HW6 Patrick Neyland

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$$H(t) = \left[-u(t)^2 + \theta\delta u(t) - \alpha\theta - \theta\Psi\varepsilon(t)\right]\cdot M_e(E-t)$$
$$+ \lambda(t)\cdot r\left[\alpha - \delta u(t) - (1-\Psi)\varepsilon(t)\right]$$

Partial derivative with respect to u

$$\frac{\partial H}{\partial \varepsilon} = -\theta\Psi M_e(E-t) - (1-\Psi)\gamma\lambda(t) \stackrel{set}{=} -\frac{d\lambda(t)}{dt}$$

Given the adjoint condition:

$$\frac{\partial H}{\partial \varepsilon} = -\theta\Psi M_e(E-t) - (1-\Psi)\gamma\lambda(t) \stackrel{set}{=} -\frac{d\lambda(t)}{dt}$$

Solve like a normal ODE:

$$\lambda(t) = e^{\int^t r(1-\Psi)\,ds}\cdot\left\{Z_1 + \int^t \theta\Psi M_e(E-s)\cdot e^{-\int^s r(1-\Psi)\,dj}\,ds\right\}$$

G.S. $\lambda(t) = e^{r(1-\Psi)t}\cdot\left\{Z_1 + \int^t \theta\Psi M_e(E-s)\cdot e^{-r(1-\Psi)s}\,ds\right\}$

$$\lambda(E) = \cancel{e^{r(1-\Psi)t}}\cdot\left\{Z_1 + \int^E \theta\Psi M_e(E-s)\cdot e^{-r(1-\Psi)s}\,ds\right\} = 0$$

$$Z_1 = -\int^E \theta\Psi M_e(E-s)\cdot e^{-r(1-\Psi)s}\,ds$$

Sub back into G.S.

$$\lambda(t) e^{r(1-\Psi)t}\cdot\left\{-\int^E \theta\Psi M_e(E-s)\cdot e^{-r(1-\Psi)s}\,ds + \int^t \theta\Psi M_e(E-s)\cdot e^{-r(1-\Psi)s}\,ds\right\}$$

P.S. $\lambda(t) = e^{r(1-\Psi)t}\cdot\int_E^t \theta\Psi M_e(E-s)\cdot e^{-r(1-\Psi)s}\,ds$

Now we can sub the P.S. into the maximum condition

$$\left[-2u(t) + \theta\delta\right]\cdot M_e(E-t) - \gamma\delta\boxed{\lambda(t)} = 0$$

$$\left[-2u(t) + \theta\delta\right]\cdot M_e(Et) - \gamma\delta\, e^{r(1-\Psi)t}\int_E^t \theta\Psi M_e(E-s)\cdot e^{-r(1-\Psi)s}\,ds = 0$$

Rearrange to isolate u(t)

$$\frac{\left[-2u(t) + \theta\delta\right]\cdot \cancel{M_e(E-t)}}{\cancel{M_e(E-t)}} =$$

$$-2u(t) + \cancel{\theta\delta} = \frac{\gamma\delta\, e^{r(1-\Psi)t}}{M_e(E-t)}\cdot\int_E^t \theta\Psi M_e(E-s)\cdot e^{-r(1-\Psi)s}\,ds - \theta\delta$$
$$\cancel{-\theta\delta}$$

$$-2u(t) = -\theta\delta + \frac{\gamma\delta\, e^{r(1-\Psi)t}}{M_e(E-t)}\cdot\int_E^t \theta\Psi M_e(E-s)\cdot e^{-r(1-\Psi)s}\,ds$$

$$u^*(t) = \frac{\theta\delta}{2}\left[1 + \frac{r\Psi\, e^{r(1-\Psi)t}}{M_e(E-t)}\cdot\int_t^E M_e(E-s)\cdot e^{-r(1-\Psi)s}\,ds\right]$$

now Sub in $e^{-\rho(E-t)}$ in for $M_e(E-t)$

$$u^*(t) = \frac{\theta\delta}{2}\left[1 + \frac{r\Psi\, e^{r(1-\Psi)t}}{e^{-\rho(E-t)}}\cdot\int_t^E e^{-\rho(E-s)}\cdot e^{-r(1-\Psi)s}\,ds\right]$$

now evaluate the integral and Simplify

$$\int_t^E e^{-\rho(E-s)}\cdot e^{-r(1-\Psi)s}\,ds = \int_t^E e^{-(1-\Psi)\gamma s - \rho(E-s)}\,ds$$

u Substitution

$$U = -(1-\Psi)\gamma s - \rho(E-s) \qquad \frac{du}{ds} = \rho - (1-\Psi)\gamma$$

$$ds = \frac{1}{\rho - (1-\Psi)\gamma}\,du$$

$$\int \frac{e^u}{\rho-(1-\Psi)\gamma}\,du = \frac{1}{\rho-(1-\Psi)\gamma}\int e^u\,du$$
$$= \frac{e^u}{\rho-(1-\Psi)\gamma}$$

add bounary values t and E

$$U = -(1-\Psi)\gamma t - \rho(E-t)$$
$$U = -(1-\Psi)\gamma E - \cancel{\rho(E-E)} = -(1-\Psi)\gamma E$$

Sub U's back in

$$\frac{e^{-(1-\Psi)\gamma E} - e^{-(1-\Psi)\gamma t - \rho(E-t)}}{\rho - (1-\Psi\ \gamma)}$$

$$u^*(t) = \frac{\theta\delta}{2}\left[1 + \frac{r\Psi\, e^{(1-\Psi)t r - \rho E}}{e^{-\rho(E-t)}\cdot(\rho-(1-\Psi)\gamma)}\left[e^{(\rho-(1-\Psi)\gamma)E} - e^{(\rho(1-\Psi)\gamma)t}\right]\right]$$