HW6 Patrick Neyland

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$$H(t) = [-u(t)^2 + \theta \delta u(t) - \alpha\theta - \theta\psi\varepsilon(t)]\cdot M_e(E-t) + \lambda(t)\cdot\gamma[\alpha - \delta u(t) - (1-\psi)\varepsilon(t)]$$

Partial derivative with respect to u

$$\frac{\partial H}{\partial \varepsilon} = -\theta\psi M_e(E-t) - (1-\psi)\gamma\lambda(t) \overset{set}{=} -\frac{d\lambda(t)}{dt}$$

Given the adjoint condition:

$$\frac{\partial H}{\partial \varepsilon} = -\theta\psi M_e(E-t) - (1-\psi)\gamma\lambda(t) \overset{set}{=} -\frac{d\lambda(t)}{dt}$$

Solve like a normal ODE:

$$\lambda(t) = e^{\int^t r(1-\psi)\,ds}\cdot\left\{Z_1 + \int^t \theta\psi M_e(E-s)\cdot e^{-\int^s r(1-\psi)\,dj}\,ds\right\}$$

$$\text{G.S.}\quad \lambda(t) = e^{r(1-\psi)t}\cdot\left\{Z_1 + \int^t \theta\psi M_e(E-s)\cdot e^{-r(1-\psi)s}\,ds\right\}$$

$$\lambda(E) = \cancel{e^{r(1-\psi)t}}\cdot\left\{Z_1 + \int^E \theta\psi M_e(E-s)\cdot e^{-r(1-\psi)s}\,ds\right\} = 0$$

$$Z_1 = -\int^E \theta\psi M_e(E-s)\cdot e^{-r(1-\psi)s}\,ds$$

Sub back into G. S.

$$\lambda(t) e^{r(1-\psi)t}\cdot\left\{-\int^E \theta\psi M_e(E-s)\cdot e^{-r(1-\psi)s}\,ds + \int^t \theta\psi M_e(E-s)\cdot e^{-r(1-\psi)s}\,ds\right\}$$

$$\text{P.S.}\quad \lambda(t) = e^{r(1-\psi)t}\cdot\int_E^t \theta\psi M_e(E-s)\cdot e^{-r(1-\psi)s}\,ds$$

Now we can sub the P. S. into the maximum condition

$$[-2u(t) + \theta\delta]\cdot M_e(E-t) - \gamma\delta\boxed{\lambda(t)} = 0$$

$$[-2u(t) + \theta\delta]\cdot M_e(Et) - \gamma\delta\, e^{r(1-\psi)t}\int_E^t \theta\psi M_e(E-s)\cdot e^{-r(1-\psi)s}\,ds = 0$$

Rearrange to isolate u(t)

$$\frac{[-2u(t) + \theta\delta]\cdot \cancel{M_e(E-t)}}{\cancel{M_e(E-t)}} =$$

$$-2u(t) + \cancel{\theta\delta} = \frac{\gamma\delta\, e^{r(1-\psi)t}}{M_e(E-t)}\cdot\int_E^t \theta\psi M_e(E-s)\cdot e^{-r(1-\psi)s}\,ds - \theta\delta$$

$$\cancel{-\theta\delta}$$

$$-2u(t) = -\theta\delta + \frac{\gamma\delta\, e^{r(1-\psi)t}}{M_e(E-t)}\cdot\int_E^t \theta\psi M_e(E-s)\cdot e^{-r(1-\psi)s}\,ds$$

$$u^*(t) = \frac{\theta\delta}{2}\left[1 + \frac{\gamma\psi\, e^{r(1-\psi)t}}{M_e(E-t)}\cdot\int_t^E M_e(E-s)\cdot e^{-r(1-\psi)s}\,ds\right]$$

now Sub in $e^{-\rho(E-t)}$ in for $M_e(E-t)$

$$u^*(t) = \frac{\theta\delta}{2}\left[1 + \frac{\gamma\psi\, e^{r(1-\psi)t}}{e^{-\rho(E-t)}}\cdot\int_t^E e^{-\rho(E-s)}\cdot e^{-r(1-\psi)s}\,ds\right]$$

now evaluate the integral and Simplify

$$u^*(t) = \frac{\theta\delta}{2}\left[1 + \frac{\gamma\psi\, e^{(1-\psi)tr - \rho E}}{e^{-\rho(E-t)}\cdot(\rho - (1-\psi)\gamma)}\left[e^{(\rho-(1-\psi)\gamma)E} - e^{(\rho(1-\psi)\gamma)t}\right]\right]$$