

ECN 5320/6320

HW 1

Fall 2022

100 pts.

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Instructions: write derivations and answers to the assignment on separate pieces of paper.
Write neatly and clearly!

- ✓ 1. Calculate $\int 9x^{1/3} dx$. (5 points)
- ✓ 2. Calculate $\int_1^2 y^{-1} dy$. (5 points)
- ✓ 3. Calculate $\int_a^b 1 dt$. (5 points)
- ✓ 4. Calculate $\int b^{2\tau} d\tau$. (5 points)
- ✓ 5. Calculate $\int_a^b re^{-rt} dt$. (5 points)
- ✓ 6. Calculate $\int \frac{1}{1+\beta\tau} d\tau$. (10 points)
- ✓ 7. Calculate $\int x(x+1)^{1/2} dx$. (10 points)
- ✓ 8. Rewrite via integration by parts and then calculate $\int t^2 \exp[at] dt$. (20 points)
- ✓ 9. Rewrite via integration by parts and then calculate $\int_1^b x \ln[x] dx$. (20 points)
10. Calculate the derivative $\frac{\partial V}{\partial x}$ of $V = \int_a^{5x} t^2 \ln[x] dt$. (15 points)

Scan your derivations into a single pdf file and upload it to Canvas by **Monday 9/26/2022 at 11:59pm**.

$$1. \int 9x^{1/3} dx$$

$$\frac{9x^{4/3}}{4/3} \quad 9x^{4/3} \cdot \frac{3}{4}$$

Power rule

$$= \boxed{\frac{27x^{4/3}}{4} + C}$$

$$2. \int_1^2 y^{-1} dy$$

$$\ln(|y|) + C$$

$$\ln(2) - \ln(1)$$

$$\boxed{\ln(2)}$$

$$3. \int_a^b 1 dt$$

$$t + C$$

$$\boxed{b - a}$$

Sub limits

$$4. \int b^{2T} dT$$

U-sub

$$2T = u$$

$$\frac{1}{2} \int b^u du$$

$$\frac{b^u}{\ln(b)}$$

$$\boxed{\frac{b^{2t}}{2\ln(b)} + C}$$

$$5. \int_a^b r e^{-rt} dt \quad \text{U-Sub}$$

$$= \int e^u du \quad u = -rt$$

$$= -e^u$$

$$= -e^{-rt} + Z \quad \text{u-Sub}$$

$$\boxed{e^{-ar} - e^{-br}}$$

Sub limits for t

$$6. \int \frac{1}{1+\beta T} dT$$

U-Sub

$$u = \beta T + 1$$

$$\frac{1}{u}$$

$$\frac{\ln(u)}{b}$$

$$\boxed{\frac{\ln(1+\beta T)}{b} + Z}$$

$$7. \int x(1+x)^{1/2} dx \quad \text{U-Sub}$$

$$u = x+1$$

$$\int u^{3/2} du = \frac{2u^{5/2}}{5}$$

$$\int u^{1/2} du = \frac{2u^{3/2}}{3}$$

$$\frac{2u^{5/2}}{5} - \frac{2u^{3/2}}{3}$$

$$\boxed{\frac{2(x+1)^{5/2}}{5} - \frac{2(x+1)^{3/2}}{3} + C}$$

$$8. \int \frac{t^2 e^{at}}{f g'} dt$$

$$\int f g' = f g - \int f' g$$

$$f' = 2t \quad g = \frac{e^{at}}{a}$$

$$\frac{t^2 e^{at}}{a} - \int \frac{2t e^{at}}{a} dt$$

$$\downarrow$$

$$\frac{2}{a} \int \frac{t e^{at}}{f g'} dt$$

$$f' = 1 \quad g = \frac{e^{at}}{a}$$

$$\frac{t e^{at}}{a} - \int \frac{e^{at}}{a} dt$$

$$\downarrow$$

$$\frac{1}{a^2} \int e^u du \quad \begin{array}{l} \text{U-Sub} \\ u = at \end{array}$$

$$= \frac{e^u}{a^2}$$

$$\frac{t^2 e^{at}}{a} - \frac{2t e^{at}}{a^2} + \frac{2e^{at}}{a^3} + C$$

$$9. \int x \ln(x) dx \quad \int fg' = fg - \int f'g$$

$$f' = \frac{1}{x} \quad g = \frac{x^2}{2}$$

$$\left[\frac{x^2 \ln(x)}{2} - \int \frac{x}{2} dx \right]$$

$$\frac{1}{2} \int x dx$$

$$\left[\frac{1}{2} \cdot \frac{x^2}{2} \mid \frac{x^2}{4} \right]$$

$$\frac{x^2 \ln(x)}{2} - \frac{x^2}{4} + C$$

$$\left(\frac{b^2 \ln(b)}{2} - \frac{b^2}{4} + \frac{1}{4} \right) \quad \text{Sub limits}$$

10. $\frac{\partial V}{\partial x}$ of $V = \int_a^{5x} t^2 \ln(x) dt$ $V = f(t; x)$

$$f(5x, x) \cdot \frac{d(5x)}{dx} - f(a, x) \cdot \frac{da}{dx}$$

becomes 0
because 'a' is
not dependent on x

$$+ \int_a^{5x} \frac{\partial f(t; x)}{\partial x} dt$$

$$(25x^2 + \ln(x)) \cdot 5 - 0$$

$$+ \int_a^{5x} \frac{t^2}{x} dt$$

$$= \frac{t^3}{3x}$$

$$\frac{125x^3 - a^3}{3x}$$

$$\left((25x^2 + \ln(x)) \cdot 5 \right) + \frac{125x^3 - a^3}{3x}$$