ECN 5320/6320

HW 2

Fall 2022
100 pts.

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Instructions: write derivations and answers to the assignment on separate pieces of paper. Write neatly and clearly!

✓1. Provide the general and particular solutions to the ODE $\frac{dy}{dt} = 2y(t)$ given $y(0) = 9$. **(5 points)**

✓2. Provide the general and particular solutions to the ODE $\frac{dy}{dt} = 23$ given $y(0) = 1$. **(5 points)**

✓3. Provide the general and particular solutions to the ODE $\frac{dy}{dt} = -10y(t) + 15$ given $y(0) = 0$. **(5 points)**

✓4. Provide the general and particular solutions to the ODE $\frac{dy}{dt} = y(t) + 4$ given $y(0) = 0$. **(5 points)**

✓5. Provide the general and particular solutions to the ODE $\frac{dy}{dt} = -2ty(t) + t$ given $y(0) = 3/2$. **(5 points)**

✓6. Provide the general solution to the ODE $\frac{dq}{dt} = -3t^2 q(t)$. **(5 points)**

✓7. Verify exactness and solve via four-step procedure $dF(y,t) = 3ty^2\, dy + (y^3 + 2t)\, dt = 0$. **(15 points)**

✓8. Verify exactness and solve via four-step procedure $\frac{dy}{dt} = -\frac{2y^4 t + 3t^2}{4y^3 t^2}$. **(15 points)**

9. Identify the integrating factor and find the general solution to $dF(y,t) = 4y^3 t\, dy + (2y^4 + 3t)\, dt = 0$. **(20 points)**

✓10. Identify the integrating factor and find the general solution to $\frac{dy}{dt} = 2ty(t) + bt$. **(20 points)**

Scan your derivations into a single pdf file and upload it to Canvas by **Wednesday 10/5/2022 at 11:59pm**.

1. $\frac{dy}{dt} = 2y(t)$ given $y(0) = 9$

Seperable & autonomous

$\frac{1}{y(t)}\, dy = 2\, dt$

$\int \frac{1}{y}\, dy = \int 2\, dt$

$\int \frac{1}{y}\, dy = 2 \int 1\, dt$

$\ln y + z_1 = 2t + z_2$

$\ln y = 2t + z$

$\boxed{\begin{array}{l} y(t) = e^{2t+z} \text{ general solution} \quad e^{z} \cdot e^{2t} \\ y(t) = 9e^{2t} \text{ particular solution} \\ \quad 9e^{2(0)} = 9 \end{array}}$

2. $\frac{dy}{dt} = 23$ given $y(0) = 1$

$\int 1\,dy = \int 23\,dt$

$y(t) = 23t + z$ general solution

$y(0) = 1 = 23(0) + z$ $y(0) = 23t + 1$ particular solution

3. $\frac{dy}{dt} = -10y(t) + 15$ given $y(0) = 0$

$\frac{y'}{-2y+3} = \frac{-10y(t)+15}{-2y+3}$

$\int \frac{1}{-2y+3}\,dy = \int 5\,dt$

$-\frac{1}{2}\ln(-2y+3) + z_1 = 5t + z_2 \longrightarrow -\frac{1}{2}\ln(-2(0)+3) = 5(0) + z$

$-2\left(-\frac{1}{2}\ln(-2y+3)\right) = (5t + z)\,-2$ $\qquad -\frac{1}{2}\ln(3) = z$

$\ln(-2y+3) = -10t - 2z$ $\qquad \left(-\frac{1}{2}\ln(-2y+3) = 5y - \frac{1}{2}\ln(3)\right) -2$

$\frac{-2y+3}{-2} \; -3 = e^{-10t-2z}$ $\qquad e^{\ln(-2y+3)} = e^{-10y + \ln(3)}$

$y = -\frac{e^{-10t-2z}}{2} + \frac{3}{2}$ $\qquad \frac{-2y+3}{-2} \; -3 = 3e^{-10y}$

$y(t) = -\frac{e^{-10(t)-z}}{2} + \frac{3}{2}$ general solution $\qquad y = -\frac{3e^{-10y}}{2} + \frac{3}{2}$

particular solution

4. $\dfrac{\frac{dy}{dt}}{y+4} = \dfrac{y(t)+4}{y+4}$ given $y(0)=0$

$\int \frac{1}{y+4}\,dy = \int 1\,dt$

$\ln(y+4) + z_1 = t + z_2$

$ - z_1 \qquad\quad - z_1$

$y + 4 = e^{t+z}$

$ - 4$

$\ln(y(0)+4) = 0 + z$

$\ln(4) = z$

$\ln(y+4) = t + \ln(4)$

$y + 4 = 4e^{t} - 4$

$y = e^{t+z} - 4$ general solution $y = 4e^{t} - 4$ particular solution

5. $\frac{dy}{dt} = -2ty(t) + t$ given $y(0) = 3/2$

general solution

$$y = e^{-\int 2(t)\,dt}\left\{ Z + \int t\, e^{\int 2(t)\,dt}\, dt \right\}$$

particular $y = e^{-t^2} + \frac{1}{2}$

6. $\frac{dq}{dt} = -3t^2 q(t)$

general solution $q(t) = e^{-t^3 + z}$

7. $dF(y,t) = \underbrace{3ty^2}_{M}dy + \underbrace{(y^3+2t)}_{N}dt = 0$

1. $F(y,t) = \int 3ty^2\, \partial y + \sigma(t)$

$F(y,t) = ty^3 + \sigma(t)$

2. $\dfrac{\partial F(y,t)}{\partial t} = y^3 + \sigma'(t)$

$\cancel{y^3} + 2t = \cancel{y^3} + \sigma'(t)$

3. $\sigma'(t) = 2t$

$\sigma(t) = t^2$

4. $F(y,t) = ty^3 + t^2 + z$

<u>Verify exactness</u>

$M = 3ty^2$ $\qquad \dfrac{\partial M}{\partial t} = 3y^2$

$N = y^3 + 2t$

$\dfrac{\partial N}{\partial y} = 3y^2$

8. $dF(y,t) = \underbrace{4y^3t^2}_{M}\, dy + \underbrace{2y^4t + 3t^2}_{N}\, dt$

1. $F(y,t) = \int 4y^3t^2\, dy + \sigma(t)$

$= y^4t^2 + \sigma(t)$

2. $\dfrac{\partial F(y,t)}{\partial t} = 2y^4t + \sigma'(t)$

3. $\cancel{2y^4t} + 3t^2 = \cancel{2y^4t} + \sigma'(t)$

$3t^2 = \sigma'(t)$

$t^3 = \sigma(t)$

4. $F(y,t) = y^4t^2 + t^3 + Z$

Verify exactness

$M = 4y^3t^2$ $\quad \dfrac{\partial M}{\partial t} = 8y^3t$

$N = 2y^4t + 3t^2$ $\quad \dfrac{\partial N}{\partial y} = 8y^3t$

9. $I[4y^3t]\,dy + I[2y^4 + 3t]\,dt$

$M = 4y^3t \quad N = 2y^4 + 3t \quad \frac{\partial M}{\partial t} = 4y^3 \quad \frac{\partial N}{\partial t} = 8y^3$

$I = e^{\int \frac{8y^3 - 4y^3}{4y^3t}} = e^{\int \frac{1}{t}dt} = e^{\ln t} = \boxed{t}$

$dF(y,t)\quad \underbrace{4y^3t^2}_{M}\,dy + \underbrace{2y^4t + 3t^2}_{N}\,dt = 0$

1. $F(y,t) = \int 4y^3t^2\,dy + \sigma(t)$

$= y^4t^2 + \sigma(t)$

2. $\frac{\partial F(y,t)}{\partial t} = 2y^4t + \sigma'(t)$

3. $\cancel{2y^4t} + 3t^2 = \cancel{2y^4t} + \sigma'(t)$

$3t^2 = \sigma'(t)$

$t^3 = \sigma(t)$

4. $F(y,t) = y^4t^2 + t^3 + Z$

10. $\frac{dy}{dt} = 2ty(t) + bt = -1\,dy + \left[2ty(t) + bt\right]dt$

$I \cdot -1\,dy + I \cdot \left[2ty(t) + bt\right]dt$

$\frac{dI(t)}{dt} = 2t\,I(t) \rightarrow \int \frac{1}{I(t)}\,dI(t) = t^2 + z$

$\ln I(t) \underset{-z}{+z} = t^2 + z - z \rightarrow I(t) = e^{-z} \cdot e^{t^2} = \boxed{e^{-t^2}}$

$dF(y,t) = \underbrace{-e^{-t^2}}_{M}\,dy + \underbrace{2ty(t)e^{-t^2} + bte^{-t^2}}_{N}\,dt$

1. $F(y,t) = \int -e^{-t^2}\,dy + \sigma(t)$

$= -e^{-t^2}y + \sigma(t)$

2. $\frac{\partial F(y,t)}{\partial t} = 2yte^{-t^2} + \sigma'(t)$

3. $\cancel{2ty(t)e^{-t^2}} + bte^{-t^2} = \cancel{2y(t)e^{-t^2}} + \sigma'(t)$

$\sigma(t) = -\frac{be^{-t^2}}{2}$

4. $F(y,t) = ye^{-t^2} - \frac{be^{-t^2}}{2} + z$