & (2) Considered



$$oP$$

$$oP = w_{21} x + w_{20}$$

slope

Bias is intercept

(i) Representation of Error:-

let $E^P \rightarrow$ error for particular point P

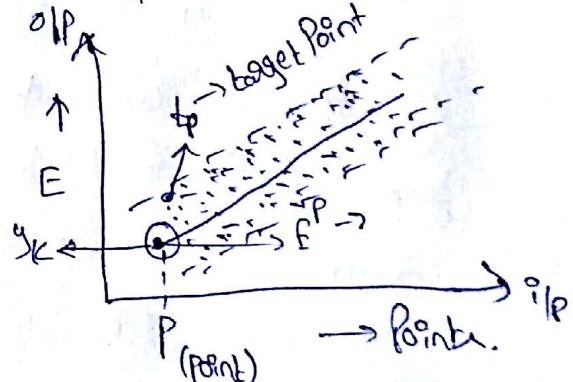
$t^P \rightarrow$ target op for point 'P'

$y^P \rightarrow$ fitted op (or) neural op response

$$\therefore E^P = (t^P - y^P) \rightarrow ①$$

\therefore total error of all set up points are .

$$E = \sum_P E^P = \sum_P (t^P - y^P)^2 \rightarrow ②$$



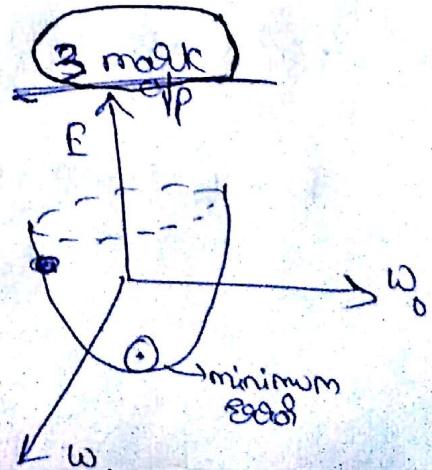
1 mark

(ii) Gradient Descent algorithm.

(Derivation)

$$\Delta w_i = \eta (t_o - y_o) \cdot x_i$$

1.



4(a)

Single layer Perceptron :-

(2 marks)

(5 marks)

(8)

→ Perceptron learning is of
Separating a set of patterns
the weights until the desired

supervised type, which is trained by
to its QP, one at a time and adjusting
it occurs for each of them.

→ For discrete Perceptron the

activation function should be hard limited

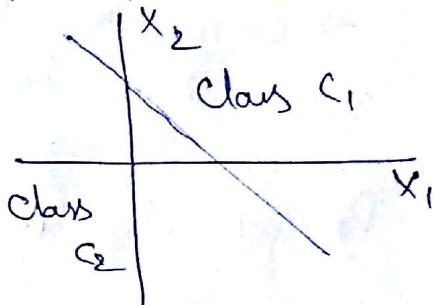
Cool sign(1) function.

→ The popular application of discrete Perceptron is pattern classification.

Let the decision region in n-dimensional space, spanned by
n-input variables.

The two regions separated by hyper plane is

$$\sum_{i=0}^n w_i x_i = 0 \rightarrow \textcircled{a}$$



The QP variable of Perceptron originates from two
linearly separable classes.

Let Ω_1 be subset of training vectors $x_1(1), x_2(2) \dots$ that belong 'C1' class
and Ω_2 be subset of training vectors $x_2(1), x_2(2) \dots$ that belong 'C2' class.

The weight vector 'w' is written as.

$wx > 0$ for every vector x belongs class C1

$wx \leq 0$ for every vector x belongs to class C2.

Algorithm for updating weights formulated as

i) If k^{th} neuron of training set x_k is correctly classified by weight vector
 $w(k)$ computed at k^{th} iteration of algorithm.

$$w^{k+1} = w^k \quad \text{if } w^k x_k > 0 \text{ and } x_k \text{ belongs to class C1}$$

$$w^{k+1} = w^k \quad \text{if } w^k x_k \leq 0 \text{ and } x_k \text{ belongs to class C2}$$

2) weight vector of Perceptron is updated with rule.

$$w^{(k+1)i} = w^{ki} - \eta x_k \quad \text{if } w^k x_k > 0 \text{ and } x_k \text{ belongs to class C2}$$

$$w^{(k+1)i} = w^{ki} + \eta x_k \quad \text{if } w^k x_k \leq 0 \text{ and } x_k \text{ belongs to class C1}$$

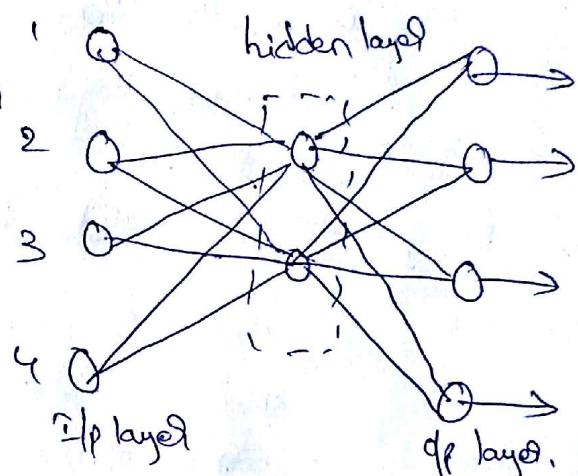
Multilayer Perceptron:-

(3 marks)

A multilayer neural network is a feed forward neural network with at least one hidden layer. It can deal with non-linear classification problems because of more complex decision regions.

- Each node in first layer can create hyper plane
- Each node in second layer can combine hyper plane to create concave region.
- Each node in third layer can combine convex region to form concave region.
- Learning rule used for multilayer neural network is generalized delta rule.
(or) back propagation rule. (or) Back Propagation algorithm.

* Back Propagation algorithm:-



↳ Diagram → (1 mark)

↳ Steps → (1 mark)



(5 marks)

Q(b):-

Ans

The error collecting algorithm to tackle an multicategory classification of single layer networks. The assumption needed is that classes are linearly pairwise separable,
(or) that each class is linearly separable from each other class.

The assumptions exists R linear discriminant functions.

$$g_i(x) > g_j(x) \quad \text{for } i, j = 1 \rightarrow R; i \neq j$$

Let weights vector w_q

$$w_q = [w_{q1}, w_{q2}, \dots, w_{q,n+1}] \xrightarrow{\text{eqn. 1}} \text{Ans}$$

(9)

The ' R '-decision functions $w_1^t y, w_2^t y \dots w_R^t y$ are evaluated.

If $w_i^t y$ is larger than any remaining $R-1$ discriminant function no adjustment of weights needed.

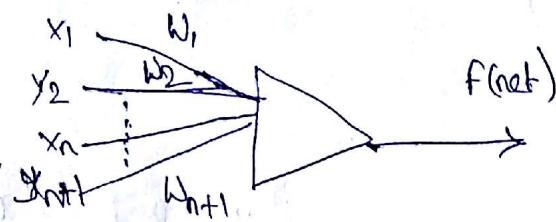
Indicates $w'_1 = w_1 ; w'_2 = w_2 \dots w'_R = w_R$.

If for some m -values we have $w_i^t y \geq w_m^t y$ then update weights vector become.

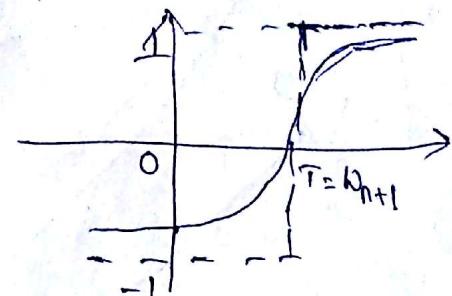
$$w'_i = w_i + c_y$$

$$w'_m = w_m - c_y$$

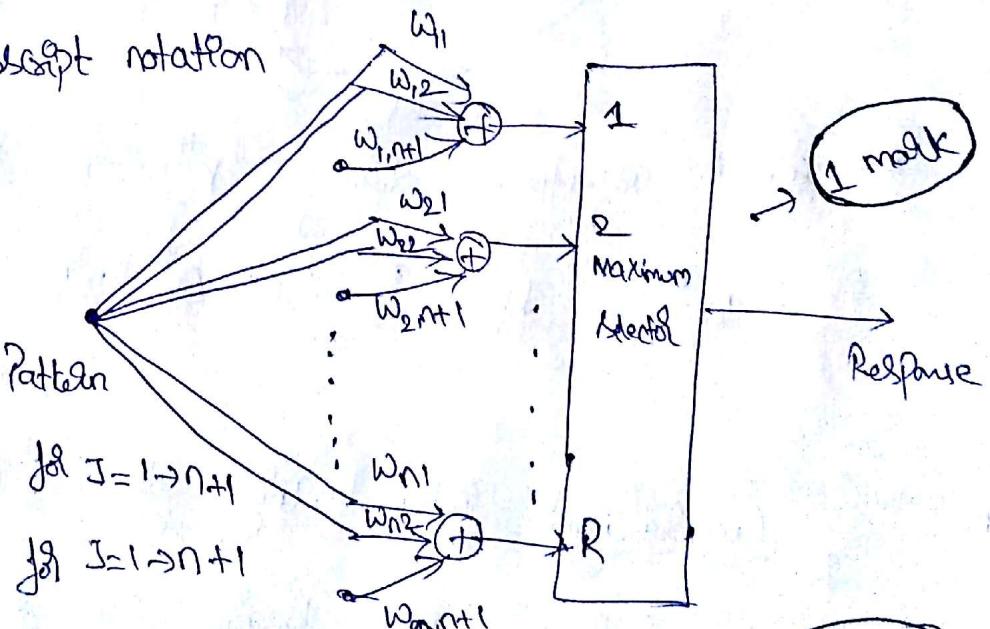
$$w'_k = w_k \text{ for } k = 1, 2 \dots R \text{ } k \neq i, m.$$



→ 1 mark



The double subscript notation for weight is



→ 1 mark

Response

$$w'_{ij} = w_{ij} + c_{y_j} \text{ for } j = 1 \rightarrow n+1$$

$$w'_{mj} = w_{mj} - c_{y_j} \text{ for } j = 1 \rightarrow n+1$$

$$w'_{kj} = w_{kj} \text{ for } k = 1, 2 \dots R \\ \text{where } k = i; m, j = 1 \rightarrow n+1.$$

→ 1 mark

By eliminating maximum selector in a R-class linear classifier and replacing it with R -discrete perceptions.

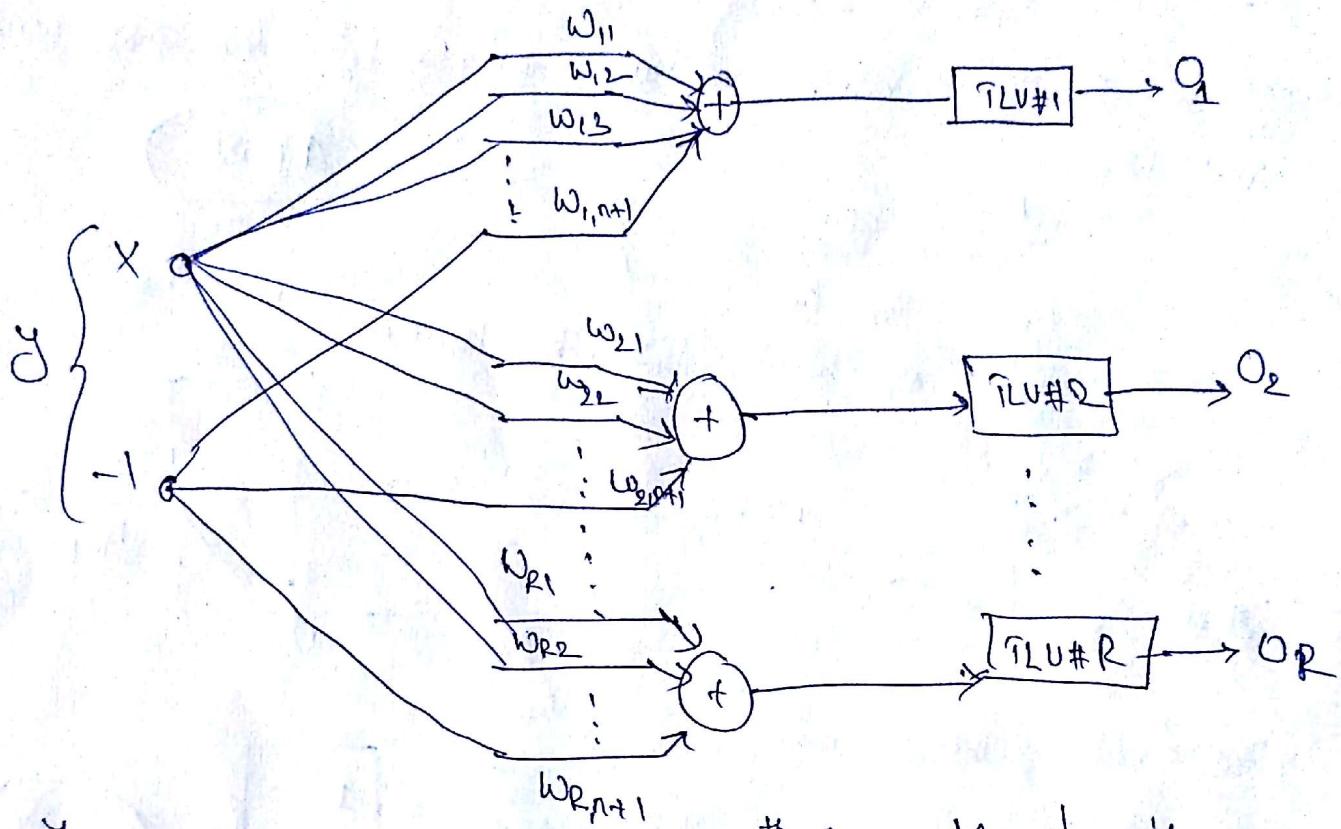
→ The properly trained classifier should respond $\delta_0 = 1$. When

$g_i(x)$ is larger than discriminant function $g_0(x)$, for $i = 1 \rightarrow R$

Instead of signaling output maximum selected responding with $O_i = 1$
a threshold limit of '1' may be used and output $O_i = 1$ and

$$O_2 = O_3 = \dots = O_R = -1.$$

(9 Marks)



The weight adjustment during k^{th} step for n^{th} is

$$w_i^{k+1} = w_i^k + \frac{\eta}{2} (d_p^k - O_i^k) y_i^k \quad \text{for } i=1 \rightarrow R.$$

5(a)
Ans
Generalized Delta learning rule:-

(6 Marks)

Generalized delta learning rule for feedforward layered neural networks.

Let us consider an multi layered network for representing an delta learning rule. to minimize the error function.

Diagram \rightarrow 3 marks

The negative gradient descent formulae for hidden layer

$$\Delta w_{ji} = -\eta \frac{\partial E}{\partial w_{ji}} \text{ for } j=1 \rightarrow J \text{ and } i=1 \rightarrow I$$

$$\therefore \frac{\partial E}{\partial w_{ji}} = \frac{\partial E}{\partial (\text{net}_j)} \cdot \frac{\partial (\text{net}_j)}{\partial w_{ji}}$$

2 marks
derivation

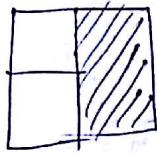
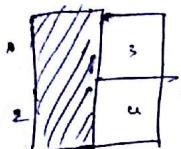
\therefore The weight adjustments in hidden layer

$$\Delta w_{ij} = \eta f'_j(\text{net}_j) z_i \sum_{k=1}^K \delta_k \text{ or } w_{kj} \quad \text{for } j=1 \rightarrow J \text{ and } i=1 \rightarrow I$$

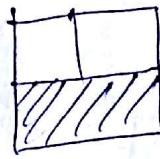
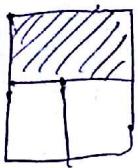
1 mark

5(B)
Ans

Given pattern



class -I



class -II

(Q)

Let Pattern vector be of order 2×2 and each white column be represented as '1'.

$$\therefore P_1 = \begin{bmatrix} 1 \\ -1 \\ -1 \end{bmatrix} \text{ and } P_2 = \begin{bmatrix} -1 \\ 1 \\ 1 \end{bmatrix} \quad \text{for class -I}$$

vertical line.

For class -II horizontal lines be

$$P_3 = \begin{bmatrix} 1 \\ 1 \\ -1 \end{bmatrix} \text{ and } P_4 = \begin{bmatrix} -1 \\ 1 \\ 1 \end{bmatrix}$$

→ 1 m

The weight of matrix w linearly separable by a hyperplane b/w two categories.

$$wP_1 + b > 0 ; wP_2 + b < 0 ; wP_3 + b < 0 ; wP_4 + b > 0$$

$$\Rightarrow \begin{bmatrix} w_{1,1} & w_{1,2} & w_{1,3} & w_{1,4} \end{bmatrix} \begin{bmatrix} 1 \\ -1 \\ -1 \end{bmatrix} = \begin{bmatrix} w_{1,1} + w_{1,2} - w_{1,3} - w_{1,4} \end{bmatrix} > 0$$

$$\Rightarrow \begin{bmatrix} -w_{1,1} - w_{1,2} + w_{1,3} + w_{1,4} \end{bmatrix} > 0, \quad \left\{ \begin{bmatrix} w_{1,1} - w_{1,2} + w_{1,3} - w_{1,4} \end{bmatrix} < 0 \right. \\ \left. \begin{bmatrix} -w_{1,1} + w_{1,2} - w_{1,3} + w_{1,4} \end{bmatrix} < 0. \right\} \rightarrow 1m$$

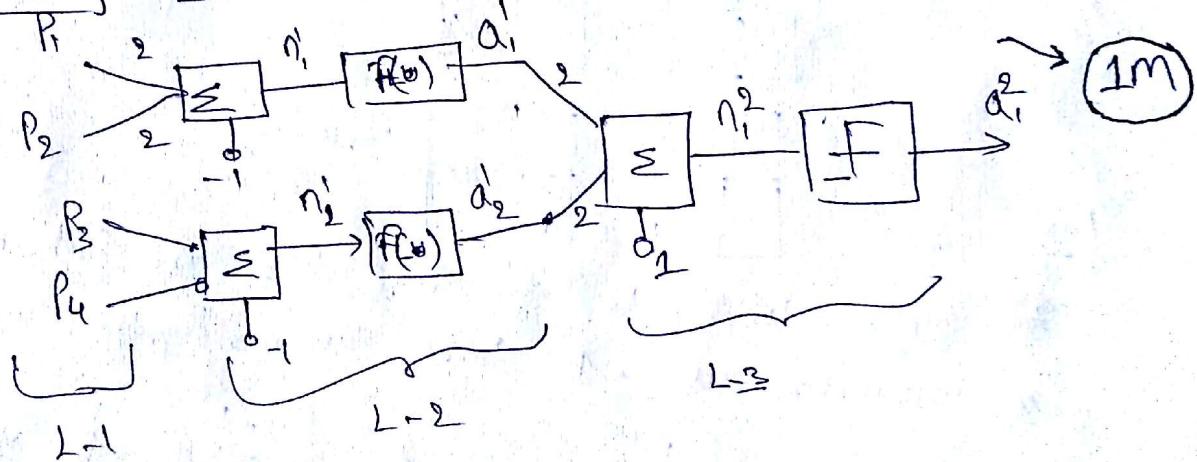
~~(*)~~ the first condition reduced to

$$w_{1,1} + w_{1,2} > w_{1,3} + w_{1,4} \text{ and } w_{1,3} + w_{1,4} > w_{1,1} + w_{1,2}$$

$$w_{1,1} + w_{1,2} > w_{1,3} + w_{1,4} \text{ and } w_{1,2} + w_{1,4} > w_{1,1} + w_{1,3}.$$

$$\therefore \Rightarrow w_{1,1} + w_{1,3} > w_{1,2} + w_{1,4}$$

(00) Multilayer Network:-



6(a):- Recurrent Associate Memory:-

6 Marks

An.

→ (i) Expanded view of Hopfield network → 2 m
↳ Diagram

→ (ii) Simplified view of Hopfield network → 2 m
↳ Diagram

→ Performance Considerations → 2 m

Problem :- (4 marks)

6(b) :- Given vector sequences $s^{(1)}, s^{(2)}, s^{(3)}$

$$s^{(1)} = \begin{bmatrix} 1 & -1 & -1 & 1 & -1 \end{bmatrix}^T, s^{(2)} = \begin{bmatrix} 1 & 1 & -1 & -1 & 1 \end{bmatrix}^T$$

$$s^{(3)} = \begin{bmatrix} 1 & -1 & 1 & -1 & 1 \end{bmatrix}^T$$

The memory weight matrix w is written as

$$w = \sum_{i=1}^{P-1} s^{(i+1)} \cdot s^{(i)T} + s^{(1)} \cdot s^{(P)T}$$

where $s^{(i+1)} = \Gamma [w \ s^{(P)}]$

$$\Rightarrow w = s^{(2)} \cdot s^{(1)T} + s^{(3)} \cdot s^{(2)T} + s^{(1)} \cdot s^{(3)T}$$

$$\therefore w = \begin{bmatrix} 3 & -1 & -1 & -1 & 1 \\ -1 & 3 & -1 & 1 & -1 \\ -1 & -1 & 3 & -1 & -1 \\ -1 & 3 & -1 & 1 & 1 \\ 1 & 1 & -1 & -1 & -1 \end{bmatrix}$$

To recall encoded sequence 's' with input $s^{(1)}$ the layer

response is 'a'.

$$a = \Gamma [w \ s^{(1)}] = P$$

where $a = \Gamma \left[s^{(2)} \cdot s^{(1)T} + \dots + s^{(K+1)} \cdot s^{(K)T} + \dots + s^{(P)} \cdot s^{(P)T} \right]$

$$\therefore a = \Gamma [w \ s^{(1)}] = \Gamma [3 \ 7 \ -5 \ -5 \ 5]^T$$

\therefore the noise vector n is given as

$$n = \sum_{i=K}^P (s^{(i+1)})(s^{(i)T} \cdot s^{(K)})$$

$$\Rightarrow n_1 = \begin{bmatrix} 3 \\ -7 \\ -5 \\ -5 \\ 5 \end{bmatrix} - 5 \cdot s^{(2)}$$

$$= \begin{bmatrix} 3 \\ 7 \\ -5 \\ -5 \\ 5 \end{bmatrix} - \begin{bmatrix} 5 \\ 5 \\ -5 \\ -5 \\ 5 \end{bmatrix} = \begin{bmatrix} -2 \\ 2 \\ 0 \\ 0 \\ 0 \end{bmatrix}^T$$

(11)

19

If magnitude of each component of η_1 is smaller than magnitude of corresponding of $S^{(2)}$ which recall further update in memory

$$\Pi [w S^{(2)}] = \Pi [5 -7 5 -3 3]^T$$

the recall required with noise term.

$$\eta_2 = [0 -2 0 2 -2]^T$$

now $S^{(1)}$ is recalled due to thresholding operation

$$\Pi [w S^{(3)}] = \Pi [5 -7 -3 5 -5]^T$$

with the noise vector

$$\eta_3 = [0 2 20 0]^T$$

\therefore The recall sequence of $S^{(1)}, S^{(2)}, S^{(3)}$ terminated.



7(a):-

Bi-directional

Associative memory:-

6 marks

(12)

Architecture :-

2 marks

Mathematical modeling of BAM:-

4 marks

The additive dynamic of BAM can be written as

$$\dot{x}_i(t) = -A_i x_i(t) + \sum_{j=1}^m f_j(y_j(t)) w_{ij} + I_i, \quad i = 1 \rightarrow N \quad \rightarrow (a)$$

$$\dot{y}_j(t) = -A_j y_j(t) + \sum_{i=1}^N f_i(x_i(t)) w_{ji} + I_j, \quad j = 1 \rightarrow N \quad \rightarrow (b)$$

where I_i & I_j are net external ip to units i & j

A_i and $f_i(\cdot)$ could be different for each other.

If neuron reached equilibrium state $\dot{x}_i(t) = 0$ and $\dot{y}_j(t) = 0$ the output of network is computed.

Let us consider X-layer as ip layer and Y-layer as output layer.

(i) Computing net input values of nodes on Y-layer

$$\text{Net}_j^Y = W_k \rightarrow (c)$$

$$\Rightarrow \text{Net}_j^Y = \sum_{i=1}^N w_{ji} x_i \rightarrow (d) \quad \rightarrow 1m$$

(ii) Computing the outputs (new values of Y-layer units) of output layer

$$y_j = f_j \left(\sum_{i=1}^N w_{ji} x_i \right), \rightarrow (e)$$

where $f_j \rightarrow$ is non linear activation function of neuron j .

∴ typical activation function is

$$y_j(l+1) = \begin{cases} +1 & \text{net}_j^Y > 0 \\ y_j(l) & \text{net}_j^Y = 0 \\ -1 & \text{net}_j^Y < 0 \end{cases} \rightarrow (f)$$

where l is step number and $j = 1 \rightarrow M$.

(iii) Compute the net input value of nodes on X-layer

$$\text{Net}^X = wY \rightarrow \textcircled{g} \rightarrow \textcircled{1M}$$

$$\Rightarrow \text{Net}_i^X = \sum_{j=1}^M Y_j w_{r,j} \rightarrow \textcircled{b}$$

(iv) Compute the outputs (new values of X-layer units) of output layer nodes

$$x_i = f_o \left(\sum_{j=1}^M w_{ij} Y_j \right) \rightarrow \textcircled{g} \rightarrow \textcircled{1M}$$

where $f_o(\cdot)$ is non linear activation function of neuron i on X-layer.

$$x_i(l+1) = \begin{cases} +1 & \text{net}_i^X > 0 \\ x_i(l) & \text{net}_i^X = 0 \\ -1 & \text{net}_i^X < 0. \end{cases} \quad l \rightarrow \text{step number} \quad i = 1 \rightarrow N.$$

7(b)

Improved Coding memory :- 4 Marks

The generic bidirectional associative memory described in defining error-free retrieval of stored pairs.

Let the stability of update within bidirectional associative memory (a, b) .

The performance can be improved by possibly redefining the energy function.

During decoding energy function decreases until reaches minimum

at (a^f, b^f)

$$\therefore E(a, b) = -a^T W.b \rightarrow \textcircled{a} \rightarrow \textcircled{1M}$$

The energy E has local minimum at (a^f, b^f) when no vector at HD = 1 from a^f and b^f .

The gradient energy function at m^{th} association pair is

$$\nabla_a E(a^{(m)}, b^{(m)}) = - \sum_{i=1}^P a^{(i)} \cdot b^{(i)T} \cdot b^{(m)} \rightarrow \textcircled{b}$$

the corresponding energy change ΔE_{ak} is

$$\Delta E_{\text{ak}}(a^{(m)}, b^{(m)}) = - \left(\sum_{i=1}^P a^{(i)} b^{(i)T} \cdot b^{(m)} \right) \Delta a_k^{(m)} \rightarrow \textcircled{c}$$

$$E(a^{(m)}, b^{(m)})$$

1M

To ensure local minima of energy multiple encoding has been proposed
order 'q' for $(a^{(m)}, b^{(m)})$ pair

$$\Delta w = (q-1) a^{(m)} \cdot b^{(m)T} \rightarrow \textcircled{d}$$

$\rightarrow 2M$

∴ weight matrix obtained for q training is

$$W = q a^{(m)} \cdot b^{(m)T} + \sum_{i \neq m}^P a^{(i)} \cdot b^{(i)T} \rightarrow \textcircled{e}$$

$$\Rightarrow W \approx q a^{(m)} \cdot b^{(m)T} \quad \text{II.}$$

8(a):
Ans.

Relations between fuzzy and crisp sets:-

5 marks

(i) Crisp Relations:-

→ 2 marks

↳ Cardinality of crisp relations

↳ Operations on crisp

↳ Union; intersection; complement; containment

↳ Composition of Relations.

(ii) fuzzy relations:-

→ 3 marks

↳ Cardinality of fuzzy relations

↳ Operations

↳ Union; intersection; complement; containment.

↳ Cartesian Product and Composition.

Problem :- 5 Marks

8(b):- Let fuzzy set A be the region in (P, T) space for material surface 'D'.

using merit function $M_A = f((P^2 + 4T^2)^{1/2})$. → 1m

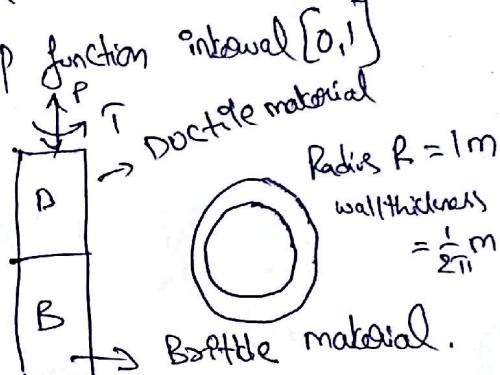
Let fuzzy set 'B' to be the region in (P, T) space for which material 'B' is.

using merit function $M_B = g(P - \beta T)$ → 1m

Let the function f and g be membership function interval $[0, 1]$

Problems defined below:- 3m

1) $A \cup B$ is set of loading for one expects either B (or) D materials is "safe"



2) $A \cap B$ is set of loading that both material B and D are "safe"

3) \bar{A} and \bar{B} are set loading for D and B are "unsafe"

4) $A \setminus B$ is set of loading for D is safe and B is unsafe

5) $B \setminus A$ is set of " for B is safe but D is unsafe

6) $\overline{A \cap B} = \overline{A} \cup \overline{B}$ loadings are not safe for both 'B' and 'D'

7) $\overline{A \cup B} = \overline{A} \cap \overline{B}$ both are safe. either B (or) D.

Fuzzy sets :- 5m

Definition, notations → 2m

fuzzy set operations → 1m

Properties of fuzzy sets → 2m

Q(a)
Ans.

9(b):- Classical Sets:-

5marks

Definition:-

Operation on Classical Sets

} $\rightarrow 2m$

Properties:-

- (i) Associative
- (ii) Distributive
- (iii) Idempotency
- (iv) Identity
- (v) Transitivity
- (vi) Involution.

} $\rightarrow 3m$

Venn diagrams:-

5marks

Given

$$\tilde{A} = \left\{ \frac{1}{1.0} + \frac{0.65}{1.5} + \frac{0.4}{2.0} + \frac{0.35}{2.5} + \frac{0}{3.0} \right\}$$

$$\tilde{B} = \left\{ \frac{0}{1.0} + \frac{0.25}{1.5} + \frac{0.6}{2.0} + \frac{0.25}{2.5} + \frac{1}{3.0} \right\}$$

$$\tilde{C} = \left\{ \frac{0.5}{1.0} + \frac{0.25}{1.5} + \frac{0}{2.0} + \frac{0.25}{2.5} + \frac{0.15}{3.0} \right\}$$

10(a):-
Ans

$$(i) \tilde{A} \cap \tilde{B} = \left\{ \frac{0}{1.0} + \frac{0.25}{1.5} + \frac{0.4}{2.0} + \frac{0.25}{2.5} + \frac{0}{3.0} \right\}$$

$$(ii) \tilde{A} \cup \tilde{B} = \left\{ \frac{1}{1.0} + \frac{0.65}{1.5} + \frac{0.6}{2.0} + \frac{0.35}{2.5} + \frac{1}{3.0} \right\}$$

$$(iii) \tilde{A}^c = \left\{ \frac{0}{1.0} + \frac{0.35}{1.5} + \frac{0.6}{2.0} + \frac{0.65}{2.5} + \frac{1}{3.0} \right\}$$

$$(iv) \tilde{B}^c = \left\{ \frac{1}{1.0} + \frac{0.75}{1.5} + \frac{0.4}{2.0} + \frac{0.75}{2.5} + \frac{0}{3.0} \right\}$$

$$(v) \tilde{A}^c \cup \tilde{B}^c = \left\{ \frac{1}{1.0} + \frac{0.75}{1.5} + \frac{0.6}{2.0} + \frac{0.75}{2.5} + \frac{1}{3.0} \right\}.$$

10(b)

Membership function :- 5 marks

Definition :-

→ 1m

features of the membership function :-

- (i) Core of membership :-
- (ii) Support of membership :-
- (iii) Boundary of membership :-
- (iv) Normal fuzzy set :-
- (v) Convex fuzzy set :-

3m

various form of membership functions :- → 1m

~~Ques:-~~

11(a) :-

Process of fuzzification and De-fuzzification :- 6m

(i) fuzzification :- (Explanation)

2m

↳ membership function representation :-
(Explanation)

(ii) De-fuzzification :- methods :-

1m

- (i) maximum membership principle
- (ii) centroid method
- (iii) weighted method
- (iv) mean maximum membership
- (v) center of sum
- (vi) center of largest area

II(b)

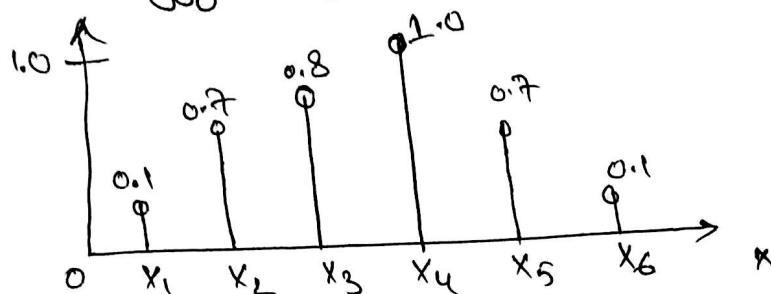
Problem :-
Given

$\mu(x_i)$	x_1	x_2	x_3	x_4	x_5	x_6
A	0.1	0.7	0.8	1.0	0.7	0.1
B	1.0	0.9	0.5	0.2	0.1	0

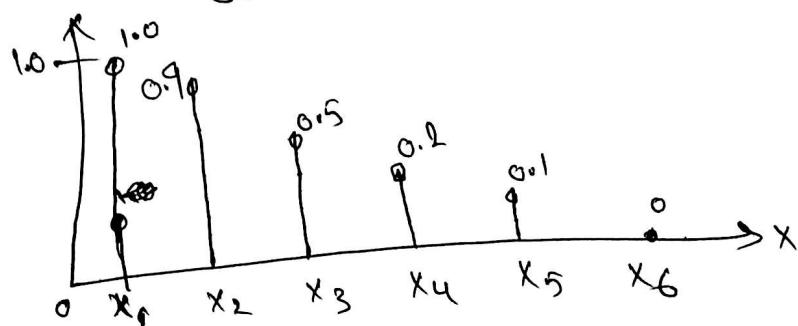
4m

To express ~~subset~~ set for given fuzzy sets using Zadeh's relation

(a) A discrete fuzzy set of 'A' represented as



(b) A discrete fuzzy set of 'B' represented as



2m

2m

Given to find :-

$$(i) (\tilde{A})_{0.6} = \left\{ \frac{1}{x_1} + \frac{0}{x_2} + \frac{0}{x_3} + \frac{0}{x_4} + \frac{1}{x_5} + \frac{0}{x_6} \right\}_{\tilde{A}}^{\Rightarrow \{x_1, x_5\}}$$

$$(ii) (\tilde{B})_{0.5} = \left\{ \frac{1}{x_1} + \frac{1}{x_2} + \frac{1}{x_3} + \frac{0}{x_4} + \frac{0}{x_5} + \frac{0}{x_6} \right\}_{\tilde{B}}^{\Rightarrow \{x_1, x_2, x_3\}}$$

$$\begin{aligned} (iii) (\tilde{A} \cup \tilde{B})_{0.8} &= \left\{ \frac{1}{x_1} + \frac{1}{x_2} + \frac{1}{x_3} \right\}_A \cup \left\{ \frac{1}{x_1} + \frac{1}{x_2} \right\}_B \\ &= \left\{ \frac{1}{x_1} + \frac{1}{x_2} \right\}_{\tilde{A} \cup \tilde{B}} \end{aligned}$$

$$\begin{aligned}
 {}^o_N (\tilde{A} \cap \tilde{B})_{\text{0.9}} &= \left\{ \frac{1}{x_1} + \frac{1}{x_2} + \frac{1}{x_3} + \frac{1}{x_4} \right\}_n \cap \left\{ \frac{1}{x_1} + \frac{1}{x_2} \right\}_{\text{B}^2} \\
 &= \left\{ \frac{1}{x_1} + \frac{1}{x_2} \right\}_{\tilde{A} \cap \tilde{B}}
 \end{aligned}$$