Signals & Systems:-

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Unit - I (Signal Analysis)

1) **Signal:-** A signal is a function of independent variables that carry some information.

→ A signal is a physical quantity that varies with time, space (or) any other independent variable by which information can be conveyed.

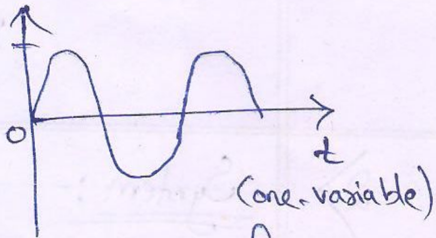
$$x(t) = F(x_1, x_2, x_3 \ldots x_n)$$

x_i = time, space, temperature ..etc., → (independent variables)

Eg:- Audio, video, ECG (Electro cardio gram), AC Power supply signal.

→ **One Dimensional signal:-**

If a signal depends on one-variable is called one-dimensional signal. Eg:- voice, AC signal.

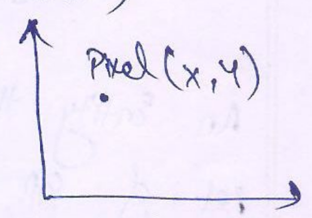


→ **Two Dimensional signal:-**

Signal depends on two variables is called two dimensional

Eg:- Pixel has two variable (x-direction & y-direction)

Eg:- Picture, video signal



→ **Multi Dimensional signal:-**

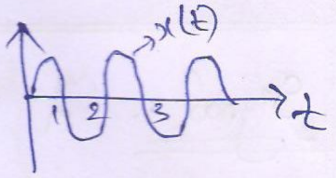
If the signal depends 1 (or) more than two variables is called multi dimensional signal.

Eg:- speed of the winds.

1.1) Classification of Signals :-

(a) Continous time Signals :- (or) Analog Signal → $x(t)$

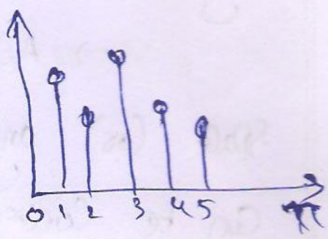
It is defined at Every instant of time (or) at continous range of time



Eg: $A \sin(\omega t)$ and $a+bt$

(b) Discrete time signals :-

defined at discrete instant of time



Eg: Sampling Period

$$x(nT) = x(t)\Big|_{t=nT}$$

T = Sampling interval

n = integer range ($-\infty$ to ∞)

(c) Digital signals :-

It is also an discrete time signal with take the infinite values '0' & '1'

digital signal (or) Binary signal

$x(n) = 0$ (or) 1 $\quad n = (-\infty$ to $\infty)$

$n = -2, -1, 0, 1, 2 \ldots$

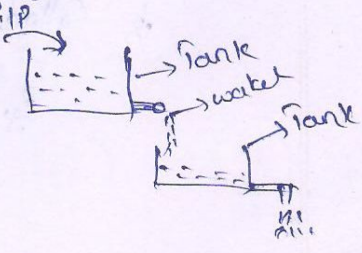
2) System :-

It is a set of Elements (or) functional blocks that are Connected together and Produce an o/p in response to i/p signal

(or)

An Entity that Processes a set of i/p signal to yield another set of o/p signal.

Eg: An audio amplifier, Attenuator, TV set.



Analogy:-

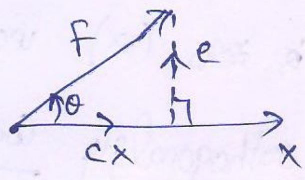
→ Signals are represented in terms of orthogonal functions.

(1) orthogonality Concept in vectors:-

All signals are basically vectors.

vector is represented in terms of its Co-ordinates.

Let us Consider vector 'f' and another vector 'x' then Projection of vector along other vector is



The dot Product of vectors f and x is given

$$f \cdot x = |f|\,|x| \cdot \cos\theta \rightarrow (a)$$

θ → angle b/w f & x

cx → Component of vector f along x (or) Projection of 'f' on 'x'

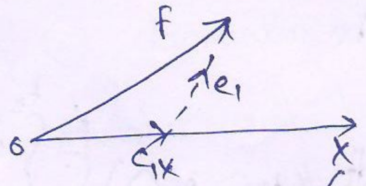
Using vector addition

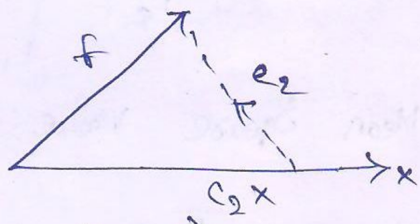
$$f = cx + e \rightarrow (b)$$

'e' → user vector it is minimum only when it is Perpendicular to 'x'.

$f = c_1x + e_1$ $\qquad$ $f = c_2x + e_2$

Eg:-





(Here e_1 & e_2 are > than e)

The Component of f along 'x' is cx which is given as $|f|\cos\theta$

$$\therefore c|x| = |f|\cos\theta \longrightarrow (c)$$

Multiplying both the sides by |x|

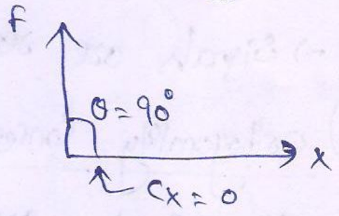
$$c|x|^2 = \underbrace{|f|\cdot|x|\cdot\cos\theta}_{\text{dot Product of vector f \& x}} \longrightarrow (d)$$

$$c|x|^2 = f \cdot x$$

$$c = \frac{f \cdot x}{|x|^2} = \frac{f \cdot x}{x \cdot x} \longrightarrow (e)$$

∵ x.x & f.x are vector Product x and x Cannot get Cancelled.

→ when 'F' is ⊥ to x, F will not have component along x - because $\theta = 90^\circ$



$$f \cdot x = |f| \cdot |x| \cdot \cos\theta \rightarrow (f)$$

$$= |f| \cdot |x| \cos 90^\circ \quad \{\because \cos 90 = 0\}$$

$$f \cdot x = 0$$

∴ The vector f & x are said to be orthogonal if their dot product is zero (or) vector are orthogonal if they are ⊥r.

(2) orthogonality concept in signals :-

Consider a signal $f(t)$ to be represented in terms of $x(t)$ over an interval t_1 & t_2

From vector addition.

Eq (1.b) → $f(t) = c\,x(t) + e(t) \qquad t_1 \le t \le t_2 \rightarrow (1)$

$$e(t) = f(t) - c\,x(t)$$

Energy of $e(t)$ will be

$$E_e = \int_{t_1}^{t_2} e^2(t)\,.dt \rightarrow (2)$$

Mean square value of $e(t)$ will be

$$\overline{e^2(t)} = \frac{1}{t_2 - t_1} \int_{t_1}^{t_2} e^2(t)\,.dt \quad (\text{from Eq } (1)) \quad \left(\because \int_{t_1}^{t_2} e^2(t)\,.dt = E_e\right)$$

$$\overline{e^2(t)} = \frac{E_e}{t_2 - t_1}$$

$$\therefore E_e = \int_{t_1}^{t_2} [f(t) - c\,x(t)]^2 \cdot dt. \rightarrow (3)$$

Value of 'c' should be selected such that E_e will be minimum

it can be obtained by differentiating E_e w.r.t to c and equating it to zero

→ for minimum E_e, $\frac{dE_e}{dc} = 0 \rightarrow (4)$

From Eq (3) $E_e = 0$ so.

i.e $$\frac{d}{dc}\left[\int_{t_1}^{t_2}\big[\underbrace{f(t) - C\,x(t)}\big]^2\right].dt = 0$$

$\hookrightarrow (a-b)^2 \Rightarrow a^2 - 2ab + b^2$

$$\Rightarrow \underbrace{\frac{d}{dc}\int_{t_1}^{t_2} f^2(t).dt}_{\text{independent of 'c' so it will be zero}} - \frac{d}{dc}\int_{t_1}^{t_2} 2c\, f(t).x(t).dt + \frac{d}{dc}\int_{t_1}^{t_2} c^2 x^2(t).dt = 0 \rightarrow (5)$$

$$\Rightarrow \quad -2\int_{t_1}^{t_2} f(t).x(t).dt + 2c\int_{t_1}^{t_2} x^2(t).dt = 0$$

$$\Rightarrow \quad \cancel{2}c\int_{t_1}^{t_2} x^2(t)\,dt = \cancel{2}\int_{t_1}^{t_2} f(t).x(t).dt$$

$$c = \frac{\int_{t_1}^{t_2} f(t).x(t).dt}{\int_{t_1}^{t_2} x^2(t).dt} \rightarrow (6)$$

The same expression can be obtained for minimum value of $\overline{\epsilon(t)}$.

from Eq (6) denominator represents Energy of $x(t)$. which cannot be zero. Hence numerator must be zero to make 'c' zero. if c is zero, there will be no component of $f(t)$ along $x(t)$.

$\rightarrow$ $f(t)$ and $x(t)$ are said to be orthogonal over interval (t_1, t_2) i.e

$$\int_{t_1}^{t_2} f(t).x(t).dt = 0$$

similarly if $f(t)$ and $x(t)$ are complex signal then they are orthogonal over an interval $[t_1, t_2]$ if.

$$\int_{t_1}^{t_2} f(t).\underbrace{x^*(t)}_{\text{Complex Conjugate of } x(t)}.dt = 0 \quad (or) \quad \int_{t_1}^{t_2} \underbrace{f^*(t)}_{\text{Complex conjugate of } f(x)}.x(t).dt = 0$$

Problems:-

(i) Show that the following signals are orthogonal over an interval [0, 1]

$$f(t) = 1 \;;\; x(t) = \sqrt{3}(1-2t)$$

Sol w.k.t orthogonal if

$$\int_{t_1}^{t_2} f(t).x(t).dt = 0$$

$$\Rightarrow \int_{t_1}^{t_2} f(t).x(t)\,dt = \int_0^1 1(\sqrt{3})(1-2t).dt$$

$$\int_0^1 t.dt \Rightarrow \frac{t^2}{2}$$

$$\frac{a^{n+1}}{n}$$

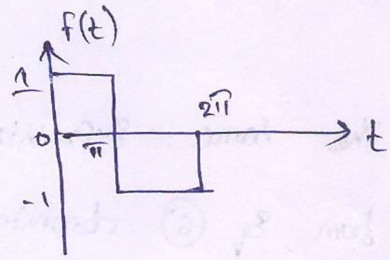
$$= \int_0^1 \sqrt{3}.dt - \int_0^1 2\sqrt{3}t.dt$$

$$= \sqrt{3}[t]_0^1 - 2\sqrt{3}\left[\frac{t^2}{2}\right]_0^1$$

$$\sqrt{3}[1-0] - 2\sqrt{3}\left(\frac{1}{2}\right)$$

$$\sqrt{3} - \sqrt{3} = 0 \rightarrow \text{orthogonal}$$

(ii) Fig shows a square wave. Represent this signal by sint. plot an energy in this representation.



Sol. Square wave be f(t) and sine wave be

$x(t) = \sin t$ then

what e(t) = ?

from $e(t) = f(t) - cx(t)$

$$f(t) = c.x(t)$$

$$= c.\sin t \rightarrow (a)$$

value of 'c' is given by $c = \dfrac{\int_{t_1}^{t_2} f(t).x(t).dt}{\int_{t_1}^{t_2} x^2(t).dt}$

$$\int_{t_1}^{t_2} f(t).x(t).dt = \int_0^{2\pi} f(t).\sin t.dt$$

(where f(t) = -1 to 1)

$$= \int_0^{\pi} 1.\sin t\,.dt + \int_{\pi}^{2\pi} (-1).\sin t.dt$$

$$= [-\cos t]_0^{\pi} - [-\cos t]_{\pi}^{2\pi} = \left[-\{\cos\pi - \cos 0\}\right] + [\cos 2\pi - \cos\pi].$$

$$= -(-1-1) + (1-(-1))]$$

$$= 4.$$

$$\int_{t_1}^{t_2} x^2(t)\,dt = \int_0^{2\pi} \sin^2 t\,dt \qquad \left(\text{where } \sin^2 a = \frac{1-\cos 2a}{2}\right)$$

$$= \int_0^{2\pi}\left(\frac{1-\cos 2t}{2}\right)dt = \frac{1}{2}\int_0^{2\pi} dt - \frac{1}{2}\int_0^{2\pi} \cos 2t\,dt$$

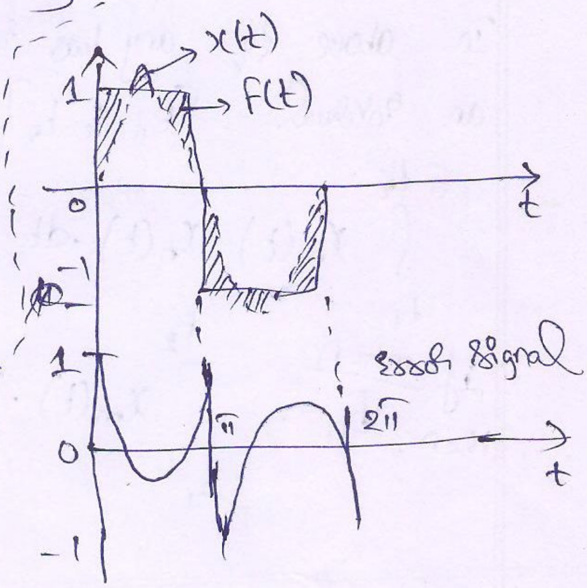
$$= \frac{1}{2}\Big[t\Big]_0^{2\pi} - \frac{1}{2}\left[\frac{\sin 2t}{2}\right]_0^{2\pi} \qquad \left(\text{where } \int_0 \cos 2t\,dt = \frac{\sin t}{2t}\right)$$

$$= \frac{1}{2}\Big[2\pi - 0\Big] - \frac{1}{4}\Big[\sin 2(2\pi) - \sin 2(0)\Big]$$

$$= \pi - \frac{1}{4}\Big[\sin 4\pi - 0\Big]$$

$$\Rightarrow \pi - 0 \Rightarrow \pi$$

$$\therefore C = \frac{\int_{t_1}^{t_2} f(t)\cdot x(t)\,dt}{\int_{t_1}^{t_2} x^2(t)\,dt} = \frac{0}{\pi} \Rightarrow C = 0$$

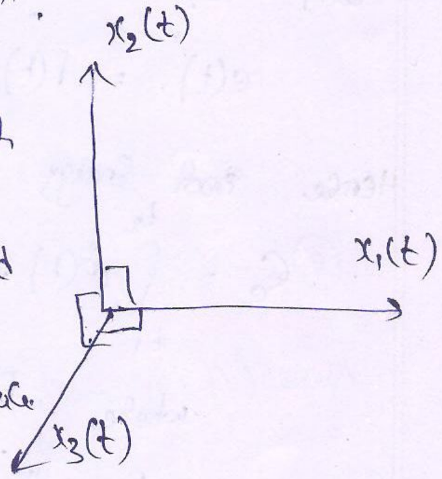


and Error $e(t) = f(t) - C\cdot x(t)$

$$e(t) = 1 - 0\,(\sin t)$$

$$e(t) = 1$$

(3) Orthogonal Signal Space :-



Let $x_1(t)$, $x_2(t)$, $x_3(t)$ be orthogonal each other i.e 3-signals are mutually ⟂r which forms 3-dimensional signal space which is also called orthogonal signal space.

→ which can represent any signal lying in that space

Note:- There are N-such mutually orthogonal signals i.e $x_1(t), x_2(t) \ldots x_N(t)$ then they form N-dimensional orthogonal signal space.

4) Signal Approximation using orthogonal functions :-

→ Consider a set of signals which are mutually orthogonal over an interval $[t_1, t_2]$. f(t) can be given as

$$f(t) \cong C_1 x_1(t) + C_2 x_2(t) + C_3 x_3(t) + \cdots + C_N \cdot x_N(t)$$

$$f(t) \cong \sum_{n=1}^{N} C_n \cdot x_n(t) \rightarrow (a)$$

In above Eq any two signal $x_m(t)$ & $x_n(t)$ are orthogonal over an interval $[t_1 \;\&\; t_2]$ i.e

$$\int_{t_1}^{t_2} x_m(t) \cdot x_n(t) \cdot dt = \begin{cases} 0 & \text{for } m \neq n \\ E_n & \text{for } m = n \end{cases} \rightarrow (b)$$

if $\underline{m=n}$ then

$$E_n = \int_{t_1}^{t_2} x_m(t) \cdot x_n(t) \cdot dt = \int_{t_1}^{t_2} x_n(t) \cdot x_n(t)\, dt$$

$$E_n = \int_{t_1}^{t_2} x_n^2(t) \cdot dt \rightarrow (c) \rightarrow \text{Energy signal.}$$

Error e(t) in the approximation of Eq is given as

$$e(t) = f(t) - \sum_{n=1}^{N} C_n x_n(t). \rightarrow (d)$$

Hence Error Energy

$$E_e = \int_{t_1}^{t_2} e^2(t) \cdot dt = \int_{t_1}^{t_2} \left[f(t) - \sum_{n=1}^{N} C_n \cdot x_n(t) \right]^2 \cdot dt \rightarrow (e)$$

where E_e is fun of $C_1, C_2, C_3 \ldots C_N$

→ Hence E_e will be minimized w.r.t to C_i if

$$\frac{\partial E_e}{\partial C_i} = 0.$$

now :- $$\frac{\partial}{\partial C_i} \left[\int_{t_1}^{t_2} \left[f(t) - \sum_{n=1}^{N} C_n x_n(t) \right]^2 \cdot dt \right\} = 0 \rightarrow (f)$$

$$\frac{\partial}{\partial C_i}\left[\int_{t_1}^{t_2} f^2(t)\,dt - \int_{t_1}^{t_2}\sum_{n=1}^{N} 2C_n f(t)\cdot x_n(t)\,dt + \int_{t_1}^{t_2}\sum_{n=1}^{N} C_n^2\cdot x_n^2(t)\,dt\right\} = 0 \rightarrow (g)$$

For $i = 1,2,3,\ldots N$ the Eq is executed. The first integration term is independent of C_i so the derivative is zero

$$\frac{\partial}{\partial C_i}\left[-\int_{t_1}^{t_2} 2\cdot C_i f(t)\cdot x_i(t)\,dt + \int_{t_1} C_i^2\cdot x_i^2(t)\,dt\right\} = 0 \rightarrow (h)$$

$$\therefore -2\int_{t_1}^{t_2} f(t)\cdot x_i(t)\,dt + 2C_i\int_{t_1} x_i^2(t)\,dt = 0$$

$$C_i = \frac{\int_{t_1}^{t_2} f(t)\cdot x_i(t)\,dt}{\int_{t_1}^{t_2} x_i^2(t)\,dt} \quad \text{where } i = 1,2,\ldots N \rightarrow (i)$$

w.k.t $\int_{t_1}^{t_2} x_i^2(t)\,dt = E_i \rightarrow$ Energy

$$\Rightarrow C_i = \frac{1}{E_i}\int_{t_1}^{t_2} f(t)\cdot x_i(t)\,dt \rightarrow (j)$$

5) <u>Mean Square Error</u> :- <u>for orthogonal functions</u> :-

The Error Energy is given by Eq no:- (4.e)

$$E_e = \int_{t_1}^{t_2}\left[f(t) - \sum_{n=1}^{N}(C_n\cdot x_n(t))\right]^2 dt \rightarrow (1)$$

$$E_e = \int_{t_1}^{t_2} f^2(t)\,dt - 2\sum_{n=1}^{N}\int_{t_1}^{t_2} f(t)\; C_n\cdot x_n(t)\,dt + \int_{t_1}^{t_2}\sum_{n=1}^{N} C_n^2\cdot x_n^2(t)\,dt \rightarrow (2)$$

Integration & summation order if we interchange (from $m=n$) concept

$$E_e = \int_{t_1}^{t_2} f^2(t)\,dt - 2\sum_{n=1}^{N} C_n\int_{t_1}^{t_2} f(t)\cdot x_n(t)\,dt + \sum_{n=1}^{N} C_n^2\int_{t_1}^{t_2} x_n^2(t)\,dt \rightarrow (3)$$

$$\left\{\begin{array}{l}\because \int_{t_1}^{t_2} f(t)\cdot x_n(t)\,dt = C_n E_n \\ \int_{t_1}^{t_2} x_n^2(t) = E_n\end{array}\right\} C = \frac{\int_{t_1}^{t_2} f(t)\cdot x_n}{\int_{t_1}^{t_2} x_n^2(t)}$$

$$E_e = \int_{t_1}^{t_2} f^2(t)\,dt - 2\cdot\sum_{n=1}^{N} C_n\cdot(C_n E_n) + \sum_{n=1}^{N} C_n^2\cdot E_n \rightarrow (4)$$

$$= \int_{t_1}^{t_2} f^2(t)\,dt - 2\sum_{n=1}^{N} C_n^2 \cdot G_n + \sum_{n=1}^{N} C_n^2 \cdot G_n$$

$$= \int_{t_1}^{t_2} f^2(t)\,dt - \sum_{n=1}^{N} C_n^2 \cdot G_n \rightarrow (5)$$

The mean square Error and Error Energy are related as

$$\overline{e^2(t)} = \frac{E_e}{t_2-t_1} \Rightarrow \frac{1}{t_2-t_1}\left[\int_{t_1}^{t_2} f^2(t)\,dt - \sum_{n=1}^{N} C_n^2 G_n\right] \rightarrow (6)$$

$\therefore \sum_{n=1}^{N} C_n^2 \cdot G_n$ is alway positive so if $E_e \to 0$ as $N \to \infty$. $\rightarrow (7)$

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6) Closed (or) Complete set of orthogonal functions:-

→ Mean square error approaches to zero as number of terms $C_n^2 E_n$ are made infinite. under below condition.

with $\overline{e^2(t)} = 0$ as $N = \infty$

from Eq (5.6) $\overline{e^2(t)} = \frac{1}{t_2-t_1}\left[\int_{t_1}^{t_2} f^2(t)\,dt - \sum_{n=1}^{N} C_n^2 E_n\right]$

$$0 = \frac{1}{t_2-t_1}\left[\int_{t_1}^{t_2} f^2(t)\,dt - \sum_{n=1}^{N} C_n^2 \cdot E_n\right] \rightarrow (a)$$

$$\Rightarrow \int_{t_1}^{t_2} f^2(t)\,dt = \sum_{n=1}^{N} C_n^2 \cdot E_n \rightarrow (b)$$

from $f(t) = \sum_{n=1}^{N} C_n x_n(t)$ when $N \to \infty \rightarrow (c)$

$\therefore f(t) = \sum_{n=1}^{\infty} C_n \cdot x_n(t) \rightarrow (d)$ {Generalized Fourier series}

→ It is said to be closed (or) complete set if there exists no other function $P(t)$ for which $\int_{t_1}^{t_2} P(t) \cdot x_n(t)\,dt = 0 \rightarrow (e)$

→ if $P(t)$ exists and above integral is zero then $P(t)$ must be member of set $\{x_n(t)\}$

$f(t) = C_1 x_1(t) + C_2 x_2(t) + C_3 x_3(t) + \ldots \rightarrow (f)$

$\therefore$ for Complete set function

$$C_i = \frac{\int_{t_1}^{t_2} f(t) \cdot x_i(t)\,dt}{\int_{t_1}^{t_2} x_i^2(t)\,dt} = \frac{1}{E_i}\int_{t_1}^{t_2} f(t) \cdot x_i(t) \rightarrow (g)$$

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7) Orthogonality in Complex functions:-

Let set of signals $x_1(t), x_2(t), x_3(t) \dots$ are complex then those signals are mutually orthogonal if

$$\int_{t_1}^{t_2} x_m(t) \cdot x_n^*(t)\,dt = \int_{t_1}^{t_2} x_m^*(t) \cdot x_n(t) \cdot dt = \begin{cases} 0 & \text{for } m \neq n \\ E_n & \text{for } m = n \end{cases}$$

$\therefore$ $f(t)$ can be expressed as

$$f(t) = \sum_{n=1}^{\infty} C_n x_n(t) \quad \text{where } C_n = \frac{1}{E_n}\int_{t_1}^{t_2} f(t) \cdot x_n^*(t) \cdot dt$$

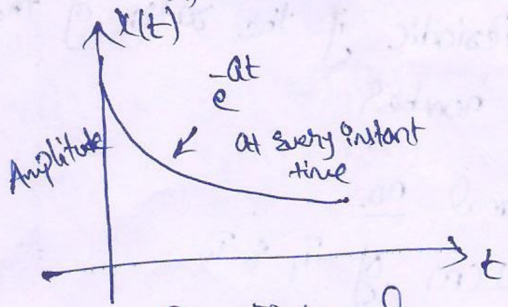
$$E_n = \int_{t_1}^{t_2} x_n(t) \cdot x_n^*(t) \cdot dt$$

8) Classification of Signals :-

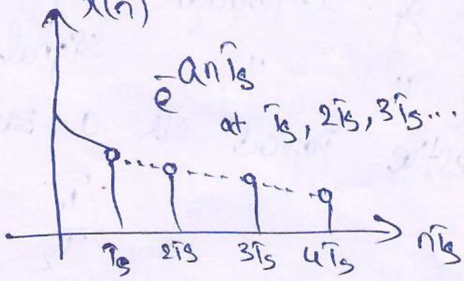
2-types depending on independent variable time.

(i) Continuous time signal/s (C.T)

(ii) Discrete time (or) signals (D.T)



(a) C.T signal



(b) D.T Signals

(1) Periodic & Non-Periodic signals:-

→ A signal is said to be periodic if it repeats at regular intervals

(or)

→ A signal which repeats after every time interval 'T' is called periodic

$x(t)$ is called periodic if and only if

$$x(t+T) = x(t) \quad \text{for all } 't'$$

t → time, T → Constant time interval

→ The smallest value of 'T' that satisfies this condition is called fundamental period in sample period of $x(t)$.

→ The reciprocal of fundamental period 'T' is called fundamental F of $x(t)$

$$f = \frac{1}{T} \quad ; \text{angular freq} = \omega = 2\pi f = \frac{2\pi}{T}$$

$$T = \frac{2\pi}{\omega}.$$

→ A signal $x(t)$ for which there is no value of T satisfying the condition $x(t+T) = x(t)$ is called non-periodic (or) aperiodic signal.

Similly for discrete time signal, $x(n)$ is said to be periodic if it satisfies $x(n+N) = x(n)$ ∀ integer 'n'.

N → +ve integer

$$\text{Angular freq } \Omega = \frac{2\pi}{N} \Rightarrow N = \frac{2\pi}{\Omega}$$

Note:- ① Sum of 2 - Continous time periodic signals may not be periodic

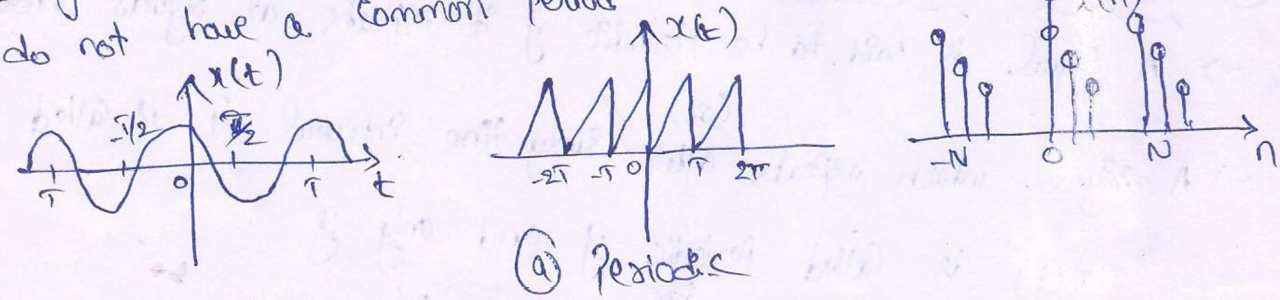
② Sum of 2 - Periodic sequences is always periodic

③ " " signals is periodic if the ratio of their respective period is a rational number

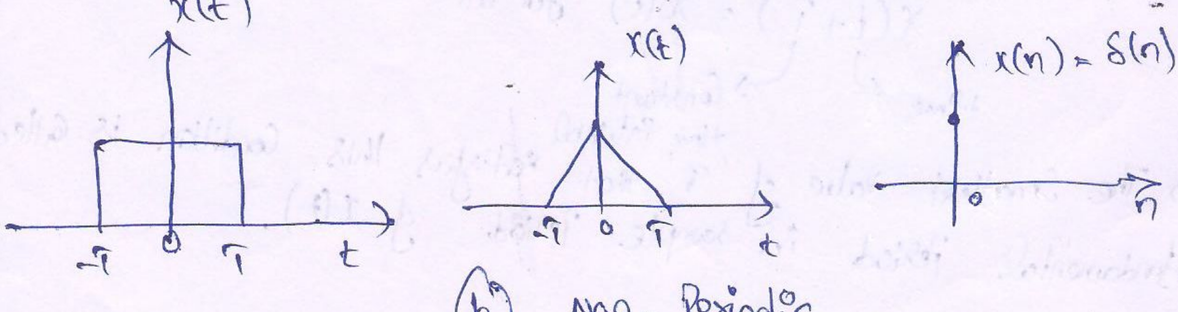
$$\frac{T_1}{T_2} = \text{rational no:-}$$

④ fundamental period is the LCM of T_1 & T_2

⑤ if the ratio T_1/T_2 is an irrational no:- then signals $x_1(t)$ & $x_2(t)$ do not have a common period and $x(t)$ cannot be periodic.



(a) Periodic



(b) Non - Periodic

Even and Odd Signals:-

→ A signal $x(t)$ (or) $x(n)$ is said to be an Even signal if it satisfies the condition

$$x(-t) = x(t) \quad \forall\, t$$
$$x(-n) = x(n) \quad \forall\, n$$

→ A signal $x(t)$ (or) $x(n)$ is said to be odd signal if it satisfies the condition

$$x(-t) = -x(t) \quad \forall \text{ all } t$$
$$x(-n) = -x(n) \quad \forall\, n$$

→ Even signals are symmetrical abt vertical axis (or) time origin where as odd signals are asymmetric
→ cosine
↳ sine

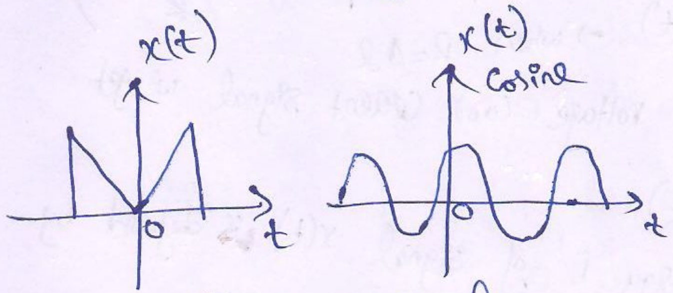
→ A signal $x(t)$ (or) $x(n)$ can be expressed as sum of two signals i.e one odd & one even.

$$x(t) = x_e(t) + x_o(t)$$
$$x(n) = x_e(n) + x_o(n)$$

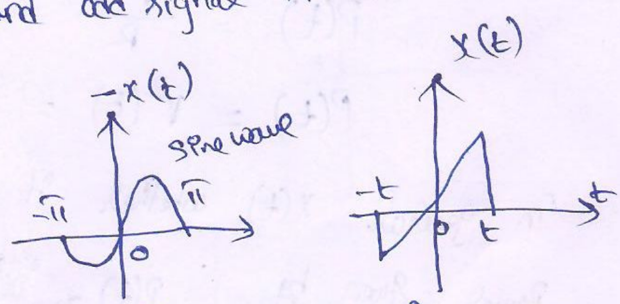
where

$$x_e(n) = \frac{1}{2}\{x(n) + x(-n)\} \quad \text{Even Part}$$
$$x_o(n) = \frac{1}{2}[x(n) - x(-n)]$$
$$x_e(t) = \frac{1}{2}\{x(t) + x(-t)\}$$
$$x_o(t) = \frac{1}{2}\{x(t) - x(t)\}.$$

→ The Product of two even (or) odd signal is an even signal and product of even signal and odd signal is an odd "



(a) Even signal.



(b) Odd signal

③ Deterministic and Random Signals:-

→ A Deterministic signal is the one where no uncertainity occur w.r.t its value at only time

$$x(t) = 100 \sin 50t \text{ (Continous)}$$

$$x(n) = 100 \sin 50n \text{ (Discrete)}$$

→ Random signal is the one about which there is some degree of uncertainity before it actually occurs.

Eg:- The o/p of TV/radio receivers when tuned to frequency where there is no broadcast.

④ Real and Complex Signals:-

→ x(t) is real signal if its value is real number and is a Complex signal if its value is a complex number.

$$\text{Eg:- } x(t) = x_1(t) + j\, x_2(t)$$

real signal and $J = \sqrt{-1}$

⑤ Energy and Power Signal:-

In electrical systems signals may represent current/voltage.

~~Then Power dissipated in~~ Consider a voltage signal v(t) accross Resistor 'R' producing current i(t) than Power dissipated in resistor is

$$P(t) = \frac{v^2(t)}{R} = i^2(t) \cdot R \quad \left(\because V = iR,\ P = \frac{v^2(t)}{R} = \frac{i^2(t) \cdot R^2}{R} \right)$$

$$P(t) = v^2(t) = i^2(t) \rightarrow \text{when } R = 1\Omega$$

In general x(t) whether it is voltage (or) current signal we get Power given by $P(t) = x^2(t)$

Total energy (or) normalized energy 'E' of signal x(t) is defined by

$$E = \lim_{T \to \infty} \int_{-T/2}^{T/2} |x(t)|^2 dt$$

$$\therefore E = \int_{-\infty}^{\infty} |x(t)|^2 dt$$

...e avg Power (or) normalized avg Power P:- of signal $x(t)$ is

$$P = \lim_{T \to \infty} \frac{1}{2T} \int_{-T/2}^{T/2} |x(t)|^2 \, dt$$

→ in case of discrete-time signal $x(n)$ integrals replaced by summation

$$E = \sum_{n=-\infty}^{\infty} |x(n)|^2$$

$$P = \lim_{N \to \infty} \frac{1}{2N+1} \sum_{n=-N}^{N} |x(n)|^2 .$$

Note:- The signal for which total energy is finite $(0 < E < \infty)$ are called Energy signals. They have zero avg Power.

Eg:- deterministic & non-Periodic signals.

→ The signals for which the avg Power is finite $(0 < P < \infty)$ are called Power signals. They have infinite Energy.

Eg:- Random, Periodic signal

→ Both Energy & Power signal are mutually exclusive.

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9) Elementary Signals:-

(i) Unit step function:- Important signal used in many cases.

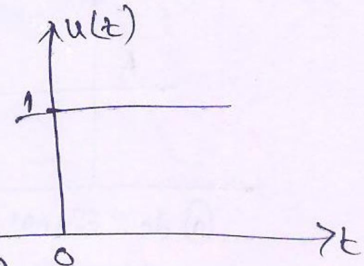
Eg:- Applying brake to an automobile we are applying constant force.

→ If a step function has unity magnitude then it is called unit step

it is defined as $u(t)$

$u(t) = 1$ for $t \geq 0$

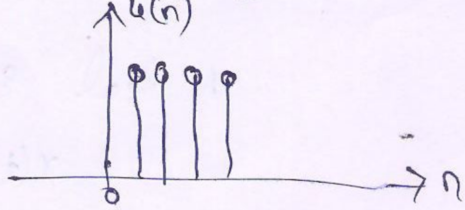
$u(t) = 0$ for $t < 0$



Discrete:-

$u(n) = 1$ for $n \geq 0$

$u(n) = 0$ for $n < 0$



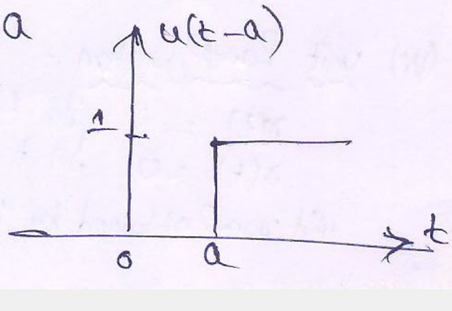
Shifted unit step function:-

$u(t-a)$ is zero if $t-a < 0$ (or) $t < a$

$u(t-a)$ is one if $t-a > 0$ (or) $t > a$

$u(t-a) = 1$ for $t > a$

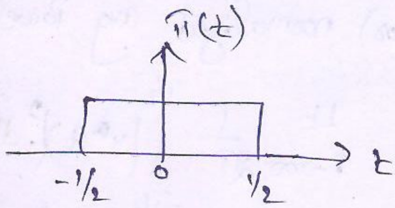
$u(t-a) = 0$ for $t < a$



(ii) Rectangular Pulse function:-

$\Pi(t) = 1$ for $|t| \le \frac{1}{2}$

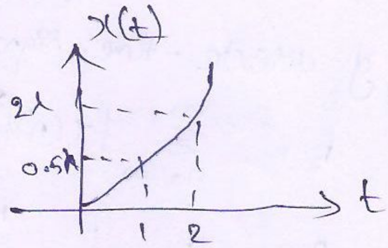
$= 0$ for otherwise



(iii) Parabolic signal:-

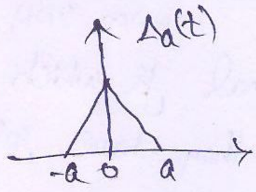
$x(t) = \frac{\lambda t^2}{2}$ for $t > 0$

$= 0 \quad ; \quad t < 0$



(iii) Triangular Pulse function:-

$$\Delta_a(t) = \begin{cases} 1 - \frac{|t|}{a} & |t| \le a \\ 0 & |t| > a \end{cases}$$



(iv) Sinusoidal signal:-

$x(t) = A \sin(\Omega t + \theta)$

$\theta \rightarrow$ phase angle in radians

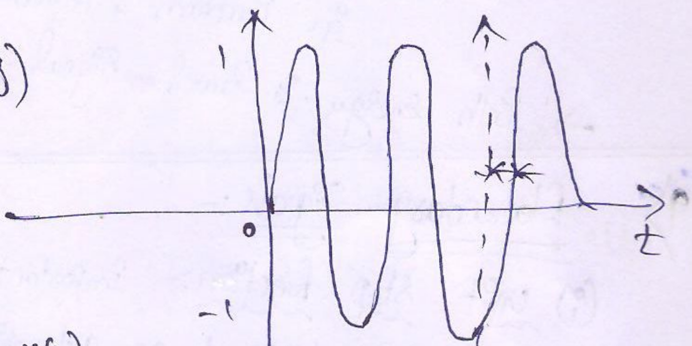
$\Omega \rightarrow$ freq in radians per sec

$A \rightarrow$ amplitude.

$A = 1 \; ; \; \theta = \frac{-\pi}{3}$

Cosinusoidal signal:-

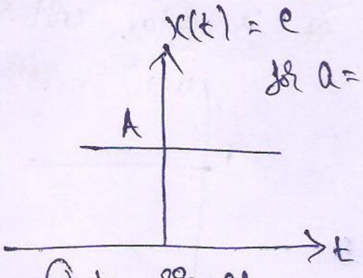
$x(t) = A \cos(\Omega t + \phi)$



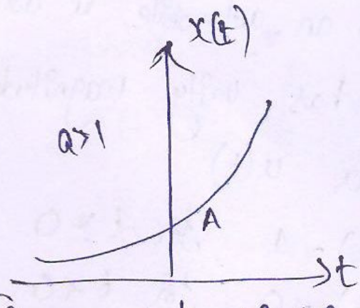
(v) Real Exponential signal:-

$x(t) = e^{at}$

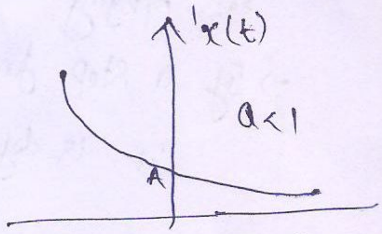
for $a = 0$



ⓐ dc - signal



ⓑ Exponentially growing



ⓒ Exponentially decaying

A Real Exponential is defined as

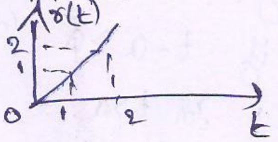
$x(t) = A \cdot e^{at}$

A & a are real.

(vi) unit Ramp function:-

$r(t) = t$ for $t \ge 0$

$r(t) = 0$ for $t < 0$



unit ramp obtained by integrating unit step

$r(t) = \int u(t)\,dt = \int t\,dt = t$

$r(t) = t$ //.

Complex Exponential Signal:-

General form of complex Exponential is

$$x(t) = e^{st} \rightarrow (a) \quad s = \text{Complex variable}$$
$$s = \sigma + j\Omega$$

$$\Rightarrow x(t) = e^{st} = e^{(\sigma + j\Omega t)} = e^{\sigma t} \cdot e^{j\Omega t} \rightarrow (b)$$

using Eulers identity

$$e^{j\Omega t} = \cos \Omega t + j \sin \Omega t \rightarrow (c)$$

Substitute (c) in eq (b)

$$x(t) = e^{\sigma t} [\cos \Omega t + j \sin \Omega t]$$

→ Depending on the values of σ & Ω we get different signal

(i) if $\sigma = 0$; $\Omega = 0$; $x(t) = 1$; Pure DC signal

(ii) if $\Omega = 0$ then $s = \sigma$; $x(t) = e^{\sigma t}$ which decay Exponentially for $\sigma < 0$ & grows Exponentially for $\sigma > 0$.

(iii) if $\sigma = 0$ then $s = \pm j\Omega$ gives $x(t) = e^{j\Omega t}$ sinusoidal signal with $\phi = 0$

(iv) if $\sigma < 0$ then finite Ω we get Exponentially decaying sinusoidal

(v) if $\sigma > 0$ then finite Ω we get " growing sinusoidal.

(vii) Gaussian signal:-

Defined as

$$x(t) = g_a(t) = e^{-at^2} \; ; \; -\infty < t < \infty$$