Find the weather the system is stable or unstable?

$y(n) = 2^n u(-n)$

$y(n) = 2^n u(-n)$

$\sum_{n=-\infty}^{\infty} 2^n u(-n)$

$$\boxed{\sum_{n=-\infty}^{\infty} h(n) < \infty}$$

$= \sum_{n=-\infty}^{0} 2^n$

$= 1+2+4+ = 2^0 + 2^{-1} + 2^{-2} + \cdots \infty$

$= 1 + \frac{1}{2} + \frac{1}{2^2} + \frac{1}{2^3} + \cdots \infty$

$= \frac{1}{1-a} = \frac{1}{1-\frac{1}{2}} = \frac{2}{2-1} = 2 < \infty$

$\therefore$ This system is stable

$h(n) = \sin \frac{n\pi}{2}$

$= \sum_{n=-\infty}^{\infty} \sin \frac{n\pi}{2}$ $\qquad \sum_{n=-\infty}^{\infty} h(n) < \infty$

$= 0 + 1 + 0 + 1 + \cdots \infty$

$\therefore$ This system is unstable

$h(n) = e^{2n} u(n-1)$

$= \sum_{n=-\infty}^{\infty} h(n) < \infty$

$= \sum_{n=-\infty}^{\infty} e^{2n} u(n-1) < \infty$

$= e^2 + e^4 + e^6 + \cdots \infty$

$\therefore$ This system is unstable

formula for energy of a signal, power of a signal

energy of a signal is defined as

$$\boxed{E = \lim_{T\to\infty} \frac{1}{2T}\int_{-T}^{T} \left[[x(t)]^2\right] dt}$$

power of a signal is defined as

$$\boxed{P = \lim_{T\to\infty} \frac{1}{2T}\int_{-T}^{T} [x(t)]^2 dt}$$

for discreet channels energy and power of a signal is defined as.

$$\boxed{E = \sum_{n=-\infty}^{\infty} |x(n)|^2}$$

$$\boxed{P = \lim_{N\to\infty} \frac{1}{2N+1} \sum_{n=-N}^{N} |x(n)|^2}$$

find which of the following signals are energy signals power signal.

1) $x(t) = e^{-3t}u(t)$

$$E = \lim_{T\to\infty} \int_{-T}^{T} |x(t)|^2 dt$$

$$= \lim_{T\to\infty} \int_{0}^{T} (e^{-3t})^2 dt$$

$$= \lim_{T\to\infty} \int_{0}^{T} e^{-6t} dt$$

$$\lim_{T\to\infty} \left[\frac{e^{-6t}}{-6}\right]_0^T$$

$$\frac{-1}{6} \lim_{T\to\infty} \left[e^{-6T} - e^{0}\right]$$

$$\frac{1}{6}\left[\frac{e^{-6\infty}}{-6} + \frac{1}{6}\right] = 0 + \frac{1}{6} = \frac{1}{6}$$

Power of a signal =

$$P = \lim_{T\to\infty} \frac{1}{2T}\int_{-T}^{T} [x(t)]^2 dt$$

$$= \lim_{T\to\infty} \frac{1}{2T}\int_{0}^{T} e^{-6t} dt$$

$$= \lim_{T\to\infty} \frac{1}{2T}\left[\frac{e^{-6t}}{-6}\right]_0^T$$

$$= \lim_{T\to\infty} \frac{1}{2T}\left[\frac{e^{-6T}}{-6} + \frac{e^{0}}{-6}\right]$$

$$= \lim_{T\to\infty} \frac{1}{2T}\left[\frac{e^{-6T}}{-6} + \frac{e^{0}}{-6}\right]$$

$$= \frac{1}{2T}\left[\frac{e^{-\infty}}{-6} + \frac{e^{0}}{-6}\right]$$

$$= \cancel{\frac{1}{2(\infty)}} \frac{1}{2T}\left[0 + \frac{1}{6}\right]$$

$$= \lim_{T\to\infty} \frac{1}{6\times 2T} = \frac{1}{12T} = 0$$

$x_2(t) = e^{j(2t+\frac{\pi}{4})}$

$e^{j(2t+\frac{\pi}{4})} \doteq 1$

$$E = \lim_{T\to\infty} \int_{-T}^{T} |x(t)|^2 dt$$

$$= \lim_{T\to\infty} \int_{-T}^{T} \left[e^{j(2t+\frac{\pi}{4})}\right]^2 dt$$

$$= \lim_{T\to\infty} \int_{-T}^{T} (1)^2 dt$$

$$= \lim_{T\to\infty} [T+T]$$

$$E = \lim_{T\to\infty} 2T = 2(\infty)$$ $\boxed{E = \text{Not finite}}$

$$P = \lim_{T\to\infty} \frac{1}{2T} \int_{-T}^{T} |x(t)|^2 dt$$

$$= \lim_{T\to\infty} \frac{1}{2T} \int_{-T}^{T} e^{2j(2t+\frac{\pi}{4})} dt$$

$$= \lim_{T\to\infty} \frac{1}{2T} \int_{-T}^{T} (1) dt$$

$$= \lim_{T\to\infty} \frac{1}{2T} [T+T] dt$$

$$= \lim_{T\to\infty} \frac{1}{2T} [2T] dt$$

$\boxed{P = 1}$

$x(t) = \cos t$

$\cos t = e^{2t}$.

$$E = \lim_{T\to\infty} \int_{-T}^{T} |x(t)|^2 dt$$

$$= \lim_{T\to\infty} \int_{-T}^{T} |\cos t|^2 dt$$

$$= \lim_{T\to\infty} \int_{-T}^{T} \left[\frac{1+\cos 2t}{2}\right]^2 dt$$

$$= \lim_{T\to\infty} \frac{1}{2}[T]_{-T}^{T} + \int_{-T}^{T} \cos 2T \, dt$$

$$= \lim_{T\to\infty} \frac{1}{2} \cdot [T+T] dT$$

$$= \lim_{T\to\infty} \frac{1}{2} [2T] dt$$

$\boxed{E = \infty ; \text{ Not finite}}$

$$P = \lim_{T\to\infty} \frac{1}{2T} \int_{-T}^{T} |x(t)|^2 dt$$

$$= \lim_{T\to\infty} \frac{1}{2T} \int_{-T}^{T} \cos^2 t \, dt$$

$$= \lim_{T\to\infty} \frac{1}{2T} \int_{-T}^{T} \frac{(1+\cos 2t)}{2} dt$$

$$= \lim_{T\to\infty} \frac{1}{4T} \int_{-T}^{T} 1 dt + \int_{-T}^{T} \cos 2t \, dt$$

$$= \lim_{T\to\infty} \frac{1}{4T} [T]_{-T}^{T} + 0$$

$$= \lim_{T\to\infty} \frac{1}{4T} [T+T]$$

$$= \lim_{T\to\infty} \frac{1}{2 \times T} [2T]$$

$\boxed{P = \frac{1}{2}}$

State and proove schwartz in equality :-

proof :-

$f(t), g(t)$ are two real signals then schwartz in equality can be defined as

$$\boxed{\int_a^b f^2(t)dt \cdot \int_a^b g^2(t)dt \geq \left[\int_a^b f(t)g(t)\,dt\right]^2}$$

w.k.T To proove the above eqn we require a condition having a value greater than zero.

we can write as

$$\int_a^b [f(t)+xg(t)]^2 > 0$$

$$\int_a^b f^2(t)dt + x^2\int_a^b g^2(t)dt + 2x\int_a^b f(t)g(t)dt > 0$$

then we k.T

$a = \int_a^b f^2(t)dt$ $\quad c = x^2\int_a^b g^2(t)dt$ $\quad b = 2x\int_a^b f(t)g(t)dt$

$a^2 + x^2c + 2xb > 0$

$\cancel{a^2 + 2bx +}$

$cx^2 + 2bx + a > 0$

This is a Quadratic eqn of form having value

$cx^2 + 2bx + a \leq a$

$ax^2 + bx + c = 0$

$[2b]^2 - 4[c](a) \leq 0$

condition $b^2 - 4ac \leq 0$

$4b^2 - 4ac \leq 0$

$$4\left[\int_a^b f^2(t)g^2(t)dt\right] - 4\left[\int_a^b g^2(t)dt \cdot \int_a^b f^2(t)dt\right] \leq 0$$

$$\cancel{4}\int_a^b f^2(t)\,g^2(t)\,dt \leq \cancel{4}\int_a^b g^2(t)dt \int_a^b f^2(t)dt$$

$$\boxed{\int_a^b f^2(t)dt \int_a^b g^2(t)dt \geq \int_a^b f^2(t)\,g^2(t)\,dt}$$

ⓒ Explain distortion less transmission through systems:-

for distortionless transmission of a signal a system must. 1. Attunate all frequency components equally i.e $H(\omega)$ should have constant Magnitude for all frequencies.

② The phase shift of each component must also satisfy certain relations which are as follows.

① for distortion less Transmission the response must be an exact Replica on i/p signal. This replica may have different magnitudes. They may also be some time delay associated with this replica.

③ we can say that the signal $f(t)$ is transmitted without distortion if the response $kf(t-t_0)$, The response is an exact replica of the input.

Scanned with CamScanner

with magnitude k times the original signal and delayed by t_0 seconds.

④ if $f(t)$ is the input signal for the distortion less transmission then the response $r(t)$ is given by

$$\boxed{r(t) = k f(t-t_0)} \quad \text{——①}$$

⑤ Applying Time shifting property to eqn ① we get

$$\boxed{R(\omega) = k F(\omega)\, e^{-j\omega t_0}} \quad \text{——②}$$

⑥ w.r.t $\boxed{R(\omega) = F(\omega)\cdot H(\omega)}$ ——③

Then we can write as comparing eqn ② & ③

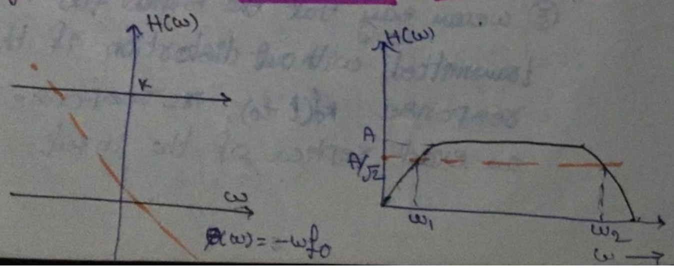
$$k F(\omega)\, e^{-j\omega t_0} = F(\omega)\cdot H(\omega)$$

$$\boxed{H(\omega) = k\cdot e^{-j\omega t_0}} \quad \text{——④}$$

⑦ To achive a distortion less transmission the signal must be in the form.

$$\boxed{H(\omega) = k\cdot e^{-j\omega t_0}}$$

Magnitude and phase characteristics of a system:-



⑧ It is evident that the magnitude of a transfer function is 'k' and is constant for all frequencies

⑨ The phase shift on the other hand is directly proportional to frequency.

$$\boxed{\theta(\omega) = -\omega t_0} \quad \text{——⑤}$$

⑩ The reason for this is that if two different frequency components are shifted by the same interval their corresponding phase changes are proportional to frequency.

for example:-

if signal $\cos \omega t$ is shifted by t_0 seconds resulting signal can be expressed as

$$\boxed{\cos \omega (t-t_0) = \cos(\omega t - \omega t_0)} \quad \text{——⑥}$$

⑪ It is clear that phase shift of a new signal $-\omega t_0$ is to a frequency of ω. from eqn ⑤ we can write as

$$\boxed{\theta(\omega) = n\pi - \omega t_0}$$

where $n = \pm$, integergers.

⑫ The addition of excess phase of $n\pi$ radians may almost change the sign of a signal hence the phase function of distortion less signal is in the form of

$$\boxed{\theta(\omega) = n\pi - \omega t_0}$$

(13) This eqn will clearly describe the characteristic of a distortion less system and its nature

Explain causality and poly wiener criterion for physical realization

1. Poly weiner criteria is a test b/w physically realizable characteristic from an in realizable one. distance

M - (magnitude)

M → magnitude = 0

A - amplitude $e^{-\alpha|\omega|}$

S - smaller values can estimate

(2) A physically realizable system shouldn't have a response before the driving function input function is applied. This is known as causality function.

(3) A causality condition can also be expressed as "A unit impulse response $H(t)$ of a physically realizable system must be causal."

(4) A signal is said to be causal if it has time less than zero.

(5) Thus the impulse response $h(t)$ of a physically realizable system for $T<0$

(6) This is a time domain criteria of physical realizability. In frequency domain this criteria implies that a necessary and sufficient condition for a magnitude function $H(\omega)$. If it is physically realizable.

$$\boxed{\int_{-\infty}^{\infty} \frac{|\ln| \, |H(\omega)|}{1+\omega^2} \, d\omega < \infty} \quad \text{— (1)}$$

(7) Magnitude function $H(\omega)$ must how ever be a square integrable before the poly wiener criteria is valid.

$$\boxed{\int_{-\infty}^{\infty} |H(\omega)|^2 \, d\omega < \infty} \quad \text{— (2)}$$

(8) A system whose magnitude function violates poly wiener criteria as given by eqn (1) has a non causal impulse response. i.e the response exist prior to application of driving function.

Significant conclution drawn from poly wiener criteria

(9) The magnitude function $H(\omega)$ may be zero at some discreet frequency but it can't be zero over a finite band of frequencies. Since this will cause the integral in eqn (1) to become infinity.

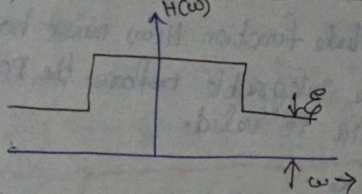
(10) We can conclude from (1) that the amplitude function can't fall off to zero faster than a function of exponential order

$H(\omega) = k e^{-\alpha|\omega|}$ is permissible.

(11) The realizable Magnitude characteristics can't have high total attenuation.

(12) The physically realizable low pass filter can be realized for arbitrary small values of ε (epsilon)

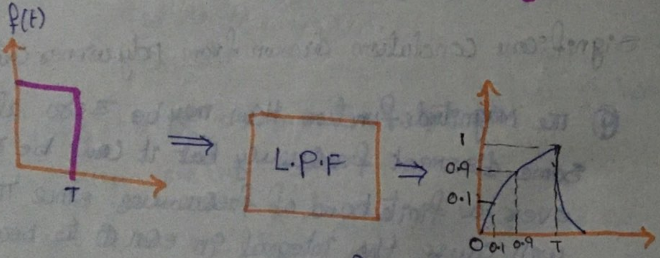
$$H(\omega) = k e^{\alpha|\omega|}$$



Relationship between Rise time and Band width

1. The time taken for a pulse to rise from 10% of initial value to 90% of a final value is called Rise time

$$\boxed{Y(t) = \left(1 - e^{-t_{0.1}/RC}\right)} \quad ①$$



The output of the low pass filter is defined as

$$y(t) = 1 - e^{-t_{0.1}/RC}$$

Then we get

$$y(t) = 1 - e^{t_{0.1}/RC} \text{ at } 0.1 \text{ time}$$

Then

$$0.1 = 1 - e^{-t_{0.1}/RC}$$

$$e^{-t_{0.1}/RC} = 1 - 0.1$$

$$e^{-t_{0.1}/RC} = 0.9$$

Applying $\log_e$ on B.S

$$\log e^{-t_{0.1}/RC} = \log_e 0.9$$

$$\frac{-t_{0.1}}{RC} = \log_e 0.9$$

$$-t_{0.1} = RC \log_e 0.9$$

$$\boxed{t_{0.1} = -RC \log_e (0.9)} \quad ①$$

$$y(t) = 1 - e^{-t_{0.9}/RC}$$

$$0.9 = 1 - e^{-t_{0.9}/RC}$$

$$e^{\frac{-t_{0.9}}{RC}} = 1 - 0.9$$

$$e^{-t_{0.9}/RC} = 0.1$$

Apply log on B.S

$$\log e^{-t_{0.9}/RC} = \log_e (0.1)$$

$$\frac{-t_{0.9}}{RC} = \log_e (0.1)$$

$\boxed{t_{0.9} = -R_C \log_e (0.1)}$ —— (3)

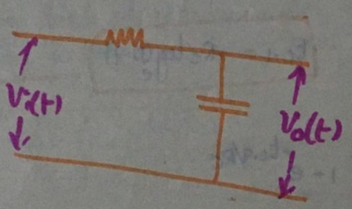
$t_r = t_{0.9} - t_{0.1} =$

$$= -R_C \log(0.1) + R_C \log(0.9)$$

Risetime t_r of an R_C low pass filter is $\frac{0.35}{f_{3dB}}$

where $f_{3dB} = \frac{1}{2\pi R_C}$ which is 3dB Band width of a filter

Low pass filter can be drawn as



$$y(t) = h(t) * x(t)$$

unit step response of a low pass filter is obtained by convolution integral $y(t) = h(t) * x(t)$ we write as $h(t)$ is a unit impulse response $h(t) = \frac{1}{R_C}\left\{e^{-t/R_C}\right\}$

$$h(t) = \int_{-\infty}^{\infty} \frac{1}{R_C} e^{-t/R_C}\, dt$$

$$= \int_0^t \frac{1}{R_C} e^{-t/R_C}\, dt$$

$$= \frac{1}{R_C} \int_0^t e^{-t/R_C}\, dt$$

$$= \frac{1}{RC}\left[\frac{e^{-t/RC}}{-1/RC}\right]_0^t$$

$$= \frac{1}{RC} \times -RC \left[e^{-t/RC}\right]_0^t$$

$$= -\left[e^{-t/RC} - e^{0/RC}\right]$$

$$= -\left[e^{-t/RC} - 1\right]$$

$$\boxed{y(t) = 1 - e^{-t/RC}}$$

$$0.1 = 1 - e^{-t_{0.1}/RC}$$

$$e^{-\frac{t_{0.1}}{RC}} = 1 - 0.1$$

$$e^{-\frac{t_{0.1}}{RC}} = 0.9$$

Apply log on B.S

$$\frac{-t_{0.1}}{RC} = \log(0.9)$$

$$-t_{0.1} = RC \log(0.9)$$

$$t_{0.1} = -RC \log(0.9)$$

$$0.9 = 1 - e^{-t_{0.9}/RC}$$

$$e^{-\frac{t_{0.9}}{RC}} = 1 - 0.9$$

$$e^{-\frac{t_{0.9}}{RC}} = 0.1$$

Apply log on B.S

$$\frac{-t_{0.9}}{RC} = \log 0.1$$

$$t_{0.9} = -R_C \log(0.1)$$

$$\frac{①}{②} = \frac{e^{\frac{-t_{0.1}}{RC}}}{e^{\frac{-t_{0.9}}{RC}}} = \frac{0.9}{0.1} = 9$$

$$= e^{\frac{-t_{0.1}}{RC} + \frac{t_{0.9}}{RC}} = 9$$

$$= e^{\left[\frac{t_{0.9}}{RC} - \frac{t_{0.1}}{RC}\right]} = 9$$

$$= e^{\frac{1}{RC}\left[t_{0.9} - t_{0.1}\right]} = 9$$

Apply log on B.S

$$\frac{1}{RC}\left[t_{0.9} - t_{0.1}\right] = \log 9$$

$$t_{0.9} - t_{0.1} = RC \log 9$$

$$t_r = RC \log(9)$$

$$t_r = RC\, 2.197$$

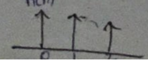
w.k.T $t_r = RC \times \frac{1}{2\pi f_{3dB}}$

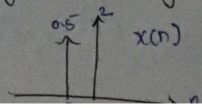
$$\boxed{t_r = \frac{0.35}{f_{3dB}}}$$

Consider an LTI system with impulse response h(n) & x(n)

a) find the response (or) ecos ~~...~~ non zero values of input

b) The overall response of y(n), h(n) is given by the wave form shown as





$$\boxed{\begin{aligned} y(n) &= x(n) * h(n) \\ y(n) &= \sum_{k=-\infty}^{\infty} x(k)\, h(n-k) \end{aligned}}$$

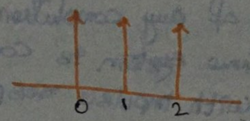
$$y(n) = \sum_{k=-\infty}^{\infty} x(k)\, h(n-k)$$

$$= \sum_{k=0}^{1} x(k)\, h(n-k)$$

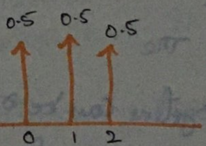
$$= x(0)\, h(n-0) + x(1)\, h(n-1)$$

$$= 0.5\, h(n) + 2\, h(n-1)$$

h(n) =

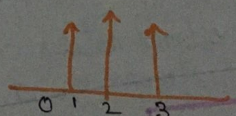


0.5 h(n)

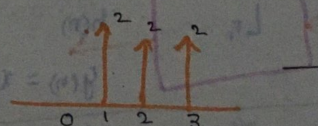


— ①

h(n-1)

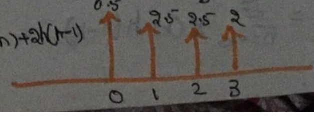


2h(n-1)



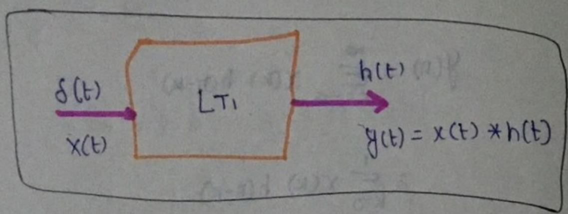
— ②

y(n) = 0.5h(n)+2h(n-1)



Convolution and co-relation of signals

concept of convolution in time domain and frequency domain represented as the block diagram

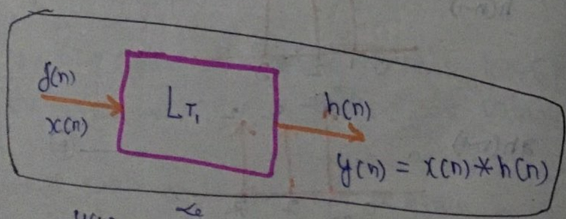


$$y(t) = \int_{-\infty}^{\infty} x(\tau) \cdot h(t-\tau)\, d\tau \quad \text{—} ①$$

$$y(t) = x(t) * h(t) = \int_{-\infty}^{\infty} x(\tau)\, h(t-\tau)\, d\tau \quad \text{—} ②$$

The output of any convolution time or continuous time system is convolution of input $x(t)$ with impulse response $h(t)$ of the system.

similarly The discreet system can be represented as convolution for discreet signals.



$$y(n) = \sum_{n=-\infty}^{\infty} x(k) \cdot h(n-k)$$

$$y(n) = x(n) * h(n) = \sum_{n=-\infty}^{\infty} x(k)\, h(n-k)$$

o/p of any discreet time LTI system is convolution of input $x(n)$ with impulse response we can obtaine

$$\boxed{x(t) * x_2(t) \longleftrightarrow X_1(\omega) \cdot X_2(\omega)}$$

The Laplace LTI play important role in convolution The linear time invariant system can be represented as $X_1(t) * X_2(t)$

$$\boxed{x_1(t) * x_2(t) \longleftrightarrow \frac{1}{2\pi} |X_1(\omega)| \, |X_2(\omega)|}$$

If the multiplication property and is often reffered as the frequency convolution the The Multiplication in time domain will become convolution in frequency domain.

The Co-rrelataion function:-

① It gives the similarity b/w Two signals. The corelation functions for energy signals and power signals can be represented seperately.

Problem :-

convolute $x(n)$, $h(n)$ find $y(n)$ where $x(n) = \{1, \underset{\uparrow}{4}, 3, 2\}$

$h(n) = \{1, 3, 2, 1\}$

Step 1 :-

$n_1 \rightarrow$ starting time of $x(n) = -1$

$n_2 \rightarrow$ starting time of $h(n) = 0$

$$\boxed{n = n_1 + n_2}$$

$$n = -1 + 0 = -1$$

$$\boxed{n = -1}$$

step 2 :- $h(n) = \{1, 3, 2, 1\}$ represent in k terms

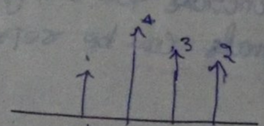
$h(k) = \{1, 3, 2, 1\}$

step 3 :- $h(-k)$

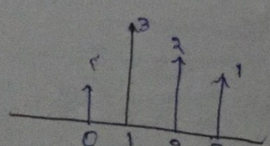
step 4 :- $h(-k-1)$

step 5 :-

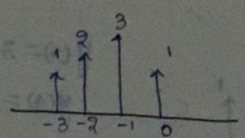
$y(n) = x(n) * h(n)$



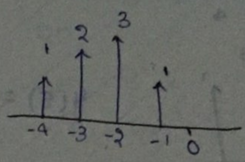
x(n)



h(k)

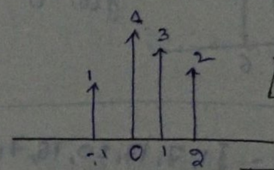


h(-k)



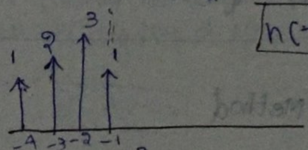
h(-k-1)

(or) directly we can write as

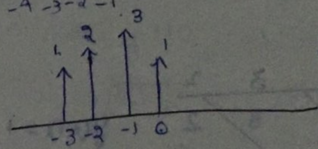


x(k)

h(-k-1)



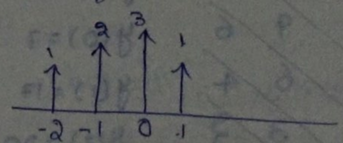
$y(-1) = 1$



$y(0) = 4+3 = 7$

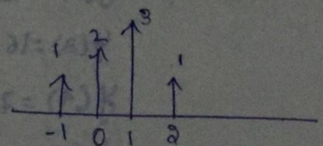
y(0) ~~y(0) = 3+6+2~~

$y(0) = 7$



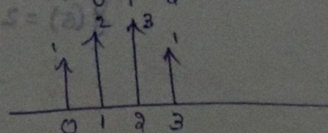
$y(1) = 12+3+2$

$y(1) = 17$



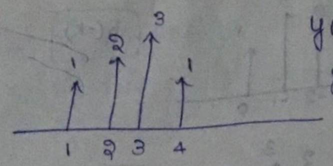
$y(2) = 8+9+2+1$.

$y(2) = 20$



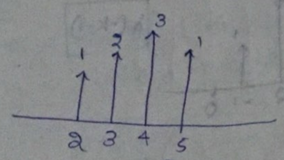
$y(3) = 4+6+6$

$y(3) = 16$

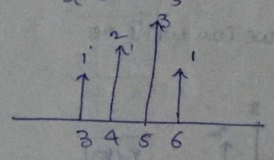


$y(4) = 3+4$

$y(4) = 7$



$y(5) = 2$



$y(6) = 0$

output $\boxed{y(n) = \{1, 7, 17, 20, 16, 7, 2\}}$

second method

h(n) \ x(n)	1	4	3	2
1	1	~~2~~ 4	3	2
3	3	12	9	6
2	2	8	6	4
1	1	4	3	2

$y(-1) = 1$
$y(0) = 7$
$y(1) = 17$
$y(2) = 20$
$y(3) = 16$
$y(4) = 7$
$y(5) = 2$

$x(n) = \{1, \underset{\uparrow}{5}, 3, 2\}$ $h(n) = \{1, 4, 3, 2\}$

	1	5	3	2
1	1	5	3	2
4	4	20	12	8
3	3	15	9	6
2	2	10	6	4

$y(-1) = 1$
$y(0) = 9$
$y(1) = 26$
$y(2) = 31$
$y(3) = 27$
$y(4) = 12$
$y(5) = 4$

$y(n) = \{1, 9, 26, 31, 27, 12, 4\}$

step 1:-

n_1 :- starting time of $x(n)$
n_2 :- starting time of $h(n)$

$\boxed{n = n_1 + n_2}$

$n = -1 + 0 = -1$

$\boxed{n = -1}$

step 2:-

$h(n) = \{1, 4, 3, 2\}$ represent in k terms.

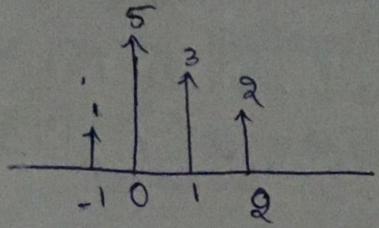
$h(k) = \{1, 4, 3, 2\}$

step 3:- $h(-k)$

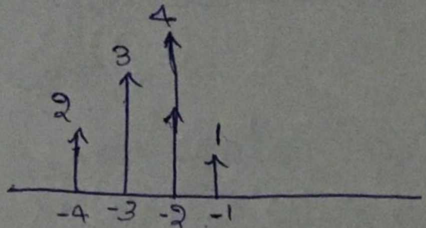
step 4:- $h(-k-1)$

step 5:- $y(n) = x(n) * h(n)$

$x(n) = \{1, \underset{\uparrow}{5}, 3, 2\}$ $\quad h(n) = \{\underset{0}{1}, \underset{1}{4}, \underset{2}{3}, \underset{3}{2}\}$

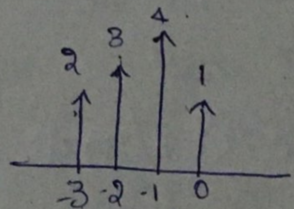


$x(k)$

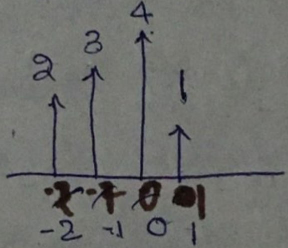


$h(-k-1)$

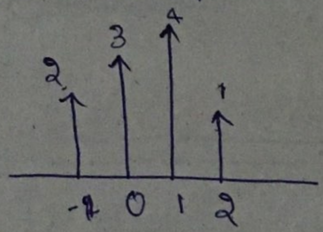
$y(-1) = 1$



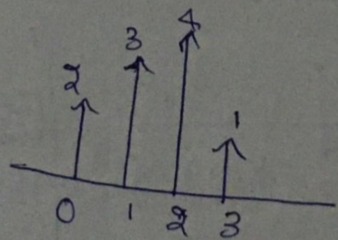
$y(0) = \cancel{2} 5 + 4$
$= 9$



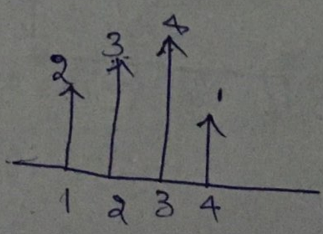
$y(1) = 3 + 20 + 3$
$= 26$



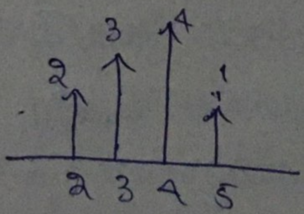
$y(2) = 2 + 15 + 12 + 2$
$= 31$



$y(3) = 10 + 9 + 8$
$= 27$



$y(4) = 6 + 6$
$= 12$



$y(5) = 4$

$y(6) = 0$

Energy signals:-

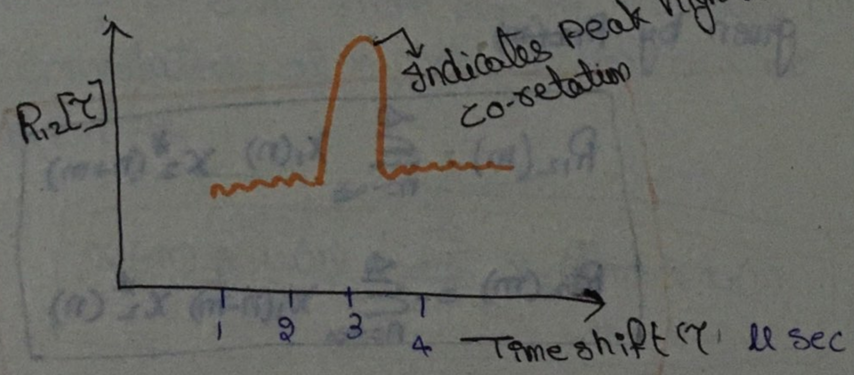
let us now define co-relation for energy signal it is given by the Mathematic following expression.

$$\boxed{R_{12}[\tau] = \int_{-\infty}^{\infty} x_1(t) * x_2(t-\tau)\, d\tau}$$

$x_1(t), x_2(t)$ are complex valued signal of finate energy.

$R_1[\tau]$ is co-relation function of $x_1(t)$ & $x_2(t)$ in the above eqn we have defined a co-relation function. Here note that co-relation function is a function of τ. and τ is time delay or time shift of one of the Two signals. i.e one of the time signal is time shifted $R_{12}[\tau]$ is calculated for various values of τ. This co-relation function has many values.

Explanation of co-relation function:-



In the above figure observe that co-relation function has highest values at $\tau = 3\mu sec$ and the rest of the other values of τ. is near Zero. The values of the co-relation function or minimum. i.e Two signals have highest co-relation for delay $\tau = 3$ µsec for other delay's the two signals are not much co-related.

(2) if the two signals are ~~&~~ real then the Correlation function $R_{12}[\tau]$ Then

$$R_{12}[\tau] = \int_{-\infty}^{\infty} x_1(t) * x_2(t-\tau)\, d\tau$$

$$R_{12}[\tau] = \int_{-\infty}^{\infty} x_1(t)\, x_2^*(t+\tau)\, d\tau$$

$$R_{12}[\tau] = \int_{-\infty}^{\infty} x_1(t-\tau)\, x_2(t)\, dt$$

$$R_{12}[\tau] = \int_{-\infty}^{\infty} x_1(t+\tau)\, x_2^*(t)\, dt$$

The above eqn's produce similar results.

co-relation function of discreet time energy signals:-

let us define co-relation function of discreet time signals given by ~~$R_{12}(m)$~~

$$R_{12}(m) = \sum_{n=-\infty}^{\infty} x_1(n)\, x_2^*(n+m)$$

$$R_{12}(m) = \sum_{n=-\infty}^{\infty} x_1(n-m)\, x_2^*(n)$$

In the above eqn's one signal is delaye'd or advanced by m samples. The co-relation function $R_{12}[m]$ is the samples of m.

Relation ship b/w co-relation and convolution:-

consider the convolution of two sequences $x_1(n)$ & $x_2(n)$

i.e

$$x_1(n) * x_2(n) = \sum_{k=-\infty}^{\infty} x_1(n-k)\, x_2(k) \quad \text{—(1)}$$

~~$x_1(m) * x_2$~~

In the above eqn 'n' is just index. hence it can be replaced by any other variable let replace n by 'm' then we get

$$x_1(m) * x_2(m) = \sum_{k=-\infty}^{\infty} x_1(m-k)\, x_2(k) \quad \text{—(2)}$$

In the above eqn k is just an index. replace it with any other index 'n' then the eqn becomes.

$$x_1(m) * x_2(m) = \sum_{n=-\infty}^{\infty} x_1(m-n)\, x_2(n) \quad \text{—(3)}$$

In the above eqn $x_1(m)$ ~~can~~ be inverted time then convolution of $x_1(-m)$, $x_2(m)$ can be written as.

$$x_1(-m) * x_2(m) = \sum_{n=-\infty}^{\infty} x_1(n-m) * x_2(n), \quad \text{—(4)}$$

after 3rd eqn

$$x_1(-m) * x_2(m) = \sum_{n=-\infty}^{\infty} x_1(-m-n)\, x_2(n)$$

$$x_1(-m) * x_2(m) = \sum_{n=-\infty}^{\infty} x_1 -(m+n)\, x_2(n)$$

The above eqn (comparing R.H.S)

$$\boxed{x_1(-m) * x_2(m) = \sum_{n=-\infty}^{\infty} x_1(n-m) * x_2(n)} \quad \text{--- (4)}$$

Thus the co-relation of two signals is obtained by convolution of one signal with time folded version of another signal. This similar principle is ~~true~~ true for continuous time signals can be stated as.

$$\boxed{x_1(-m) * x_2(m) = R_{12}(m)} \quad \text{--- (5)}$$

$$R_{12}(m) = x_1(-m) * x_2(m)$$
$$R_{12}(\tau) = x_1(-\tau) * x_2(\tau) \quad \text{--- (6)}$$

Properties of Fourier transform:-

① $x^*(t) \longrightarrow X^*(\omega)$

$X(-t) \longrightarrow X^*(\omega)$

$R_{12}(m) \longrightarrow X_1'(\Omega) \cdot X_2(\Omega)$

$R_{12}(\tau) \longrightarrow X_1^*(\omega) \cdot X_2(\omega)$

Thus co-relation can be calculated by multiplying fourier transform of one function and complex conjugate of fourier transform of another function. Inverse fourier transform of product gives co-relation function.

Power signals:-

(f) The co-relation function of periodic power signals is given by

$$\boxed{R_{12}[\tau] = \lim_{T\to\infty} \frac{1}{2T}\int_{-T}^{T} x(t) \cdot x_2^*(t-\tau)\, dt} \quad \text{--- (1)}$$

If we compare above Eqn with the ~~eqn~~ function of energy signals is given as

$$R_{12}[\tau] = \int_{-\infty}^{\infty} x_1(t)\; x_2^*(t-\tau)\, d\tau \quad \text{--- (2)}$$

If we compare above eqn with co-relation function of energy signals given by above eqn we find that integration result is divided by $\frac{1}{2T}$ and limitting condition as $T \to \infty$ when x_1, x_2 both are real, we can write the eqns as

$$R_{12}(\tau) = \lim_{T\to\infty} \frac{1}{2T}\int_{-T}^{T} x(t)\, x_2(t-\tau)\, d\tau \quad \text{--- (3)}$$

$$R_{12}(\tau) = \lim_{T\to\infty} \int_{-T/2}^{T/2} x_1(t)\, x_2^*(t+\tau)\, d\tau \quad \text{--- (4)}$$

$$R_{12}(\tau) = \lim_{T\to\infty} \int_{-T/2}^{T/2} x(t-\tau) x_2^*(t)\, dt \quad \text{--- (5)}$$

for discreet values we can write as

$$\boxed{R_{12}[m] = \lim_{N\to\infty} \frac{1}{N}\sum_{n=\langle N\rangle} x_1(n)\, x_2^*(n+m)} \quad \text{--- (6)}$$

Scanned with CamScanner

$$\boxed{R_{12}(m) = \lim_{N\to\infty} \frac{1}{N} \sum_{n=<N>} x_1(n-m)\, x_2^*(m)}$$ — 7

1 — Auto co-relation:-

1. when we calculate co-relation of a signal with it self then it is called ~~ortho~~ so Auto Co-relation
2. Thus if $x_1(t) = x_2(t)$ then Auto co-relation function can be written as [Auto co-relation function for energy signals] is

$$\boxed{R[\tau] = \int_{-\infty}^{\infty} x(t)\, x^*(t-\tau)\, dt} \quad \text{— (1)}$$

In the above eqn if the complex variables is delayed in other words the real valued signal is delayed then the above function can be written as

3 — $$\boxed{R[\tau] = \int_{-\infty}^{\infty} x(t+\tau)\, x^*(t)\, dt}$$

4 — similarly Auto co-relation of a discreet time signal is given by ~~R~~

2 — ~~R[m]~~=

$$R[m] = \sum_{n=-\infty}^{\infty} x(n)\, x^*(n-m)$$

2 — $$= \sum_{n=-\infty}^{\infty} x(n+m)\, x^*(n)$$

Auto corelation of Power signals:-

It can be represented as

$$\boxed{R(\tau) = \frac{1}{T_0} \int_{-T_0/2}^{T_0/2} x(t)\, x^*(t-\tau)\, dt} \quad \text{— (1)}$$

τ is in negative direction then we can write as

$$\boxed{R[\tau] = \frac{1}{T_0} \int_{-T_0/2}^{T_0/2} x(t+\tau)\, x^*(t)\, dt} \quad \text{— (2)}$$

for any values of 't' we can write as

$$\boxed{R[\tau] = \frac{1}{T} \int_{-T/2}^{T/2} x(t)\, x^*(t-\tau)\, d\tau} \quad \text{— (3)}$$

Auto co-relation function for discreet

$$\boxed{R(m) = \lim_{N\to\infty} \frac{1}{N} \sum_{n=<N>} x(n)\, x^*(n+m)}$$

explain the Relation to signal energy and signal Power

(or)

explain the relation ship b/w signal energy and signal Power

Proof:- we define Auto corelation function of energy signals and Power signals now let us relate Auto co-relation to energy and power of a signal

Relation of Auto Corelation with energy:-

consider the Auto corelation of energy signal as

$$R[\tau] = \int_{-\infty}^{\infty} x(t)\, x^*(t-\tau)\, dt$$

$$= \int_{-\infty}^{\infty} x(t)\, x^*(t)\, dt \quad \because \tau = 0$$

$$= \int_{-\infty}^{\infty} |x(t)|^2\, dt$$

$$\boxed{R(\tau) = E}$$

Relation of Auto co-relation with Power

$$R(\tau) = \lim_{T\to\infty} \frac{1}{T} \int_{-T/2}^{T/2} x(t)\, x(t-\tau)\, dt$$

$$= \lim_{T\to\infty} \frac{1}{T} \int_{-T/2}^{T/2} x(t)\, x(t)\, dt \quad \tau = 0$$

$$= \lim_{T\to\infty} \frac{1}{T} \int_{-T/2}^{T/2} |x(t)|^2\, dt$$

$$\boxed{R(\tau) = P}$$

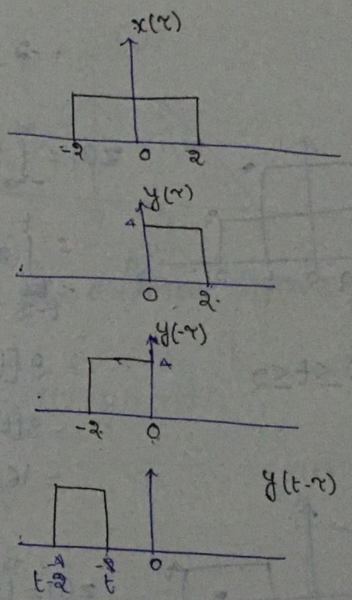
problems:-

① Obtain $x(t)$ convoluted with $y(t)$ and sketch the results.

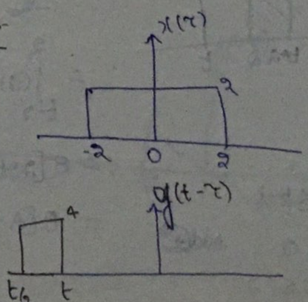
$$x(t) = 2 \quad \text{for } \{-2 \le t \le 2\}$$
$$= 0 \quad \text{else where}$$

$$y(t) = 4 \quad \text{for } 0 \le t \le 2$$
$$= 0 \quad \text{else where}$$

$$z(t) = x(t) * y(t)$$



step 1:-

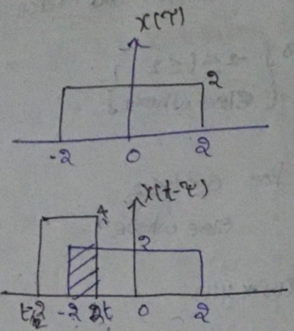


$$z(\tau) = \int_{-\infty}^{\infty} x(\tau)\, y(t-\tau)\, d\tau$$

$$= \int_{-\infty}^{t-2} 0 \cdot 0 + \int_{t-2}^{-2} 0 + \int_{-2}^{0} 2 \cdot 0\, d\tau + \int_{0} 2 \cdot 0\, d\tau$$

$$= 0$$

Step:-2



$z(t) = \int_{-\infty}^{\infty} x(\tau)\, y(t-\tau)\, d\tau$

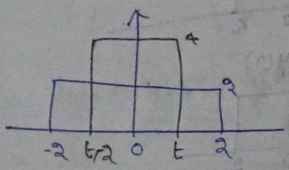
$= \int_{-2}^{t} (2)(4)\, d\tau$

$= \int_{-2}^{t} 8\, d\tau$

$= 8[t+2]$

$= -2 \le t \le 0$

Step 3:-



$z(t) = \int_{-\infty}^{\infty} x(\tau)\, y(t-\tau)\, d\tau$

$= \int_{t-2}^{t} (2)(4)\, d\tau$

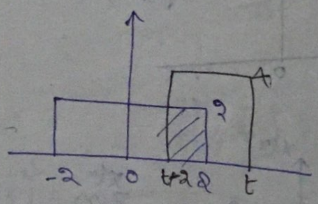
$0 \le t \le 2$

$= 8[t-(t-2)]$

$= 8[t-t+2]$

$= 16$

Step:-4



$z(t) = \int_{-\infty}^{\infty} x(\tau)\, y(t-\tau)\, d\tau$

$= \int_{t-2}^{2} (2)(4)\, d\tau$

$t-2 \le 2 < t$

$\cancel{-t}+\cancel{t}-2 \le 2-t \le \cancel{t}-\cancel{t}$

$= 8[2-t+2]$

$-2 \le 2-t \le 0$ Add(-2)

$= 8[4-t]$

$-4 \le -t \le -2$

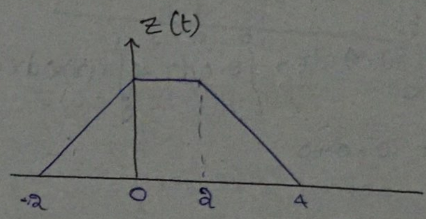
$2 \le t \le 4$

$z(\tau) = 0 \; : \; t \le 0$

$8t+16 \quad -2 \le t \le 0$

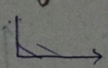
$16 \quad 0 \le t \le 2$

$-8t+32 \quad 2 \le t \le 4$



* Convolute Two different signals $y(t) = t \; ; \; 0 < t < 2$

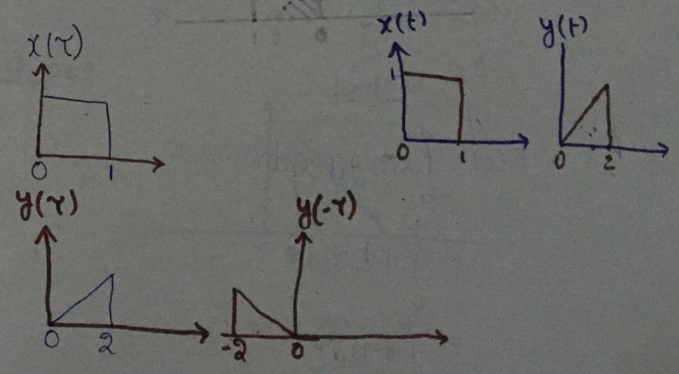
$= 0 \; ;$ else where



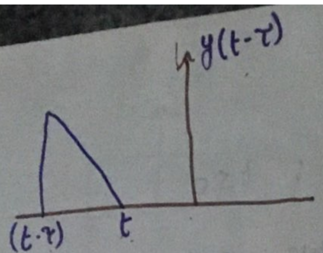
$x(t) = 1 \quad 0 < t < 1$

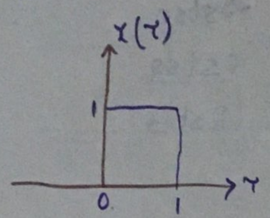
$= 0$ else where

$z(t) = \int_{-\infty}^{\infty} x(\tau)\, y(t-\tau)\, d\tau$



Step:1





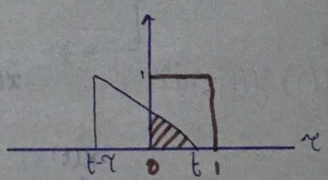
$\boxed{t<0}$

$$z(t) = \int_{-\infty}^{t} y(t-\tau)\, 0 d\tau + \int_{t}^{0} 0 \cdot 0\, d\tau + \int_{0}^{1} x(\tau) \times 0\, d\tau$$

$$= 0+0+0$$

$$= 0$$

step2:-



$0 \leq t \leq 1$

$$z(t) = \int_{-\infty}^{\infty} x(\tau)\, y(t-\tau)\, d\tau$$

$$= \int_{0}^{t} 1 (t-\tau)\, d\tau$$

$$= \int_{0}^{t} (t-\tau)\, d\tau$$

$$= t\, (\tau)_0^t - \left(\frac{\tau^2}{2}\right)_0^t$$

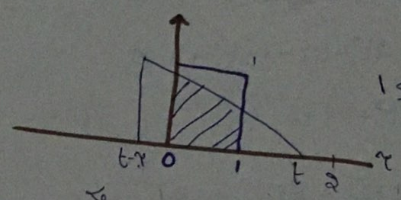
$$t(t) - \frac{t^2}{2}$$

$$\frac{2t^2 - t^2}{2}$$

$$\frac{t^2}{2}$$

$\boxed{0 \leq t \leq 1}$

step3:-



$1 \leq t \leq 2$

$$z(t) = \int_{-\infty}^{\infty} x(\tau)\, y(t-\tau)\, d\tau$$

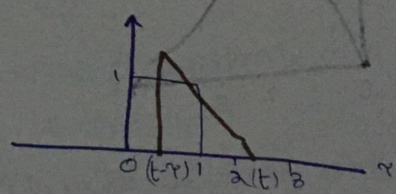
$$= \int_{0}^{1} 1\, (t-\tau)\, d\tau$$

$$= t\, (\tau)_0^1 - \left(\frac{\tau^2}{2}\right)_0^1$$

$$= t - \frac{1}{2}$$

$$= \frac{2t-1}{2} \Rightarrow 1 \leq t \leq 2$$

step4:-



$$z(t) = \int_{-\infty}^{\infty} x(\tau)\, y(t-\tau)\, d\tau$$

$$= \int_{t-2}^{1} (t-\tau)\, d\tau$$

$t[\tau]_{t-2}^{1} - \left[\frac{\tau^2}{2}\right]_{t-2}^{1}$

$t[1-(t-2)] - \frac{1}{2}\left[1^2-(t-2)^2\right]$

$t-t^2+t2-\frac{1}{2}[1-t^2-4+2t]$

$\cancel{t-t^2+t\tau+\frac{t}{2}}\,t\,2$

$2t-2t^2+t\tau-1+t^2+4-2t$

$-t^2+t\tau+3$

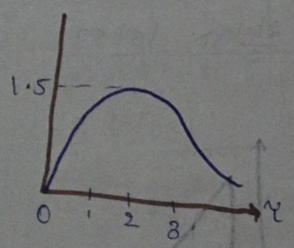
$t[1-t+2]-\frac{1}{2}[1-(t-2)^2]$

$t[3-t]-\frac{1}{2}[-t^2-4+2t-1]$

$3t-t^2-\frac{1}{2}[-t^2+2t-5]$ $\boxed{2\leq t\leq 3}$

$[3t-t^2-\frac{1}{2}+\frac{t^2}{2}+2-2t]$

4M **Energy spectral density :-**



Energy s

4M write a note on Energy spectral density?

The spectral density function gives the distribution of energy of a signal in a frequeny domain.

it can be represented as

$$E=\int_{-\infty}^{\infty}|x(t)|^2\,dt \quad \text{— (1)}$$

$$=\frac{1}{2\pi}\int_{-\infty}^{\infty}|X(\omega)|^2\,d\omega \quad \text{— (2)}$$

$$=\frac{1}{2\pi}\int_{-\infty}^{\infty}|X(2\pi f)|^2\,2\pi\cdot df \quad \text{— (3)}$$

$$E=\int_{-\infty}^{\infty}|X(f)|^2\,df$$

$$\psi(f)=|X(f)|^2$$

$$E=\int_{-\infty}^{\infty}\psi(f)\,df \quad \text{— (4)}$$

This EQn's shows that Total energy of a signal is given by the total area under the curve $\psi(f)$

$\psi(f)$ represents energydensity of a signal.

units is Jouls/Hz.

$\psi(f)$ is called as energy spectral density of a signal $x(t)$.

Explain conceptional block diagram of energy spectral density?

① Normally ESD spectrum is plotted for possitive as well as -ve frequencies.

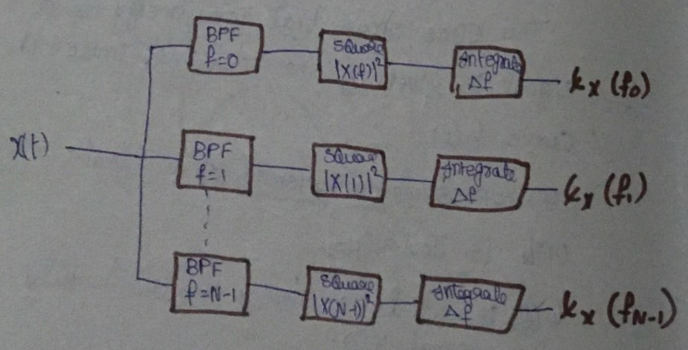
② The single sided ESD of signal is twice the Double side ESD.

③ ESD is symmetric around $f=0$

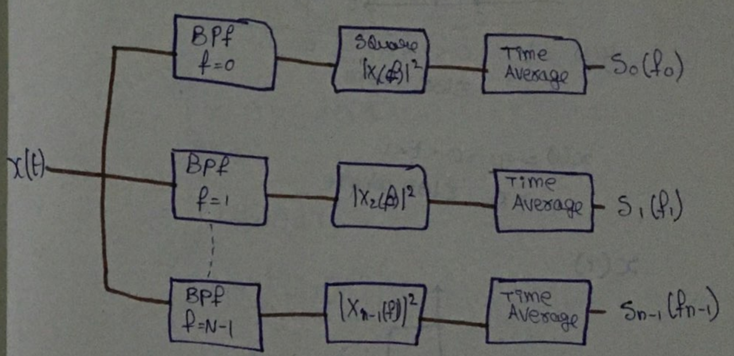
④ $x(t)$ is applied to various Band width filters having the same pass Band of Δf. and centered frequencies $f_0, f_1, f_2 - - - - f_{n-1}$.

⑤ Then signal of each section is squared and then integrated over Δf. which gives the spectral density $\psi(f)$ for different frequencies $f_0, f_1, f_2, \ldots f_{n-1}$

Block diagram of Conceptional ESD.



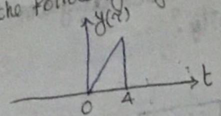
Draw the conceptional block diagram of Power spectrum density?



Explain the differences b/w ESD and PSD?

ESD	PSD
1. It gives the distribution of energy of a signal in frequency domain	1. It gives the distribution of power of signal in frequency domain
2. ESD is defined as $\psi(f) = \lvert X(f)\rvert^2$, $E = \int_{-\infty}^{\infty} \lvert X(f)\rvert^2 = \int_{-\infty}^{\infty} \psi(f)df$	2. PSD is defined as $S(f) = \frac{1}{T^2}\sum_{k=-\infty}^{\infty} \lvert X(kf_0)\rvert^2 \delta(f-kf_0)$, $S(t) \rightarrow \int_{-\infty}^{\infty} S(f)\, df$
3. Auto corelation function for energy signal and ESD form a fourier transform pair	3. Auto co-relation of Power signal and PSD form a fourier transform pair

convolute the following signal and sketch the results



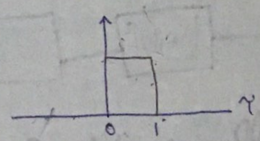
$y(t) = t \quad 0<t<4$

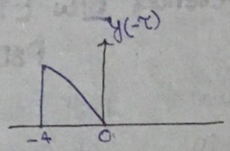
$\quad = 0 \quad$ Else where

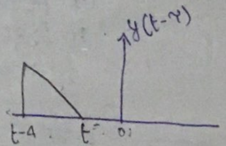
$x(t) = 1 \quad 0<t<1$

$\quad = 0 \quad$ Else where

$x(\tau)$







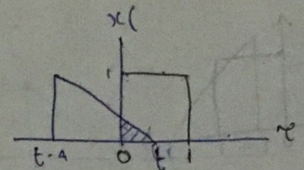
$$z(t) = \int_{-\infty}^{\infty} x(\tau)\, y(t-\tau)\, d\tau$$

Step 1:-

$$= \int_{t-4}^{t} y(t-\tau)\times 0 + \int_{t}^{0} 0\cdot 0 + \int_{0}^{1} x(\tau)\times 0 + \int_{1}^{\infty} 0\times 0$$

$= 0$

Step 2:-



$$z(t) = \int_{-\infty}^{\infty} x(\tau)\, y(t-\tau)\, d\tau \qquad 0 \le t \le 1$$

$$= \int_{0}^{t} 1\cdot(t-\tau)\, d\tau$$

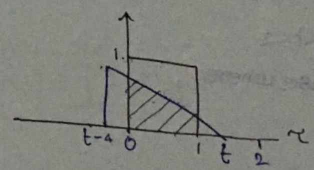
$$= t[\tau]_0^t - \left[\frac{\tau^2}{2}\right]_0^t$$

$$= t^2 - \frac{t^2}{2}$$

$$= t^2\left[1-\frac{1}{2}\right]$$

$$= t^2/2$$

Step 3:-



$$z(t) = \int_{-\infty}^{\infty} x(\tau)\, y(t-\tau)\, d\tau$$

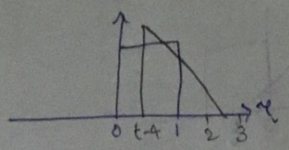
$$= \int_{0}^{1} 1\cdot(t-\tau)\, d\tau$$

$$= t[\tau]_0^1 - \left[\frac{\tau^2}{2}\right]_0^1$$

$$= t[1] - \frac{1}{2} \qquad 1 \le t \le 2$$

$$= \frac{2t-1}{2}$$

step 4:-



$$z(t) = \int_{-\infty}^{\infty} x(\tau)\, y(t-\tau)\, d\tau$$

$$= \int_{t-4}^{1} (t-\tau)\, d\tau$$

$$= t[\tau]_{t-4}^{1} - \left[\frac{\tau^2}{2}\right]_{t-4}^{1}$$

$$= t[1-t+4] - \frac{1}{2}\left[1-(t^2+16-8t)\right]$$

$$= t - t^2 + 4t - \frac{1}{2}\left[-t^2+8t-15\right]$$

Problem:-

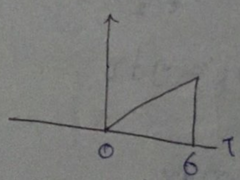
$y(t) = t \quad 0<t<6$

$= 0$ Else where

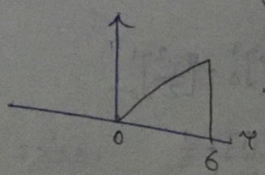
$x(t) = 1 \quad 0<t<2$

0 Else where

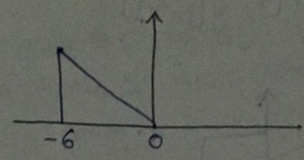
$y(t)$



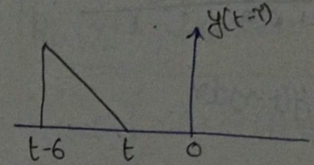
$y(\tau)$



$y(-\tau)$



$y(t-\tau)$



Step 1:-

$$z(t) = \int_{-\infty}^{\infty} x(\tau)\, y(t-\tau)\, d\tau$$

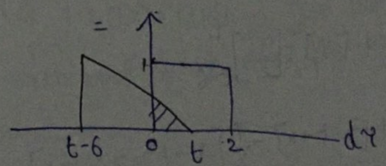
$$= \int_{t-6}^{t} (0) * (t-\tau) + \int_{t}^{0} 0 \times 0 + \int_{0}^{6} 1 * 0$$

$= 0 //$

Step 2:-

$$z(t) = \int_{-\infty}^{\infty} x(\tau)\, y(t-\tau)\, d\tau$$

$=$

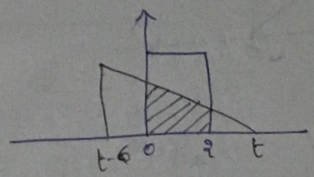


$$= \int_{0}^{t} 1 \cdot (t-\tau)\, d\tau$$

$$= t[\tau]_0^t - \left[\frac{\tau^2}{2}\right]_0^t$$

$$= t[t-0] - \frac{1}{2}[t^2-0] = t^2 - \frac{t^2}{2} = t^2/2$$

Step 3:- $z(t) = \int_{-\infty}^{\infty} x(\tau)\, y(t-\tau)\, d\tau$



$$z(t) = \int_{0}^{\infty} x(\tau) y(t-\tau) d\tau$$

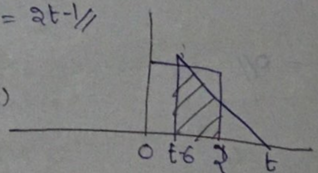
$$= \int_{0}^{2} 1\,(t-\tau)\, d\tau$$

$$= t[\tau]_0^2 - \left[\frac{\tau^2}{2}\right]_0^2$$

$$= t[2] - [1]$$

$$= 2t - 1$$

Step 4:- $z(t)$



$$\int_{t-6}^{2} (t-\tau) d\tau$$

$$t[\tau]_{t-6}^{2} - \left[\frac{\tau^2}{2}\right]_{t-6}^{2}$$

$$t[2-t+6] - \frac{1}{2}\left[4 - t^2 - 36 + 12t\right]$$

$$2t - t^2 + 6t - \frac{1}{2}\left[-t^2 + 12t - 32\right]$$

Cross corelation of energy signals and properties:-

1. Two signals $x_1(t)$ & $x_2(t)$ is a pair of complex valued signals of finite energy the cross co-relation of two signals is given by $R_{12}[\tau] = \int_{-\infty}^{\infty}$

$$\boxed{R_{12}[\tau] = \int_{-\infty}^{\infty} x(t)\, x_2^*(t-\tau)\, dt} \quad \text{—— ①}$$

If $\int_{-\infty}^{\infty} x(t)\, x_2^*(t) = 0$ Then $R_{12} = 0$

$x_1(t), x_2^*(t)$ are orthogonal signals.

Properties:-

1. The cross co relation function exibits symmetry

$$\boxed{R_{12}[\tau] = R_{21}^*[\tau]} \text{—— ①}$$

cross corelation is not a general commutative

i.e $\boxed{R_{12}[\tau] \neq R_{21}[\tau]}$ —— ②

② property 2:-

If $R_{12}[0] = 0$ Then the signals can be represented i.e $\int_{-\infty}^{\infty} x_1(t) x_2^*(t) dt = 0$ Then the signals are orthogonal signals over the entire interval.

Property 3:-

The cross co-relation of two energy signals corresponds to multiplication of fourier transform of one signal by complex conjugate of by fourier transform of second signal.

$$R_{12}[\tau] \longrightarrow X_1(f)\, X_2^*(f)$$

This is known as co-relation theorm.

LAPLACE TRANSFORM

Laplace transform is a function of $f(t)$ is defined for $t>0$ and can be represented as

$f(s) = L[f(t)]$

$$\boxed{F(s) = L[f(t)] = \int_0^\infty f(t)\, e^{-st}\, dt}$$

problem :-

find the Laplace transform of a standard function k?

$$= \int_0^\infty f(t) e^{-st} dt$$

$$= \int_0^\infty k e^{-st} dt$$

$$= k \int_0^\infty e^{-st} dt$$

$$= k \left[\frac{e^{-st}}{-s}\right]_0^\infty$$

$$= k\left[0 - \left(\frac{1}{-s}\right)\right]$$

$$= k/s$$

$$\boxed{L[f(t)] = k = \frac{k}{s}}$$

find the Laplace transform of e^{at}?

$$\int_0^\infty e^{at} e^{-st} dt$$

$$\int_0^\infty e^{-t(s-a)} dt$$

$$\left\{\frac{e^{-t(s-a)}}{-(s-a)}\right\}_0^\infty$$

$$= \frac{e^{-\infty}}{-(s-a)} - \left\{\frac{e^{0(s-a)}}{-(s-a)}\right\}$$

$$= \frac{1}{s-a}$$

$$\boxed{\therefore L[e^{at}] = \frac{1}{s-a}} \quad s-a>0$$

find the Laplace transform of e^{-at}?

$$\int_0^\infty f(t) e^{-st} dt = \int_0^\infty e^{-at} e^{-st} dt$$

$$= \int_0^\infty e^{-t(s+a)} dt$$

$$= \left\{\frac{e^{-t(s+a)}}{-t(s+a)}\right\}_0^\infty$$

$$= \frac{e^{-\infty}}{-(s+a)} - \left[\frac{e^{0(s+a)}}{-(s+a)}\right]$$

$$= \frac{1}{s+a}$$

Scanned with CamScanner

find L.[sin at]

$$\int_0^\infty f(t)\, e^{-st}\, dt \qquad \therefore \sin at = \frac{e^{iat} - e^{-iat}}{2i}$$

$$\int_0^\infty \sin at\, e^{-st}\, dt$$

$$\int_0^\infty \frac{e^{iat} - e^{-iat}}{2i}\, e^{-st}\, dt$$

$$\frac{1}{2i}\left[\int_0^\infty e^{iat} e^{-st}\, dt - \int_0^\infty e^{-iat} e^{-st}\, dt\right]$$

$$\frac{1}{2i}\, L[e^{iat}] - L[e^{-iat}]$$

$$\frac{1}{2i}\left[\frac{1}{(s-ia)} - \frac{1}{s+ia}\right] \qquad (s-ia)(s+ia)$$

$$\frac{1}{\cancel{2i}}\left[\frac{\cancel{2}ia}{s^2+a^2}\right] \qquad s^2 + \cancel{sia} - \cancel{sia} + a^2$$

$$\boxed{L[\sin at] = \frac{a}{s^2+a^2}}$$

find L[cos at]

$$\int_0^\infty f(t)\, e^{-st}\, dt \qquad \cos at = \frac{e^{iat} + e^{-iat}}{2}$$

$$\int_0^\infty \frac{e^{iat} + e^{-iat}}{2}\, e^{-st}\, dt$$

$$\frac{1}{2}\left[\int_0^\infty e^{iat} e^{-st}\, dt + \int_0^\infty e^{-iat} e^{-st}\, dt\right]$$

$$\frac{1}{2}\left[\int_0^\infty e^{-t(s-ia)}\, dt + \int_0^\infty e^{-t(s+ia)}\, dt\right.$$

$$\frac{1}{2}\left[\frac{e^{-t(s-ia)}}{-(s-ia)}\right]_0^\infty$$

$$\frac{1}{2}\left[L\, [e^{iat}\right.$$

$$\boxed{L[\cos at] = \frac{s}{s^2+a^2}}$$

L [cosh at]

$$\int_0^\infty f(t)\, e^{-st}\, dt$$

$$\int_0^\infty \frac{e^{at} + e^{-at}}{2}\, e^{-st}\, dt$$

$$\frac{1}{2}\left[\int_0^\infty e^{at} e^{-st}\, dt + \int_0^\infty e^{-at} e^{-st}\, dt\right]$$

$$\frac{1}{2}\left[L[e^{at}] + L[e^{-at}]\right]$$

$$\frac{1}{2}\left[\frac{1}{s-a} + \frac{1}{s+a}\right]$$

$$\frac{1}{\cancel{2}}\left[\frac{\cancel{2}s}{s^2-a^2}\right]$$

$$\boxed{L[\cosh at] = \frac{s}{s^2-a^2}}$$

L [sinh at]

$$\int_0^\infty f(t)\, e^{-st}\, dt$$

$$\int_0^\infty \frac{e^{at} - e^{-at}}{2}\, e^{-st}\, dt$$

$$\frac{1}{2}\left[\int_0^{\infty} e^{at}e^{-st}dt - \int_0^{\infty} e^{-at}e^{-st}dt\right]$$

$$\frac{1}{2}\left[L[e^{at}] - L[e^{-at}]\right]$$

$$\frac{1}{2}\left[\frac{1}{s-a} - \frac{1}{s+a}\right]$$

$$\frac{1}{2}\left[\frac{2a}{s^2-a^2}\right]$$

$$\boxed{L[\sinh at] = \frac{a}{s^2-a^2}}$$

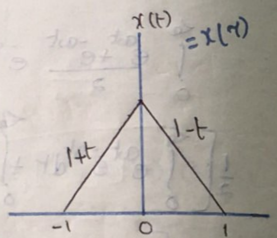
Convolute the following functions

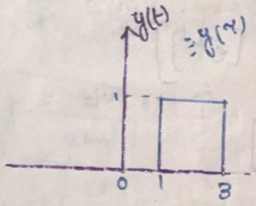
$y(t) = 1 \quad 1<t<3$

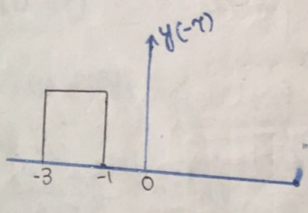
$\quad = 0$ Else where

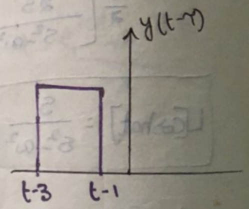
$x(t) = 1+t \quad -1 \le t \le 0$

$\quad = 1-t \quad 0 \le t \le 1$

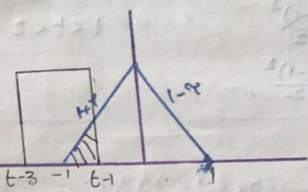








step1:-



$$z(t) = \int_{-\infty}^{\infty} x(\tau)\, y(t-\tau)\, d\tau$$

$$= \int_{-1}^{t-1} (1+\tau)\cdot 1\, d\tau$$

$$= \left[\tau + \frac{\tau^2}{2}\right]_{-1}^{t-1}$$

$$= \left[\cancel{t-1+1+\frac{t^2}{2}}\right]$$

$$= t + \frac{(t-1)^2}{2} + \frac{1}{2}$$

$$= t + \frac{(t-1)^2}{2} + \frac{1}{2}$$

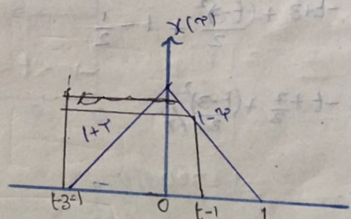
①

$-1 \le t-1 \le 0$

add one on B.S

$\boxed{0 \le t \le 1}$

step2:-



$$z(t) = \int_{-\infty}^{\infty} x(\tau)\, y(t-\tau)\, d\tau$$

$$= \int_{-1}^{0} (1+\tau)\cdot 1\, d\tau + \int_{0}^{t-1} (1-\tau)\cdot 1\, d\tau$$

$$= [\tau]_{-1}^{0} + \left[\frac{\tau^2}{2}\right]_{-1}^{0} + [\tau]_0^{t-1} - \left[\frac{\tau^2}{2}\right]_0^{t-1}$$

$-1+\frac{1}{2}+(t-1)-\frac{(t-1)^2}{2}$

$\frac{-3}{2}\frac{(t-1)^2}{2}$

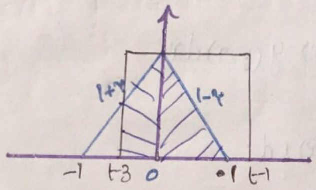
$t+\frac{1}{2}-\frac{(t-1)^2}{2}$ //

$0 < t-1 < 1$

add ① on B.S

$1 < t < 2$

step 3:-



$$z(t) = \int_{-\infty}^{\infty} x(\tau)\, y(t-\tau)\, d\tau$$

$$= \int_{t-3}^{0} (1+\tau)\, d\tau + \int_{0}^{1} (1-\tau)\, d\tau$$

$$= [\tau]_{t-3}^{0} + \left[\frac{\tau^2}{2}\right]_{t-3}^{0} + [\tau]_{0}^{1} - \left[\frac{\tau^2}{2}\right]_{0}^{1}$$

$$= -t+3 + \frac{(t-3)^2}{2} + 1 - \frac{1}{2}$$

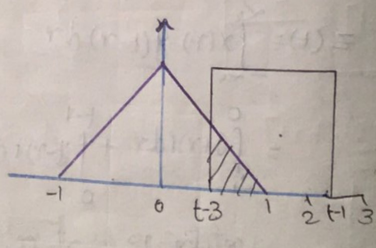
$$= -t + \frac{7}{2} + \frac{(t-3)^2}{2} \;//$$

$1 < t-1 < 2$

add one on B.S

$\boxed{2 < t < 3}$

step 4:-



$$z(t) = \int_{-\infty}^{\infty} x(\tau)\, y(t-\tau)\, d\tau$$

$$= \int_{t-3}^{1} (1-\tau)\, d\tau$$

$$= [\tau]_{t-3}^{1} - \left[\frac{\tau^2}{2}\right]_{t-3}^{1}$$

$$= [1-t+3] - \left[\frac{1}{2} - \frac{(t-3)}{2}\right]$$

$$= 4-t-\frac{1}{2}[-t-2]$$

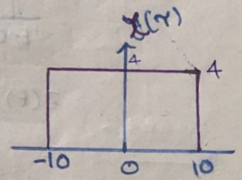
$$= \frac{8-2t-t-2}{2}$$

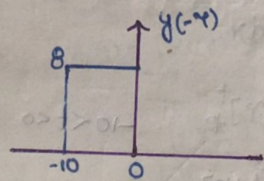
$$= \frac{6-3t}{2} \;//$$

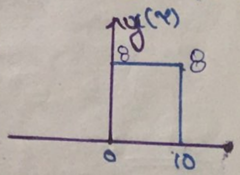
find the convolution of

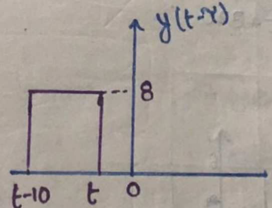
$x(t) = 4 \qquad -10 \le t \le 10$

$y(t) = 8 \qquad 0 \le t \le 10$

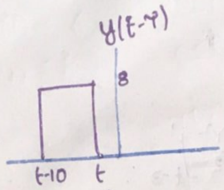


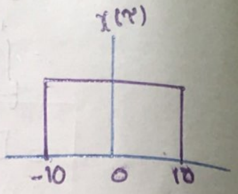






Step 1 :-

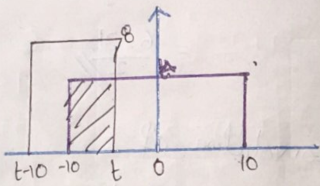




$$z(t) = \int_{-\infty}^{\infty} x(\tau)\, y(t-\tau)\, d\tau$$

$$= 0$$

Step 2 :-



$$z(t) = \int_{-\infty}^{\infty} x(\tau)\, y(t-\tau)\, d\tau$$

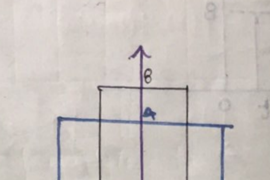
$$= \int_{-10}^{t} 32 \cdot d\tau$$

$$= 32\,[\tau]_{-10}^{t} \qquad -10 < t < 0$$

$$= 32\,[t+10]$$

$$= 32t + 320$$

Step :- 3



$$z(t) = \int_{-\infty}^{\infty} x(\tau)\, y(t-\tau)\, d\tau$$

$$= \int_{t-10}^{0} 32\, d\tau + \int_{0}^{t} 32\, d\tau$$

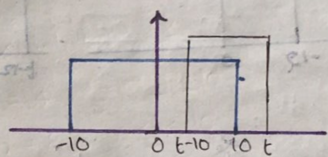
$$= 32\,[\tau]_{t-10}^{0} + 32\,[\tau]_{0}^{t}$$

$$= 32\,[-t+10] + 32\,[t-0]$$

$$= -\cancel{32t} + 320 + \cancel{32t}$$

$$= 320$$

Step 4 :-



$$z(t) = \int_{-\infty}^{\infty} x(\tau)\, y(t-\tau)\, d\tau$$

$$= \int_{t-10}^{10} 32\, d\tau$$

$$= 32\,[\tau]_{t-10}^{10}$$

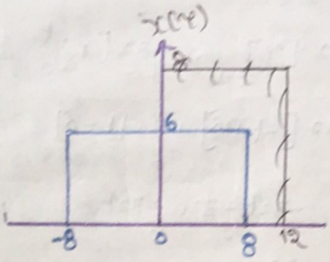
$$= 32\,[10-t+10]$$

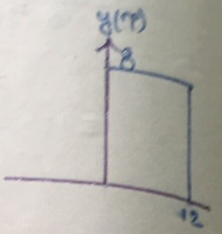
$$= \cancel{32t}\; 32\,[20-t]$$

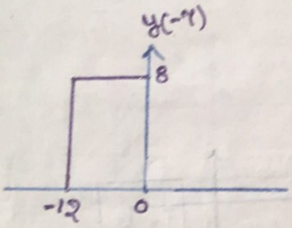
$$= 640 - 32t$$

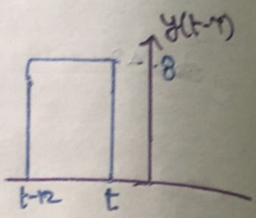
$x(t) = 6 \qquad -8 \le t \le 8$

$y(t) = 8 \qquad 0 \le t \le 12$







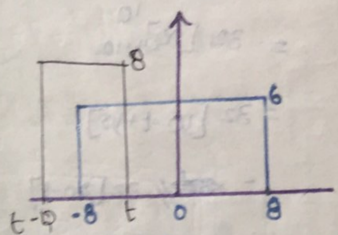


step1 :-

$$z(t) = \int_{-\infty}^{\infty} x(\tau)\, y(t-\tau)\, d\tau$$

$$= 0$$

step2 :-



$$z(t) = \int_{-\infty}^{\infty} x(\tau)\, y(t-\tau)\, d\tau$$

$$= \int_{-8}^{t} 48\, d\tau$$

$$= 48\, [\tau]_{-8}^{t}$$

$$= 48\, [t+8]$$

$$= 48t + 384$$

find $L[t^n]$ $= \frac{n!}{s^{n+1}} \mid \frac{\sqrt{n+1}}{s^{n+1}}$

$\int_0^\infty t^n \cdot e^{-st} \cdot dt$

$st = x$

$dt = \frac{dx}{s}$

$\int_0^\infty \frac{x^n}{s^n} e^{-x} \frac{dx}{s}$

$\frac{1}{s^{n+1}} \int_0^\infty x^n e^{-x} dx$

$\frac{1}{s^{n+1}} \int_0^\infty x^n e^{-x} dx$

$\boxed{\sqrt{n} = \int_0^\infty x^{n+1} e^{-x} dx}$

$= \frac{1}{s^{n+1}} \int_0^\infty x^{n+1-1} \cdot e^{-x} dx$

$= \frac{\sqrt{n+1}}{s^{n+1}} = \frac{n!}{s^{n+1}}$ n = integers

Find the Laplace transform from the following function:-

$L\left[t^{3/2} + \frac{1}{\sqrt{t}}\right]$

$L[t^n] = \frac{n!}{s^{n+1}}$ n is integer

$\checkmark L[t^n] = \frac{\sqrt{n+1}}{s^{n+1}}$ n is fraction.

$L[t^{3/2}] + L[t^{-1/2}]$

$\frac{\sqrt{\frac{3}{2}+1}}{s^{3/2+1}} + \frac{\sqrt{-\frac{1}{2}+1}}{s^{-1/2+1}}$

$\frac{\sqrt{5/2}}{s^{5/2}} + \frac{\sqrt{1/2}}{s^{1/2}}$

$\sqrt{n+1} = n\sqrt{n}$

$\sqrt{\tfrac{1}{2}} = \sqrt{\pi}$

$\frac{\frac{3}{2}\sqrt{\frac{3}{2}}}{s^{5/2}} + \frac{\sqrt{\frac{1}{2}}}{s^{1/2}}$

$\frac{\frac{3}{2} \times \sqrt{\frac{1}{2}+1}}{s^{5/2}} + \frac{\sqrt{\pi}}{s^{1/2}}$

$\frac{\frac{3}{2} \times \frac{1}{2}\sqrt{\frac{1}{2}}}{s^{5/2}} + \frac{\sqrt{\pi}}{s^{1/2}}$

$\boxed{\frac{\frac{3}{4}\sqrt{\pi}}{s^{5/2}} + \frac{\sqrt{\pi}}{s^{1/2}}}$

find $L[\sin^2 3t]$

$\sin^2\theta = \frac{1-\cos 2\theta}{2}$

$L\left[\frac{1-\cos 2(3t)}{2}\right]$

$= \frac{1}{2}\left[L[1] - L[\cos 6t]\right]$

$= \frac{1}{2}\left[\frac{1}{s} - \frac{s}{s^2+6^2}\right] = \frac{1}{2}\left[\frac{1}{s} - \frac{s}{s^2+36}\right]$

find $L[\cos^3 5t]$

$\cos 3\theta = 4\cos^3\theta - 3\cos\theta$

$\cos^3\theta = \dfrac{\cos 3\theta + 3\cos\theta}{4}$

$\frac{1}{4} L\left[\cos 3(5t) + 3\cos(5t)\right]$

$\frac{1}{4}\left[L[\cos 15t] + 3L[\cos 5t]\right]$

$$\boxed{\frac{1}{4}\left[\frac{s}{s^2+15^2} + 3\left[\frac{s}{s^2+5^2}\right]\right]}$$

find $L[\sin t \cos 2t]$

$\sin A \cos B = \dfrac{\sin(A+B) + \sin(A-B)}{2}$

$L\left[\dfrac{\sin(t+2t) + \sin(t-2t)}{2}\right]$

$\frac{1}{2}\left[L[\sin 3t] + L[\sin(-t)]\right]$

$\frac{1}{2}\left[L[\sin 3t] - L[\sin(t)]\right]$

$$\boxed{\frac{1}{2}\left[\frac{3}{s^2+3^2} - \frac{1}{s^2+1^2}\right]}$$

find $L[\underbrace{\sin t \cdot \sin 2t}\ \sin 3t]$

$\dfrac{\cos(2t-2t) - \cos(t+2t)}{2}\sin 3t$

$\frac{1}{2}[\cos(-t) - \cos(3t)]\sin 3t$

$\frac{1}{2}[\cos t \sin 3t - \cos 3t \sin 3t]$

$\frac{1}{2}\left[\dfrac{\sin(t+3t) - \sin(t-3t)}{2}\right] - \dfrac{\sin 2(3t)}{2}$

$\frac{1}{2}\cdot\frac{1}{2}[\sin 4t + \sin 2t] - \sin 6t]$

$\frac{1}{4} L[\sin 4t] + L[\sin 2t] - L[\sin 6t]$

$\frac{1}{4}\left[\dfrac{4}{s^2+4^2} + \dfrac{2}{s^2+2^2} - \dfrac{6}{s^2+6^2}\right]$

$\cos(A+B) = \cos A \cos B - \sin A \sin B$
$\cos(A-B) = \cos A \cos B + \sin A \sin B$
$\sin(A+B) = \sin A \cos B + \cos A \sin B$
$\sin(A-B) = \sin A \cos B - \cos A \sin B$
$\sin 2\theta = 2\sin\theta\cos\theta$
$2\sin A \cos B = \sin(A+B) + \sin(A-B)$
$2\cos A \sin B = \sin(A+B) - \sin(A-B)$
$2\cos A \cos B = \cos(A+B) + \cos(A-B)$
$2\sin A \sin B = \cos(A-B) - \cos(A+B)$

$L[\sin 3t \sin 4t \cos 6t]$

$L\left[\dfrac{\cos(3t-4t) - \cos(3t+4t)}{2} \times \cos 6t\right]$

$\frac{1}{2} L[[\cos t - \cos 7t] \cos 6t]$

$\frac{1}{2} L[\cos t \cdot \cos 6t - \cos 7t \cos 6t]$

$\frac{1}{2} L\left[\dfrac{\cos(t+6t) + \cos(t-6t)}{2}\right] - L\left[\dfrac{\cos(7t+6t) + \cos(7t-6t)}{2}\right]$

$\frac{1}{4}\left[L[\cos 7t] + L[\cos 5t] - L[\cos 13t] - L[\cos t]\right]$

$\frac{1}{4}\left[\dfrac{s}{s^2+7^2} + \dfrac{s}{s^2+25} - \dfrac{s}{s^2+13^2} - \dfrac{s}{s^2+1}\right]$

Scanned with CamScanner

find $L[\cos(at+b)]$

$\cos(A+B) = \cos A\cos B - \sin A\sin B$

$L[\cos at\cos b - \sin at\sin b]$

$\cos b\, L[\cos at] - \sin b\, L[\sin at]$

$\cos b \cdot \frac{s}{s^2+a^2} - \sin b \cdot \frac{a}{s^2+a^2}$

find the Laplace Transform for the following functions.

$e^{-at}\sin bt$, $e^{-at}\cos bt$, $e^{at}t^n$, $e^{-t}\cos t$, $\sin t$, $e^{-2t}\cos^2 t$

1) $L[e^{-at}\sin bt]$

here e^{-at} multiply with some function gives solution which is one of the property of laplace.

$L[\sin bt]_{s\to s+a}$

$\frac{b}{s^2+b^2} = \frac{b}{(s+a)^2+b^2}$

2) $L[e^{-at}\cos bt]$

$L[\cos bt]_{s\to s+a}$

$\left[\frac{s}{s^2+b^2}\right]_{s\to s+a}$

$\frac{s+a}{(s+a)^2+b^2}$

3) $L[e^{at}t^n]$

$L[t^n]_{s\to s-a}$

$\left[\frac{n!}{s^{n+1}}\right]_{s\to s-a}$ n is integer

$\frac{n!}{(s-a)^{n+1}}$ simillarly $\frac{\Gamma(n+1)}{(s-a)^{n+1}}$ n is fraction

4) $L[e^{-2t}\cos^2 t]$

$\cos^2 t = \frac{1+\cos 2t}{2}$

$L[\cos^2 t]_{s\to s+2}$

$L\left[\frac{1+\cos 2t}{2}\right]$

$\frac{1}{2}\left[\frac{1}{s} + \frac{s}{s^2+2^2}\right]_{s\to s+2}$

$\frac{1}{2}\left[\frac{1}{s+2} + \frac{s+2}{(s+2)^2+2^2}\right]$

Properties of Laplace transformations:-

1. Linearity property:-

It can be stated as $L[a f(t) + b g(t)] = a L[f(t)] + b L[g(t)]$

$$\boxed{L[a\cdot f(t)+b\cdot g(t)] = a\cdot L[f(t)] + b\cdot L[g(t)]}$$

$\int_0^\infty f(t)e^{-st}dt$

$\int_0^\infty [a\cdot f(t) + b\, g(t)]e^{-st}dt$

$a\int_0^\infty f(t)e^{-st}dt + b\int_0^\infty g(t)e^{-st}dt$

$a \cdot L[f(t)] + b\, L[g(t)]$

2. Scaling property:-

If Laplace of $f(t) = f$

If $L[f(t)] = f(s)$ Then $L[f(at)] = \frac{1}{a} F\left[\frac{s}{a}\right]$

$= \int_0^\infty f(at) e^{-st} dt$

Put $at = x$

$t = x/a$

$dt = dx/a$

$\int_0^\infty f(x) e^{-s(x/a)} \frac{dx}{a}$

$\frac{1}{a} \int_0^\infty f(x) e^{-s(x/a)} dx$

$= \frac{1}{a} F\left(\frac{s}{a}\right)$

3. Differenciation of Transforms:-

$L[f(t)] = F(s)$ Then $L[t f(t)] = -\frac{d}{ds} F[s]$

$F[s] = L[f(t)]$ — ①

$F[s] = \int_0^\infty f(t) e^{-st} dt$

D.w.r.to s.

$\frac{d}{ds} F[s] = \int_0^\infty f(t) \frac{d}{ds} e^{-st} dt$

$\frac{d}{ds} F[s] = \int_0^\infty f(t) - t e^{-st} dt$

$\frac{d}{ds} F[s] = -t \int_0^\infty f(t) e^{-st} dt$

$-\frac{d}{ds} [F[s]] = t \int_0^\infty f(t) e^{-st} dt$

$L[t f(t)] = -\frac{d}{ds} F[s]$

Note:- $-\frac{d}{ds} F[s] = L[t f(t)]$

$L[t^2 f(t)] = (-1)^2 \frac{d^2}{ds^2} F[s]$

$L[t^n f(t)] = (-1)^n \frac{d^n}{ds^n} F[s]$

Integration of Transforms:-

If Laplace transformation of a function exists then the integral of the corresponding transform w.r.to s over u in the complex frequency domain is equal to its division by in the time domain

$L\left[\frac{f(t)}{t}\right] = \int_s^\infty F(u) du$ (or) $\int_s^\infty F(s) ds.$

$F(s) = \int_0^\infty f(t) e^{-st} dt$

replace $s \to u$

$F(u) = \int_0^\infty f(t) e^{-ut} dt$

Apply $\int_s^\infty$ on both sides

$\int_s^\infty F(u)\, du = \int_s^\infty \int_0^\infty f(t) e^{-ut} dt\, du$

$\int_s^\infty F(u) du = \int_0^\infty f(t) \left[\int_s^\infty e^{-ut} du\right] \cdot dt$

$= \int_0^\infty f(t) \left[\frac{e^{-ut}}{-t}\right]_s^\infty dt$

$= \int_0^\infty f(t) \left[0 - \frac{e^{-st}}{-t}\right] dt$

$$\int_0^\infty F(u)du = \int_0^\infty \frac{f(t)}{t} e^{-st} dt$$

$$= \int_0^\infty F(u)du = L\left[\frac{f(t)}{t}\right]$$

ROC of Laplace transformations:-
(Region of Convergence)

steps to be followed

① ROC consists of strips of area parallel to $j\omega$ axis in s-plane.

Proof:-

① ROC [X(s)] consists of values of s for which fourier transform $x(t)e^{-\sigma t}$ This is possible when $x(t)e^{-\sigma t}$ is fully integrable con condition depends on σ. ROC is strips which is only in the terms of poles of 'σ'.

step2:-

ROC doesn't contain any poles values of X(s) has finite value's in ROC is infinite and X(s) has at one. hence ROC shouldn't contain any poles.

② ROC is entire s-plane for absolutely integrable and finite duration signals.

Proof:- x(t) lies in an interval ~~rate~~ $\int_{t_1}^{t_2} x(t)e^{-\sigma t}dt<\infty$

This integration is finite for any value of σ

ROC is entire s-plane.

Integration property:-

$$L\left\{\int_0^t f(t)dt\right\} = \frac{F(s)}{s}$$

The function f(t) is continuous then the

$$L\left\{\int_0^t f(t)dt\right\} = \frac{F(s)}{s}$$

$$\int_0^\infty f(t)e^{-st}dt \qquad \int u.v = u\int v dt - \int u'\int v dt$$

$$\int_0^\infty \left[\int f(t)dt\right] e^{-st} dt$$

$$\int_0^t f(t)dt \int_0^\infty e^{-st}dt - \int_0^\infty \frac{d}{dt}\left(\int_0^t f(t)dt\right)\int_0^\infty e^{-st}dt$$

$$\int_0^t f(t)dt \left[\frac{e^{-st}}{-s}\right]_0^\infty - \int_0^\infty f(t)dt\left[\frac{e^{-st}}{-s}\right]_0^\infty dt$$

$$0 + \frac{1}{s}\int_0^\infty f(t)\, e^{-st}dt$$

$$\frac{F(s)}{s}$$

Differentiation property:-

If a function f(t) is piecewise continuous then the Laplace transform of the derivative $\frac{d}{dt}f(t)$ is given by

$$L\{f'(t)\} = sF(s) - f(0)$$

$$f'(t) = \frac{d}{dt}f(t)$$

$$\int_0^\infty f(t)e^{-st}dt$$

$$\int_0^\infty \frac{d}{dt}f(t)\, e^{-st}dt$$

$$\int_0^\infty \underbrace{e^{-st}}_{①}\underbrace{\frac{d}{dt}f(t)}_{②} dt$$

Scanned with CamScanner

$$e^{-st}\int_0^{\infty}\frac{d}{dt}f(t)dt - \int\frac{d}{dt}e^{-st}\int\frac{d}{dt}f(t)\,dt$$

$$\{e^{-st}.f(t)\}_0^{\infty} - \int(-s)e^{-st}dt \cdot f(t)dt$$

$$\cancel{0} - f(0) + s\int_0^{\infty} f(t)e^{-st}dt$$

$$= sF(s) - f(0)$$

Note:-

$$\boxed{\begin{aligned} L\{f'(t)\} &= sF(s) - f(0) \\ L\{f''(t)\} &= s^2F(s) - sf(0) - f'(0) \\ L\{f'''(t)\} &= s^3F(s) - s^2f(0) - s'f(0) - f''(0) \\ L\{f^n(t)\} &= s^nF(s) - s^{n-1}f(0) - s^{n-2}f(0) - \dots f^{n-1}(0) \end{aligned}}$$

Imp

first shifting theorm:-

If the function $f(t)$ has transformed $f(s)$ then the Laplace transform of $\boxed{e^{-at}f(t) \longrightarrow F(s+a)}$

Proof:- $F(s) = \int_0^{\infty} f(t)e^{-st}dt$

$$F(s+a) = \int_0^{\infty} f(t)e^{-(s+a)t}dt$$

$$= \int_0^{\infty} f(t)e^{-st}e^{-at}dt$$

$$= \int_0^{\infty} e^{-at}\int_0^{\infty} f(t)e^{-st}dt$$

$$= e^{-at}L[f(t)]$$

$$= e^{-at}.f(t)-$$

similarly

$$\boxed{f(s-a) = L[e^{at}f(t)]}$$

Second shifting theorm:-

Imp

If the function $f(t)$ has the ~~...~~ function $f(s)$ Then

Laplace Transform $f(t-a)u(t-a)$ is $e^{-as}f(s)$

Proof:- $L\{f(t-a)u(t-a)\} = e^{-as}F(s)$

$$\int_0^{\infty} f(t)e^{-st}dt$$

$$\int_0^{\infty} f(t-a)u(t-a)e^{-st}dt$$

$f(t-a)u(t-a) < 0$

$t-a = \tau$

$t = \tau + a$

$$\int_0^{\infty} f(t-a)e^{-st}dt$$

$$\int_0^{\infty} f(t-a)e^{-st}dt$$

$t-a = \tau$

$t = \tau + a$

$$\int_0^{\infty} f(\tau)e^{-s(\tau+a)}d\tau$$

$$\int_0^{\infty} f(\tau)e^{-s\tau}e^{-as}d\tau$$

$$e^{-as}\int_0^{\infty} f(\tau)e^{-s\tau}d\tau$$

$$e^{-as}F(s)$$

Imp:

Initial value theorm:-

If the function $f(t)$ and its derivative are Laplace transformable then we can write as

$$\boxed{\lim_{t\to 0} f(t) = \lim_{s\to\infty} sF(s)}$$

Proof:- w.k.T $L[f'(t)] = s[L(f'(t))] - f(0)$

$$\int_0^{\infty} f'(t)\, e^{-st}\, dt = s(F(s)) - f(0)$$

Taking limits as $\oint$ $\lim_{s\to\infty}$

$$\lim_{s\to\infty} \int_0^{\infty} f'(t)\, e^{-st}\, dt = \lim_{s\to\infty} [sF(s) - f(0)]$$

L.H.S

$$\lim_{s\to\infty} \int_0^{\infty} f'(t) e^{-st} dt = 0$$

$$0 = \lim_{s\to\infty} [sF(s) - f(0)]$$

$$0 = \lim_{s\to\infty} sF(s) - f(0)$$

$$\lim_{s\to\infty} sF(s) = f(0)$$

$$\lim_{s\to\infty} s.F(s) = \lim_{t\to 0} f(t)$$

Imp

Final value theorem :-

if $f(t)$ and $f'(t)$ are Laplace transformable then

$$\boxed{\lim_{t\to\infty} f(t) = \lim_{s\to 0} sF(s)}$$

Proof :-

W.K.T — By diff property.

$$L[f'(t)] = sF(s) - f(0)$$

applying

$$\int_0^{\infty} f'(t) e^{-st} dt = sF(s) - f(0)$$

Taking $\lim_{s\to 0}$ on B.S

we have

$$\lim_{s\to 0} \int_0^{\infty} f'(t)\, e^{-st} dt = \lim_{s\to 0} [sF(s) - f(0)]$$

here $\lim_{0\to\infty}$ and $\frac{d}{dt}$ cancled out eachother

$$\int_0^{\infty} f'(t)\, dt = \lim_{s\to 0} [sF(s)] - \lim_{s\to 0} f(0)$$

$$[f(t)]_0^{\infty} = \lim_{s\to 0} [sF(s)] - \lim_{s\to 0} f(0)$$

$$[f(\infty) - f(0)] = \lim_{s\to 0} sF(s) - \lim_{s\to 0} f(0)$$

$$\lim_{t\to\infty} f(t) - \cancel{\lim_{t\to 0} f(t)} = \lim_{s\to 0} sF(s) - \cancel{\lim_{s\to 0} f(t)}$$

$$\boxed{\lim_{t\to\infty} f(t) = \lim_{s\to 0} sF(s)}$$

properties of Roc :-

* let $x(t)$ is Right sided signals and line real value of $s = \rho_0$ is in Roc Then all values of 's' for which real value of is greater than ρ_0 will also be in Roc.

of $= \rho_0$

Find the Laplace transform of unit step function :- Draw its ROC

$$u(t) = 1 \quad t>0$$
$$= 0 \quad t<0$$

W.K.T

$$u(t) = \int_0^{\infty} f(t)\, e^{-st} dt$$

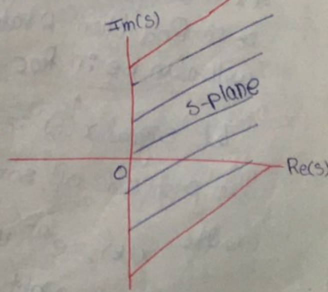
$$= \int_0^{\infty} 1\, e^{-st} dt$$

$$= \left[\frac{e^{-st}}{-s}\right]_0^{\infty}$$

$$= \left[0 - \left(-\frac{1}{s}\right)\right]$$

$$= \frac{1}{s}$$

R.O.C



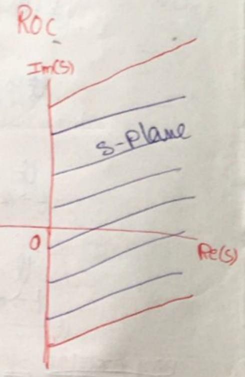
⑤ Laplace transform of unit Ramp function draw its Roc.

w.k.T Ramp function can be defined as

$$x(t) = t \quad t>0$$
$$= 0 \quad t<0$$

$$\int_0^{\infty} f(t)e^{-st}dt$$

$$\int_0^{\infty} t\, e^{-st}dt$$



$$\boxed{\int te^{at} = \frac{e^{at}}{a^2}(at-1)}$$

$$\left\{\frac{e^{-st}}{s^2}(-st-1)\right\}_0^{\infty}$$

Multiply with '−'.

$$\left\{\frac{-e^{-st}}{s^2}(st+1)\right\}_0^{\infty}$$

$$-\frac{1}{s^2}\left\{e^{-st}(st+1)\right\}_0^{\infty} \Rightarrow \frac{-1}{s^2}[0-1] = \frac{1}{s^2} \quad \boxed{s>0}$$

⑤ let x(t) is right sided signal and line real value of $s = P_0$ is in Roc then R values of 's' for which real values $s > P_0$ will also be in Roc.

Proof:- let signal x(t) is right sided then its Roc is also right side of some line parallel to jω axis.

example $x(t) = e^{-at}.u(t)$ is a right sided signal then its laplace transform is $S(x) = \frac{1}{s+a}$ with Roc real value of $s > -a$ which is right sided hence if the line at real value of $s = P_0$ then this function falls in Roc for all the values of 's', real value of $s > P_0$ will also present in Roc.

⑥ let x(t) be left sided signal and the line real value of $s = P_0$ is in Roc then all values of 's' for which real values of $s < P_0$ will also in Roc.

proof

for left hand sided signal the Roc is also left side of some line parallel to jω axis hence if real value of $s = P_0$ falls in Roc then all the values of 's' for which real values of $s < P_0$ will lies in Roc

⑦ if x(t) is two sided signal and real value of $s = P_0$ is in Roc then Roc will be strip in s-plane that includes line real value of $s = P_0$

Proof:- for both sided sequences Roc lies in the region $\sigma_1 < Re(s) < \sigma_2$. This Roc is the strip parallel to jω axis in s-plane.

⑧ The Roc is bounded by poles (or) extends to infinity

Proof

If the function has two poles then Roc the area b/w these two poles parallel to jω axis for two sided sequence, for single sided sequence the area extends from one pole to infinity but it doesn't include any pole.

⑨ if x(t) is right sided then Roc is region to right of right most pole and if x(t) left sided then Roc is region in s-plane to left side of left most pole.

Proof:- It is clear from fact Roc should not any pole left most pole has all the

Poles on its right side and Roc is on left side of left most pole for left sided signal.

(10) List out all the properties of Roc Laplace transform?

① The Roc of $F(s)$ includes the string lines parallel to imaginary axis in the s-plane. This property has the validity for the Roc of $F(s)$ that consist of value's of both real and imaginary parts

i.e $s = \sigma + j\omega$

where σ = real part

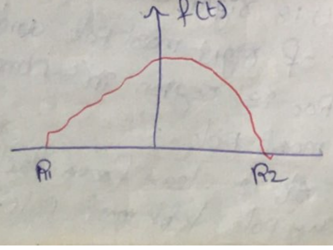
$j\omega$ = imaginary part

and it can be represented as

$$\int_{-\infty}^{\infty} |f(t)|\, e^{-\sigma t}\, dt < \infty$$

② The Roc doesn't contain any poles for the values of Rational Laplace transform. This is because $F(s)$ is infinite at poles and integral can't converge at this point hence Roc can't contain the values of 's' that are poles.

③ If $f(t)$ is absolutely integral and of finite duration then the Roc is the entire s-plane. The logic behind this property is that zero out side an interval of finite duration as shown in figure.



if $f(t)$ is right sided and if Roc of real values of $s = \sigma_0$ then all the values of complex variable for which real value of s will also be in Roc

④ This property will obtain from the two fundamental points for a right hand side signal the Roc will lie in right half of the s-plane.

⑤ The Roc doesn't contain any poles.

⑥ If $f(t)$ is left sided and if Roc of real value of $s = \sigma_0$ then all values of complex variable s for which real value of $s < \sigma_0$ will also be in Roc

⑦ If the function $f(t)$ is rational then its Roc extends to infinity in addition no poles of $F(s)$ are involved in Roc region

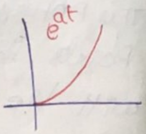
Problem :-

Find the Laplace transform of derivative of signal $x(t)$ invert the transform to find signal derivative

Sol :- Differenciation property proof.

(5)

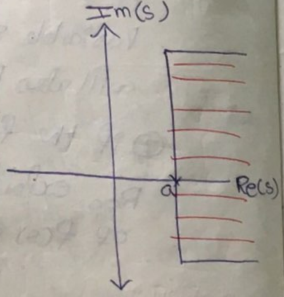
④ Laplace transform of growing exponential function
(or)
find the Laplace transform for the signal shown.



$$= \int_0^\infty f(t)\, e^{-st}\, dt$$

$$= \int_0^\infty e^{+at} e^{-st}\, dt$$

ROC: $s-a>0$, $s>a$



$$= \int_0^\infty e^{-(s-a)t}\, dt$$

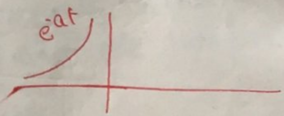
$$= \left[\frac{e^{-(s-a)t}}{-(s-a)}\right]_0^\infty$$

$$= 0 - \frac{1}{-(s-a)}$$

$$= 0 + \frac{1}{s-a} = \frac{1}{s-a}$$

Problems:-

find Laplace transform of decaying function



$$= \int_0^\infty f(t)\, e^{-st}\, dt$$

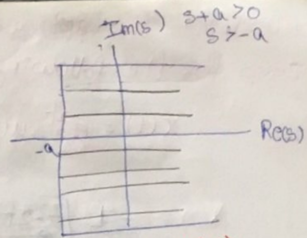
$$= \int_0^\infty e^{-at} e^{-st}\, dt$$

$$= \int_0^\infty e^{-t(s+a)}\, dt$$

$$= \left[\frac{e^{-t(s+a)}}{-(s+a)}\right]_0^\infty$$

$$= 0 + \frac{1}{s+a}$$

$$= \frac{1}{s+a}$$



if $x(t)$ is even function, prove that $X(s) = X(-s)$ and if $x(t)$ is odd prove that $X(s) = -X(s)$

(or)

Prove that L [even & odd functions] is even and odd functions respectively.

Sol:-

$$X(s) = \int_{-\infty}^{\infty} x(t)\, e^{-st}\, dt$$

$$X(-s) = \int_{-\infty}^{\infty} x(t)\, e^{st}\, dt$$

$t = -p$
$dt = -dp$

$$= \int_{-\infty}^{\infty} x(-p)\, e^{s(-p)}\, (-dp)$$

$$= -\int_{-\infty}^{\infty} x(-p)\, e^{-sp}\, dp$$

$$= -X(-s)$$

$$= -X(s)$$

Conditions:-

1. if $x(t) = x(-t)$
   $X(-s) = \int_{-\infty}^{\infty} x(t) e^{-st} dt = X(s)$
2. if $x(t) = -x(-t)$, $X(-s) = -\int_{-\infty}^{\infty} x(t) e^{-st} dt = -X(s)$

Then we can write as

$$X(s) = \int_{-\infty}^{\infty} x(t)\, e^{-st}\, dt$$

Determine the Laplace transform Roc, Pole and zero for the following function.

$$x(t) = e^{-2t}u(t) + e^{-3t}u(t)$$

$$= L\left[e^{-2t}u(t) + e^{-3t}u(t)\right]$$

$$= L[e^{-2t}u(t)] + L[e^{-3t}u(t)]$$

$$= \int_0^{\infty} e^{-2t}e^{-st}dt + \int_0^{\infty} e^{-3t}e^{-st}dt$$

$$= \int_0^{\infty} e^{-(s+2)t}dt + \int_0^{\infty} e^{-(s+3)t}dt$$

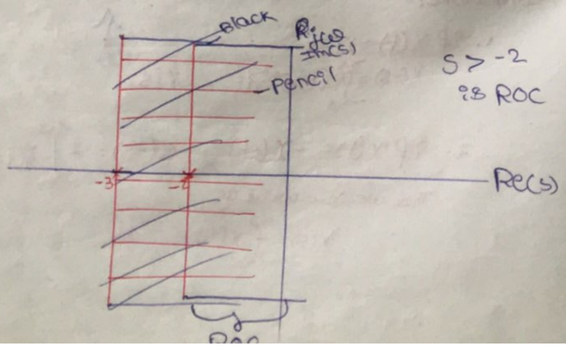
$$= \left\{\frac{e^{-(s+2)t}}{-(s+2)}\right\}_0^{\infty} + \left\{\frac{e^{-(s+3)t}}{-(s+3)}\right\}_0^{\infty}$$

$$= \left\{0 - \left(\frac{1}{-(s+2)}\right)\right\} + \left\{0 - \frac{1}{-(s+3)}\right\}$$

$$= \frac{1}{s+2} + \frac{1}{s+3}$$

$\therefore s+2>0$ $\quad$ $s+3>0$

$s>-2$ $\quad$ $s>-3$



$s>-2$ is ROC

Find the Laplace transform, Roc, Poles for the function

$$x(t) = e^{-6t}u(t) + e^{9t}u(t)$$

$$= L\left[e^{-6t}u(t) + e^{9t}u(t)\right]$$

$$= L[e^{-6t}u(t)] + L[e^{9t}u(t)]$$

$$= \int_0^{\infty} e^{-6t}e^{-st}dt + \int_0^{\infty} e^{9t}e^{-st}dt$$

$$= \int_0^{\infty} e^{-(s+6)t}dt + \int_0^{\infty} e^{-(s-9)t}dt$$

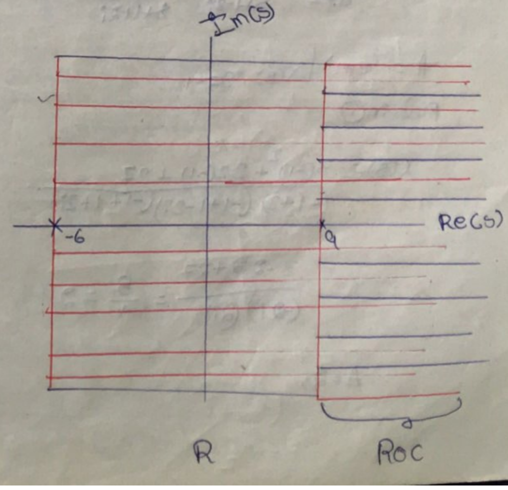
$$= \left\{\frac{e^{-(s+6)t}}{-(s+6)}\right\}_0^{\infty} + \left\{\frac{e^{-(s-9)t}}{-(s-9)}\right\}_0^{\infty}$$

$$= \left\{0 - \left[\frac{1}{-(s+6)}\right]\right\} + \left\{0 - \left[\frac{1}{-(s-9)}\right]\right\}$$

$$= \frac{1}{s+6} + \frac{1}{s-9}$$

$\therefore s+6>0$ $\quad$ $s-9>0$

$s>-6$ $\quad$ $s>9$



Q find signal $x(t)$ that a Laplace given

(5) $X(s) = \dfrac{3s^2+22s+27}{(s^2+3s+2)(s^2+2s+5)}$

$X(s) = \dfrac{3s^2+22s+27}{(s+2)(s+1)(s^2+2s+5)}$

s^2+3s+2
$s^2+2s+s+2$
$s(s+2)+1(s+2)$
$(s+1)(s+2)$

$s^2+2s+5 = ax^2+bx+c$

$\dfrac{-b\pm\sqrt{b^2-4ac}}{2a}$

$= \dfrac{-2\pm\sqrt{4-20}}{2}$

$= \dfrac{-2\pm\sqrt{-16}}{2}$

$= \dfrac{-2\pm 4i}{2}$

$= -1\pm 2i$

$s = -1+2i, \ -1-2i$

$X(s) = \dfrac{3s^2+22s+27}{(s+1)(s+2)(s+1-2i)(s+1+2i)}$

$= \dfrac{A}{s+1} + \dfrac{B}{s+2} + \dfrac{C}{s+1-2i} + \dfrac{D}{s+1+2i}$

$A = (s+1)|X(s), \ s=-1$

Put in (1)

$X(s) = \dfrac{3(-1)^2+22(-1)+27}{(-1+2)(-1+1-2i)(-1+1+2i)}$

$= \dfrac{3-22+27}{(-2i)(2i)} = \dfrac{8}{4} = 2$

$A = 2$

Now $B = (s+2)|X(s); s=-2$

Put $s=-2$ in eqn (1)

$\dfrac{3(-2)^2+22(-2)+27}{(-2+1)(-2+1-2i)(-2+1+2i)}$

$= \dfrac{3(4)-44+27}{(-1)(-1-2i)(-1+2i)}$

$= \dfrac{12-44+27}{(-1)(1-4(i^2))}$

$= \dfrac{-5}{-5} = 1$

$\therefore B = 1$

Now $C = (s+1-2i)|_{s=-1+2i}$

Put in eqn (1)

$\dfrac{3(-1+5i)^2+22(-1+2i)+27}{(-1+2i+1)(-1+2i+2)(-1+2i+1+2i)}$

$\dfrac{3(1+4i^2-4i)-22+44i+27}{2i(2i+1)(4i)}$

$\dfrac{3-12-12i-22+44i+27}{(-4+2i)(4i)}$

$\dfrac{4+32i}{-16i+8i^2} = \dfrac{4+32i}{-16i-8}$

Now $D = (s+1+2i)|_{X(s)}; \ s=-1-2i$

Put in eqn (1)

$\dfrac{3(-1-2i)^2+22(-1-2i)+27}{(-1-2i+1)(-1-2i+2)(-1-2i+1-2i)}$

$= \frac{3(1-4+4i) - 22 - 44i + 27}{(-2i)(-2i+1)(-4i)}$

$= \frac{3(-3-4i) - 22 - 44i + 27}{-8(-2i+1)}$

$= \frac{3(-7) - 44i + 5}{-8(-2i+1)}$

$\frac{-21 - 44i + 5}{-16i - 8}$

Q. find the inverse Laplace of $X(s) = \frac{1+e^{-2s}}{3s^2+2s}$

$\frac{1}{s(3s+2)} = \frac{A}{s} + \frac{B}{(3s+2)}$

$1 = A(3s+2) + B(s)$

$s = 0$

$1 = 2A$

$A = \frac{1}{2}$

equate ~~constant~~ on co-effic of s.

$0 = 3A + B$

$0 = 3(\frac{1}{2}) + B$

$B = -\frac{3}{2}$

$\frac{1/2}{s} + \frac{-3/2}{(3s+2)} + e^{-2s}\frac{1/2}{s} + e^{-2s}\frac{-3/2}{(3s+2)}$

$\frac{\frac{1}{2}}{s} + \frac{-3/2}{3(s+2/3)} + e^{-2s}\frac{1/2}{s} + e^{-2s}\frac{-3/2}{3(s+2/3)}$

Inverse laplace Transform

$\left\{\frac{1}{2}u(t) - \frac{1}{2}e^{-2/3 t}\right\} + e^{-2s}u(t-2) + (-\frac{1}{2})e^{-2/3 t}u(t$

V-IMP

(6) The inverse Laplace transform of a signal is given by $X(s) = \left\{\frac{\omega_n^2}{s^2 + 2s\zeta\omega_n + \omega_n^2}\right\}$

where ω_n or ζ is a natural frequency and damping factor for what value of $\zeta = -1 < \zeta < 1$ can be fourier transformed be completed Laplace Transform.

$X(s)$ can be solved as

$X(s) = \frac{as^2}{(s-P_1)(s-P_2)}$

where P_1, P_2 are the Two poles and ω_n = natural frequency and ξ = damping factor.

(i) both poles P_1 & P_2 lies on the real axis for damping factor $\xi > 1$ This factor is essentially a product of first Two order Terms as a result in which $X(j\omega)$ decreases monotonically and magnitude of ω increases Thus $X(j\omega)$ varies from 0 to ∞.

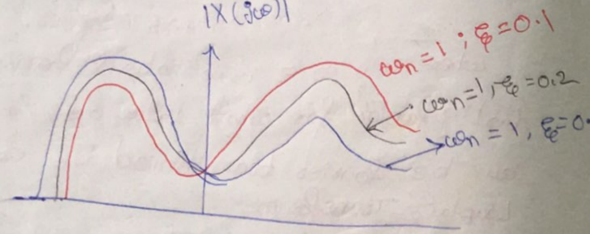
(ii) $s = j\omega$ increases monotonically as ω increases from 0 to 1 and the angle of these vectors also increases from 0 to $\pi/2$ By absorbing the length of vectors from each of the Two poles.

(iii) consider the pole vector for damping ratio $\xi > 1$ Then the length and angle for the pole from the

imaginary axis and less effective to the changes
5 in ω. Then the length and angle of the vector
for the pole close to imaginary axis.

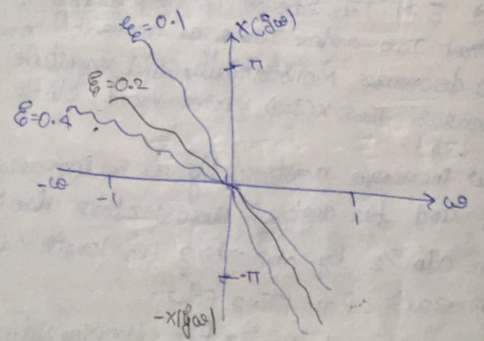
(iv) consider P_1 & P_2 are complex for damping factor $0<\xi<1$ as a result the step response and impulse response of this function have oscillating parts.

(v) The frequency response is drawn for several values of damping factor ξ and natural frequency $\omega_n = 1$



Magnitude and phase of frequency response for second order system which damping factor $0<z<1$ the range of the frequency is

$$\omega_n\sqrt{1-\xi^2} - \xi\omega_n < \omega < \omega_n\sqrt{1-\xi^2} + \xi\omega_n$$



Z - Transforms:- unit VIII

There are two types of signals.

1. Continuous signal $f(t)$
2. Discreet signal $f(n)$

where $n=0,1,2,3$

Definition:-

Let $f(n)$ be a sequence defined for $n=\pm 0, \pm 1, \pm 2, \pm 3$

Then the z-transform of $f(n)$ is defined as

$$\boxed{\sum_{n=-\infty}^{\infty} f(n)\, z^{-n}}$$

and is known as bilateral or two sided z-transform and is denoted by $z[f(n)]$ (or) $\bar{f}(z)$ (or) $F(z)$

where 'z' is a complex variable.

If the $f(n)$ is a causal sequence for which $f(n)=0$ when $n<0$ the z transform of $f\{n\}$ known as unilateral or one sided z-transform.

$$\therefore \boxed{Z\{f(n)\} = \sum_{n=0}^{\infty} f(n)\, z^{-n}}$$

z-transform of a sequence is defined only for certain values of 'z' and the region in z-plane in which z-transform is defined is known as region of convergence.

for example

$$1 + \frac{1}{z} + \frac{1}{z^2} + \cdots$$

for example

$$\boxed{\frac{1}{|z|} < 1} \text{ or } \boxed{|z| > 1}$$

Properties of z transforms:-

5 **Linearity Property:-**

- z-transform is linear or prove

or

prove the Z-transform property

$$z\{a.f(n)+b.g(n)\} = a.z\{f(n)\}+b.z\{g(n)\}$$

R.H.S $\Rightarrow$ $\sum_{n=-\infty}^{\infty} f(n) z^{-n}$

$$z\{\sum_{n=-\infty}^{\infty} \{a.f(n)+b g(n)\} z^{-n}$$

$$\sum_{n=-\infty}^{\infty}\left[\{a.f(n)\}z^{-n} + \{b.g(n)\}z^{-n}\right]$$

$$a\left[\sum_{n=-\infty}^{\infty} f(n) z^{-n}\right] + b\left[\sum_{n=-\infty}^{\infty} g(n) z^{-n}\right]$$

$$a\, z\{f(n)\} + b\, z\{g(n)\}$$

Time shifting property:-

$$\boxed{z\{f(n-n_0)\} = z^{-n_0}\bar{f}(z)}$$

It can be written as

$$z\{f(n-n_0)\} = z^{-n_0}\bar{f}(z)$$

$$\sum_{n=0}^{\infty} f(n-n_0) z^{-n}$$

assume $\boxed{m = n-n_0}$

$\boxed{n = m+n_0}$

$$\sum_{m=n-n_0}^{\infty} f(m) z^{-(m+n_0)}$$

$$\sum_{m=-n_0}^{\infty} f(m) z^{-m} z^{-n_0}$$

$$\left[\sum_{m=0}^{\infty} f(m) z^{-m}\right] z^{-n_0}$$

$$z\{f(z)\} z^{-n_0}$$

$$= \bar{f}(z) z^{-n_0} //$$

Hence proved//

Frequency shifting property:-

$$\boxed{\begin{array}{l} z\{a^n f(n)\} = \bar{f}(z/a) \\ z\{a^n f(t)\} = \bar{f}(z/a) \\ z\{e^{-at} f(t)\} = \bar{f}(z.e^{at}) \end{array}}$$

$$\sum_{n=0}^{\infty} a^n f(n) z^{-n} \quad z\{a^n f(n)\} = \bar{f}(z/a) \text{ — } ①$$

$$\left[\sum_{n=0}^{\infty} f(n)\left(\frac{z}{a}\right)^{-n}\right]$$

$$z\{\bar{f}(z/a) \text{ — } ①$$

$$z\{a^n f(t)\} = \bar{f}(z/a)$$

$$\sum_{n=0}^{\infty} a^n f(nt) z^{-n}$$

$$\sum f(nt) \left(\frac{z}{a}\right)^{-n} = \bar{f}(z/a)$$

$Z\{e^{-at}f(t)\} = \bar{f}(z \cdot e^{at})$

$\sum_{n=0}^{\infty} e^{-at} f(t) z^{-n}$

$\sum_{n=0}^{\infty} f(nt) (z \cdot e^{at})^{-n}$

$\boxed{Z\{e^{-at}f(t)\} = \bar{f}(z \cdot e^{at})}$ — ③

prooved

Time reversal property for Bilateral transform.

$Z\{f(-n)\} = \bar{f}(1/z)$

$\sum_{n=-\infty}^{\infty} f(-n) z^{-n}$

put $m = -n$

$\sum_{m=-n}^{\infty} f(n) z^{m}$

$\sum_{m=-n}^{\infty} f(n) \left(\frac{1}{z}\right)^{-m}$

$\boxed{Z\{f(-n)\} = \bar{f}(1/z)}$

Hence prooved

Differenciation property

$$\boxed{\begin{aligned} Z\{n \cdot f(n)\} &= -z \frac{d}{dz} \bar{f}(z) \\ Z\{n \cdot f(t)\} &= -z \frac{d}{dz} \bar{f}(z) \end{aligned}}$$

$\bar{f}(z) = Z\{f(n)\} = \sum_{n=0}^{\infty} f(n) z^{-n}$

$\bar{f}(z) = \sum_{n=0}^{\infty} f(n) z^{-n}$ — ①

D. w.r.to dz on B.S

$\frac{d}{dz}\bar{f}(z) = \sum_{n=0}^{\infty} \frac{d}{dz} f(n) z^{-n}$

$\frac{d}{dz}(\bar{f}(z)) = \sum_{n=0}^{\infty} f(n) \; -n z^{-n-1}$

$\frac{d}{dz}(\bar{f}(z)) = \sum_{n=0}^{\infty} f(n)(-n) z^{-n} z^{-1}$

$\frac{d}{dz}(\bar{f}(z)) = \frac{-1}{z} \sum_{n=0}^{\infty} f(n)(n) z^{-n}$

Multiply with '$-$'

$-z \frac{d}{dz}(\bar{f}(z)) = \sum_{n=0}^{\infty} n f(n) z^{-n}$

$\boxed{-z \frac{d}{dz} \bar{f}(z) = Z\{n \cdot f(n)\}}$ Hence prooved.

$Z\{n \cdot f(t)\} = -z \frac{d}{dz} \bar{f}(z)$

$\bar{f}(z) = \sum_{n=0}^{\infty} f(nt) z^{-n}$ — ①

D.w.r.to dz on B.S

$\frac{d}{dz}\bar{f}(z) = \sum_{n=0}^{\infty} f(nt) \frac{d}{dz} z^{-n}$

$= \sum_{n=0}^{\infty} f(nt)(-n) z^{-n-1}$

$\frac{d}{dz}\bar{f}(z) = \frac{-1}{z} \sum_{n=0}^{\infty} f(nt)(-n) z^{-n}$

Multiply with '$-$' on B.S

(5)

$-z\{n f(t)\} =$

$$\boxed{-z\frac{d}{dz}\bar{f}(z) = z\{n\cdot f(t)\}}$$

Hence prooved

Proove the initial value Theorem:- in z-transform

It can be written as

$z\{f(n)\} = \bar{f}(z)$; Then $f(0) = \underset{z\to\infty}{Lt}\,\bar{f}(z)$

$$\sum_{n=0}^{\infty} f(n) z^{-n}$$

$$\sum_{n=0}^{\infty} f(n)\left(\frac{1}{z}\right)^n$$

if $n=0$

$= f(0)$

$$\boxed{\underset{n\to\infty}{Lt}\,\bar{f}(z) = f(0)}$$

final value Theorm of z-transform if $z\{f(n)\} =$

if $z\{f(n)\} = \bar{f}(z)$, Then $\underset{z\to1}{Lt}(z-1)\bar{f}(z) = \underset{n\to\infty}{Lt} f(n)$

$z\{f(t)\} = \bar{f}(z)$ then $\underset{z\to1}{Lt}(z-1)\bar{f}(z) = \underset{t\to\infty}{Lt} f(t)$

Assume

$$z\{f(n+1)\} = \sum_{n=0}^{\infty} f(n+1) z^{-n}$$

$m = n+1$

$n = m-1$

$$= \sum_{n=m+1}^{\infty} f(m) z^{-(m-1)}$$

$$= \sum_{m=1}^{\infty} f(z) z^{-m} z^{1}$$

$$z\{f(n+1)\} = z\sum_{m=1}^{\infty} f(m) z^{-m} \pm f(0) z^{0} - f(0) z^{0}$$

$z\{f(n+1)\} = z\{\bar{f}(z)\} - f(0)$ —— (1)

add $-\bar{f}(z)$ on B.S

$z\{f(n+1)\} - \bar{f}(z) = z\{\bar{f}(z)\} - f(0) - \bar{f}(z)$

$\bar{f}(z) = z\{f(n)\}$

$z\{f(n+1)\} - z\{f(n)\} = z\{\bar{f}(z)\} - f(0) - \bar{f}(z)$

$z\{f(n+1) - f(n)\} = \bar{f}(z)[z-1] - f(0)$

Taking $\underset{z\to1}{Lt}$ on B.S we get

$\underset{z\to1}{Lt}\, z\{f(n+1) - f(n)\} = \underset{z\to1}{Lt}\{\bar{f}(z)(z-1) - f(0)\}$

$\underset{z\to1}{Lt}\, \bar{f}(z)(z-1) - f(0) = \underset{N\to\infty}{Lt}\sum_{n=0}^{\infty}\{f(n+1) - f(n)\}$

$$\boxed{\underset{z\to1}{Lt}\,\bar{f}(z)(z-1)\,\bar{f}(z) = \underset{N\to\infty}{Lt} f(n)}$$

Hence prooved

proove the convolution Theorm in z-transform:-

It states that if $z\{f(n)\} = \bar{f}(z)$; $z\{g(n)\} = \bar{g}(z)$

Then $z\{f(n) * g(n)\} = \bar{f}(z)\,\bar{g}(z)$

Scanned with CamScanner

5

$$f(n) * g(n) = \sum_{r=-\infty}^{\infty} f(r)\, g(n-r) \quad \text{—①}$$

$$\sum_{n=-\infty}^{\infty} f(n) z^{-n}$$

$$\sum_{n=-\infty}^{\infty} f(n) * g(n) z^{-n}$$

$$\sum_{n=-\infty}^{\infty} \left\{ \sum_{r=-\infty}^{\infty} f(r)\, g(n-r) \right\} z^{-n}$$

Interchanging the integral

$$\sum_{r=-\infty}^{\infty} f(r) \left\{ \sum_{n=-\infty}^{\infty} g(n-r) \right\} z^{-n}$$

$$\sum_{r=-\infty}^{\infty} f(r) \left\{ \sum_{n=-\infty}^{\infty} g(n-r) \right\} z^{-n}$$

$m = n - r \qquad n = m + r$

$$\sum_{r=-\infty}^{\infty} f(r) \left\{ \sum_{n=-\infty}^{\infty} g(m) \right\} z^{-(m+r)}$$

$$\sum_{r=-\infty}^{\infty} f(r) z^{-r} \sum_{m=-\infty}^{\infty} g(m) z^{-m}$$

$f(n) * g(n)$

or

$$\boxed{Z\{f(n) * g(n)\} = F(z) * G(z)}$$

Hence Proved

* write the differences between L, fourier, Z-Transforms:-

1. Laplace Transformation is an extention of continuous time fourier transform.
2. The ~~Border~~ Broad class of signals, there are many signals for which fourier transform doesn't converge but laplace Transform does converge.
3. The laplace transform allow us to perform transform Analysis of unstable systems and to develope additional inside and tool for LTI system Analysis.
4. Z-Transform is discreet time counterpart of Laplace Transform.
5. Laplace Transform is meant for continuous time signals and z-transform is meant for discreet time signal.
6. To co-relate Z-transform and Laplace Transform sampling Theorm is used as a bridge b/w continuous and discreet signals.
7. In continuous time system laplace Transformation is considerds as ageneralisation of f.T similarly it is possible to generate f.T for discreet time signals and systems
8. Z-transform play a vital role in Analysis of representation of discreete time LTI system
9. Discreet time f.T can be applied only to stable system since it exists only in the impulse response is absolutely summable. where as z-transform of impulse exist for unstable system Thus Z-transform can be used to study much larger class of systems and signals.
10. As a continuous time the discreet time complex exponentials are eigen functions of LTI system which provide the Z-transform with powerful set of properties for Analysis of signals and systems
11. The primary role of Z-transform in engg. study of system characteristics and derivation

⑤

of computational structure for implementing discreet time signal applications

12. unilateral z-transform is used to solve d.e subject to initial conditions.

13 Limits are one side in L.T. but Two sided in F.T.

14. L.T of any function f(t) equals to a $f[f(t)]$ multiplied by convergence factor $e^{-\sigma t}$, if if $f(t)=0$ where $t<0$ Then we can write all the eqn's as

$$L[f(t)] = \oint f[f(t)\, e^{-at}] \quad \text{if } f(t)=0;\ t<0$$

$$\text{F.T } F[j\omega] = \int_{-\infty}^{\infty} f(t)\, e^{-j\omega t}\, dt$$

$$\text{L.T } F(s) = \int_{-\infty}^{\infty} f(t)\, e^{-st}\, dt$$

$$z\{f(z)\} = \sum_{n=-\infty}^{\infty} f(n)\, z^{-n}$$

Similarly

Inverse F.T

$$f(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} F(j\omega)\, e^{j\omega t}\, dt$$

Inverse L.T

$$f(t) = \frac{1}{2\pi j}\int_{\sigma_1 - j\infty}^{\sigma_1 + j\infty} F(s)\, e^{st}\, dt$$

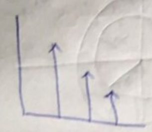
Inverse integral of Z.T

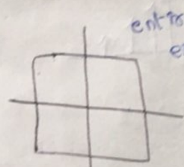
$$Z^{-1}\{X(z)\} = x(kT) = x(k) = \frac{1}{2\pi j}\oint_C X(z)\, z^{k-1}\, dz$$

Sequences, Their ROC in z-transforms

Sequences | ROC

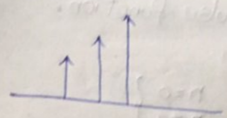
casual

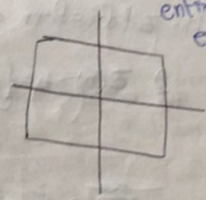




entire z-palne except $z=0$

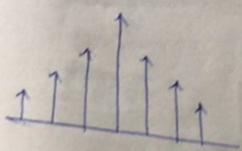
non casual

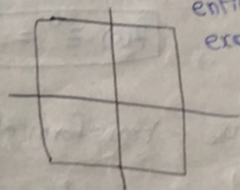




entire z-plane except $z=\infty$

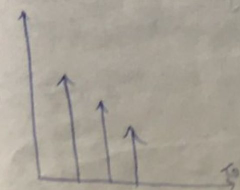
Two sided signal

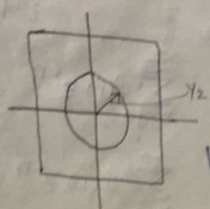




entire z-plane except $z=0$ $z=\infty$

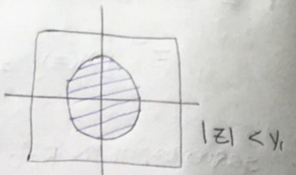
Infinate eqns.

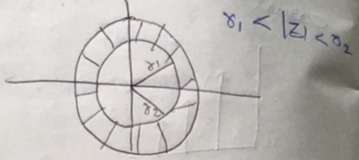




$|z| > 1/2$

⑤





find the Z-transform of unit impulse function.

$\cancel{Z(n)} = \left\{ \delta(n) = \begin{cases} 1 & n=0 \\ 0 & n\neq 0 \end{cases} \right\}$

$\sum_{n=0}^{\infty} f(n)\, z^{-n}$

$\sum_{n=0}^{\infty} \delta(n)\, z^{-n}$

$\boxed{f(0)\cdot z^{-0} = 1}$

find the z-transform of unit step function

$u(n) = \begin{cases} 0 & n<0 \\ 1 & n\geq 0 \end{cases}$

$\sum_{n=0}^{\infty} f(n)\, z^{-n}$

$\sum_{n=0}^{\infty} u(n)\, z^{-n}$

$\sum_{n=0}^{\infty} 1\left(\frac{1}{z}\right)^{n}$

$= 1 + \frac{1}{z} + \left(\frac{1}{z}\right)^2 + \left(\frac{1}{z}\right)^3 + \cdots$

$= \frac{1}{1-\left(\frac{1}{z}\right)} = \frac{z}{z-1} \quad |z| > 1$

find the z-transform of constant function k.

$\sum_{n=0}^{\infty} f(n)\, z^{-n}$

$\sum_{n=0}^{\infty} k\, z^{-n}$

$k \sum_{n=0}^{\infty} z^{-n}$

$k\left\{1 + \frac{1}{z} + \frac{1}{z^2} + \cdots\right\}$

$k\left\{\frac{z}{z-1}\right\}$

find the z-transform of a^n

$\sum_{n=0}^{\infty} f(n)\, z^{-n}$

$\sum_{n=0}^{\infty} a^n z^{-n}$

$\sum_{n=0}^{\infty} \left(\frac{a}{z}\right)^n$

$1 + \frac{a}{z} + \left(\frac{a}{z}\right)^2 + \left(\frac{a}{z}\right)^3 + \cdots$

$= \frac{1}{1-\left(\frac{a}{z}\right)} = \left|\frac{z}{z-a}\right| ; \ |z| > a$

find the z-transform of $z\{(-1)^n\}$

$\sum_{n=0}^{\infty} f(n)\, z^{-n}$

$\sum_{n=0}^{\infty} (-1)^n z^{-n}$

⑤ $\sum_{n=0}^{\infty} \left(\frac{-1}{z}\right)^n$

$-1+\left(\frac{-1}{z}\right)+\left(\frac{-1}{z}\right)^2+\left(\frac{-1}{z}\right)^3+\cdots$

$\frac{1}{1-(-1/z)} = \left|\frac{z}{z+1}\right| = |z|$

$z\{e^{at}\}$

$\sum_{n=0}^{\infty} f(n)\, z^{-n}$

$\sum_{n=0}^{\infty} e^{at}\, z^{-n}$

$\sum_{n=0}^{\infty} e^{nat}\, z^{-n}$

$\sum_{n=0}^{\infty} \left(\frac{e^{at}}{z}\right)^n$

$1+\frac{e^{at}}{z}+\left(\frac{e^{at}}{z}\right)^2+\cdots$

$\frac{1}{1-\frac{e^{at}}{z}} = \left|\frac{z}{z-e^{at}}\right|$

$z\{a^{n-1}\}$

$\sum_{n=0}^{\infty} f(n)\, z^{-n}$

$\sum_{n=1}^{\infty} a^{n-1}\, z^{-n}$

$\sum_{n=1}^{\infty} \frac{z}{a}\left(\frac{a}{z}\right)^n a$

$\sum_{n=1}^{\infty} a^{n-1}\, z^{-n}$

$a^0 z^{-1}+a^1 z^{-2}+a^2 z^{-3}+\cdots$

$\cancel{\frac{a}{z}+\frac{a}{z^2}}$ $\frac{1}{z}+\frac{a}{z^2}+\frac{a^2}{z^3}+\cdots$

$\frac{1}{z}\left\{1+\frac{a}{z}+\frac{a^2}{z^2}+\cdots\right\}$

$\frac{1}{z}\left\{\frac{1}{1-a/z}\right\} = \frac{1}{z}\left\{\frac{z}{z-a}\right\}$

$=\left|\frac{1}{z-a}\right| \qquad |z|>a$

Find $z\{n\}$

$\sum_{n=0}^{\infty} f(n)\, z^{-n}$

$\sum_{n=0}^{\infty} n\, z^{-n}$

$\sum_{n=0}^{\infty} n\, z^{-n}$

$\frac{1}{z}+\frac{2}{z^2}+\frac{3}{z^3}+\cdots$

$\frac{1}{z}\left\{1+\frac{2}{z}+\frac{3}{z^2}+\cdots\right\} \qquad \because 1+2x+3x^2+\cdots = (1-x)^{-2}$

$\frac{1}{z}\left\{1-\frac{1}{z}\right\}^{-2}$

$\cancel{\frac{1}{z}\left\{\frac{z-1}{z}\right.}$

$\frac{1}{z}\left\{1-\frac{1}{z}\right\}^{-2}$

$\frac{1}{z}\left\{\frac{1}{\left(1-\frac{1}{z}\right)^2}\right\}$

$\frac{1}{z}\left\{\frac{z^2}{(z-1)^2}\right\} = \left|\frac{z}{(z-1)^2}\right| \qquad |z|>1$

find $z\{n^2\}$

$\sum_{n=0}^{\infty} f(n) z^{-n}$

5 W.K.T $z\{n^2\} = -z \frac{d}{dz}\left(\frac{\cancel{z}}{\cancel{z-1}}\right)^2 \left\{\frac{z}{(z-1)^2}\right\}$

$$= -z \left\{\frac{(z-1)^2(1) - z(2)(z-1)1}{(z-1)^4}\right\}$$

$$= -z \left\{\frac{\cancel{(z-1)}\{(z-1) - 2z\}}{(z-1)^{\cancel{4}3}}\right\}$$

$$= -z \left\{\frac{(z-1) - 2z}{(z-1)^3}\right\}$$

$$= -z \left\{\frac{-z-1}{(z-1)^3}\right\}$$

$$= z \left\{\frac{\cancel{(z-1)}}{\cancel{(z-1)^{3}}2}\right\}$$

$$= \frac{z}{\cancel{(z-1)}^2}$$

$$\boxed{z\{n^2\} = z\left\{\frac{z+1}{(z-1)^3}\right\}}$$

☆ find the $z\{r^n \cos n\theta\}$

consider $z\{(re^{i\theta})^n\} = \left[\frac{z}{z - re^{i\theta}}\right]$

$$z\left[r^n(\cos\theta + i\sin\theta)^n\right] = \frac{z}{z - r(\cos\theta + i\sin\theta)}$$

$$z\left[r^n(\cos n\theta + i\sin n\theta)\right] = \frac{z}{(z - r\cos\theta) - ri\sin\theta}$$

rationalise on R.H.S

$$= \frac{z}{(z - r\cos\theta) - ir\sin\theta} \times \frac{(z - r\cos\theta) + ir\sin\theta}{(z - r\cos\theta) + ir\sin\theta}$$

$$= \frac{z\left[(z - r\cos\theta) + ir\sin\theta\right]}{(z - r\cos\theta)^2 + r^2\sin^2\theta}$$

$$= \frac{z\left[z - r\cos\theta\right] + z\, ir\sin\theta}{z^2 + r^2\cos^2\theta - 2zr\cos\theta + r^2\sin^2\theta}$$

$$= \frac{z\left[z - r\cos\theta\right] + z(ir)\sin\theta}{z^2 - 2zr\cos\theta + r^2}$$

$$z\{r^n\cos n\theta + r^n \sin n\theta\} = \frac{z\left[z - r\cos\theta\right] + zir\sin\theta}{z^2 - 2zr\cos\theta + r^2}$$

compare real and imaginary parts

Put $r = 1$

$$z\left[r^n\cos n\theta\right] = \frac{z\left[z - r\cos\theta\right]}{z^2 - 2zr\cos\theta + r^2} = \frac{z\left[z - \cos\theta\right]}{z^2 - 2z\cos\theta + 1}$$

$$z\left[r^n\sin n\theta\right] = \frac{zir\sin\theta}{z^2 - 2zr\cos\theta + r^2} = \frac{z\, i\sin\theta}{z^2 - 2z\cos\theta + 1}$$

~~Put r=~~

Put $\theta = \pi/2$

$$z\left[r^n\cos n\theta\right] = \frac{z\left[z - \cos\frac{\pi}{2}\right]}{z^2 - 2z\cos(\frac{\pi}{2}) + 1} = \frac{z^2}{z^2 + 1}$$

$$z\left[r^n\sin n\theta\right] = \frac{iz\sin(\frac{\pi}{2})}{z^2 - 2z\cos\frac{\pi}{2} + 1} = \frac{z}{z^2 + 1}$$

Scanned with CamScanner

find $z\{e^{-at}\cos bt\}$

⑤ $z\{f(t)e^{-at}\} = \bar{f}(ze^{at})$

$$z\{\cos bt\} = \left|\frac{z[z - \cos bt]}{z^2 - 2z\cos bt + 1}\right| \quad z \to ze^{at}$$

$$\frac{ze^{at}(z\cdot e^{at} - \cos bt)}{(ze^{at})^2 - 2ze^{at}\cos bt + 1}$$

find Z-transform, sketch ROC for the sequence

$x(n) = u(n)$

$$\sum_{n=0}^{\infty} u(n)z^{-n}$$

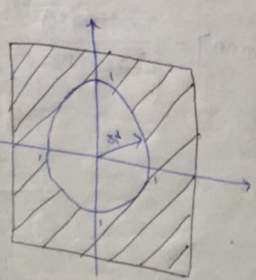
$$\sum_{n=0}^{\infty} 1\,z^{-n}$$

$$\sum_{n=0}^{\infty} \left(\frac{1}{z}\right)^n$$

$$1 + \frac{1}{z} + \frac{1}{z^2} + \cdots$$

$$\frac{1}{1-\left(\frac{1}{z}\right)} = \left|\frac{z}{z-1}\right| = |z| > 1$$

ROC:-



Inverse Z-transform:-

If $Z\{f(n)\} = \bar{f}(z)$ Then $z^{-1}[\bar{f}(z)]$ can be found out by the following methods.

There are four methods to find z^{-1}

1. expansion method
2. long division method
3. partial fraction method
4. Residue Theorem (or) cauchi Counter integral method

find z^{-1} for the following using expansion method:-

① $z^{-1}\left\{\frac{1}{z+2}\right\}$

In expansion method we need to have the problem in terms of specific format then only we can Analyse.

$$\left(\frac{1}{z+2}\right)$$

$$\frac{1}{z(1+2/z)}$$

$$\frac{1}{z}\left(1+\frac{2}{z}\right)^{-1} \qquad (1+x)^{-1} = 1 - x + x^2 - x^3 + x^4$$

$$\frac{1}{z}\left[1 - \left(\frac{2}{z}\right) + \left(\frac{2}{z}\right)^2 - \left(\frac{2}{z}\right)^3 + \cdots (-1)^{n-1}\left(\frac{2}{z}\right)^{n-1} + (-1)^n\left(\frac{2}{z}\right)^n\right]$$

$$\frac{1}{z}(-1)^{n-1}\left(\frac{2}{z}\right)^{n-1}$$

$$(-1)^{n-1}\cdot\frac{2^{n-1}}{z^{n-1}}\times\frac{1}{z} = (-1)^{n-1}\frac{2^{n-1}}{z^n}$$

$$= (-1)^{n-1}2^{n-1}z^{-n} \qquad \text{Co--}$$

Co-efficient of $z^{-n} = (-1)^{n-1}2^{n-1}$

⑤ find $Z^{-1}\left\{\frac{1+2z^{-1}}{1-z^{-1}}\right\}$ by Long division method

$$\begin{array}{r|l} & 1+3z^{-1}+3z^{-2}+3z^{-3} \\ \hline 1-z^{-1} & 1+2z^{-1} \\ & 1-z^{-1} \\ \hline & 3z^{-1} \\ & 3z^{-1}-3z^{-2} \\ \hline & 3z^{-2} \\ & 3z^{-2}-3z^{-3} \\ \hline & 3z^{-3} \\ & 3z^{-3}-3z^{-4} \end{array}$$

find $Z^{-1}\left\{\frac{3z^2-18z+26}{(z-2)(z-3)(z-4)}\right\}$ using partial fraction method

$3z^2-18z+26 = \frac{A}{z-2}+\frac{B}{z-3}+\frac{C}{z-4}$

$3z^2-18z+26 = A(z-3)(z-4)+B(z-2)(z-4)+C(z-2)(z-3)$

put $z=2$

$12-36+26 = A(-1)(-2)+0+0$

$2=2A$

$\boxed{A=1}$

put $z=3$

$27-54+26 = B(1)(-1)$

$-1=-B$

$\boxed{B=1}$

put $z=4$

$48-72+26 = C(2)(1)$

$2C=2$

$\boxed{C=1}$

$= z\left[\frac{1}{(z-2)}+\frac{1}{(z-3)}+\frac{1}{(z-4)}\right]$

$Z\{a^{n-1}\} = \frac{1}{z-a}$

$a^{n-1} = Z^{-1}\left\{\frac{1}{z-a}\right\}$

$= 2^{n-1}+3^{n-1}+4^{n-1}$ //

Cauchy residue Theorem:-

It states that

$\oint_C f(z)\cdot dz = 2\pi i$ (sum of Residues of poles lying inside)

There are two types.

Residues:-

① if $z=a$ is a simple pole i.e a pole of order 1

Then residue of $\{f(z)\}_{z=a}$ is given by

$\boxed{\text{order 1}}$ $\quad Res\,[f(z)]_{z=a} = \lim_{z\to a}(z-a)\,f(z)\,z^{n-1}$

② if $z=a$ is a pole of order n then

$\boxed{\text{order n}}$ $\quad \lim_{z\to a}\frac{1}{(m-1)!}\frac{d^{m-1}}{dz^{m-1}}(z-a)^m f(z)\,z^{n-1}$

Scanned with CamScanner

Find z^{-1} using residue Theorem:-

(5) $z^{-1}\left\{\frac{z^2}{(z-a)(z-b)}\right\}$

$Res[f(z)]_{z\to a} = \lim_{z\to a} \cancel{(z-a)} \frac{z^2}{\cancel{(z-a)}(z-b)} z^{n-1}$

$= \lim_{z\to a} \frac{z^{n+1}}{(z-b)}$

$= \frac{a^{n+1}}{(a-b)}$ —— (1)

$Res[f(z)]_{z=b} = \lim_{z\to b} \cancel{(z-b)} \frac{z^2}{(z-a)\cancel{(z-b)}} z^{n-1}$

$= \lim_{z\to b} \frac{z^{n+1}}{(z-a)}$ —— (2)

$= \frac{b^{n+1}}{b-a}$

$$\boxed{\oint_C f(z)dz = 2\pi i \left\{\frac{a^{n+1}}{a-b} + \frac{b^{n+1}}{b-a}\right\}}$$

Find z^{-1} using counter integral method.

$z^{-1}\left\{\frac{2z^2+4z}{(z-2)^3}\right\}$

$m=3$

$Res(f(z))_{z=2} = \lim_{z\to 2} \frac{1}{(m-1)!} \frac{d^{m-1}}{dz^{m-1}} (z-a)^m f(z) z^{n-1}$

$= \lim_{z\to 2} \frac{1}{(3-1)!} \frac{d^2}{dz^2} \cancel{(z-2)^3} \left[\frac{2z^2+4z}{\cancel{(z-2)^3}}\right] z^{n-1}$

$= \lim_{z\to 2} \frac{1}{2!} \frac{d^2}{dz^2} [2z^2+4z] z^{n-1}$

$\lim_{z\to 2} \frac{1}{2!} \frac{d^2}{dz^2} [2z^{n+1} + 4z^n]$

$\lim_{z\to 2} \frac{1}{2!} \frac{d}{dz} [2(n+1)z^n + 4z^{n-1}]$

$\lim_{z\to 2} \frac{1}{2!} \{2(n+1)\, n . z^{n-1} + 4 . n . (n-1) z^{n-2}\}$

$\frac{1}{2!} [2(n+1) n . 2^{n-1} + 4 . n (n-1) 2^{n-2}]$

$\frac{1}{2!} \left[\cancel{2}(n+1) n . \frac{2^n}{\cancel{2}} + \cancel{4} . n(n-1) \frac{2^n}{\cancel{2^2}}\right]$

$\frac{1}{2!} [n(n+1)2^n + n(n-1)2^n]$

$\frac{1}{2!} n 2^n [n+\cancel{1}+n-\cancel{1}]$

$\frac{2^n n (2n)}{2}$

$= 2^n . n^2$ //

Find the z^{-1} for the following partial fraction

$\frac{4z^2-24z+28}{(z-4)(z-6)(z-8)}$

$\frac{4z^2-24z+28}{(z-4)(z-6)(z-8)} = \frac{A}{(z-4)} + \frac{B}{(z+6)} + \frac{C}{(z-8)}$

$4z^2-24z+28 = A(z+6)(z-8) + B(z-4)(z-8) + C(z-2)(z-4)$

Put $z=4$

$64-96+28 = A(-2)(-4)$

$-4 = 8A$

$A = -1/2$

(5)

put $z=6$

$4z^2-24z+28 = B(z-4)(z-8)$

$144-144+28 = B(2)(-2)$

$-4B = 28$

$B = -7$

put $z=8$

$4z^2-24z+28 = C(z-4)(z-6)$

$256-192+28 = C(4)(2)$

$92 = 8C$

$C = \frac{23}{2}$

Find $Z^{-1}\left\{\frac{4z}{(z-1)^3}\right\}$ long division method

$= \frac{4z}{(z-1)^3}$

$= \frac{4z}{z^3(1-\frac{1}{z})^3}$

$= \frac{4}{z^2}(1-\frac{1}{z})^{-3}$

$= \frac{4z^{-2}}{(1-z^{-1})^3}$

$(a-b)^3 = a^3 - 3a^2b + 3ab^2 - b^3$

$= \frac{4z^{-2}}{1-3z^{-2}+3z^{-1}-z^{-3}}$

$\cancel{1-3z^{-2}+3z^{-1}-z^{-3}}$) $\cancel{4z^{-2}+12z^{-4}}$

$4z^{-2}$

$\cancel{4z^{-2}}-12z^{-4}$

$1+3z^{-1}-3z^{-2}-z^{-3}$) $4z^{-2}$ ($4z^{-2}+12z^{-3}+24z^{-4}+$

$4z^{-2}+12z^{-3}-12z^{-4}-4z^{-5}$

$-\quad -\quad +\quad +$

$12z^{-3}+12z^{-4}+4z^{-5}$

$12z^{-3}+36z^{-4}-36z^{-5}-12z^{-6}$

$(-)\quad (-)\quad (+)\quad (+)$

$\lim_{z=1} \frac{1}{2!}\frac{d^2}{dz^2}\frac{4z}{(z-1)^3}(z-1)^3 z^{n-1}$

$\frac{1}{2!}\frac{d^2}{dz^2} 4z^n$

$\frac{1}{2!}\frac{d}{dz} n z^{n-1}$

$\lim_{z=1} \frac{1}{2!} \frac{d}{dz} n(n-1)z^{n-2}$

$\frac{1}{2!} n(n-1)(1)^{n-2}$

$2n(n-1) \quad n \neq 0$

$0 \quad n=0$

Scanned with CamScanner