HW 5 Patrick Neyland

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1. Given: $\frac{dk(t)}{dt} = rk(t) + sw(0) + \gamma[w(t) - w(0)] \quad \forall t \in [0, T] \qquad (5)$

$\frac{dk(t)}{dt} = rk(t) - A \quad \forall t \in [T, \overline{T}] \qquad (6)$

$k(0) = 0 \qquad k(\overline{T}) = 0$

$$\boxed{\begin{array}{ll} \text{G.S. for (5):} & k(t) = e^{rt} \cdot \left\{ Z_1 + \int^t sw(0) + \gamma[w(t) - w(0)] e^{-rv} dv \right\} \\ \text{G.S. for (6):} & k(t) = e^{rt} \cdot \left\{ Z_2 + \int^t A e^{-rv} dv \right\} \end{array}}$$

P.S. for (5) $\quad k(0) = \frac{\cancel{e^{rt}}}{\cancel{e^{rt}}} \cdot \left\{ Z_1 + \int^0 sw(0) + \gamma[w(t) - w(0)] e^{-rv} dv \right\} = \frac{0}{\cancel{e^{rt}}}$

$Z_1 = -\int^0 sw(0) + \gamma [w(t) - w(0)] e^{-rv} dv$

$k(t) = e^{rt} \cdot \left\{ -\int^0 sw(0) + \gamma [w(t) - w(0)] e^{-rv} dv + \int^t sw(0) + \gamma[w(t) - w(0)] e^{-rv} dv \right\}$

$$\boxed{k(t) = e^{rt} \cdot \left\{ \int_0^t sw(0) + \gamma [w(t) - w(0)] e^{-rv} dv \right\}}$$

P.S. for (6): $\quad k(t) = \cancel{e^{rt}} \cdot \left\{ Z_2 + \int^T A e^{-rv} dv \right\} = 0$

$Z_2 = -\int^T A e^{-rv} dv$

$k(t) = e^{rt} \cdot \left\{ -\int^T A e^{-rv} dv + \int^t A e^{-rv} dv \right\}$

$$\boxed{k(t) = e^{rt} \cdot \left\{ \int_T^t A e^{-rv} dv \right\}}$$

2. $\frac{\cancel{e^{rt}} \cdot \left\{ \int_T^{\overline{T}} A e^{-rv} dv \right\}}{\cancel{e^{rt}}} = \frac{\cancel{e^{rt}} \cdot \left\{ \int_0^T sw(0) + \gamma [w(t) - w(0)] e^{-rv} dv \right\}}{\cancel{e^{rt}}}$

$\int_T^{\overline{T}} A e^{-rt} dt = \int_0^T sw(0) + \gamma [w(t) - w(0)] e^{-rt} dt$

$\frac{A \cancel{\int_T^{\overline{T}} e^{-rt} dt}}{\cancel{\int_T^{\overline{T}} e^{-rt} dt}} = \frac{\int_0^T sw(0) + \gamma [w(t) - w(0)] e^{-rt} dt}{\int_T^{\overline{T}} e^{-rt} dt}$

$$\boxed{A = \frac{\int_0^T sw(0) + \gamma [w(t) - w(0)] e^{-rt} dt}{\int_T^{\overline{T}} e^{-rt} dt}}$$